INVESTIGATION OF A METHOD OF SOUND PRESSURE MEASUREMENT WITH A MACH-ZEHNDER INTERFEROMETER

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This is to certify that the

thesis entitled

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SOUND PRESSURE MEASUREMENT WITH
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INVESTIGATION OF A METHOD OF SOUND PRESSURE MEASUREMENT WITH A MACH-ZEHNDER INTERFEROMETER

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good resolution. The averaging effects caused by the size of previously employed instruments and the effective distortion of the sound field from the presence of a detector in its path is eliminated.

The purpose of this work was to ascertain that measurements of this type could be made and perhaps further developments in the method. The interferometer employed is that designed and constructed by Macy (2) in the Michigan State College Physics Laboratory. A general schematic diagram of the interferometer and the associated optical train follows:

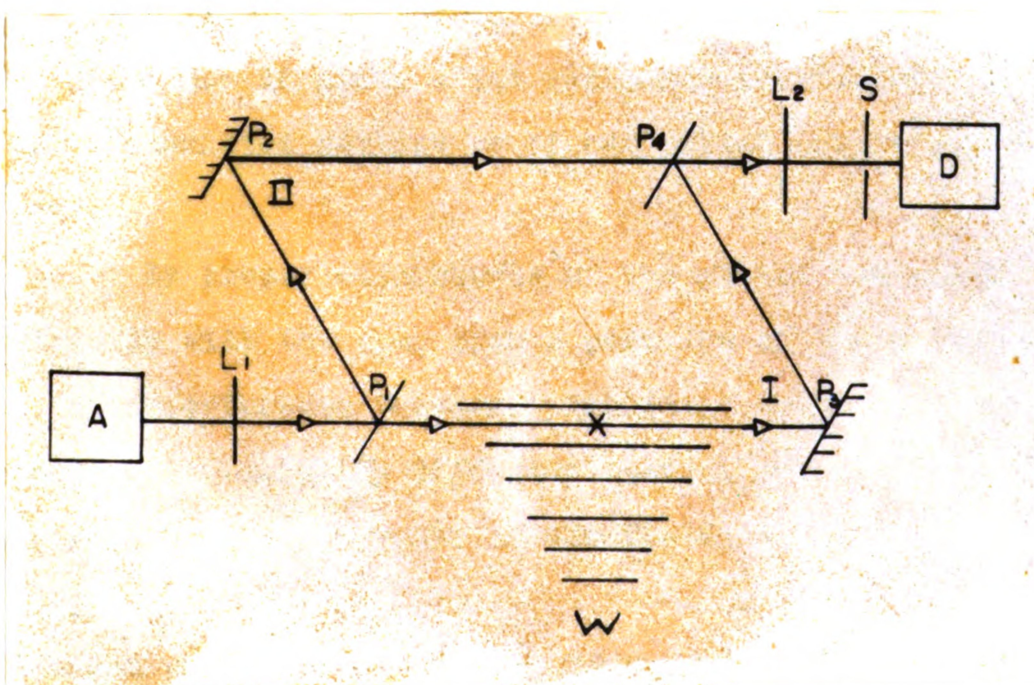

Figure 1

The interferometer consists of four glass plates whose planes are normal to a plane connecting their centers. Their mountings permit small adjustments of position and orientation from their positions as shown. Plates 1 and 4 are plane parallels, half silvered or aluminized mirrors, while plates 2 and 3 are plane front surface mirrors.

Light from source A rendered parallel by lens L1 travels to plate 1. The light beam is divided, part (I) is transmitted through plate 1 to plate 3 and then reflected to plate 4, while the other part (II) of the original beam is reflected to plate 2 and again reflected to plate 4. At plate 4 the reflected portion of I and the transmitted portion of II are united into a single beam.

By proper adjustments of the plates one can observe directly with the eye, from a position of the detector D, an interference pattern. The interference pattern is produced by slightly different optical path lengths of coherent rays.[a] A point in the field is bright or dark

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[a] Light producing a vertical fringe pattern can be thought of as coming from two separated horizontal images of the same point in the real source. In a broad source the light may be thought of as coming from many such points. However, any particular pair of virtual images that are chosen to be discussed are called coherent points, producing coherent rays.

dependent upon whether the rays unite in or out of phase. It suffices to say that one can produce a pattern of alternate light and dark fringes which are perpendicular to a plane connecting the centers of the four glass plates. Furthermore, with a broad source of light and a circular aperture, this pattern can be localized in a plane normal to the light path at a designated place such as indicated in Figure 1 by X.

A variation of the optical path of beam I in the region of X will cause a variation in the fringe pattern. A sound wave propagated perpendicular to the light path will produce a time variation in pressure which is uniform along a line through X from plate 1 to plate 3. This local pressure change is accompanied by a density change which in turn varies the index of refraction that produces the optical path variation and a subsequent shift in the interference pattern.

The variation in fringe pattern is projected upon D by lens L2. L2 simultaneously serves the purpose of projecting an image of the slit S at the position X. This image causes no diffraction or distortion of the light beam within the interferometer proper or produces no distortion in the sound wave. The slit is provided with an adjustable opening such that narrow sampling, of the order of .01 to .001 part of a wave length of the sound wave may be obtained. The shift in the fringe pattern across the slit in its fixed position produces a light signal of periodic intensity variation. A photo-multiplier tube placed behind the slit receives the signal. The signal is amplified and the output observed with an oscilloscope or measured by a vacuum tube voltmeter.

The calibration of the instrument for absolute measurement lies beyond the scope of this report, but will be discussed briefly in the appendix.

CHAPTER II

INSTRUMENTATION

Interferometer

An immediate problem to one unfamiliar with the interferometer is the obtaining of the desired fringe pattern. Each plate has a rotational degree of freedom about a vertical axis passing through its center and a rotational degree of freedom about a horizontal axis passing through its center, shown as H's and V's respectively in Figure 2. In addition plates 2 and 3 have translational degrees of freedom T. The translational motions are restricted to a horizontal plane such that the centers of the plates will move along straight lines connecting the centers of plates 4 and 1 respectively.

The multiple degrees of freedom allow a diversified choice of patterns, auxiliary cells and compensation for such things as minute warping and temperature variations. They also present a problem in the initial alignment of the instrument to produce fringes. Ideally each controlled movement of a plate in one of its degrees of freedom would be independent of all other movements. The construction of this instrument closely approximates the ideal case.

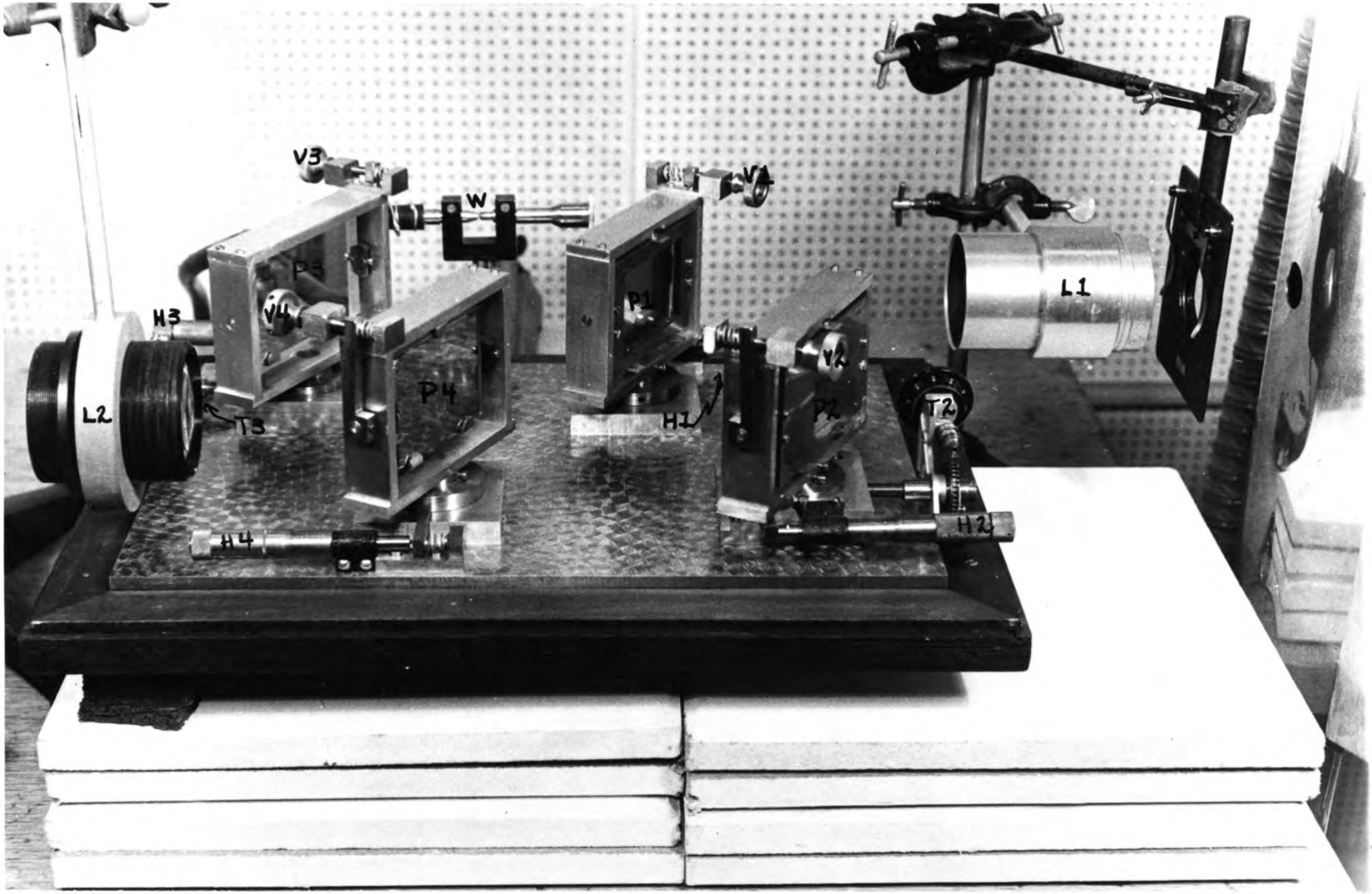

Figure 2. Mach-Zehnder Interferometer

Price (3) reports a new auxiliary attachment for making the initial alignment of the instrument.[b] Procedures for obtaining fringes are reported by Macy (2), Winckler (4), Winkler (5) and others. Following are some remarks intended to be supplementary and not a complete outline of the adjustment of the interferometer.

