APPLICATION OF THE PRINCIPLES OF THE STRUCTURAL GOLDEN MEAN AND THE FIBONACCI SERIES OF INTEGERS TO SEVERAL OF BELA BARTOK'S MUSICAL COMPOSITIONS WRITTEN AFTER 1930

Thesis for the Degree of M. M.

MICHIGAN STATE UNIVERSITY
LAWRENCE ANTHONY MAHER

1974

ABSTRACT

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By

Lawrence Anthony Maher

The mathematical concepts of the golden mean and the Fibonacci series of numbers, which are manifest in many natural phenomena, can be applied to the later compositions of Béla Bartók. Though the composer was not known to discuss these concepts in relation to music openly, there is evidence that he was exposed to them due to his keen interest in nature, and that he consciously applied these ideas to some of his musical works. Analysis of a number of his compositions written after 1930 reveals that frequently a correlation can be made between these concepts and the music's structural format; many movements are divided into sections after sixty-two percent of their total length is completed, in adherence to the golden mean ratio. This division point may correspond to the beginning of a recapitulation or second theme area, or it may occur at the point of a strong

climax in the movement. A movement may be further divided into sub-sections according to the golden mean arrangement. The number of bars or beats, rather than the performance time, is used as the basis of length in the works examined.

The Fibonacci series of integers (1,1,2,3,5,8,13,21, etc.) is by mathematical design a manifestation of the golden mean ratio. It can be shown that in many of Bartók's compositions bar numbers or beat numbers that correspond to these integers frequently have structural significance.

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A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF MUSIC

1974

ACKNOWLEDGMENTS

The author wishes to express his gratitude to the following people for their help in the various stages of preparing this report: Dr. Russell Friedewald, for his constructive criticism concerning the organization and the actual writing of the paper; Ms. Barbara Lawrence, for checking the validity of the mathematical aspect of the report; and Jill Pendergast, for her help in the preparation of the diagrams for the rough draft.

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CHAPTER I

INTRODUCTION

Introduction to the Golden Mean

It is the main purpose of this study to make applications of mathematical principles to several of Béla Bartók's later musical compositions. Before attempting to look at the actual music, one must present these principles so that their musical application may be meaningful.

The first main concept to be treated here is that of the golden mean, or golden section. The golden mean refers to the dividing of a segment in such a way that the ratio of lengths between the whole segment and the larger sub-section is equal to the ratio of the lengths between the larger sub-section and the smaller sub-section. The ratio that is achieved is known as the golden mean, and either of the segments that results from this division may be called a golden section. This idea may be illustrated with the help of a geometric diagram and this diagram will serve as a starting point for a mathematical proof. The theorem to be proved is that the ratio between the lengths of the whole segment and the larger part will be the same as the ratio between the lengths of the larger and smaller parts only if its numerical value is equal to approximately 1.618. Secondly, it will be shown that this ratio can only be achieved when the segment is divided at approximately sixty-two percent of its total length.

The diagram in Figure 1 represents a segment whose end points are A and B, with a dividing point C. Point C must be placed so that the ratio between the whole segment (AB) and the larger part (AC) is equal to the ratio between the larger segment (AC) and the smaller segment (CB).

Figure 1.

Since this study would be meaningless if the segments to be examined had no length, the stipulation will be made that both of the parts and the entire segment cannot be equal to zero. Furthermore, the numerical value for the golden ratio must be greater than zero, since a zero or negative proportion would again be meaningless in this context.

Figure 2.

$$\frac{AB}{AC} = \frac{AC}{CB} \qquad AB \neq 0 \qquad \frac{AB}{AC} > 0$$

$$AC \neq 0$$

$$CB \neq 0$$

The desired ratio will be designated as the unknown x. This would be equal to both $\frac{AB}{AC}$ and $\frac{AC}{CB}$. The starting premise for the proof, then, is given in Figure 3.

Figure 3.

$$x = \frac{AB}{AC}$$

Since AB is equal to the sum of AC and CB (see Figure 1), the above equation can be rewritten as follows:

Figure 4.

$$x = \frac{AC + CB}{AC}$$

Using the law of addition of numerators over a common denominator, this equation is further transformed as seen below.

Figure 5.

$$x = \frac{AC}{AC} + \frac{CB}{AC}$$

Since $\frac{AC}{AC}$ is equal to 1, the next equation in the proof is as follows:

Figure 6.

$$x = 1 + \frac{CB}{AC}$$

Due to the law of the invertibility of fractions, the further transformation shown below is obtained.

Figure 7.

$$x = 1 + \frac{1}{\frac{AC}{CB}}$$

However, it was a starting assumption that $\frac{AC}{CB}$ was equal to $\frac{AB}{AC}$. Therefore, an exchange can be made to obtain the following equality.

Figure 8.

$$x = 1 + \frac{1}{\frac{AB}{AC}}$$

Furthermore, it was given at the outset of the proof that x was equal to $\frac{AB}{AC}$. The exchange will now be made, producing the following result.

Figure 9.

$$x = 1 + \frac{1}{x}$$

By multiplying both sides of the equation in Figure 9 by x, the equation in Figure 10 is produced.

Figure 10.

$$x^2 = x + 1$$

Likewise, if x + 1 is subtracted from both sides of the equation of Figure 10, the result is:

Figure 11.

$$x^2 - x - 1 = 0$$

The above equation is now applicable to the quadratic formula of mathematics. This formula states that in any equation of the structure seen in Figure 12, x can be computed by using the formula in Figure 13.

Figure 12.

$$ax^2 + bx + c = 0$$

Figure 13.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

± means plus or minus

In Figure 12, a, b, and c are variables: "a" corresponds to the number of times x^2 is multiplied; "b" is the number of times x is multiplied; and "c" is the number that is added to ax^2 and bx to equal zero. Now the original equation as seen in Figure 11 will be listed together with the quadratic equation of Figure 12 so that it can be seen what the values of a, b, and c are in the original problem (Figure 14).

Figure 14.

$$x^2 - x - 1 = 0$$

$$ax^2 + bx + c = 0$$

It is clear that in the original equation "a" must be equal to one; "b" a negative one; and "c" a negative one. Now the formula from Figure 13 will be used to calculate x, or the desired golden ratio (Figure 15).

Figure 15.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2\ (1)}$$

Simple computation produces the following result:

Figure 16.

$$x = \frac{1 \pm \sqrt{5}}{2}$$

Since upon calculation, $\frac{1 - \sqrt{5}}{2}$ is found to be less than zero, it will be discarded. It was previously stated that x must be greater than zero to be meaningful. This leaves the sole solution to the original problem to be as follows:

Figure 17.

$$x = \frac{1 + \sqrt{5}}{2}$$

It is known that the square root of five is an irrational number and cannot be given an exact digital value, but is approximately equal to 2.236. Therefore, the next step is shown in the equation of Figure 18.

Figure 18.

$$x = \frac{1 + 2.236}{2}$$

Hereafter it will be assumed that the equalities are not exact, but close approximations. Adding the numerators of the equation of Figure 18 produces the following result:

Figure 19.

$$x = \frac{3.236}{2}$$

This division yields the following solution:

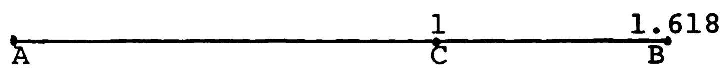
Figure 20.

$$x = 1.618$$

Thus 1.618 is a close approximation of the numerical value of the golden mean. In other words, when the ratio of lengths between the whole segment and larger part is 1.618, the ratio of lengths between the larger and smaller part is also 1.618.

Next, it must be determined where the dividing point C should be placed along the segment so that this golden ratio is achieved. It is known that the ratio of $\frac{AB}{AC}$ is equal to approximately 1.618, so Figure 1 can be redrawn with values assigned to the various lengths to conform to this ratio.

Figure 21.



To find what percentage of the entire segment is included in the larger part, AC, the following equation will be used: Z is equal to the percentage (Figure 22).

Figure 22.

$$\frac{1}{1.618} = \frac{Z}{100}$$

By using the formula for equating fractions with a variable the following result is obtained:

Figure 23.

$$100 \times 1 = Z(1.618) \text{ or}$$

$$100 = Z(1.618)$$

When both sides of the second equation in Figure 23 are divided by 1.618, the equation below is produced:

Figure 24.

$$\frac{100}{1.618} = Z$$

The long division will now be worked out to one decimal place to find the numerical value for the percentage Z (Figure 25).

Figure 25.

```
               61.8
1.618  |100.0000
         9708
          2920
          1618
          13020
          12928
```

Therefore, the value of Z is approximately equal to 61.8 percent. This means that the dividing point C from the original diagram must be placed at 61.8 percent of the total length of the segment. To put it another way, it could be stated that AC must be 61.8 percent of the total lenth and CB must be 38.2 percent of the total length.

The final phase of this proof will be to apply this percentage to some arbitrary length and see if it does indeed satisfy the requirements of the golden ratio. A segment that is 200 units long will be considered:

Figure 26.

Firstly, it will be stated that for convenience here and in the rest of this study, 61.8% will be rounded off to 62%. In the segment shown in Figure 26, therefore, the dividing point F must be placed at 62% of the length of the entire segment, or at unit 124. Lengths can then be assigned to the segments, as shown in Figure 27.

Figure 27.

DF = 124 EF = 76 DE = 200

The ratio of the whole segment over the larger part should equal the ratio of the larger part over the smaller part. This equality is mathematically stated in Figure 28. Assigning the numerical values to the segments produces the results given in Figure 29.

Figure 28.

$$\frac{DE}{DF} = \frac{DF}{EF}$$

Figure 29.

$$\frac{200}{124} = \frac{124}{76}$$

By cross multiplying the above fractions, the following equation is obtained:

Figure 30.

$$15,200 = 15,376$$

Considering the slight inaccuracy caused by the use of estimations, the above equation is close enough to be considered correct. No whole integer other than the one used (124) to place the dividing point F would have produced better results. Since the numerical value of the golden mean can only be approximated, the use of estimates must suffice in determining how to divide a segment into sections. Therefore, hereafter the value of 62% will be used to determine the location of the dividing point when the golden mean ratio is at work.

Introduction to the Fibonacci Series

The second mathematical concept to be presented and later applied to the music of Bartók is that of the Fibonacci series. This is a special series of positive integers starting with the number 1. The next member of the series is also 1, and the sum of these first two members equals the third Fibonacci number, which is 2. The subsequent members of the series can be found by computing the sum of the last two known previous integers. For example, since 2 + 1 = 3, the fourth Fibonacci number is 3, the next Fibonacci number is equal to 2 + 3, or 5, and so on. Figure 31 lists the first twenty numbers in the Fibonacci series.

Figure 31.[1]

Order in Series	Integer
1	1
2	1
3	2
4	3
5	5
6	8
7	13
8	21
9	34
10	55
11	89
12	144
13	233
14	377
15	610
16	987
17	1597
18	2584
19	4181
20	6765

The Fibonacci series has several interesting applications in nature, especially having to do with living animals and plants. For example, the scales of a pine cone grow outward from where it is attached to the branch in a spiral pattern. The number of spirals present always represents numbers in the Fibonacci series. Often there are thirteen spirals to the left side, and eight spirals to the right side. The same Fibonacci numbers also apply to the number of spirals in the seed patterns of sun flowers.[2]

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[1] Verner Hoggatt, Fibonacci and Lucas Numbers (Boston: Houghton Mifflin Co., 1969), p. 83.

[2] *Ibid.*, p. 81.

Furthermore, the Fibonacci series is adhered to by the arrangement of leaves on the stems of many plants. If one leaf is selected as a starting point and a counting is done until another leaf directly above the starting one is reached, the number of leaves will probably correspond to a Fibonacci number. Also, the number of turns the leaves make about the stem in the process will likewise be equal to a Fibonacci integer. Figure 32 is taken from Fibonacci and Lucas Numbers and is a diagram of a pear plant that has eight leaves counting from the base leaf to the first one directly above it, and has three revolutions about the stem in the process.[3] Both these numbers are Fibonacci numbers.

Besides having numerous applications in nature, the Fibonacci series of numbers has many unique mathematical properties. The property that is of concern here is the ratio that is produced when any two consecutive numbers of the series are put into division. Figure 33 shows a calculation of the ratios of consecutive Fibonacci numbers, starting with the first two numbers and proceeding to the fourteenth and fifteenth members. Upon examination of the ratios in Figure 33 it becomes evident that as the numbers of the series go higher, the ratio between two consecutive Fibonacci numbers gets closer and closer to the golden ratio. In other words, the Fibonacci series is an inherent

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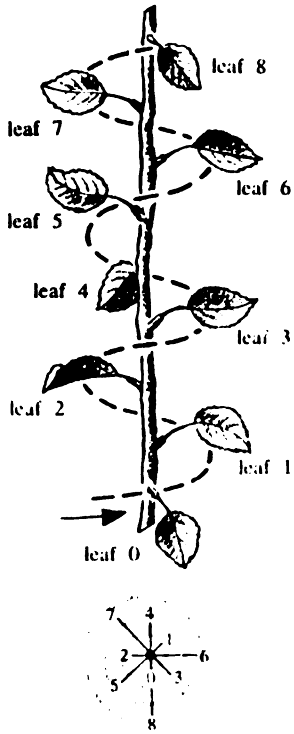
[3] Ibid., p. 80.

Figure 32.



3 Turns — Ratio $\frac{3}{8}$

Figure 33.[4]

$$\frac{1}{1} = 1.0000$$

$$\frac{2}{1} = 2.0000$$

$$\frac{3}{2} = 1.5000$$

$$\frac{5}{3} = 1.6667$$

$$\frac{8}{5} = 1.6000$$

$$\frac{13}{8} = 1.6250$$

$$\frac{21}{13} = 1.6154$$

$$\frac{34}{21} = 1.6190$$

$$\frac{55}{34} = 1.6176$$

$$\frac{89}{55} = 1.6182$$

$$\frac{144}{89} = 1.6180$$

$$\frac{233}{144} = 1.6181$$

$$\frac{377}{233} = 1.6180$$

$$\frac{610}{377} = 1.6180$$

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[4] Ibid., p. 29.

manifestation of the golden mean principle. If a segment were 89 units long, for example, it is known without multiplying by .62 that the closest integer that represents the dividing point in the golden ratio would be 55, the previous member of the Fibonacci series. If a segment were 233 units long, the dividing point would be 144 and the two parts formed would be 144 and 89 units in length. Thus the Fibonacci series is closely related to and is a constant illustration of the golden mean principle.

