THE RADIATION PATTERN OF CIRCULAR APERTURES

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

Robert Edgar Houston Jr.

1951

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THE RADIATION PATTERN OF CIRCULAR APERTURES

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Robert Edgar Houston, Jr.

has been accepted towards fulfillment
of the requirements for

M.S. degree in Physics

Robert H. Noble
Major professor

Date 13 September 1951

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THE RADIATION PATTERN OF CIRCULAR APERTURES

by

Robert Edgar Houston Jr.

A Thesis

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Department of Physics

1951

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A problem such as this requires the help of many persons. To this end I wish to thank Dr. R. D. Spence, Mr. Lee Bell, and my wife Barbara Houston. Special appreciation is due Dr. Robert H. Noble for his invaluable suggestions and direction toward its successful completion.

Robert E. Houston Jr.

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I. INTRODUCTION

Any discussion of the pattern created by an incident plane wave on a circular aperture immediately brings into focus three cases of particular interest. Kirchoff's integral and boundary conditions enable a prediction of the general aspects of the pattern provided the radius of the aperture is large compared to the wave-length of the incident wave.[1] Should the radius of the aperture be small in comparison with the wave-length of the incident wave, the general features of the pattern may be explained by the Rayleigh-Lamb method.[2]

It is well known, however, that neither of these methods pretends to give the exact solution when the wave-length is of the same order of magnitude as the radius of the aperture. Recently Bethe[3] has developed a theory which may be extended to cover this case. Bethe's theory has the advantage of being vectorial in character. Since an electromagnetic field is essentially vectorial this would seem to suit the conditions of the problem more closely. The theory depends upon the use of fictitious magnetic charges and currents in the diffracting aperture. The conclusion drawn clearly shows that the field near the edge of the aperture will tend toward infinity. This is the problem to be investigated experimentally.

The overall approach to the problem was to construct equipment which would allow an electromagnetic field detection probe to be small in comparison with the wave-length and yet retain definite limits to the physical size of the apparatus.

Andrews[4,5] has made a very through investigation of the diffraction pattern of apertures in a manner similar to the method used in this paper. However, he made no close study of the edge effects of the aperture.

II. THEORY

Development of electromagnetic radiation in the microwave region led to theoretical considerations of the effect of a small hole in a cavity upon the oscillation of that cavity. The solution of this problem is valid for a circular aperture in a perfectly conducting infinite plane screen. The theory was developed for the case of a hole small in comparison with the wave-length. Extension of the theory to the condition where the wave-length and radius of the aperture are the same order of magnitude has not been completed. However, the type of solution to be expected is indicated.

Previous theoretical methods[1,2] in the treatment of diffraction phenomenon are inadequate in a problem involving electromagnetic radiation. This is immediately apparent since an electromagnetic field is essentially vectorial in character while the previous theories were scalar in form. A theory involving a vector formulation has the distinct advantage of fulfilling the divergence conditions:

$$\mathrm{div}\ \vec{E} = \mathrm{div}\ \vec{H} = 0 \qquad (1)$$

This is identical with saying that the vectorial theory gives transverse waves in the wave zone which would not necessarily be the case in the scalar form.

In his approach to the solution Bethe finds an expression which satisifies Maxwell's equations as well as certain of the boundary conditions. To fulfill the remainder of the boundary conditions and to allow for the simplest solution the concept of fictitious magnetic surface charge and density in the aperture was introduced. The magnetic and electric fields in the aperture are then given by the expressions

$$H_{aperture} = -\frac{2}{\pi}\frac{\vec{H}_0 \cdot \vec{r}}{(a^2 - r^2)^{1/2}} \tag{2}$$

$$E_{aperture} = \frac{1}{\pi}\frac{\vec{r}}{(a^2 - r^2)^{1/2}}\vec{E}_0 + \frac{2}{\pi}(a^2 - r^2)^{1/2}\,\vec{n} \times \vec{H}_0 \tag{3}$$

where n is the unit vector in the direction of the inward normal to the surface, a is the radius of the aperture, E_0 and H_0 are known field components, and r is the varying radius of the aperture. E_0 and H_0 are usually of the same order of magnitude and consequently as $r \rightarrow a$, $E_{aperture}$ approaches infinity. It is quite apparent that $H_{aperture}$ approaches infinity as $r \rightarrow a$. Hence it is to be expected that at the edge of an aperture the field will tend toward infinity. Such a result is shown experimentally in this dissertation.

The theory has not been extended to include apertures of the order of magnitude of the wave-length. However, the method of solution has been indicated and the $(a^2 - r^2)^{-\frac{1}{2}}$ dependency remains as an integral part of the solution.

III. EXPERIMENTAL PROCEDURE AND EQUIPMENT.

The experimental equipment consisted of a source of electromagnetic radiation, a cylindrical parabolic reflector, a plane conducting screen containing the aperture, and a detection system.

The circuit diagram for the source of electromagnetic radiation is shown in figure 1. The power supply contained a rectifier tube and two 'L' filter sections. This reduced the ripple in the output voltage to a satisfactory level. The operating values were: input voltage, 110 volts; oscillator plate current, 65 milliamps at 250 volts; and oscillator grid current, 8 milliamps. The oscillator and power supply are shown in figure 2.

In the oscillator circuit there were two Westinghouse 368A.S. door-knob tubes in a push-pull arrangement with tuned grid and tuned plate circuits. A push-pull oscillator produced more output than could have been attained by use of the same tubes in a parallel arrangement. Tuning of plate and grid tank circuits resulted in an additional increase in power output at the desired frequency.

The wave-length of the oscillator was set for 50 cm. This wave-length was chosen because it represents a compromise between the two conditions

