INTENSITY OF THE CONTINUOUS X-RAY SPECTRUM FROM ANTIMONY TARGETS AS A FUNCTION OF WAVELENGTH

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INTENSITY OF THE CONTINUOUS X-RAY SPECTRUM FROM ANTIMONY TARGETS AS A FUNCTION OF WAVELENGTH

by

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R. H. Esling

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Introduction

The difficulties inherent in any experimental technique known to date for determining the absolute values of the intensities of continuous x-ray emission (Bremsstrahlung) have influenced most early investigators in this field to make only theoretical studies of the problem[1], and of the later experimental studies made, most of the measurements of intensity have been measurements only of relative intensity[2]. It is the purpose of this paper to present an experimental study of the Bremsstrahlung by the measurement of the absolute intensities under specific conditions and to compare the results obtained with the theoretical intensities calculated for the same conditions according to one of the later theoretical studies of the subject.

H. R. Kelly[3] and A. F. Reno[4] have each given a rather complete review of the work done, in this field. It is sufficient therefore to remark here that of the later theories proposed some agreement of experimental data and theoretical calculations has been obtained for a relativistic equation developed by F. Sauter[5]. The experimental data mentioned was obtained from studies of the energy $J_\nu d\nu$ as a function of the azimuthal angle of emission[6] or as a function of the energy of the incident electron[7].

In this report a comparison between experimental and theoretical values of $J_\nu d\nu$ as a function of wavelength will be presented.

Theory

In the quantum theory of the radiation processes the probability function ω, of the transition probability for a process in which n quanta are emitted or absorbed, will be proportional to e^{2n} where e is the electronic charge[8]. It may also be shown that if we consider the Coulomb interaction between two free particles as a small perturbation causing a deflection of the particles, this deflection is equivalent in the power of e involved to the emission or absorption of two light quanta. Then if we classify the radiation processes according to the values of e^{2n} we have:

1st order--$\omega \sim e^2$; one light quantum is involved in the interaction.

2nd order--$\omega \sim e^4$; two light quanta are involved.

3rd order--$\omega \sim e^6$; three light quanta, or one quantum and a Coulomb deflection are involved.

The Bremsstrahlung is a particular example of third order processes. It is the case of the interaction between a free electron and a free nucleus. In this case the Coulomb interaction is proportional to e^2Z so that the expansion mentioned above is really an expansion in terms of e^2Z. The first term of this approximation represents Born's approximation.

Most of the theories written for the Bremsstrahlung have been based on Born's approximation and are, in that respect, only valid for the lighter elements and high primary velocities of the bombarding electron since it may be shown that Born's approximation gives correct results only if

$$\frac{4\pi^2 Ze^2}{hv_0} \ll 1 \quad \text{and} \quad \frac{4\pi^2 Ze^2}{hv} \ll 1$$

where v_0 and v represent the velocities of the electron before and after collision. From these conditions we would expect the theory to break down for high atomic numbers even though the primary energy were high. For example, for lead, $Z = 82$, the value of $\frac{4\pi^2 Ze^2}{hc} = 3.76$.[9]

Sauter's (loc. cit) early work resulted in a non-relativistic expression for $J_\nu d\nu$, the intensity in a frequency range $d\nu$, per electron, per atom, per unit area of target for observations made at a distance R from the target and at an azimuthal angle of θ. His later work resulted in a relativistic expression for $J_\nu d\nu$. Both of these expressions were developed using Born's approximation and consequently were limited in their range of validity. He pointed out, however, that Sommerfeld's (loc. cit.) rigorously valid formula could be obtained by multiplication of the earlier non relativistic expression by

$$\frac{\dfrac{(2\pi\alpha Z)^2}{\beta_0\beta}}{\left(e^{\frac{2\pi\alpha Z}{\beta_0}}-1\right)\left(1-e^{-\frac{2\pi\alpha Z}{\beta}}\right)} \qquad (1)$$

where $\alpha = \frac{2\pi e^2}{hc}$ = the fine structure constant,

Z = the atomic number of the nucleus,

$\beta = \frac{v}{c}$

v = the velocity of the electron after collision,

v_0 = the velocity of the electron before collision,

c = the velocity of light,

e = the Naperian base

and concluded it would seem reasonable that multiplication of tha later relativistic expression by this same factor would yeild an expression for $J_\nu d\nu$ which would be valid over the entire spectral range. This assumption he judged to be correct on the basis of some calculation made by Maue, which though then unpublished, were confided to Sauter.

For high values of β_0, (i.e., for high primary velocities), the correction factor (equation 1) deviates from unity chiefly at the short wavelength limit. For small values of β_0 the value of equation 1 becomes smaller than unity except in the immediate neighborhood of the short wavelength limit. In this study we are interested in the latter situation as may be seen by the theoretical calculations on page 20. There, the value of the correction factor approaches unity only near the short wavelength limit.

Some mention should be made of the screening of the pure Coulomb field by the charge distribution surrounding the nucleus. Sauter's work does not take into account this screening effect -- not because the effect is negligible but probably because the mathematics which account for the screening are too unwieldly to handle. Heitler[10] gives a brief discussion of the effect for high energy quanta but we have no information regarding the magnitude of the effect for the experimental energies which we shall use. For lack of information, therefore, the effect of screening shall of necessity be neglected in this study.

The relativistic expression developed by Sauter is

$$
\begin{aligned}
J_\nu d\nu = \frac{2e^6Z^2}{R^2c^3}\frac{p\,d\nu}{p_0^3}\Bigg\{ & \frac{(3E_0^2-c^2p_0^2)m^2c^4\sin^2\Theta}{u^4} - \frac{2E_0^2-c^2p_0^2}{u^2} - \frac{Ecp_0\cos\Theta}{2u^2} \\
& - \frac{cp_0^2}{2pu}\ln\frac{E+cp}{E-cp} + \frac{cp_0^3(h\nu\cos\Theta-cp_0)}{2P_0^2u^2} + \Bigg(\frac{m^2c^4}{2u^2} \\
& - \frac{h\nu}{u} + \frac{(h\nu)^2(h\nu-cp_0\cos\Theta)}{2c^2P_0^2u}\Bigg)\frac{p_0^2}{pP_0}\ln\frac{P_0+p}{P_0-p} \\
& + \ln\frac{E_0E-m^2c^4+c^2p_0p}{E_0E-m^2c^4-c^2p_0p}\Bigg[\frac{(3E_0h\nu m^2c^4-E_0Ec^2p_0^2)m^2c^2\sin^2\Theta}{2p_0pu^4} \\
& + \frac{3E_0^2c^2p_0^2-4E_0h\nu m^2c^4+c^4p_0^2p^2}{4c^2p_0pu^2} - \frac{(E_0E+c^2p_0^2)h\nu\cos\Theta}{4cpu^2}\Bigg]\Bigg\}
\end{aligned}
\qquad (2)
$$

where e = the electronic charge,

Z = the atomic number of the nucleus,

R = the distance to the defining aperature from the target,

c = the velocity of light,

p = the momentum of the electron after collision,

p_0 = the momentum of the electron before collision,

ν = the frequency of the emitted quantum,

E_0 = the initial total energy of the bombarding electron,

E = the total energy of the electron after collision,

m = the rest mass of the electron,

Θ = the angle, with respect to the v_0 direction, at which the quantum is emitted,

$u = E_0(1-\beta_0\cos\theta)$,

h = Plank's constant,

q = the momentum of a quantum,

$P_o = [p_o^2 + q^2 - 2pq\cos\theta]^{1/2}$.

Upon multiplication of eq. 2 by eq. 1 we have

$$J_{\nu_{corr}}\,d\nu = J_\nu\,d\nu\,\frac{\dfrac{(2\pi\alpha Z)^2}{\beta_0\beta}}{\left(e^{\frac{2\pi\alpha Z}{\beta_0}}-1\right)\left(1-e^{-\frac{2\pi\alpha Z}{\beta}}\right)} \qquad (3)$$

which is assumed to be valid in the whole spectral range. This equation (eq. 3) is the one in which we are most interested since the experimental results obtained shall be compared with theoretical values of J_ν calculated by eq. 3 for the wavelengths used in the experimental study.

Experimental Requirements

The ideal method of experimentally checking any theory of the Bremsstrahlung would be to make studies of the quanta produced simultaneously with the Coulomb deflection of a single electron in the field of a single nucleus. To experimentally isolate a single electron and a single nucleus as free particles is of course difficult. Possibly the required conditions could be obtained with a cloud chamber but it is doubtful whether the quantum associated with a given Coulomb deflection could be studied with any such apparatus. To approximate these ideal experimental conditions we must have a constant electron source supplying electrons at a constant rate, all of these electrons having equal initial energies. These electrons must then be made to pass through a screen of atoms and the quanta emitted in a known time are the quanta studied. This screen of atoms should be thin enough that all the electron-nucleus interactions approximate the ideal conditions. The thickness of the screen of atoms should therefore be only sufficient to produce an accurately measureable number of quanta.

When the foregoing conditions are satisfied as nearly as possible it is necessary to have the quanta-measuring apparatus as sensitive as possible in order that the quanta from the very thin atomic screens may be accurately measured.

