THE MEASUREMENT OF THE DE HAAS-VAN ALPHEN EFFECT BY THE GOUY METHOD

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ABSTRACT

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by Richard Norman Wagner

A torsion-type, magnetic susceptibility, beam balance capable of making differential weighings of 30 micrograms on samples weighing up to 50 grams was built. Force measurements were made on a single crystal of bismuth with magnetic fields up to 17,700 gauss at a temperature of $4.2^{o}K$ using the Gouy method. The results of these measurements illustrate the utility of the Gouy method in detecting De Haas-Van Alphen oscillations.

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I INTRODUCTION

A torsion beam balance was constructed to measure the magnetic susceptibility of metals at low temperatures. The balance follows the general design of previous balances.[1,2] Samples weighing up to 50 grams weight can be used. The uncertainty in a given weighing is about 30 micrograms at present. Restoring force is supplied by a current carrying coil interacting with a permanent magnet. An optical-electrical servosystem is used to keep the beam in null position, minimizing eddy current effects and increasing the effective stiffness.

The balance is sensitive enough to use the Faraday method, but to avoid the necessity of obtaining properly shaped pole faces as are used to reduce sample positioning errors, the Gouy method was used. Further, it was decided to investigate the suggestion, to the author, by Dr. J. F. Cochran and Dr. M. Garber that De Haas-Van Alphen (here after referred to as DH-VA) oscillations might be studied using the Gouy method.

Previous studies of DH-VA oscillations have been carried out with the sample suspended in a nearly uniform field. Shoenberg used the Faraday method in his inital studies of the DH-VA effect in bismuth.[3] This method measures some average value of the susceptibility when the

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[1]M. Garber, W. G. Henry, and H. G. Hoeve, Can. J. Phys. <u>38</u>, 1595, (1956).

[2]F. T. Hedgcock, Rev. Sci. Instr. <u>31</u>, 390, (1960).

[3]D. Shoenberg and M. Zakie Uddin, Proc. Roy. Soc. <u>156</u>, 701, (1935).

susceptibility is field dependent, thereby making the Faraday method useless in resolving closely spaced DH-VA oscillations. For the above reason Shoenberg used the torsion method in a later experiment on bismuth.[4] The torsion method is simple and sensitive, but it has the disadvantage that it measures the difference in the susceptibilities along the principal axes. Present day DH-VA studies are frequently made with pulsed very high fields using a pick-up method for measuring the susceptibility.

The balance was constructed for use in investigating the susceptibility of alloys but it was decided to try and use it to detect DH-VA oscillations in bismuth. The absolute susceptibility of bismuth was not accurately determined because the orientation of the bismuth crystal was not well determined. The measurements, however, show that DH-VA oscillations can be detected easily.

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[4]D. Shoenberg, Proc. Roy. Soc. 170, 341, (1939).

II THEORY

The magnetic force on a long rod of material suspended in a magnetic field gradient can be calculated by considering the virtual work required to make a virtual displacement, δz, of the rod. This virtual work corresponds to the change in the magnetic free energy of the rod when the displacement is an isothermal reversible process. Calculation of the virtual work can be simplified if the following assumptions, readily realizable with the Gouy method, are made:

1) The rod material is homogeneous.
2) The cross sectional area, A, of the rod in the plane perpendicular to the displacement, δz, is a constant.
3) The ends of the rod are in regions of uniform field.

Now, the virtual work required to displace the entire rod a distance δz can be considered as the work done in taking a small volume, $A\,\delta z$, from one end of the rod to the other.

With no essential loss of generality it will be further assumed that the permeability of the rod material is isotropic, i.e., a scalar. The medium surrounding the rod is a vacuum, and one end of the rod is in a region of zero field.

The difference in the magnetic free energy of the rod is equal to the magnetic free energy of the small volume of material, $A\,\delta z$, in the high field region. This energy is the difference between the energy of the field, due to fixed current sources, without the presence of the small piece of magnetic material, and, if the magnetic material is present, the amount of work done by the current sources necessary to re-

store the currents from zero to their initial value.[5] The additional work necessary to build up the currents in the presence of the body is

$$U = \frac{1}{2}\int_V \left[H_1 B - HB_1 - HB + 2\int_0^B H\,dB\right] dV$$

where V is the volume of the magnetic material, H and B are the fields in the presence of the magnetic material, and H_1 and B_1 are the fields in the absence of the magnetic material. By substituting

$$B_1 = \mu_0(H_1 + M_1) = \mu_0 H_1\,, \qquad B = \mu_0(H + M) = \mu_0 H(1 + \chi)$$

into the above equation we have (μ_0 = permeability of free space)

$$U = \frac{1}{2}\mu_0\int_V \left[\chi(H_1 - H)H + 2\int_0^H H\,dM\right] dV \;.$$

If the high field region is at the bottom of the sample and the mechanical force, $\mathfrak{F}$, on the sample and the displacement, δz, are taken in the upward direction

$$\mathfrak{F}\,\delta z = \frac{1}{2} A\,\delta z\,\mu_0\left[\chi(H_1 - H)H + 2\int_0^H H\,dM\right]$$

$$\mathfrak{F} = \frac{1}{2} A\,\mu_0\left[\chi(H_1 - H)H + 2\int_0^H H\,dM\right] .$$

[5] J. A. Stratton, <u>Electricity and Magnetism</u> (McGraw-Hill, New York, 1941), p. 128.

The force is proportional to the free energy density at the high field end of the rod and to the cross sectional area of the rod. Therefore fluctuations in the force on the rod correspond directly to fluctuations of the free energy density at the high field end of the rod.

It is often assumed that the presence of the magnetic material does not affect the current sources, in which case $H_1 = H$, and the usual expression for the force on the rod is written

$$\mathfrak{F} = A\mu_0 \int_0^H H\,dM \,.$$

For materials in which $M/H = \chi \simeq$ constant

$$\mathfrak{F} = \frac{1}{2}\mu_0 A \chi H^2 \,.$$

In cases where the magnetic susceptibility varies with the applied magnetic field the theoretical analysis is usually based on the free energy expression. This corresponds directly to the measured force on the sample. This simple and direct feature of the Gouy method seems hitherto to have been overlooked.

In the case of the De Haas-Van Alphen effect the expression for the free energy may be written[6]

$$F \propto T H^{\frac{3}{2}} \, e^{-\frac{2\pi k T m^* c}{e\hbar H}} \cos \frac{\hbar c A_0}{e H}$$

where constant terms and the quadratic background rise of F with H are

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[6] R. G. Chambers, Can. J. Phys. 34, 1395 (1956).

omitted. This expression is characterized by a periodic variation in 1/H with the period given by

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c A_o}$$

where A_o refers to the extremal cross sectional area in momentum space of the Fermi surface, taken perpendicular to the magnetic field. Thus, the measurement of the period of the oscillations gives topological information about the Fermi surface of metals. The amplitude variation which gives information about the effective masses, is rather more complicated. It has not been analyzed in this work. The main object is to show the utility of the Gouy method for De Haas-Van Alphen effect measurements. The data obtained using the Gouy method will be compared with data obtained using the magnetothermal method.

III APPARATUS

A Gouy-type apparatus was built utilizing a torsion beam balance. An optical-electrical system is used to detect the position of the beam and an electro-mechanical system is used to restore the beam to null position.

The beam and sample are surrounded by a vacuum enclosure including a long Pyrex tube, called a cold finger, in which the sample is suspended. The cold finger is surrounded by a partially silvered dewar with provision for pumping on liquid helium.

A commerical electromagnet which can be rotated about a vertical axis was used. In an attempt to reduce the magnetic field at the low field end of the sample, a cylindrical iron shield was placed around the top end of the sample.

Magnet and Magnetic Shield

The magnet was a Harvey-Wells model L-158. It was used with 12" dia. x 2.375" cylindrical pole faces which were machined from Armco Magnetic Ingot Iron.[1] With a gap width of 3 inches and with the maximum obtainable current of 200 amperes through the magnet coils, a field of 17,700 gauss was obtained.

The magnet power supply is a Harvey-Wells model HS-10200 capable of delivering 200 amperes at 100 volts, to the magnet, with a stability of one part in 10^5.

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[1]Composition: C, 0.015; Mn, 0.028; P, 0.005; S, 0.025; Si, 0.003; Fe, 0.924.

The magnet U-frame support is mounted on a pedestal by thrust and alignment bearings so that the magnet can be rotated 360° about a vertical axis. The pedestal is mounted on a rolling rail truck which allows the magnet to be rolled into place under the dewar. It can be rolled away from the dewar allowing the dewar to be removed in order to gain access to the sample.

The top of the sample is in the region of the magnet windings. To reduce the magnetic field in this region, a magnetic shield was made. It consists of two semi-cylindrical pieces of iron, such that when bolted together, they form a cylinder around the dewar (see Figure 1). Insertion of the magnetic shield reduced the magnetic field in the region near the top of the sample for low fields, i.e., for fields of less than 10,000 gauss between the pole faces. For high fields, however, the reduction was negligible (see Figure 2). At high fields the shield may still give the advantage of a reduced field gradient, dH_x/dz, in the region near the top of the sample (z = vertical direction, x = magnetic axis).