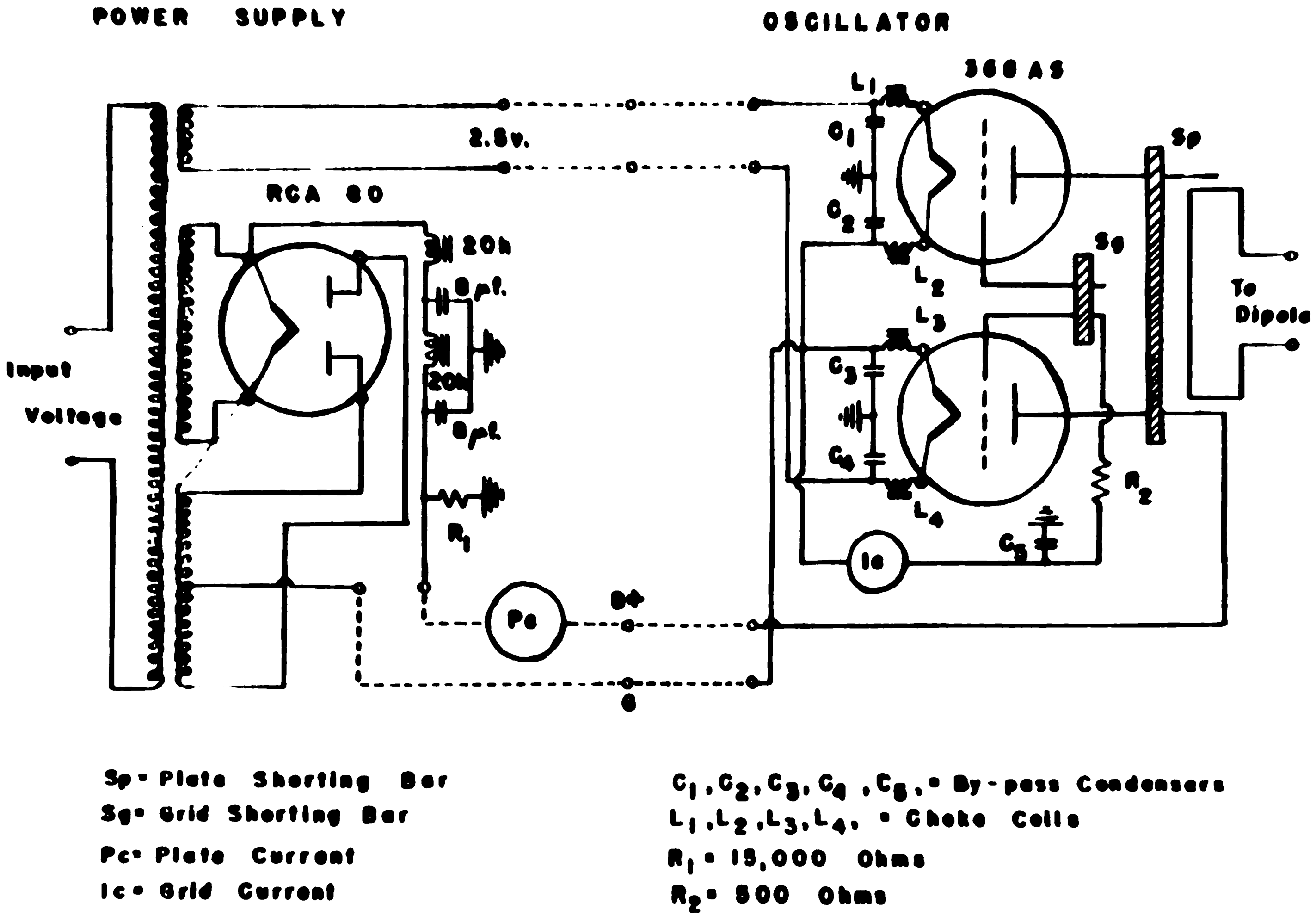

POWER SUPPLY
OSCILLATOR
368 AS
RCA 80
2.5v.
20h
8 pf.
20h
8 pf.
R_1
Pc
B+
L_1
C_1
C_2
L_2
L_3
C_3
C_4
L_4
Ic
C_5
R_2
Sg
Sp
To Dipole
Input Voltage
Sp = Plate Shorting Bar
Sg = Grid Shorting Bar
Pc = Plate Current
Ic = Grid Current
C_1, C_2, C_3, C_4, C_5, = By-pass Condensers
L_1, L_2, L_3, L_4, = Choke Coils
R_1 = 15,000 Ohms
R_2 = 500 Ohms

Figure 1

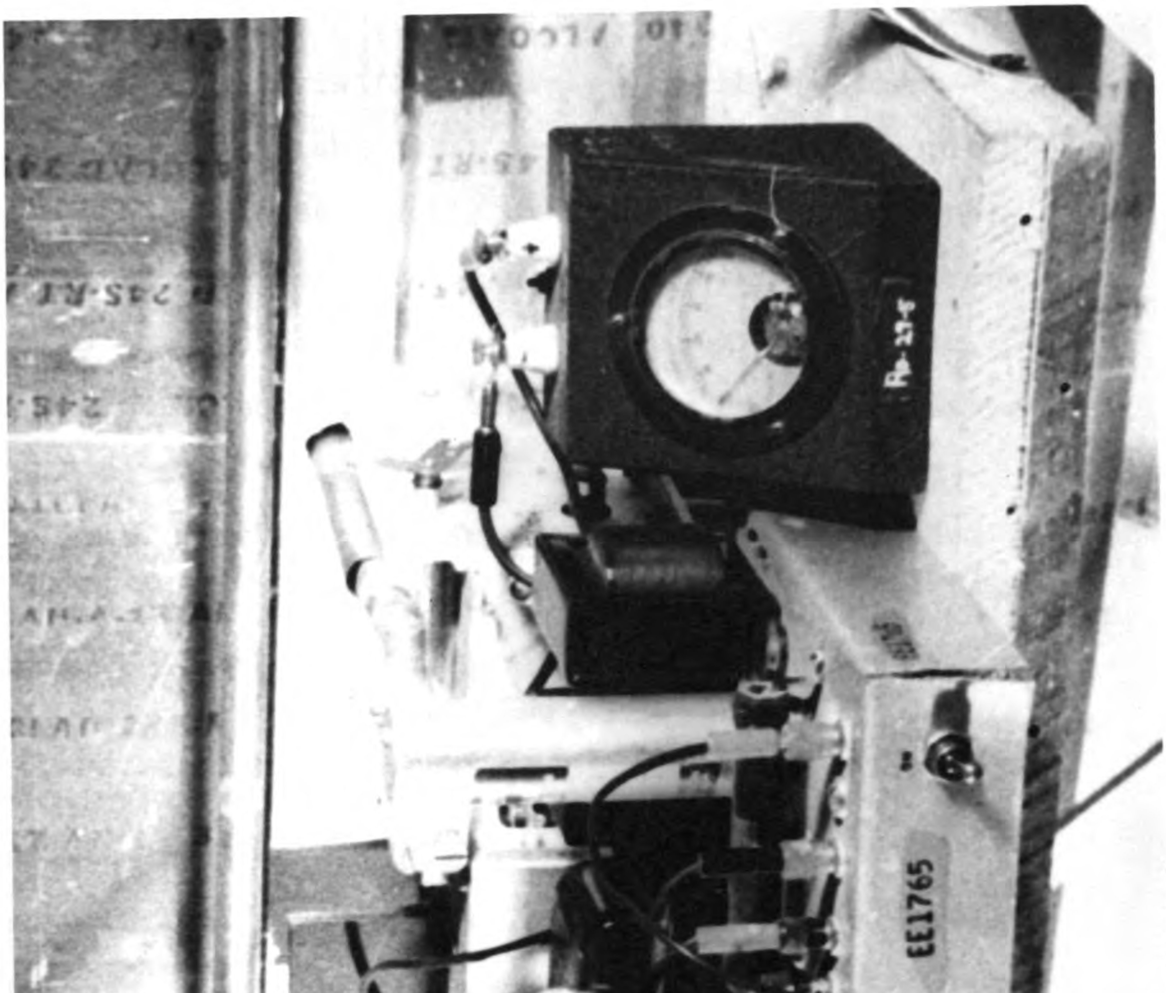


Figure 2

which must be satisified. Certainly the equipment could not be so large that moving and handling could not be accomplished easily by a few persons. Secondly, to insure meaningful results the detector of the probe circuit had to be small compared to the wave-length. This insured that the effect upon the radiation pattern would be negligible. A wave meter was utilized to check the frequency and from the well known relation

$$f\lambda = c \qquad (4)$$

the wave-length was determined. A system of standing waves could also be utilized in a wave-length calibration.

The oscillator output was connected by coaxial cable to a half-wave dipole antenna located at the center of the parabolic reflector. The coaxial cable ran through a brass tube one-half inch in diameter which supported the antenna.

Radiation from the antenna alone was not sufficiently powerful to be detected with the available equipment. Hence it was necessary to construct a reflector to direct most of the energy to the conducting screen as well as produce a plane wave. Due to the radiation pattern of a dipole antenna a cylindrical parabolic reflector presented no more distortion than that of a paraboloidal reflector. This choice also enabled a greatly simplified construction. A view of the reflector is shown in figures 3 and 4.