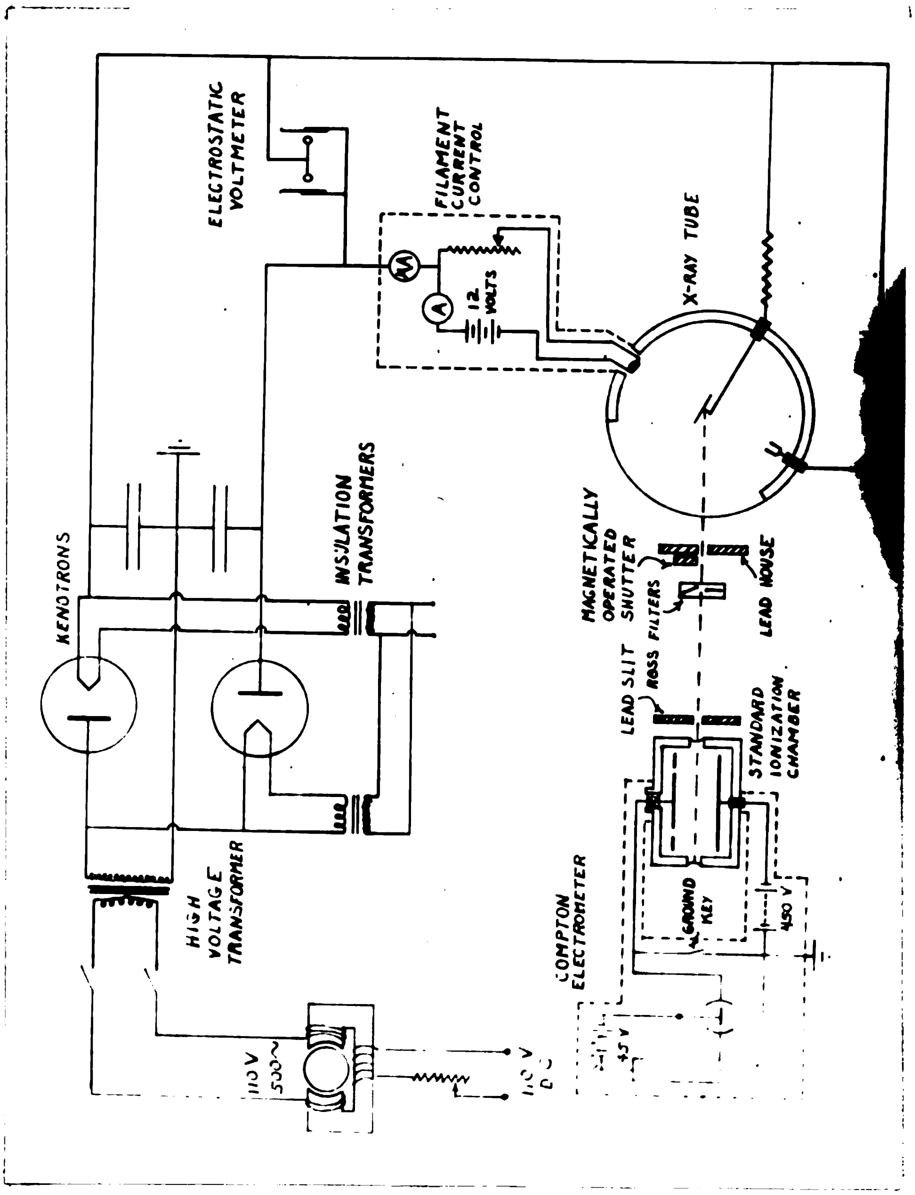

KENOTRONS
INSULATION TRANSFORMERS
HIGH VOLTAGE TRANSFORMER
110 V 500~
110 V D.C.
ELECTROSTATIC VOLTMETER
FILAMENT CURRENT CONTROL
M
A
12 VOLTS
X-RAY TUBE
MAGNETICALLY OPERATED SHUTTER
ROSS FILTERS
LEAD HOUSE
LEAD SLIT
STANDARD IONIZATION CHAMBER
COMPTON ELECTROMETER
GROUND KEY
450 V
45 V

Fig. 1

Experimental Apparatus

The Constant Electron Scource:

The actual electron source was a spiral tungsten filament made of 8 mil wire, and was electrostatically shielded so that the electrons given off were collimated into a beam which had an elliptical cross section of about 1.2 square cm. at the target of the experimental x-ray tube (the atomic screen). As may be seen from the complete wiring diagram of the apparatus (figure 1) the constant high voltage was supplied by a voltage-doubler circuit and high-voltage 500~ transformer. This voltage supply was controlled by adjusting the current through the field coils of a 500~ alternator which was connected to the primary of the high-voltage transformer. This particular voltage supply has been described by Pettitt[11].

To supply electrons at a constant rate the space current through the experimental tube must be accurately controlled. This was done by means of a rheostat, manually operated, in series with the filament, while the entire current-control unit was electrostatically shielded to eliminate corona loses from the measured current. This arrangement has been described by Kelly and Reno (loc. cit.) except that the meter which they used has been replaced with a General Electric galvanometer # 32C236G1 shunted to read 9.3×10^{-9} amperes per division.

Whenever measurements were made to obtain data for this study two operators were required, one to control the voltage and current, to keep them constant, and one to make measurements on the x-radiation studied.

The Screen of Atoms:

The targets used in the experimental x-ray tube were formed by evaporating antimony onto a cellophane backing in the manner described by Kelly and Reno. The thicknesses of the targets were measured by the interferometer method[12] and were of the order of magnitude of 150A to 250 A. It will be observed from the sample target shown below that the thin targets used were transparent. Little is known about the atomic arrangement in these very thin films, so, in order to reduce the experimental data we shall have to assume a uniform distribution of the atoms throughout the thin film.

Kelly and Reno made intensity measurements on targets of plain cellophane, with no metallic film evaporated onto them, and decided that the intensity of x-radiation from the cellophane backing was negligible. Consequently, no correction in their reports was made for the backing material. Fritz[13] later made measurements on the effect of the backing material by using different thicknesses of backing material and determining the corresponding intensities of x-radiation from the targets. His work indicates

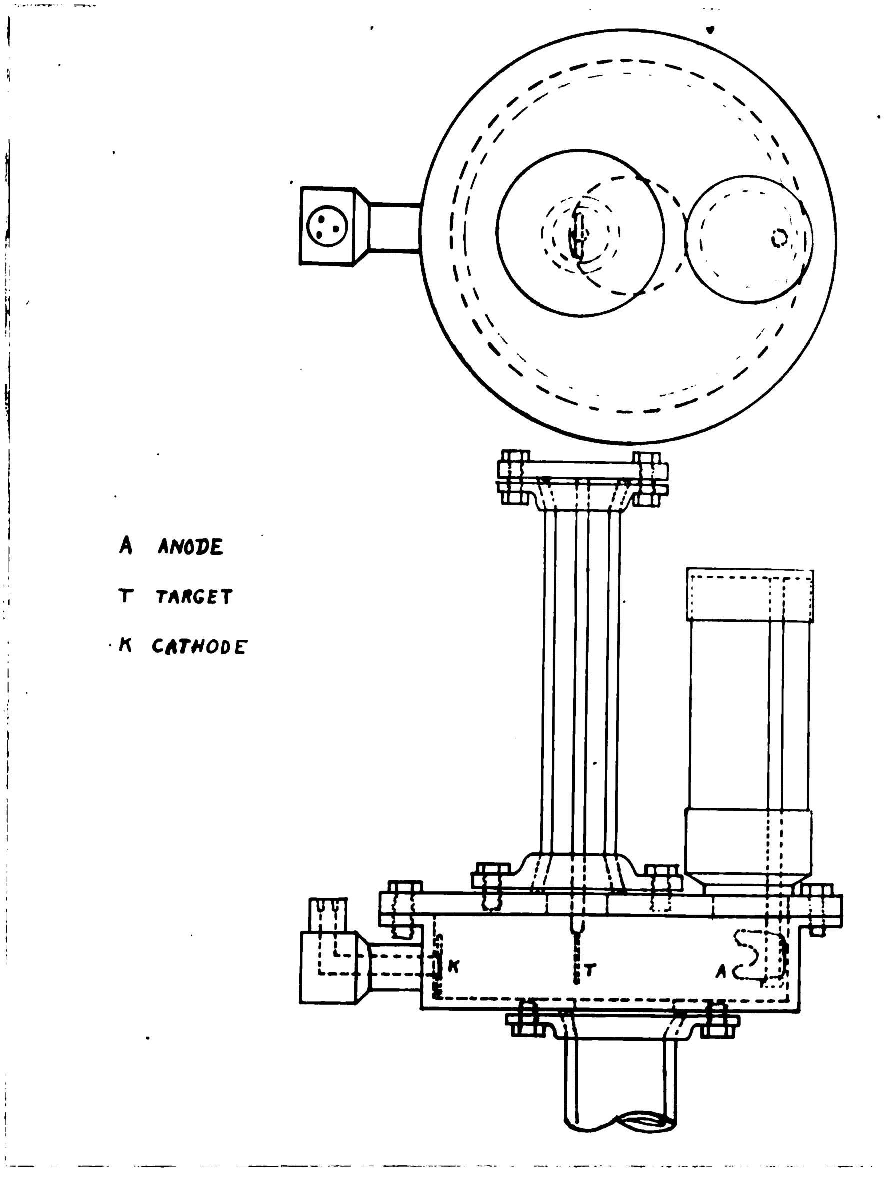


Fig. 2

that for the cellophane backing used in this work (thickness .002 cm.) the backing effectively increases the true intensity of radiation by about 32%. The fact that the cellophane alone gives a negligible amount of radiation whereas the cellophane in conjunction with a metallic film contributes appreciably to the number of quanta radiated is possibly explained by a rediffusion of the electrons through the metallic foil thus causing extra collisions and yielding extra quanta. As the thickness of the backing is increased the number of electrons slowed down and rediffused through the metallic film would be increased.

In this report the figure developed by Fritz will be used as a correction factor to take account of the target backing.