Golden Sections and Fibonacci Series in Music of Bartók

It is known that Bartók was always fascinated by nature, and inspiration from nature was often a basis for his compositional undertakings. "Bartók was inspired by the beauty of nature, always captivated by natural scenery."[5] The close association between nature and the golden mean and Fibonacci series make Bartók's keen interest in nature noteworthy. The composer wrote of his music:

> I like to keep in mind all sorts of regions, and I like to clothe them in accordance with the changing seasons, just as I like to invest songs with the colours of the countryside that has preserved them.[6]

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[5]Bence Szabolcsi, "Man and Nature in Bartók's World," <u>New Hungarian Quarterly</u> 2 (Oct.-Dec. 1961), p. 94.

[6]<u>Ibid</u>., p. 95.

It has been previously stated that the seed patterns of the sunflower illustrate the Fibonacci series. Bartók considered the sunflower his favorite plant, and was always interested in many living plants and animals of nature.

Hungarian folk songs greatly influenced Bartók's compositions. He said, "Folk music is a phenomenon of nature. Its formation developed as spontaneously as the other natural organisms; the flowers, animals, etc."[7]

It is this close association between Bartók and natural phenomena that adhere to these concepts that suggest that the composer may have used these ideas in his music after being exposed to their natural applications. Examples of Bartók actually discussing the use of these concepts could not be found, but the composer made no secret of his close alliance with nature and how he considered natural forces to play a part in his compositions. Bence Szabolcsi states:

> It is moreover the 'law of nature' itself, which Bartók evidently felt he could also bring in his works, thereby reaching the 'pure spring' of which he had been dreaming all his life.[8]

A second reason for applying the golden section and the Fibonacci series to Bartók's music is provided by the

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[7]Ernö Lendvai, Bela Bartók--An Analysis of His Music, Alan Bush (London: Kahn & Averill, 1971), p. 29.

[8]Szabolcsi, p. 100.

work of theorist Ernö Lendvai. In his book _Béla Bartók--An Analysis of His Music_, Lendvai applies these concepts quite convincingly to sections of _Music for Strings, Percussion and Celesta_ and the _Sonata for Two Pianos and Percussion_. It is Lenvai's writing on the golden mean and the Fibonacci series in relation to the music of Bartók that forms the only source of published material in English on this subject to date. The theories expounded by Lendvai, including the ones being dealt with here, have received increased exposure and considerable sympathy among numerous musical theorists since the book was published, but this topic of mathematics in the music of Bartók remains relatively new and unexplored. Even though some of Lendvai's reasoning and observations can be criticized, his writings form a very important part of the material that has been presented in regard to the music of Béla Bartók.

Lastly, the golden section and the Fibonacci series are applied to the music of Bartók because of the positive results that this type of analysis of the actual music yields. Whether one chooses to believe that Bartók was using these methods consciously or not, the fact remains that application of these devices can be traced in a considerable amount of his work. It is the purpose of this study to examine music of the composer written after 1930, and point out applications of these principles.

Method of Analysis

In order that the meaning of the golden section be better understood in regard to musical composition, a piece of music will be likened to a line segment. This line segment could represent an entire movement of a concerto, a small piano piece, or anything between these two poles. Since a musical work does not have length in the same sense that a line has length, the criteria for musical measurement lies in the actual number of bars, or beats, and the time duration of the piece. For example, if the piece to be analyzed is 150 bars long and has a constant meter throughout, it may be visualized as having a length of 150.

Figure 34.

To find the golden section of this length, it is necessary to multiply by .62, or to take 62 percent of 150. The computed solution of 93 indicates the location of the dividing point of the sections. In other words, after 93 bars, or approximately 93 bars, there will be some kind of main structural division or climax in the music if the golden ratio is in operation.

The determination of length as discussed above was made on the basis of a constant meter. If, however, metric switches are often encountered in a piece of music, such as

$\frac{2}{4}$ to $\frac{3}{4}$ to $\frac{4}{4}$, and so forth, the number of quarter-notes, rather than the number of bars, provides a more logical measuring stick. After counting the number of quarter-notes, one multiplies their total by .62 to determine where the golden section occurs.

The computation of length in terms of actual time duration was pursued at the outset but soon discarded. As different performances of the same piece varied considerably in length, due to the many tempo changes, both gradual and abrupt, it appeared futile to attempt to compute a specific golden section in terms of durational length, since this length, in all probabilities, would never be the same in any two performances. Lendvai negates using the passing of time as a basis of length:

> At first glance it may appear contradictory that the points of section determined by the laws of GS (golden section) can remain unaffected by the changing tempi. This phenomenon is easy to understand if we consider that music breathes in metric pulsation and not in the absolute measurement of time. In music, passing time is made realisable by beats or bars whose role is more emphatic than the duration of performance.[9]

These points being considered, in a piece of music employing changing meters, a unit value such as the quarter-note or eighth-note would be used as the basis of length, regardless of whether the metronomic value of that length was constant in the work.

The distinction that Lendvai makes between the positive golden section and the negative golden section deserves

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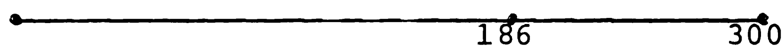
[9]Lendvai, p. 26.

consideration here. A musical segment of 300 bars or beats is represented below, with the normal golden section calculated and placed along the line at unit 186. In this figure a larger section of 186 units is followed by a smaller section of 114 units or 300 units minus 186 units. This arrangement of a large segment followed by a small segment is referred to as the positive golden section. If the segments are reversed, i.e., small followed by large,

Figure 35.



the arrangement is referred to as the negative golden section. In this illustration (Figure 36) the smaller segment of 114 units is now placed in front of the larger segment of 186 units.

Figure 36.

Furthermore, it is possible to divide a piece more than once, thereby calculating auxiliary golden sections as well as the main golden section. Figure 37 illustrates a 144-unit piece with the main golden section arrangement being positive. Each of the resultant two sections may be

Figure 37.

found to divide further into more positive golden section arrangements. In Figure 38 the first larger segment of 89 units breaks down into smaller segments of lengths of 55 and 34.

Figure 38.

The second section of 55 units is also broken into smaller segments of lengths of 34 and 21. The main golden section and the two smaller golden sections pictured in Figure 38 are all said to be positive.

Moreover, it is possible that both the positive and negative golden sections can be used simultaneously. Figure 39 illustrates a length of 144 beats or bars. While the main golden section is positive, the golden section that divides the first segment of 89 bars is negative.

Figure 39.

It is now possible to determine how the golden section principle actually applies to the music. After the beat or bar number of the golden section is determined, that location is examined. Often it will occur at a main structural division, such as a recapitulation or a contrasting thematic entry. This dividing point may also be found at a textural climax, a tempo change, or a change of register.

In all of the musical compositions treated, several different applications of the golden section were searched for, both on the large level and the subordinate level. If the overall golden section structure appeared to apply to the music, a diagram was made of that particular piece of music showing what application or applications were found.

The use of the Fibonacci numbers was also examined in relation to the music studied. Mention is made of any instance where the number of bars or beats within a certain well-defined section corresponds exactly to a Fibonacci number. Both beats and measures were examined, because it appears that Bartók placed emphasis on this mathematical series in relation to bar numbers as well as beats. In the Divertimento for String Orchestra, Bartók uses Fibonacci integers in relation to bars and beats simulataneously, achieving a double application of the series. Therefore, both the number of bars and number of beats will be related to the series in the course of this study.

The limitation of this project to the works of Béla Bartók written after 1930 is imposed for the following reasons: firstly, many earlier works were examined, but did not yield as many applications as his later works. Secondly, Lendvai is of the opinion that Bartók did not begin formulating and using these concepts in his music until he was between thirty and forty years of age. The results of the observations made for this study are evidence in favor of Lendvai's contention.

The works to be discussed will be presented according to their classifications as listed in the table of contents. For the sake of comprehensiveness, a mention of the works that do not appear to adhere to these principles will be made in their appropriate sections. Furthermore, analysis of the first and third movements of <u>Music for Strings, Percussion, and Celesta</u> and the same movements of <u>Sonata for Two Pianos and Percussion</u> will be omitted due to the fact that Lendvai has already studied these sections with a fair amount of detail. Those wishing to investigate his analyses of these movements in regard to the golden mean or Fibonacci series are referred to pages 18-28 of his book.

CHAPTER II

SOLO CONCERTOS WITH ACCOMPANIMENT

Third Piano Concerto (1945)

The concepts of the golden section and the Fibonacci series can be observed in the first movement of the Third Concerto for Piano and Orchestra, completed in 1945. The basis of the calculation of length is that of the number of bars. Since the $\frac{3}{4}$ meter is nearly constant throughout the movement, the relatively few $\frac{2}{4}$ bars in the 187 bar segment are negligible; if quarter-notes had been used the location of the golden section would have changed, by less than one percent of the length of the entire movement, or by no more than four quarter-notes.

The length of 187 bars is used in the calculation to locate the positive golden section.

Figure 40.

$$
\begin{array}{r}
187 \\
\times\ .62 \\
\hline
3\ 74 \\
112\ 2 \\
\hline
115.94
\end{array}
$$

In cases such as this where the calculated golden section number includes a fraction (115.94), the solution will be rounded off to the nearest whole integer, or 116. Scrutiny reveals that the recapitulation of the opening material in the original key begins at bar 117, forming an important structural division point within the movement. In the beginning of the movement and again at bar 117, the piano states the first theme, after a one and one-half bar introduction in the strings. Example 1 contains two reproductions from the score: the first is the opening of the movement, and the second is the section starting at bar 117.

Further examination of the movement reveals that its segments can be broken up into auxiliary golden sections. As the negative golden section arrangement simply dictates that the smaller part of 38 percent comes before the larger part of 62 percent, the negative GS of the first 116 bars is calculated by multiplying the length in question by .38.

Figure 41.

$$\begin{array}{r} 116 \\ \times\ .38 \\ \hline 9\ 28 \\ 34\ 8 \\ \hline 44.08 \end{array}$$

At bar 44 of the movement a new contrasting melodic idea is introduced in the solo instrument; this is later developed and recapitulated with some variation.

Example 1.[10]

The second section of the movement, from bar 117 to the end, has a total length of 71 bars and a negative GS of 26.98 bars, which is rounded off to 27.

Figure 42.

$$\begin{array}{r} 71 \\ \underline{\text{X}\ .38} \\ 5\ 68 \\ \underline{21\ 3} \\ 26.98 \end{array}$$

Thus, the negative GS occurs at bar 144, 27 bars after the beginning of the second main section of the movement. A main textural change from the orchestral color to the piano occurs one bar later at measure 145, at which time the piano begins a transition from the first subject to second subject material. The actual calculation of the theoretical negative GS of this segment is to bar number 144, which is a member of the Fibonacci series.

A calculation for the positive GS of the length of the first subject area, bars 1 through 43, gives a figure of 26.66.

Figure 43.

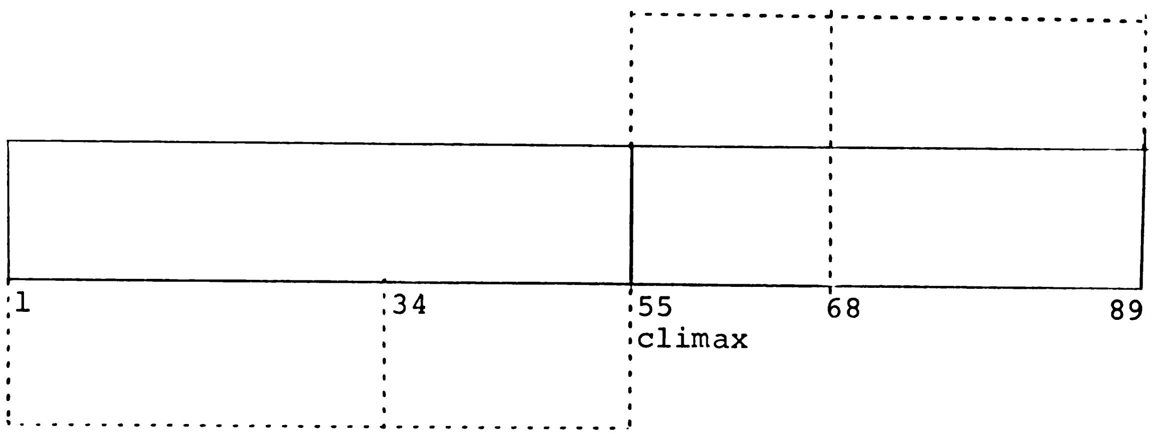
$$\begin{array}{r} 43 \\ \underline{\text{X}\ .62} \\ 86 \\ \underline{25\ 8} \\ 26.66 \end{array}$$

At bar 27, a change in texture from the orchestral tutti climax to transitional material in the piano occurs. This change in texture is similar to the one that occurs at bar 145, previously determined to be the negative GS of the second large segment of the movement.

The Fibonacci series of numbers has further applications to this movement. If 34 bars are counted backwards from the end of the movement, it is found that this location corresponds to bar 154, where the recapitulation of the second melodic idea appears in the piano. Furthermore, if this counting is stopped at various Fibonacci-series numbers, it is found that there are significant textural and orchestrational changes after 8, 13, and 21 bars from the end. Specifically, the orchestra enters the texture at bar 180 (8 bars from the end), the piano takes up a melodic pattern after the orchestra has been prominent at bar 175 (13 bars from the end), and the same kind of textural change takes place from bar 166 to 167 (21 bars from the end). Thus the section from bar 154 to the end is an entity structured by the Fibonacci series; the fact that it is set off from the rest of the movement by the final $\frac{2}{4}$ bar might be regarded as a clue of its structural independence from the first 153 bars of the movement. A guide to the format of the golden section diagrams is given in Diagram 1, followed by the actual GS diagram for the first movement of the piano concerto (Diagram 2). See pages 29 and 30.