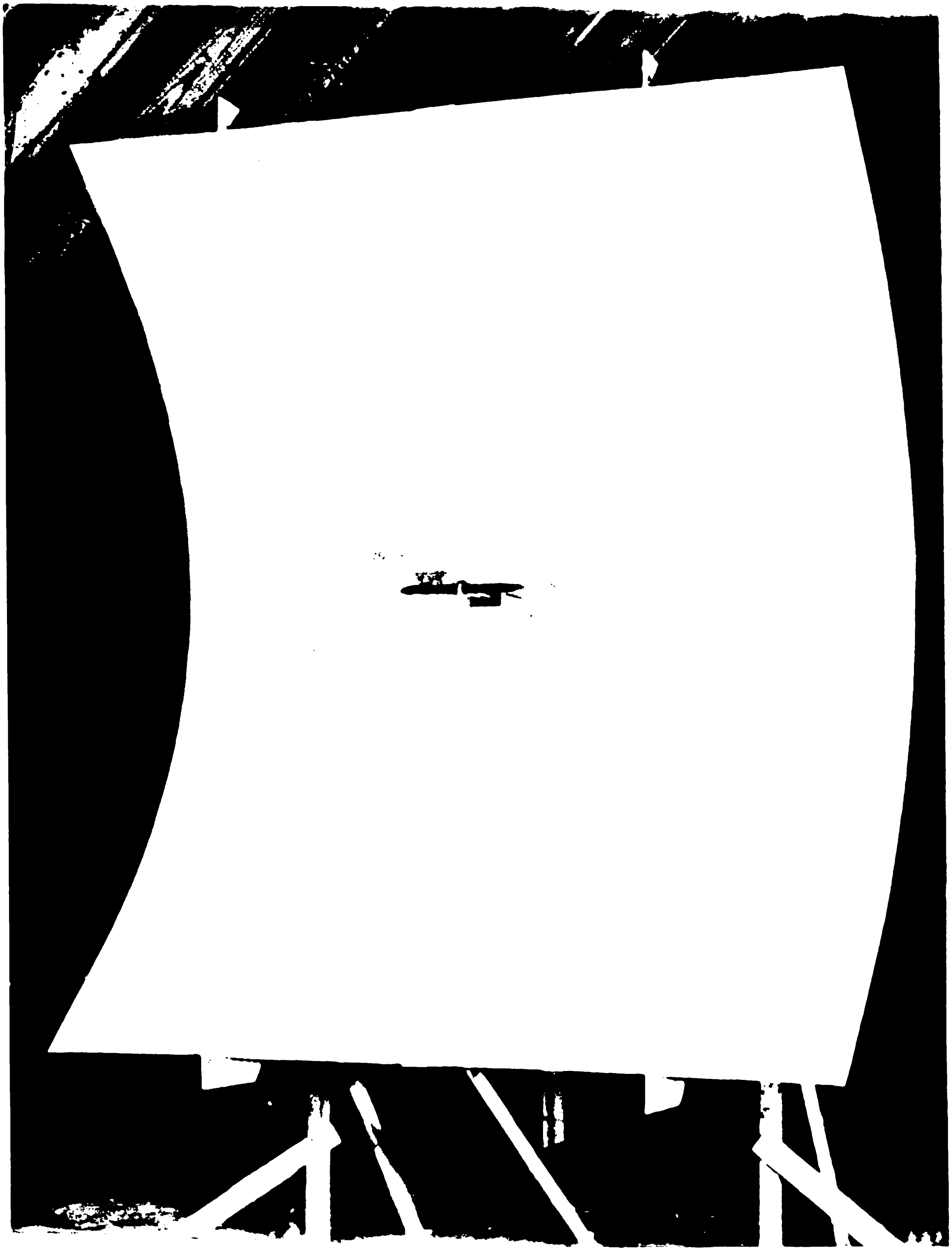


Figure 3

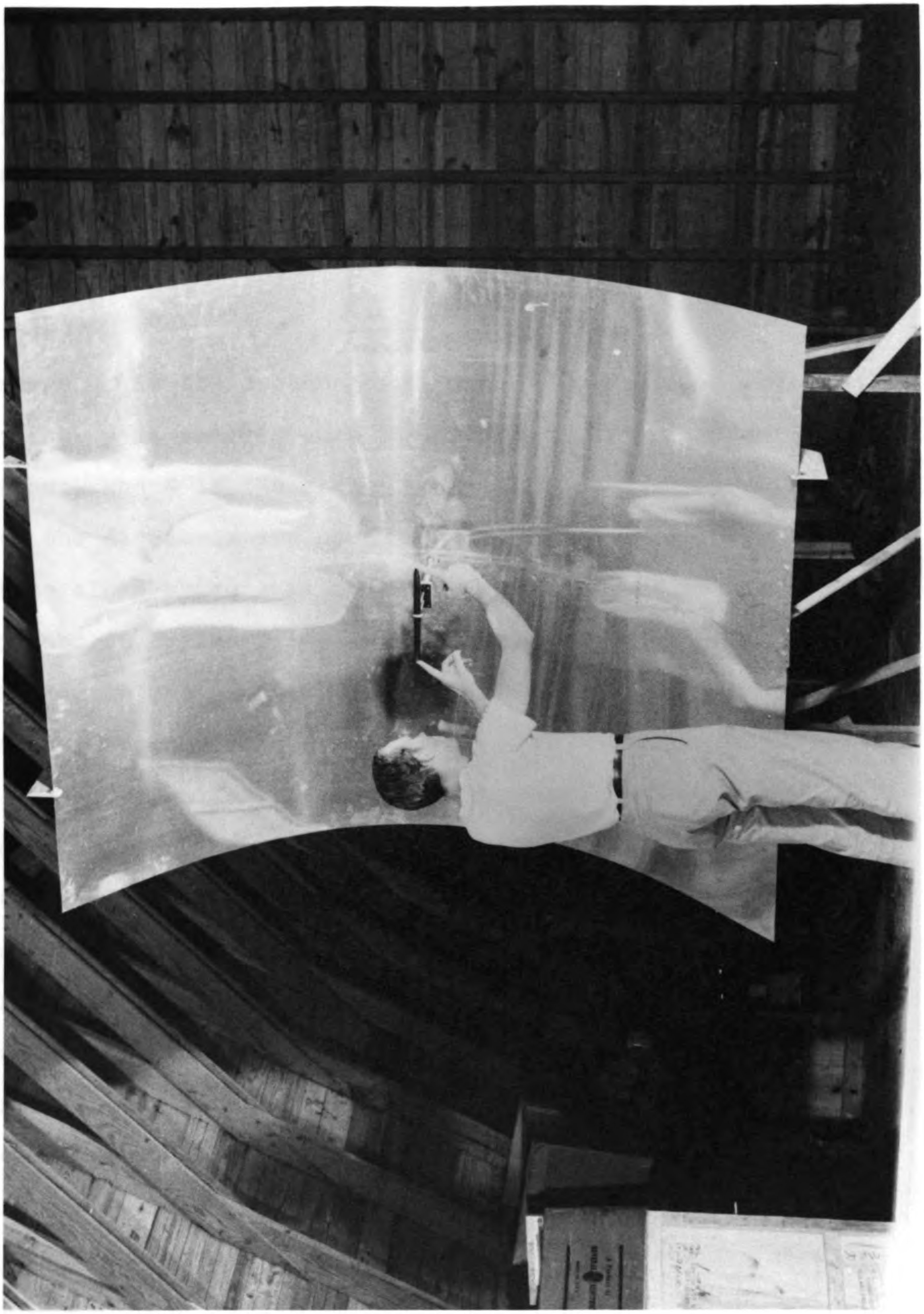


Figure 4

The reflector size was dictated by the fact that energy loss is great if the reflector dimensions are less than four times the wave-length.