The possible orientation of the sound source with the given instrument and the method of detection, dictated that vertical localized fringes were to be used in this investigation. The desire for maximum intensity contrast in the fringe pattern requires that one should work with or near the zero order fringe, the so called white light fringes.

Briefly, this adjustment may be obtained as follows. The interferometer is illuminated with a monochromatic source of such intensity that if L2 is removed, one may look with the naked eye directly into the interferometer from the position of D. It is assumed that proper initial adjustment of the instrument has been completed thereby aligning the four plates into parallel planes. A slight movement of any H adjustment, H3 for instance, would then bring a vertical fringe pattern into view.

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[b] Price's method untried by this author.

One can think of vertical fringes as being produced by coherent points lying in horizontal planes. If the fringes are not vertical, and the controls are ideal, then proper adjustment of the individual V controls will produce vertical fringes. In actuality, small changes of the controls conform with the ideal case of independent adjustments but if large adjustments are to be made, then compensations with a plate's own conjugate controls must be made.

In general an adjustment of plate 2 may be compensated by a corresponding adjustment of plate 3 while the fringes remain in the desired localization and vertical orientation. The same correspondence holds true between plates 1 and 4. It was found that after proper manipulation of V3 and V4, while compensated by V2 and V1, that a rotation of H3 or H4 respectively, would allow the fringes to remain vertical. The establishment of this adjustment allows the operator to control his fringe pattern during operation from plates 3 and 4 most easily.

With the previous adjustments made one is now ready to examine the change of fringe locality and the number of fringes in view. Suppose the left and right orientations of coherent points are such that a clockwise H4 rotation spreads the fringe pattern out, that is to say fewer fringes appear in the field. Furthermore, a shift toward the source or toward the observer will simultaneously take place and, for the sake of the example, let it be toward the source. This movement may be checked by the parallax method of placing a fixed object in the vicinity of the fringes. As the eye is moved from side to side the fixed object will appear to move in a fixed relationship to the fringes only if it is at the same place in space as the fringes. The original number of fringes may now be restored without further change in the locality of fringes by the appropriate counter-clockwise movement of H3. The process is just reversed if the right-left sense is reversed. The above type of procedure is easily mastered by a few repeated and recorded trials of a beginning operator.

The outline of a method for obtaining white light fringes follows. It is assumed that vertical fringes from a monochromatic source are in the view of the operator. H3 is turned in the direction that produces a broadening of the fringes. If the rotation is continued a curvature will appear in the nearly straight fringes, a position of parallelism of all the plates is passed through and beyond. The straight fringes will then reappear. The above process is the reversal of left to right or right to left of coherent points as the case may be. At the position of curvature or "flipping over" one can observe a direction along which the center of curvature would seem to lie. One mirror can then be translated in a direction to equalize paths I and II such that the fringes appear to move across the field of view in the direction of the center of curvature. A white light should then be placed to illuminate either the upper or lower half of the field of view. As the zero order fringes approach, colored fringes will appear as extensions of the monochromatic fringes but in the white field of view. If one suspects that he has passed beyond the position of equal optical path length he may again try the center of curvature test. Observe carefully that the right-left orientation does not reverse during the process. The reversal may be detected since as one continues to translate in the same direction, the fringes in the field of view will appear to move in the opposite direction.

The region of white light fringes is used for greater light intensity contrast, maximum contrast being obtained when the coloring is symmetrically arrayed about the central black fringe. If this is not the case then the light rays in beams I and II are traveling through different thicknesses of glass. This is true because glass has a much different dispersive power than that of air. The pattern may be corrected by compensating rotations in the same direction of H1 and H4. H2 and H3 have no effect because of the front surface reflection at plates 2 and 3.

One may note in Figure 2 the sponge rubber supports underneath the tripod mount of the interferometer. This type of mount is one attempt to isolate the instrument in any manner possible from all mechanical shocks or vibrations. The sponge rubber in turn rests upon several layers of fiber blocks which are of the type used to damp sound waves. A discussion of the isolation of the sound field in a particular arm of the interferometer will be found in the appendix.

Detector

The normal fringe pattern has a spatial distribution of light intensity that can be represented by a cosine squared curve. When the sound is on, the signal as seen from the photo-tube is the same as though the slit were moved back and forth across a small portion of the light intensity curve at the frequency of the sound wave.

Suppose the fringe shift to be very small and the slit is arranged such that its relative central position is at the most nearly linear slope of the intensity curve. Further assume that the fringe shift is a sinusoidal function of time, then relative motion between the two would generate a sinusoidal signal of light intensity for the tube. If a mercury arc, operated by a 60 cycle alternating voltage, is used as a light source, then the resulting signal is not a simple sine wave, but rather an amplitude modulated sine wave because of the light variation.

It is this type of signal which it is necessary to detect. The 931A photo-multiplier tube was chosen because of its good response to the proposed use of the mercury green line and for its availability. Other characteristics such as fatigue, amplification and voltage requirements, that were reported by Marshall, Coltman and Bennett (6), also by Kessler and Wolfe (7), indicated that this tube would be acceptable.

The tube was mounted as a unit with the adjustable bilateral slit (see Figure 3). The mounting is light tight. The upright barrel containing the tube is machined at the bottom in a stepwise, slip fit fashion to set into its base; this preserves the light seal and allows the easy removal of the tube. It also permits a rotation of the slit about a vertical axis passing through the center of the tube. The slit proper is mounted on a short cylinder whose axis is normal to the main upright assembly. Again the connection is in the form of a machined slip fit with screw lock, which allows a fine adjustment of the slit jaws parallel to the fringes.

The face of the slit holder is covered with a white screen. A black scribe mark on the screen indicates the position of the slit opening and allows a visual inspection of the relative slit fringe position. The slit jaw length is approximately 15 mm. which corresponds to the height of the sensitive part of the photo-tube cathode. The opening is controlled by a fine micrometer screw, one complete turn of which provides an opening of 0.5 mm. A photograph of the slit and tube mounting is shown in Figure 4.

Horizontal movement of the assembly by the micrometer screw in a direction normal to the light ray can be accomplished, while translational movement parallel to