Vacuum Enclosure

The beam, optical system, and sample are in a vacuum enclosure so that the sample may be surrounded by a suitable atmosphere (see Figure 3). To provide magnetic shielding for the voice coil the vacuum can was constructed with iron and soldered together. Flange D was also machined from iron. The inside of the vacuum can was painted with General Electric Glyptal No. 1201 to prevent rusting.

The transfer tube receptacle is a german silver tube soldered at the bottom to flange D and sealed at the top with a "quick coupling".

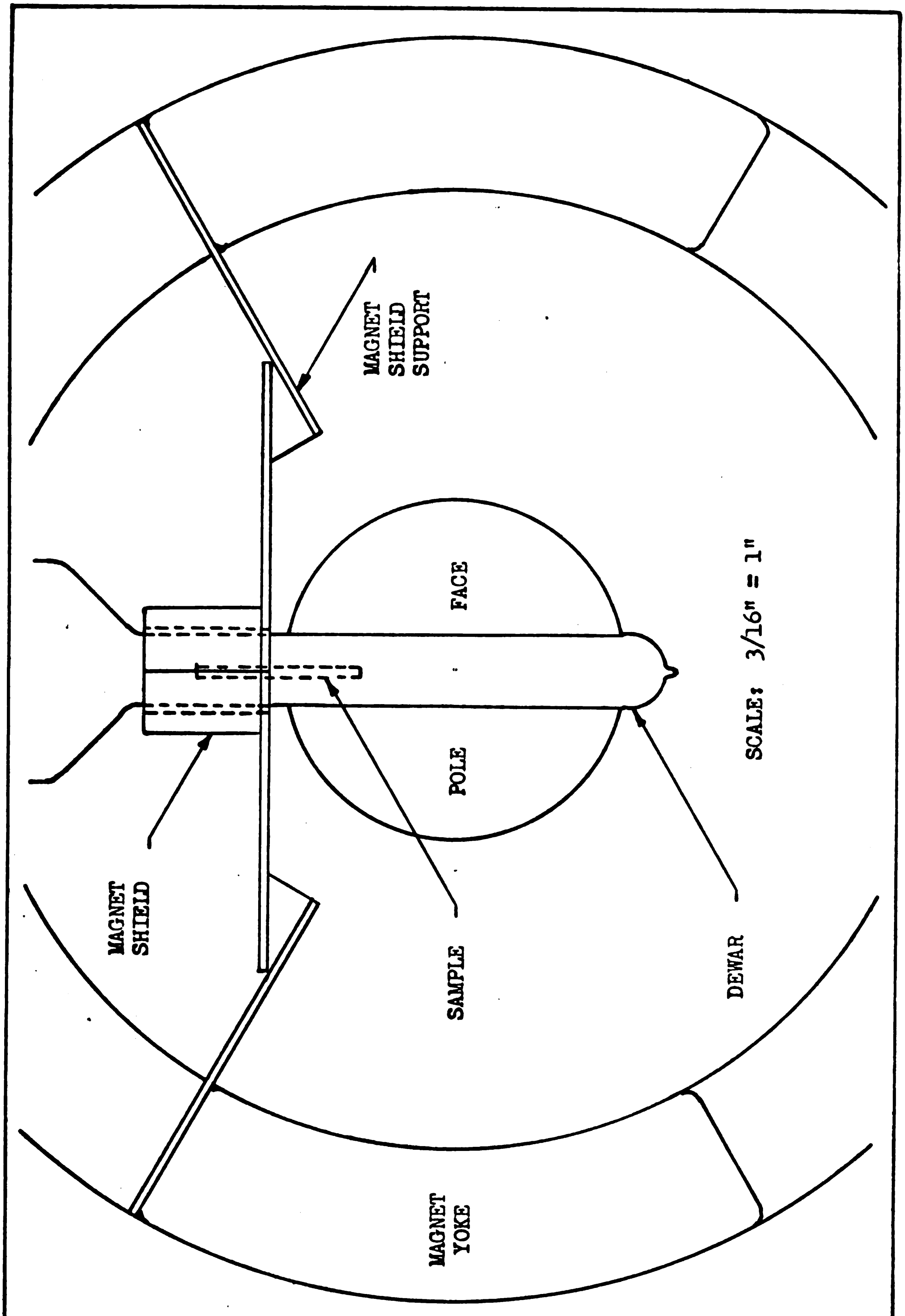

FIGURE 1 MAGNET SHIELD

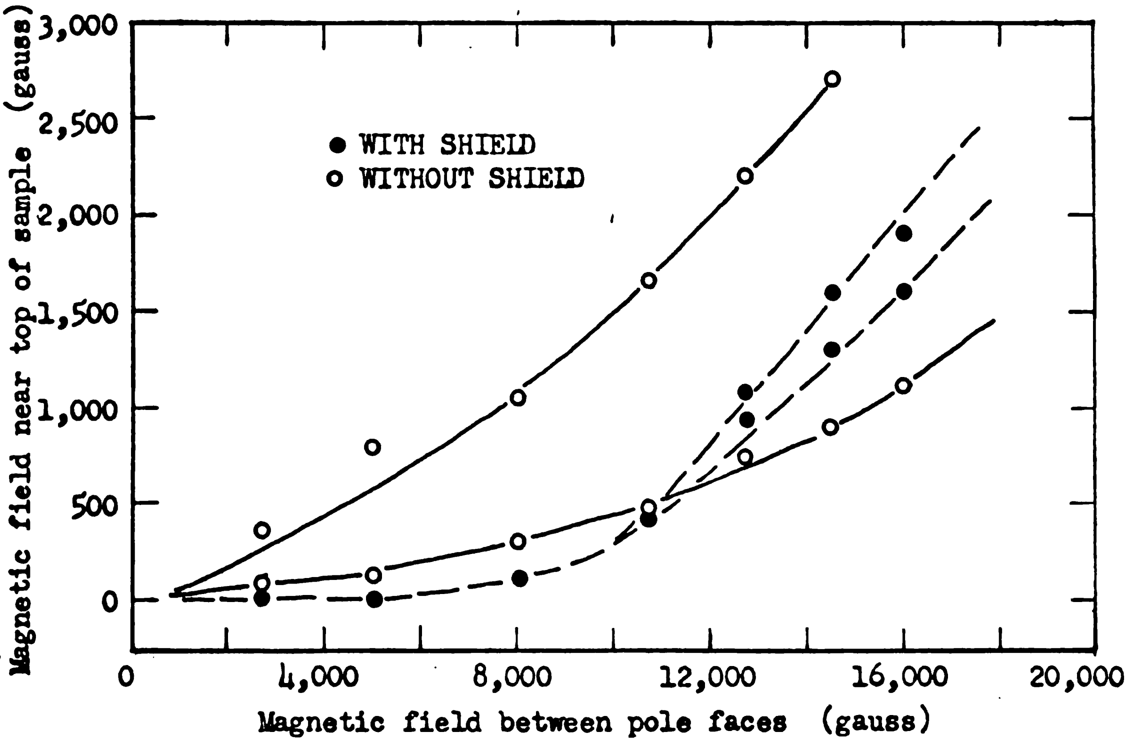

FIGURE 2

EFFECT OF MAGNET SHIELD

NOTE: Measurements were made about 1 inch above and 1 inch below the top of the sample.

