AN INTERFERENCE METHOD OF MEASURING LONG DISTANCES

THESIS FOR THE DEGREE OF M. S.

Stuart Hay Chamberlain

1930

Interference (Light)
Interferometer
Distances - Measurement

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I desire to express my appreciation to my teacher, Dr. C. W. Chamberlain, who suggested the solution of the problem of employing waves of light for measuring long distances, gave invaluable aid during the progress of the research, and provided most of the instrumental equipment required. I wish to thank Professor C. W. Chapman, who made possible the undertaking of this problem, for his unfailing kindness, encouragement, and assistance. I also wish to thank Professor W. E. Laycock for his valuable assistance in the preparation of photographs.

The purpose of this thesis is to outline the present status of the art of measuring distance and the limitations of the instruments and methods employed. These limitations, determined by the physical properties of the materials from which the standards of length are constructed and the finite length of the waves of light employed in all optical instruments, cannot be overcome by the methods now employed.

Advance can be made along the lines described for the first time in this thesis. The finite length of the light wave, a handicap in all instruments employing lenses, is turned to practical account by making it the unit of measurement; thus securing an ideal standard of length, possessing none of the limitations of present standards.

The thesis describes an original type of instrument, capable of measuring distances, small and great, with an accuracy and convenience of operation hitherto unattained.

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Present Methods of Measuring Distance and their Limitations

Chaining

In making linear measurements with great accuracy rods of glass or metalhave been employed; their lengths being known at a given temperature and corrections applied for variations from the standard temperature. Occasionally compensated rods made of two metals have been used. The most approved method (especially in the United States) makes use of steel tapes, which are much more convenient to use than rods. Tapes have been constructed of invar steel, with a temperature coefficient of expansion one tenth that of steel or even zero, but such alloys have a critical point at a temperature near that of the operating temperature and undergo a molecular rearrangement which causes undue growth of the metal with age. This fact, in addition to temperature hysterisis, which makes the temperature coefficient depend upon the previous history of the tape, i.e., whether the temperature is rising or falling, limits the use of invar steel.

An accurate measurement of a "base line" necessitates the supporting of the tape 60 to 100 meters long on stakes 10 to 20 meters apart. The measurements should be made at night, so as to be subject to the smallest possible range of temperature. The services of 12 men are required to make a measurement. An observer at each end of the tape, 2

thermometer observers, 3 stretchers, 1 recorder, and five men to carry lamps and bring forward the tape after each length is measured. As there is a tendency for the temperature of the tape to lag behind that of the air, the base line is measured first with a rising and then with a falling thermometer. This procedure is repeated several times in opposite directions, and a mean taken. If the ground is not level the measured length must be corrected and reduced to the true horizontal length. The measurement of distance by means of a tape is a laborious and expensive operation. A base line thus obtained is fixed and is of use for the solution of but one problem.

Traversing

In surveying by traversing, the position of a point is determined by one linear and one angular measurement, requiring the use of tape and theodolite. The accuracy of the latter instrument depends upon the accuracy of division of the circle and the resolving power of the telescope, which in turn depends on the diameter of the objective lens. The limitations therefore of a theodolite, small enough to be portable, are serious and these combined with the difficulties and expense of chaining, already described, make an improved method of measuring long distances most desirable.

Minor and Grand Triangulation

In triangulation but one lineal measurement, that of the base line, is required. From such a base line the relative distances of numerous points and their coordinates are calculated. When a survey of a great country is made, the position of a number of points at distances from 20 to 100 miles from each other. In such Grand Triangulation the most powerful instruments are employed, the work is executed with the highest degree of accuracy possible, and the curvature of the earth is taken into account. Upon the completion of the grand triangulation the topography of the country is determined by breaking up the great triangles into a number of smaller triangles, having an average side of one mile or less. Less powerfil instruments are employed and the curvature of the earth is neglected. The great cost of measuring the single base line in the grand triangulation prevents the accurate measurement of any other line in either grand or minor triangulation, although such measurements would greatly increase the accuracy of the final result. Engineering science awaits the development of an instrument and method which will make possible the use of a portable base line , the length of which has been determined with great accuracy.

The Telescope as a Distance Measurer

A telescope fitted with a special diaphragm, may be used for measuring distances with the aid of a graduated staff.

A Tacheometer is essentially a transit theodolite, the diaphragm of which is furnished with two or four stadia webs, wires, lines, or points. The object of Tacheometry is to enable horizontal and vertical distances to be computed from readings upon a stadia rod, thus rendering chaining operations unnecessary.

Let D = distance from axis of instrument to the stadia rod.

S = intercept on stadia rod.

f = principal focal length of object glass.

L = distance apart of the stadia webs.

d = distance of object glass from axis of instrument.

Then

$$D = S.\frac{f}{L} + f + d .$$

The ordinary theodolite tacheometer is limited to distances not exceeding 400 feet and in the hands of a skilled observer can determine such distances with an accuracy of one part in two thousand; approximately the accuracy of good chaining.

The Subtense Theodolite

The principle of this instrument is the same as that of the tacheometer described above, except that for each reading the cross-hairs are adjusted by means of finely divided micrometer screws to intercept some constant distance S on the stadia rod.

The same formula

$$D = S \cdot \frac{f}{L} + f + d$$

holds, but S has a constant value, usually 10 feet, while L or $\frac{f}{L}$ is variable.

The <u>Subtense Theodolite</u> is capable of measuring limited distances with an accuracy slightly greater than that obtained by a tacheometer. For distances between 800 and 1000 feet, however, the accuracy is about 1 part in 650. The accuracy of chaining over the same distance is approximately 1 part in 1440.

The Cleps

The <u>Cleps</u> was invented by Professor Porro, of Milan. It differs considerably from the tacheometer. The latter has its horizontal and vertical circles uncovered, while the Cleps encloses them in a cubical box. In the tacheometer the telescope and the angles are read by verniers; in the Cleps the telescope is eccentric and angles are read by micrometer wires applied to microscopes. The instrument was used on the Mont Cenis Tunnel Survey and is capable of an accuracy of 1 part in 2000. It is extremely delicate and its accuracy of adjustment cannot be tested.

The Omnimeter

The Omnimeter consists of an ordinary theodolite but with the graduated vertical scale replaced by a horizontal scale read by a powerful microscope mounted vertically at right angles to the telescope. The latter is focused first on one end of the distant stadia rod, then on the other.

Let X = distance from omnimeter to stadia rod.

S = length of stadia rod.

h = distance from axis of telescope to horizontal scale.

r'' and r' = readings on horizontal scale.

Then

$$X = \frac{h\,s}{r'' \pm r'} .$$

The instrument is somewhat more refined than the subtense theodolite, but does not greatly exceed it in accuracy.

Range Finding

The Telemeter

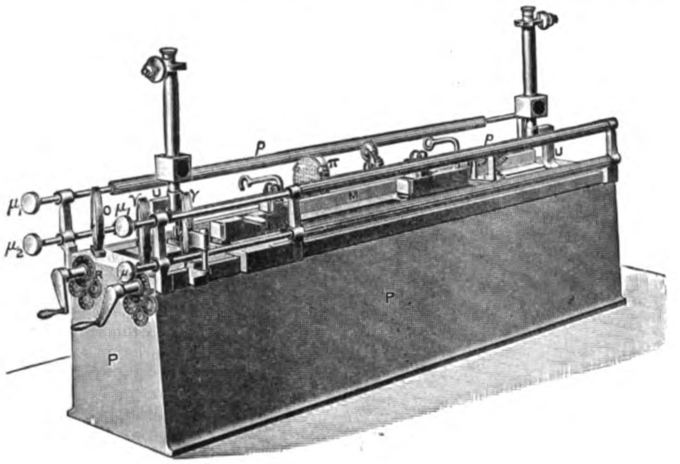
The telemeter is a small and convenient instrument, less than five inches long, operating on the principle of the sextant. The observer determines the angle subtended by the distant object at opposite ends of a base line which he "steps off". The scale is marked to read direct in feet. The accuracy is small, about 1 part in 100.

The Mekometer

The *Mekometer* consists of two instruments, similar to the telemeter, connected by a silk covered hemp cord about 150 feet long. It is very portable, rapid in use, reads direct in feet, and has an accuracy for small distances of about 1 part in 1000.