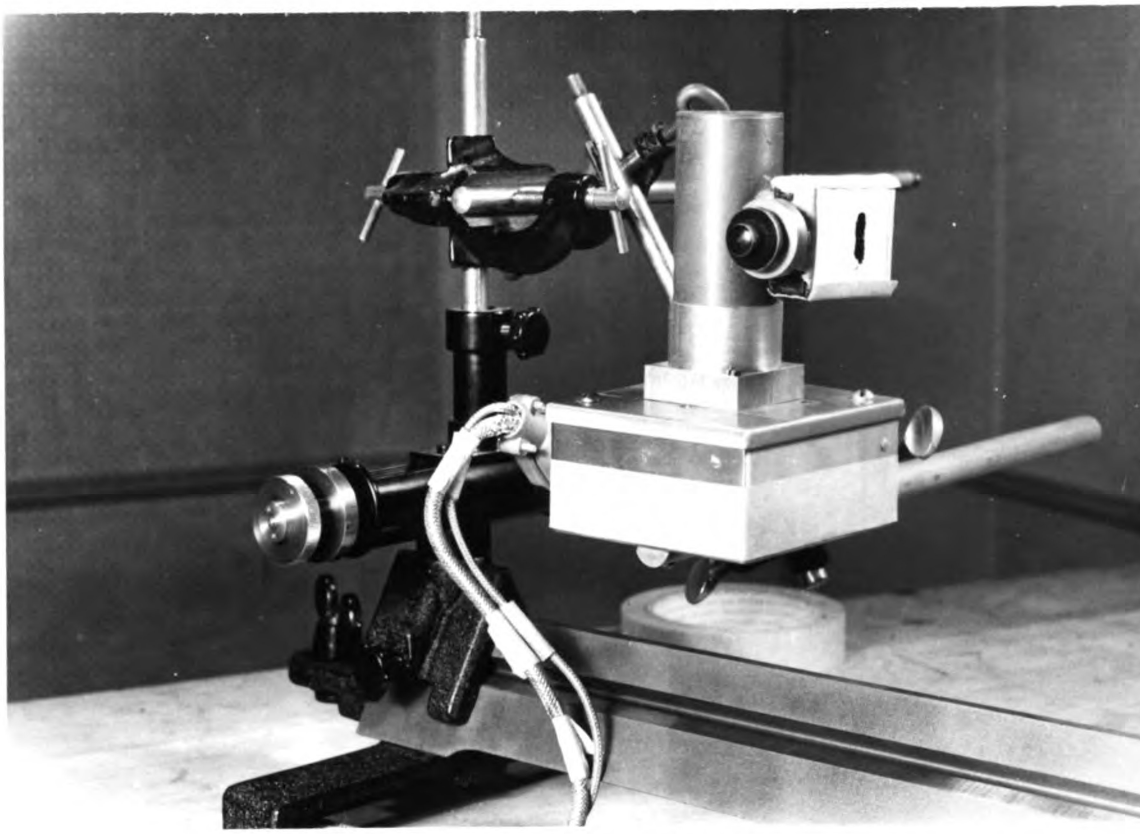


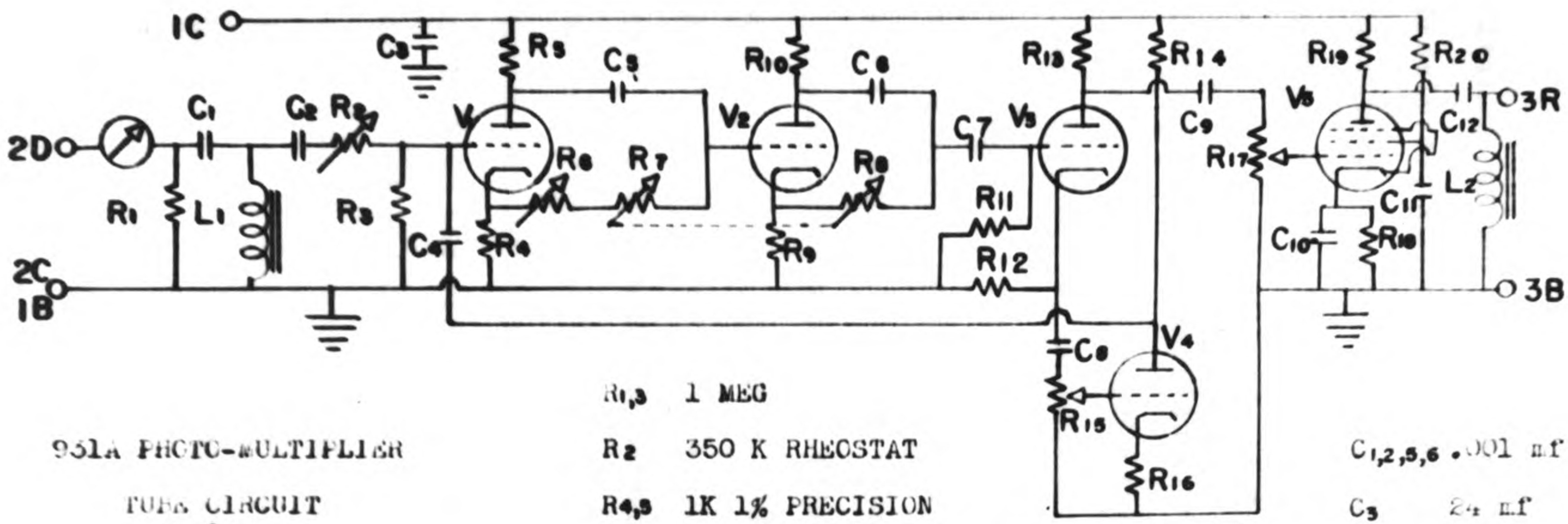
Figure 3. Photo-Tube

Figure 4. Photo-Tube and Selective Amplifier

the light may be made along the optical bench. Various heights and angles may be readily obtained with the system of clamps and rods as shown. The entire photo-chassis is constructed of copper and aluminum providing an electrical shield which is adequately grounded to reduce stray signal pickup.

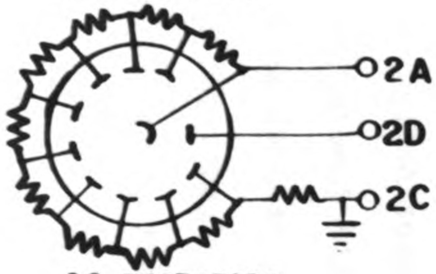
In any photo-tube work there remains a great problem of dark current or random noise (see appendix.) The random noise of a photo-multiplier appears well distributed over a large frequency range. It was therefore believed that the amplification of the photo-tube signal should be restricted as closely as possible to the frequency of the signal. Since the work was to be carried out in the range of 5500 c/s. and up, one could place a high pass filter in the plate lead of the photo-tube (see diagram of circuit, Figure 5). The cut-off or critical frequency is calculated at about 1200 cycles. The pre-filter will then cut out the fundamental and first nine harmonics of the 120 cycle arc noise and will damp the immediate harmonics above this frequency.

The "Selectoject" (8, 9) which is used by many radio amateurs to replace a variable crystal filter in phone work, etc., was used in the next stage of detection. While the circuit is most often used in rejecting a particular frequency, it is readily adaptable to the



951A PHOTO-MULTIPLIER

TUBE CIRCUIT



10 RESISTORS

100K EACH

$R_{1,3}$	1 MEG			$C_{1,2,5,6}$	.001 mf
R_2	350 K RHEOSTAT			C_3	24 mf
$R_{4,5}$	1K 1% PRECISION				
$R_{6,15}$	50K RHEOSTAT	R_{17}	.5 MEG RHEO.	$C_{4,7,9}$	.01 mf
$R_{7,8}$	350K GANGED RHEO.	R_{18}	600	$C_{8,11}$	.06 mf
$R_{9,10}$	2K 1% PRECISION	R_{19}	100K	C_{10}	8 mf
R_{11}	5 MEG	R_{20}	.5 MEG	C_{12}	.006 mf
$R_{12,16}$	2K	$V_{1,2,3,4}$	6SL7½	L_1	15 hy
$R_{13,14}$	50K	V_5	6SJ7	L_2	2.5 hy

FIGURE 5

amplification of the same narrow band. Figure 4 shows the chassis containing the pre-filter and amplifier as connected to the photo-tube and slit assembly. This selective amplifier circuit was chosen to obtain amplification of the signal without including the amplification of all the random frequencies of the photo-tube noise.

The amplifier is easily tuned to the frequency of the signal by a coarse frequency control, CFC indicated in the photograph. This control actuates R7 and R8 (see Figure 5, wiring diagram) which are ganged rheostats that regulate the frequency of the feed-back signal which is to be in phase with the input signal, while others are fed back at 180° out of phase. R6 is a rheostat in series with R7 and acts as a fine frequency control, FFC, such that very accurate matching between the sound frequency and the pass frequency of the amplifier can be made. R15 is a potentiometer that controls the amount of feed-back or sharpness of the pass band and is called selectivity S. Caution must be observed in the S adjustment since, if the feed-back becomes equal to or greater than the signal, oscillation will occur at the tuned frequency. Oscillation however is easily detected by the sharp increase in output signal.

Isolation or matching of impedence between the photo-tube and the selective amplifier is accomplished by the variable resistor R2, denoted as IM on the photograph. While operating the system with the photo-tube as a signal source, the value of R2 should be near zero, however if one desires to check the amplifier with another source of input signal, there may be an adjustment necessary in order that the amplifier remain selective. The isolation is accomplished at the output end of the selective amplifier by a standard stage of audio frequency amplification. R17 denoted as FAC, final amplification, controls the gain in this last stage.

The tube sockets were mounted on a suspended rubber sheet one-eighth inch in thickness providing a shock mounting to minimize the microphonic effect of the electron tubes. A common ground buss throughout, liberal wire shielding and a metal bottom for the chassis were used to minimize stray signal.

The performance of the amplifier is shown by the following curves. In Figure 6, curve A, the output is shown as a function of frequency, final amplification and selectivity being fixed but arbitrary. A 5 hy. choke was across the final output. The input voltage was a constant, from a Hewlett-Packard variable audio-oscillator, at 0.5 volts. The frequency of the amplifier was tuned at each point for a maximum signal. Curve B is the same as A except for a lower final gain but a slightly greater

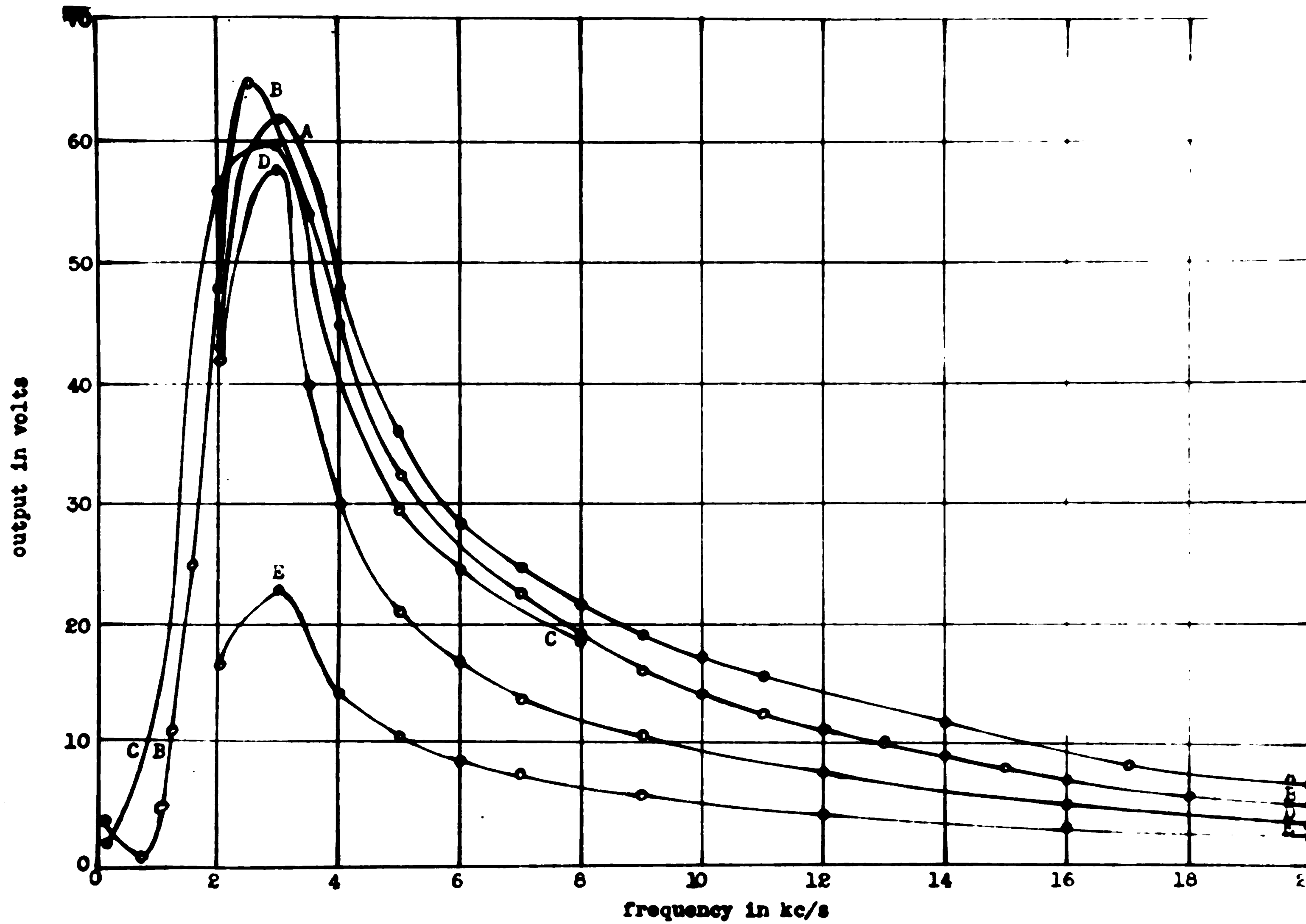


Figure 6. Frequency Response Curves at Constant Selectivity

selectivity. Both remain fixed however throughout. Curve C is identical with B except that the choke was removed.[c] E is identical with A except that the selectivity was lowered. D shows the input voltage reduced to 0.3 volts, the selectivity is in the neighborhood of A but arbitrary, as is the final amplification. The five typical curves at constant selectivity all show an output voltage drop of about 6 Db. from 5 to 10 kc/s.

