MEASUREMENT OF DIELECTRIC CONSTANTS BY MEANS OF A MICROWAVE INTERFEROMETER

Thesis for the Degree of M. Sc
MICHIGAN STATE UNIVERSITY
Yen Fu Bow
1956

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AN ABSTRACT

Submitted to the College of Science and Arts
Michigan State University of Agriculture and
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Department of Physics and Astronomy

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YEN FU BOW ABSTRACT

A simple Fabry-Perot interferometer in the microwave region is discussed. By means of this interferometer, the ratio of dielectric constants of polystyrene and teflon has been determined as 0.793, which is comparable with the accepted value 0.735.

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I. INTRODUCTION

The measurement of dielectric constants in the microwave region is of interest because information about the electric polarizability and relaxation times can be obtained, which, in turn, leads to a better understanding of liquids and solids. If one defines a complex dielectric constant $\epsilon = \epsilon' - j\epsilon''$, then the usual dielectric constant is $k = \epsilon'/\epsilon_0$, where ϵ_0 is the permitivity of vacuum and the loss angle δ is defined by $\tan\delta = \epsilon''/\epsilon'$. Thus in order to specify the dielectric properties, one should have values of k and the loss tangent.

In the measurement of dielectric constants in the microwave region, both waveguide and freespace techniques are applicable. Along with the usual shorted line and resonant cavity methods, one may also use microwave interferometers based on optical techniques. The principles of the Fabry-Perot and Michelson interferometers have been adapted by several authors to these measurements.(1,2,3) In order to make reliable absolute determinations of the complex dielectric constant, carefully designed instruments with well collimated microwave beams and accurately dimensioned reflectors are necessary. Culshaw, in particular, has carefully analyzed the Fabry-Perot interferometer from a theo-

retical standpoint and has designed an instrument for use at 8 mm. wavelengths which gives results with less than one percent deviation.

The purpose of the present investigation is to test the possibility of determining relative values of the dielectric constant with a somewhat less carefully designed instrument.

II. THEORETICAL CONSIDERATIONS

The detail theoretical development of the Fabry-perot interferometer in the microwave region can be found in the paper presented by Culshaw.[3] A brief review with certain clarifications is given here.

(i) Transmitted Intensity and Sharpness of Fringes

The operation of this interferometer is due to multiple reflections between two surfaces or films. Consider incident plane waves, and let P and R be the transmission and reflection coefficients of intensity of the films; then the resultant intensity transmitted through both films is given by the wellknown Airy formula[4]

$$T = \frac{P^2}{1-2R\ \mathrm{Cos}\phi + R^2} \tag{1}$$

where $\phi = (2\pi/\lambda)2\mu t\ \mathrm{Cos}\theta$ is the phase difference between consecutive beams, t is the distance apart of the films, μ the refractive index of the medium between the films, and θ the angle of incidence on the films. For maximum transmitted intensity $\cos\phi = 1$ or

$$2\mu t\ \cos\theta = n\lambda \tag{2}$$

where n is the order of interference, and

$$T_{max} = \frac{P^2}{(1-R)^2} \tag{3}$$

For zero loss in the films, $P = 1-R$, then T_{max} becomes unity.

By equation (1), the phase difference between points of half maximum intensity either side of resonance can be found as

$$\Delta = 2\phi_1 \tag{4}$$

where $\cos\phi_1 = \left\{2R - (1-R)^2\right\}/2R$. Defining the Q-value measured on the interferometer as the wavelength divided by $\frac{\lambda\Delta}{2\pi}$ gives

$$Q = 2\pi/\Delta = \pi/\phi_1 \tag{5}$$

Since ϕ_1 is a decreasing function of R, thus it is clear that to obtain sharp fringes reflectors of high reflectivity are required. Actually, this is the very important fact which led to the construction of the Fabry-Perot interferometer in the optical region, as pointed out by Boulouch in 1906.[4]

(ii) Development of Reflectors

If the electric field in a plane wave traveling in the x-direction is given by the real part of $E = E_0 \exp(i\omega t - \gamma x)$, where $\gamma = \alpha + j\beta$ is the propagation constant, and if $R_{n+1,n}$ is the amplitude reflection coefficient at the boundary be-

tween infinite media n and n-1 (see Fig. 1), it can be shown by the method of multiple reflections, or by impedance considerations, that the amplitude reflection coefficient due to the layer of medium n is given by[5]

$$A_{n+1} = \frac{R_{n+1,n} + \left[\exp(-2j\gamma_n d_n)\right] R_{n,n-1}}{1 + R_{n+1,n} R_{n,n-1} \exp(-2j\gamma_n d_n)} \tag{6}$$

where d_n is the thickness of the layer n. This is the fundamental formula for the transfer of reflection coefficients through dielectric layers.

