THEORY OF HEAT CONDUCTION AND CONVECTION FROM A HOT VERTICAL CYLINDER

THESIS FOR THE DEGREE OF M. A.

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HEAT CONDUCTION AND CONVECTION

FROM A

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To Doctor William Scribner Kimball without whose suggestions and aid this thesis would not have been completed.

I. INTRODUCTION. GENERAL HEAT LAWS

The purpose of this investigation is to extend the work of a recent paper * on heat conduction and convection from the plane case to that of a cylinder. The treatment is the same as that in the above mentioned paper, with modifications and restrictions to adapt it to the cylindrical case. This in brief, is as follows:

A cylinder heated to the temperature of boiling water stands vertically in an atmosphere of air at ordinary room temperature. Surrounding this cylinder, in its immediate neighborhood, is a thin film thru which heat flows by conduction and convection. The development and discussion of the laws by which heat flows thru this film comprises this paper.

Two empirical laws ** and a simplified hydrodynamics equation make possible the present treatment.

LAW I. The locus of the maxima of the velocity curves is an isothermal surface whose temperature is the mean of the temperatures of the hot plate and the ambient air.

LAW II. Half the heat is convected up away inside the film and half outside. This thin film is bounded by the cylinder and the isothermal surface defined by Law I.

* W.S.Kimball and W.J.King "Theory of Heat Conduction and Convection from a Hot Vertical Plate". Unpublished - Presented at the meeting of the American Mathematical Society April 4, 1931.

** Kimball and King (Same as above) p. 2

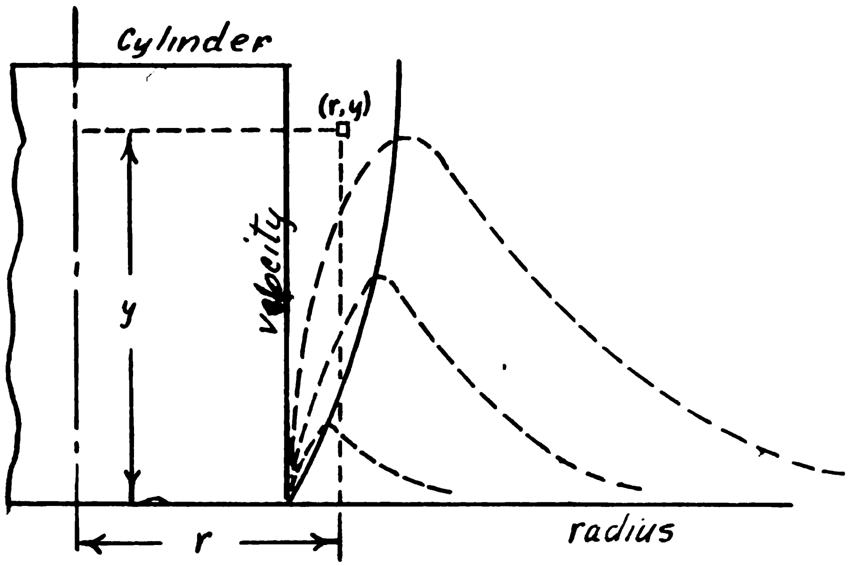

Figure 1.

Since the gas in this thin film is transporting heat by convection as well as by conduction, it is in motion. Hence there are viscous and buoyancy forces which must be taken into consideration. In order to fully appreciate the effect of these viscous forces in a gas and the relation existing between buoyancy and the rate of doing work, let us state Kimball's Theorems *.

THEOREM I. The rate of doing work per unit volume by the buoyancy is proportional to the rate of heat transfer thru unit area.

THEOREM II. The viscous forces operating in a gas in a steady state form a mechanical couple, and hence their sum is always zero. Viscosity's sole mechanical effect is that of a couple: it passes on the reactions from place to place within the gas or to the walls of the container together

* Kimball and King p.4

with a corresponding torque.

There is also the illuminating corollary: The total buoyancy is always balanced by inertia effects plus a downward pull by the walls of the container which are together equal and pppposite to the viscous drag.