Figure 7 shows the output as a function of input or amplification at frequencies at 6, 8 and 10 kc/s. The final amplification was well below saturation. The selectivity was constant and just below the point of oscillation. The input signal strengths obtained from the photo-tube are in general well below 0.2 volts and therefore lie in a region of constant amplification and relative signals may be compared most easily.

Figure 8 gives an indication of the selectivity of the amplifier. Each curve was made at a constant 0.3 volt input. The amplifier was tuned to the frequency of the peak of the curve and held fixed for the readings of the particular curve. The frequency of the H. P. audio-oscillator was varied to obtain the frequency readings and the output measured. Curves A, B, C and D with peaks

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c 2.5 hy was the final value decided upon.

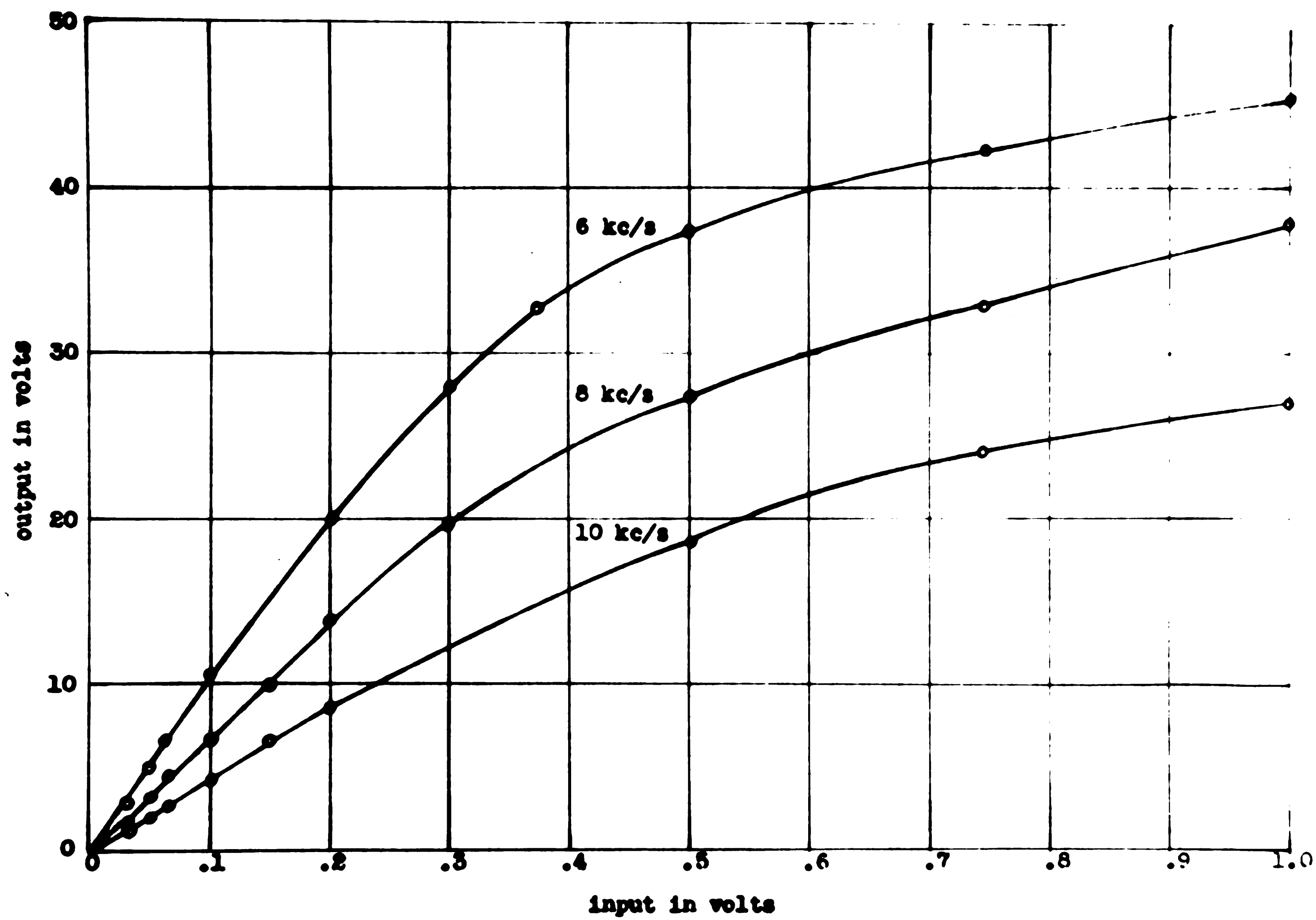


Figure 7. Amplification

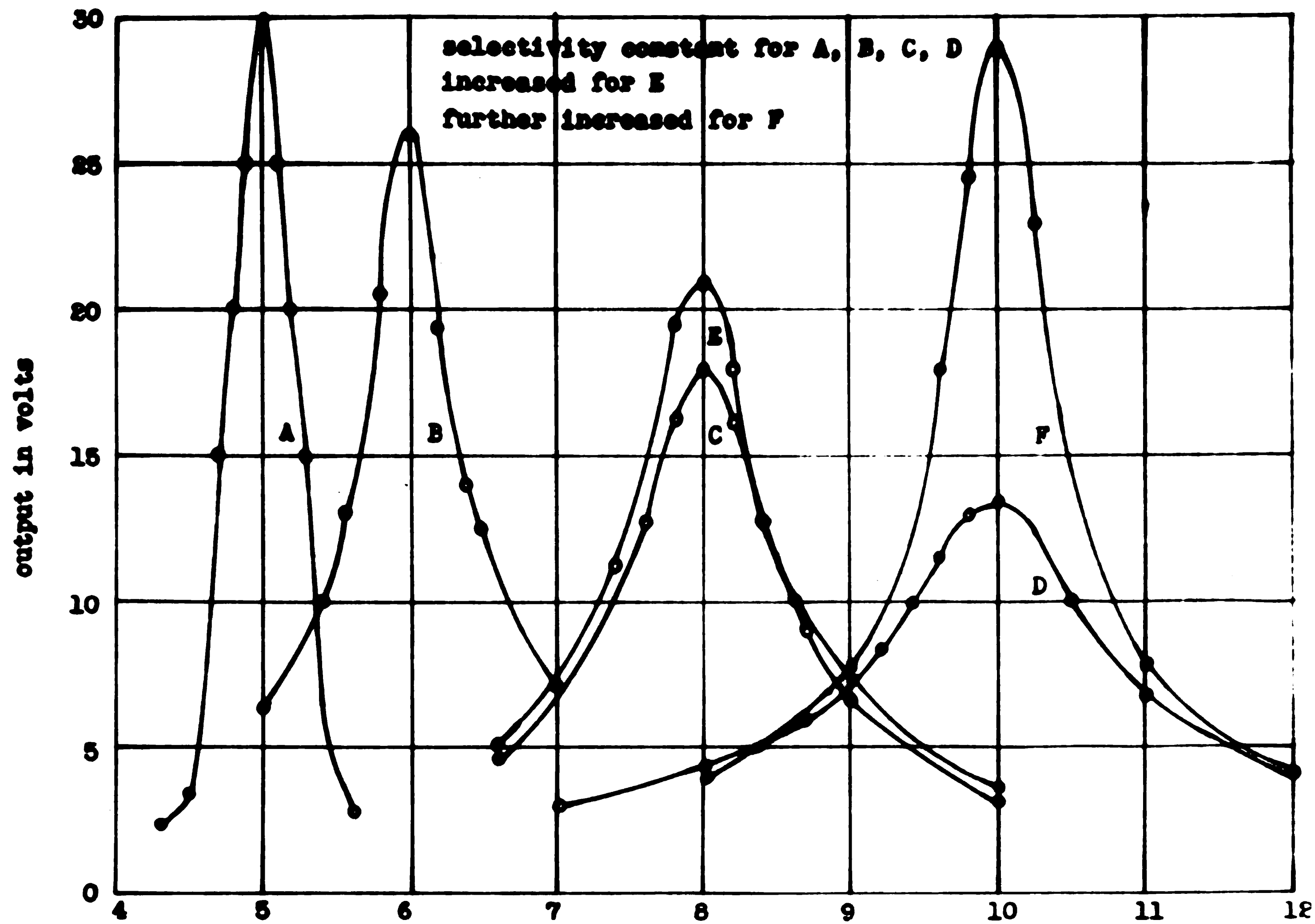


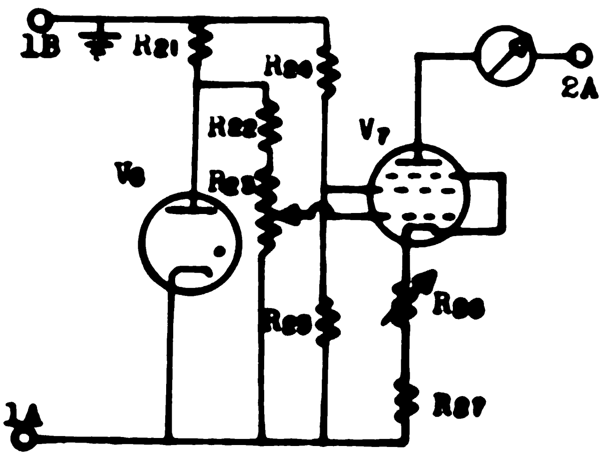
Figure 8. Effect of Selectivity with Constant Input

at 5, 6, 8 and 10 kc/s., were taken with the same settings of final amplification, impedence matching and selectivity. Curve E was obtained by increasing S only. Curve F was obtained by increasing S slightly more than for E. The sharp selectivity as shown reduces the interference of the random noise so common in photo-tube work. A non-linear response was noted in Figure 6, however Figure 8 shows that by increasing the selectivity as the frequency is increased that a linear response in the range of 5 to 10 kc/s. can be obtained.