The Experimental X-Ray Tube:

The filament and thin target were used in the experimental x-ray tube described by Kelly and were arranged as shown in figure 2. The tube is essentially a short steel cylinder which has an inside diameter of 8 inches and an inside height of 2.5 inches. The target support and anode are insulated from the steel tank by means of heavy pyrex tubes so that the tank may be kept at a negative potential. This arrangement eliminates the production of x-ray by electrons bombarding the walls of the tank and thus all the radiation studied comes from the target. (The series of defining aperatures prevent radiation from the anode from entering the ionization chamber.) The window through which the x-rays leave the tube is made of aluminum, .04 mm. thick, and is large enough to permit the study of radiant energy at all angles between 12° and 160° with respect to the direction of the incident electron. The tube was evacuated by means of a Cenco-Hyvac fore pump in conjunction with an oil diffusion pump.

The Quanta-measuring Apparatus:

The variables in Sauter's theory are J_ν, $d\nu$, ν, E_0, Z, and Θ. In this report J_ν was studied as a function of ν alone. It was necessary, therefore, to have some experimental means of effectively monochromatizing the radiation to be studied and it was decided that Ross filters[14a] would serve the purpose well. These filters give effectively monochromatic results without absorbing so much of the energy that they make the accurate readings difficult.

Ross filters are a combination of two filters made of elements near each other, preferably adjacent, in the periodic table. The thickness of these filters are adjusted so that both filters have the same absorption in all parts of the spectrum except between the K absorption limits of the two elements. A monochromatizing effect is produced by allowing the filters to alternately intercept a beam of non-monochromatic radiation while measurements are made of the intensity of unabsorbed radiation. Any difference in the effect of the two filters must then be due to the unbalanced region between the two K absorption limits. For elements adjacent in the periodic table this "pass band" is quite narrow (.018A,

.021A, and .034A for the filters used in this work). The mean wavelength of each pass band is considered to be the wavelength of the monochromatic radiation studied.

There is an optimum thickness for the foils used in Ross filters. The expression for transmitted band intensity has a maximum value when the filter thickness is [14b]

$$t_o = \frac{\log r}{\mu_L (r-1)}$$

where t_o = the optimum filter thickness,

μ_L = the linear absorption coefficient on the long wavelength side of the pass band,

$r = \mu_s / \mu_L$

μ_s = the linear absorption coefficient on the short wavelength side of the pass band.

This expression is obtained in the following manner: Using the well known expression for the absorption of x-radiation,

$$I = I_o e^{-\mu t}$$

where I_o = the intensity of the radiation incident on the filter, I = the intensity of the radiation passed by the filter, μ = the linear absorption coefficient for wavelength λ, and t = the thickness of the absorbing foil we may write:

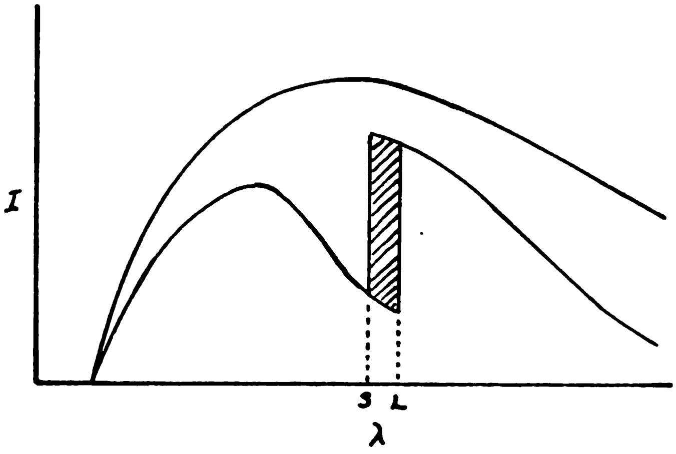


Fig. 3

$$I_L = I_o e^{-\mu_L t} \quad \text{and} \quad I_s = I_o e^{-\mu_s t}$$

Now the pass band intensity is proportional to the area of the pass band (see figure 3) which in turn is proportional to

$$\Delta I \Delta\lambda = I_o (e^{-\mu_L t} - e^{-\mu_s t})\Delta\lambda$$

Then to find the optimum thickness,

$$\frac{d}{dt}(\Delta I \Delta\lambda) = I_o \Delta\lambda (\mu_s e^{-\mu_s t} - \mu_L e^{-\mu_L t}) = 0$$

whence

$$\mu_s e^{-\mu_s t_o} = \mu_L e^{-\mu_L t_o}$$

We define

$$r = e^{(\mu_s - \mu_L) t_o} = \frac{\mu_s}{\mu_L}$$

Then

$$\begin{aligned} \log r &= (\mu_s - \mu_L) t_o \\ &= \mu_L \left(\frac{\mu_s}{\mu_L} - 1\right) t_o \\ &= \mu_L (r-1) t_o \end{aligned}$$

or

$$t_o = \frac{\log r}{\mu_L (r-1)}$$

It is preferable, though not necessary, that the filters used be of optimum thickness. In practice the two filters are mounted in a carrier frame and a condition of "balance" obtained by rotating one of the filters slightly with respect to the other in order to adjust the effective thickness of the rotated filter.

Kirkpatrick (loc. cit.) had discussed Ross filters in detail and points out that it is sometimes advisable to use a third element in conjunction with one of the two basic filters as a means of obtaining a better balance than would otherwise be possible. This was done with one of the filters used in this work.

The filters used were:

Filter	Mean λ of pass band	Calculated t_0
Antimony	.415 A	------------
Tin		6.27×10^{-3} cm.
Cadium	.474 A	3.37×10^{-3} cm.[15]
Silver		3.35×10^{-3} cm.[15]
Molybdenum	.635 A	2.53×10^{-3} cm.
Columbium		2.90×10^{-3} cm.[16]

A thin aluminum foil (.001 cm. thick) was used in conjunction with the molybdenum filter.

The wavelengths available through the use of Ross filters are limited in number. The condition that the two filters in a given filter system must be near each other, preferably adjacent in the periodic table makes only a few elements readily usuable as filters since only a few elements available in foil form meet this condition. In an effort to overcome this limiting characteristic of the filters experimenters have made filters of metallic salts evenly distributed in a beeswax or other binder, but this technique has the disadvantage of introducing other elements into the absorbing foil.

For the experimental study reported in this paper it was decided that at least three filter systems should be used, that is, that the theory should be checked at at least three points. Only two Ross filters, however, were readily available for a study in the region where our equipment made it convenient to work. Cadium, silver, molybdenum, and columbium were readily available in foil form, rolled to the desired thickness, from Baker and Company, Incorporated. Tin was also available so it was observed that if an antimony filter were prepared and balanced with the tin foil a third wavelength would be available. Antimony, however, was brittle and could not be rolled into a foil by any known method. A new method was therefore developed of evaporating the antimony onto cellophane backing so as to form a foil of the desired thickness. This method, discussed in the appendix, produced a filter of pure antimony and was therefore considered to be better than a filter formed by distributing an antimony salt in a binder.

The absolute intensity of the quanta passed by the Ross filters was measured by determining the number of ion pairs formed in an ionization chamber. The charge produced, due to the formation of these ion pairs, was recorded by a Compton electrometer. The ionization chamber used was the one used and described by Kelly and Reno. It was made of brass with mica windows at the ends and was filled with CH_3Br at a pressure of about 68 cm. The collector plates were of thin aluminum sheet and were equiped with electrostatic guard rings to eliminate end effects due to irregularities of the field at the ends of the collector plates. Sufficient voltage (450 volts) was applied across the collector plates to obtain practically saturation current. It is estimated from previous work (see Kelly, loc. cit., fig. 10) that 96% of the ions formed were collected before recombination. A small correction was also made for the fluorescent effect (see pp. 24).

The electrometer used was a new one made at the Stanford University instrument shop by Mr. B. G. Stuart and had a sensitivity of about 5700 mm. per volt. In using the electrometer the "strictly ballistic" method discussed by Webster and Yeatman[17] was employed.

Theoretical Calculations

The theoretical values of J_ν with which the experimental results of this work are compared were calculated from equation 3 after dividing equation 3 by $d\nu$. The values of the various terms in the formula as well as the final corrected values of J_ν are listed on the following pages and the theoretical values of J_ν are plotted as a function of λ in figure 6 where they are compared with the experimental results.

Values of constants used in calculating J_ν by Sauter's equation:

$m = 9.11780 \times 10^{-28}$ gm.

$e = 4.80650 \times 10^{-10}$ esu

$h = 6.63428 \times 10^{-27}$ erg-sec

$c = 2.99776 \times 10^{10}$ cm./sec.

$Z = 51$

$R = 34.0$ cm

$\Theta = 60^\circ$

$E_o = 8.8351 \times 10^{-7}$ ergs

$\beta_o = .37405$

$u = 7.1827 \times 10^{-7}$ ergs

$p_o = 1.1024 \times 10^{-17}$ gm-cm/sec.

Terms in Sauter's equation and calculated theoretical values of J_ν!

