

ACOUSTIC BIREFRINGENCE
OF LIQUID POLYSTYRENE

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

Donald Alan Hall

1954

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# ACOUSTIC BIREFRINGENCE OF LIQUID POLYSTYRENE

by

Donald Alan Hall

#### A Thesis

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Department of Physics

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19.24

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Donald a. Hall

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#### I. INTRODUCTION

#### A. Historical

Brewster<sup>1</sup> was the first to investigate the effect of a transparent solid on polarized light. It was shown later by Fresnel<sup>2</sup> that the observed effects could be attributed to double refraction, or, as it is also known, birefringence. This double refraction was found to be due to strains in the solid. In 1866, James C. Maxwell<sup>3</sup> attempted to determine if a state of strain existed in a viscous liquid that was in motion by observing the effect on polarized light when passed through the liquid. He was unable in the first attempts to detect an occurrence of double refraction. At a later date the experiment was repeated and Maxwell<sup>14</sup> observed a relation between the indicaes of refraction in, the viscosity  $\gamma$ , and the velocity gradient. This relation is

$$\Delta n = Ne - No = M \cdot \frac{\partial u}{\partial z} \cdot \gamma \tag{1}$$

where

M = the Maxwell constant of the liquid

ou = the velocity gradient.

The phenomena of double refraction has also been observed when a liquid is subjected to sound energy. The frequency of the sound is in the ultrasonic range. The effect was observed in solutions of iron oxide and vanadium pentoxide by Cookson and Osterberg<sup>5</sup> and by Lucas<sup>6</sup> in liquids such as linseed oil and castor oil.

Several theories of acoustic double refraction have been proposed, each differing significantly from the others. Lucas bases his macroscopic theory on the stress in the liquid and relates the acoustic double refraction to the dynamic or flow double refraction of Maxwell. Peterlin gives a microscophic theory based on particle orientation. The particles are oriented by the velocity gradient giving rise to the whole liquid becoming amisotropic. A third theory is given by  $0 \text{ka}^8$ . This is also a microscopic theory which attributes the double refraction to an orientation of colloidal suspensions by the sound pressure in the liquid. Due to the nature of the liquid used in this experimental work, the theory given by Lucas, which does not depend upon colloidal suspensions or particles, will be outlined.

# B. Theory

Lucas<sup>6</sup> makes use of the normal and tangential stress relations for viscous liquids as given by the Navier-Stokes hydrodynamic equations. These are:

$$N_{1} = \lambda \theta + 2 \eta \frac{\partial u}{\partial x} - P \qquad T_{1} = \eta \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial y} \right)$$

$$N_{2} = \lambda \theta + 2 \eta \frac{\partial w}{\partial y} - P \qquad T_{2} = \eta \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial x} \right) \qquad (2)$$

$$N_{3} = \lambda \theta + 2 \eta \frac{\partial w}{\partial y} - P \qquad T_{3} = \eta \left( \frac{\partial v}{\partial x} + \frac{\partial w}{\partial y} \right)$$

where N = the Normal stresses,

T = the Tangential stresses,

p = the pressure,

A = are two viscosity coefficients from Stokes

theory,

O = 34 + 34 + 34 + 34 + 34 velocity.

Applying these equations to a plane progressive longitudinal wave with velocity v traveling in the OX direction, the expressions for the normal and tangential stresses become:

$$N_1 = (\lambda + 2\eta) \frac{\partial u}{\partial x} - \rho$$

$$N_2 = N_3 = \lambda \frac{\partial u}{\partial x} - \rho$$

$$T_1 = T_2 = T_3 = 0$$
(3)

where the velocity components in the OY and OZ direction are zero. This is then equivalent to a liquid which is under an isotropic pressure given by

$$p - \lambda \stackrel{>}{>}_{\sim} . \tag{4}$$

The liquid is also experiencing a normal force acting in the OX direction as shown in Figure 1.

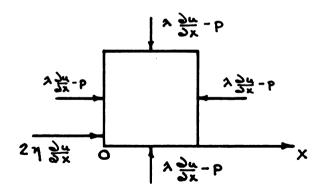


Fig. 1. Pressure and Force Acting on Liquid.

The normal force can be expressed as

where  $\eta$  is the usual value for shear viscosity. This force gives rise to an anisotropic condition in the liquid and can be found if the velocity gradient,  $\frac{\lambda u}{\lambda x}$ , in the direction of propagation can be determined.