The power supply for the amplifier was a commercial plate voltage supply unit. The regulation was checked and found to vary less than two volts in 310 for currents of 0 to 50 milli-amps. The plate voltage required for the amplifier was 250 volts and the total plate current was well below the 50 ma. limit. The ripple factor was negligible. The filaments were heated in series by a 24 volt battery. This method was used for convenience since a 24 volt supply was also needed in the light system.

The high voltage needed for the 931A was obtained from a commercial supply. The supply was well filtered but the regulation was below the standard required. A regulator for the high voltage was therefore included in the amplifier chassis. The wiring diagram (Figure 9) may be inspected for details of the regulator and filament

HIGH VOLTAGE REGULATOR



R_{21}	250K
$R_{22,24}$	1 MEG
R_{23}	.5 MEG POT.
R_{25}	50K
R_{26}	20K RHEOSTAT
R_{27}	5K
V_6	VR75
V_7	6SJ7

FILAMENT CIRCUITS

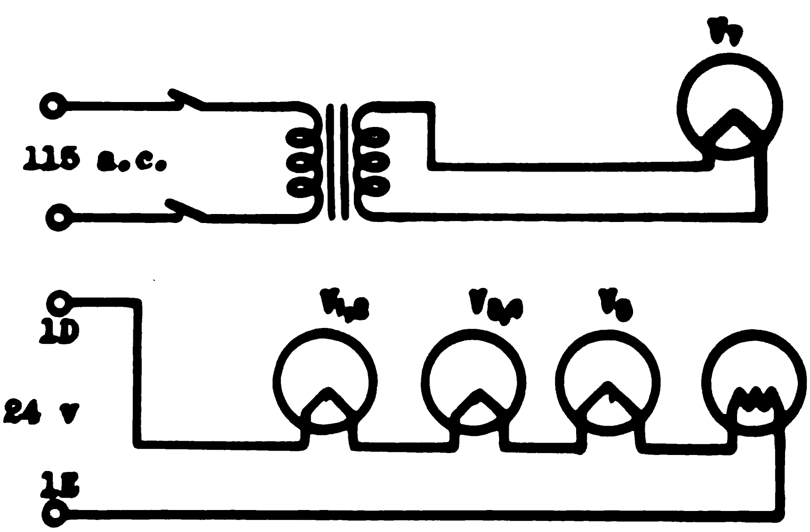


FIGURE 9

circuit. The filament power supply of the 6SJ7 in the regulator circuit was provided by a separate transformer. This was felt to be a necessary protection for the other tubes, in the event that the 6SJ7 would somehow short out and place a high voltage on the amplifier tubes. R23 and R26 are rheostats that may be used as coarse and fine voltage controls, respectively, of the voltage that is applied across the photo-tube from cathode to plate.

A milli-ammeter is in series with the line from the regulator to the photo-tube. One milli-amp. corresponds to about 1200 volts. This voltage should not be exceeded to avoid harming the photo-tube. A value of 750 to 900 volts was found satisfactory to produce sufficient amplification of the signal and yet keep the noise level at a minimum. The micro-ammeter mounted in the amplifier measures the d.c. anode current of the photo-tube. The larger this current becomes the greater the fatigue of the tube and consequent loss of sensitivity. Therefore, the tube should be operated at all times with the least current possible and still obtain the signal strength desired. Two-hundred and fifty micro-amps. is a maximum value of anode current quoted by the manufacturer. In measurement work one should try to keep the current well below 10 micro-amps.

There are four connections or plugs associated with the amplifier (see Figure 4). The first is a five plug connection carrying a negative 2000 volts on 1A, a positive 250 on 1C with 1B as their common ground. 1D and 1E complete the 24 volt filament circuit. Plug 2 is the connection between the amplifier and the photo-tube chassis. 2A is the regulated negative high voltage, 2D carries the signal, while 2C is the common ground. The third connection is provided for the signal output and black is the common ground, while the red terminal is a connection for the signal proper. The fourth plug must be connected to a 110 volt supply for the regulator tube filament power.

The constancy of the frequency of the sound source is critical because of the sharp selectivity of the amplifier. This was checked continuously during measurements by applying the output of a microphone on the vertical input of an oscilloscope. The horizontal input of the oscilloscope was connected to a H. P. audio-oscillator and the lissajous figure was then observed. In this manner appropriate frequency adjustments of the sound could then be made.

Optical System

The optical system consists of two compound lenses L1 and L2, as shown in Figure 10, together with a mercury green filter placed as shown in Figure 17. The lenses are high quality projection lenses; L1 has a focal length of approximately 25 cm. and L2 has a focal length of about 20 cm. The lenses are mounted on ring stands for ease in adjustment and alignment.

L1 should be placed such that parallel or slightly convergent light enters the interferometer. The focus and height alignment of the light source may be checked without disarrangement of the apparatus by observing the image of the aperture on the room wall as it is reflected from the side opposite the detector of plate 4. L2(repeated for emphasis) simultaneously focuses the fringe pattern from X to the slit, and the slit to the position of X. The image of the slit acts as the limiting porthole to sample a very small part of the sound wave at a time. Therefore, the fringes are localized at X in order that L2 may play this double role. The position of L2 controls the magnification of the fringe pattern at the slit and the reduction of the slit image at X. The measurements taken were at approximately unit magnification.

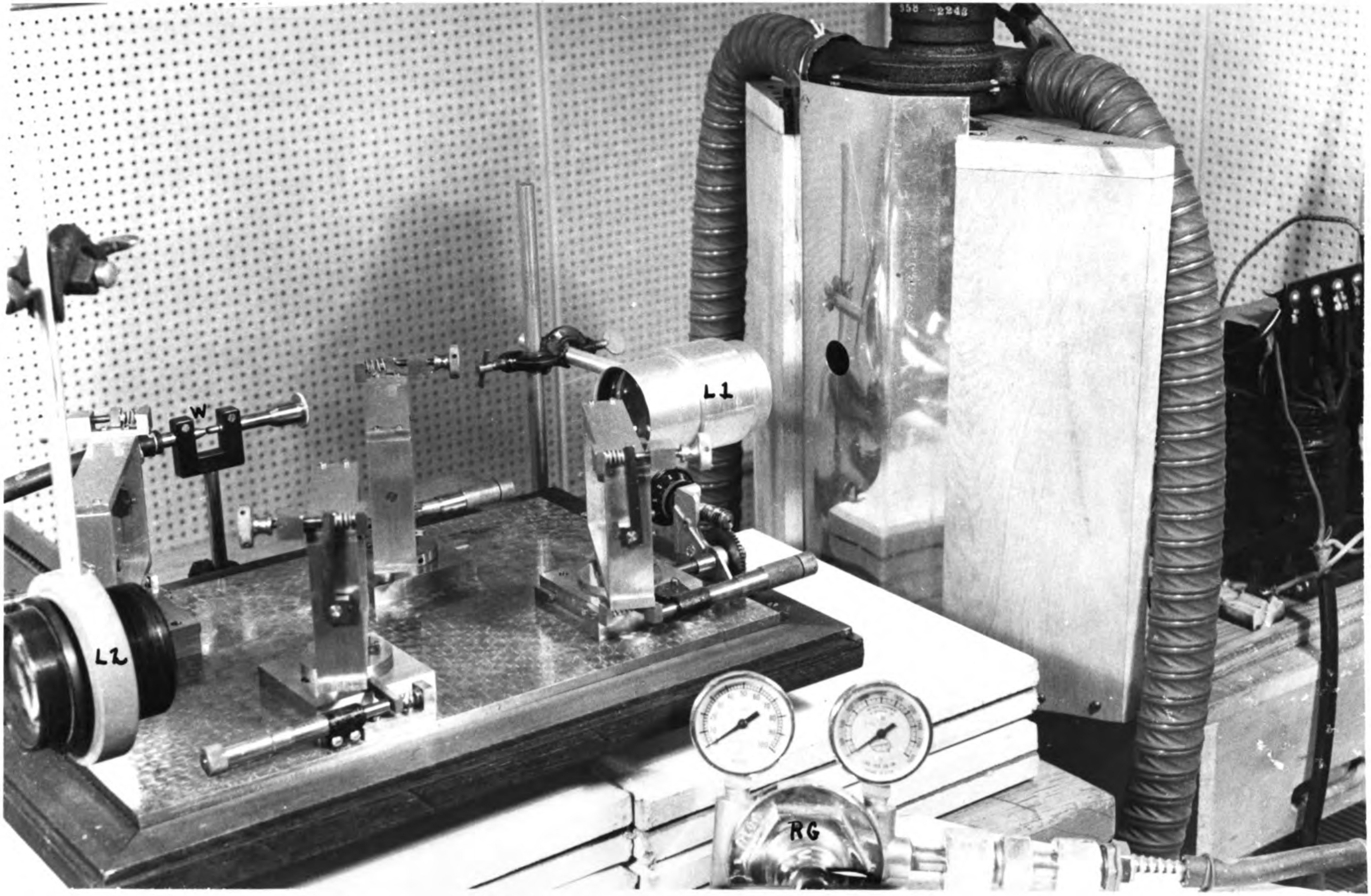


Figure 10. Light and Sound Source and Optical System

Light Source

The light source was a 750 watt mercury vapor arc. The tube itself is about thirteen inches long and one and one-half inches in diameter, and contained within the upright structure as shown in Figure 10. The upright is made in the form of a double walled chimney. A 24 volt d.c. exhaust fan is mounted at its top and air is drawn from below and flows around the arc. Two flexible hoses carry the heat from the fan and it may be fed to the building ventilating system.