		.320Å	.350Å	.415Å	.500Å	.550Å	.635Å
ν	$\times 10^{-18}$	9.3680	8.6530	7.2235	5.9955	5.4505	4.7209
$h\nu$	$\times 10^{9}$	6.2143	5.6810	4.7923	3.9776	3.6160	3.1320
$q = \frac{h\nu}{c} = \frac{h}{\lambda}$	$\times 10^{18}$	2.0732	1.8955	1.5986	1.3269	1.2062	1.0448
$\frac{A}{d\nu} = \frac{2e^6Z^2}{R^2c^3} = 2.3809 \times 10^{-97}$	(constant factor)						
$E = E_0 - h\nu$	$\times 10^{7}$	8.2137	8.2670	8.3559	8.4373	8.4735	8.5219
$p = \left(\frac{E^2}{c^2} - m^2c^2\right)^{1/2}$	$\times 10^{18}$	1.9060	3.6624	5.4642	6.6916	7.2028	7.8124
$P_0 = (p_0^2 + q^2 - 2p_0 q\cos\Theta)^{1/2}$	$\times 10^{17}$	1.0148	1.0209	1.0318	1.0424	1.0473	1.0541
$\frac{A}{d\nu}\cdot\frac{p}{p_0^3}$	$\times 10^{54}$	3.3873	6.5086	9.7106	11.8919	12.8004	13.8838
$\kappa = \frac{(3E_0 - c^2p_0^2)m^2c^4\sin^2\Theta}{u^4} = 4.2235$	(constant factor)						
$\gamma = \frac{2E_0^2 - c^2p_0^2}{u^2} = 2.8143$	(constant factor)						
$\epsilon = \frac{Ecp_0\cos\Theta}{2u^2}$		1.3153	1.3239	1.3356	1.3512	1.3570	1.3647
$\zeta = \frac{cp_0^2}{2pu}\ln\frac{E+cp}{E-cp}$		.1854	.1850	.1846	.1838	.1835	.1831
$\eta = \frac{cp_0^3(h\nu\cos\Theta - cp_0)}{2u^2P_0^2}$		-.1132	-.1128	-.1124	-.1113	-.1109	-.1103
$k = \frac{m^2c^4}{2u^2} - \frac{h\nu}{u} + \frac{(h\nu)^2(h\nu - cp_0\cos\Theta)}{2uc^2P_0^2}$		.5612	.5690	.5859	.5939	.5992	.6075
$\mu = \frac{p^2}{pP_0}\ln\frac{P_0+p}{P_0-p}$		2.3387	2.4265	2.5417	2.6533	2.7501	2.8131
$k\mu$		1.3405	1.3802	1.4892	1.5757	1.6477	1.7089
$\rho = \ln\frac{E_0E - m^2c^4 + c^2p_0p}{E_0E - m^2c^4 - c^2p_0p}$		.7261	1.4340	2.2511	2.8962	3.2302	3.6473

		.320A	.350A	.415A	.500A	.550A	.635A
$\sigma = \frac{(3E_0 h\nu m^2c^4 - E_0 E c^2 p_0^2) m^2 c^2 \sin^2\Theta}{2 p_0 p u^4}$		1.5697	.5561	.0814	-.1516	-.2308	-.3238
$\tau = \frac{3E_0^2 c^2 p_0^2 - 4E_0 h\nu m^2 c^4 + c^4 p_0^2 p^2}{4c^2 p_0 p u^2}$		2.7884	1.6330	1.2977	1.2117	1.1887	1.1735
$\chi = \frac{(E_0 E + c^2 p_0^2) h\nu \cos\Theta}{4cpu^2}$		.2200	.1053	.0601	.0411	.0259	.0209
$\rho(\sigma + \tau - \chi)$		3.0047	2.9883	2.9693	2.9532	3.0103	3.0231
$J_\nu = \frac{A}{d\nu} \frac{p}{p_0^3} \left[\text{[illegible]} - \gamma - \epsilon - \zeta + \eta + k\mu + \rho(\sigma + \tau - \chi) \right]$	$\times 10^{53}$	1.4025	2.7049	4.1123	5.1059	5.6526	6.2242
$\beta = \left(1 - \frac{m^2c^4}{E^2}\right)^{1/2}$		.0693	.1326	.1959	.2385	.2547	.2747
$\frac{2\pi\alpha Z}{\beta_0} = 6.2527$	(constant factor)						
$\frac{2\pi\alpha Z}{\beta}$		33.7682	17.6319	11.9371	9.8081	9.1611	8.5126
$\frac{\frac{(2\pi\alpha Z)^2}{\beta_0 \beta}}{\left(e^{\frac{2\pi\alpha Z}{\beta_0}} - 1\right)\left(1 - e^{-\frac{2\pi\alpha Z}{\beta}}\right)}$		.4273	.2127	.1440	.1183	.1108	.1027
$J_{\nu_{corr}}$	$\times 10^{54}$	5.9921	5.7524	5.9208	6.0378	6.2607	6.3919

Experimental Results

The working equation:

The quantities measured experimentally are not the values of $J_\nu\, d\nu$ but are related to $J_\nu\, d\nu$ by a relationship which shall be termed as the "working equation".

To begin with, only a fraction of the intensity of the x-radiation leaving the target is measured. Between the target and the effective portion of the ionization chamber are several absorbing materials which diminish the measured intensity. The fact that the absorption coefficients vary with wavelength does not complicate the problem as it would seem at first, because, since we are interested only in the radiation of wavelength equal to the mean wavelength of the pass band we may consider the absorption to be that of monochromatic radiation whose wavelength is the mean wavelength of the pass band. More strictly, we shall consider what happens to the intensity of radiation having initial intensity $I_{o\nu}\, d\nu$ at the target where ν is the mean frequency of the pass band and $d\nu$ is the frequency range of the pass band. The small variation of the linear absorption coefficients shall be neglected over the range of the pass band. Figure 4 shows the positions of the absorbing materials which diminish the intensity of the measured radiation.

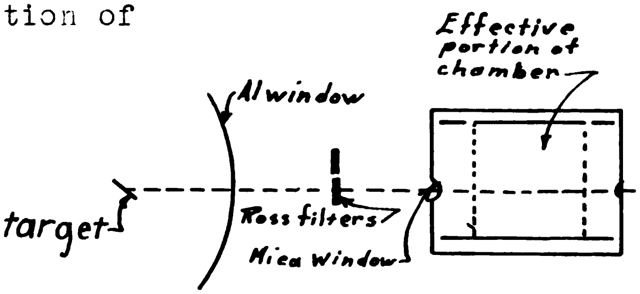


Fig. 4

The aluminum window of the x-ray tube was .004 cm. thick and the fraction of the intensity transmitted by the window at the wavelengths used in this problem was:

λ =	.415	.474	.635	A
$I_{1\nu}/I_{0\nu}$	.9874	.9818	.9582	

The absorption of the silver and cadmium filters were calculated according to Reno's work with these filters. Since, however, the values of the linear absorption coefficients were not readily available for antimony and columbium the fraction of initial intensity transmitted by each of the antimony, tin, molybdenum, and columbium filters was measured directly by placing each filter in the path of a Bragg spectrometer. The fractions transmitted by the different filters are listed below:

Filter		$I_{2\nu}/I_{1\nu}$ at wavelength λ
Sb	.415 A	.7010
Sn	.415 A	.1807
Cd	.474 A	.7410
Ag	.474 A	.1280
Mo	.635 A	.6800
Cb	.635 A	.1431

Compton and Allison[18] give an expression whereby one is able to calculate the absorption of the .0043 cm. mica window. Using this expression we find that the fractions transmitted by the mica window are:

λ =	.415	.474	.635	A
$I_{3\nu}/I_{2\nu}$ =	.9862	.9804	.9561	

Next, between the mica window and the effective portion of the ionization chamber, the radiation studied passes through 7.8 cm. of CH_3Br at a pressure of 68.3 cm. of Hg. A chamber of known length containing CH_3Br having the same density as the CH_3Br in the ionization chamber was placed in the path of a Bragg spectrometer and the absorption measured. From data thus obtained it was found that the fractions of radiation transmitted by the first part of the ionization chamber were:

λ =	.415	.474	.635	A
$I_{4\nu}/I_{3\nu}$ =	.666	.545	.279	

Now combining the effects of all the absorbers in the path the fraction of the initial intensity arriving at the front of the collector plates with different filters in the path was calculated to be:

Filter		$I_{4\nu}/I_{0\nu}$
Sb	.415 A	.4546
Sn	.415 A	.1172
Cd	.474 A	.3887
Ag	.474 A	.0671
Mo	.635 A	.1738
Cb	.635 A	.0366

Finally, the radiation actually studied was that absorbed in the effective portion of the chamber. This portion was 10 cm. long so from the data previously taken on the absorption of CH_3Br we calculated the fraction absorbed in this part of the chamber to be:

λ =	.415	.474	.635 A
$\frac{I_{4\nu} - I_{5\nu}}{I_{4\nu}}$	.406	.543	.806

and upon taking the product $\frac{I_{4\nu} - I_{5\nu}}{I_{4\nu}} \cdot \frac{I_{4\nu}}{I_{4\nu}}$ we get the fraction of the initial intensity absorbed in the effective part of the ionization chamber.

The deflection of the electrometer when any given filter is in place is proportional to the amount of radiation absorbed in the effective portion of the ionization chamber. So, if δ_a and δ_b represent the respective deflections of the electrometer when the two foils of a Ross filter system are placed in the path then $(\delta_a - \delta_b)$ is proportional to the measured intensity $I_{m_\nu} d\nu$.

$$(\delta_a - \delta_b) \propto I_{m_\nu} d\nu = (.96)(.68)K\left[\left(\frac{I_{4\nu}}{I_{o\nu}}\right)_a - \left(\frac{I_{4\nu}}{I_{o\nu}}\right)_b\right]\left(\frac{I_{4\nu} - I_{5\nu}}{I_{4\nu}}\right) I_{o\nu}\, d\nu$$

or

$$\delta_{Sb} - \delta_{Sn} = .0790\, I_{o\nu}\, d\nu$$

$$\delta_{Cd} - \delta_{Ag} = .1004\, I_{o\nu}\, d\nu$$

$$\delta_{Mo} - \delta_{Cb} = .0635\, I_{o\nu}\, d\nu$$

where the figure .96 is the correction for the saturation current (see page 16), the figure .68 is the correction for the target backing (see page 10) and K = .881 represents the correction for the fraction of the incident radiation lost in fluorescent radiation from excited Br ions in the ionization chamber. Thus:

$$K = 1 - k_f ;$$

k_f is the ratio of the average intensity of incident radiation. Clark[19], working with an identical ionization chamber found the value of k_f to be .119 whence K = .881.