The second movement of the concerto also has applications to the golden mean and the Fibonacci series. Since the meter is not constant in the movement, the number of quarter-note values, 551, is considered to be the total length. The calculation for the positive GS dividing point

Diagram 1. Format for golden section diagram showing main and auxiliary golden section arrangements. Numbers used are picked arbitrarily.



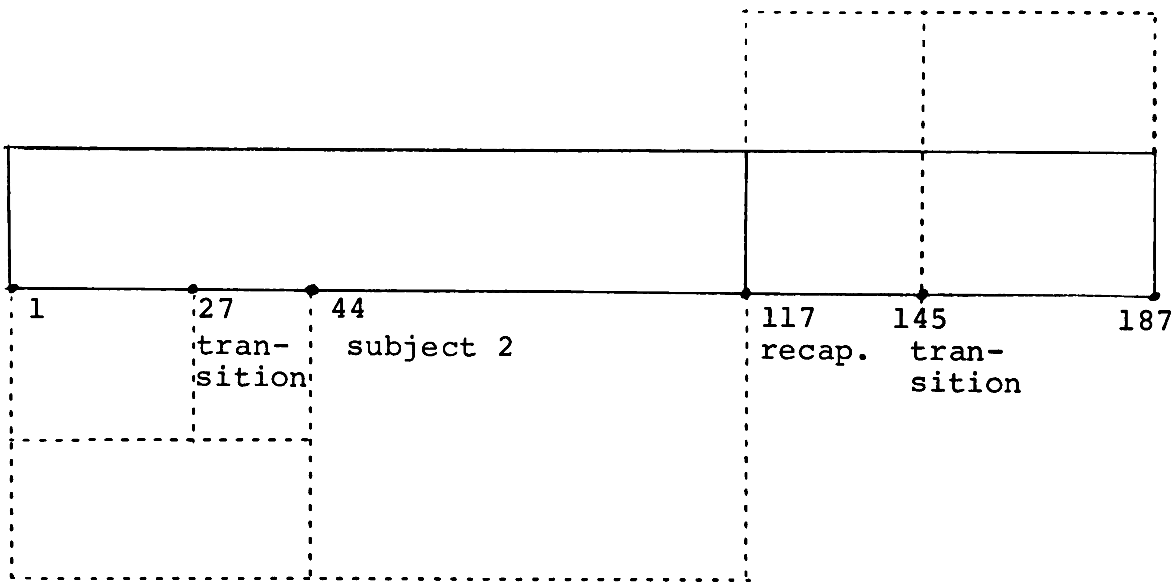
Description of GS arrangements

overall GS positive
first section GS positive
second section GS negative

Overall GS denoted by solid line and divider over numbered segment

Auxiliary GS's denoted by dotted line and divider above or below numbered segment

Diagram 2. Third Piano Concerto 1945 Movement One
GS diagram



overall GS positive
GS bars 1-44 positive
first section GS negative (bar 1-116)
second section GS negative (bar 117-end)

is given in Figure 44. The 342nd quarter-note occurs on the third beat of bar 87, five beats before a major structural division takes place, and where the composer has placed a double bar (between bars 88 and 89). Noteworthy is the bar number at the beginning of this Tempo I section, since it is a number in the Fibonacci series.

Figure 44.

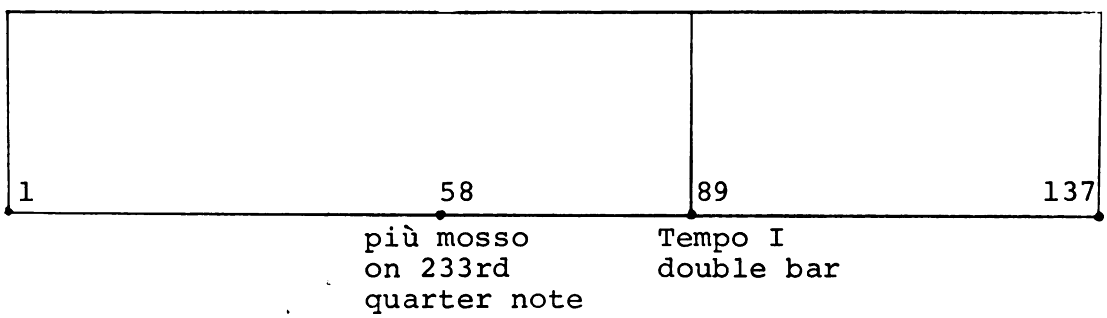
$$\begin{array}{r} 551 \\ \underline{\times\ .62} \\ 11\ 02 \\ \underline{330\ 6} \\ 341.62 \end{array}$$

The first 57 bars of the music are separated from the rest of the movement by a different tempo, different melodic material, and a double bar. The first beat of the poco più mosso section (starting at bar 58) occurs on the 233rd quarter-note value of the movement.

Diagram 3 (page 32) is a golden section diagram of the second movement of the concerto. Even though the length of this movement was determined by the number of quarter-note values, sections are divided and referred to by the use of bar numbers: this procedure of using bar numbers in the GS diagram, whether or not they are used to calculate length, will be adhered to in the subsequent sections of this study.

The last movement of the concerto appears to contain only one application to the Fibonacci series and no

Diagram 3. Third Piano Concerto 1945 Movement Two
GS diagram



overall GS positive

applications to the golden mean ratio. The recapitulatory segment starting at bar 589 is separated from the Presto at bar 644 by a grand pause of two bars. This section (bars 589 through 643) contains 55 measures.

In the <u>Third Piano Concerto</u>, applications to the golden mean and Fibonacci series were most numerous in the first movement, but became less and less frequent in subsequent movements. Several of the multi-movement works analyzed followed this general pattern of declining applications.

<u>Violin Concerto (1937-38)</u>

The next work to be studied will be the composer's reduction (for violin and piano) of the later violin concerto, written in the years 1937 and 1938. Even though the overall GS of the first movement does not correspond to a major structural point or climax, numerous other applications can be made to smaller golden sections, and the Fibonacci series can be traced. Performance timings that Bartók indicates in his musical scores subdivide movements into smaller sections, several of which contain GS applications. Even though these timings are important in that they separate whole movements into subdivisions, the composer did not intend for them to be exact:

> It is not suggested that the durations be exactly the same at each performance; both these and the metronomic indications are suggested only as a guide for the executants.[11]

The first performance timing marked in this movement (fifty-one seconds) coincides with a tempo change after bar 21, a Fibonacci number. The first segment is clearly separated from the rest of the movement by a change in texture, tempo, and melodic material.

The next timing (forty seconds) that is found in the music is at another structural division point, before the Quasi tempo I marking at bar 43. Again the length of the segment enclosed by these timings, that is, from bar 22 through 42, is equal to 21 bars.

The Quasi tempo I section that begins at bar 43 extends to the Risoluto marking at bar 56, and its length in bars is equal to the Fibonacci integer 13. Furthermore, this segment is divided into the positive golden section arrangement due to the fact that there is a textural and motivic contrast after eight bars.

The Risoluto section which begins at bar 56 extends for 17 bars, continuing through measure 72. Since neither the texture nor the motivic material in the solo part or the accompaniment changes within this section, it would suggest that some type of pitch or dynamic climax might

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[11] Béla Bartók, Note from score of Violin Concerto, reduction for violin and piano, Boosey & Hawkes, Ltd., 1941.

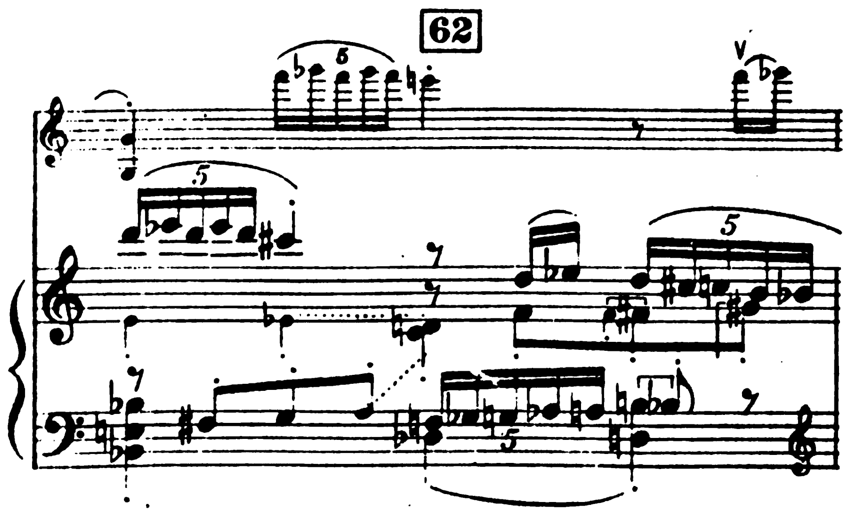
correspond to the location of the golden section. In pursuit of this possibility, the negative golden section, which is given in Figure 45, is determined. Since in this

Figure 45.

$$\begin{array}{r} 17 \\ \times\ .38 \\ \hline 1\ 36 \\ 5\ 1 \\ \hline 6.46 \end{array}$$

section, as in the previous segments of the movement, the meter is a constant $\frac{4}{4}$, the number of bars is used to determine length. After six full bars from the starting point are counted, bar number 62 is reached. It is in this

Example 2.[12]



[12] Copyright 1941 by Hawkes & Son, Ltd. Reprinted by permission, Boosey & Hawkes, Ltd.

measure that both the solo part and the accompaniment reach their highest pitch, the piano reaching E flat^{3}, and the violin imitating the idea one beat later, reaching G flat^{3}. At no other point within this section does either part extend to this extreme range.

The next section of the first movement (bars 73 through 91) has an initial marking of Calmo and a length of 19 bars. As the meter within this section is constant, this number (19) is used as the basis of length, and the positive golden section is determined to be 11.78.

Figure 46.

$$\begin{array}{r} 19 \\ \underline{\times\ .62} \\ 38 \\ \underline{11\ 4} \\ 11.78 \end{array}$$

After twelve bars, a new melodic idea is introduced in the solo violin (bar 85) and a tempo marking of sempre più lento is indicated.

A later sub-section of 34 bars, which gives the Fibonacci series further application, begins at bar 160 and extends through bar 193. It is characterized by the Vivace tempo, the essential melodic material being that of the sixteenth-note passage in the solo violin.

In the next section for consideration, bars 213 through 247, frequent changes of meter were encountered.

For this reason, the number of quarter-note values, rather than the number of bars, was used to calculate its length. On the basis of the 142 quarter-note values present, the positive GS of 88.04 was determined. The 88th quarter-note

Figure 47.

$$\begin{array}{r} 142 \\ \underline{\times\ .62} \\ 2\ 84 \\ \underline{85\ 2} \\ 88.04 \end{array}$$

duration of this segment occurs on the second beat of bar 234, where a volume climax is reached as a crescendo from bar 233 leads to the only double forte marking of the segment in the accompaniment. Furthermore, at bar 234, the piano takes up a new eighth-note pattern in octaves (Example 3).

The next sub-section of the movement has a constant meter throughout and extends from bar 248 through 279, a length of 32 bars. The 20th bar of this portion corresponds to measure 267, where a division is made by the return to the Risoluto marking, and the use of the quintuplet sixteenth-note pattern in both of the parts. This material

Figure 48.

$$\begin{array}{r} 32 \\ \underline{\times\ .62} \\ 64 \\ \underline{19\ 2} \\ 19.84 \end{array}$$

had been used previously, notably in bar 62 (Example 2).

Example 3.[13]

The final segment to be examined in this movement extends from bar 280 through 301, and is given a performance timing of forty-one seconds. It begins with a forte tremelo in the accompaniment followed by a scalar pattern in the low register of the violin. Again the meter is constant, and the length is 22 bars. After 14 bars have elapsed (positive GS) a textural change occurs when the solo part drops out,

Figure 49.

$$\begin{array}{r} 22 \\ \underline{\times\ .62} \\ 44 \\ \underline{13\ 2} \\ 13.64 \end{array}$$

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and the piano takes up the melodic interest (bar 294). Besides this textural change, a structural division is emphasized by the tempo marking of Più mosso, ♩ = 140.

The second movement of the concerto contains an application to the overall positive golden section arrangement. The unit of the eighth-note value was used to determine length because of the fluctuations in meter. This length is equal to 1,049 units and is used in the positive GS calculation seen in Figure 50. The 650th eighth-note

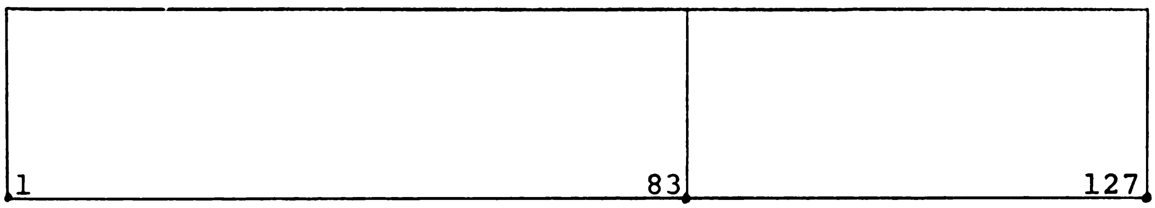
Figure 50.

$$\begin{array}{r} 1049 \\ \underline{\times\ .62} \\ 20\ 98 \\ \underline{629\ 4} \\ 650.38 \end{array}$$

value occurs at the end of bar 81, one bar before a major formal division takes place. At bar 83, a new tempo and meter are introduced as well as a different vocabulary of melodic materials. A listening to the movement supports the idea that a major division within the movement does occur at this point. Furthermore, Bartók places a double bar as well as a performance timing immediately before the new segment begins at bar 83 (Example 4). The GS diagram for this movement is given in Diagram 4 (page 40).