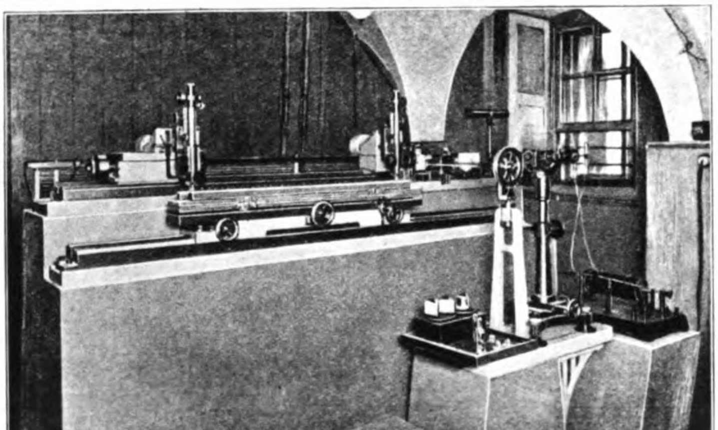
The Range Finder

As this instrument is its own base line and is about one meter long, it is somewhat awkward to use and transport. The distant object is viewed simultaneously from both ends by means of totally reflecting prisms. The varying angle subtended by objects at varying distances is compensated by moving in the line of sight of one of the two paths a prism of small angle. The movement of the prism is calibrated to read direct in feet. The accuracy of the instrument for distances up to 3000 feet is approximately 1 part in 250. For distances up to 6000 feet the accuracy is only 1 part in 125.



Michelson's Measurement of the Meter

PLATE I



Wave length Comparator at National Physical Laboratory

PLATE II

Light Waves as Standards of Length

Prof. A. A. Michelson of Case School of Applied Science and Prof. W. A. Morley of Western Reserve University first proposed the use of a wave of light as a standard of length. In 1892 - 3 the former, working at the Bureau International des Poids et Mesures, Sevres, determined the value of the meter in terms of the wave length of certain lines in the cadnium spectrum, thus affording the first satisfactory approach to a natural standard of length. He found 1,553,163.5 red wave lengths in the meter. In 1906 MM. Benoit, Fabry, and Perot, employing a more refined method reported 1,553,164.13 red wave lengths in the meter. By international agreement the length of the red cadnium wave has been made the ultimate standard of length.

Plate I illustrates the apparatus employed by Prof. Michelson. While the wave of light meets all requirements for an ideal standard of length, it will be noted that the apparatus, well suited to the laboratory, would be useless in the field or workshop.

Plate II illustrates the wave length comparator designed by Professor Tutton and used at the National Physical Laboratory of England. It is admirably adapted for comparing standards in terms of light waves, but is massive and limited to a single use.

The length of a visible light wave is not much greater than the smallest object which can be observed in the best compound miccroscope. The limit of visibility of the theoretically perfect microscope is one half a wave of light. To render light waves serviceable for purposes of measurement they must be made to interfere, i.e., the light intensity must be crowded out of a portion of the field of view, otherwise uniformly illuminated, and crowded into other portions. In such a case the field, when illuminated with monochromatic light, is crossed by alternate light and dark bands. A movement of the bands in the field over a distance corresponding to the width of one band is produced by a change in the optical path of one of the two interfering beams an amount equal to the length of a wave of light.

Conditions Necessary for the Production of Interference of Light

1. Light must be taken from a single source.
2. The light must be divided into two equal parts.
3. The two beams must be reunited at a small angle.

Methods of Producing Interference

Interference methods and interference apparatus are always concerned with two wave fronts which are parallel or nearly parallel and which are located one behind the other a distance varying from zero to a few centimeters. These two wave fronts are equivalent to light reflected from the front and back surfaces of an air wedge of corresponding thickness and angle. This will be referred to as the "air plate".

The character of the interference fringes obtainable from such an air plate depends upon the method of illumination and the method of observation. For convenience of discussion they will be divided into five classes.

Class I. If the air plate is illuminated by a broad source of monochromatic light and enters the eye, placed at a convenient distance, the rays which reach the eye from different points of the air plate are reflected from the surfaces of the latter at varying angles. The path difference between the rays reflected from the two surfaces at any point - $2\ n\ t \cos r$, where r is the angle of incidence on the back surface of the air plate, n the index of refraction, and t the thickness of the plate. The fringes will be the loci of points for which $2\ n\ t \cos r$ is constant. They indicate variations of n, t, and r, jointly or separately.

When the air plate is extremely thin, and t is very small, the difference of phase due to the variation of r at different points of this air "film" may be negligible, so that if the optical thickness $n\ t$ is uniform the fringes will be very broad and the air film will appear uniformly dark or bright all over, as the air film is made to increase or decrease in thickness. If either n or t vary, fringes will appear which are the loci of equal optical thickness.

When the thickness of the air plate is appreciable the change of phase due to the varying angle of incidence becomes effective and for all air plates except very thin films the fringes are loci of points from which the light reaching the eye meets the plate at equal angles of incidence. With an air plate of uniform thickness the fringes are circles concentric with the normal from the eye to the air plate and are in focus at infinity. If the air plate or film is wedge shaped the fringes are arcs of circles whose centers are displaced from the normal. If the surfaces of the air film are not regular, or if the air is not homogenous, causing local variations of n t, the circular fringes will be locally distorted at these points. Such irregularities are most noticeable near the center of the system where the fringes are broadest. With any but the thinnest air films the fringes are closely packed as the angle of incidence increases and are visible only in neighborhood of normal incidence.

It is important to note that fringes of Class I are formed when light corresponding to each point in the fringe system reaches the eye from separate points (or very small regions) of the film, and is reflected from the film at varying angles. Except for extremely thin films, the fringes are mainly loci of equal incidence angles, and are very slightly affected in shape and position by variations in film thickness.

Class II. Instead of viewing the fringe system at infinity with the naked eye, one may use a telescope of large aperture focused for infinity, so that for all points of the field of view light from the whole surface of the air film is received by the telescope. The fringes are now purely loci of equal illumination and are not affected in shape or position by variations in the optical thickness of the film. When the telescope is directed normally to the film circular fringes concentric with the axis are seen in the focal plane of the eyepiece. Since at any point in this plane, light from the whole of the film is focused, the phase at any such point depends on the <u>mean</u> film thickness and the angle of incidence of the ray. Any variations in film thickness contribute their effect equally to all points in the fringe system, which therefore indicates the inclination of the rays only, and tells us nothing about the flatness, homogeneity, or parallelism of the air film.

If, however $n\ t$ varies much, the distinctness of the fringes will be impaired; for we can regard the resultant illumination at the focal plane of the telescope as due to the superposition of a number of exactly similar fringe systems, of which some differ in phase from others. This results in diminishing the contrast between the bright and dark parts of the field.