These laws and theorems must be kept in mind and incor- ~~por~~ porated in the present treatment together with the fundamental equations of heat *. Also the Langmuir film theory** is a convenient check upon this investigation.

II. THE FILM THICKNESS; THE TEMPERATURE EXPRESSION

If we denote the temperature of the gas at a point within the film by T , the temperature of the cylinder by T_1 , the radius of the cylinder by r_0, the distance from the axis of the cylinder to the point by r , and the height of the point above the plane of the base by y , (see figure 1) we may form an empirical temperature expression

$$T = T_1 - a(1 + be^{-\alpha y}) \log \frac{r}{r_0} , \tag{1}$$

where a, b, and α are constants, two of which are evaluated by the present treatment. If we use the symbol T_m for the temperature of the film boundry, and T_0 for the temperature of the ambient air, we have by Law I

$$T_m = \frac{T_1 + T_0}{2} . \tag{2}$$

Substituting this expression for temperature in (1) we

* J.G.Coffin "Vector Analysis" pp. 104 - 116

** Phys. Rev., 34, 401 (1912)

obtain the relation

$$(3) \qquad \log \frac{r_m}{r_o} = \frac{T_1 - T_o}{2a\,(1+be^{-ay})} \; ,$$

where r_m is the radius of the film boundry. Now from (3) it is readily seen that

$$(4) \qquad r_m = r_o \, e^{\frac{T_1 - T_o}{2a\,(1+be^{-ay})}}$$

which gives the film thickness

$$(5) \qquad r_m - r_o = r_o \left\{ e^{\frac{T_1 - T_o}{2a\,(1+be^{-ay})}} - 1 \right\}$$

This shows that the film thickness increases with the height y.

III. THE FORCE EQUATION

Inside the film the empirical temperature expression (1) satisfies the differential equation

$$(6) \qquad -K\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial y^2}\right\} = C\frac{\partial T}{\partial y} \; ,$$

where K is the coefficient of conductivity and $C = \alpha K$. This equation differs from the simplified classical hydrodynamic equation for combined conduction and convection only in that $C = \alpha K$, approximately a constant, instead of the convection energy flux density. This latter is given by $C = \frac{3}{2}knv$ where k is Boltzmann's constant, n is the molecular concentration, and v is the convection velocity, and it varies from zere at the hot cylinder to a maximum at the film boundry. However this constancy of C seems to be experimentally justified even though the theory here is somewhat incomplete *.

* Kimball and King p. 3

It may be easily seen that for a constant value of y the left member of (6) vanishes. This indicates pure conduction outward from the cylinder thru the film for a constant height with no loss of heat en route. This is a refinement of the Langmuir film concept, a theory which represents resultant effects accurately in terms of pure conduction thru practically stationary gas in an equivalent film of constant thickness surrounding a hot body.

Within the film the fundamental equations of hydrodynamics are simplified by neglecting inertia effects, second order velocity effects, horizontal velocities, and by using the gas law. The downward viscous force F on an annular surface of dimensions $2\pi r \times 1$ will then be (see figure 2)

(7) $$F = -2\pi\eta r \frac{\partial v}{\partial r} \quad ,$$

where η is the coefficient of viscosity of the gas and v is its convection velocity. The difference between the force outside the ring and the force inside is a resultant downward force which is the viscous drag *. This is given by

(8) $$dF = -2\pi\eta \frac{\partial}{\partial r}\left(r \frac{\partial v}{\partial r}\right) dr \, .$$

If there is no acceleration, this force must be balanced by the buoyancy. Let m be the mass of a molecule of the gas g the acceleration due to gravity, n the molecular density of the hot gas and n_o that of the cooler gas. Then mgn is the weight in absolute units per unit volume of hot gas and mgn_o the corresponding weight of the displaced unit volume of the cooler gas. From Archimedes principle, the buoyant