In the first movement of the <u>Violin Concerto</u>, applications to the Fibonacci series and the golden mean

Diagram 4. <u>Violin Concerto</u> 1937-38 Movement Two
GS diagram



allegro
scherzando
9/8

overall GS positive

Example 4.[14]

are numerous at the auxiliary levels, but the main golden section application cannot be made. In the second movement, on the other hand, the main golden section works well, but all other smaller applications are missing. This leads one to speculate if Bartók intended the applications of the two movements to supplement each other, together resulting in complete adherence to these concepts.

In the third movement of the concerto, the section starting with the Risoluto marking at bar 29 and extending

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through bar 86 contains 58 bars in a constant $\frac{3}{4}$ meter. Its positive GS is calculated to be 35.96. The 36th bar of this

Figure 51.

$$\begin{array}{r} 58 \\ \underline{\times\ .62} \\ 1\ 16 \\ \underline{34\ 8} \\ 35.96 \end{array}$$

section corresponds to bar 64, where the culmination of a four-bar crescendo is reached in the piano part. Also, the texture is changed at bar 64, with the left hand playing block chords on beats one and three against a disjunct melodic line. This is in contrast to the triplet figures that were prevalent in both of the parts before this divisional point at bar 64 is reached.

The following portion, that which lasts from bar 87 through 125, is set off by a performance timing of thirty-six seconds. It begins with triplet figures in the violin and the piano, with a marking of Un poco sostenuto. The meter is constant throughout this segment, which has a length of 39 measures; the positive GS calculation is 24.18.

Figure 52.

$$\begin{array}{r} 39 \\ \underline{\times\ .62} \\ 78 \\ \underline{23\ 4} \\ 24.18 \end{array}$$

After 24 bars of this section are completed, bar 111 is reached. Here the violin has a dynamic climax to fortissimo, and the accompaniment has a simultaneous sforzando attack. A tempo marking of 𝅗𝅥. = 66 is also placed in the score at this point.

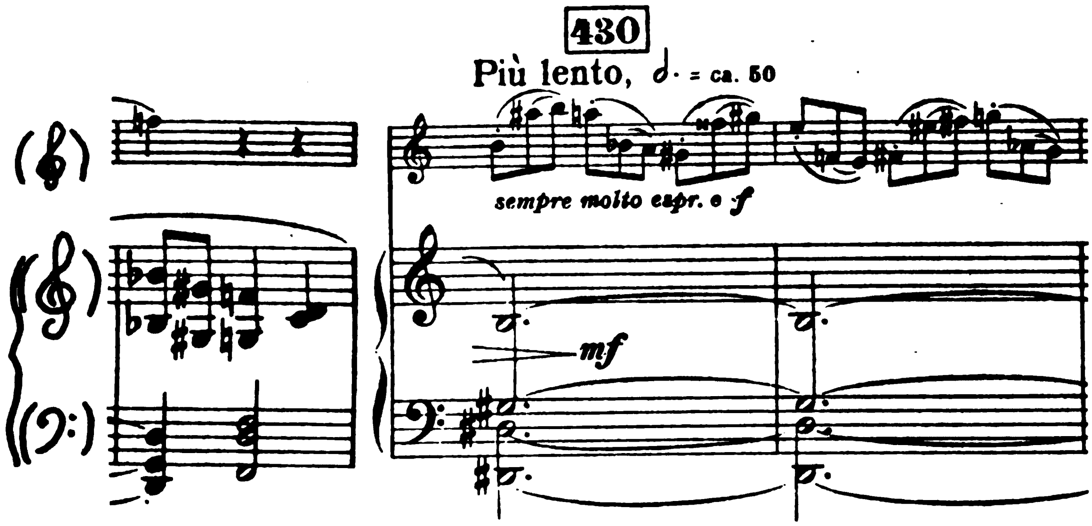
One additional segment within the third movement merits mention. It begins at bar 400 and extends through bar 450. (In the 1941 edition of the Boosey & Hawkes score an error was made in the bar numbering; measure 415 should be number 416, and thereafter 1 should be added to all of the printed bar numbers.) The segment has a constant meter signature and is 51 bars long, with a positive GS of 31.62. Figure 53.

$$\begin{array}{r} 51 \\ \underline{\times\ .62} \\ 1\ 62 \\ \underline{30\ 6} \\ 31.62 \end{array}$$

The 32nd bar of this section corresponds to bar 431 (erroneously marked in the score as 430). At this point new melodic material is heard in the solo part over a sustained harmony in the piano. A tempo change also serves to emphasize the division between bars 430 and 431 (Example 5).

Two other concertos of Bartók from the period after 1930 were examined, and found to have no application to the overall golden mean in any of their movements. They were the Second Piano Concerto and the Viola Concerto, dating from 1931 and 1945 respectively. This does not imply that no small-scale applications exist in these works, however. A work was chosen for discussion here only if one or more of its movements adhered to the golden section arrangement in the overall context. Since this condition was not met in either of these concertos, they were not examined further.

Example 5.[15]



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CHAPTER III

ORCHESTRAL WORKS

Concerto for Orchestra (1943)

The first movement of the concerto has a length of 1,787 eighth-note values and a negative GS of 679. This

Figure 54.

$$\begin{array}{r} 1787 \\ \underline{\times\ .38} \\ 142\ 96 \\ \underline{536\ 1} \\ 679.06 \end{array}$$

point corresponds to bar 153, one bar before the oboe introduces the second subject, which is based on the melodic interval of the major second (Example 6).

The first texture change and performance timing of the movement (one minute and thirty-eight seconds) occurs after 34 bars, a Fibonacci integer.

The second small section, from bar 35 through 75 contains 41 bars in a constant meter, and has a negative GS of 16. This corresponds to bar 50, which directly precedes a change of tempo, texture, and melodic material.

Example 6.

Figure 55.

```
      41
   X .38
    3 28
   12 3
   15.58
```

Counting from the Allegro vivace at bar 76, one finds that a contrasting melodic idea begins on the 55th eighth-note value at bar 95, after a bar of total silence. Furthermore, the second theme (negative GS of movement) occurs on the 233rd eigth-note value, counted from the same location (bar 76).

The Tempo I section that extends from bar 221 through 271 has a length of 123 eighth-note values and a negative GS of 47. This point occurs at bar 246, five eighth-note values before a climax is reached at bar 248 (Example 7).

Figure 56.

$$\begin{array}{r} 123 \\ \underline{\times\ .38} \\ 9\ 84 \\ \underline{36\ 9\ \ } \\ 46.74 \end{array}$$

Example 7.[17]

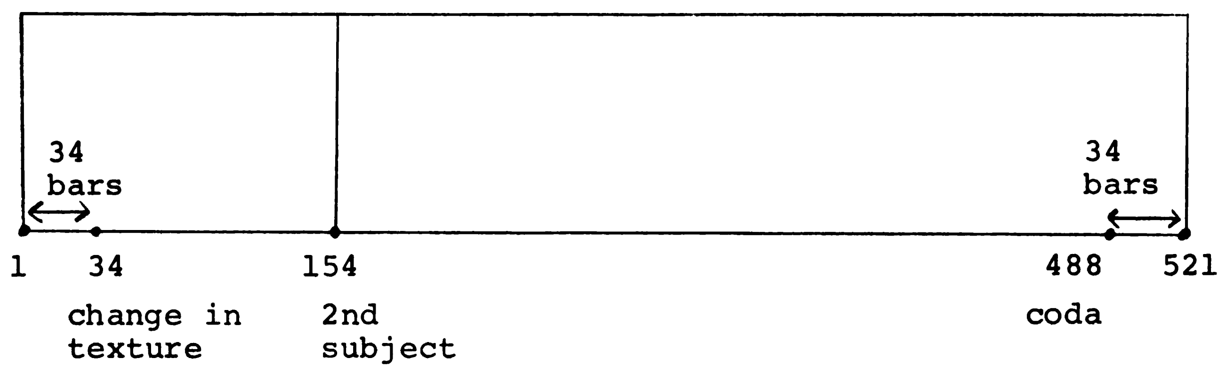
The section beginning at the Tranquillo marking at bar 272 and extending through bar 312 also contains 123 eighth-note values and has a negative GS of 46. This GS corresponds to bar 288, the location at which the English horn restates the motive that was played in the clarinet at the opening of this segment.

The Tempo I section of bars 313 through 395 is 83 bars long in a constant $\frac{3}{8}$ meter. Its positive GS is 51.46, which corresponds to bar 364. (Inasmuch as several examples of the actual GS calculations have already been presented, these calculations will be omitted in the subsequent sections of this report, with the exception of two or three introductory figures for each chapter.) At this point the brass instruments begin an imitative pattern with the fanfare motive, preparing the climax of bar 386.

Since the section starting at bar 488 uses the same motivic material as the section starting at bar 76, the segment from bars 76 through 487 may be considered a large portion of the movement. Having a length of 1,240 eighth-note values, its negative GS is equal to 471. The 471st eighth-note value occurs at bar 231; this is the first bar of a Tempo I section characterized by sixteenth-note patterms in the strings and woodwinds.

The section extending from bar 488 to the end of the movement contains 34 measures, and its length is determined to be 97 eighth-note values. The positive GS of this is 60, which corresponds to bar 509; this is the point at which the scalar sixteenth-note patterns cease, and the preparation for the climax at bar 514 begins. The golden section application for this movement is given in Diagram 5.

Diagram 5. <u>Concerto for Orchestra</u> 1943 Movement One
GS diagram



overall GS negative

The second movement of the concerto contains 263 bars in a constant $\frac{2}{4}$ meter, and has a positive GS of 163. This point is one bar before the dividing double vertical slashes appear in the score, immediately followed by the recapitulation (Example 8).

Example 8.[18]

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The opening section of the movement, which lasts through bar 24, has a negative GS of 9. Bar 9 is the first full bar of the initial subject in the bassoons.

The following timed section (twenty-five seconds) through bar 44 has a length of 20 bars and a negative GS of 8. The eighth bar corresponds to bar 32, where a dynamic climax is reached and a new melodic phrase is begun in the oboes.

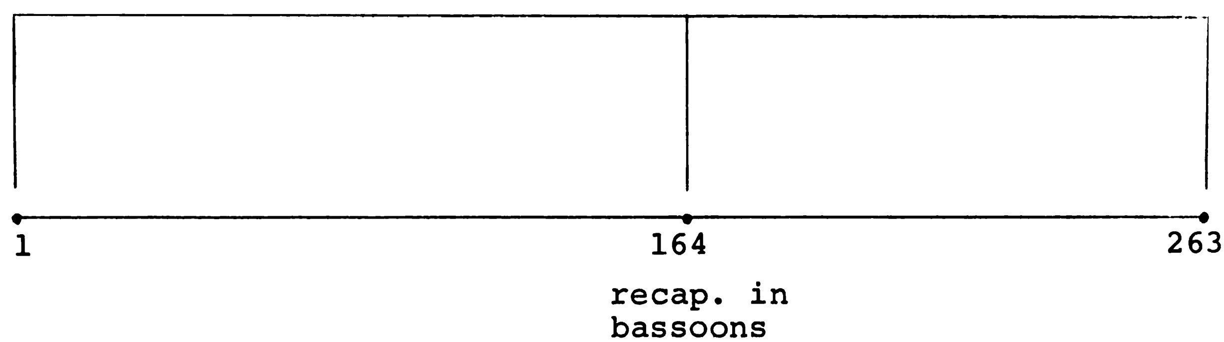
There is a textural change and structural division after 89 bars of the movement, giving the Fibonacci series application.

The section beginning at bar 181, characterized by the melodic interest in the clarinets and the oboes, lasts through bar 211. It has a length of 31 bars and a positive GS of 19. After 19 bars there is a tempo change (bar 200) where the melody is carried by the flutes and the clarinets.

Bartók indicates a performance timing of six minutes and seventeen seconds for this movement. This figure is equal to 377 seconds, which is a Fibonacci integer. The GS diagram for the second movement of the concerto is seen in Diagram 6.

In the third movement of the concerto, the first performance timing (one minute) and texture change occurs after 21 bars. The 34th bar of the movement is the first bar of an a tempo section, where scalar patterns are introduced in the woodwinds and strings. The number of quarter-note

Diagram 6. *Concerto for Orchestra* 1943 Movement Two GS diagram



overall GS positive

values that are contained in the section starting at bar 62 and extending through bar 72 is 34. Furthermore, counting from bar 73, one finds that the imitative section at bar 84 begins on the 34th quarter-note value.

Like movement three, the fourth movement of the *Concerto for Orchestra* contains application to the Fibonacci series while having no adherence to the overall GS structure. The oboe starts the initial theme on the 13th eighth-note value of the movement, and after 144 additional eighth-note values it states the theme an octave lower (bar 32). Bar number 89 contains the first complete silence in all of the parts, followed by forte glissandi in the trombones. Bar number 144, furthermore, is the first bar of the coda, preceded by the flute cadenza.