The parabolic curve was plotted from the polar equation

$$\rho = \frac{p}{1-\cos\theta} \qquad (5)$$

where ρ is the distance from the focus to the plotted point, p is the distance from the directrix to the focus, and θ is the angle formed by the line perpendicular to the directrix and passing through the focus with the line along which ρ is measured. The surface of the parabolic cylinder was one-sixteenth inch aluminum fitted to the parabolic wooden frame. Details of construction of the frame can be seen in figure 5.

One half-wave dipole constructed by the author and a commercially built antenna were utilized in the experiment. Due to matching problems between the oscillator output and the dipole more power was available from the commercial antenna. Consequently, the commercial antenna was used exclusively to obtain the final data although the shape of the patterns obtained varied only a few percent when the results of each antenna were compared.

The frame of the conducting screen, which contained the aperture, was formed by four pieces of lumber 1"x2"x 12' and some diagonal bracing as shown in figure 6a.

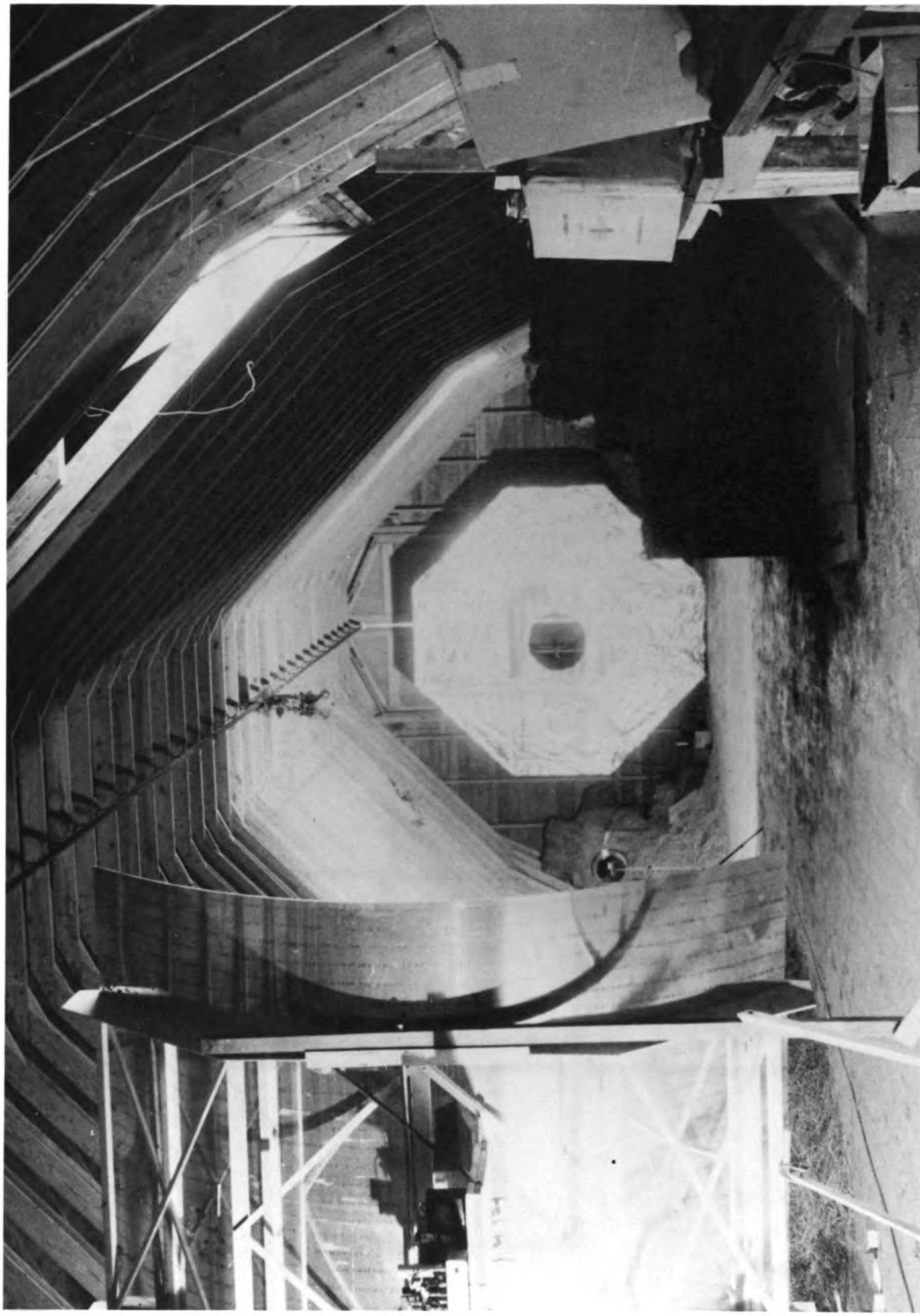


Figure 5

The hinged section allowed for removal from the place of construction. Binder twine was run around the edge of the frame to add stability and to provide support for the covering. The center of the frame contained a composition material $\frac{1}{2}$"x4'x4' with a hole in the center slightly larger than 100 cm. in diameter. Hence the covering could be given rigidity around the aperture and assurance that the aperture was planar resulted. A schematic diagram of this is shown in figure 6b.

The covering of the frame consisted of strips of ordinary wrapping paper glued together to form a single sheet about 12'x12'. Then strips of aluminum foil were laid on this with one-half inch overlapping joints perpendicular to the joints of the wrapping paper. The aluminum foil was overlapped to provide a coupling between the strips. The entire surface was rolled smooth for two reasons. The first was to reduce interference in the aperture due to reflection from surface unevenness and the second to try and make contact between adjacent strips of aluminum foil. It wasn't absolutely necessary to make contact due to the overlap of the strips of foil. The overlap allows a capacitative coupling between each particular foil strip and consequently the current induced in one strip would not be restricted but could flow to any other section of the screen. Capacitative coupling has a $1/\omega c$ dependency where ω is 2π multiplied by the frequency of oscillation and c is the capacitance. Obviously at a frequency of 600 megacycles/second coupling losses were very small. The screen is shown in figure 7.

The different size apertures were obtained by placing a one-sixteenth inch sheet of aluminum with the desired size aperture over the 100 cm. aperture in the screen. Obviously this thickness does not approach the desired 'infinitely thin' category to the extent of the aluminum foil. However, compared to the wave-length it provides a good approximation of an infinitely thin screen.

One serious difficulty occurred with the larger of these apertures. They were constructed of a soft aluminum sheeting which had been rolled rather tightly. Despite every effort they resisted becoming exactly a plane but deviated from this ideal by a few percent.