It is important to note that fringes of Class II are

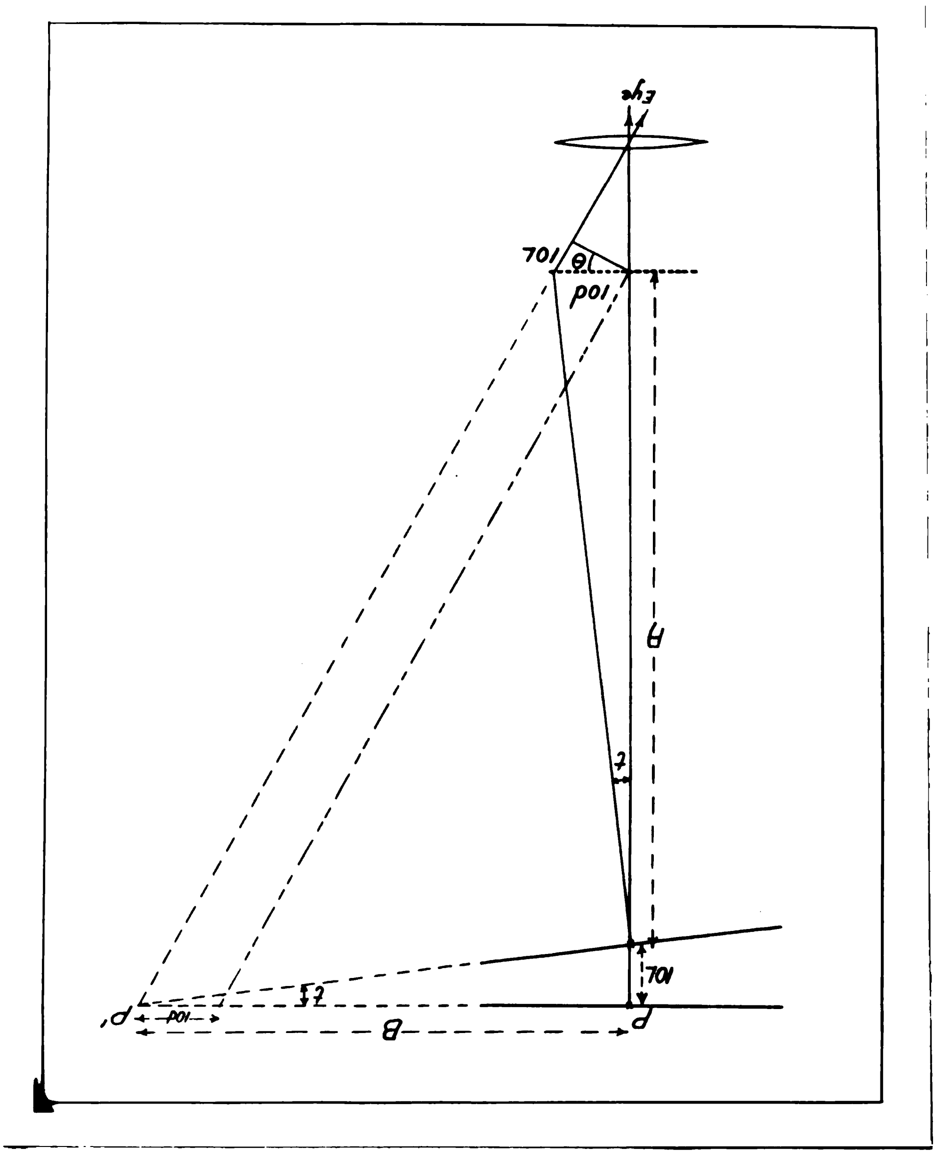
formed when light corresponding to each point of the fringe system comes from the whole of the film (or from some part of it), but meets it at varying angles of incidence. The fringes are bands of equal illumination. If sharp and distinct we infer that the film is of fairly uniform optical thickness over the region utilized; but no information of the character or position of any variations which may actually be present can be gained.

Class III. If the source is a point located at infinity, such as a small illuminated pinhole at the principal focus of a collimating lens, all rays strike the air film at the same angle of incidence. There are no variations in phase from one part of the film to another except such as may be due to variations of $n\,t$. To observe a fringe system of Class III it is necessary to employ a telescope with the eyepiece removed. The object glass collects the parallel rays after they leave the air film and produce an image of the source in the focal plane. If the eye is placed at this image, the whole surface of the air film is seen illuminated and traversed by fringes which are true contours of the optical thickness $n\,t$.

It is important to note that fringes of Class III are not located at a definite distance from the eye, like those of Classes I and II. They are visible at <u>all</u> distances, and appear to coincide with any surface on which the eye is focused. If the film is thick the fringes will be most distinct at normal incidence. A theoretical point source is not available. To secure secure brightness the hole must have an appreciable area, and its image in the focal plane of the telescope lens may be considered a small portion of the Class II fringe system produced by a broad source. If this portion is in the center, where the phase varies slowly with the angle of incidence, there will be no phase difference between the light from one part of the pinhole and another; but if it is at an outer part of the system where the Class II fringes are closely packed, the small cone of rays which reach the eye from any point of the air film comprises an appreciable range of phase retardation, and the contour fringes are rendered indistinct.

The interferometers designed by Young, Fizeau, Rayleigh, Michelson, Barus, and Tutton, all employ interference fringes of the three classes described above, which are generally known as Fizeau fringes. In 1910 Dr. C. W. Chamberlain designed the Compound Interferometer with which he accurately measured magnitudes 200 times beyond the limit of the compound microscope. The fringes employed are compound Fizeau fringes.

A thorough study of all known types of interference led to the conclusion that none were capable of solving the problem of measuring a distance along a beam of light. The Director of the United States Coast and Geodetic Survey, as late as 1920, seemed to have reached the conclusion that no such solution was possible and sought to improve the method of Grand Trianulation by measuring longer base lines over rough terrain and undertook, at great expense, the measurement of a base line from a point on Mt. Wilson to Mt. San Antonio peak as a distance over which Prof. Michelson should make an accurate determination of the velocity of light, "realizing that it might lead to the determination of distance in terms of the velocity of light and thus might furnish a means of measuring base lines in mountainous regions or on archipelagoes". As the velocity of light is 186,420 miles per second the above suggestion would involve the accurate measurement of exceedingly small intervals of time, and does not seem to offer much promise, but indicates the pressing need for better methods of measuring large distances.



The Diffractometer

PLATE III

A New Type of Interference : Fourth Class
The Diffractometer

Any type of interferometer employing Fizeau fringes is most useful when illuminated with monochromatic light. Only five or six fringes can be observed in white light, when the air film is wedge-shaped and the optical paths are equal. In 1911 Dr. C. W. Chamberlain invented an instrument called the Diffractometer, which combines an interference with a diffracting or refracting system. Plate III is a line drawing illustrating the principle of the instrument. M and N are the rear and front faces of the reflecting air film of a Fizeau interference system, inclined at an angle t. The difference in optical path is 10 waves of light, or 10L. Let M and N produced meet at P', at a distance B from P, the foot of the perpendicular from the point of observation to the mirror M. Place a grating at a distance A from N, and parallel to N, so that 10 grating spaces lie between the point where the ray from M meets the grating and where the ray from N meets it. To an observer, looking through the grating at an angle t, the retardation of 10L produced by the grating in the light from N brings the two beams into phase and interference in white light destroyed at P is restored at P'. As the difference in optical path of the two interfering beams is increased, the fringes may be restored at P' by increasing the distance A. <u>The Diffractometer produces white light</u>

fringes at the intersection of the surfaces M and N. Draw a line from the intersection of the ray from M with the grating, making an angle with the grating equal to the diffracting angle θ.

Then

$$\tan t = \frac{10\ L}{B}$$

and

$$\tan \theta = \frac{10\ d}{A} .$$

Neglecting quantities of the third and higher orders

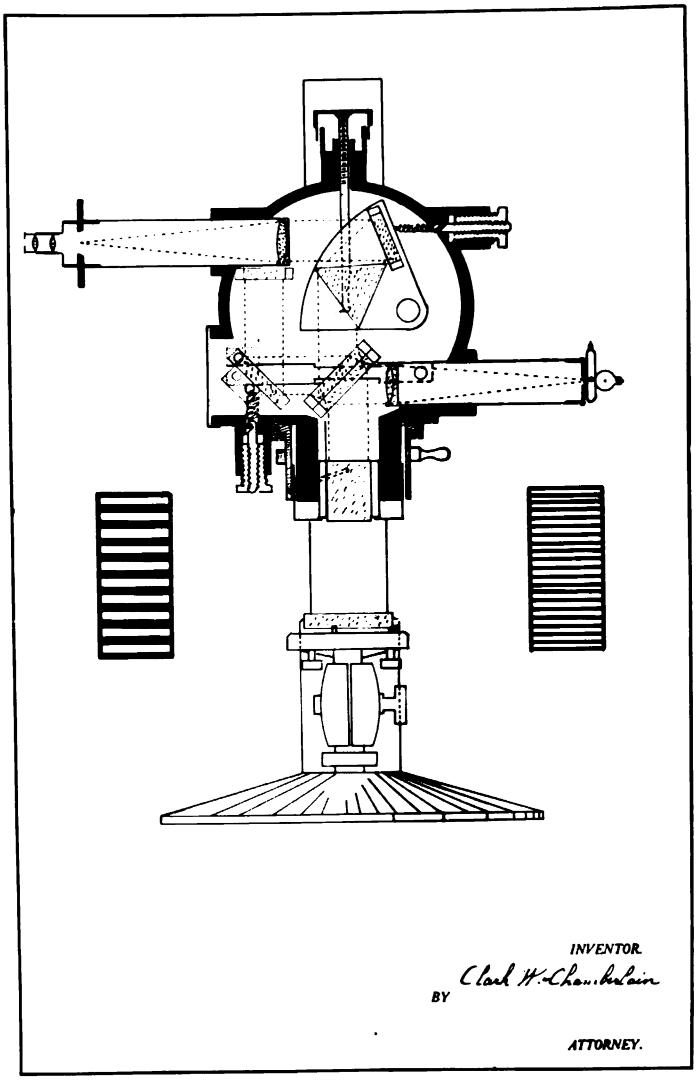
$$\tan t = .943\ \frac{10\ d}{A} .$$

Therefore

$$A = B\frac{d}{L}\ .943 .$$

As d and L are known, A can be found if B can be determined. This is accomplished by counting the fringes in monochromatic light as the intersection is moved back from P' to P, and measuring the width of a fringe. The product of the fringe width by the number of fringes equals the distance B.