Applying equation (6) to determine the reflection from a quarter wave sheet of dielectric bounded on either side by free space, and changing the notation so that A_1 refers to one sheet, gives

$$A_1 = R_{01}(1+\psi) \;/\; (1+R_{01}^2\psi) \tag{7}$$

where $\psi = \exp(-\pi \tan\delta')$, $\tan\delta' = \beta/\alpha$, and R_{01} the amplitude reflection coefficient at freespace-dielectric boundary. For no dielectric loss, $\tan\delta' = 0$, equation (7) reduces to

$$A_1 = 2R_{01}/(1+R_{01}^2) \tag{8}$$

Noting that $R_{01} = \dfrac{1-\sqrt{k}}{1+\sqrt{k}}$,[5] we have

$$A_1 = \frac{1-k}{1+k} \tag{9}$$

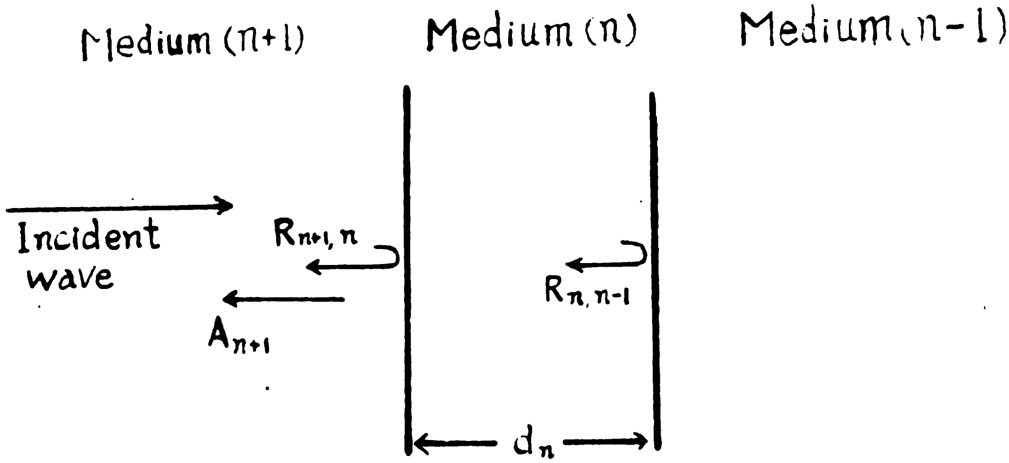

Fig. 1. Arrangement for calculation of reflection coefficients.

It is not difficult to prove by using equation (8) and mathematical induction that

$$A_n = \frac{1 - k^n}{1 + k^n} \tag{10}$$

Thus, neglecting dielectric loss, the amplitude reflection coefficient of n quarter wave dielectric sheets spaced quarter wavelength in free space is equal to that of a simple quarter wave sheet having a dielectric constant of k. It is thus clear that the amplitude reflection coefficient increases with the number of quarter wave sheets used. Equation (7) also indicates that the dielectric loss tends to decrease the reflectivity.

(iii) Diffraction Consideration

Since the dimensions of the instrument are much smaller in terms of the wavelength than those normally used in the optical region, a consideration of the diffraction which inevitably occurs is very important. As pointed out by Gooker and Clemmow,[6] the most useful approach to this problem is to make use of the fact that the field at all points in front of a plane aperture of any field distribution may be regarded as arising from the interference of plane waves in various directions. The amplitude and phase of these waves expressed as a function of their direction of travel, constitute an angular spectrum of radiation which, appropriately expressed, is the Fourier transform of the aperture distribution. The theory developed in (i) shows that the reflectors will exert a selective action on these plane waves of the angular spectra. From equations (1) and (2), it can be shown that the angular width θ of the spectrum transmitted between points of half maximum intensity is given by the following formula

$$\cos\theta = 1 - 1 / 2nQ \tag{11}$$

where n is the order of interference and Q is the Q-value defined above. Thus the higher the Q-value and order, n, of interference, the greater this selection, the process being strictly analogous to what occurs in a resonant cavity which can propagate a number of modes. Consequently, the diffraction effect may be neglected in an interferometer with very high Q-value. Also because of diffraction effects energy will be lost outside the reflector system, the amount lost increasing with reflector separation. Thus the Q-value should decrease as the separation of the reflectors increases.