The detection system consisted of two parts; (1) the probe circuit and (2) the physical means of moving the probe in the plane of the aperture. The antenna of the probe circuit was a 1N23A crystal. This was connected by a twisted pair of fine #36 wires to a galvanometer as in figure 8. Part (2), also shown in figure 8, consisted of an optical bench with a 1"x2"x 8' piece of wood mounted vertically on two

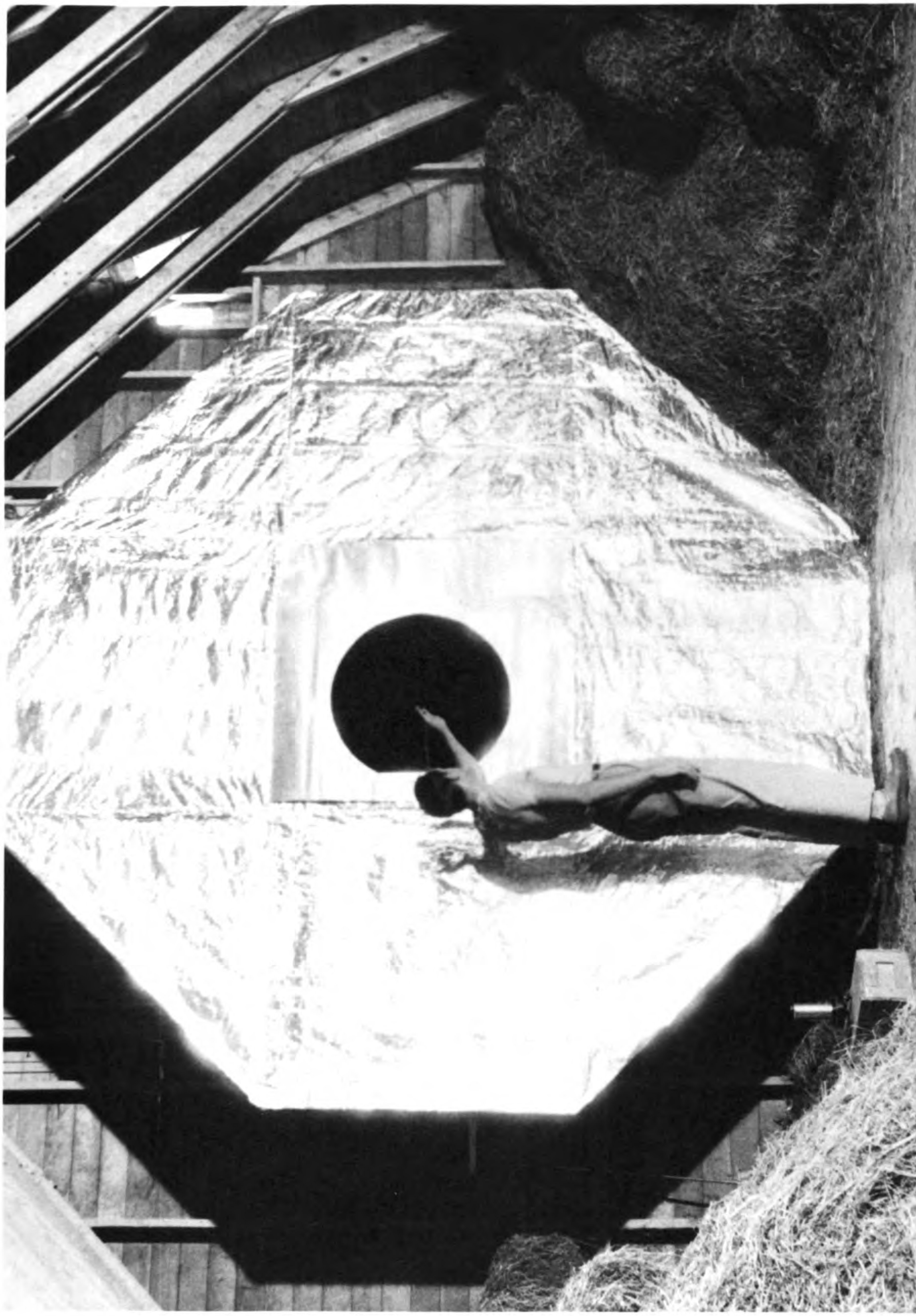
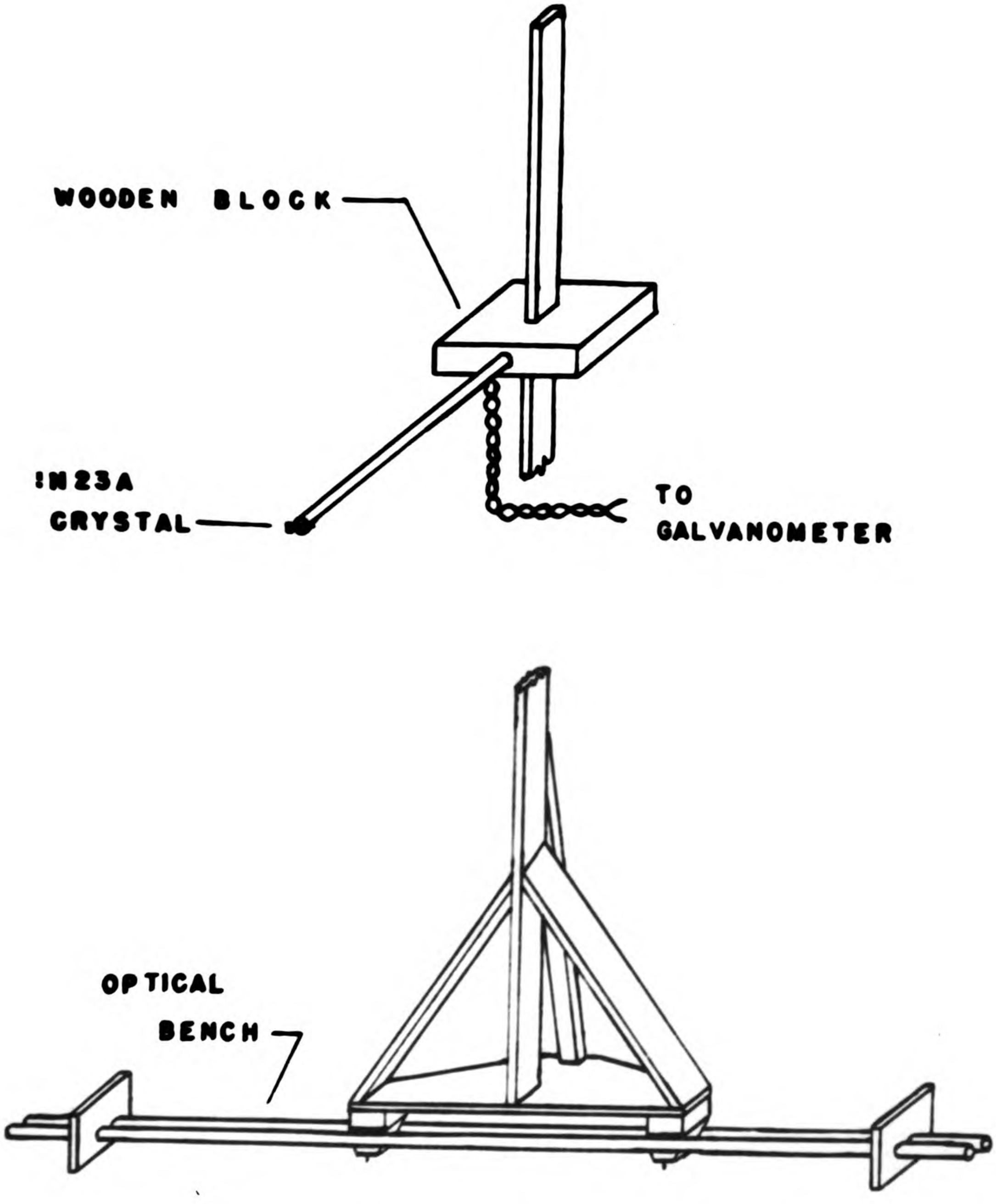


Figure 7



PROBE

Figure 8

bench slides. This gave transverse movement. A second piece of wood with a 1"x2" hole would slide up and down the vertical pole, but with a small amount of torque applied to the block it held its position. A three foot dowel with the crystal mounted on one end was inserted in the block of wood. The dowel provided the necessary torque to hold the block of wood in any set position. This allowed an easily adjustable vertical motion. The dowel also supported the twisted pair of wires in the direction of propogation. In this position a minimum disturbance of the field was obtained.