In the chimney are apertures of suitable size (10, 11, 5), shape and alignment to allow the light to pass to the interferometer. The design of the upright is such that different sized or shaped apertures can easily be incorporated into the framework.

The power is obtained from an ordinary 115 volt a.c. line, but the auto transformer can be adapted to work from a variety of the common line sources available. (See appendix, Figure 18, for a wiring diagram.) A selection switch enables one to use either 500, 625, or 750 watts at the operator's desire.

Not shown in the photograph is a screen constructed of fiber squares the same as those upon which the interferometer rests. This screen was used between the source and the interferometer to reduce temperature gradients within the interferometer which cause anomalous fringe shifts.

Sound Source

A Galton type of whistle was used for a sound source. The whistle was operated by a 45 lbs./sq. in. source of compressed air. It was mounted on an optical bench and is shown in position in Figure 10. The pressure at the whistle was adjustable by a standard welding regulator valve RG. A metal tank approximately three liters in volume was placed in the pressure line as a moisture collector. The frequency of the whistle was adjustable by a micrometer screw and has a range of about 5 to 15 kc/s.

The sound level was checked by a General Radio Company Type 759 Sound Level Meter and found to have a maximum of approximately 125 Db. at about 8,000 c/s.

Not shown in the photograph was a thin flexible plastic (opened refrigerator bag) screen placed between the whistle and the interferometer to reduce the cool air blast from entering the interferometer, thus causing temperature gradients as mentioned above.

CHAPTER III

CALCULATIONS AND RESULTS

Calculations of Fringe Shift from Sound Pressure

Calculations relating sound pressure of plane waves to fringe shifts are made by use of the Lorentz-Lorentz (12) formula for the relationship between the index of refraction n and density of the medium ρ.

$$(1) \quad \frac{n^2 - 1}{n^2 + 2} \cdot \frac{1}{\rho} = \text{const.}$$

Since n = 1.0003 for air at room temperature; we can then write to a good approximation

$$(2) \quad (n - 1) = C\rho$$

where C is a new constant.

By two assumptions, that air obeys the ideal gas law and that the transmission of sound in air is an adiabatic process, we have the well known relationship

$$(3) \quad \rho \cdot \text{const.} = p^{-\gamma}$$

where p is the pressure and γ is the ratio of the specific heat at constant pressure to specific heat at constant volume. By rewriting equation (2) we have

$$(4) \quad n - 1 = kp^{-\gamma} .$$

If we denote the extreme values of pressure along a given light path during a time interval, then the difference in index at corresponding times is found by equation (3) to be

$$(5) \quad n_1 - n_2 = k\,(p_1^{-\gamma} - p_2^{-\gamma}).$$

We now proceed in a manner similar to that of R. B. Kennard (13). From equation (4)

$$(6) \quad k = (n_1 - 1)\, p_1^{\gamma}$$

and substituting into (5)

$$(7) \quad n_1 - n_2 = (n_1 - 1)\left(1 - \left(\frac{p_1}{p_2}\right)^{\gamma}\right).$$

Let L be the length of the light path, which is subjected to a uniform pressure change along its length (this assumes a plane sound wave), λ_0 is the wave length of the monochromatic light in a vacuum. The number of wave lengths N_0 is given by

$$(8) \quad N_0 = \frac{L}{\lambda_0}$$

If we say that N_1 and N_2 correspond to n_1 and n_2 respectively, then

$$(9) \quad n_1 = \frac{N_1}{N_0} \quad ; \quad n_2 = \frac{N_2}{N_0}$$

Let $\Delta N \equiv N_1 - N_2$ and substitute equations (8) and (9) into (7) giving

$$(10) \quad \Delta N = (n_1 - 1) \frac{L}{\lambda_0} \left(1 - \left(\frac{p_1}{p_2}\right)^{\gamma}\right)$$

ΔN represents the number of fringe shifts expected as the pressure swings about the atmospheric pressure p_a. Now let

$$(11) \quad p_1 = p_a - \Delta p \quad ; \quad p_2 + \Delta p$$

where $\pm \Delta p$ is the peak compressional and rarefactional pressure produced above and below p_a by the sound wave. $|\Delta p|$ was found from the relationship

$$(12) \quad \frac{Db}{20} + \log 2.8 \cdot 10^{-4} + \log \Delta p$$

where Db is the decibel rating above the standard $2.8 \cdot 10^{-4}$ dy./cm.^{2} level.

Following is a table prepared from typical values of sound levels, wave length of light and length of optical path involved.

$p_a = 1.013 \cdot 10^6$ dy./cm.^{2}

$n_1 - 1 = 3 \cdot 10^{-4}$

L = 8 cm.

$\lambda_0 = 5.46 \cdot 10^{-5}$ cm.

$(n_1 - 1) \frac{L}{\lambda_0} = 44$

$\gamma = 1.4$

Db	$1 - \left(\frac{p_1}{p_2}\right)^{\gamma}$	ΔN
100	$8.4 \cdot 10^{-5}$	0.0035
120	$7.72 \cdot 10^{-4}$	0.0340
125	$1.37 \cdot 10^{-3}$	0.0603
130	$2.45 \cdot 10^{-3}$	0.108

It is apparent from the above table that high sound intensities are required to produce an appreciable fringe shift, Although 125 Db. in the audible range represent a very high intensity (considered as equivalent to the sound level in an airplane engine test room), values of this magnitude are common in the ultrasonic range.

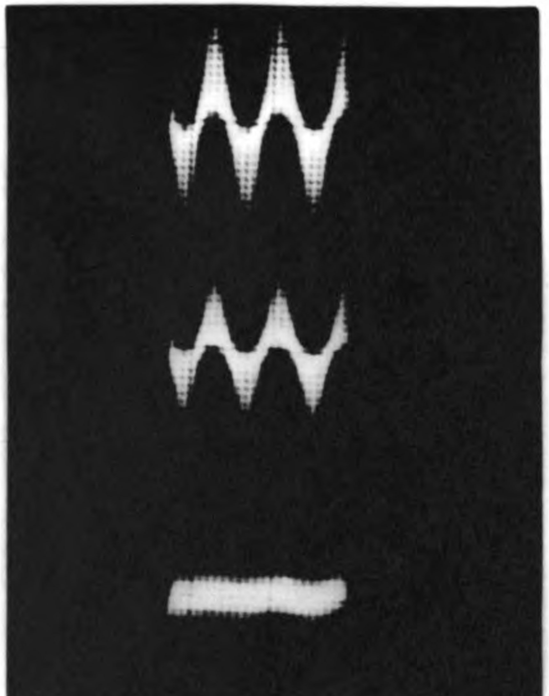
Results

Figures 11, 12, 13 and 14 are some typical pictures showing the signal obtained with the whistle radiating one arm of the interferometer. The photographs were made from the screen of the oscilloscope by a DuMont type 297 Oscillograph-Record camera.

The top and middle print of Figure 11 was taken at 9 kc/s., a slit width of 0.12 mm. and with 40 and 20 lbs./sq. in. pressure on the whistle respectively, while the bottom print shows the noise level without the sound. Figure 12 is a similar series taken at 5.6 kc/s., a slit width of 0.20 mm. and pressures at 40, 30 and 20 lbs./sq. in. reading from top to bottom.

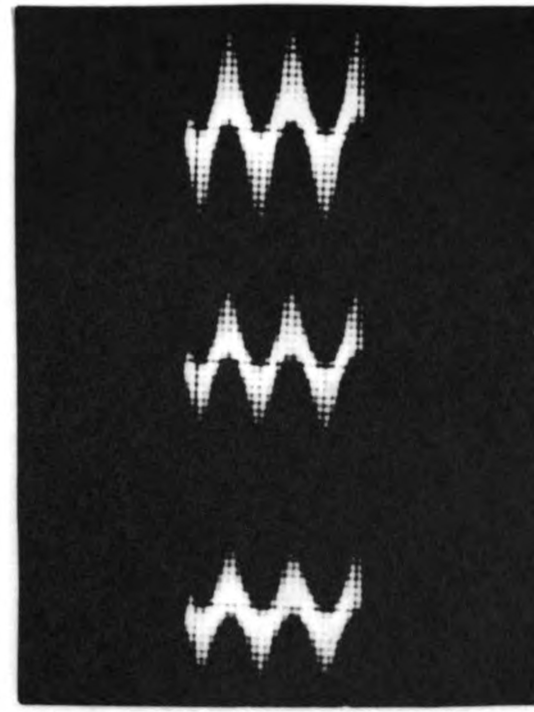
Figure 13 indicates how the signal is modulated. The frequency of the signal was 5.6 kc/s. In the upper line the horizontal sweep was at one-half of the whistle frequency and internal synchronization was used. It shows a family of sine curves at the whistle frequency, the amplitudes of which vary due to the 120 cycle light source intensity variations. The bottom is a picture of the same signal but the horizontal sweep is set at 120 c/s. and the 60 cycle line signal was used for synchronization. This may be thought of as the whistle frequency modulated by the light variation as in the ordinary type of amplitude modulation encountered in AM radio.