* Leigh Page "Introduction to Theoretical Physics" p. 230

force per unit volume is

(9) $$mgn_0 - mgn = mg(n_0 - n).$$

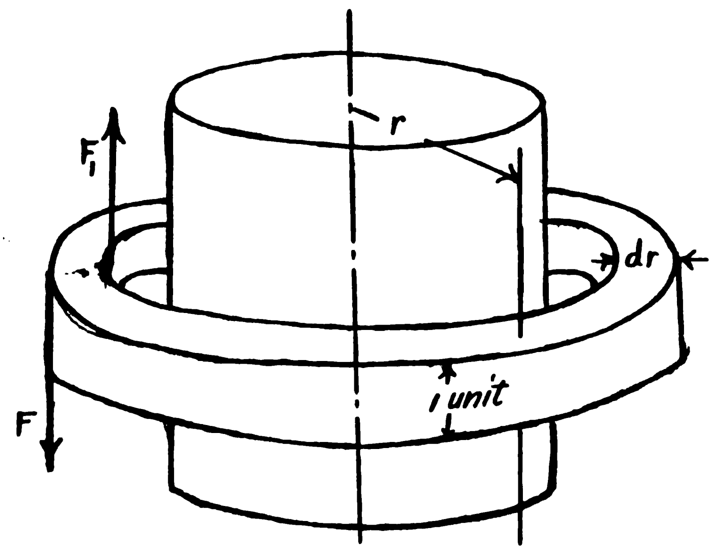

Figure 2.

The condition that the buoyancy balance the drag is therefore

$$-2\pi\eta \frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)dr = 2\pi mg(n_0 - n)\,r\,dr\,,$$

from which we obtain the fundamental force equation

(10) $$-\eta \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) = mg(n_0 - n)$$

It is illuminating to compare this fundamental force equation with the classical laws of Poiseuille, for viscous flow in capillary tubes, and LaPlace for pressure distribution in isothermal atmosphere. For the first case we have simply to replace the right hand member of (10) by its equivalent in terms of pressure, that is,

$$mg(n_0 - n) = \frac{1}{L}(p_0 - p) = \text{constant}$$

We now integrate the resulting expression and obtain a formula for velocity. The volume V of fluid passing any cross section in unit time is found to be given by the relation

$$V = 2\pi \int_0^{r_0} v r\,dr = \frac{\pi r_0^4}{8\eta l}(p_0 - p). \tag{11}$$

Formula (11) is the well known formula of Poiseuille*.

For the second case we assume that there is no viscosity, hence the left hand mamber of (10) is zero. By holding T constant and using the gas law**

$$p = nkT, \tag{12}$$

we have ~~from (10)~~ instead of (10)

$$0 = mg(n_0 - n);\; -mgn\,dy = dp,$$

$$\therefore\; dp = \frac{-mgp\,dy}{kT}.$$

$$p = p_0 e^{\frac{-mgy}{kT}}. \tag{13}$$

We have then the La Place law for pressure distribution***. Thus the fundamental force equation (10) is perhaps the simplest combination of the two classical formulas of Poiseuille and La Place.

* L.B.Loeb "Kinetic Theory of Gases" p. 243

** Leigh Page p. 294

*** Hertzfeld "Kinetische Theorie der Warme" p. 21

IV. CONVECTION VELOCITY

With the application of the gas law (12) the force equation (10) becomes

$$(14) \qquad -\eta \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) = \frac{mgp}{k}\left(\frac{1}{T_0}-\frac{1}{T}\right) = \frac{mgp}{kT_0}\left(\frac{T-T_0}{T}\right).$$

For simplicity, let us make the following definitions:

$$(15) \qquad \begin{aligned} &x = \log\frac{r}{r_0} \;; \quad x_m = \log\frac{r_m}{r_0} = \frac{T_1-T_0}{2a(1+be^{-ay})}, \text{ from (3);} \\ &x_0 = \log\frac{r_0}{r_0} = 0 \end{aligned}$$