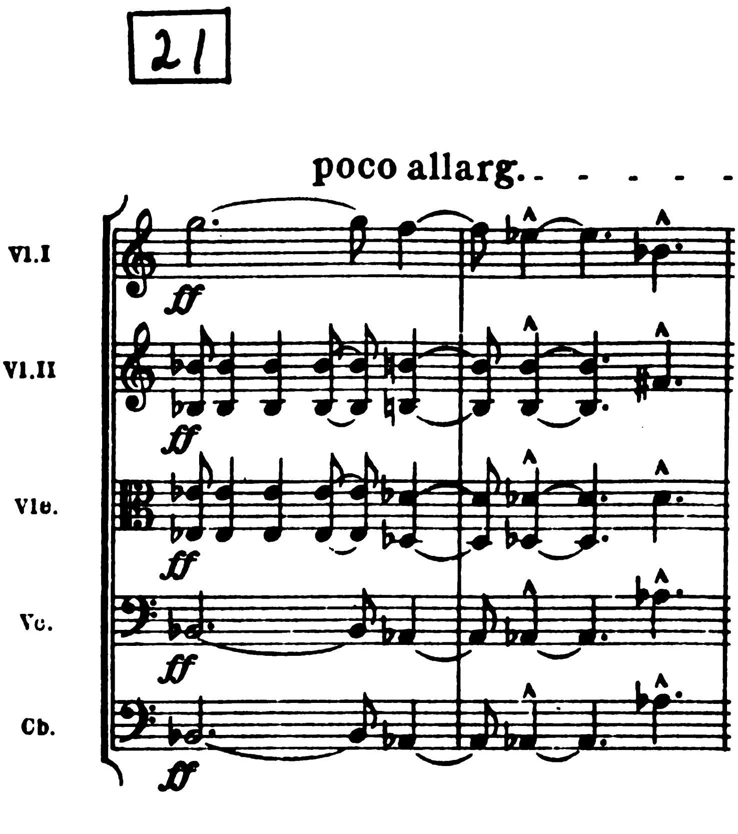
The Finale of the concerto contains only two applications to the Fibonacci series. The portion that begins at bar 148 with the bassoon solo has a length of 13 bars, and the Tempo I presto section that begins at bar 384 (extending through measure 417) contains 34 measures.

Divertimento for String Orchestra (1939)

The *Divertimento for String Orchestra*, a three movement work with a performance duration of approximately twenty-two minutes, was completed in 1939. The number of dotted quarter-note values (563) is the basis of length for the first movement, and the positive GS is 349. This point occurs at bar 129, where a triple forte tutti pattern is played, two bars before a recapitulatory segment begins at bar 131.

The first main portion of the movement, as determined by the overall GS (bars 1-129), has a length of 349 dotted quarter-note values. Its positive GS (216) occurs at bar 80, which is the first bar of the Più tranquillo segment.

Example 9.[19]



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The second section (bar 130 through the end), on the other hand, has a length of 214 dotted quarter-notes and a positive GS of 133, which occurs at bar 178. This measure is followed by a timing (one minute and seven seconds) and a contrasting segment.

The first portion from the opening of the movement lasts 13 bars and contains 34 dotted quarter-note values; furthermore, bar number 21 is a double forte climax point (Example 9) that occurs after 55 dotted quarter-note values from the beginning of the movement.

Example 10.[20]

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Bartók places the first performance timing (one minute and thirty-two seconds) after 41 bars. The length of this segment (bar 1-41) is 111 dotted quarter-note units, and the positive GS is 69. This is located at bar 25, the first measure of the Un poco più tranquillo segment.

A counting from the GS point of the first section (bar 80) reveals that the only $\frac{4}{8}$ meter of the movement (bar 94) occurs on the 34th dotted quarter-note value; furthermore, Fibonacci's application can be made by counting units of length (♩.) from the imitative section at bar 95. Figure 57 lists where various points in the music are located, and their significance.

Figure 57. ♩. units after measure 95

After	Occurs at bar	Significance
21	101	beginning of imitative section
34	105	f climax, homorhythmic
55	113	beginning of più mosso, agitato
89	125	ff, at a più mosso marking
144	144	climax, similar to bar 21
233	178	contrasting segment (a tempo) overall GS of movement

The final portion of the first movement extends from bar 179 through the end, and since all but two of the bars within it are in $\frac{9}{8}$ time, the segment is considered to have a length based on the number of bars, or 26. The positive GS is calculated to be 16 bars. The two bars of $\frac{6}{8}$ time appear after 16 bars within this segment; also, this meter change (bar 195) occurs 144 eighth-note values after the beginning of this section. Diagram 7 on page 58 gives the GS applications for the entire first movement of the work.

The second movement of the Divertimento, which is 74 bars long in a constant $\frac{4}{4}$ meter, has a positive GS of 46. This point is two bars after a climax and change in texture is made (Example 11).

Example 11.[21]

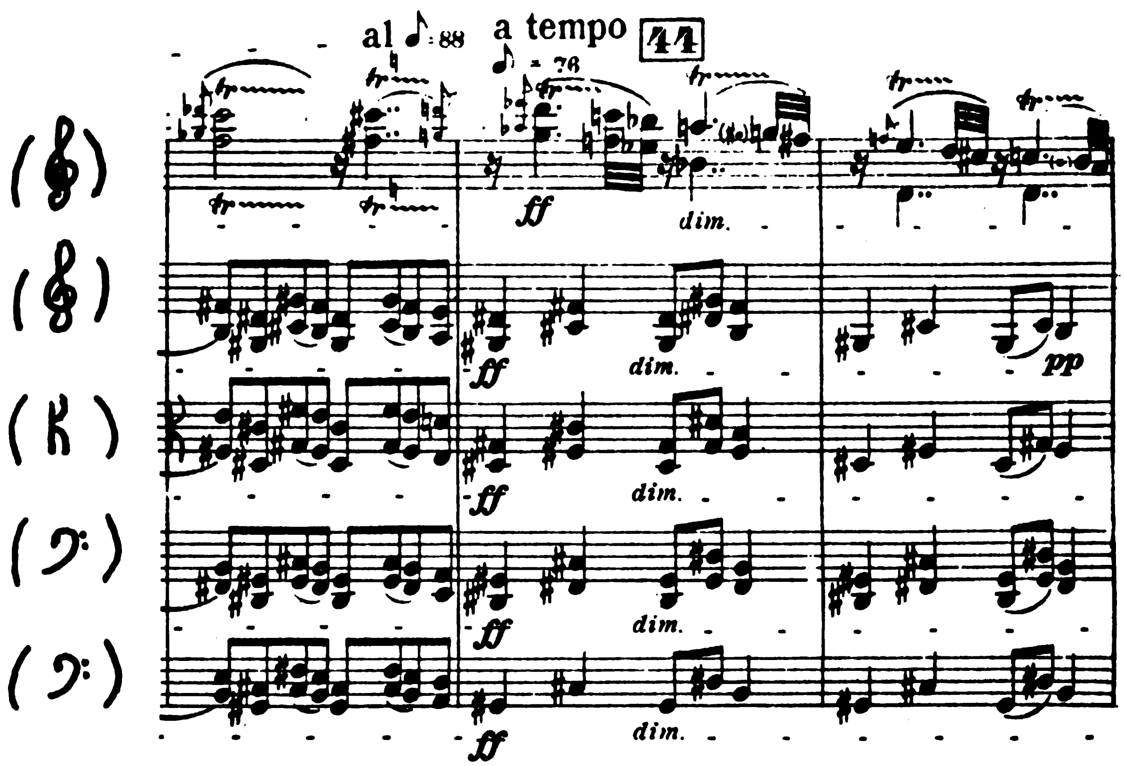

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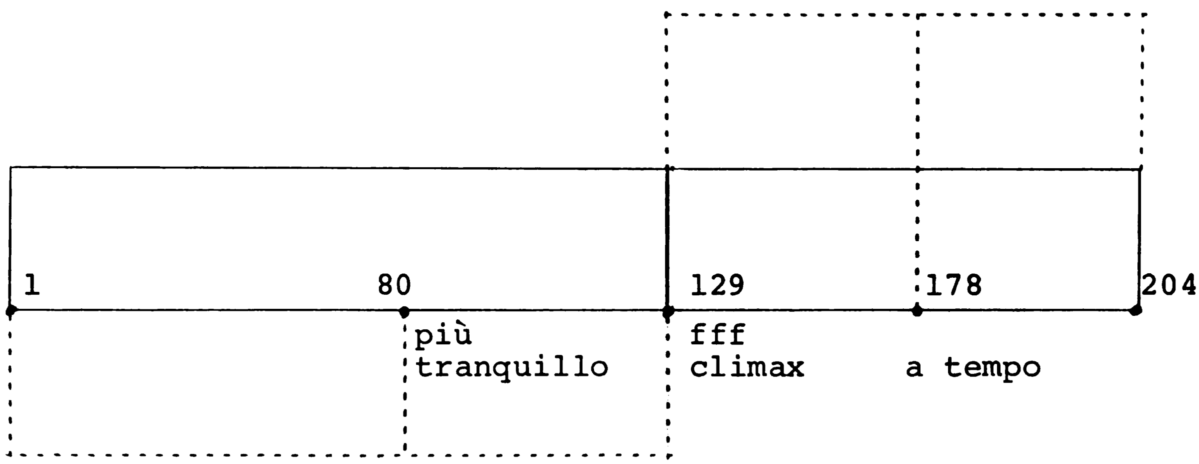
Diagram 7. <u>Divertimento for String Orchestra</u> 1939
Movement One
GS diagram



overall GS positive
first section GS positive
second section GS positive

Bar 44 through the end, a 31 measure segment, has a negative GS of 12. After 12 bars, there is a performance timing of thirty-seven seconds followed by a contrast of tempo and texture (bars 55-56).

The timed section of bars 33 through 49 (one minute and forty-seven seconds) has a length of 17 bars, and a positive GS of 11. Bar 44, the overall GS of the entire movement (Example 11) occurs after 11 measures within this small segment. The diagram of GS applications to the second movement is given in Diagram 8.

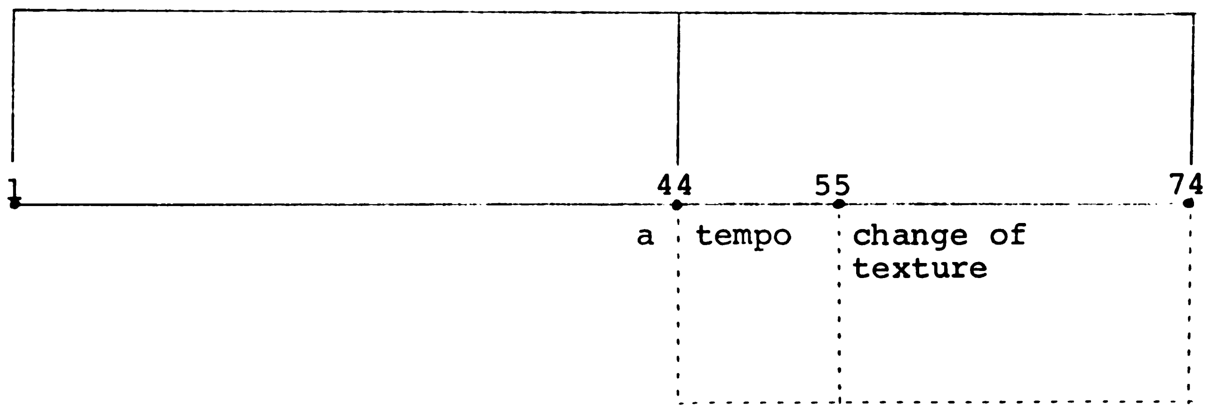
Though no overall golden section application can be made to the last movement of the work, numbers of the Fibonacci series are used in its structure. First, there are several thirteen bar phrases, each of which is listed by inclusive bar numbers in the following Figure.

Figure 58.

1-13	49-61
13-25	290-302
36-48	533-545

Moreover, in the segment that begins at bar 513, the 21st bar corresponds to a change in texture and the beginning of a Vivace section.

Diagram 8. <u>Divertimento for String Orchestra</u> 1939
Movement Two
GS diagram



overall GS positive
second section GS negative

Music for Strings, Percussion, and Celesta (1936)

The second movement of this work is a 520-bar segment including many metric changes. The number of quarter-note values, 1,010, is used as the basis of length to calculate the positive GS of 626. This occurs at bar number 305, seven beats before the fugal section is begun (bar 309). This fugal technique is similar in pattern to the opening of the first movement.

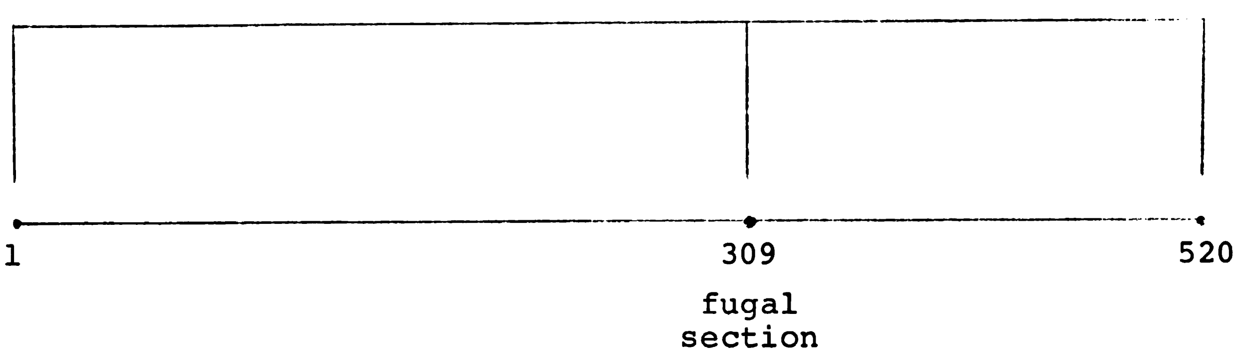
Bars 1 through 76 of the second movement form a segment which ends at a grand pause, and contain 123 quarter-note values. The positive GS of 82 occurs in bar 41, where the motive built on the melodic interval of the perfect fourth first appears; this motive is developed through the remainder of the section.

The portion beginning at the fugal section at bar 309 and extending through the end of the movement contains 377 quarter-note values, 377 being a Fibonacci number. The GS diagram of the entire second movement is given in Diagram 9 (page 62).

The fourth movement of the work contains 1,152 quarter-note values. The positive GS of 714 occurs at bar 178, two bars before a textural change takes place between bar 180 and 181.

The second segment of the movement, bar 181 through the end, contains 429 quarter-note values. The positive GS

Diagram 9. <u>Music for Strings, Percussion and Celesta</u> 1936
Movement Two
GS diagram



overall GS positive

of this length (265) occurs at bar 244, the first bar of a Calmo section, where a change in texture is made.

The segment between rehearsal letters C and D contains 124 quarter-note values and has a positive GS of 77. This corresponds to bar 102, one measure before a textural and dynamic climax is reached.