The experiment was performed in the loft of the Sheep Barn on the Michigan State College Campus. This site was chosen because it offered several distinct advantages. There was no steel framework in the structure to distort or cause reflections of the electromagnetic radiation. A distance of 68 feet between the screen and reflector could be obtained still leaving sufficient work space. When the loft doors were open there were no structures behind the aperture to cause reflections into the aperture and invalidate the results. The apparatus was completely protected so that during inclement weather the apparatus did not have to be taken down with the resulting problem of realignment before data could be again taken. Certain aspects were negative but these will be mentioned later.

The screen was erected and light from the aperture was allowed to fall on the reflector. By this means the antenna could be located exactly on the focal line of the reflector. The screen was checked to make certain the plane of the aperture was parallel to the antenna. A plumb bob was used to check the vertical plane of the conducting screen. Preliminary readings were obtained to ascertain the symmetries of the patterns. It was extremely poor. After many hours of checking and rechecking it was discovered that a local radio station with a nearby transmitter was affecting the pattern. Consequently, subsequent readings were taken as a difference between the readings with the 600 megacycle/second oscillator off and then on. The exceptions to this occurred when the radio station was not in operation as evidenced by a zero effect in the aperture.

Since automatic recording devices were not utilized it was necessary to take frequent successive readings to approximate a continuous pattern. The effect which the experiment was designed to investigate occurs very close to the edge of the aperture. Hence the carriage on the optical bench was moved only two millimeters between readings near the edge. This continued for five or six readings when an interval of one cm. or one-half cm. was chosen. When the probe had traversed the entire aperture the crystal was then raised or lowered depending upon which chord or diameter of the aperture was to be plotted.

IV. RESULTS AND INTERPRETATION

As has been stated previously the experiment reported in this dissertation was performed in the Sheep Barn. In addition to the advantages previously listed there were certain drawbacks. Possibly the most serious was the hay sling track suspended from the roof rafters as may be seen in figure 5. Every effort was made to insure that the pattern was symmetrical in all respects. However, it will be noted that the values on the right side of the aperture were always slightly lower than on the left. Interference from the hay sling track may have been a contributing factor to this asymmetry.

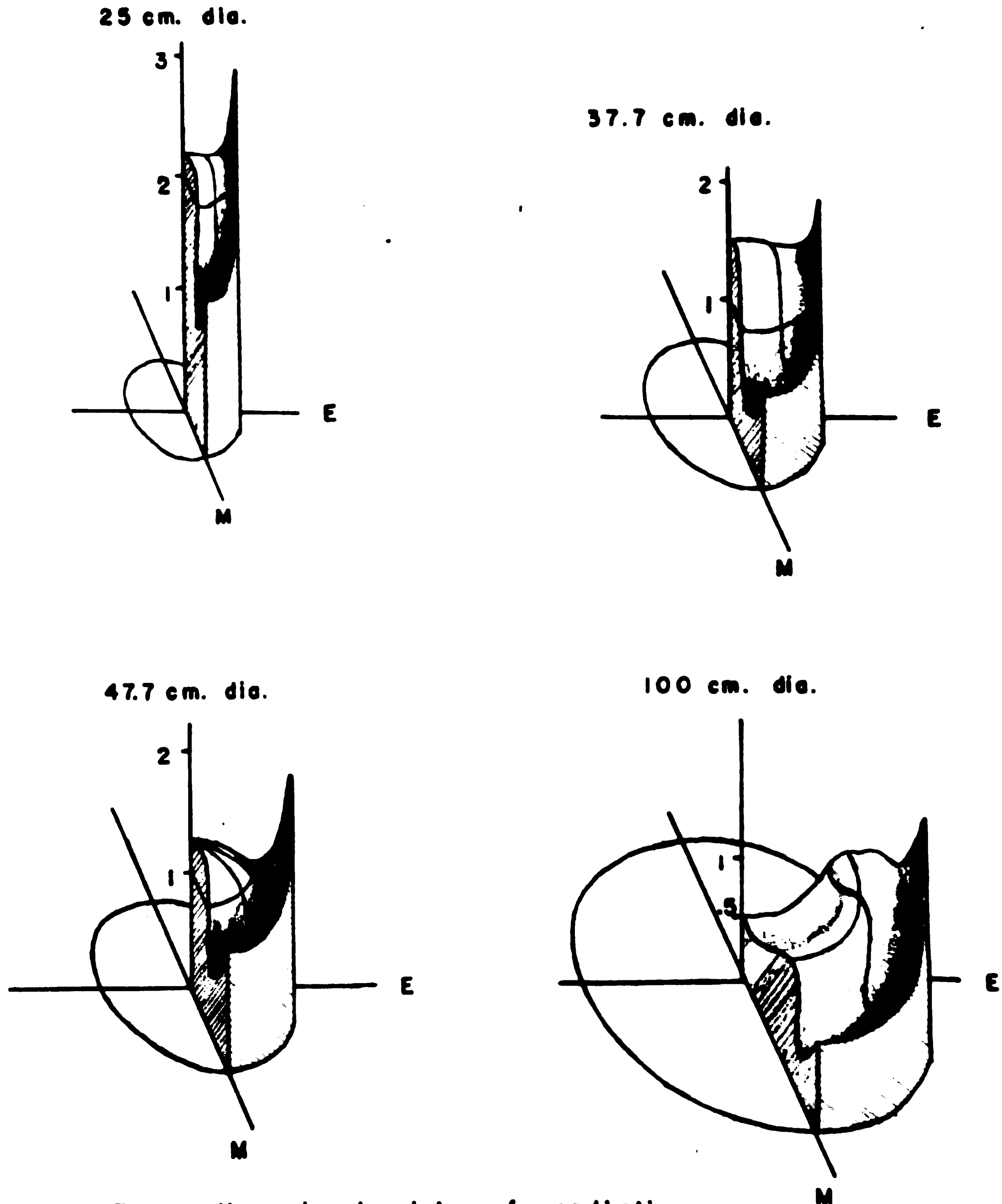
The terms electric and magnetic diameters have been used at various points in this paper. Due to the construction of the reflector it was only possible to orient the antenna horizontally. The electric field lines, which run from one end of the antenna to the other, may be represented by one vector oriented as is the antenna. Thus, the electric diameter is the horizontal diameter of the aperture. The magnetic field lines, which encircle the antenna in a vertical plane, may be represented by a vector perpendicular to the antenna in a vertical direction. Hence, the magnetic diameter is the vertical diameter of the aperture.

The three dimensional plots shown in figure 9 allows an insight into the form of the pattern. In all four diagrams the important item to notice is the sharp rise near the edge of the aperture. This is the effect which Bethe has predicted.

Certain other aspects also bear investigation. All of the small apertures, i.e., diameters of 12.5 cm., 15.9 cm., and 25 cm., have essentially the same form as shown in part a of figure 9. The value along the electric diameter drops only slightly from the maximum value occurring at the center until it approaches the aperture edge. However, along the magnetic diameter the value drops until approaching the edge where it again takes a sharp turn upward.

For the next two apertures, i.e., diameters of 37.7 cm. and 47.7 cm., the maximum value in the center drops slightly and there is a greater proportional drop along the electric diameter. The magnetic diameter falls off more than the electric diameter as previously. One other investigation has obtained the identical result.[5]

The fourth diagram (d) in figure 9 shows the pattern for the largest aperture. The value at the center of the aperture is low and there is a distinct



Three dimensional plots of radiation intensity over the apertures indicated. E is the electric diameter and M is the magnetic diameter.