We must next consider the relationship between the deflection of the electrometer and the number of ions formed in the ionization chamber. The electrometer is calibrated to give a deflection that is linear with respect to the quantity of charge collected in the chamber. The deflection is then

$$\delta = kQ = kCV$$

where δ = the deflection of the electrometer in mm.

k = the charge sensitivity of the electrometer in $\frac{\text{mm.}}{\text{coulomb}}$

Q = the charge in coulombs

C = the capacity of the electrometer system in farads

V = the potential difference in volts applied to the electrometer system.

The difference in deflection for a given Ross filter is

$$\delta_a - \delta_b = kC(V_a - V_b) = kC\Delta V$$

whence

$$\frac{\delta_a - \delta_b}{k} = C\Delta V$$

Now, if the right hand side of the last equation is divided by the product of the charge on an electron and the time during which radiation is absorbed in the chamber we obtain the number, n, of ion pairs produced in the ionization chamber per second:

$$n = \frac{C\Delta V}{et} = \frac{\delta a - \delta b}{ket}$$

where e = the electronic charge in coulombs,

t = the exposure time in seconds.

Then the energy measured by the ionization chamber per unit of time is equal to the number of ion pairs multiplied by the energy per ion pair which is given by Stockmeyer^{20} to be 25.4 electron volts or 4.06 x 10^{-11} ergs.

If the radiation entering the ionization chamber enters through an aperature of cross section a the energy measured in the chamber per second per unit area at a distance R from the target for the mean wavelength of a given pass band is

$$I_{m_\nu} d\nu = \frac{4.06 \times 10^{-11}}{keta} (\delta_a - \delta_b) \frac{\text{ergs}}{\text{sec - cm.}^2}$$

But the measured intensity $I_{m_\nu} d\nu = BI_{o_\nu} d\nu$

where
$$B = (.96)(.68)(.881) \left[\left(\frac{I_{4\nu}}{I_{o_\nu}}\right)_a - \left(\frac{I_{4\nu}}{I_{o_\nu}}\right)_b \right] \left(\frac{I_{4_\nu} - I_{5_\nu}}{I_{4_\nu}}\right)$$

so that
$$I_{o_\nu} d\nu = \frac{4.06 \times 10^{-11}}{keta} \cdot \frac{(\delta a - \delta b)}{B} \frac{\text{ergs}}{\text{sec-cm.}^2}$$

We wish, however, to obtain the intensity of radiation from the interaction of a single electron and a single nucleus per unit area of target. So we must take account of the number of electrons striking the target during the exposure time and of the number of atoms per unit area of target. During the exposure time, the number of electrons incident upon the target is

$$\sigma = \frac{it}{e}$$

where i = the current through the x-ray tube in amperes,

e = the electronic charge in coulombs.

The number of atoms per square cm. of target is

$$M = \frac{N\rho x}{A} \quad \frac{\text{atoms}}{\text{cm}^2}$$

where N = Avogadro's number $= 6.06 \times 10^{23}$

ρ = the density of the target in g/cm^3

x = the thickness of the target in cm.

A = the atomic weight of the target element in g/mole.

Applying these corrections to the expression for $I_{0\nu}\, d\nu$ we get

$$J_\nu\, d\nu = \frac{4.06 \times 10^{-11}}{keta} \cdot \frac{(\delta a - \delta b)}{B} \cdot \frac{e}{it} \cdot \frac{A}{N\rho x}$$

$$= \frac{4.06A \times 10^{-11}}{kit^2 aN\rho x} \cdot \frac{(\delta a - \delta b)}{B}$$

$$= 4.165\ (\delta a - \delta b) \,/\, B \times 10^{-37}$$

The units of $J_\nu\, d\nu$ are ergs per second flowing through a square cm. at a distance R and azimuthal angle θ from the target, generated by a single electron incident upon a single atom per square cm. of target.

Determination of the charge sensitivity, k:

In order to evaluate the expression for $J_\nu\, d\nu$ it is necessary to know the charge sensitivity of the electrometer system. This was determined by measuring the capacitance of the system. We know

$$\delta_i = kQ_i = kC_EV_i$$

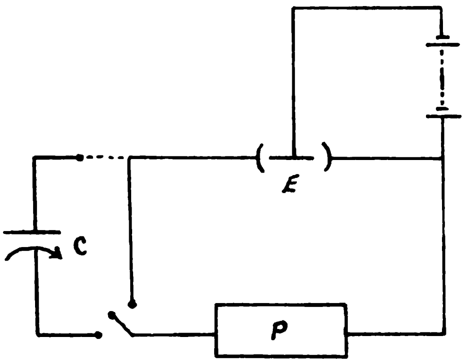
and if a known capacitance is placed in series with the electrometer system (see figure 5)

$$\delta_j = k\left(\frac{C_EC_S}{C_E+C_S}\right) v_j$$

Suppose $v_j = 2V_i$. Then if $\delta_i = \delta_j$,

$$C_EV = \frac{C_EC_S}{C_E+C_S} \cdot 2V$$

or $C_E = C_S$



E = electrometer
C = std. condenser
p = potentiometer

Fig. 5

Applying a potential difference directly to the electrometer system a certain deflection was obtained. Then the standard, variable condenser was put in series with the electrometer system and the standard capacitance varied until the same deflection was obtained with the potential difference across the series system equal to twice the original potential difference across the electrometer system alone. This method of measuring the capacitance is considered to be as accurate as the method used by Kelly and Reno and has the advantage of measuring C_E directly. The value of C_E obtained by this method was 94.0$\mu\mu f$.

Experimental data:

The technical difficulties encountered in obtaining the experimental data made the task a difficult one. The greatest difficulty arose from the fragility of the targets since sudden, large surges of current due to gas bursts in the experimental x-ray tube or to sudden discharging of the condensers often destroyed targets before any data could be obtained. The results here presented are calculated using the data from antimony targets II-3 and III-1, which data is considered to be the most reliable obtained in this study.

Target Sb II-3

Voltage: 40 kv. Current: 9.6×10^{-7} amp.

Exposure: 5 sec. Thickness: 2.35×10^{-6} cm.

$$\text{Average } \delta_{Sb} - \delta_{Sn} = 13.8 \text{ mm.}$$

$$\delta_{Cd} - \delta_{Ag} = 22.3 \text{ mm.}$$

$$\delta_{Mo} - \delta_{Cb} = 20.0 \text{ mm.}$$

The values of B and $d\nu$ are:

λ :	.415	.474	.635	A
B :	.0970	.1004	.0635	
$d\nu$:	3.1082	2.8524	2.4622	$\times 10^{17}$

so dividing the working equation for $J_\nu d\nu$ by $d\nu$ we get

$$J_\nu = 1.907 \times 10^{-52} \text{ at } .415 \text{ A}$$

$$J_\nu = 3.243 \times 10^{-52} \text{ at } .474 \text{ A}$$

$$J_\nu = 5.328 \times 10^{-52} \text{ at } .635 \text{ A}$$

The data from target III-1 was obtained at 33 kv. and only for wavelengths of .415 A and .635 A.

Target III-1

Voltage: 33 kv. Current: 9.6×10^{-7} amp.

Exposure: 5 sec. Thickness: 1.71×10^{-6} cm.

Average $\delta_{Sb} - \delta_{Sn} = 4.5$ mm.

" $\delta_{Mo} - \delta_{Cb} = 12.7$ mm.

The values of B and $d\nu$ are as previously listed so that

$J_\nu = .854 \times 10^{-52}$ at .415 A

$J_\nu = 4.649 \times 10^{-52}$ at .635 A

The constant factors used in evaluating the working equation were:

$A = 121.71$ g/mole.