The Diffractometer fringes are exceedingly useful in locating the white light fringes in the process of adjusting interferometers. As an instrument for measuring distance it will be noted that the angular width of the fringes is small when the distance A is large, and large and expensive plates are required for the measurement of great distances. The Diffractometer is not the best solution of the problem but is of great interest as the first successful attempt to



The Recording Interferometer

PLATE IV

measure distance along a beam of light.

A New Type of Interference : Fifth Class
The Recording Interferometer

This instrument was developed at Michigan State College by Dr. C. W. Chamberlain and represents a very great advanve in the science of precision measurement. It was described before the American Physical Society in February, 1930, and is at the writing of this thesis in the process of manufacture by the Bausch and Lomb Optical Co., Rochester, N. Y.

Plate IV shows the arrangement of optical parts. Light from a broad slit, illuminated by an incandescent filament, is collimated and divided into two parts which travel separate paths, one variable in length. The beams are reunited, with their wave fronts parallel, passed through a Wadsworth or constant deviation prism, and viewed through a telescope focused for infinity. With eyepiece removed circular Fizeau fringes are seen in monochromatic light, when the interfering paths are unequal. With the eyepiece in position, a new type of interference system in white light is brought into view. The continuous spectrum is crossed by sharp dark bands whose positions in the spectrum correspond with the wave lengths for which the interference system is opaque, or the path difference is an odd number of half wave lengths. The number of bands appearing in an octave is identical with the path difference of the interfering beams, measured in terms of the longest wave. Lines appearing in that portion of the spectrum between 5016 and 6678 A. U. of helium correspond to a linear movement of one ten-thousandth of a centimeter. In a similar manner, a range of spectrum may be arranged such that a change of one line in the range corresponds to a movement of one twenty-thousandth of an inch, or any desired decimal part of a centimeter or inch.

Calibration of the Recording Interferometer

The Recording Interferometer is calibrated to read exact decimal parts of an inch or centimeter without the use of standard gauges or other auxillary means, This feature makes the instrument reliable and convenient. The sensitiveness of the instrument is variable from one millionth to one thousandth of an inch: a single instrument taking the place of an entire set of instruments of any design now used. The movement of the plunger, the anvil, or the interferometer head changes the relative lengths of the interfering paths of light.

Let e = movement of plunger.

then $2e$ = change of optical path.

If $2e$ is an odd number of half wave lengths

$$2e = L(N - \tfrac{1}{2})$$

the two beams will interfere destructively, and a black band will appear in the continuous spectrum at the position of the wave length L. The optical system is opaque for this particular wave length, which does not enter the prism but is reflected back toward the source. There will be other wave lengths for which $2e$ is exactly some odd number of half wave lengths which will also be absent in the spectrum. Vertical dark bands will cross the spectrum wherever the wave length is such that $2e = L(N - \frac{1}{2})$, where N is any integer. If the wave lengths corresponding to any two of these bands are known, the wave lengths of <u>all</u> the rest of them can be calculated as follows:

Let L' and L'' represent the two known wave lengths; L is the wave length to be calculated; also let n', n'', and n be their corresponding integers in the above formula.

Then $\quad 2e = L'(n' - \frac{1}{2})$, or $2e/L' = n' - \frac{1}{2}$ $\quad$ (1)

and $\quad 2e = L''(n'' - \frac{1}{2})$, or $2e/L'' = n'' - \frac{1}{2}$. $\quad$ (2)

Subtracting (2) from (1)

$$2e(1/L' - 1/L'') = n' - n'' . \qquad (3)$$

Similarly we may derive an equation

$$2e(1/L - 1/L'') = n - n'' . \qquad (4)$$

Dividing (4) by (3)

$$\frac{1/L - 1/L''}{1/L' - 1/L''} = \frac{n - n''}{n' - n''}$$

and simplifying

$$1/L(n' - n'') - 1/L''(n' - n) + 1/L''(n - n'')$$

$$\frac{1}{L} = \frac{n' - n}{n' - n''}\frac{1}{L''} + \frac{n - n''}{n' - n''}\frac{1}{L'} . \qquad (5)$$

In applying this formula $(n' - n)$ is obtained by counting the bands between L and L"; similarly (n' - n") is the number of bands between L' and L". The number of bands, and therefore the accuracy of calibration, can be increased by increasing the difference of path between the interfering bands.

To calibrate the instrument to read .00005 of an inch, the appearance or disappearance of one band in the field should indicate a movement of the moveable jaw of the interferometer of .00005 inch. The helium tube may be conveniently employed.

Helium red	= .0000667815 cm.	or	.00002629 in.
Helium yellow	= .0000587562 cm.	or	.000023132 in.
Helium blue	= .000050477 cm.	or	.000019873 in.
Helium blue	= .0000501567 cm.	or	.00001975 in.

Placing the left jaw of the shutter eyepiece on the red helium line, the problem is to find the exact location for the right jaw of the shutter such that a change of one band in the field indicates a movement of .00005 inch.

Red:	Unknown:	Blue:
L" = .00002629	L = ?	L' = .00001975

When the plunger moves .00005

the optical path changes 2(.00005) = .00010

Number of waves passing at L" $= \frac{.0001}{.00002629}$.

" " " " " L = 1 + " .00002629

(no. of waves)x(length of wave) = distance moved by plunger

$$L\left(1 + \frac{.0001}{.00002629}\right) = .0001 .$$

Computing the above

$L = .000020819$ inch.

Having found the length of the light wave which locates the correct position of the right hand shutter, this wave length is accurately located in the spectrum in the following manner:

Suppose the optical paths are such that 50 bands lie between L'' and L. (100 bands, or any other number, may be used.) Employing equation (5)

$$n - n'' = 50$$

$$n' - n'' = 50 - X$$

$$n' - n = X$$

$$\frac{1}{L} = \frac{X}{50 - X} \cdot \frac{1}{L''} + \frac{50}{50 - X} \cdot \frac{1}{L'} .$$

Solving for X

$$X = 50 L'' \frac{(L - L')}{L'(L'' - L)} = 13 \text{ bands.}$$

We next set 50 + X = 63 bands between the red and blue helium lines, then set the left shutter on the red line and the right shutter on the 50th band. <u>The field of view will now read accurately in terms of .00005 of an inch</u>.

Further calibration is carried out in a similar manner, leaving the left shutter set on the red line and in succession setting the right shutter

		Number of light waves pass by	Number of bands pass by
on band 62 1/2	instrument reads .00004 in.	2 waves	4 bands
on band 50	instrument reads .00005 in.	2.5 waves	5 bands

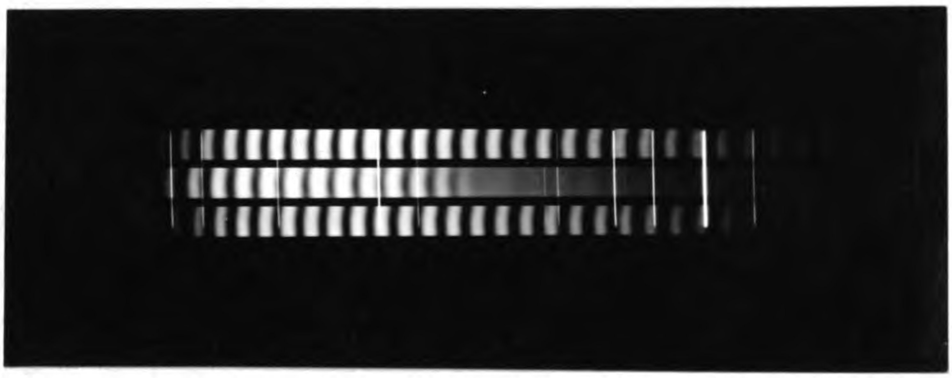
continued:

on band 41 2/3	instrument reads	.00006	in.	3 waves	6 bands
on band 31 1/4	instrument reads	.00008	in.	4 waves	8 bands
on band 25	instrument reads	.0001	in.	5 waves	10 bands
on band 12 1/2	instrument reads	.0002	in.	10 waves	20 bands
on band 8 1/3	instrument reads	.0003	in.	15 waves	30 bands
on band 6 1/4	instrument reads	.0004	in.	20 waves	40 bands
on band 5	instrument reads	.0005	in.	25 waves	50 bands
on band 4 1/6	instrument reads	.0006	in.	30 waves	60 bands
on band 3 1/8	instrument reads	.0008	in.	40 waves	80 bands
on band 2 1/2	instrument reads	.001	in.	50 waves	100 bands

The accuracy of setting the smaller ranges may be increased by increasing the number of bands employed. For example, having determined that the range 12½ bands equals .0002 of an inch, the number of bands in this range may then be increased to 50. If so, the number of bands fixing all the ranges following will then be multiplied by 4.