Equation (14) may then be written in the simplified form

$$(16) \qquad -\eta\frac{\partial^2 v}{\partial x^2} = r_0^2 e^{2x}\frac{mgp}{kT_0}\left(\frac{T-T_0}{T}\right),$$

and the temperature expression (1) becomes

$$(17) \qquad T = T_1 - ax(1+be^{-ay}).$$

By substituting for T in (16) its value as given by (17) we obtain a function of x. If we expand this as a power series in x using no terms greater than second order and integrate using law I to determine the constants of integration, that is, $\frac{\partial v}{\partial x}=0$ when $x = x_m$, we obtain as the result of the first integration the expression

$$(18) \qquad -\frac{\partial v}{\partial x} = \frac{pmgr_0^2}{\eta k T_0 T_1}\Big[(T_1-T_0)\left\{x-x_m+x^2-x_m^2+\tfrac{2}{3}(x^3-x_m^3)\right\} - \frac{T_0}{T_1}a(1+be^{-ay})\left\{\frac{x^2-x_m^2}{2}+\frac{2}{3}(x^3-x_m^3)\right\} - \frac{T_0}{3T_1^2}a^2(1+be^{-ay})(x^3-x_m^3)\Big].$$

Integrating again using the fact that $v=0$ at $r=r_0$, or in terms of the variable x we have from (15) that $v=0$ when $x=0$, we obtain for the convection velocity

(19) $$V = \frac{pmgr_o^2}{\eta k T_o T_1}\left[\frac{(T_1-T_o)^2}{2a\omega}\left\{1+\frac{A}{2a\omega}+\frac{B}{3a\omega}\right\}x - \frac{T_1-T_o}{2}x^2 - \frac{A}{3}x^3 - \frac{B}{6}x^4\right],$$

wherein $\omega = (1+be^{-ay})$, $A = T_1 - T_o - \frac{T_o a\omega}{2T_1}$,

and $B = T_1 - T_o - \frac{T_o}{T_1}a\omega - \frac{T_o}{2T_1^2}a^2\omega^2$.

By Law I the maximum convection velocity V_m occurs at the outer boundry of the film, that is, where $x = x_m$, hence from (19) we readily obtain

(20) $$V_m = \frac{pmgr_o^2(T_1-T_o)^3}{8\eta k T_o T_1 a^2\omega^2}\left(1+\frac{2A}{3a\omega}+\frac{Bx_m}{2a\omega}\right).$$

V. TRANSFER OF HEAT; THE LANGMUIR FILM.

The heat conduction per unit area* at the hot cylinder is

(21) $$q = -K\frac{\partial T}{\partial r}\bigg]_{r=r_o} = K\frac{a}{r_o}(1+be^{-ay})$$

the right hand expression being obtained by substituting for $\frac{\partial T}{\partial r}\Big]_{r=r_o}$ its value obtained from the differentiation of the temperature expression (1). The total heat convected away from a cylinder of height L is

(22) $$Q = \int_0^L 2\pi r_o q\,dy = 2\pi a K\int_0^L (1+be^{-ay})dy = 2\pi a K\left\{L + \frac{b}{a}(1-e^{-aL})\right\}.$$

Now according to Law II we may write

(23) $$\frac{Q}{2} = 2\pi a K L = 2\pi a K\frac{b}{a}(1-e^{-aL})$$

* Coffin p. 104

The heat transferred to the gas beyond the film is $2\pi a K L$ and $2\pi a K \frac{b}{a}(1-e^{-aL})$ is the excess supply of heat at the lower part of the film due to the larger temperature gradient there. This excess heat is convected up thru the film. Although the theory involved is somewhat incomplete, the fact represented by (23) is justified experimentally*.

We may interpret this equivalence of $\frac{Q}{2}$ and $2\pi a K L$ by reliance upon the Langmuir film concept, a theory which shows that actual rates of heat transfer, heat transfer coefficients, and so forth, are exactly what they would be if heat were transported away from a hot body by pure conduction across a film of stationary gas and of constant thickness. For such a case, if Q is the total heat flow for pure conduction across any cylindrical surface within the Langmuir film at r distance from the axis, then

(24) $$Q = 2\pi r L K \frac{\partial T}{\partial r}.$$

Integrating between the temperature limits T_1 and T_0, and expressing the boundry of the Langmuir film by r_L, we have

(25) $$Q = 2\pi K L \frac{T_1 - T_0}{\log \frac{r_L}{r_0}}.$$

Since Q; T_1, T_0, K, and r_0 are all measureable, we can easily compute the Langmuir film boundry r_L and thus find the film thickness.