It will be possible to obtain an expression for the

velocity gradient from the viscous decay of the wave along the OX direction. From the formula for the displacement u we have  $u = u_0 e^{-\kappa x} \sin \omega (1 - \frac{x}{2})$  (6)

or 
$$\frac{du}{dx} = u_* \omega e^{-ax} \left[ \Leftrightarrow \sin \omega (t - \Leftrightarrow) - a\cos \omega (t - \Leftrightarrow) \right]$$
 (7)

In general 
$$\frac{\omega}{V} = \frac{2\pi}{\Lambda} >> \infty = \frac{1}{\text{DISTANCE FOR DECAY TO }\frac{1}{L}}$$
 (8)

Therefore the velocity gradient as a function of the maximum displacement will be in the first but sufficient approximation

$$\frac{dv}{dx} = 4 \omega^2 e^{-\kappa x} \sin \omega (t - \frac{\kappa}{2}) \approx 4 \omega^2$$
 (9)

The sound intensity can also be expressed as a function of the maximum displacement and can be written as

 $I = \frac{1}{2} \rho \vee u_s^2 \omega^2$ where  $\rho$  is the density. Combining equation (9) and equation (10) yields the amplitude of the particle velocity gradient which is  $\left(\frac{\partial u}{\partial x}\right)_{MAY} = \frac{u_s \omega^2}{\sqrt{2}} = \frac{\omega}{\sqrt{2}} \sqrt{\frac{21}{\rho \sqrt{2}}}.$ (11)

By substituting this expression for the velocity gradient into the formula for the normal force we obtain the amplitude of the normal stresses as

amplitude of the normal stresses as  $SN_{i, \text{MAX}} = 2\eta \frac{Su}{Sx} = 2\eta \frac{\omega}{\sqrt{21}}$ (12)

which is the anisotropic force present in the progressive wave. From this we see that for a liquid of negligible viscosity or for elastic waves at low frequencies, the anisotropic stresses disappear.

Lucas shows that the double refraction that is caused by a laminar flow of the liquid can also be analyzed on the basis of an anisotropic normal force. He then compares the double refraction thus obtained with that from the acoustic case. From Stokes, the tangential stress on a liquid moving with laminar flow is  $T_3 = \eta \frac{3u}{3\eta}$ . (13)

According to elastic theory, this tangential stress can be replaced by two perpendicular normal stresses which act along OY and OZ. This is shown in Figure 2.

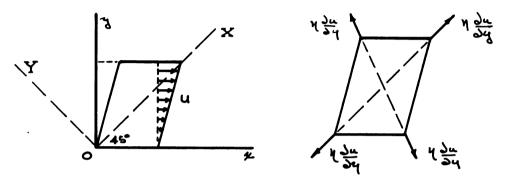


Fig. 2. Forces Acting on Liquid Moving with Laminar Flow.

These forces are of magnitude  $\eta = 0$ . The anisotropy that results from these two forces can be replaced by a single normal stress acting at  $45^{\circ}$  to the velocity of the flow u, and is of magnitude  $\int N_{45} = 2 \, \eta = 0 \, \text{magnitude} \qquad \qquad (14)$ 

Referring back to Maxwell's relation for the double refraction arising from laminar flow of a liquid, equation 1, we can obtain an expression for the double refraction in terms of the normal force given in equation (14). Or  $\Delta n = \kappa_e - \kappa_o = \frac{M}{2} \cdot S N_{46} . \tag{15}$ 

By anology, then, the acoustic double refraction can be related to the acoustic normal force of equation (12). The acoustic double refraction then becomes

$$(\Delta N)_{MAX} = \frac{M}{2} (SN)_{MAX} = V M \stackrel{\triangle}{\sim} \sqrt{\frac{21}{\rho V}}$$
(16)

M = viscosity, where M = Maxwell's constant of flow double refraction.  $\omega =$ angular-frequency  $= 2\pi f$ , f = frequency of the sound wave, v = wave velocity, I = acoustic intensity.  $\rho = density.$ 

The optical axis is given by the plane of the wave front. i.e., it coincides with the velocity of the liquid whereas in the case of laminar flow, as in Maxwell's measurements. it is at 45° to the direction of the velocity of the liquid.