Examples of Scope Patterns



frequency 9 kc/s
slit width .12 mm.
blowing pressure 40, 20 and 0 lbs./sq. in. from top to bottom

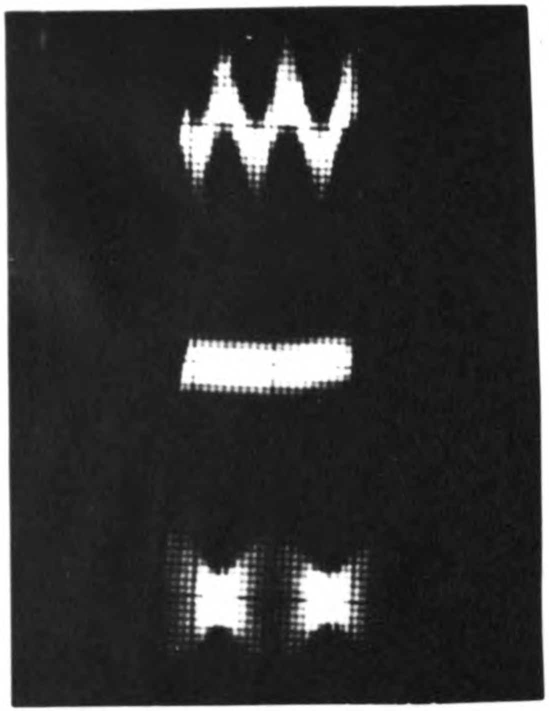
Figure 11



frequency 5.6 kc/s
slit width .20 mm.
blowing pressure 40, 30 and 20 lbs./sq. in. from top to bottom

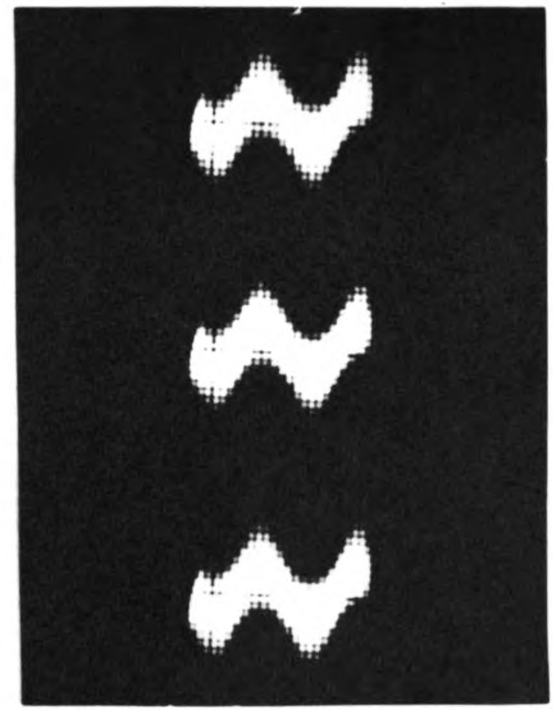
Figure 12

Examples of Scope Patterns



signal frequency 5.6 kc/s
horizontal sweep
top - 2.8 kc/s
middle - no sound
bottom - 120 c/s

Figure 13. Signal Modulation



distance of whistle to interferometer arm
top - 16 cm.
middle - 36 cm.
bottom - 56 cm.

Figure 14. Various Whistle Positions

The three exposures of Figure 14 were taken at 9 kc/s. and at 15 lbs./sq. in. blowing pressure but the whistle was placed at 16, 36 and 56 cm. from the position of the slit image in a line perpendicular to the light beam.

If one had a point sound source, emitting spherical waves, then the exposures from top to bottom should show an amplitude decrease as the inverse of the distance from the light beam to the whistle. No particular effort was made in this investigation to obtain such a source or to damp the sound wave after its passing through the interferometer arm. It was observed with the sound level meter that a very small intensity decrease occurred in a distance of 40 cm. from the source as compared to readings taken at the source.

The variation of output signal with whistle blowing pressure was further investigated. Plots were made while holding constant variables such as selectivity, amplification, photo-tube high voltage and slit width. Figure 15 shows four of these curves obtained at different sound frequencies, and the linear relationship (within anomalous fluctuations) between the output and pressure can be observed. The relative signal measurements were recorded from the oscilloscope face.

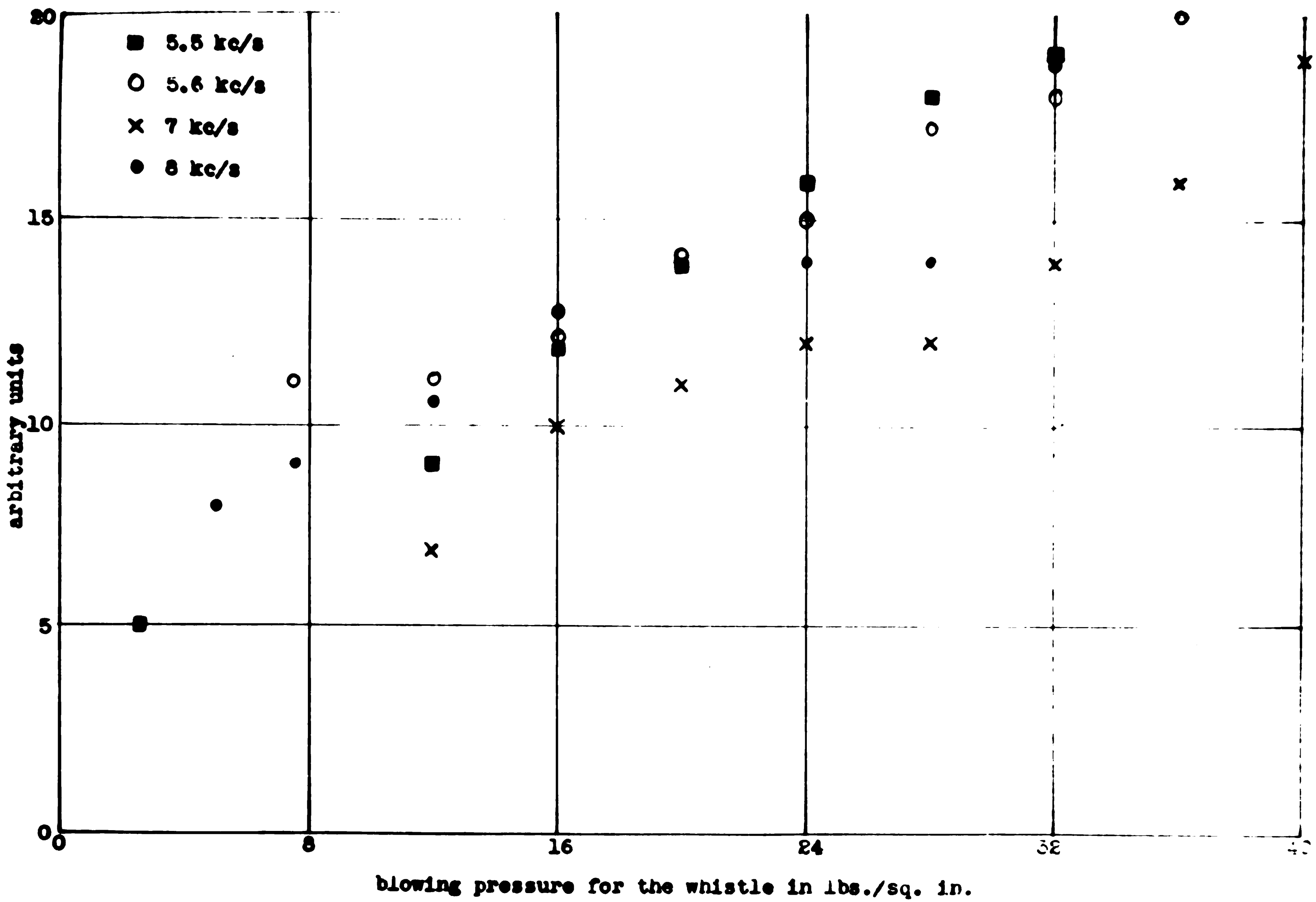


Figure 15. Signal Output as a Function of Blowing Pressure

Plots of the relative signal strength as a function of slit position across the entire fringe pattern were made to determine if a maximum signal would be obtained if the slit was located in the region of the maximum slope of the light intensity curve of a single fringe and in the region of the greatest fringe contrast. One of these plots was made with the interferometer and associated optical train producing ten dark fringes in a field 5 cm. wide at the detector. Figure 16 shows measurements taken across the central one-third of this pattern. This portion contained the zero order fringe and the strongest signals were produced in this portion of the pattern, while as in all such measurements taken, if the slit was located at the fringes nearer the edge of the pattern, a signal of lower amplitude was produced. It was also clearly seen that a maximum signal was obtained when the position of the slit was midway between a dark and a bright fringe, which is the theoretical position of maximum light intensity slope.