If we divide (25) by two we obtain

* Kimball and King p. 11

(26) $$\frac{Q}{2} = \frac{2\pi KL}{\log \frac{r_L}{r_0}} \left(\frac{T_1 - T_0}{2} \right) .$$

Equation (26) shows that if half the temperature drop takes place between the hot cylinder and the Langmuir film boundry r_L , then in the case of pure conduction , half the heat is transported beyond the film.

A comparison of (26) with (23) which we take as the mathematical equivalent of the experimental law II shows that

(27) $$a = \frac{T_1 - T_0}{2 \log \frac{r_L}{r_0}}$$

$$\text{hence} \quad r_L - r_0 = r_0 \left(e^{\frac{T_1 - T_0}{2a}} - 1 \right)$$

Thus we have the relation between the constant a and the Langmuir film thickness $r_L - r_0$. Tt is interesting to compare (27) with (5) and note that the Langmuir film thickness is the limiting value of (5) as y becomes infinite This suggests that the isothermal film thickness at the top of the cylinder is approximately the same as the Langmuir film thickness. This fact was also apparent in the case of a hot vertical plate *.

Restricting the discussion now to the isothermal film as defined by (5), we shall find the total amount of heat carried away vertically from this film. The excess energy per unit volume coming from the hot cylinder is

$$\frac{3}{2} nk (T - T_0)$$

where k is Boltzmann's constant. Hence the flux is

* Kimball and King p. 11

$$\frac{3}{2} nk(T-T_0)v \ .$$

The heat carried away inside the film, which we have shown to be $\frac{Q}{2}$ is the integral of the flux over the total surface thru which heat flows, hence

(28) $$\frac{Q}{2} = 3\pi k \int_{r_0}^{r_m} n(T-T_0)\, vr\,dr \ .$$

By use of the gas law (12) this may be written

(29) $$\frac{Q}{2} = 3\pi p \int_{r_0}^{r_m} \frac{T-T_0}{T}\, vr\,dr = 3\pi p \int_{0}^{x_m} \frac{T-T_0}{T}\, v r_0^2 e^{2x} dx \ .$$

The right hand member of (29) is obtained by substituting for r , r_0, and r_m the values defined by (15). We now express the integrand of the right member of (29) as a power series in x up to and including terms of the third order by using the expression for v from (19) and the power series expansions of $\frac{T-T_0}{T}$ and of e^{2x} . Performing the indicated integration we obtain for the heat carried away inside the film

(30) $$\frac{Q}{2} = \frac{3\pi p^2 m g r_0^4 (T_1-T_0)^5}{16 \eta k T_0 T_1^2 a^3 \omega^3}\left(C + \frac{2}{3}D + \frac{E}{4a\omega}\right) ,$$

wherein $$C = 1 + \frac{A}{2a\omega} + \frac{Bx_m}{3a\omega} \ ,$$

$$D = (T_1-T_0)\frac{C}{a\omega} - \frac{T_1 - T_0 C}{2T_1} \ ,$$

and $$E = \frac{T_0 a\omega}{2T_1} - \frac{A}{3} - \frac{T_0 a\omega C}{2T_1^2}(T_1-T_0) - \frac{(T_1-T_0)(T_1+T_0C)}{T_1} + \frac{(T_1-T_0)^2 C}{a\omega}$$

VI. EVALUATION AND SIGNIFICANCE OF CONSTANTS

To determine the constant a we equate (30) and (23) and find that

(31)
$$a = P r_0 (T_1 - T_0)^{\frac{5}{4}}$$

wherein $P = \left[\dfrac{3p^2 mg(C + \frac{2}{3}D + \frac{E}{4a\omega})}{32\eta T_0 T_1^2 k L K \omega^3}\right]^{\frac{1}{4}}$.