Summing up the predictions that can be made on the basis of the Lucas theory expressed in equation (16), we should find that the double refraction should be proportional to the following:

- 1. the frequency of the sound wave:

- 2. the Maxwell constant;3. the viscosity of the liquid;4. the reciprocal of the temperature;
- 5. the square root of the sound intensity which is proportional to the sound amplitude.

#### C. Earlier Experimental Studies

Lucas pointed out that the usual methods for the measurement of the birefringence cannot be applied in the special case of acoustic birefringence. Measurements by means of compensators or quarter-wave plates are based on the comparisons of beams of elliptically polarized light which have the same degree of ellipticity. Therefore Lucas used another method. The liquid is brought between two crossed Nicols so that no light emerges from the analyzer. If an ultrasonic field is applied to the liquid, light again emerges. Lucas derived that the intensity of the emergent light is directly proportional to the sound intensity and to the square of the ultrasonic frequency.

Although Lucas reports on some experimental observations, ne did not make any quantitative measurements. Some quantitative studies have been reported by Zvetkov, Mindlina, and Makarov9. The results of these authors appear to prove the Lucas theory. Lucas developed his theory for the case of progressive waves. The experimental set-up of the Russian authors did not exclude the possibility of stationary waves. Zvetkov, Mindlina, and Makarov used the following set-up. For a sound source they used a piezoquartz fastened on a brass plate which served as a holder and at the same time, as one of the electrodes. Aluminum foil pasted on the other face of the quartz acted as the second electrode. The quartz was placed directly into the liquid to be investigated. The cell to hold the liquid was made of glass. Windows for the light to pass through the liquid were of strain-free cover glass. The quartz was fed from an oscillator with a frequency range of 600kc/s to 4000kc/s.

Zvetkov, Mindlina, and Makarov measure the birefringence of the liquid by measuring the intensity of the emergent light after it has passed through a polarizing Nicol prism, the liquid cell, and a second Nicol prism that has been crossed with the polarizer. A schematic drawing of their apparatus is shown in Figure 3.

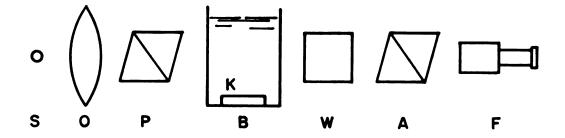


Fig. 3. Apparatus used by Zvetkov, Mindlina, and Makarov.

In Figure 3, S is the light source which was the filament of an incandescent lamp; O is a collimating lens; B the glass cell which contained the liquid under investigation and the quartz K; P is the polarizing Nicol whose principle section was set at an angle of 45° with the ultrasonic wave front; W is a Wollaston prism; and A is the analyzing Nicol. The scale of angular displacement on the analyzing Nicol was divided into 0.01° divisions. The image of the filament was observed through the telescope F.

The Wollaston prism was used in the following manner.

It was mounted so that its principle section was in the same plane as the polarizer P. This would cause one of the two images seen in the telescope to be extinguished. The analyzer was then rotated until the second of the images was extinguished. The analyzer is then said to be in the zero position. When an ultrasonic field was introduced into the liquid from the piezoquartz, the liquid became optically

anisotropic. This caused the first image, the one that had been extinguished by placing the principle section of the Wollaston parallel to the plane of polarization from P, to reappear. The brightness of this restored image could then serve as a measure of the birefringence present.

The use of the Wollaston prism to measure the birefringence is based on the assumption that for small values of  $\Delta n$  the intensity of the light beam giving the second image, the one extinguished by the analyzer, is not changed when the sound is present in the liquid. Let the intensity of this beam be  $I_0$ . When birefringence is present and the first Wollaston image has reappeared with intensity I, the analyzer can be turned until the second Wollaston image is of equal intensity, or,  $I = I_0$ . The angular displacement of the analyzer from the zero position to the point where the images are of equal brightness is designated by  $\ll$ .