$k = 6.122 \times 10^{13}$ mm./coulomb

$a = .912$ cm.^{2}

$N = 6.06 \times 10^{23}$ atoms/mole

$\rho = 6.22$ g/cm.^{3}

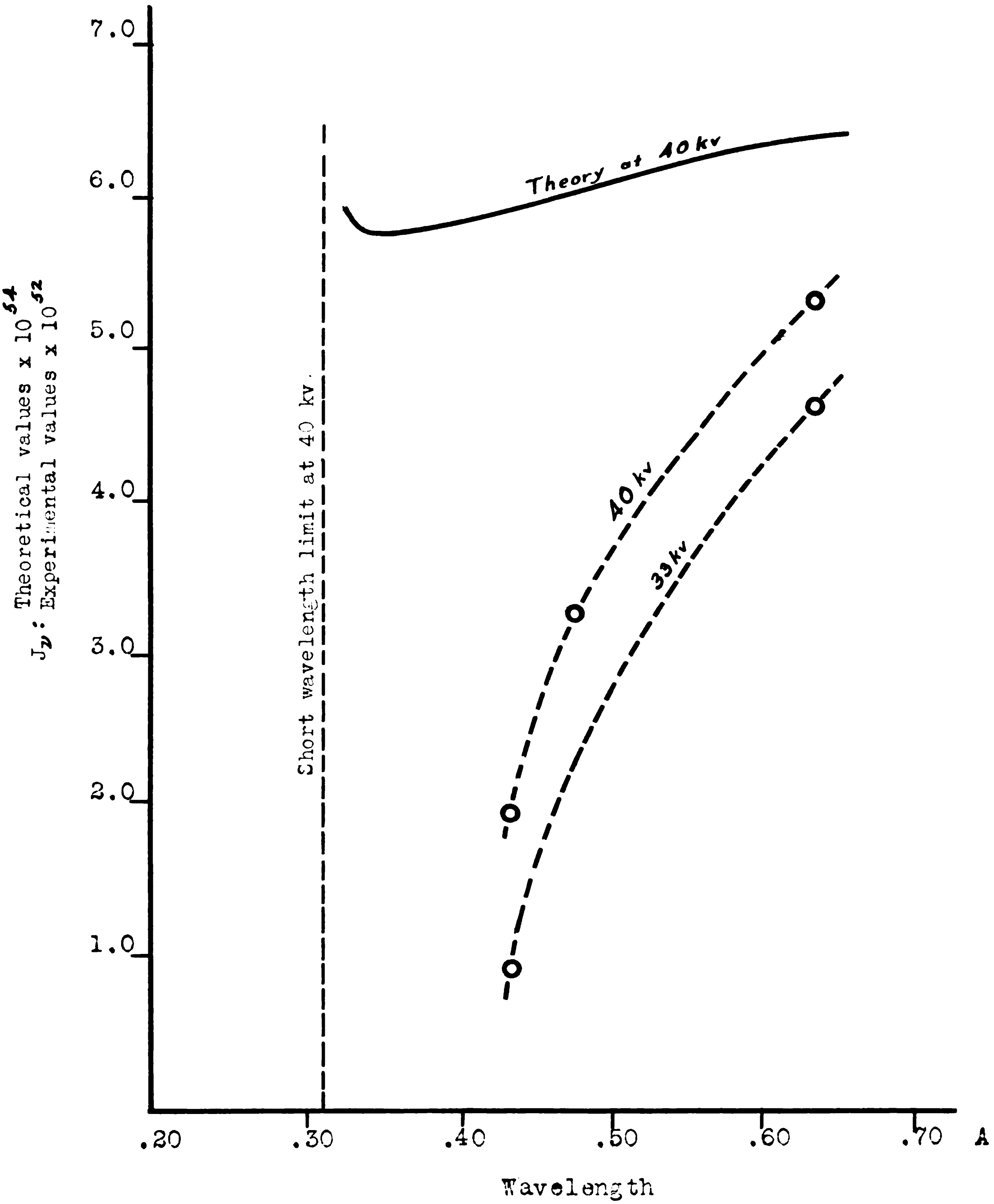


Fig. 6: Comparison of experimental results with theory.

Comparison of experiment with theory:

The experimental results are plotted with the theoretical curve in fig. 6.

Previous experimentors in this field have not obtained exact agreement with theory. Comparitively good agreement of experimental results with theory has been obtained for the spatial distribution[6] but Reno[4], studying $J_\nu d\nu$ as a function of voltage, agreed only roughly with the theory. It will be observed from the results here presented that (a) the experimental order of magnitude is greater, (b) the experimental curves do not appear to have a non-zero intensity at the short wavelength limit. Indeed, the experimental curves look very much like intensity curves from thick targets. Possibly, to approximate the ideal experimental conditions it will be necessary to use even thinner atomic screens than those used in this work. If, however, that is necessary, a new method of determing target thickness must be devised since the method used in this study was limited to 150 A[12].

On the other hand, it is possible that, while the theory accounts for the spatial distribution fairly well, the theory is not entirely accurate in other respects and consequently correlation between all theoretical calculations and experimental results are not to be expected. In view of the failure of other experimental data to check the theory except for the spatial distribution it would seem that this is the case. Before any definite statement can be made, however, with respect to this particular study some revision of apparatus is in order so that data can be taken under the most nearly ideal experimental conditions possible.

Appendix

Preparation of the Antimony Filter

The Dimensions of the Foil

An antimony foil of the proper thickness was prepared by evaporating the metal onto a backing sheet which had a negligible absorption in the wavelength region where the tin-antimony filter was to be used. Since it was difficult to obtain aluminum foil which was not only sufficiently thin to have an absorption comparable with that of commercial cellophane but also free from small holes, cellophane was used as a backing material. The cellophane used (approximately .002 cm. thick) was tested and found to have a negligible absorption in the wavelength region where the filter was to be used.

The optimum thickness of the tin foil to be used with the prepared antimony foil was calculated[14b] to be .006 cm. (see pp. 14). Although no information was available regarding the density of thin films of antimony deposited by evaporation, the tabulated density of crystalline antimony was close to that of tin so the desired thickness of the antimony foil was therefore expected to be approximately .006 cm. Considerable latitude in the thickness of one of the foils is allowed since the final condition of equal absorption by both foils is accomplished by rotating one of them to adjust its effective thickness.

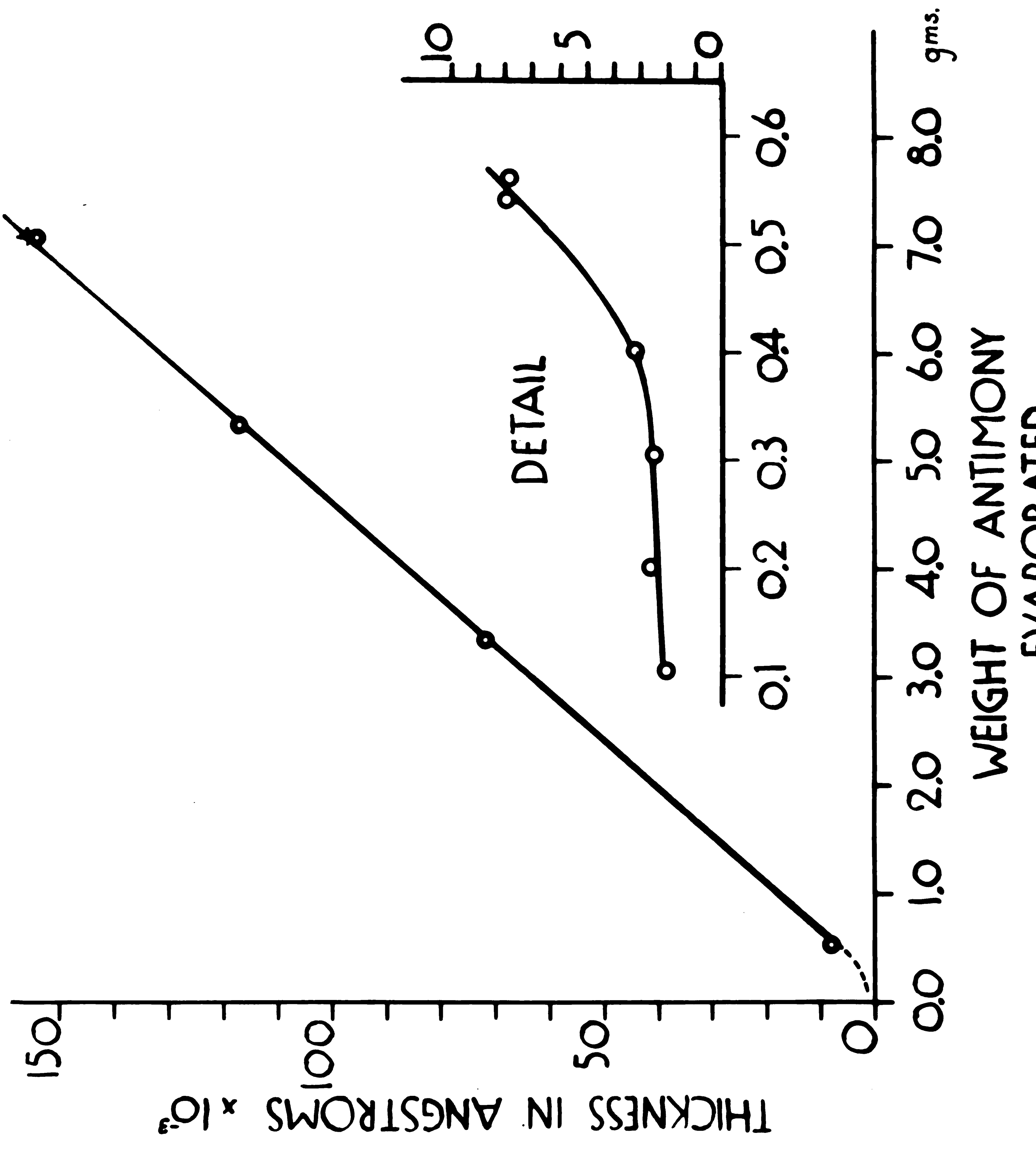

THICKNESS IN ANGSTROMS x10³
150
100
50
0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
gms.
WEIGHT OF ANTIMONY EVAPORATED
DETAIL
10
5
0
0.1
0.2
0.3
0.4
0.5
0.6

Fig A-1

In order to obtain the desired thickness of antimony deposited by evaporation it was convenient to know the relationship between the amount of metal evaporated and the corresponding thickness of deposit. To determine this relationship various amounts of pure antimony were evaporated and the thickness of the corresponding films obtained were determined. For relatively thin films (2000A to 3500A) the metal was powdered in a mortar and then evaporated from a V-shaped trough formed from a .003 x .8 x 3.0 cm. piece of molybdenum. This trough technique is in contrast to the common practice of evaporating from a filament to which the molten metal will adhere. It is desireable to use a trough to which molten metal will not adhere. Caldwell[21], evaporating different elements from filaments of various metals, furnishes data on the wetting properties of metals. The thickness of the antimony foil formed in each case was determined from interferometer measurements[12], using the mercury green line isolated from a low pressure mercury arc spectrum by suitable filters as the interferometer source rather than the ultra violet lines mentioned in reference 12. Figure A-1 (detail), shows the results of the four foils prepared in this region.