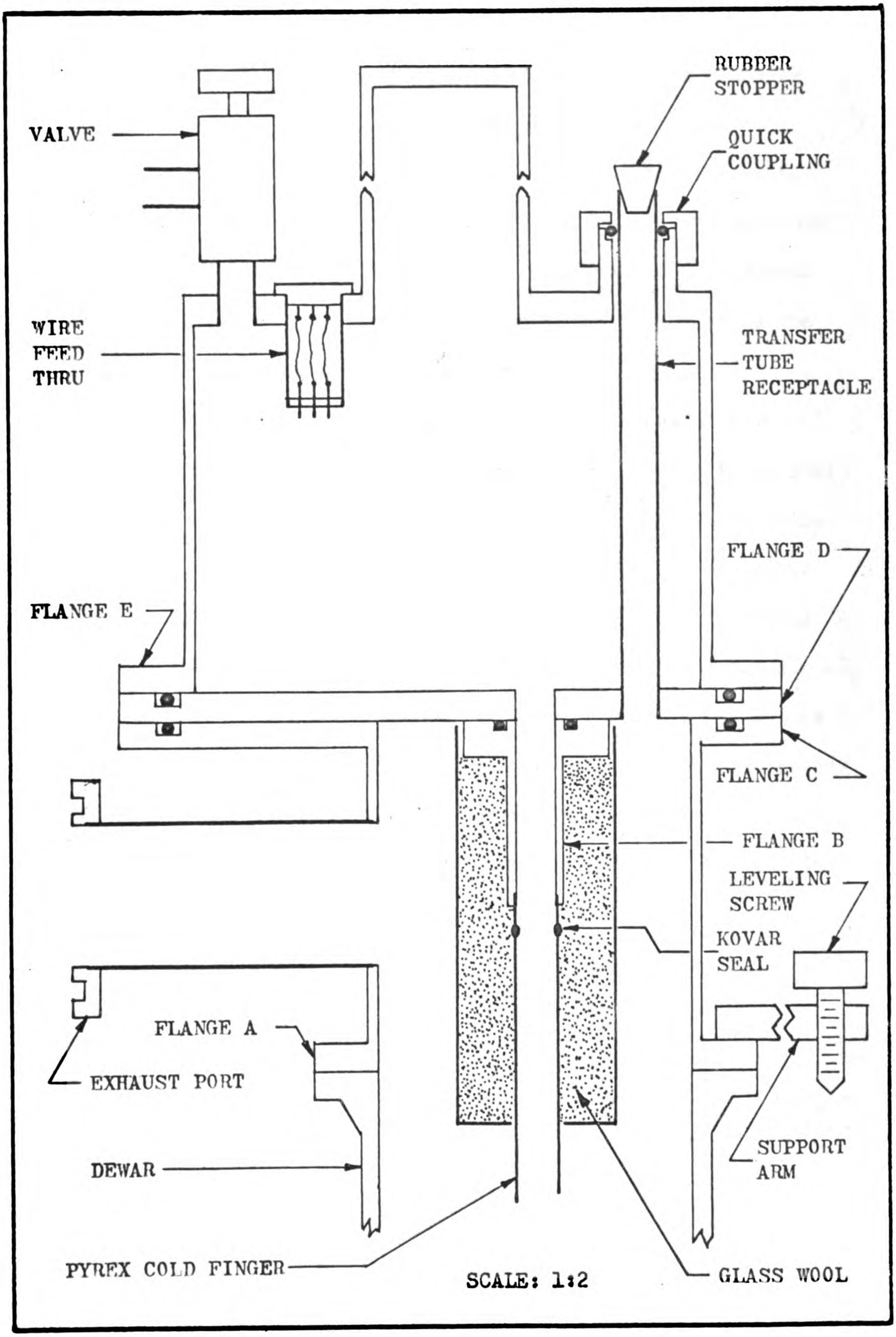

FIGURE 3 VACUUM CAN

The transfer tube is admitted through the transfer tube receptacle without disturbing the atmosphere in the vacuum can. The transfer tube receptacle can be sealed by inserting a rubber stopper.

The vacuum can, flange D, and flange C are held together with screws and are sealed with two O-rings. The vacuum can is removable so that the beam and optical system may be adjusted. It is removed by removing the screws holding it to flanges C and D, loosening the "quick coupling", and then lifting the vacuum can.

Flange B is held to flange D with screws and is sealed with an O-ring. At the top of the cold finger there is a Kovar to Pyrex seal, the Kovar being soldered to flange B. Experience has shown that the cold helium gas rushing past a Kovar seal, when transferring liquid helium, will cause the seal to fracture. To prevent this a thermal insulator, made from stainless steel tubing and packed with glass wool, proved successful. The top of the thermal insulator is slotted like a bayonet type light bulb socket. Attached to flange B are two screws which slide into the two slots in the thermal insulator so that the insulator is attached to flange B by pushing up and then twisting the thermal insulator.

The three support arms and the dewar are bolted to flange A. A leveling screw on the end of each support arm provides a means of leveling, raising, and lowering the entire apparatus.

A two inch diameter exhaust port is provided for pumping on the liquid helium.

Access to the sample is obtained by removing the dewar, thermal insulator, and the cold finger.

Balance and Optical System

The 6 cm long beam is machined from 0.10 inch thick tempered aluminum (see Illustration 1 and Figure 4). It is supported by a torsion ribbon of Elgiloy (obtained from Elgin National Watch Company), 0.001" x 0.100" x 1.625". The torsion constant of this ribbon is approximately 3,000 dyne-cm/radian. The beam is restored to its null position by adjusting the current through the voice coil which is suspended from one end of the beam into the gap of a PM speaker magnet. The top of the voice coil also serves as a weight pan. Current to the voice coil is supplied through two helical #40 wire leads. A change in current through the voice coil of 0.025 microamperes, for a change in balance load of one microgram, is required to restore the beam to null position.

Because of difficulty in placing weights on the weight pan without disturbing the helical voice coil leads and the position of the voice coil in the magnet air gap an accurate calibration of the voltage versus force was not carried out. Although the voice coil current was expected to be linear with the force recent experience has shown that this may not be so. Steps to correct this include: rearranging the voice coil leads, reducing the number of turns on the voice coil, and more careful placing of the voice coil in the magnet gap.

Position of the beam is detected by the position of a spot of light focused on two cadmium selenide photo-conductive cells (Clairex type 404SL). The lens focuses an image of the filament of the light bulb on the galvanometer mirror and in turn, the spherical galvanometer mirror focuses an image of the slit on the two photo-cells (see Illustration 2 and Figure 5). Current is supplied to the light bulb with a