From the relationship

$$\underline{\underline{I}}_{0} = \tan^{2} \alpha \tag{17}$$

it is possible to obtain a working formula that will relate the angular displacement  $\alpha$  and the birefringence  $\Delta n$ . The instantaneous brightness of the first image is equal to

$$I' = I_0 \sin^2(\pi \delta) = I_0 \sin \frac{\pi \ell}{2} (\pi_0 - \pi_0)$$
 (18)

where  $\delta$  is the optical path difference of the rays,  $\ell$  is the length of the light beam in the acoustic field,  $\lambda$  is the light wave length, and  $(n_e - n_o)$  is given by the

following equation  $N_e - N_o = \frac{8\pi^2 + AM\pi}{c} \cdot \sin 2\pi + (1 - \frac{x}{c})$ (19)

where c is the velocity of the acoustic wave,  $\rightarrow$  the frequency of the wave, and  $\triangle$  the maximum amplitude of the wave. Since the observed path difference does not exceed a few per cent, one can make the approximation that  $\sin^2(\pi \xi) \approx (\pi \xi)^2$  in equation (18) so that the mean value of the brightness I is found to be

$$I = I_o \left(\frac{\pi \ell^2}{\lambda}\right)^2 \Delta \kappa^2 \tag{20}$$

where  $\Delta n$  is the mean quadratic birefringence. Combining equation (17) and equation (20) we obtain

$$\Delta u = \frac{\lambda}{\pi I} \, t_{AN} \, \alpha. \tag{21}$$

The birefringence can thus be determined by measuring the angular displacement of the analyzer.

#### II. EXPERIMENTAL METHODS USED IN THE PRESENT STUDY

For the investigations reported in this thesis, two different methods were used. The first method was a variation of the method used by the Russian authors; the second method was based on photometric measurements.

#### A. Method Using Wollaston Prism

As the experiment of Zvetkov, Mindlina, and Makarov9 did not exclude the possibility of stationary waves, it seemed interesting to investigate the case of stationary waves. In this experimental arrangement use was made of Barium Titinate ceramics as a source of ultrasonic energy instead of piezoquartz. The choice was made so that the output characteristics of the oscillator used for the electromotive driving voltage and current could be utilized. This oscillator is a U.S. Navy radio frequency transmitter, Model ATD, with a frequency range of 540kc/s to 15,000kc/s. The output circuit of the transmitter was modified so as to conform to the Pi-section matching circuit as described in the ARRL Handbook 10. This change was necessitated due to the low impedance of the BaTi transducers and with this circuit it is possible to obtain maximum transfer of rf energy from the oscillator to the transducer. The voltage necessary to operate the transmitter is obtained from the dynamotor that was supplied with the transmitter.

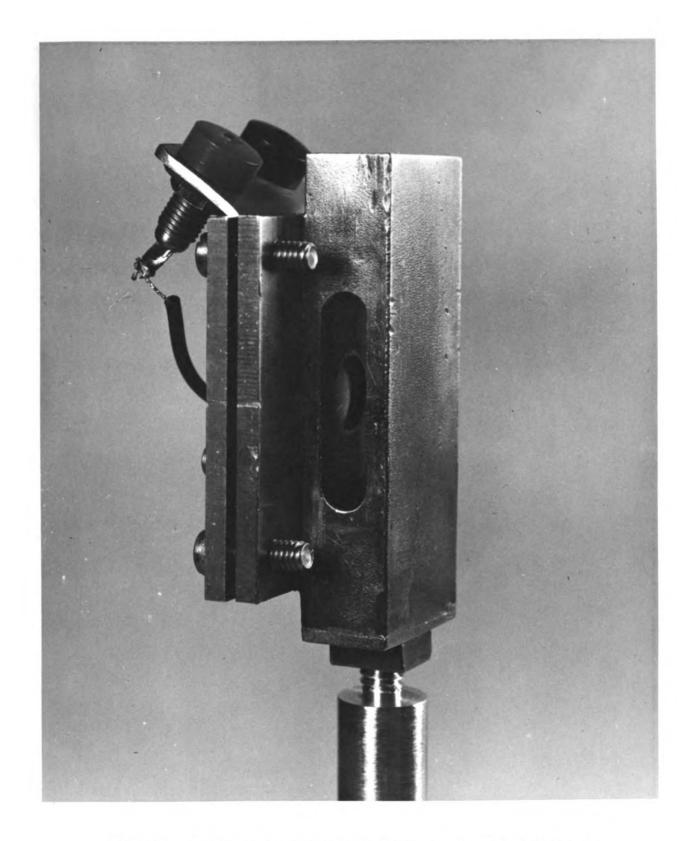


Fig. 4. Tank to Hold Liquid and Transducer.

The dynamotor in turn obtained its operating voltage from the 28 volt DC supply in the building.