To prepare thicker foils it was necessary to evaporate more antimony than could be held in a single trough. The use of a large trough was prohibited by the current capacity of the electrical system so an arrangement of three parallel troughs was used (figure A-2). Each trough was controlled

Fig. A-2

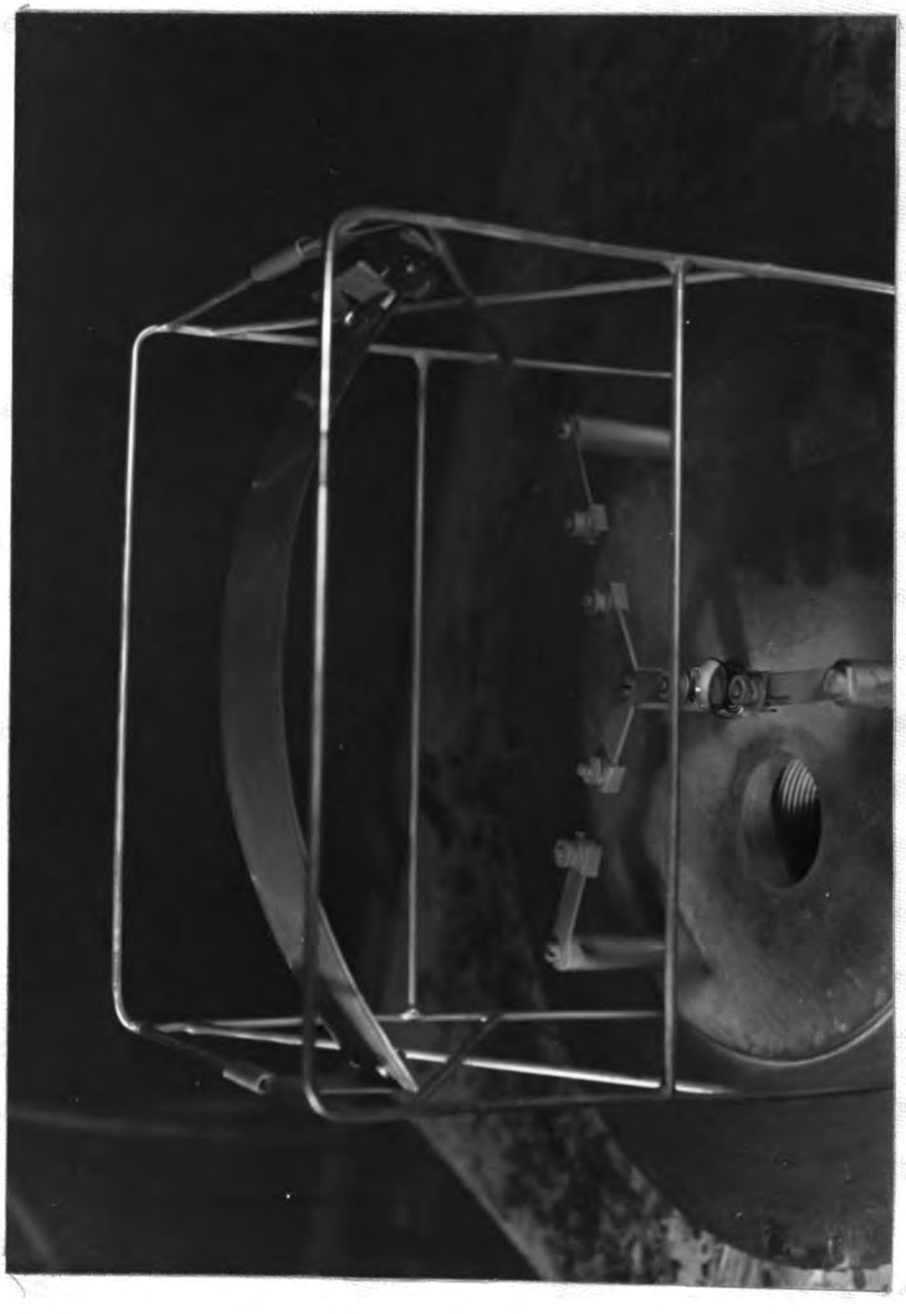


Fig A-3

by a separate switch. This arrangement could be made to handle about one gram of metal but was somewhat awkward to use. Good control was partially dependent upon the ability of the operator to see the metal in the troughs. This was rather difficult, however, since the coating of antimony on the inside of the evaporator jar became opaque before all the metal was evaporated. Nevertheless, two foils were prepared by this method and the data obtained showed that the curve definitely was not linear in the region 2000 A to 10,000 A. It appeared, however, that the curve was approaching linearity between 4,000 A and 10,000 A.

By this method sixty foils, each 10,000 A thick, would have to be prepared in order to make the finished filter. A new technique was therefore introduced with which it was possible by four successive evaporations on the same cellophane backing to simultaneously prepare four foils, each 150,000 A thick. The finely ground antimony was placed in a .75 ml. micro-analysis crucible which was nested in a spiral filament made of 15 mil tungsten wire (figure A-3). This method would easily handle 2.0 grams of antimony for each evaporation. The crucible containing the antimony was accurately weighted before and after each evaporation to determine the amount of metal evaporated. The sleight error introduced by the deposit of tungsten on the outside of the crucible was considered negligible.

In order to prepare four foils simultaneously a piece of cellophane large enough to be cut into four foils was mounted in a cylindrical frame of 13.5 cm. radius. This distance was sufficiently large to give a uniform deposit over the surface to be coated but was still within the range of high speed antimony atoms leaving the heated crucible. The crucible was placed at the center of the cylinder.

Films prepared by this technique were too thick to be conveniently measured by interferometer methods since the fringe shift would have been about 55 fringes for $\lambda = 5461$ A, but the deposit was heavy enough to make possible the determination of thickness by weight-area measurements. The density of the antimony was taken to be 6.618 grams per cm^3.

It will be observed that for the relatively thick foils the curve (figure A-1) is linear, though for the thin foils this was not true. If this effect is in any way due to a potential barrier set up about the cellophane, either by ion bombardment when a high voltage is applied to the evaporator chamber during the outgassing process or by some charge carried by the evaporated particles, then this effect should be eliminated by using aluminum foil or some other conductor as a backing material for the evaporated foil. The final single foil prepared was measured to be 154,000 A thick, 2.5 cm. wide, and 20.5 cm. long.

The Uniformity of the Foil

It was necessary that the final four-fold foil prepared be uniform over an area at least as great as that of the defining aperature of the x-ray beam with which the filter