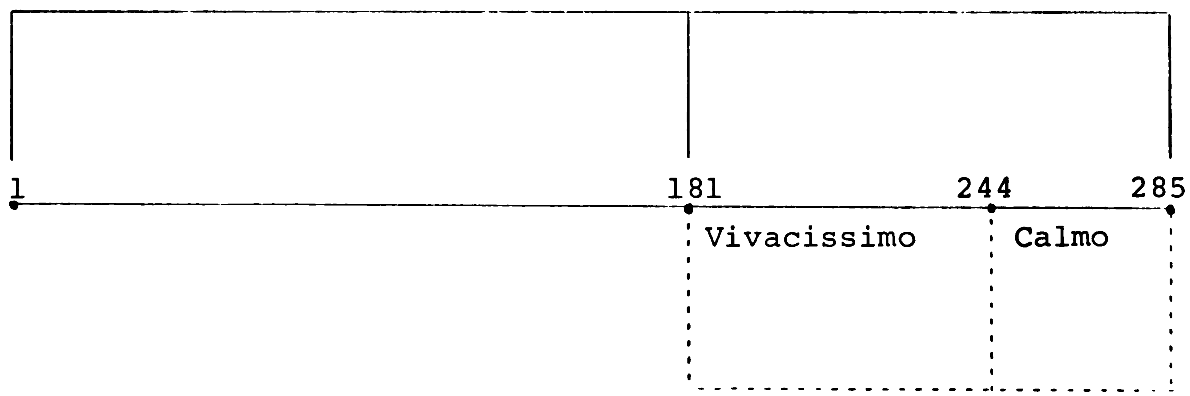
The section from rehearsal letter E to F contains 297 quarter-note values; its positive GS of 184 corresponds to bar 181, which is the positive GS for the entire movement.

The Calmo section at bar 244 (second section GS point) begins on the 987th quarter-note value of the entire movement; this integer of 987 is a member of the Fibonacci series.

The portion starting at rehearsal letter I and extending through the end of the movement contains 40 quarter-note values, and has a negative GS of 15, which occurs at bar 279. This measure is directly followed by a change of texture, where the main melodic interest moves from the strings to the piano and harp.

One further mention of the Fibonacci application will be made. Starting at rehearsal letter A, the 89th quarter-note value is the first beat of B; starting at D, the 89th quarter-note is the first beat of E; and the first beat of H is the 55th quarter-note value counting from letter G. The GS diagram for this movement is given in Diagram 10.

Diagram 10. <u>Music for Strings, Percussion and Celesta</u> 1936
Movement Four
GS diagram



overall GS positive
second section GS positive

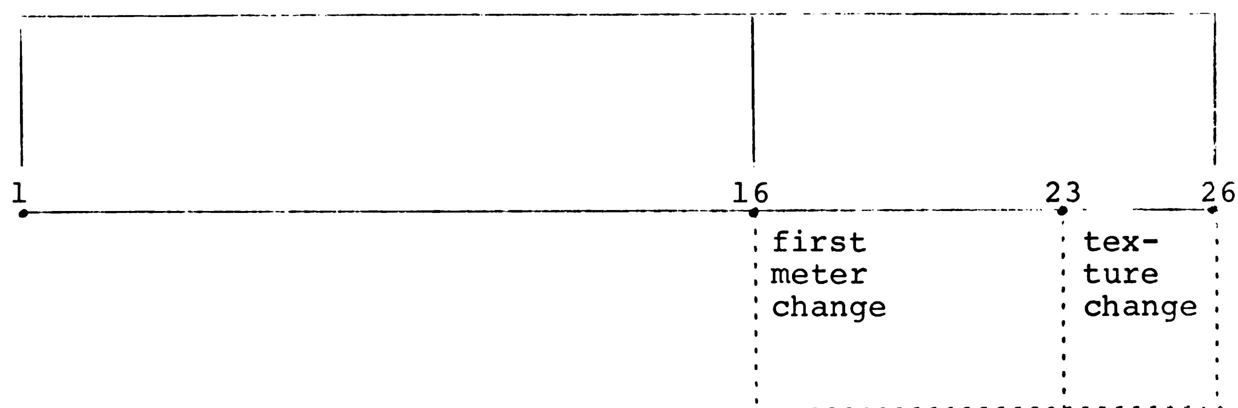
Mikrokosmos Suite for Orchestra (1942)

The *Prelude* from this suite is an orchestrated version of a Bartók piano piece first published in the album "Homage to Paderewski" in 1942. The number of half-note values (72) is the basis of length, the positive GS being 44.64. This division occurs at the end of bar 15, which corresponds to the first change of meter within the piece.

The segment from bar 16 to the end contains 27 half-note values, giving it a positive GS of 17. After 17 half-note values, there is a change in texture, accompanied by an allarg. marking at bar 23. Diagram 11 shows the GS arrangement for the *Prelude* of the suite (page 66).

None of the other movements of the *Mikrokosmos Suite* was found to conform to the overall golden section arrangement.

Diagram 11. Mikrokosmos Suite for Orchestra-Prelude 1942
GS diagram



overall GS positive
second section GS positive

CHAPTER IV

CHAMBER WORKS

Fifth String Quartet (1934)

In the first of the five movements in this quartet, the opening section to rehearsal letter A contains 13 bars, all in a $\frac{4}{4}$ meter. In bar 8, the quintuplet sixteenth-note melodic idea is first introduced. This motive is restated and developed extensively throughout the remainder of the movement.

Because the second movement contains many metric changes, the number of quarter-note values, 224, is used as the basis of length. The positive GS calculation is given in Figure 59.

Figure 59.

$$\begin{array}{r} 224 \\ \underline{\times\ .62} \\ 4\ 48 \\ \underline{134\ 4} \\ 138.88 \end{array}$$

The 139th quarter-note beat occurs in bar 35, the first bar of the Più lento section at letter C.

The section from letter C to the end of the movement contains 85 quarter-note values, and has a negative GS of 32. This point is at bar 43, which is at the beginning of the Largo segment, introduced by scalar passages in eighth notes.

Figure 60.

$$
\begin{array}{r}
85 \\
\underline{\text{X}\ .38} \\
6\ 80 \\
\underline{25\ 5} \\
32.30
\end{array}
$$

Example 12.[22]

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The beginning of the movement to letter B is a 100-unit length, with a negative GS of 38. The 38th quarter-note value is in the second part of bar 10, where a new tempo is indicated, and letter A is placed. Dividing slashes are placed in the middle of this measure. The GS for the second movement of the quartet is given in Diagram 12 (page 70).

The third movement of the quartet is divided into three main sections: Scherzo, Trio, and Scherzo da capo. In the Scherzo da capo the segment from rehearsal letter B to C contains 18 bars in a constant meter. At the 7th bar (negative GS), a climax of pitch and dynamics is reached (bar 54).

Example 13.[23]

54

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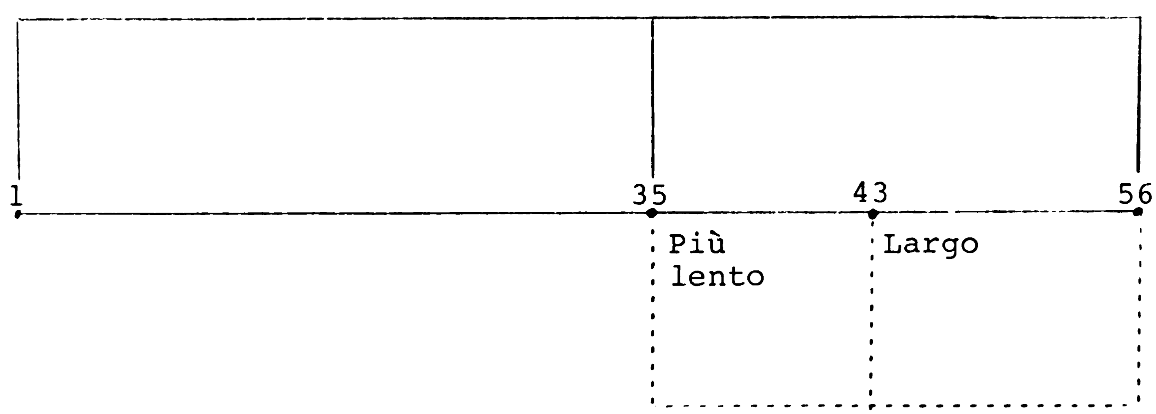
Diagram 12. <u>Fifth String Quartet 1934</u> Movement Two
GS diagram



overall GS positive
second section GS negative

The fourth movement of the quartet, Andante, contains 301 quarter-note values and has a positive GS of 187. The 187th unit occurs in bar 63, the bar before the beginning of the Più mosso, agitato section at letter C. This Più mosso segment is characterized by the chromatic thirty-second-note patterns.

Example 14.[24]

The opening section to letter A contains 22 bars, all of which adhere to the meter signature of $\frac{3}{4}$. After 14 bars (positive GS) a new tempo marking is located.

The segment from letter D to the end is also in a constant mater of $\frac{3}{4}$, and extends for 20 bars. After 8 bars (negative GS), the thirty-second-note pattern characteristic of this section is dispensed with and the coda of the movement begins (bar 90). The GS diagram of the fourth

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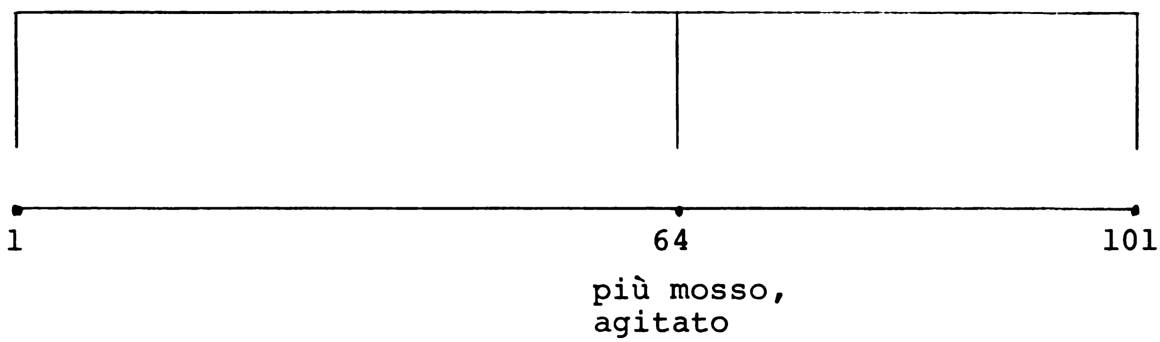
movement of the quartet is found in Diagram 13.

In the Finale of the quartet, the section from letter B to C contains 55 measures, giving the Fibonacci series application.

The portion from D to E contains 51 bars in a constant $\frac{2}{4}$ meter, and its positive GS is 32. After 32 bars the pitch and dynamic climax of bar 182 is reached.

Example 15.[25]

Diagram 13. <u>Fifth String Quartet</u> 1934 Movement Four
GS diagram



overall GS positive

Contrasts (1938)

Contrasts is a three movement work for violin, clarinet, and piano. The first movement has a length of 381 quarter-note values (excluding non-metered cadenzas), and a positive GS of 236. The 236th quarter-note value occurs at bar 59, two bars after the recapitulation begins at bar 57.

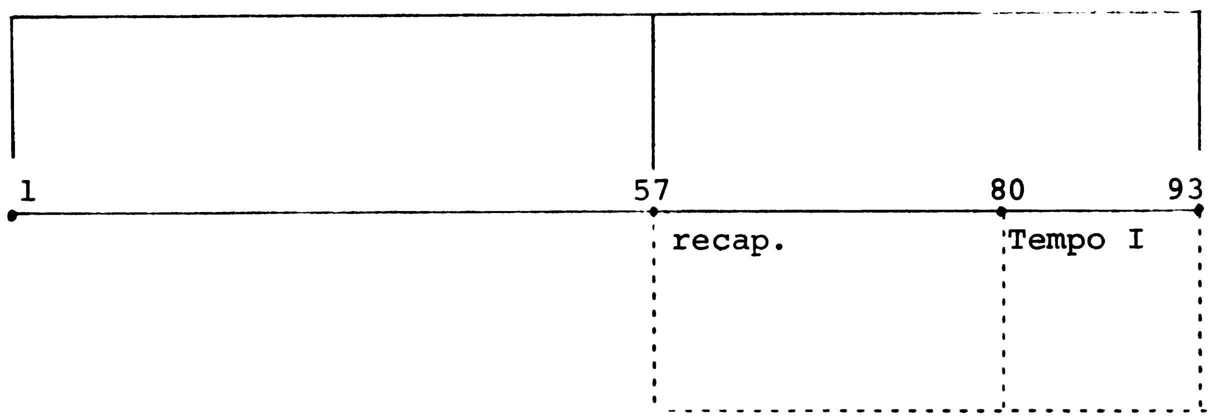
The segment from bar 57 to the end of the movement has a length of 154 quarter-note units, and its positive GS is 95, which occurs at bar 79. This is followed by a Tempo I section (bar 80), accompanied by a change of texture and melodic material.

The timed (thirty-nine seconds) section extending from bar 72 through 84 contains 13 bars and has a length of 60 quarter-note values. The positive GS (37) occurs at bar number 79, which is the GS point for the second main portion of the movement.

The coda section of the first movement, from bar 85 to the end, has a length of 36 quarter-note values. Its negative GS is 14; after 14 units the clarinet cadenza begins. Diagram 14 provides the GS diagram for the opening movement of Contrasts (page 75).

The unit of length for the second movement of the work is also the quarter-note value. The segment contains 250 units, the positive GS being 155. This point corresponds to bar number 34 (Fibonacci integer), which is followed by a change of texture and a tranquillo marking at bar 35.

Diagram 14. *Contrasts* 1938 Movement One
GS diagram



overall GS positive
second section GS positive

The second segment of movement two (bar 35 through the end) has a length of 92 units and a positive GS of 57, which occurs at bar 45. This is the first bar of a Movendo segment, preceded by a performance timing of fifty-two seconds. This Movendo segment could be considered the coda of the movement.