Figure 9

maximum in both the electric and magnetic diameters. This can be seen more clearly in figure 10. Here the electric and magnetic diameters for the 100 cm. aperture are shown in their entirety. With the exception of the end effects these very closely resemble the results obtained by Andrews.[5] Here again the magnetic diameter falls to a lower value near the edge of the aperture than does the electric diameter.

Figure 11 indicates the method by which the three dimensional plots were made. Chords parallel to the electric diameter were chosen. These were then plotted. Upon completion of these plots the magnetic diameter was then plotted. The plot of the magnetic diameter was useful in that a check on the values at the center and each of the chord intersections was obtained.

Figure 12 shows the values obtained for the 15.9 cm. aperture. As mentioned previously the smallest three apertures had the same pattern except that the maximum in the centers varied. Again the value of the magnetic diameter is lower near the edge than it is on the electric diameter.

The curve shown in figure 13 is a plot of the radiation intensity per unit of aperture area against the aperture diameter. Unfortunately, due to conditions beyond the control of the author, it was possible to

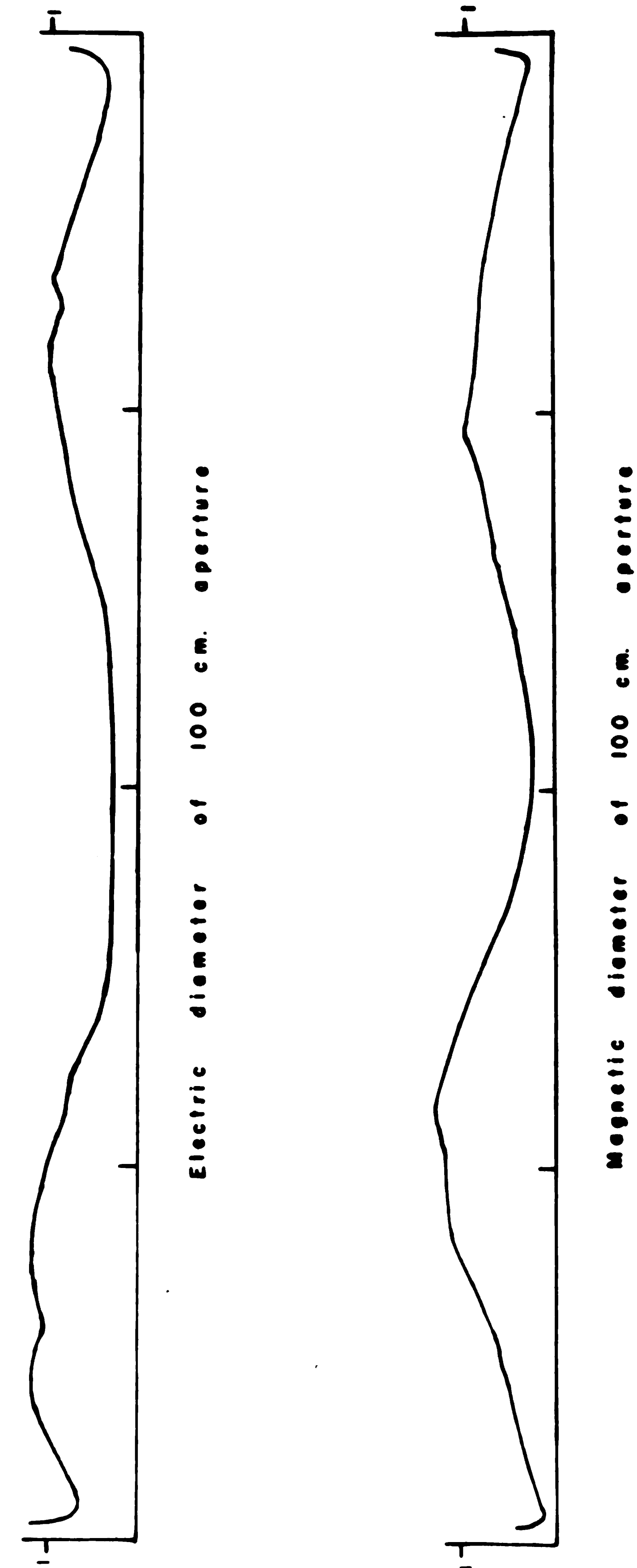
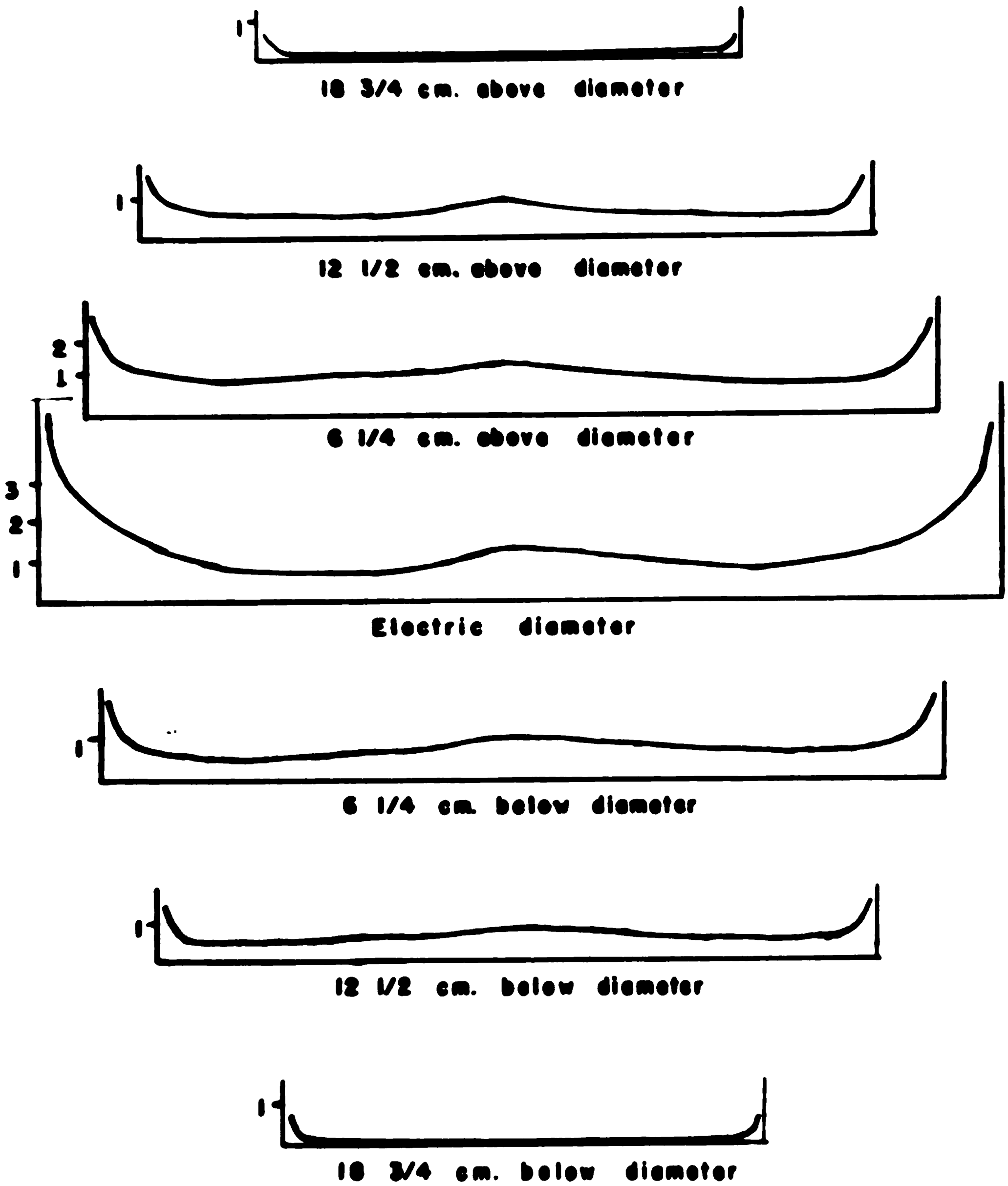
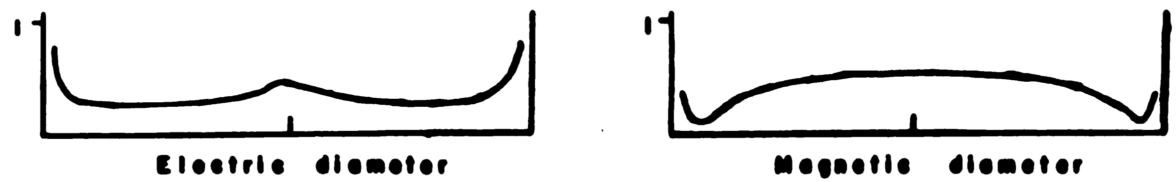