The cell or tank which was used to hold the liquid and the transducer is shown in Figure 4. The body of the tank is made from a section of 3 cm. copper wave guide. Its inside dimensions are; I cm. x 2.3 cm. x 6.5 cm. long. Two openings, 1 cm. x 4 cm. are cut in the sides of the wave guide. These are covered with optically strain-free glass which act as windows for the light beam. A piece of brass, 5 cm. x 5 cm. x 0.3 cm., is soldered on the back of the tank. A hole, 1.6 cm. in diameter, is drilled into the brass plate to a depth of 0.15 cm. A second hole is started at this depth of 1.3 cm. diameter which continues into the plate for 0.075 cm. A third hole of 1.0 cm. diameter starts at this point and continues through into the tank. Figure 5 is a cross-section through the brass plate and the wall of the copper wave guide which shows the details of the holes just described.

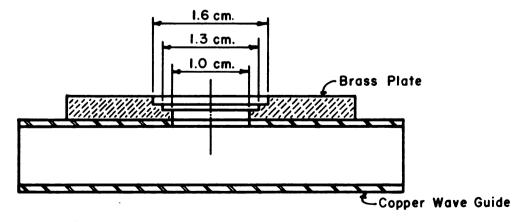


Fig. 5. Section Through Tank for Liquid and Transducer.

The BaTi ceramics used as transducers are flat circular disks, 1.28 cm. in diameter and 0.25 cm. thick. These dimensions are for the ceramics with a fundamental frequency of 1000kc/s. The ceramic disk fits into the 1.3 cm. diameter hole in the brass plate and is held in place by a Neoprene rubber "0" ring which is placed in the 1.6 cm. diameter hole. A second brass plate, with the same dimensions as the plate soldered to the copper wave guide, is then placed on top of the "O" ring. It is held to the first plate by means of four machine screws, one in each corner. A 1.24 cm. diameter hole is drilled through the second plate at its center. Through this hole is passed a spring loaded brass contact that bears against the silver plated surface of the BaTi ceramic. This acts as one of the electrical connections to the transducer. The reverse face of the ceramic is also silver plated, and, since it is in direct contact with the first brass plate, the whole metallic tank acts as the second contact. Two "bananna" plug terminals are mounted on the second brass plate. The output of the oscillator is fed to the BaTi ceramic through these terminals.

Measurement of the birefringence of the liquid was made in the same manner as described by Zvetkov, Mindlina, and Makarov<sup>9</sup>. Figure 6 is a schematic drawing of the optical arrangement that was employed.

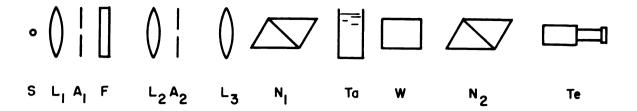


Fig. 6. Optical Arrangement for the Measurement of Acoustical Birefringence.

In Figure 6, S is the light source, a high pressure mercury are lamp or a tungsten filament lamp used for a white light source; L<sub>1</sub> is a focusing lens; L<sub>1</sub> is a limiting aperture; F is an optical filter which passes the mercury green line of 5460.7 % which is not used with the white light source; L<sub>2</sub> is a second focusing lens; L<sub>2</sub> is a circular aperture which acts as the effective source; L<sub>3</sub> is a collimating lens which renders the light parallel; N<sub>1</sub> is the polarizing Nicol prism; Ta is the tank containing the liquid and the transducer; W is the Wollaston prism; N<sub>2</sub> is the analyzing Nicol prism; and Te is the viewing telescope.

Measurements of the angular displacement of the analyzer Nicol and the corresponding transducer current are given in Table I.

### B. Method Using Photometer

For further varification of the Lucas theory, use was made of an experimental set-up of W. Gessert, working in our own laboratories. In addition to a similar optical arrangement, with exception of the Wollaston prism, as shown in

Figure 6, there was included an American Instrument Company photometer in place of the viewing telescope. The photometer used a very sensitive photo-multiplier tube. The polarizing Nicol was set at 45° to the ultrasonic wave front. Then the analyzer Nicol was rotated until a minimum reading was obtained on the photometer. This reading was then used as a "zero" point. When the ultrasound was turned on, the light reading on the photometer increased due to the birefringence in the liquid. The difference between the "zero" reading and the reading with the sound turned on is then recorded. At the same time the quartz current is measured. These readings are given in Table II.

of note is the fact that this experimental set-up used a piezoquartz as a source of ultrasonic energy, operating at 1000kc/s, as compared to the BaTi ceramics that are used in the former method. Use of the piezoquartz compares to the experimental method of Zvetkov, Mindlina, and Makarov. As a point of interest, a trial was made using the method of the Russian authors and the quartz source of ultrasonic energy. The measurements thus obtained are given in Table III.