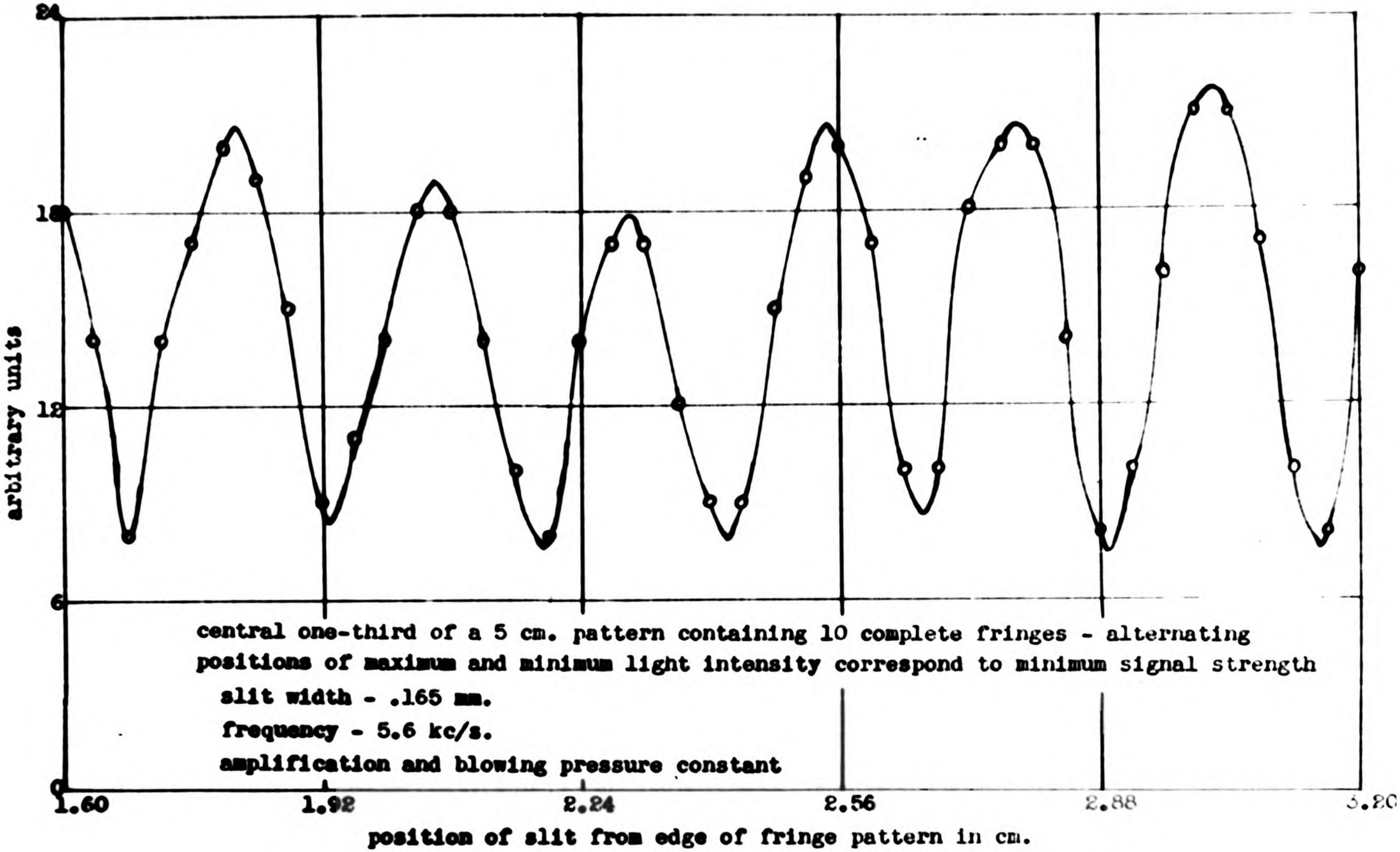


Figure 16. Signal Output as a Function of the Relative Slit Fringe Position

CHAPTER IV

SUMMARY AND CONCLUSION

Figure 17 presents a complete photograph of the apparatus used. The 750 watt mercury arc is at the extreme upper right. Its light is collimated by the lens L1 and illuminates the Mach-Zehnder interferometer. A fringe pattern is obtained and localized directly before the whistle. L2 projects the fringes through the green line filter F to the slit. A photo-tube is mounted directly behind the slit.

The changing pressure associated with a sound produces a shift in the fringe pattern. The image of the slit in the interferometer samples a small portion of the sound wave that causes the shift. A light intensity variation thus produced is presented to the phototube: the output signal which is at the frequency of the whistle and is amplitude modulated by the alternating light source, isthen amplified by the selective amplifier. The amplifier together with a vacuum-tube voltmeter and oscilloscope, either of which can be used to measure the final signal strength, are shown in the lower part of the photograph. Included in the photograph is the sound frequency measuring equipment consisting of the microphone M and the second oscilloscope.

Figure 17. Total Apparatus

Figures 11, 12 and 15 show that the final signal observed was directly proportional to the pressure exciting the whistle. Signals were obtained in the continuous frequency range of from 5.5 to 10 kc/s. and from 5 to 45 lbs./sq. in. whistle blowing pressure.

Results such as presented in Figure 16 show that two positions of minimum and two of maximum signal are observed as one locates the slit at different positions across one complete light-dark fringe. This was to be expected since the static fringe pattern intensity distribution is a cosine squared function and the signal may be thought of as being produced by a small periodic variation along the light intensity distribution function, which contains two points of maximum slope in one complete wave form.

It is therefore shown by the qualitative results as presented in the previous pages that the method investigated, employing the Mach-Zehnder type of interferometer, can be used to measure quantitatively the absolute pressure in a sound field. It has the advantages of non-distortion of the sound field and the measurement rests upon well established pressure, density and index of refraction relationships. Furthermore, the method has the advantage that a very small part of the sound field is sampled at a given time and therefore the method can even be extended to measurements of sound pressures of the shorter ultrasonic waves and still be free from averaging effects.

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APPENDIX

APPENDIX

One of the next steps to be considered as a continuation of this project would be the calibration of the instrument for absolute measurements. Timbrell (1) uses a glass windowed cell in the arm of the interferometer at a position X (see Figure 1). The windows of the cell must be parallel flats to prevent fringe distortion and they must be compensated in the other arm II of the interferometer by an equal thickness of optically parallel glass to retain fringes of sharp contrast. The cell can be pressurized to a static value and, by means of a d.c. coupled amplifier, a calibration of the fringe shift with measurable pressures can be made.

He also includes another set of windows that are part of a tube through which the sound may be propagated. This suitably constructed tube would allow one to propagate the beam (by machining a hole in the base of Macy's interferometer) in a direction perpendicular to the light beam and such that disturbance of the other arm of the interferometer would be minimized or eliminated. The second set of windows would again have to be compensated with glass in the undisturbed beam. This author would suggest trying to reduce the number of additional plates necessary, from eight to four, by the combination of the pressure cell and sound tube into one unit, or perhaps incorporate the compensating plates into a pressure tight calibrating cell.

One might investigate a method of calibrating the instrument by plotting the light intensity distribution across the fringe pattern, then calibrate the intensity change required to give a certain signal output from the photo-tube. One could then find the amount of fringe shift causing the signal and thus the pressure change causing the shift.

The instability of the signal output experienced in this preliminary work is believed to be caused by a combination of the following factors. Disturbances of both arms of the interferometer by the sound, temperature gradients in the interferometer from the heat of the light source and from the cool air supplying the whistle and thus a fringe drift, variation in the illumination of the light source, and by dark current or random noise of the photo-tube.

The first may be eliminated or reduced by construction of the sound tube described and placing it in a direction normal to the plane of the centers of the interferometer plates, such that its terminating end is highly sound absorbent. Temperature control could be more easily accomplished if the sound were thus confined.

It would allow one to enclose the interferometer in a jacket such that a constant temperature could be maintained by perhaps a flow of water about the jacket. The degree of stability of the fringe pattern may be established by the superposition of two curves similar to that of Figure 16. Variations in the illumination of the light source could be reduced by the construction of a regulated voltage supply. McDonald and Harris (14) report the design of a refrigerated photo-tube mount that greatly reduces dark current and base leakage.

If the sound tube is to be constructed, then the use of horizontal fringes will be necessary. Particular attention should then be given to the design of suitable shaped apertures for the best definition of fringes. One may consult Bennett (10, 11) or Winkler (5). Some confusion exists in Winkler's notation of the fringe orientation angle φ but it is believed by this author that he is in agreement with Bennett.

It is suggested that a careful study be made regarding the ideal relationship of slit width to the width of the fringe pattern. If the anomalous fringe drift is reduced then it is believed that a much narrower slit opening could be used and the detection of smaller fringe shifts accomplished.

The following hints are offered to anyone using the apparatus. Localize the fringes at the position of the sound wave in the interferometer by the parallax method, then place L2 in position such that the fringes are focused on the slit and the image of the slit is at the position of the real fringes. Next set the frequency of the selective amplifier at the frequency of the sound by a signal from the H. P. audio-oscillator that coincides with the microphone output of the sound source. Set the selectivity just below the point of oscillation at this frequency, then one is ready to apply the high voltage to the photo-tube. Open the slit until a desired signal is obtained and yet keep the photo-tube anode current to a minimum.

There is considerable backlash in the translating controls of the interferometer and skewing of plates 2 and 3 results when the direction of thrust is reversed. It is convenient if one needs to move the central fringe across the field, to do it in the following manner. For example, make all movements to the right with the proper rotztion of T2, and all movements of the central fringe to the left with T3. These adjustments can be reversed after some length of time if the light paths become too long or too short, as the case may be.

There were pronounced anomalous fringe shifts observed at about 8.4 kc/s. This phenomenon is observed with the naked eye and is believed to be a resonant mechanical vibration of one of the interferometer plates at this particular wave length.

As a matter of interest the original detection of the sound pressure varying the fringe pattern was seen by the naked eye in the following manner. The central fringe with chromatic light was obtained; the fringe pattern was then broadened until approximately two distorted fringes covered the entire field of view. This pattern gave a maze of color combinations intricately entwined. One could then observe particularly narrow bands of prominent colors. Upon introduction of the sound beam into one arm of the interferometer the particular color noted would change shades due to the fringe shift.

A circuit diagram follows of the light source, together with the measured values of the variable reactor. It is the variable reactor that enables one to readily select different light intensities.

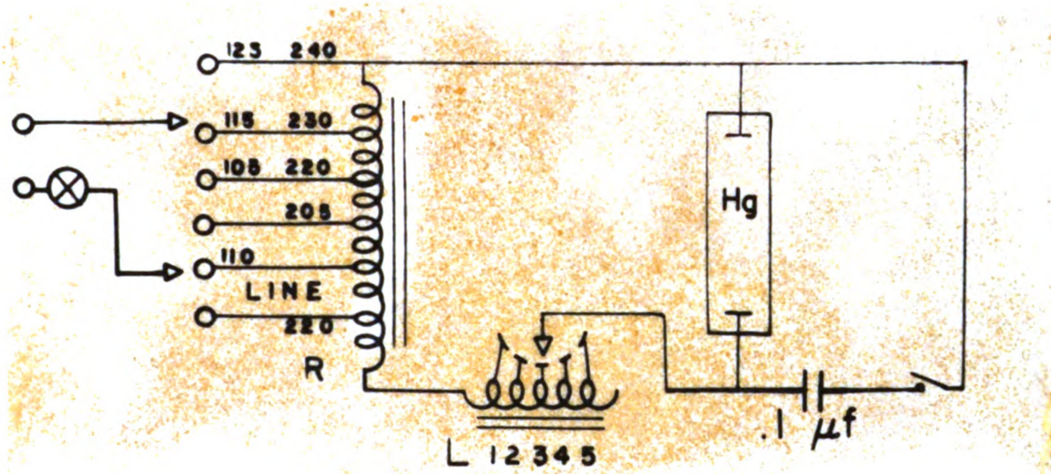

Figure 18

L1 = 0.145 hy.

L2 = 0.161 hy.

L3 = 0.177 hy.

L4 = 0.195 hy.

L5 = 0.215 hy.

If the transformer is operated on a line voltage of 115 volts with the input connected across 110-115 as shown, then the power dissipated in the mercury arc:

Reactance	Power
L5	500 watts
L4	625 watts
L3	750 watts