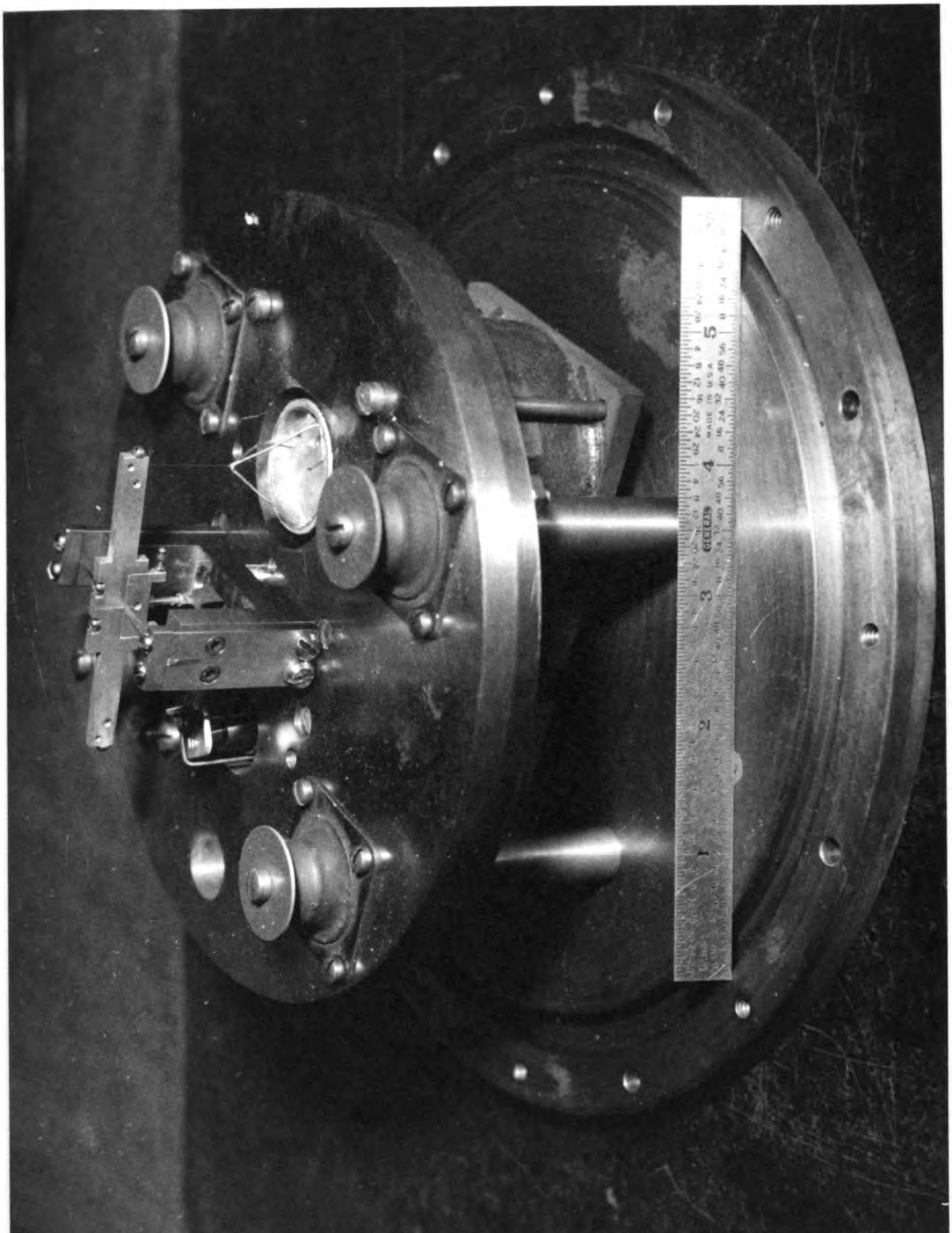

ILLUSTRATION I BALANCE

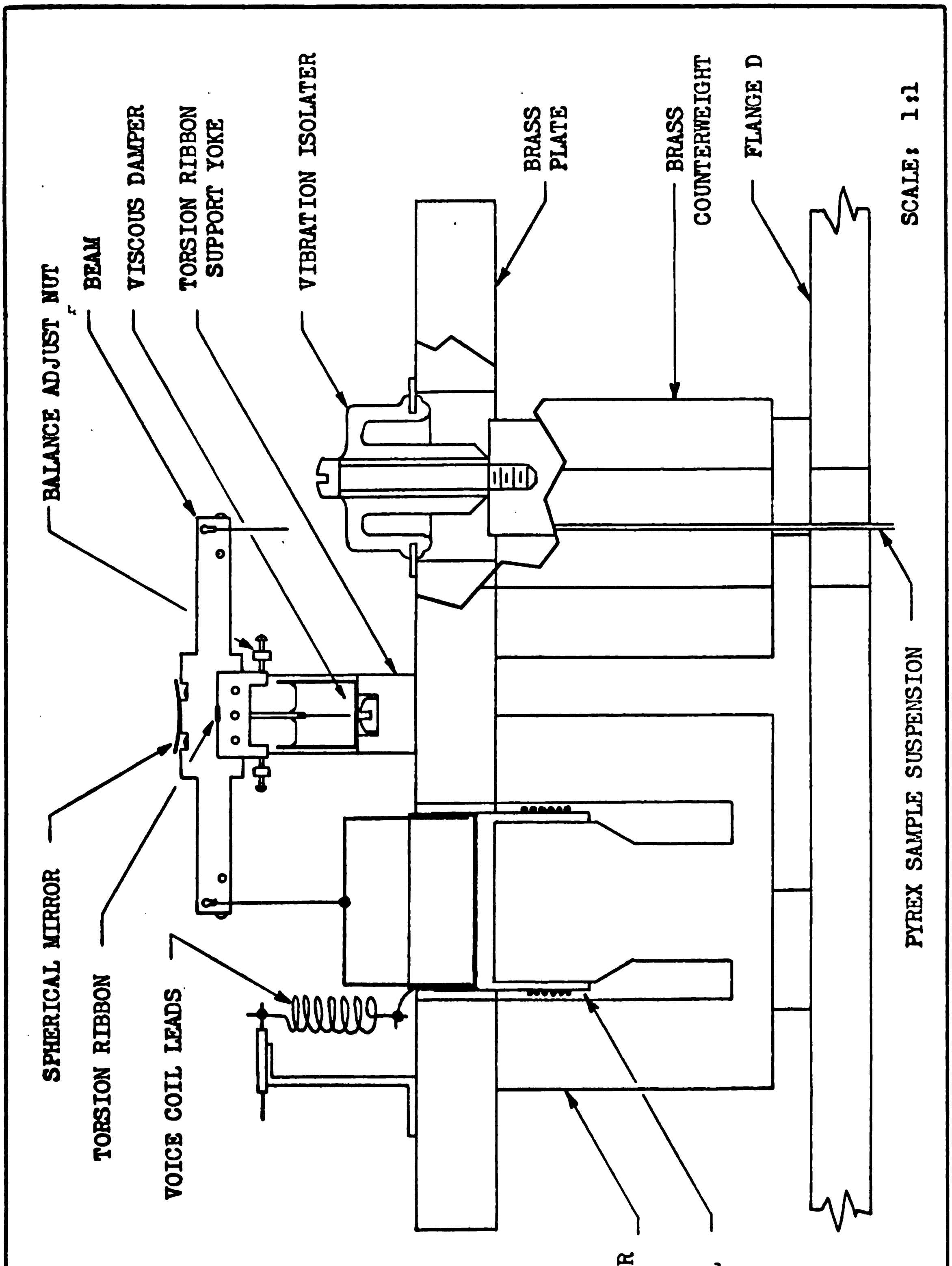

FIGURE 4 BALANCE

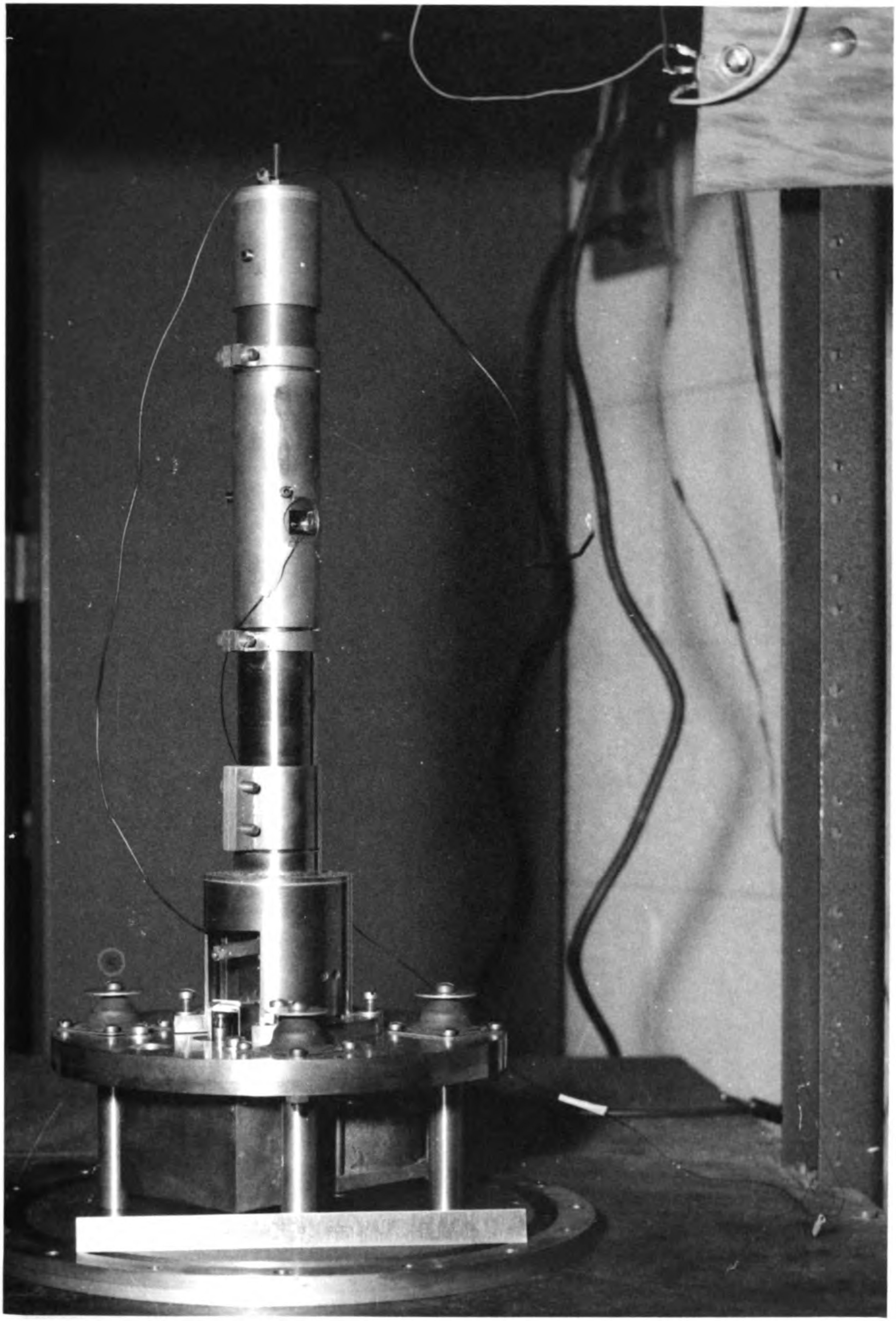

ILLUSTRATION II BALANCE AND OPTICAL SYSTEM

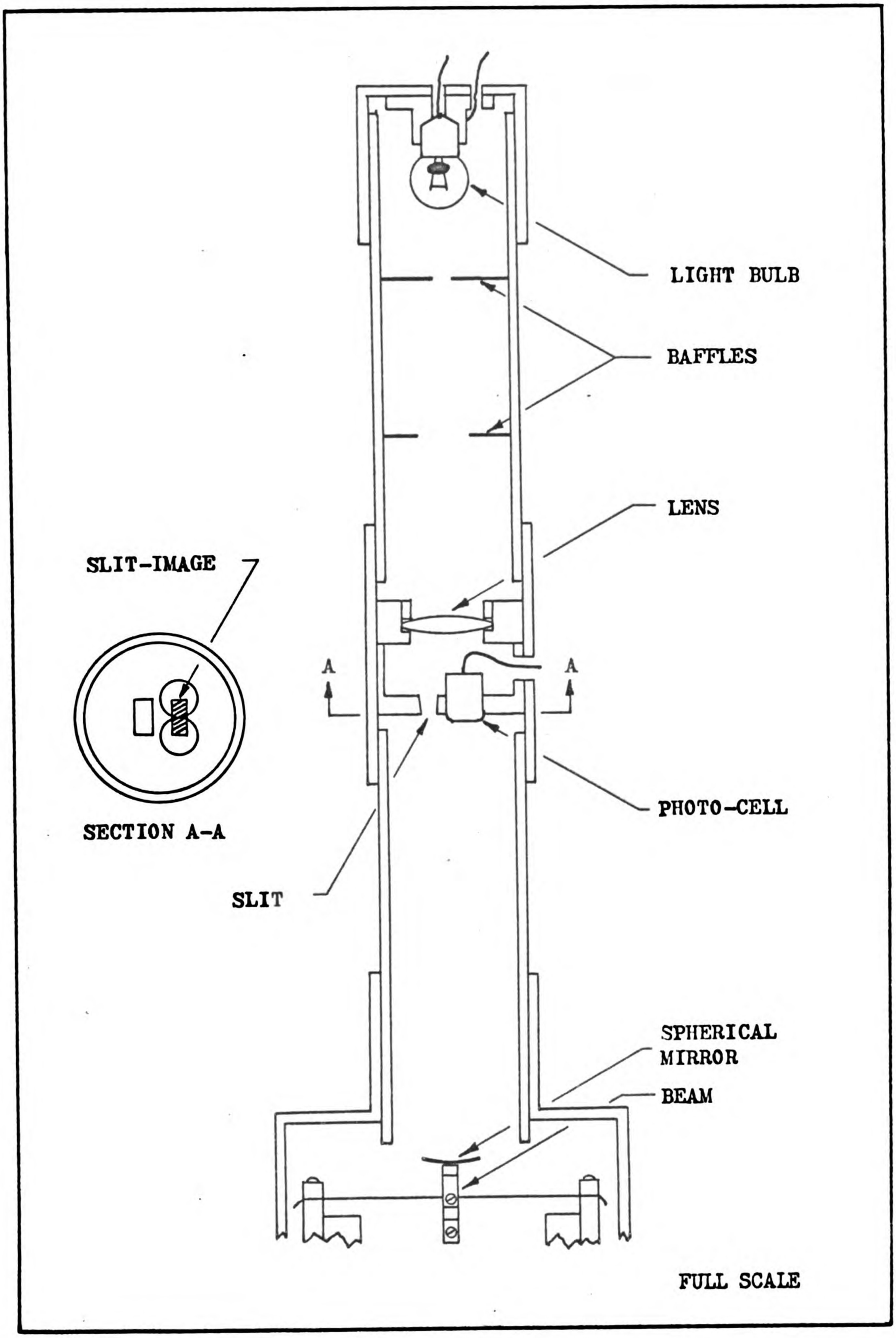

FIGURE 5 OPTICAL SYSTEM

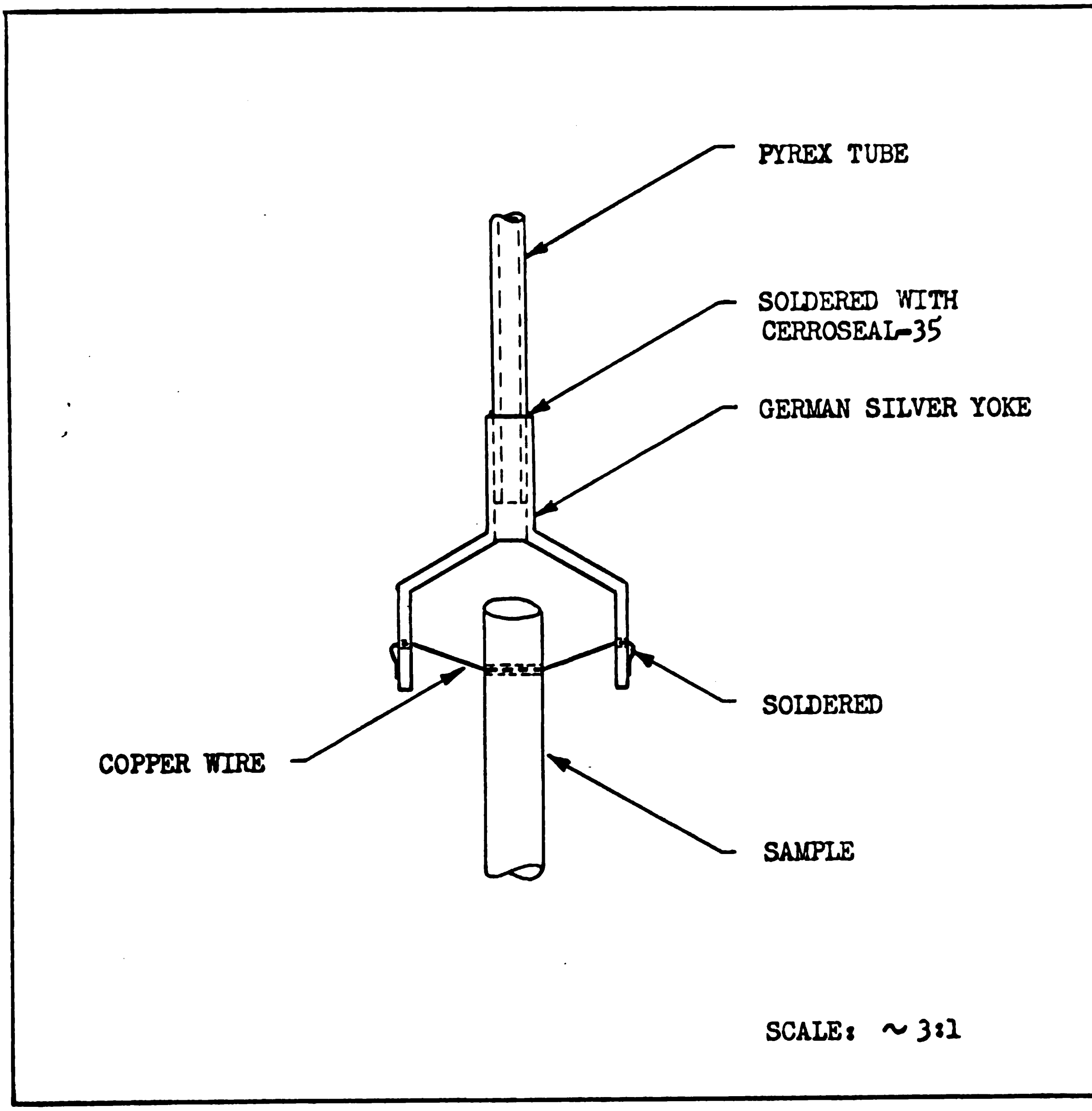

FIGURE 6

SAMPLE HOLDER

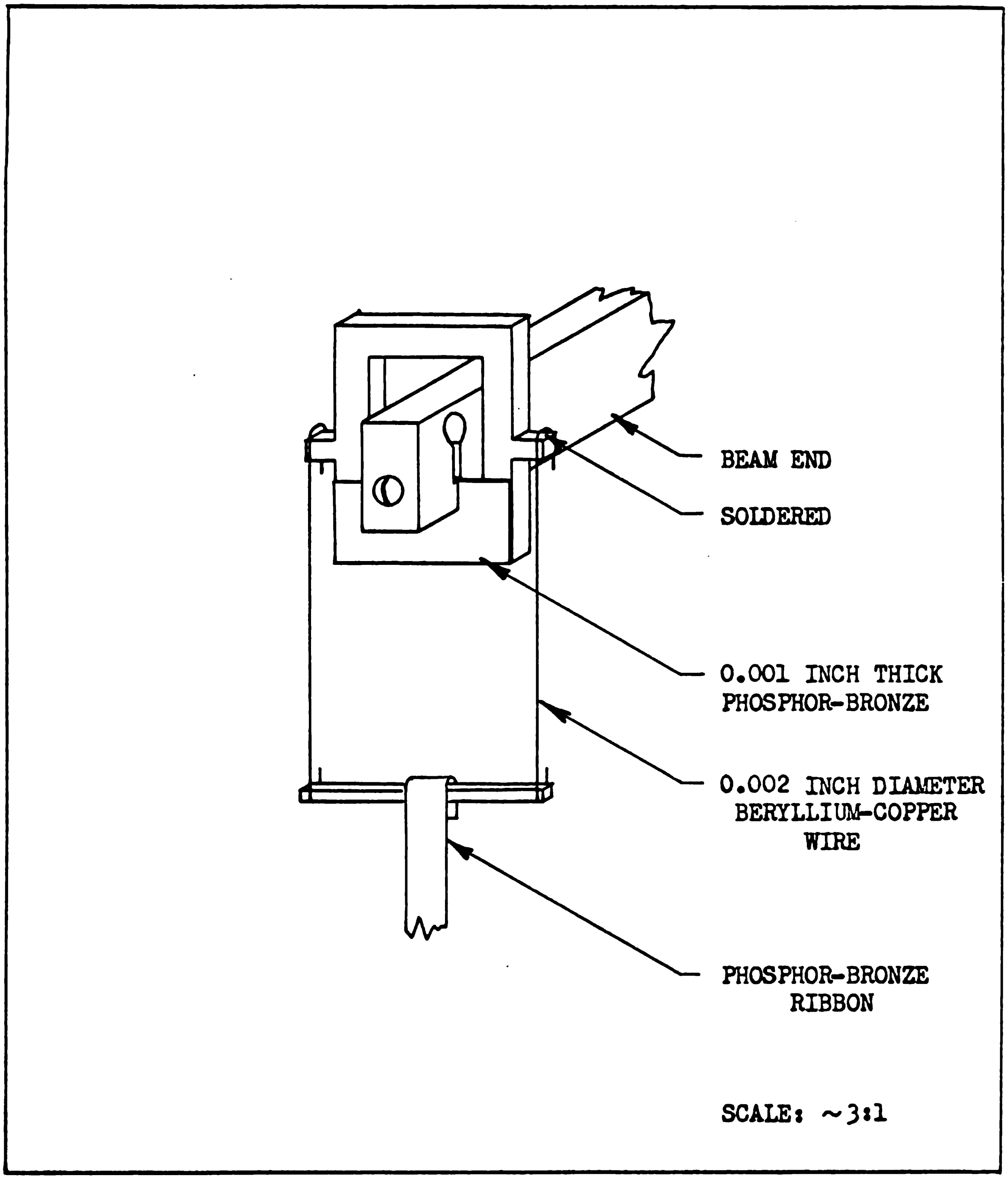

FIGURE 7

IMPROVED BEAM END COUPLING

DC power supply having a stabilization factor of 2,000. When the slit-image illuminates both photo-cells equally the beam is said to be in null position.

The sample is suspended by a 1.5 mm diameter Pyrex tube. The sample is attached to the bottom of the tube with a copper wire which goes through a small horizontal hole in the top of the sample and is soldered at both ends to a german silver yoke (see Figure 6).

The Pyrex sample suspension tube and the voice coil are attached to the beam ends by 0.75" x 0.10" x 0.001" phosphor bronze ribbons, which are pinched in slots by tightening the #0-80 screws at the beam ends. This was later found to be unsatisfactory as the stiffness of the phosphor bronze ribbon increased the effective torsion constant of the beam to about 30,000 dyne-cm/radian. After these experiments the phosphor bronze ribbons were replaced by a yoke suspended by two 0.002 inch diameter wires (see Figure 7). This new suspension, less stiff than the old, and placed so as to raise the center of mass of the beam reduced the effective torsion constant of the beam to 1,000 dyne-cm/radian.

A vane on the bottom of the beam is immersed in glycerin to provide viscous damping of the beam motion.

The beam and optical system are mounted on a brass plate which is isolated from vibration with four rubber vibration mountings (Lord # 106PL-2).

Electronics

The general features of the electronics are illustrated in Figure 8 and the details in Figure 9 and Table I. The circuit is