The eyepiece of the Recording Interferometer will be constructed with a fixed left hand shutter. The right hand shutter will move by a screw with divided head to give the above or any other desired readings.

For laboratory measurements of the highest precision the Recording Interferometer surpasses any other previous design of interferometer, as a movement of ½ wave of light may cause one band to move across the entire visible spectrum, and measurements accurate to 1/100 of a light wave can be made.



Photograph of Recording Fringes

PLATE V

The chief value of the instrument lies in its ability to leave a record of the measurement, and for shop purposes to skip a selected number of bands in the record, thus recording every fourth, tenth, fiftieth, or hundredth band. In shop use, having determined the amount of "tolerance" allowed, the Recording Interferometer may be set to read in terms of that tolerance. The operator would then set a small number of bands in the field of view. The allowed tolerance would then admit of an increase or decrease of one band.

As the Recording Interferometer renders unnnecessary the counting of bands during actual movement, the record may be photographic, from which observations may be made at any time under the most favoraole conditions. This allows the use of the comparator for determining exact whole and fractional number of bands within a given range.

Plate V is a photograph taken of recording fringes which shows the permanent record of a small movement of the mirror M. The upper portion shows the spectrum record of recording fringes before movement and the central spectrum the fringes after a small movement of the mirror M. The lower spectrum duplicates the position of M in the upper spectrum, and constitutes a check on the former. The helium spectrum is superimposed on the three and records a check and makes possible the calculation of the distance moved by the distane mirror M.

recording photographically with <u>any</u> degree of sensitivity at any predetermined intervals of small displacements, opens up whole new avenues of research in many fields of science.

Measurement of Distance by Means of Light Waves

Due to the difficulty encountered in the direct measurement of very small angles, the present practice is to measure off a base line as long as existing conditions will permit. This has led to the practice of constructing high platforms to carry tape line over such common natural obstructions as irregularities in the terrain, highways, orchards, etc. If a short, portable base line is to be used for the accurate determination of great distances, the length of this base line must be known with a high degree of accuracy requiring direct measurement of this length in wave lengths of light. Moreover,equally accurate optical methods must be devised for determining the very small angle subtended by this short base line at great distances.

An ideal tape line may be considered to possess the following advantages: It should be of unlimited length, though at the same time remaining readily portable. Its graduations should be closely and accurately spaced, and capable of being read to greater or lesser degrees of precision. Its weight should be zero for there should be no sag, eliminating the necessity of supports and stretching to secure tauntness. Its temperature coefficient of expansion should be zero and it should have no critical point near operating temperatures. Irregularities of temperature along its path should have no effect upon its accuracy in use. The effect of windage, stretching, and the curvature of the earth, should be eliminated. Best of all, fixed at one end, it should be capable of extending <u>itself</u> rapidly over inaccessible places and of returning the free end to the observer for reading.

Such an ideal tape line fulfilling all these conditions is formed when two beams of light are sent out from a source. With the velocity of light they are projected in an absolutely straight line to the distant point from whence they may return by reflection to their starting point; accurately calibrated in terms of the absolute and unvarying standard, there is no drop of the catenary, stretch, windage, or temperature coefficient. This ideal tape requires no supports and extends itself. The limiting distance it may be sent out becomes, in the end, the limit of visibility. The paths the two beams travel are not affected by curvature of the earth's surface, beyond that due to the changing index of refraction of the air through which the two beams must travel.

Before two beams of light may be utilized for measuring large distances, a number of very serious difficulties must be overcome. Were the two beams to travel parallel to each other, a change in distance will change the

the two paths exactly the same amount, leaving no optical difference which could be measured. If the two beams are caused to diverge,at great distances their optical paths will be subject to individual disturbances which will affect one beam and not the other. This difficulty may be obviated by causing the two beams to be only slightly separated and converging slightly. In this case, most of the way they travel nearly identical paths, and any slight differences due to localized inequalities of temperature tend to integrate out over long paths. It is well to remember that a beam of monochromatic light, though accurately subdivided, carries no numbers on its divisions and hence no division can be identified unless laboriously determined from the central white light fringe. Practicality demands that at great distances intense white light be used, but only a few white light fringes, eight or ten at the most, are known. In this case the use of the diffractometer fringe suggests itself. But when fringes in monochromatic light , a definite distance apart, let us say, of three fringes to the width of an interferometer end mirror, are set on an interferometer end mirror and viewed through a telescope and grating from a great distance, the angle subtended by an ordinary interferometer plate is so small that the individual fringes in focus on its surface are not resolved. This necessitates the use of large and costly plates and telescopes of high resolving power. When the image of the central white light fringe is located in some diffracted order of the grating spectrum, it is necessary to employ monochromatic light and laboriously count back fringes, occasionally substituting white light to relocate the central fringe, until the central white light fringe stands in the center of the field of view. It is difficult to accurately determine the width of fringe, so the distance traveled by the white light fringe in this process is only approximately known. In this method the base line and the observer are located at opposite ends of the distance to be measured, the difficulties of accurate counting of fringes increasing with the distance. The Diffractometer represents the first, and up to 1929 the only known, method of measuring distance along a beam of light by means of interference, but the expense and difficulty of operation limit its field of application.

The fact that the new fifth type of white light fringe, employed in the Recording Interferometer, cannot be used alone to solve the problem may be explained in the following way: The two converging beams after crossing may be thought of as forming the base and hypotenuse of a right-angled triangle. As the crossing point of the two beams moves farther away, due to a movement of the distant source, the leg of this triangle grows longer. If an interference system at the receiving end is to reunite these two diverging beams, then as the base increases the leg increases proportionally, or stated in another way, the beam representing the hypotenuse of this triangle will fall a little farther back and off to one side of that point on its mirror where it was originally incident, as the distance to be measured increases. This will result in a shift, or shearing, of the interfering wave fronts laterally, and laterally only, which, due to their very nature, produces absolutely no change in the appearance of a recording fringe system. But it is the change in length of the leg of this triangle which we wish to measure, since it is the change in length of the base line subtended by a fixed angle at this increased distance and is to be used in measuring this change in distance due to the movement of the source. The distance required of this mirror to be moved back is just that amount necessary to keep its beam always on some fixed point of its mirror, or in other words, to again reduce the shear to zero. Thus it becomes evident that, since recording fringes are not sensitive to a shearing of the interfering wave fronts laterally, they cannot be used alone to solve the problem. However, it is also evident that if some other entirely independent method were devised for detecting when, by a movement of this mirror backwards, the beam again fell on the same point, the retardation in this beam could be measured by the recording fringes in white light.

Statement of the Problem

1. To find a method, preferably an interference one, for determining when a returning beam falls always on some fixed point of its mirror in the receiving apparatus. This amounts to the problem knowing when the two interfering wave fronts are sheared on each other laterally by <u>zero</u> amount.

2. To find an interference method, using white light, for measuring the change in base line subtended between two beams of light, with fixed angle between, at varying distances.

3. To find two such interference methods <u>wholly independent of each other</u>, for an adjustment of the apparatus must not introduce in the recording fringe system a change not due to a movement of the distant source, nor must the measurement produce a change in the adjustment.