By expressing all of the factors involved in (31) in terms of the fundamental dimensions length, time, mass, and temperature, it was found that a had only temperature for its dimension. From the empirical expression (1) it is readily seen that a is a linear function of temperature, the other factors being dimensionless. Hence we have a dimensional check.

The above value of a in the expression (15) for the logarithm of the ratio of the bounding radii of the film gives

(32)
$$x_m = \frac{T_1 - T_0}{2a(1+be^{-ay})} = \frac{1}{2Pr_0(1+be^{-ay})(T_1-T_0)^{\frac{1}{4}}} .$$

By expanding $x_m = \log\frac{r_m}{r_0}$ into a power series in $r_m - r_0$ we find that to a first approximation

$$x_m = \frac{r_m - r_0}{r_0} .$$

Substituting this value of x_m in (32) gives the relation

(33)
$$r_m - r_0 = \frac{1}{2P(1+be^{-ay})(T_1-T_0)^{\frac{1}{4}}} ,$$

which shows that to a first approximation the film thickness $r_m - r_0$ varies inversely as the fourth root of the temperature difference and is independent of the radius of the cylinder. These facts check with the given data.

In terms of P from (31) the heat equation (30) may be written

$$Q = \frac{4\pi r_0^4}{a^3} KLP^4 (T_1 - T_0)^5.$$

Now substituting the value of a found above, the expression for Q becomes

$$Q = 4\pi r_0 KLP(T_1 - T_0)^{\frac{5}{4}},$$

which may be expressed in terms of x_m from (32) in the form

(34) $$Q = \frac{2\pi KL (T_1 - T_0)}{x_m (1 + be^{-ay})}.$$

Comparing (34) with (26) we note that the expressions are equivalent when

(35) $$x_m (1 + be^{-ay}) = \log \frac{r_L}{r_0} = x_L ,$$

where x_L is defined for the Langmuir film in a manner to correspond with x_m for our film bounded by the isothermal surface.

From (23) and (26) we find that

(36) $$a = \frac{T_1 - T_0}{2 \log \frac{r_L}{r_0}}.$$

Expanding the denominator in a power series in $r_L - r_0$ and neglecting all powers of $r_L - r_0$ except the first, we obtain as a first approximation

(37) $$\frac{a}{r_0} = \frac{T_1 - T_0}{2(r_L - r_0)}.$$

In other words, $\frac{a}{r_0}$ is to a first approximation one half the temperature gradient of the equivalent Langmuir film. This corresponds to the value a as derived for the plane case *.

We know that the heat conduction per unit area at the hot cylinder may be expressed in terms of the heat transfer coefficient h as

(38) $$q = h(T_1 - T_0) = K\frac{a}{r_0}(1 + be^{-ay}).$$

from (21). Hence from (31)

(39) $$h = \frac{Ka(1+be^{-ay})}{r_0(T_1 - T_0)} = KP(1+be^{-ay})(T_1 - T_0)^{\frac{1}{4}}$$

This expresses the heat transfer coefficient as being independent of the radius of the cylinder and proportional to the fourth root of the temperature difference, a fact which seems to be in accord with known data.

With the aid of (31) we may now express (20), the maximum convection velocity at the top of the cylinder as

(40) $$V_m = \left[\frac{mgLK(T_1 - T_0)}{6\eta k T_0(1+be^{-ay})}\right]^{\frac{1}{2}} \frac{\left(1 + \frac{2A}{3a\omega} + \frac{Bx_m}{2a\omega}\right)}{\left(C + \frac{2}{3}D + \frac{E}{4a\omega}\right)^{\frac{1}{2}}}$$

We note here that the maximum convection velocity is proportional to the square root of the temperature difference and is independent of the radius of the cylinder.