As a qualitative check on the Lucas theory, Measurements of acoustic birefringence were attempted by the Wollaston prism method at four frequencies; IOOOkc/s, 2000kc/s, 2500kc/s, and 5000kc/s on solutions of IOO%, 80%, and 60% concentration. The polystyrene was diluted, by volume, with carbon tetrachloride in order to reduce the dynamic viscosity.

The results of these observations are given in Table IV.

The liquid that was investigated is poly-alphamethyl-styrene. The sample was obtained from the Dow Chemical Company and is designated by them as Dow Resin 276-V2. This resin is primarly composed of tri-polymer alphamethyl-styrene.

## III. EXPERIMENTAL DATA

Table I

BIREFRINGENCE MEASURED WITH WOLLASTON PRISM ULTRASONIC SOURCE, Bati TRANSDUCER

Frequency: 1000kc/s

Source	Current (rf amps)	Rotation (degrees)
white white white white white white white green	46533333334442222288 00110000011100000011	1.0 4.0 6.25 10.0 2.5 2.0 2.0 5.5 11.25 16.0 11.0 1.0 1.0 1.0 2.0 2.0

l"White" indicates that a white light source was used and "green" indicates that a filtered mercury source was used.

Table II

BIREFRINGENCE MEASURED WITH PHOTOMETER
ULTRASONIC SOURCE, PIEZOQUARTZ

Frequency: 1000kc/s

Code	Light.	Crystal Current (rf ma)	Current Squared (x10 <sup>-1</sup> amps)
G-1	2.0	75	56
	1.0	95	90
	3.0	12 <b>5</b>	156
	5.0	145	209
	5.0	165	271
	6.0	175	305
G-2	1.0 2.0 3.0 4.0 6.0 7.0 8.0	80 110 140 170 195 220 245 250	64 121 195 288 378 482 598 612
G <b>-</b> 3 <b>a</b>	1.5	130	169
	2.5	146	213
	3.0	162	262
	4.0	178	315
	6.0	210	440
	7.5	232	538
	9.0	247	606
	9.0	247	606
	13.0	265	700
	15.0	280	782
	17.0	300	900
	19.0	310	960
G-+a	1.5	100	100
	5.0	150	225
	7.0	180	322
	9.0	180	322
	9.5	200	400
	11.5	220	482
	11.0	255	624

Table II continued

Code	Light	Crystal Current (rf ma)	Current Squared (x10 <sup>-1</sup> amps)
G_)+a	7 - 8 10 - 12 11.5 16.5	200 235 233 265	400 550 542 700
G <del>_1</del> c	10.0 12.0 29.0 36.0 14.0 54.0	120 142 190 220 240 267	144 202 360 485 575 713
	14.0 20.0 33.0 31.0 35.0 42.0 46.0 49.0	160 160 208 215 215 250 265	256 2 <b>56</b> 406 455 455 625 700 <b>70</b> 0
G <b>-</b> 5	2.0 3.0 4.0 5.5 7.5 10.0	145 170 195 210 228 240 265	210 288 380 480 540 575 <b>7</b> 00

Table III

# BIREFRINGENCE MEASURED WITH WOLLASTON PRISM ULTRASONIC SOURCE, PIEZOQUARTZ

Frequency: 1000kc/s
Light Source: High Pressure Mercury
Arc Filtered for 5460.7 A