Electric diameter of 100 cm. aperture
Magnetic diameter of 100 cm. aperture

Figure 10



Measurements in the electric direction of the aperture with a diameter of 47.7 cm.

Figure 11



Aperture with 15.9 cm. diameter

Figure 12

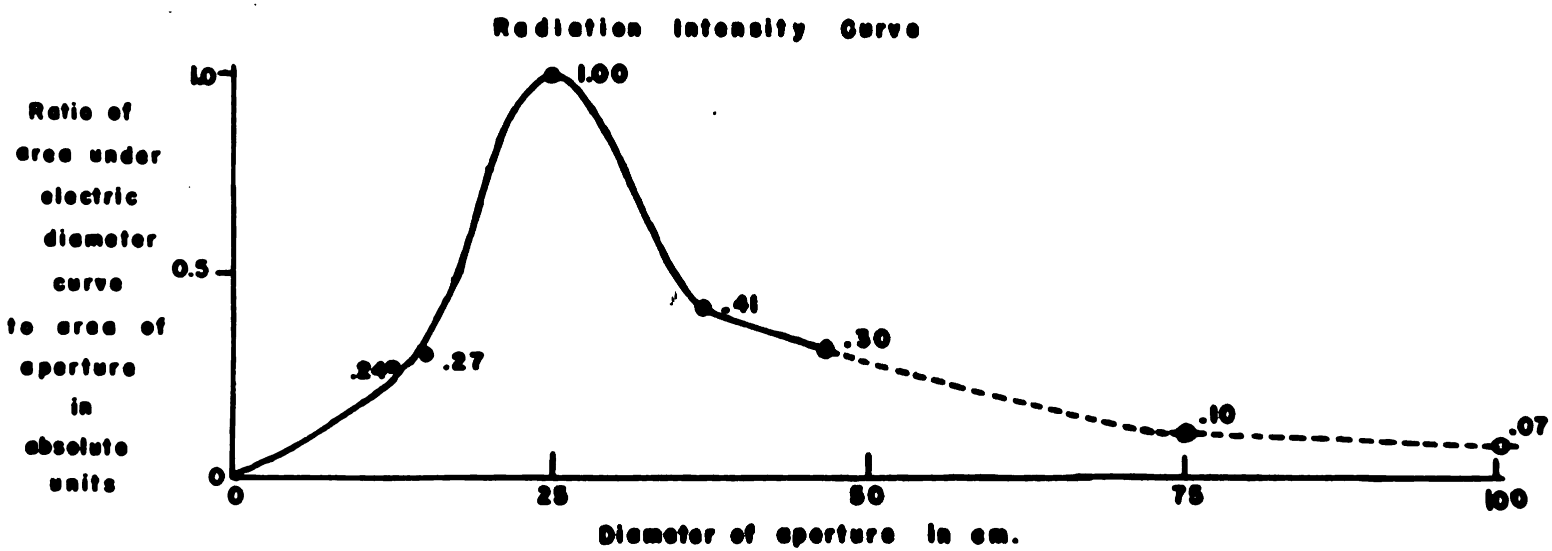


Figure 13

obtain only the values of the electric diameter of the 75 cm. aperture. Consequently, the area under the electric diameter curve was used since this was the only curve common to all apertures. To be complete and accurate the volume under the entire radiation pattern should be determined. It was thus assumed that the area under the electric diameter would give approximate results since the pattern was similar in most cases. The greatest error will occur in the two largest apertures.

To obtain the ordinate values in figure 13 all the electric diameters were plotted on graph paper with the same scale. The squares under the curves were then counted. The total number of squares for each aperture were then divided by the actual area of the aperture. The results of this division were then divided by the largest of these values to make comparison easier. The predicted maximum should occur in the aperture whose diameter is one-half the wave-length. This is clearly shown.

The choice of aperture diameter size is anything but consistent. Five of them are fractions or multiples of the wave-length, i.e., 12.5 cm., 25 cm., 37.5 cm., 75 cm., and 100 cm. Two apertures were chosen to determine if there was similarity between the acoustic and electromagnetic cases. For the two apertures 15.9 cm. ($\epsilon = ka = \frac{2\pi}{\lambda} a = 1$) and 47.7 cm. ($\epsilon \simeq 3$), the electric diameters resemble the figures shown by Spence[1] for the corresponding ϵ.

As has been previously been indicated in section II one expects a $(a^2 - r^2)^{-\frac{1}{2}}$ dependency. Obviously, for small r the expression will be negligible compared to contributions from other terms. However, as $r \rightarrow a$ the entire result is dependent upon this factor.

Probe position was difficult to determine in a precise manner. Horizontal movement was accurate to ± 0.5 mm. since the entire probe moved along the optical bench which had a scale on it. However, when the probe was moved in a vertical direction the accuracy was reduced since the interval variation was ± 1 mm. For the two millimeter intervals this is quite large, but compared to the aperture diameters it is not too great. The crystal occupied some volume instead of being a point and consequently the center of the crystal was the point from which the measurements were made. Since the crystal was two cm. in length the readings started 11 mm. from the aperture edge. With an antenna of this length the possibility exists that the apparatus was incapable of detecting effects which might have occurred in intervals of less than two cm.

The frequency of oscillation was determined by a frequency meter which was accurate to $\pm \frac{1}{2}\%$. This means that the wave-length was 50 $\pm$ 0.25 cm. The radii of the smaller apertures were accurate within one percent while for the larger apertures the error is one-half percent.

One other construction error appeared and remained constant throughout the experiment. This was the deviation of the reflector from a true parabolic curve. At the edges the deviation was 2% while close to the center it was about 1%. As enumerated so far the sources of these errors act independently of one another. Consequently they may or may not add to give a larger error.

There exists an error which must be considered at all times however. With the conducting screen removed the field varied 10% over the area in which the apertures were placed. This may have been caused by the preceding errors or it may have been due to turning the oscillator on and off between the readings.

Theoretical values for the $(a^2 - r^2)^{-\frac{1}{2}}$ dependency were computed and compared with the values obtained in the experiment. The variation was 10% or less. To procure a more accurate description of the curves automatic recording devices could be used. However, the percentages are within the experimental error and the effect which Bethe predicted has been shown to exist.

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