Example 16.[26]

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The opening section through bar 18 contains 72 quarter-note values, its positive GS being 48. This point occurs at bar 11, where a change from the tempo and texture of bar 10 is made.

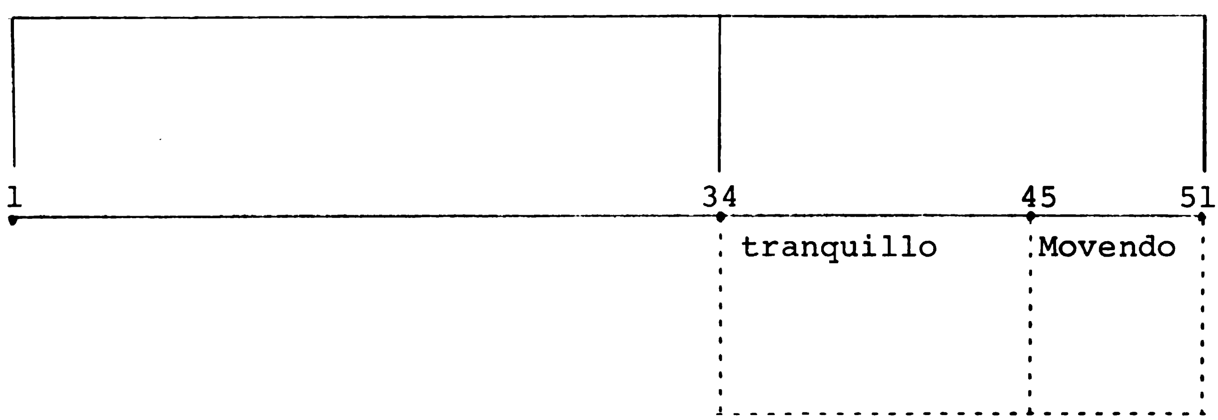
The following portion, beginning at bar 19 and extending through bar 35, contains 81 quarter-note values, and its positive GS is 50. This corresponds to bar 29, the outset of a Tempo I section characterized by the omission of the piano tremelos in octaves that were prevalent in the preceding measures. Diagram 15 provides the chart of GS applications to this movement.

The final movement of this chamber work has a length of 1,610 eighth-note values; its positive GS is 998. This occurs at bar 168, which is the last bar of the segment in $\frac{5+8}{8}$ time. This bar serves as a major division point within the movement, being followed by the recapitulatory Tempo I portion, which begins at bar 169.

The first Tempo I segment of the movement that begins at bar 59 occurs on the 233rd (Fibonacci) eighth-note value of the movement.

The timed segment (one minute and ten seconds), beginning at bar 169 and extending through bar 213, contains 45 bars, all in a constant meter of $\frac{2}{4}$ (excluding the violin cadenza), and its negative GS is 17. After 17 bars the change in texture at the Più mosso marking occurs (bar 186).

Diagram 15. <u>Contrasts</u> 1938 Movement Two
GS diagram



overall GS positive
second section GS positive

The segment from bar 214 through 247 is 34 bars long in a constant meter. The 21st bar corresponds to bar number 234, where the glissandi in the piano are introduced and a Molto tranquillo marking is located. Diagram 16 provides the GS diagram for the third and last movement of Contrasts.

Sixth String Quartet (1939)

In the first movement of this work, Fibonacci application can be made to the opening viola melody. This solo segment is 13 bars long; the highest pitch and strongest dynamic is reached in the 8th bar.

Example 17.[27]

The Vivace section (bars 24 through 80) contains 57 bars in a constant meter of $\frac{6}{8}$. After 35 bars (positive GS), bar 59 is reached, which is followed by a contrast in texture and melody at bar 60.

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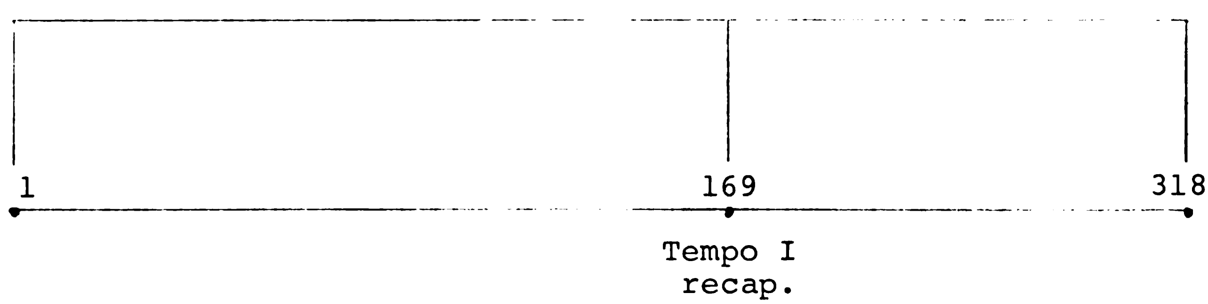
Diagram 16. Contrasts 1938 Movement Three
GS diagram



overall GS positive

The section that starts at bar 137 and extends through bar 157 (twenty seconds) has a length of 21 bars.

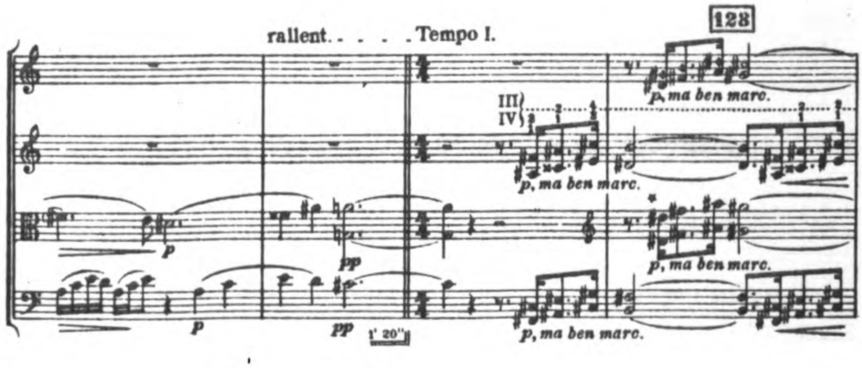
The next section to be dealt with includes bars 180 through 221, a scope of 42 measures. The positive GS of 26 corresponds to bar 205, where an ascending melodic line is introduced in the first violin. This motivic idea is developed later in the segment in the second violin and the viola.

Since there are many metric variations within the second movement of the quartet, the number of eighth-note values, 1,513, is considered to be the length of the segment. The positive GS is 938, which occurs at bar 120. This is two bars before a major structural division takes place, with the beginning of the recapitulatory segment at bar 122 (Example 18).

The final timed section (forty-four seconds) from bar 173 to the end of the movement contains 148 eighth-note values, its positive GS being 92. This corresponds to the last part of measure 184, which is followed by a change of tempo and texture. The GS diagram for the second movement of the quartet is given in Diagram 17 (page 83).

The third movement of the work has a length of 1,160 eighth-note values, and a positive GS of 719. This occurs at bar 97, where a double forte climax is reached after a double bar is marked (Example 19).

Example 18.[28]



Example 19.[29]

The first segment of the movement, as divided by the overall GS, extends from the opening through bar 96, and has a length of 712 eighth-note values. Its positive GS (441) occurs at bar 60, a tutti climax point that is similar to bar 97 (Example 19) due to the repeated eighth-note pattern in all of the voices.

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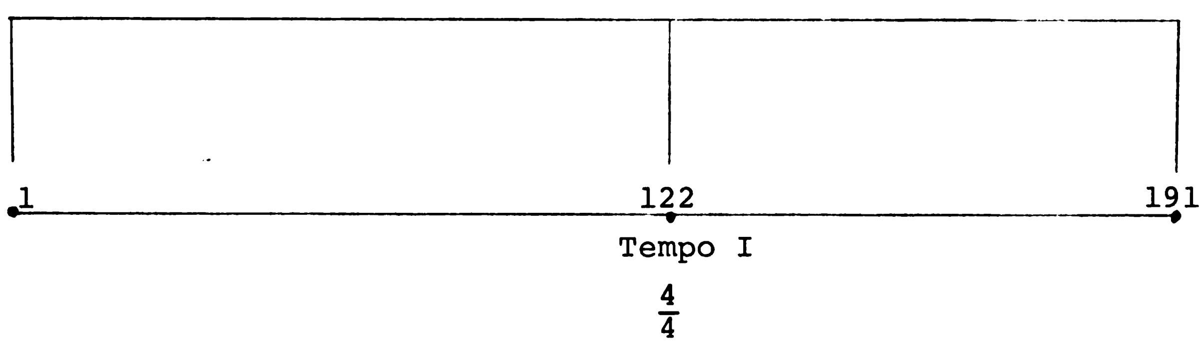
29 *Ibid*.

Diagram 17. Sixth String Quartet 1939 Movement Two
GS diagram



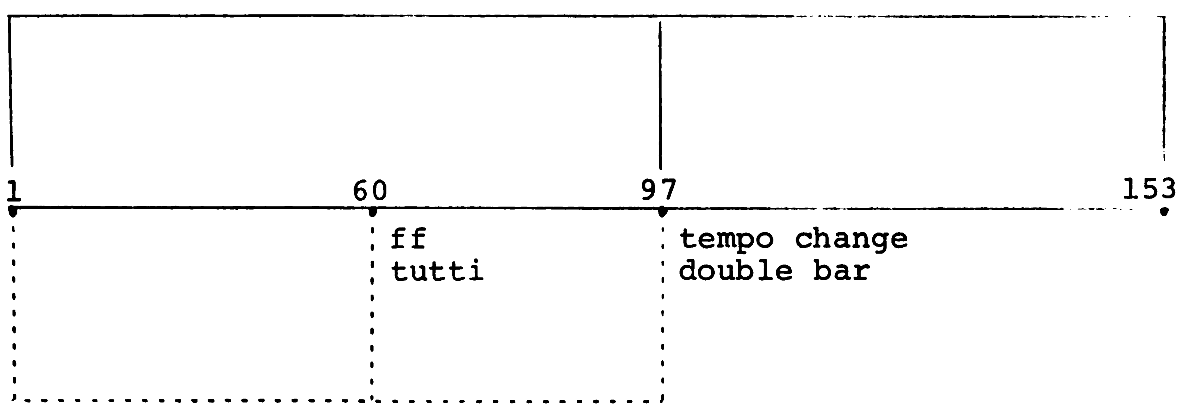
overall GS positive

The small portion which includes bars 123 through 134, all in $\frac{4}{4}$ time, has a length of 12 bars and a negative GS of 5. The fifth bar (127) of the segment is the location at which the triplet sixteenth-note figure is heard; this pattern is subsequently developed and restated throughout the remainder of the segment. Diagram 18 (page 85) provides the GS diagram for the third movement of the quartet.

The final movement of this chamber work, which uses the metric signatures of $\frac{6}{8}$ and $\frac{9}{8}$, contains applications to the Fibonacci series of integers. A crescendo that starts at bar number 8 results in a forte climax at bar 13. The 89th dotted quarter-note value occurs at bar 45, which is directly followed by a performance timing (three minutes) and a contrasting Molto tranquillo segment. Furthermore, the 144th dotted quarter-note value occurs at bar 72, the beginning of a Tempo I section.

Neither the second movement of the <u>Sonata for Two Pianos and Percussion</u> (1937) nor any movement of the <u>Sonata for Solo Violin</u> (1944) was found to adhere to the main golden section arrangement.

Diagram 18. Sixth String Quartet 1939 Movement Three
GS diagram



overall GS positive
first section GS positive

CHAPTER V

MIKROKOSMOS PIANO PIECES

(1930-1937)

Volume Four

Piece number 99, Crossed Hands, contains 23 bars in a constant $\frac{3}{4}$ meter. After 14 bars, a section using the inversion of the primary motive begins. Bars 1 and 2, and 15 and 16, are reproduced in Example 20. Diagram 19 shows the GS application for Crossed Hands.

Figure 61.

$$\begin{array}{r} 23 \\ \underline{\times\ .62} \\ 46 \\ \underline{13\ 8} \\ 14.26 \end{array}$$

Number 106, Children's Song, contains 44 bars with a constant $\frac{2}{4}$ meter. At the positive GS point (27th bar), a dividing slash (|) appears, followed by a Più lento section. This slash appears in a number of Bartók's scores and is characteristically placed at formal division points.

From bar 27 to the end of Children's Song has a length of 18 bars, with a positive GS of 11. This corresponds to measure 37, where a Tempo I section begins after a

Example 20.[30]

definite cadence on the pitch A has been reached. The GS application for Children's Song is shown in Diagram 20 (page 89).

Piece number 116, Melody, is a 43 bar segment in a constant $\frac{4}{4}$ time. After 27 bars, the positive GS point, an a tempo recapitulatory section begins. Diagram 21 is the GS diagram for this piece (page 90).

Number 119, Dance in $\frac{3}{4}$ Time, contains 29 bars, and has a positive GS of 18. After 18 bars, there is a cadence followed by a marking of a tempo. Diagram 22 provides the golden section arrangement for Dance in $\frac{3}{4}$ Time (page 91).