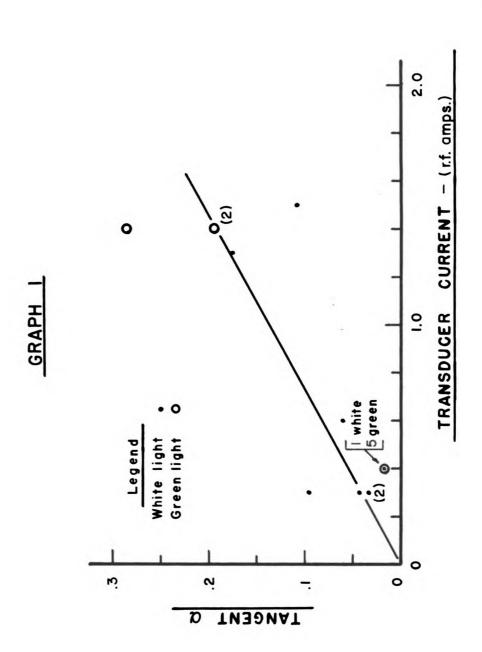
Current (rf ma)	Rotation (degrees)
120 139 152 167 182 230	0.75 1.50 1.50 1.75 1.75

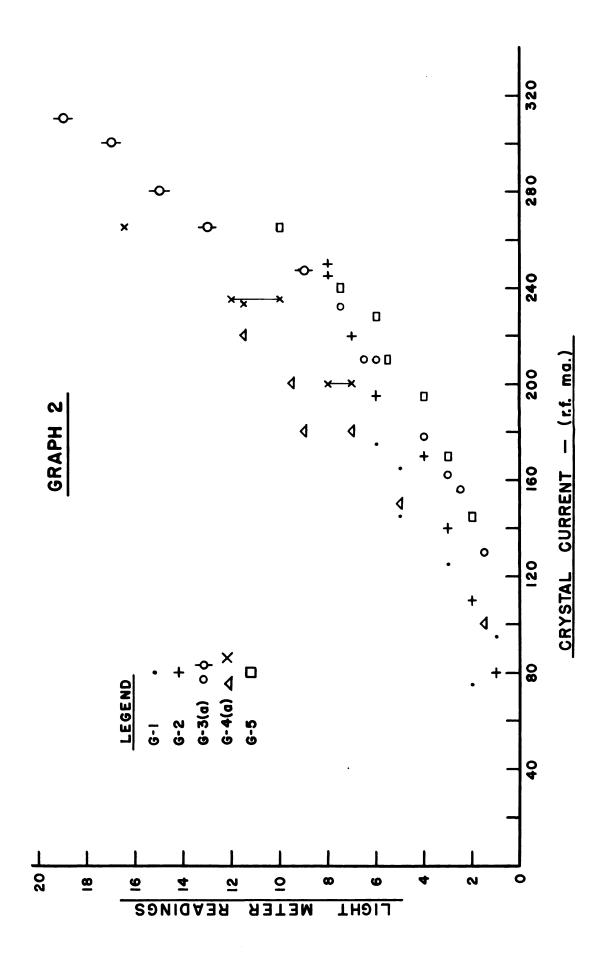
#### Table IV

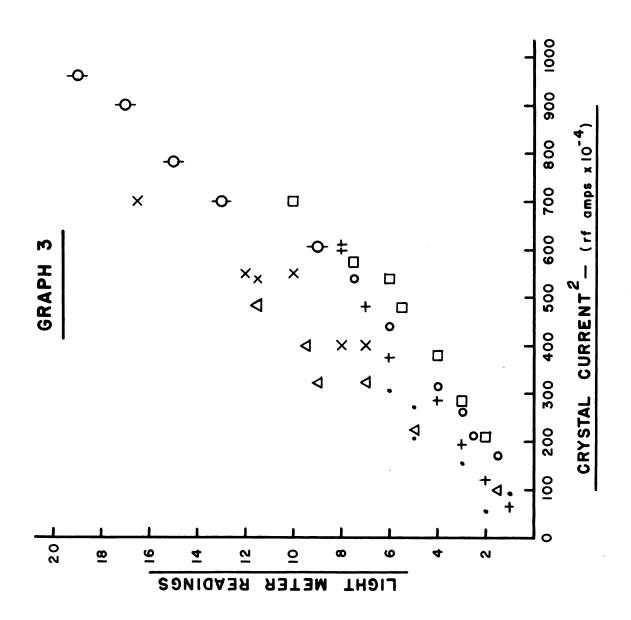
QUALITATIVE MEASUREMENTS OF BIREFRINGENCE AT VARIOUS FREQUENCIES AND CONCENTRATIONS ULTRASONIC SOURCE, Bati TRANSDUCER