III. DESIGN AND OPERATION OF THE INTERFEROMETER

A schematic diagram of the interferometer is shown in Figure 2. It consists of the microwave source, reflectors and detector. A 723A/B klystron generating radio frequency at a wavelength of 3.2 cm. is used as the source. The klystron is coupled directly to a short section of rectangular waveguide which is terminated in a circular plate. This arrangement gives a fairly uniform phase across the aperture plane (E-plane). Each reflector is constructed with four quarter-wave sheets of polystyrene spaced quarter-wave length in air. Figure 3 indicates the construction and dimensions of the reflectors. Polystyrene was used for its low dielectric loss. Since high Q-value could not be expected for this simple interferometer, in order to compensate for the diffraction effect, the detecting system must have as much directivity as conveniently possible; i.e., the receiving horn should not be too small. Several types of rectangular horns were tried. The dimensions of the one which was used are indicated in Figure 4.

Fundamentally, the operation of this interferometer is the same as Fabry-Perot interferometer in the optical region. Because of the multiple reflection between the reflectors with high reflectivity, interference fringes can be ob-

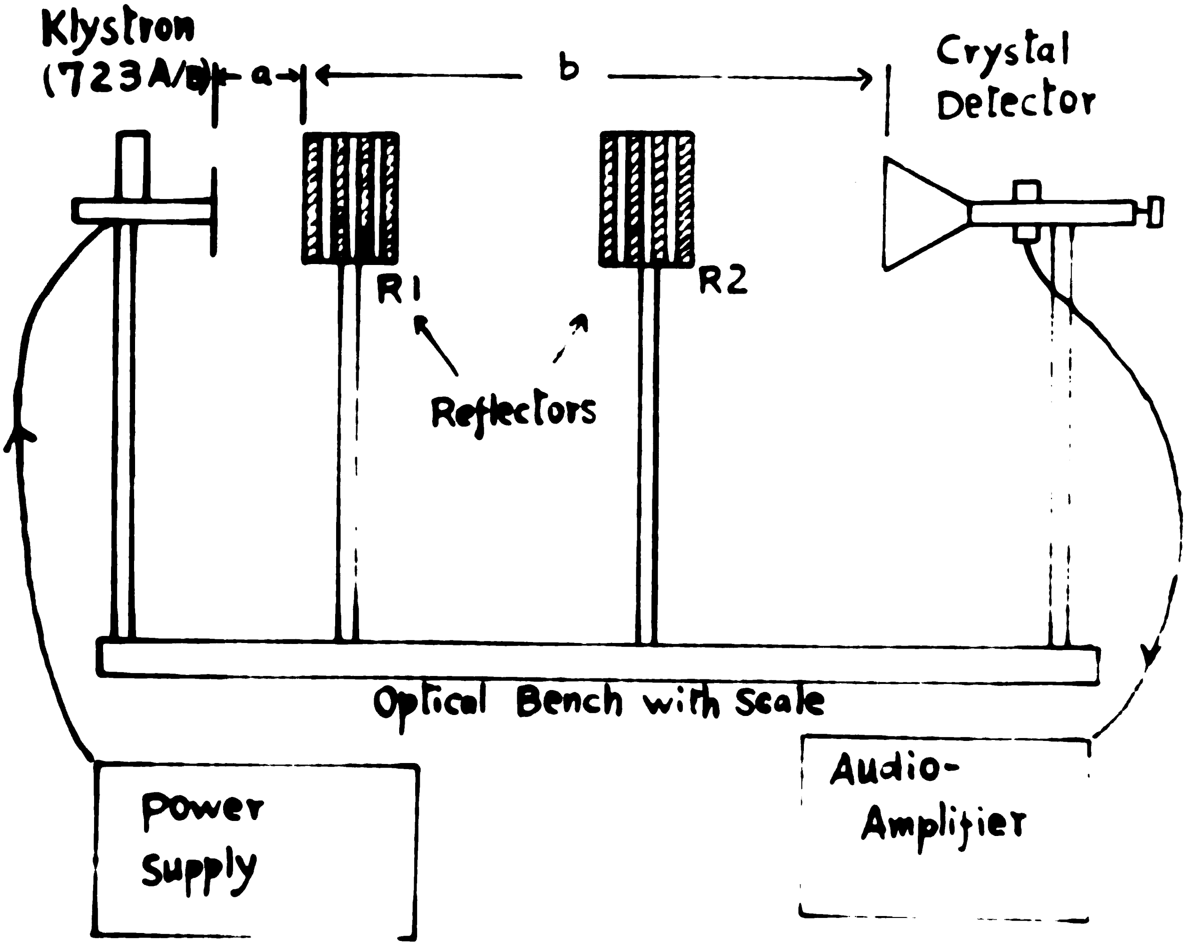

Fig. 2. Schematic diagram of the interferometer
(A 12 cm., B 46 cm.)