None of the remaining pieces from the fourth volume or any of the pieces in volume five was found to comply with the overall GS structure; however, piece number 128

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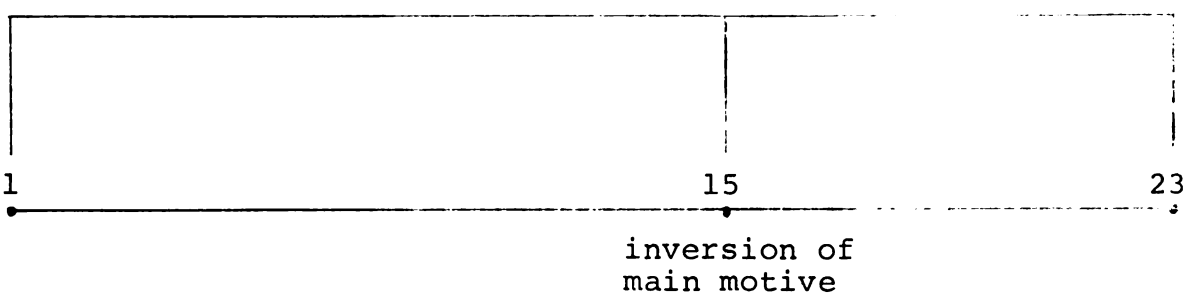
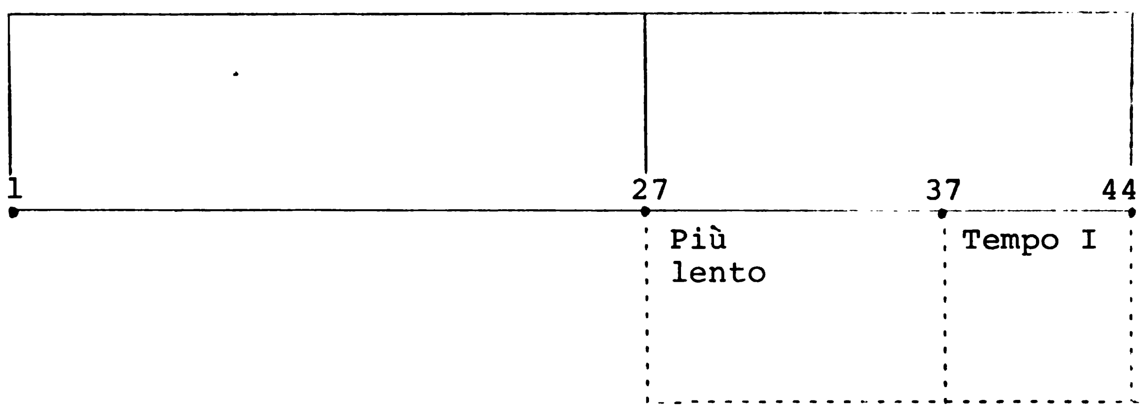
Diagram 19. Mikrokosmos Number 99 Crossed Hands
GS diagram



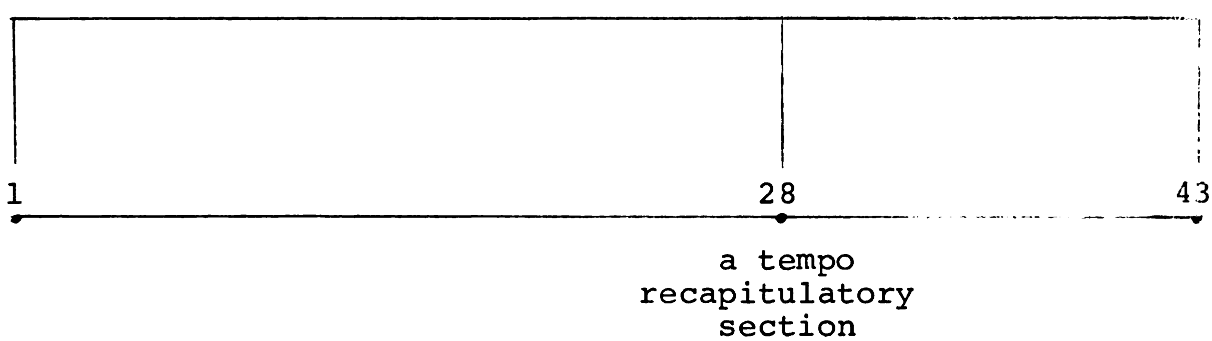
overall GS positive

Diagram 20. Mikrokosmos Number 106 Children's Song
GS diagram



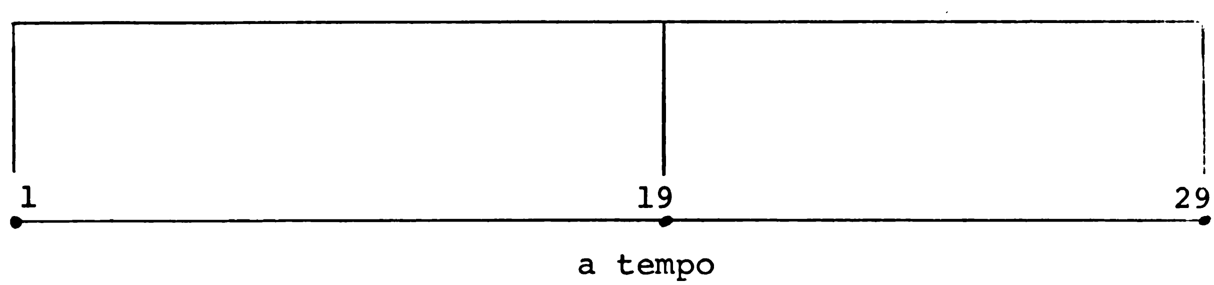
overall GS positive
second section GS positive

Diagram 21. <u>Mikrokosmos</u> Number 116 <u>Melody</u>
<u>GS diagram</u>



overall GS positive

Diagram 22. *Mikrokosmos* Number 119 *Dance in $\frac{3}{4}$ Time*
GS diagram



overall GS positive

(volume five) contains 55 quarter-note values within its first main portion.

Volume Six

Number 143, Divided Arpeggios, contains 80 bars, all but one of which is in the same meter. The positive GS is at bar 50, where an a tempo section begins with a sixteenth-note pattern divided between the hands.

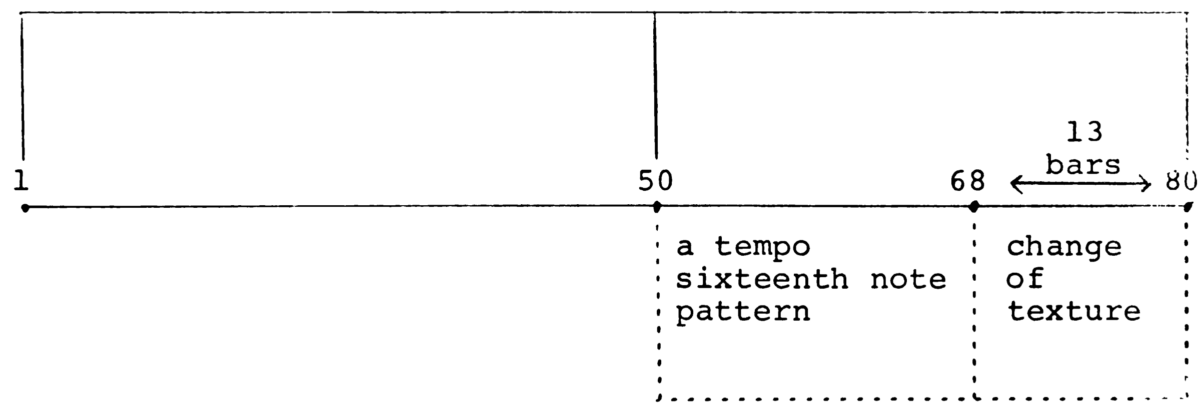
The portion from bar 50 to the end has a positive GS of 19 bars. This point corresponds to the change in texture that occurs at bar 68. This final section, from bar 68 to the end of the piece, has a length of 13 measures. GS applications in Divided Arpeggios are given in Diagram 23 (page 93).

Number 144, Minor Seconds, Major Sevenths, contains many metric changes, so the number of half-note values, 144 (a Fibonacci number), is used as the basis of length. This equality between the number of the piece and the number of beats does not occur in any of the other pieces of volume four, five, or six.

The positive GS point of 89 half-note values occurs at bar 43, the opening bar of the Più andante section.

The Fibonacci series of integers has other applications to this piece; the first $\frac{2}{4}$ meter signature appears directly before the 21st half-note value. Furthermore, counting from the Più andante marking reveals that a dynamic

Diagram 23. *Mikrokosmos* Number 143 *Divided Arpeggios*
GS diagram



overall GS positive
second section GS positive

climax occurs at the 13th measure--bar number 55. The GS scheme for piece number 144 is given in Diagram 24 (page 95).

Example 21.[31]

Piece number 150, Dance in Bulgarian Rhythm, has a length of 94 bars with a constant meter signature. The positive GS is 58, where there is a dynamic climax accompanied by a change in texture from the moving eigth-note motion to more sustained patterns. This GS number is made up of two Fibonacci integers, and these same integers appear in the $\frac{5}{8}$ meter of the piece.

The first section through bar 58 has a negative GS of 22. After 22 bars, the first restatement of the opening motive is made at one octave below the original pitch level.

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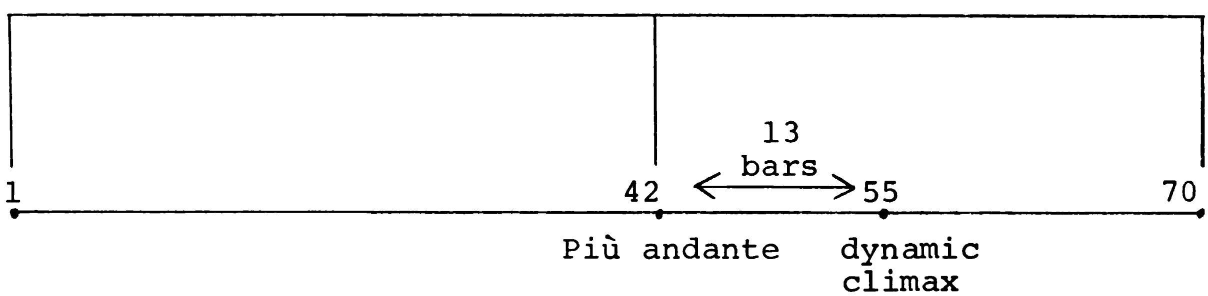
Diagram 24. Mikrokosmos Number 144 Minor Seconds, Major Sevenths
GS diagram

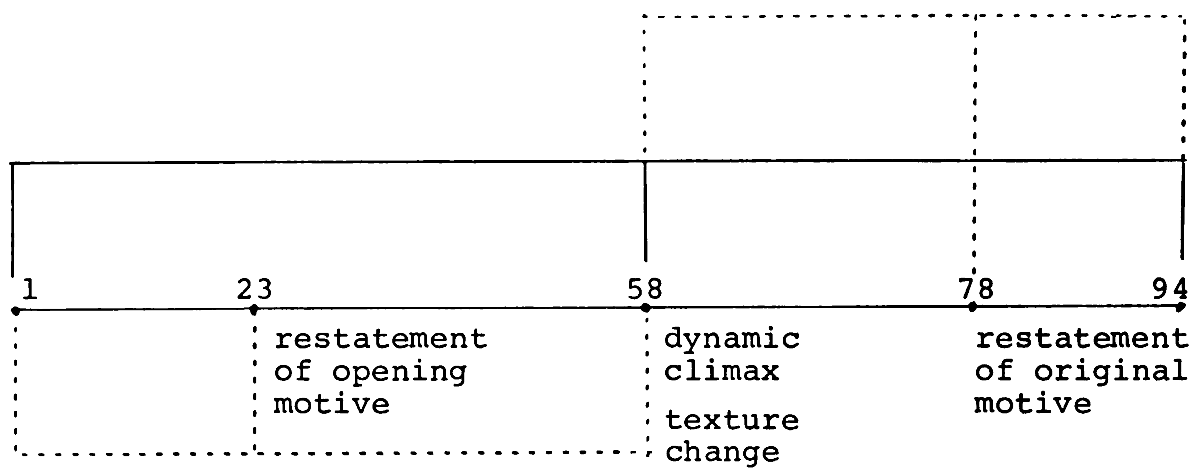


overall GS positive

The second section of the piece has a length of 36 bars and a positive GS of 22. The 22nd bar occurs one bar after another restatement of the initial motive is begun at bar 79, this time at a pitch level a perfect fifth below the original. The various GS applications to Dance in Bulgarian Rhythm are illustrated in Diagram 25 (page 97).

None of the remaining pieces of volume number six of Mikrokosmos was found to have any applications to the overall GS structure.

Diagram 25. <u>Mikrokosmos</u> Number 150 <u>Dance in Bulgarian Rhythm</u>
GS diagram



overall GS positive
first section GS negative
second section GS positive

CHAPTER VI

SUMMARY AND CONCLUSION

Seventeen separate movements of large works and seven Mikrokosmos piano pieces were found to adhere to the overall golden section arrangement. Of these twenty-four movements, only the first movement of the Concerto for Orchestra was found to have a negative overall golden section, all other main golden section arrangements being positive.

Nine of these twenty-four segments contained one auxiliary GS in addition to the main GS, and in eight of these nine cases, the auxiliary GS was of the first main section. Most of the auxiliary GS arrangements were positive.

Three examples contained auxiliary golden sections in both of the sections as divided by the main GS: the first movement of the Divertimento for String Orchestra; the first movement of the Third Piano Concerto; and Mikrokosmos piece number 150, Dance in Bulgarian Rhythm. In this category there were three instances of both the positive and negative auxiliary golden section arrangements.

Two multi-movement works were found to contain application to the main golden section in every one of their movements: Contrasts and Music for Strings, Percussion, and Celesta (movements one and three of the latter work were not covered in this report).

Eleven movements were found to contain one or more examples of Fibonacci series usage without the overall golden section arrangement: two additional movements contained GS arrangements within isolated segments without any structural usage of the Fibonacci series or overall GS arrangement.

Analysis of the musical works treated in this study reveals that some movements do have several applications to the golden section principle and the Fibonacci series, notably the first movement of the Divertimento for String Orchestra, and the first movement of the Third Piano Concerto. Noteworthy is the fact that Mikrokosmos piece number 144, Minor Seconds, Major Sevenths, contains 144 half-note values.

With the exception of the string quartets, the first movement of a multi-movement work usually exhibited most applications to these principles, and the number of applications declined in the subsequent movements.

Several examples showed some adherence to these principles, though some movements were not as consistent in their application as were others.

All of the applications made in the previous chapters in this report are presented as evidence for the contention that Bartók may have used the principles of the golden mean and the Fibonacci series consciously in relation to the structural format of some of his later musical compositions.

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