Light Source: High Pressure Mercury Arc Filtered for 5460.74

Frequency (kc/s)	Concentration (per cent)	Birefringence Present
2000	100	Yes
2500	100	Yes
5000	100	No
1000	80	Yes
2000	80	Faint
5000	80	No
1000	60	Yes
2000	60	No.
5000	60	No







#### IV. DISCUSSION OF RESULTS

From the development of Zvetkov, Mindlina, and Makarov9, the tangent of the angular rotation of the analyzer Nicol should be proportional to the birefringence present. This is expressed in equation 21. Then from the Lucas theory. the birefringence should also be proportional to the square root of the sound intensity which is proportional to the sound amplitude. This is given in equation 16. It can be shown that the acoustic power is proportional to the voltage across the transducer squared 11 or the square of the transducer current. This means that the angle of rotation can be plotted against the current through the transducer which has been done in Graph 112. Due to the difficulties involved in making measurements of the birefringence in the polystyrene by the method of Zvetkov, Mindlina, and Makarov, the data do not clearly show this relation. It is significant though, that the birefringence does increase with increasing transducer current.

The data in Table III was compiled using the method of the Russian authors but the sound source was a piezoquartz instead of a BaTi transducer. Here again the birefringence shows an increase with increasing transducer current.

The difficulties encountered in the Zvetkov, Mindlina, and Makarov method consist of the following: determination of equal brightness of the Wollaston images; small angular

rotation for low values of birefringence; and long time required to make a measurement during which the liquid heats due to absorption.

Although the data in Table II were compiled by using a more elaborate experimental arrangement, namely, the American Instrument Company photometer, the ease with which a large amount of measurements may be made justifies its use. The determination of the minimum light position of the analyzer Nicol, with no sound field present, is reduced to the reading of a meter. The resulting increase in the light when the ultrasound is present in the liquid is also easily determined, again by a simple meter reading. This makes it possible to reduce the time required for a particular observation to be made and thus reduces the error introduced by the subsequent heating of the liquid by sound absorption. The difficulties inherent in this method are due to line voltage variations and the dark current of the photomultiplier tube. Both of these introduce errors in the measurements for low values of birefringence since they affect the meter reading by £ 1 scale division. Thus for the low light value readings, the observations could be adjusted by £ 50%. If the same variation is true for the higher values of birefringence, the observations would be affected by only £ 5% and would decrease for even higher light values.

The plot of light meter readings versus transducer current is made in Graph 2. The observations have been

plotted directly without any adjustment of the readings for the possible errors previously mentioned. Even without an attempt at a best-fit curve, it is possible to see a relationship between the light values and their corresponding transducer current. The Lucas theory predicts that the intensity of the emergent light, due to birefringence, will be proportional to the intensity of the ultrasound. Since the ultrasonic intensity is proportional to the square of the current, it should be possible to show the proportionality between the emergent light and the square of the transducer current. This has been done in Graph 3. Here again the data have not been adjusted for a best-fit curve but even so the relationship between the light values and the square of the current is clear. The photometric measurements agree therefore with the general results of the less reliable method of measuring birefringence by rotation of the analyzer.

A quantitative study of the frequency dependence of the acoustic birefringence was not made. Some preliminary measurements were made at various frequencies which seemed to indicate a decrease of the effect with increasing frequency. This is given in Table IV. However, no reliable conclusion can be drawn from these results as the sound intensities at the various frequencies were not measured.

It may be mentioned here that a modification of the Lucas theory by Peterlin<sup>7</sup> predicts a decrease of acoustical birefringence with frequency above an optimum frequency.

It is also noteworthy that the Russian authors did not find acoustic birefringence in solutions of polyisobutylene or polystyrene in the frequency range of 600kc/s to 4000kc/s for the strongest ultrasonic fields that they were able to produce. The problem of the frequency dependence of acoustic birefringence appears very interesting for future studies.

#### V. SUMMARY

The Lucas theory predicts that the acoustic birefringence produced by progressive waves should be proportional to the frequency of the sound wave, the Maxwell constant, the viscosity of the liquid, the reciprocal of the temperature, and the square root of the sound intensity. This experimental study of acoustical birefringence of polystyrene showed that the following predictions of Lucas can be observed also in the case of stationary ultrasonic waves; the proportionality between the birefringence and the viscosity of the liquid, the proportionality between the amplitude of the ultrasonic wave and the resulting birefringence, and that the emergent light measured by the photometer is proportional to the sound intensity.

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