4. To use an interference method in calibration of the instrument.

Problem Solved by a Combination of Fizeau and Recording Fringes

The circular, or straight line, interference bands in monochromatic light, first described by Fizeau and used in most interference systems, form a most excellent method for determining when a beam of light falls on some fixed point of its mirror. In monochromatic light when zero shear exists, the eye sees the center of the Fizeau Circular Fringe system, hereafter described as the F. C. F. system. A shearing of the interfering wave fronts <u>laterally</u> exposes a new portion of the F. C. F. system. The radii of the circles is determined as follows:

Let

$2t$ = distance between interfering wave fronts

r = radius of successive rings

P = distance from front surface of air plate to point of observation

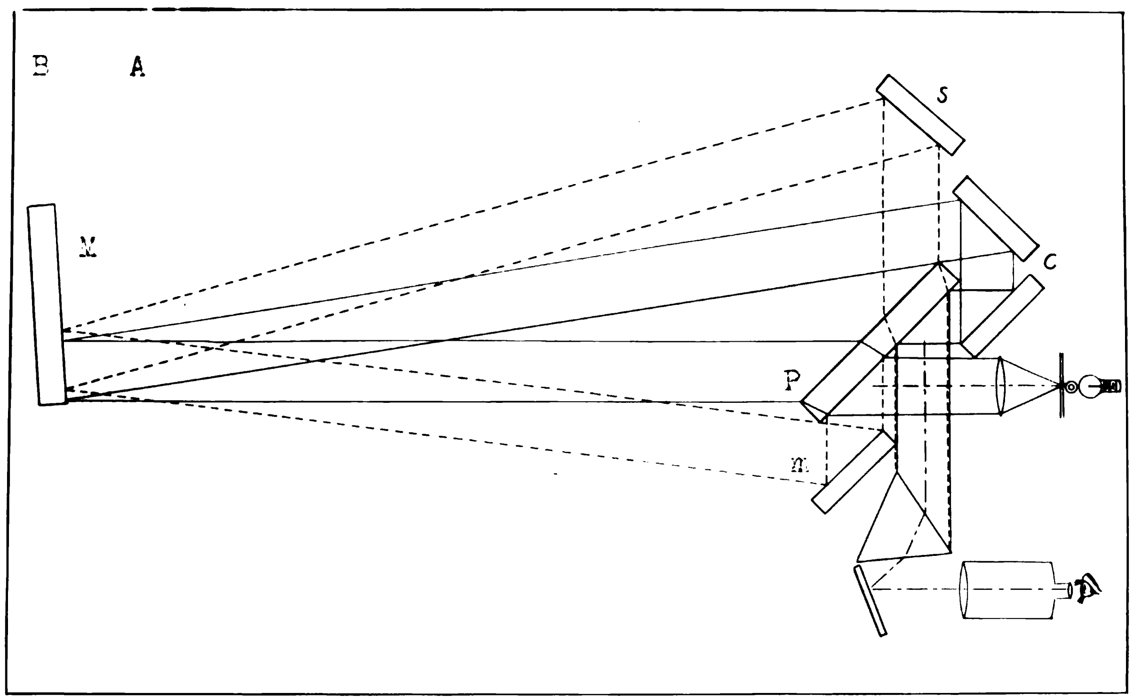
n = number of ring; any integer

L = wave length of light

then

$$r = P\sqrt{\frac{nL}{2t - nL}}$$

If a perpendicular passed through two congruent points on the air plate is not the pole P, we are not looking at the center of the system of circles, but off to one side. Due to the small size of the ordinary interferometer plate only sections of the larger rings are seen at any one time, and appear curved toward the center of the system. The effect is exactly the same if congruent points are sheared on each other. The location of the center of the F. C. F. system is extremely sensitive, and watching for the center of this system of circles in monochromatic light, the setting of a mirror so that a beam falls always on some fixed point may be reproduced with great precision. When a setting has been made for zero shear <u>before</u> <u>and</u> <u>after</u> <u>the</u>



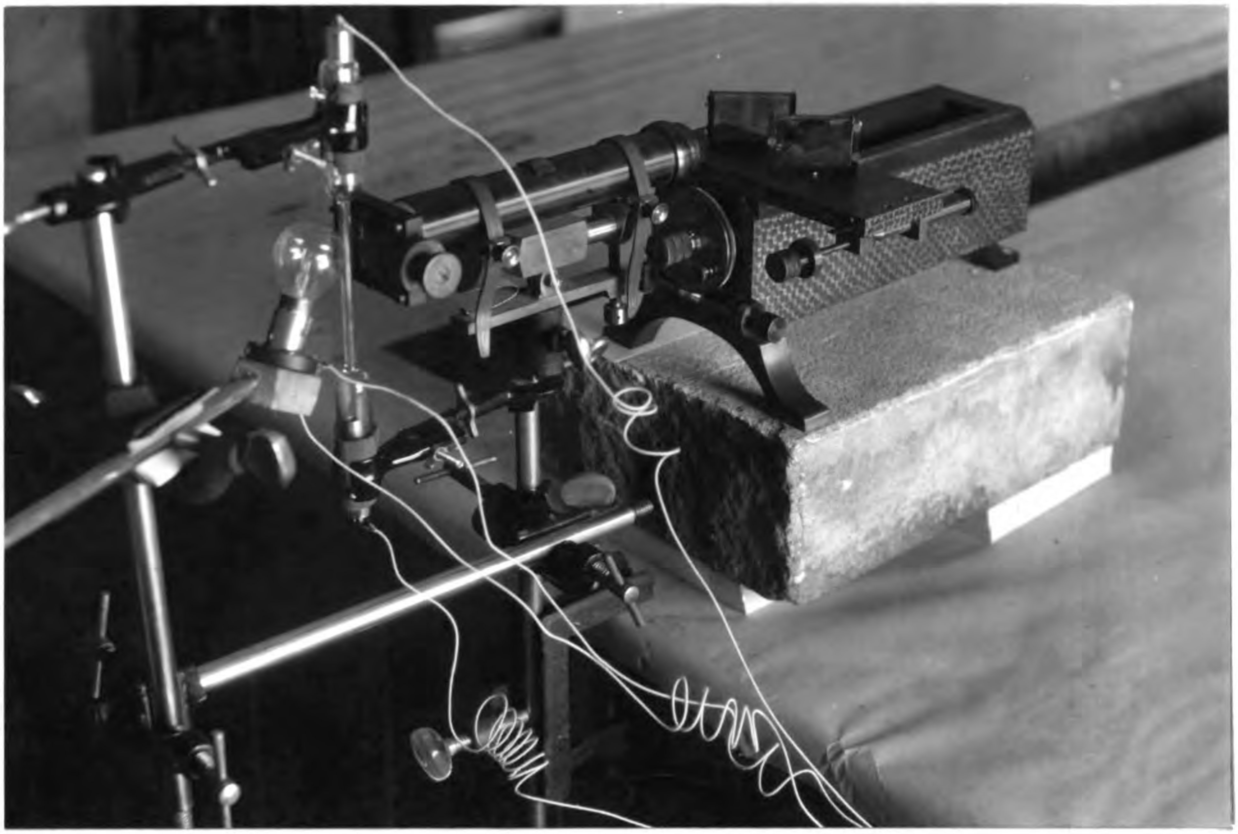
Interference Method of Measuring Distance
Along a Beam of Light

PLATE VI

change in distance, the difference in path of one beam over the other resulting from the change is the change in base line subtended by this change in distance, and is readily measured by the recording fringes in white light.