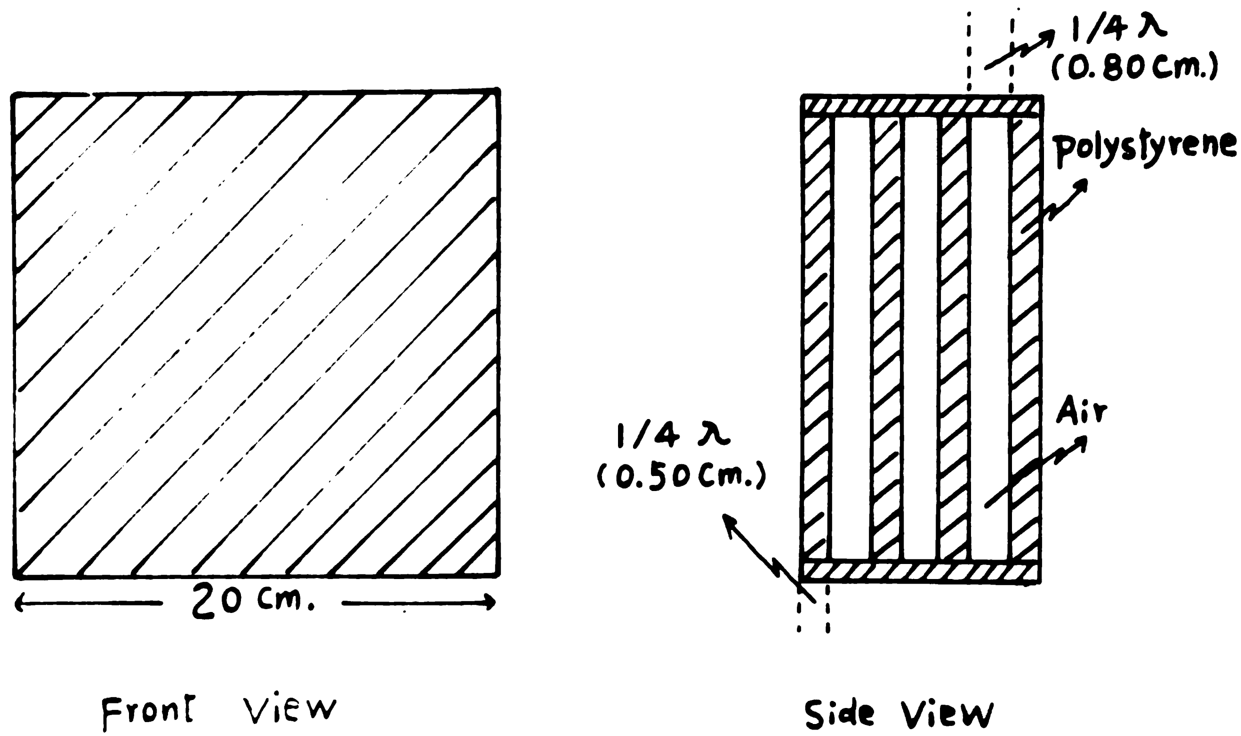

Fig. 3. Structure and dimensions of the reflector

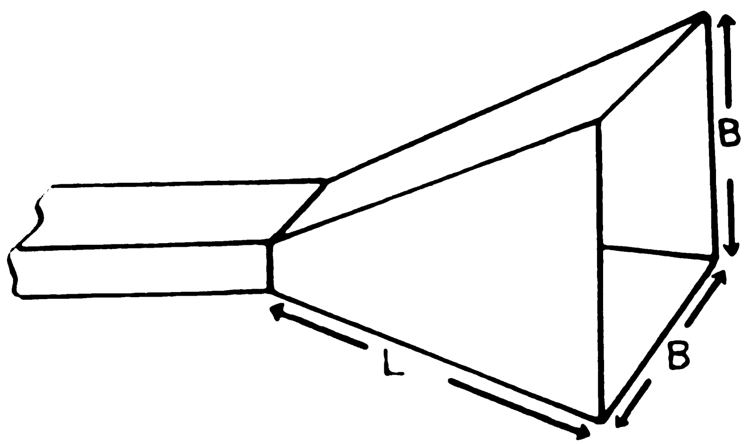

Fig. 4. Dimensions of the receiving horn
(L 18.4 cm., B 13.8 cm.)

served for the transmitted radiation. However, since our interferometer operates with essentially parallel beam, the circular fringe system of the optical model is not obtained; instead, the fringe is observed by altering the distance between the reflectors. For convenience, the reflector R1 (Figure 2) is fixed and the reflector separation is changed just by moving R2. A system of fringes obtained in this way is shown in Figure 5. The configuration of source, reflectors and detector is indicated in Figure 2 and the transmitted intensity is observed on the scale of the audio-amplifier of the detecting system. It is noted that the distance between the fringes is half wavelength as predicted by equation (2). The Q-value estimated from this plot is approximately 20. Because of the diffraction effect the transmitted intensity decreases with increasing reflector separation.