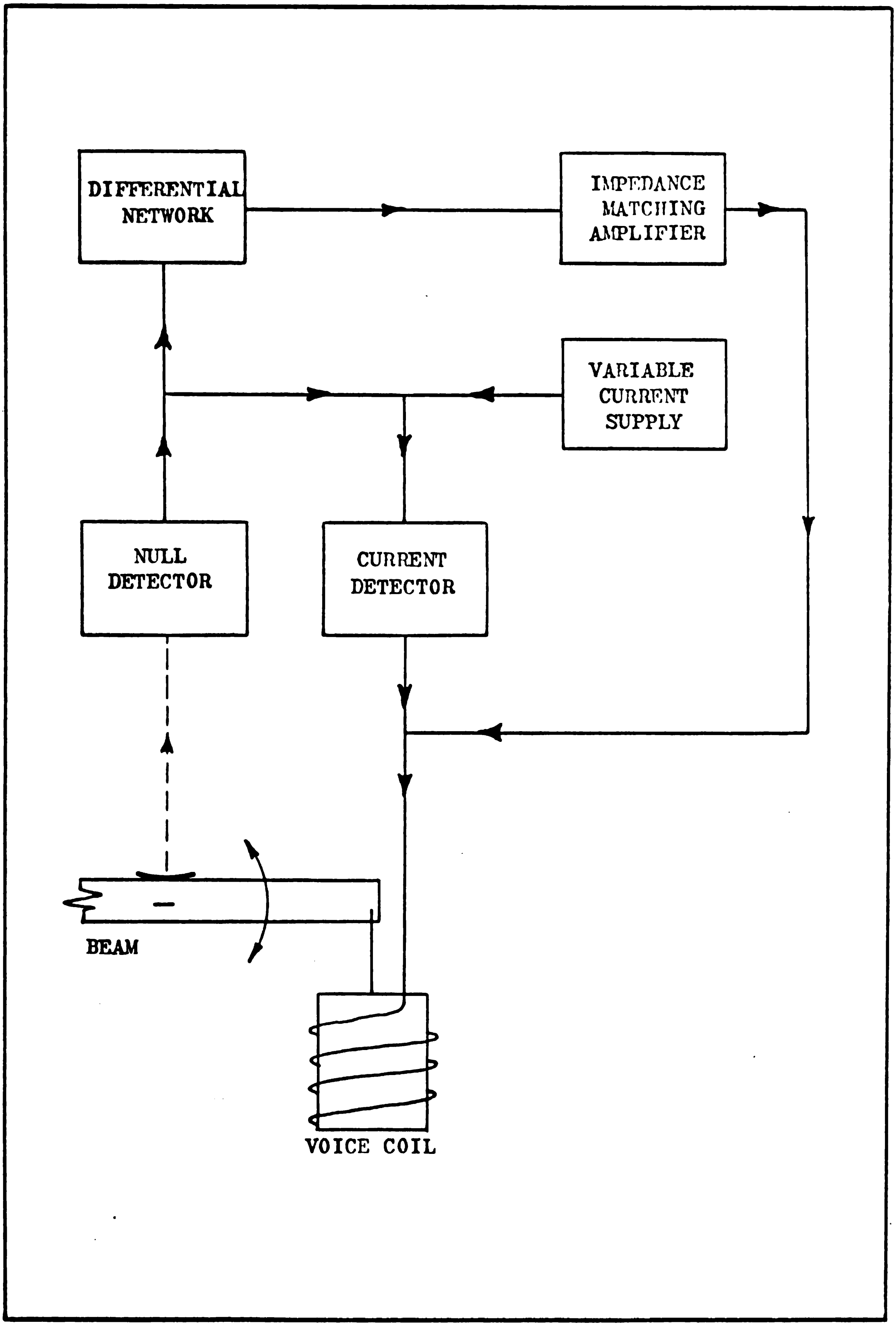

FIGURE 8 SERVOSYSTEM

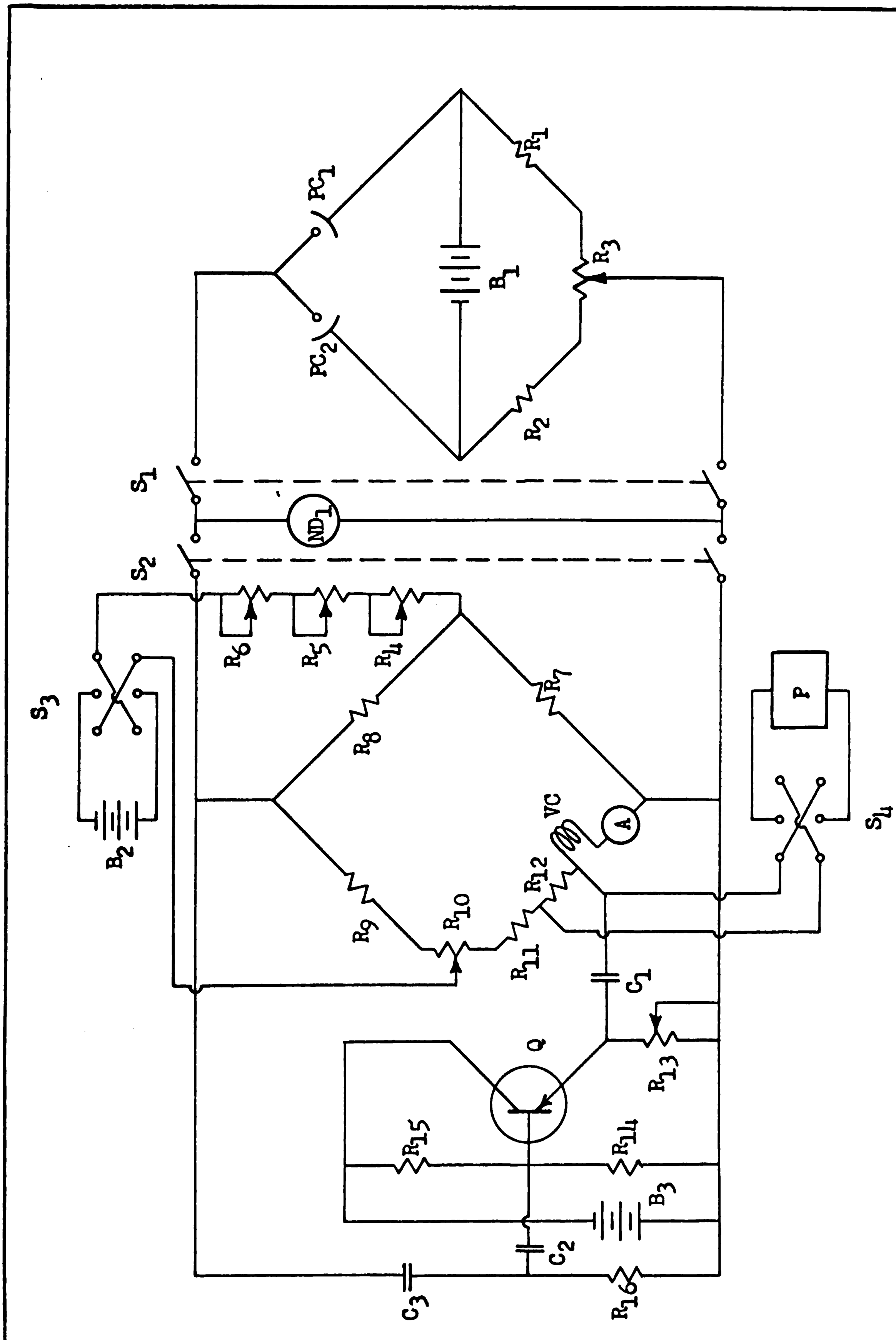

FIGURE 9 CIRCUIT

TABLE I

ELECTRICAL COMPONENTS

Resistors	
R_1, R_2	1 K ohm 1% 1/2 watt wire wound precision
R_3, R_{10}	100 ohm helipot
R_4	300 K ohm helipot
R_5	10 K ohm helipot
R_6	200 ohm helipot
R_7, R_8, R_9	1 K ohm 1% 1/8 watt metal film
R_{11}	800 ohm 1% 1/8 watt metal film
R_{12}	100 ohm 1% 1/2 watt wire wound precision
R_{13}	1 K ohm wire wound potentiometer
R_{14}, R_{15}	39 K ohm 1% 1/8 watt metal film
R_{16}	22 K ohm 1% 1/8 watt metal film
Capacitors	
C_1	300 mfd 10 WVDC
C_2, C_3	50 mfd 10 WVDC
Batteries	
B_1, B_2	Two series connected 6 VDC automobile batteries.
B_3	6 VDC automobile battery
Switches	
S_1, S_2	DPST toggle switch
S_3, S_4	DPDT toggle switch
Transistor	
Q	RCA type 2N247
Photo-cells	
PC_1, PC_2	Clairex type 404SL
Voice coil	
VC	About 75 turns of #40 wire wound in one layer on an aluminum foil form.
Ammeter	
A	5 - 0 - 5 ma (zero center)
Null Detector	
ND_1	Leeds and Northrup DC microvolt amplifier, used on 1 - 0 - 1 mv (zero center) scale
Potentiometer	
P	Leeds and Northrup type K-3, used with a Rubicon galvanometer, catologue no. 3417 sensitivity .005 a/mm

almost identical to one used by Henry.[2]

Null detection is achieved by means of a Leeds and Northrup DC microvolt amplifier attached across a Wheatstone bridge (called the input bridge) which has two photo-cells on adjacent arms.

The DC component of the unbalance voltage goes directly across another Wheatstone bridge (called the output bridge) which contains the voice coil in one arm. This current through the voice coil interacts with a PM speaker magnet partially restoring the beam to null position. This restoring current adds a torsion constant of about 5,000 dyne-cm/radian to the effective mechanical torsion constant of 30,000 dyne-cm/radian; a negligible increase. An operational amplifier is now being tried as a stage of amplification between the input bridge and the output bridge in an effort to increase the electronic torsion constant.

The unbalance voltage of the input bridge is also impressed across a differential network, C_3 and R_{16}. This velocity dependent signal is impressed on the base of the emitter follower connected transistor. The transistor has a current gain of about 35 and affords impedance matching between the differential network and the voice coil. Gain is adjusted by R_{13} for most effective damping. A too high gain setting results in an unstable system and the beam oscillates. The electronic damping is effective in reducing oscillations of the beam caused by mechanical disturbances such as room vibrations and a varying magnetic force on the sample.

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[2]W. G. Henry, Private communication with the author, National Research Council, Ottawa, Canada.

A constant current is supplied through the voice coil by battery, B_2, connected across the output bridge. This current is adjustable by means of the series connected potentiometers R_4, R_5, and R_6. The current direction can be reversed with switch S_3. The purpose of the output bridge is to isolate the battery, B_2, from the input bridge. Current through the voice coil is measured roughly with the ammeter, A, and accurately by measuring the voltage drop across the 100 ohm wire wound resistor, R_{12}, with a Leeds and Northrup type K-3 potentimeter,P.

The input bridge is balanced by replacing the photo-cells with two matched resistors and adjusting R_3 until the null detector, ND_1, reads zero. This is done with S_2 open and S_1 closed. Likewise, the output bridge is balanced by adjusting R_{10} for a zero reading on ND_1 with S_2 closed and S_1 open.

During an experiment the current through the voice coil was adjusted with R_4, R_5, and R_6 so that ND_1 would read zero with S_1 and S_2 closed and the voltage across R_{12} measured with the potentiometer.

IV DESCRIPTION OF EXPERIMENTS

The experiments that will be described were carried out using the same bismuth crystal.

The rotational and absolute susceptibility experiments were done to get the "feel" of the apparatus besides any useful data that might come from them. The DH-VA experiment was done to test the utility of the Gouy method in detecting DH-VA oscillation.

Because the balance was not calibrated the data presented is the voltage drop across R_{12}, called the restoring voltage, necessary to balance out the magnetic force on the sample (see Figure 9). This voltage is the difference between the voltage necessary to balance the gravitational torque on the beam when the magnetic field is zero, called the field-off voltage, and the voltage necessary to balance the beam when the magnetic field is on, called the field-on voltage. Because the field-off voltage drifted at the rate of about two microvolts per minute it was necessary either to take a field-off voltage reading between each field-on reading as done in the absolute susceptibility experiment or to take field-off voltage readings at the beginning and the end of the run and interpolate between by assuming a linear drift with time as done in the rotational and DH-VA experiments.

The dial readings on the magnet current supply were calibrated against the magnetic field with a Rawson type 820 rotating coil fluxmeter (see Table II). The magnet was cycled to about 18,000 gauss two times before the calibration and before each series of measure-

TABLE II

MAGNET CALIBRATION WITH THREE INCH GAP
USING RAWSON TYPE 820 FLUXMETER

Magnet current control dial readings		Measured magnetic field (gauss)	
Course	Fine	(field increasing)	(field decreasing)
0	0	58	68
0	300	124	135
0	500	---	177
0	800	235	241
6	500	305	317
10	500	390	410
23	500	682	700
35	500	951	967
45	500	1,175	1,196
100	500	-----	2,430
135	500	3,196	3,214
165	500	-----	3,886
180	500	4,197	4,220
240	500	5,550	5,561
330	500	7,559	7,577
390	500	8,881	8,912
500	500	11,268	11,296
650	500	14,076	14,100
800	500	16,061	16,090
1,000	500	17,685	------
1,000	1,000	17,723	------

NOTE: The magnet was cycled to maximum field twice before making the measurements.

ments. The increasing field values were used in the following experiments.

Absolute Susceptibility

Measurements were made on the crystal at room temperature with the crystal in a vacuum and with the crystal in one atmosphere of oxygen. The crystal was suspended from the beam with a 0.002 inch diameter beryllium-copper wire thereby allowing the sample to rotate about the vertical axis. The sample could be seen to rotate when a field of 100 gauss was applied and, therefore, the sample was assumed to align itself with the direction of minimum (minimum absolute value of χ) susceptibility in the plane of rotation along the direction of the magnetic field.

One atmosphere of oxygen was admitted into the evacuated vacuum can directly from the storage bottle. The pressure was measured with a bellows-type gauge to an estimated accuracy of ± 0.5 inches of Hg.

Data from these measurements are presented in Tables VI and VII. It can be seen from the voltage readings repeated at the same magnetic field that the readings are reproducible to 30 microvolts making the values* for the restoring voltage with fields less than 2,000 gauss meaningless. However, the error in the restoring voltage values at 17,685 gauss due to voltage measuring errors is only 0.02%.