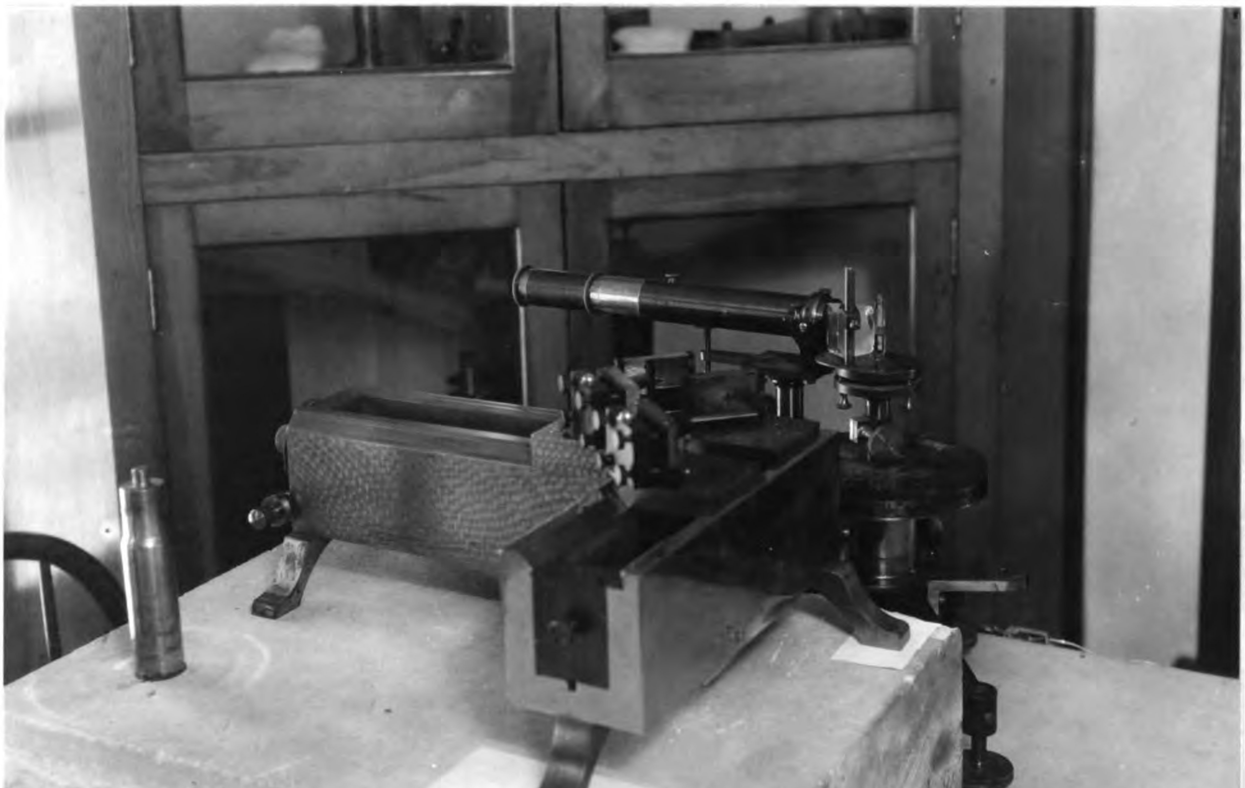
Description of Instruments

Plate VI is a line drawing illustrating the arrangement of optical parts and shows the paths of the interfering beams of light. Light from a broad slit illuminated with white light is first collimated, then by means of a 50% silver film on a plane parallel plate of glass is divided into two parts; a transmitted and a reflected portion. The latter beam falls upon a full-silvered surface from which it is reflected in such a manner as to cause the two beams to be slightly separated and converging at a slight angle toward a distant optical surface M full-silvered on the front face; the two beams crossing at or just before they strike the mirror. The returning beams, now slightly diverging at the same small angle, are received by an interference system which reunites them, renders them parallel, and permits both linear and lateral displacements of the two interfering wave fronts. The two beams thua reunited pass through a Wadsworth (or other constant deviation prism) and are viewed through a telescope, which during the entire process of adjustment and measurement remains focused for parallel rays. When in adjustment, and when the paths traveled by the two interfering beams are nearly equal, an eye placed at the telescope sees a white light spectrum crossed by a number of dark interference bands of the Recording Interferometer type. Superimposed on these distinct bands may be caused to appear the red, yellow, and blue, bright lines of helium from a discharge tube placed immediately before the slit for the dual purpose of acting as a cylindrical lens to focus white light in the slit and to furnish monochromatic light both for adjusting the instrument and for conveniently locating the range between which the recording fringes are to be counted. The S mirror moves on ways which are at right angles to the ways along which moves the carriage on which the two C mirrors are mounted. Consider for a moment that the S beam falls always on some fixed point of the S mirror. As the distance of the receiving station from the distant mirror increases, due to a movement of the distant mirror M, parallel to itself, from A to B, the S mirror must be moved back parallel to itself in order that the light may fall on the same fixed point of this S mirror. After a displacement of the distant mirror M no such movement is ever required of the mirror C as its beam always strikes the same point on it. The distance moved by the S mirror represents the increase in length of a base line subtended by the two diverging beams resulting from a movement of the distant mirror. This increase in length of the S path over the C path is compensated for by a movement of the C carriage along its ways a distance one-half the distance moved by the S mirror. The motion of the carriage on which the C mirrors are mounted, resulting from turning the divided head which turns the the calibrated screw which drives the carriage, is measured directly in wave lengths of light. In order that the change in position of the C mirrors may not affect the accuracy with which the setting of the S mirror is made, it is essential that the method for making the adjustment of the latter be entirely independent of the movement of the C carriage and mirrors. This is accomplished by using the Fizeau Circular Fringe system for setting the S mirror. With a fairly broad slit the intense yellow line of helium, appearing in the spectrum viewed through the telescope, forms an excellent monochromatic source for locating the center of this fringe system which is in focus at infinity. When the center of the system is located, white light is substituted and the C carriage is moved backwards or forwards until some definite number of recording fringes appear between the red and blue lines of helium flashed at intervals. A movement of the S mirror made necessary by a movement of the distant mirror by an unknown amount, shears the interfering wave fronts on each other laterally only, exposing a new portion of the F. C. F. system. When the S mirror, hereafter called the "shear" mirror, is moved back to the point where the center of the F. C. F. system reappears, the increase in the S path over the C



Transmitting Apparatus : 1st Arrangement

PLATE VII



Receiving Apparatus : 1st Arrangement

PLATE VIII

path represents the increase in base line subtended between the two beams at the increased distance. In white light the number of recording fringes will be seen to have increased with the displacement of the S mirror. The C mirrors, hereafter called the "compensator" mirrors, are moved back an amount necessary to reduce the number of recording fringes within the range to their original number. The C mirror moves a distance necessary to compensate for the increase in the S mirror path. The amount of movement of the carriage of the compensator mirrors along its ways in wave lengths is one-half the increase in shear mirror path, which is one-half the increase in base line subtended at the increased distance resulting from the movement n B - A. The distance moved by the carriage of the compensator mirrors is determined either directly from the recording fringes, or indirectly from the screw and divided head, which drive the compensator mirrors' carriage, which are calibrated in wave lengths of light.

Description of Apparatus

Plates VII and VIII show the first arrangement of the experimental apparatus. Plate VII shows the transmitting apparatus and Plate VIII shows the receiving apparatus, located at opposite ends of the distance to be measured. The compensator system of mirrors must be two in number in order that there may be as many reflections for the C

Final Transmitting and Receiving Apparatus

PLATE IX

beam as for the S beam. Otherwise, as the reflection takes place from a silver surface, they could not be caused to finally interfere. It is inconvenient to have the adjustments of the transmitting apparatus out of reach of the observer stationed at the receiving apparatus, especially since it is found necessary that the two plates which carry the dividing and reuniting films be adjusted to exact parallelism.

Plate IX shows the final experimental arrangement in which a single large plane parallel plate serves for both dividing and reuniting the two beams, thus eliminating the most difficult and objectionable adjustment of the first arrangement. At the opposite end of the distance to be measured, instead of placing the receiving apparatus, a plane mirror is substituted, which returns the two beams to another point on the same plate from whence they started; here they are reunited and pass through the remainder of the optical system exactly as before. In this arrangement all adjustments, save that of the distant mirror are within close reach of the observer. It is the intention that the "rodman" carry the mirror to which is mounted a telescope, adjusted so as to sight along a normal to the mirror. In this case, after the mirror is firmly mounted the rodman sights the telescope on the distant source, which brings the returning beams into correct position. The apparatus is mounted on a rotating table which enables the operator to take a measurement in any direction without readjusting.

Method of Making Measurements
Adjustment of Interference System

Plate VI shows the slit of the collimator illuminated by a small incandescent lamp and a helium discharge tube. The latter does not interfere with the former but serves rather as a cylindrical lens to focus the light of the incandescent filament on the slit. This arrangement makes it possible to view a line spectrum or to superimpose it on a continuous spectrum or view the latter alone.

For long distances a small tungsten arc lamp is recommended. The intensity is high and the lamp is easy to operate. Two small spherical tungsten electrodes are sealed in a bulb filled with gas of low ionizing potential. When an e. m. f. is applied the gas is sufficiently ionized to start the discharge, soon bringing the spheres to incandescence and maintaining a tungsten arc. A slit illuminated by such a source produces in the field of view a line spectrum between two continuous spectra; an ideal arrangement for making measurements.