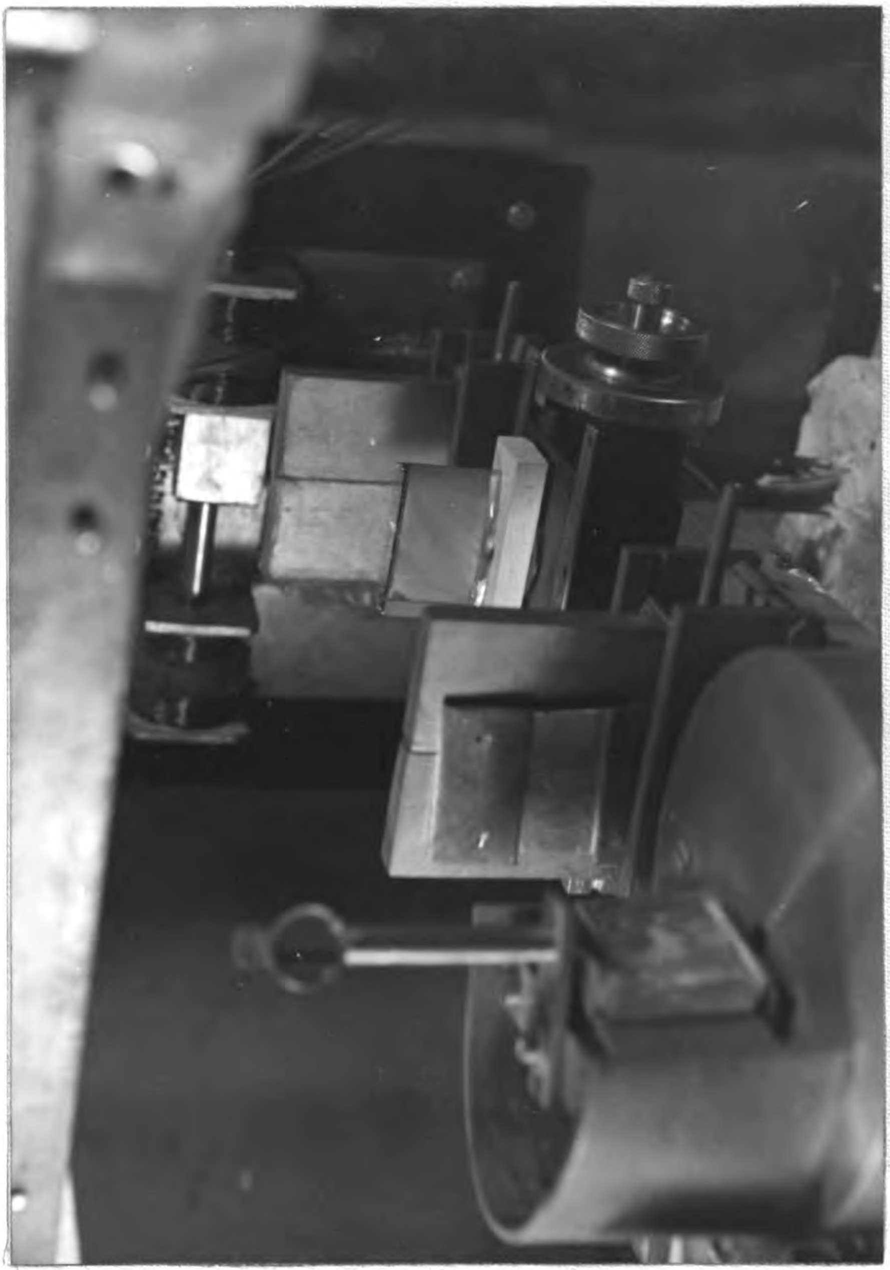


Fig. A-4

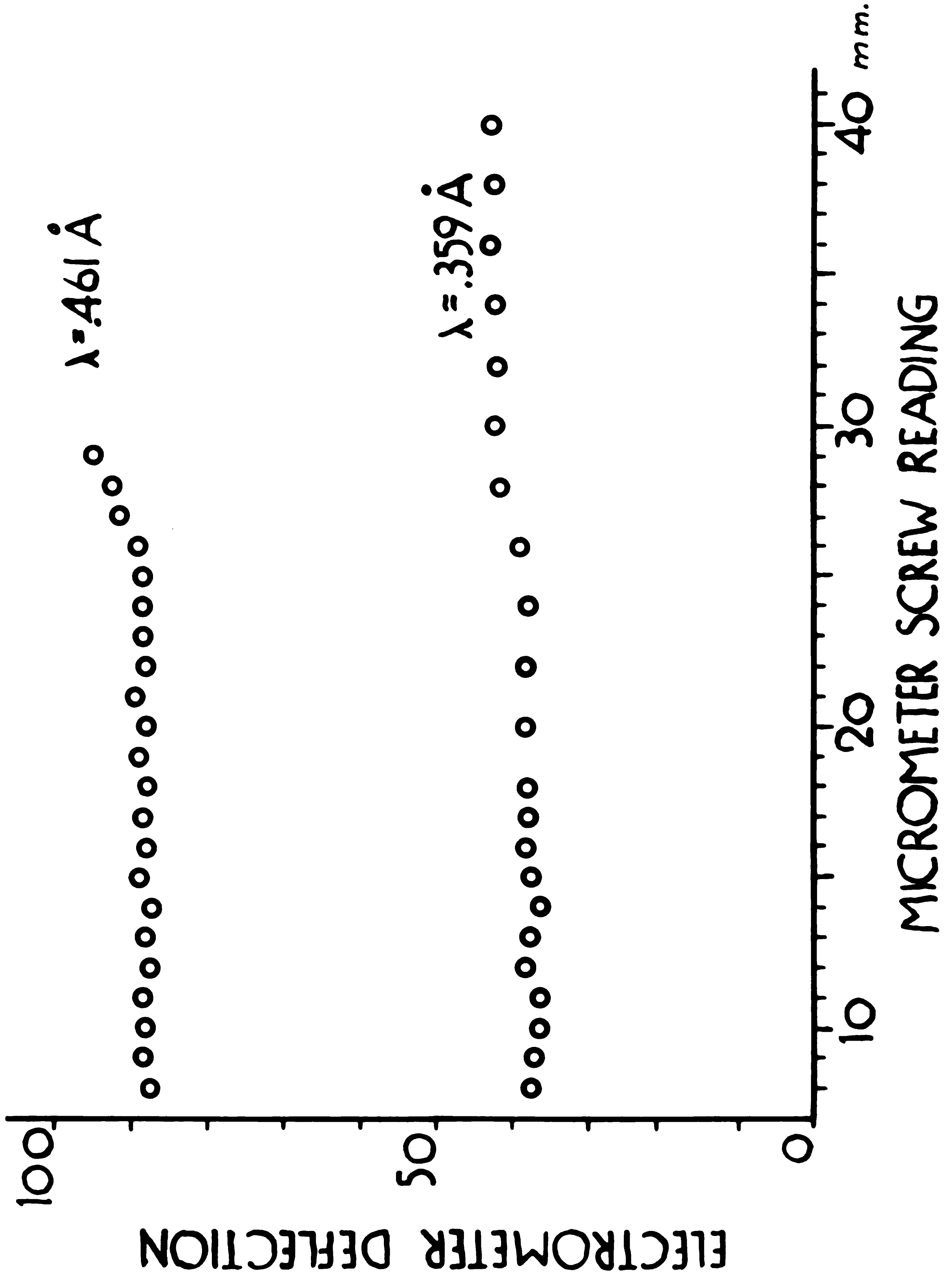
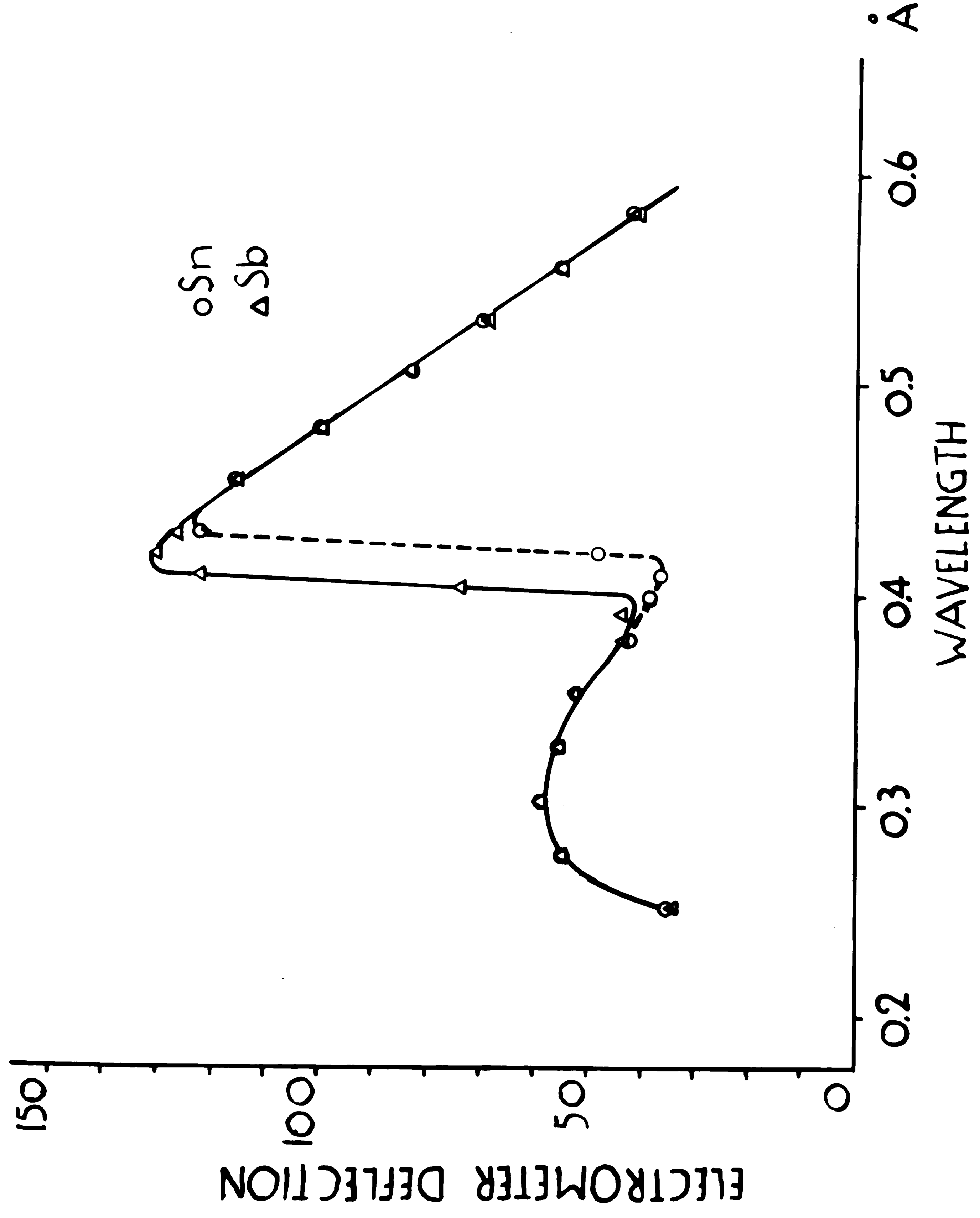

ELECTROMETER DEFLECTION
100
50
0
MICROMETER SCREW READING
10
20
30
40 mm.
λ=.461 Å
λ=.359 Å

Fig A-5

system was to be later used. To test for uniformity the single foil was wound around two vertical pins in such a manner that four thicknesses of absorbing foil might be presented to a narrow beam (less than 1.0 mm.) of x-rays. The pins were firmly fixed on the travelling base of a micrometer screw which was placed between the defining slits of a Bragg spectrometer (figure A-4) so that the four thicknesses were perpendicular to the beam. Absorption readings were taken at points one millimeter apart across the four-fold foil, the absorption being measured as a function of the ionization produced in an ionization chamber. The x-ray source was kept constant by manual control of x-ray tube current and constant potential high voltage. It will be observed (figure A-5) that the variations in electrometer deflections are less than one millimeter over a section of foil 1.4 cm. in length. The variation is within the accuracy of the apparatus.

In its final form the filter had to be at least .4 x 2.2 cm. This was easily prepared from the uniform portion of the four-fold foil. Actually, the final foil was made 1.0 cm. wide so that it might be rotated appreciably with out becoming effectively narrower than the defining aperature with which it was to be used. The pieces of foil were cut to size and cemented to an aluminum frame which was then mounted with the tin foil in the balanced filter holder.

If the four-fold had not been uniform over a sufficiently large area the four foils could have been mounted separately



o Sn
Δ Sb
150
100
50
0
ELECTROMETER DEFLECTION
0.2
0.3
0.4
0.5
0.6
Å
WAVELENGTH

Fig A-6

and a condition of uniform absorption accomplished by displacing one of them with respect to the others. That is, if the non-uniformity of each foil had been wedge-like the thickness would be changed by sliding the wedges with respect to each other.

The Balanced Tin-Antimony Filter

Once the tin and antimony foils had been mounted together in a balanced filter holder they were adjusted so that they absorbed equally at all wavelengths except between their K limits. This was accomplished by placing the foils alternately in the x-ray path of a Bragg spectrometer and rotating one of the foils until its effective absorbing thickness matched that of the other foil. In this particular case the antimony foil was perpendicular to the x-ray beam and the tin foil was rotated about 4° from the perpendicular. The absorption curves are shown in figure A-6.

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