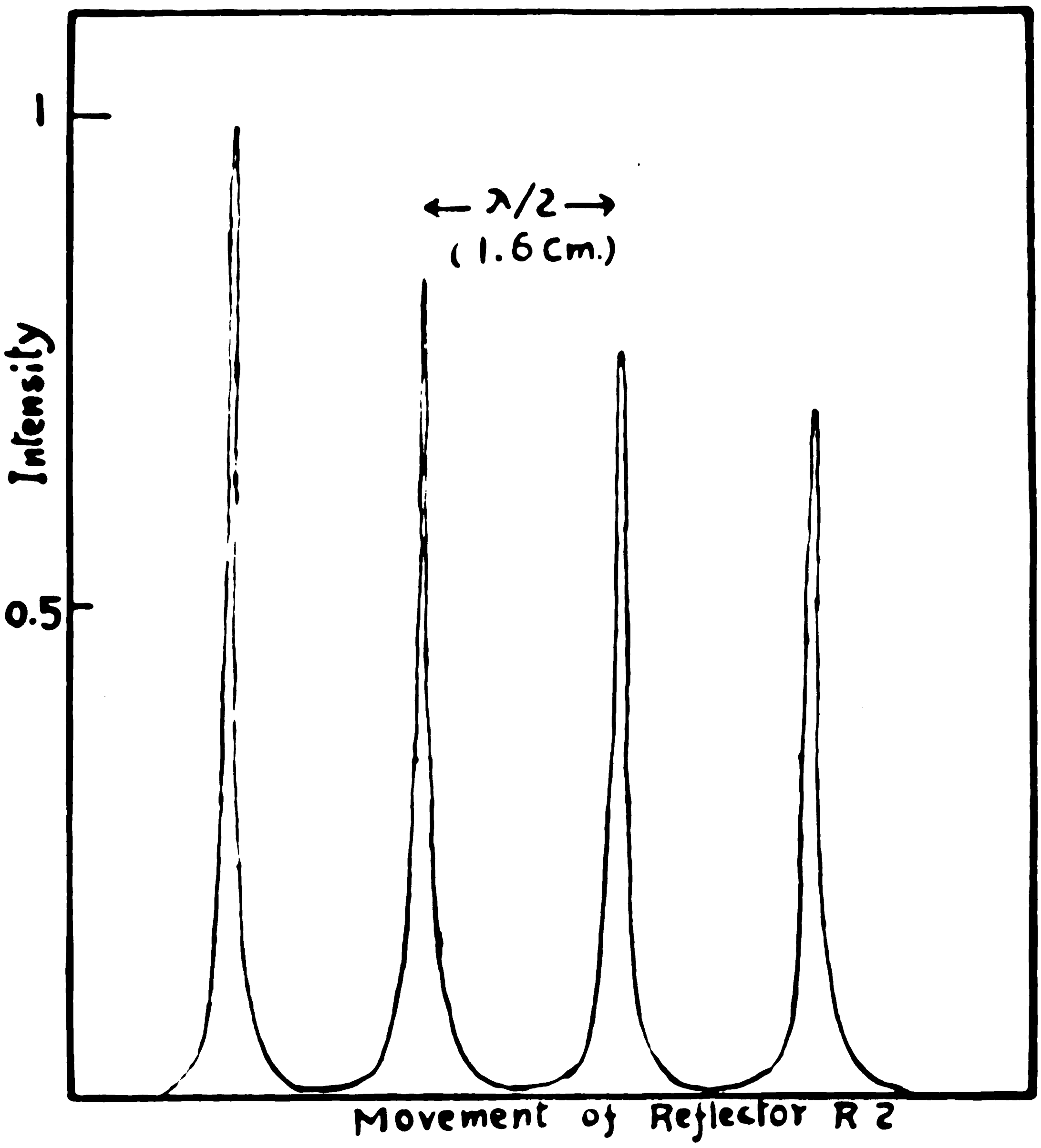

Fig. 5. Showing sharpness of fringes on the interferometer in relation to the wavelength (initial reflector separation 25 cm.)

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IV. MEASUREMENT OF DIELECTRIC CONSTANTS

By means of the interferometer discussed above, we have measured the ratio of the dielectric constants of polystyrene and teflon. With the configuration indicated in Figure 2, a maximum response (fringe) was located by adjusting R2, then a sheet of the material was inserted between the reflectors and the shift of the fringe noted. However, Culshaw pointed out that, because of the differing impedances, effective path length changes occur at the boundaries of the sheet, the amount depending on the position of the sheet between the reflectors. In order to cancel out this path length change at the boundaries, measurements must be made varying the position of the sheet by small intervals over a range of half wavelength and the mean fringe shift found. The dielectric constant k is then given by

$$\text{mean shift} = d(\sqrt{k} - 1) \qquad (12)$$

where d is the thickness of the sheet. The measured fringe shifts due to polystyrene and teflon are collected in the following table:

sample position (distance in cm. from R2)	fringe shifts (cm.)	
	polystyrene (thickness 0.70cm.)	teflon (thickness 0.45cm.)
17.0	0.25	0.05
16.7	0.35	0.12
16.4	0.50	0.30
16.0	0.60	0.35
15.7	0.30	0.12
15.4	0.25	0.05
mean	0.375	0.165

It is noted that the fringe shift is periodic of the sample position, a fact in agreement with Culshaw's prediction. From the mean fringe shift, the calculated dielectric constants for polystyrene and teflon are 2.341 and 1.856 respectively. Although these values are not in agreement with handbook values, their ratio 0.793 is comparable with the accepted value 0.785 (dielectric constants of polystyrene and teflon are 2.55 and 2.00 respectively). Furthermore, with our measured ratio and assuming 2.55 as the dielectric constant of polystyrene, we get 2.02 for the dielectric constant of teflon. This is in agreement with the accepted value within one percent. Thus, it may be clear that our simple arrangement is a very convenient method for comparing dielectric constants.

V. DISCUSSION

It should be pointed out that only the real part of the dielectric constant has been measured in this experiment. The imaginary part which is associated with the dielectric loss can be estimated by observing the change of the Q-value due to the sample; however, this can only be obtained by means of an interferometer with very high Q-value.

An obvious extension of the experiment discussed above is to compare the dielectric constants of liquids. With this idea in mind, it seems possible that more applications can be developed, namely, the titration of weak acids and the measurement of relative temperature variation of polarization.

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