Referring to Plate VI, the mirrors C, M, m, and S, optically flat to 1/10 wave of light, are heavily silvered on their front faces and polished. The plate P, with plane and parallel surfaces figured to an accuracy of 1/20 of a light wave, is silvered on the face toward m and C with a 50% film. The beam of light which is to be made to interfere is divided into equal parts; 50% is transmitted to the mirror M and 50% is reflected to the mirror m. As already explained, the plate P serves both as a dividing and reuniting plate, making it possible to secure <u>optically</u> equal paths, i.e., the two beams travel equal paths in air and glass. The beam which is first transmitted is finally reflected, and the reflected beam is transmitted. A failure to secure a film which will <u>exactly</u> divide the incident beam produces no serious results as is shown by the following table:

TABLE I

% reflected by thin film	10	40	50	60	90	50	40	10	50	50	50
% reflected transmitted ray	.009	.024	.025	.024	.009	.05	.048	.018	.10	.15	.225
% transmitted reflected ray	.09	.24	.25	.24	.09	.25	.24	.09	.25	.25	.25
% reflected ray reflected by end mirror	.10	.10	.10	.10	.10	.20	.20	.20	.40	.60	.90
% transmitted ray reflected by end mirror	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
% of total interference	.018	.048	.05	.048	.018	.1	.096	.036	.2	.3	.45
% of fogging light	.081	.216	.225	.216	.081	.2	.192	.072	.15	.10	.025
Ratio of interfering to fogging light	450	450	450	450	450	200	200	200	75	33	.051

From the table on the preceding page it will be noted that:

1. The percent of fogging, which should be a minimum, depends upon the relative intensity of the light reflected by the pair of mirrors m and S and the pair C C, and not upon the thin film.

2. Paired reflecting films of 10 and 90%, 20 and 80%, 40 and 60%, etc., produce the same results.

3. The maximum interference and minimum fogging are secured with a 50% thin film and end mirrors of equal reflecting power.

The plane parallel plate P and the mirrors C, C, m, and S, are adjusted perpendicular to the same plane and parallel to the vertical axis of the instrument. The angle between m and P, always small, depends upon whether the instrument is to be used for measuring small or great distances. The mirror M is mounted on an optical bench in the same plane and at a distance of one meter from the vertical axis. A rough adjustment will bring the transmitted reflected and reflected transmitted beams into the field of view.

By adjusting the mirrors C or S the two images of a single line in the spectrum may be made to coincide and Fizeau fringes will appear in monochromatic light. The field of view may now be enlarged if necessary by widening the slit.

Setting on a Distant Mirror

Fizeau fringes are segments of concentric circles whose center should be brought into the field of view. This is easily done by moving the S mirror so as to move the Fizeau fringes in the direction of their concavity. They will widen as the center of the Fizeau system approaches the field of view. Next by moving back the C mirrors, as the optical paths approach equality the recording fringes begin to appear in the continuous spectrum, at first very fine and in great numbers, their number indicating the distance yet to be moved.

Small movements of the shear mirror S and compensator mirrors C will reduce to zero the number of Fizeau bands in the monochromatic field and the recording fringes in the continuous spectrum. When the number of both types of fringes is reduced to zero the setting on the distant mirror M is complete, and the reflected transmitted beam falls on the shear mirror S at a point which can be duplicated with great accuracy. In actual practice it has been found preferable not to reduce the number of recording fringes to zero, but to work with a fixed number, three or four, in the field. Whether the fixed number is positive or negative is instantly determined by applying a slight pressure to the carriage of the C mirrors; a plus reading being indicated by a fringe movement toward the red end of the spectrum, and a minus reading by a movement toward the blue.

Calibration of the Instrument

The instrument is first set on the mirror M at a known distance from its vertical axis - say 1 meter. The mirror M is then moved a meter farther along the optical bench and adjusted to return the transmitted reflected beam to the field of view. This will cause the reflected transmitted, or shear, beam to fall on another point of the shear mirror S, which must be moved farther back. The mirror S is correctly set in its new position when the Fizeau bands in the monochromatic field are reduced to zero, or their original number. The correct setting on the mirror M in its second position <u>lengthens</u> the shear path. <u>This is the change in base line to be determined</u>. <u>It is found recorded by the recording fringes in the continuous spectrum</u>.

With a fixed angle between P and m, hereafter called collectively the "wedge", the recording fringes measure the movement of the mirror M. Then

$$\tan a = \frac{n\,L}{D}$$

where a = angle of the wedge

n = number of recording fringes added

L = length of longest wave in the range.

D = distance moved by mirror M.

A permanent wedge is secured by grinding a hollow quartz cylinder, approximately one inch long, and with a diameter equal to the aperture of the mirror M. It is not necessary to grind the quartz cylinder to any exact angle.

It is ground at random and the angle determined in the following way: With the plate P and mirror m clamped against the ends of the quartz cylinder, the diverging beams are directed toward the mirror M at a distance of one meter from the axis of the instrument (in this case the axis of the table). Suppose that 100 fringes are recorded when the mirror M is moved one meter farther away. If L is .00005 cm.,

$$\tan a = \frac{100 \times .00005}{100} = \frac{1}{2000}$$

Measuring a Long Distance

The wedge having been calibrated, it is reversed in the instrument, which is now ready to measure long distances. It will be noted from the photograph of the instrument that the linear separation of P and m is approximatelt 2.5 cm. With the wedge reversed the beams will converge and then diverge. As the angular separation between S and C is 2.5 cm., the mirror M cannot be placed closer than the crossing point of the rays, which in this case is 500 meters. Shorter distances can be measured with a wedge of larger angle.

When the mirror M is placed at a greater distance, the shear mirror S must be moved farther back, each recording fringe between the red and blue of helium indicating a movement of the distant mirror M by one centimeter. If the short base line of the instrument is limited to 20 cm., one wedge measures distances from 500 up to 4000 meters with an accuracy at the 4000 meter limit of 1cm. in 400,000.

A second wedge of 1/8th the angle will record 12½ fringes for a movement of one meter. The interfering beams converging from such a wedge will meet 4000 meters away, and the same short base line will measure distances from 4 to 36 kilometers with an accuracy of 8 cm. One instrument furnished with 4 wedges will measure all distances from a few meters to 100 kilometers with an accuracy hitherto thought impossible.

A greater degree of calibration of the wedges can be attained by moving the mirror M along the testing optical bench a distance greater than one meter. Such optical benches, constructed from standard steel railroad rails are part of the regular equipment of many engineering laboratories.

Data

Two test wedges were constructed in the laboratory and measured, with the following results:

TABLE II

Wedge No. 1 : Diverging

Number of observation	1	2	3	4	5
Number of Fizeau fringes	3	3	0	4	4
Distance to mirror M	600	600	600	600	600
Distance moved by mirror M	5	5	10	15	15
Number of recording fringes	16.4	16.3	32.7	49.	49.1
Movement of shear mirror S	.003073	.003054	.006128	.009183	.009201
Angle of wedge	.000615	.000611	.000612	.000612	.000613

Mean angle of wedge - .0006126 radian.

TABLE III

Wedge No. 2 : Converging

Number of observation	1	2	3	4	5
Number of Fizeau fringes	5	5	5	5	5
Distance to mirror M	600	600	600	600	600
Number of recording fringes	21.4	42.8	42.7	64.1	64.2
Movement of shear mirror S	.004010	.008021	.008002	.012012	.012031
Angle of wedge	.000802	.000802	.000802	.000801	.000802

Mean angle of wedge - .000802 radian.

In the above measurements the recording fringes were noted between the red and green lines of helium. These lines are 4861 and 6563 Angstrom Units respectively. Using the formula developed on page 20 the movement of the shear mirror corresponding to a change of one recording fringe in this range of spectrum equals

$$\frac{L' L''}{L' - L''} = \frac{.00006563 \times .00004861}{.00006563 - .00004861} = .0001874 \text{ cm.}$$

Employing wedge no. 1 the addition of one recording fringe in the field of view corresponds to a movement of the mirror M of 0.305 cm., and in the case of wedge no. 2 of 0.236 cm. With an interferometer base line limited to 20 cm., wedge no. 1 can measure distances up to 323 meters, and wedge no. 2 to 250 meters.

No added difficulties present themselves in the construction of wedges of smaller angle and greater range. No attempt was made in this direction as it would appear that the practicability of the method had been established. All measurements made in connection with this thesis were small enough to admit of checking by direct measurement.

Conclusion

The present methods of measuring long distances have been carried to their limit, and leave much to be desired.

A new method of measurement has been devised, tested, and found to be acceptable. It combines the use of Fizeau interference fringes and a new type devised by Dr. C. W. Chamberlain, and called by him recording interference fringes. The instrument can be calibrated in the laboratory in light waves with great precision, and is thus independent of subsidiary standards of length.

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