From (18) and (31) we may now determine the slope of the velocity curves at the hot cylinder. We find this to be

(41) $$\left.\frac{\partial v}{\partial x}\right]_{x=0} = \frac{\rho m g r_0 (T_1 - T_0)^{\frac{3}{4}}}{2\eta k T_0 T_1 P(1+be^{-ay})}\left[1 + x_m + \frac{2}{3}x_m^2 - \frac{T_0}{4T_1}\left(1+\frac{x_m}{3}\right) - \frac{T_0(T_1 - T_0)^{\frac{7}{4}}}{12 T_1^2 \omega}\right]$$

It is evident that the viscous drag per unit area at the hot

* Kimball and King p. 14

cylinder varies among other things as the three fourths power of the difference in temperature between the cylinder and the ambient air.

VII. STATISTICAL CHECK

Using the available data, we find that the expressions developed herein check to within a small percent of error. With the following table of constants in c.g.s. units, the constant a , the film radii and thickness, and the values of various constants involved in the development were determined.

$p = 1.013 \times 10^{6}$	$k = 1.37 \times 10^{-16}$	$\eta = 2.00 \times 10^{-4}$
$m = 28 \times 1.66 \times 10^{-27}$	$T_1 = 383$	$T_0 = 293$
$g = 980$	$K = 2720$	$L = 25$

The check was made using the approximation $\alpha = \frac{b}{L}$ which is equation (23) when the small exponential term is neglected. The expression $(1+be^{-\alpha y})$ in this case becomes $(1+be^{-b})$ for y equal to L. The tabulated results of the computations for various values of x are as follows:

$x_m = \log \frac{r_m}{r_0}$	r_0	b	$\frac{a}{r_0}$	a	$\left(C+\frac{2}{3}D+\frac{E}{4a\omega}\right)$	Film thickness $r_m - r_0$
.05	8.19	2	86.5	709	.4	.42
.05	7.98	2.2	90.6	724	.4	.43
.1	4.03	2	87.9	354	.45	.42
.2	1.86	2	95	177	.58	.41
.5	.89	2	120	71	1.33	.36

From the fact that the value of the constant $(C+\frac{2}{3}D+\frac{E}{4a\omega})$ increases somewhat as the radius of the cylinder decreased, it is evident that for more accurate results the expansion of the various power series herein involved should include terms of the third and perhaps higher order. However within the limits of experimental accuracy the expansion to the second order seems to be sufficient for values of the cylinder radius r_0 greater than one centimeter.

A comparison of the results of this paper and those of the plane case* is interesting. It is quite evident that since the underlying principles are the same in both developments, the results should check to a large extent. The most striking ckeck perhaps is the fact that the limiting value of the isothermal film thickness as y becomes infinite, that is, dropping the factor $(1+be^{-ay})$, is the same as the Langmuir film thickness. This was exactly the situation in the plane case.

The value of a as determined by Kimball is approximately the same as the value of $\frac{a}{r_0}$ determined herein. They have the same dimensions and play the same part, being the half temperature gradient of the equivalent Langmuir film.

New results of the present theory are that the maximum velocity, heat transfer coefficient, and thickness of the film, to a first approximation, are independent of the radius.

* Kimball and King p. 21

VIII. CONCLUSIONS

The theory checks the following laws:

1. The heat conducted and convected away from the cylinder is proportional to the five-fourth's power of the temperature difference.
2. The maximum convection velocity is independent of the radius of the hot cylinder and is proportional to the square root of the temperature difference.
3. To a first approximation the thickness of the film is independent of the radius of the cylinder.
4. The heat transfer coefficient is independent of the radius of the cylinder, proportional to the fourth root of the temperature difference and the square root of the pressure.
5. The thickness of the equivalent Langmuir film is the same as the thickness of the present isothermal film at y equal to infinity. Furthermore it agrees with its experimental magnitude being between .40 and .50 cms. depending upon the choice of b exactly as was the case for the hot plate film. It is independent of the radius of the cylinder. This is perhaps the most important experimental check afforded by the present treatment.
6. The theory holds accurately for cylinders of radius 1 cm. or greater. Smaller cylinders can be treated by considering additional terms in the expansions.

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