The downward magnetic force, in dynes, acting on a cylindrical sample of uniform cross sectional area, A, in a medium is

$$\mathcal{F}_{sm} = (\chi_s \rho_s - \chi_m \rho_m)(H_1^2 - H_2^2)\,\frac{A}{2} \;,$$

---

*Refers to $V_{sv} - V_{sm}$ of Table III.

where the subscripts s and m refer to the sample and the medium surrounding the sample respectively, χ is the specific susceptibility, ρ the density, H_1 the field at the bottom of the sample, and H_2 the field at the top of the sample (values of H_2 were estimated from Figure 2). The subscript v replaces m when the medium is a vacuum.

In the case where the susceptibility is independent of the field a plot of $\mathcal{F}$ versus $(H_1^2 - H_2^2)$ is a line of constant slope. However, the plot of restoring voltage versus $(H_1^2 - H_2^2)$ is a line of decreasing slope indicating a non-linear relation between the restoring voltage and the force on the sample (see Figure 10).

Using the above equation for the force the absolute susceptibility of the sample may be calculated from

$$\chi_s = \chi_m \frac{\rho_m}{\rho_s} \frac{\mathcal{F}_{sv}}{\mathcal{F}_{sv} - \mathcal{F}_{sm}} ,$$

or if the restoring voltage is directly proportional to the magnetic force

$$\chi_s = \chi_m \frac{\rho_m}{\rho_s} \frac{V_{sv}}{V_{sv} - V_{sm}} .$$

The calculations of χ_s are tabulated in Table III using, $\chi_m = 106.2 \times 10^{-6}$,[1] $\rho_m = 1.33 \times 10^{-3}$ g/cc,[2] $\rho_s = 9.86$ g/cc.[3]

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[1]Charles D. Hodgmen (ed.), Handbook of Chemistry and Physics (41st ed.), (Chemical Rubber Publishing Co., Cleveland, 1959), p. 2651.

[2]Robert W. Vance and W. M. Duke (eds.), Applied Cryogenic Engineering (John Wiley and Sons, New York, 1962), p. 440, Fig. A.6.

[3]H. M. Trent, D. E. Stone, and R. Bruce Lindsay (eds.), "Density of Solids," American Institute of Physics Handbook (1st ed.), (McGraw-Hill Book Company, New York, 1957), p. 2-17.

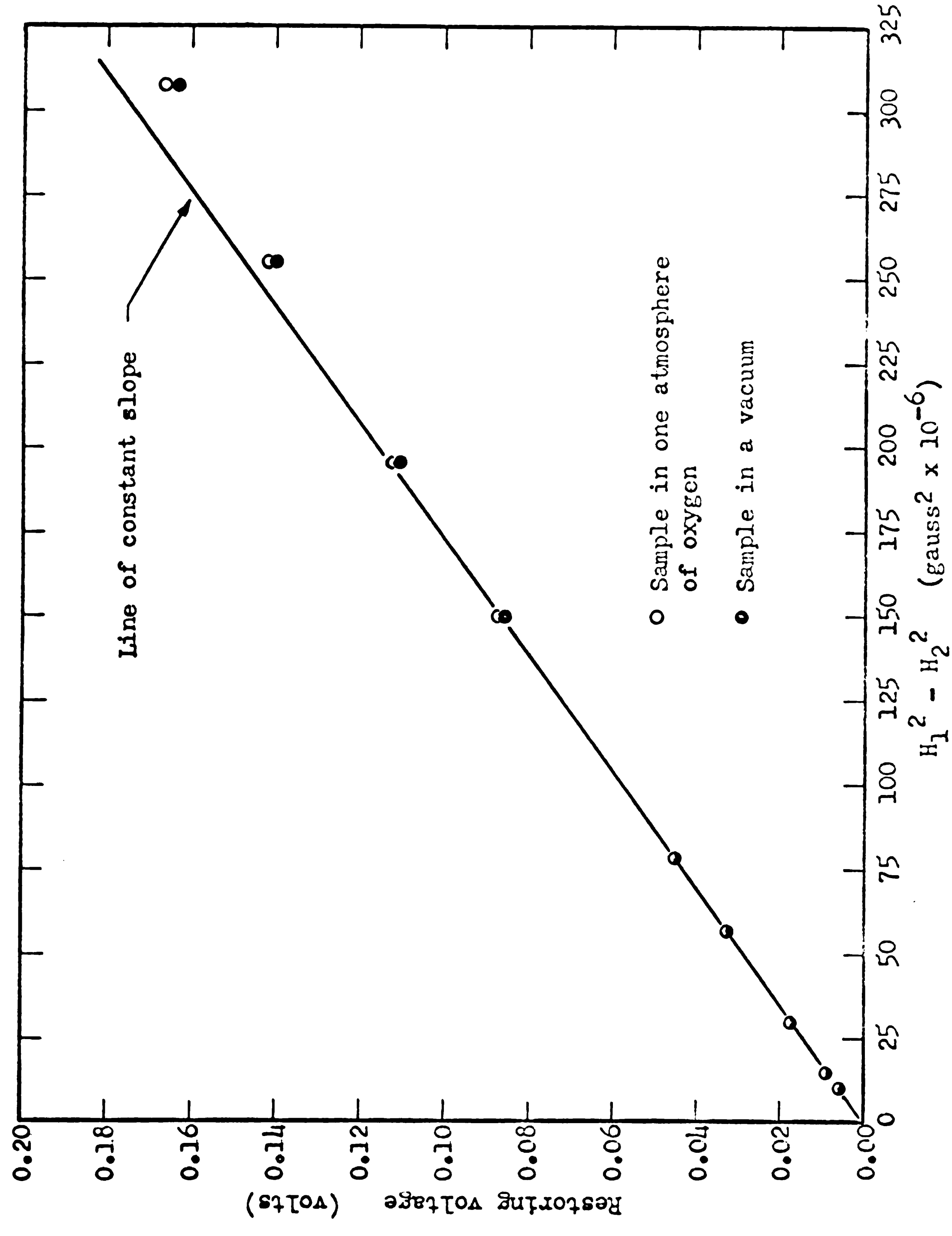

FIGURE 10

EFFECT OF SURROUNDING THE SAMPLE WITH ONE ATMOSPHERE OF OXYGEN ON THE RESTORING VOLTAGE, ROOM TEMPERATURE

TABLE III

CALCULATIONS OF THE ABSOLUTE SUSCEPTIBILITY AT ROOM TEMPERATURE

Field (gauss)	Restoring voltage V_{sv} (volts)	Restoring voltage V_{sm} (volts)	$V_{sv} - V_{sm}$ (volts)	$V_{sv}/(V_{sv} - V_{sm})$	Susceptibility χ_s (cgs x 10^{-6})
390	0.00008	0.00011	-0.00003	-2.60	-0.03
951	0.00054	0.00052	0.00004	11.2	0.16
1,175	0.00084	0.00081	0.00003	31.0	0.44
2,410	0.00333	0.00343	-0.00009	-35.8	-0.51
3,196	0.00591	0.00601	-0.00010	-59.1	-0.85
3,860	0.00867	0.00883	-0.00015	-56.3	-0.81
4,197	0.01024	0.01036	-0.00012	-88.5	-1.27
5,550	0.01784	0.01807	-0.00023	-78.3	-1.12
7,559	0.03302	0.03349	-0.00047	-70.4	-1.01
8,881	0.04567	0.04632	-0.00065	-69.8	-1.00
12,230	0.08574	0.08715	-0.00142	-60.5	-0.87
14,076	0.11066	0.11231	-0.00165	-67.1	-0.96
16,061	0.13972	0.14167	-0.00194	-72.0	-1.03
17,685	0.16361	0.16592	-0.00232	-70.6	-1.01

[a]The magnetic field at the high field end of the sample.

[b]The voltage drop across R_{12} required to balance the magnetic force on the sample with the sample in a vacuum.

[c]The voltage drop across R_{12} required to balance the magnetic force on the sample with the sample in one atmosphere of oxygen.

[d]The specific susceptibility at room temperature calculated from $\chi_s = \chi_m \rho_m V_{sv}/\rho_s(V_{sv} - V_{sm})$, where $\chi_m \rho_m/\rho_s = 1.43 \times 10^{-8}$ cgs.

Because the percentage error, due to the 30 microvolt uncertainty in voltage measurements, in the value of $V_{sv} - V_{sm}$ is the smallest ($\sim 2\%$) for the 17,685 gauss case; the susceptibility of the crystal for the previously described orientation is thought to be -1.01×10^{-6} cgs $\pm 2\%$. The values of V_{sv} and V_{sm} are so close that the non-linearity of the balance has little effect on the above calculation.

Rotational Experiment

In an effort to determine the angle between the trigonal axis of the crystal and the vertical two series of room temperature measurements were made by rotating the magnet 190 degrees about a vertical axis in 15 degree steps. Measurements were made at fields of 4,200 gauss and 11,270 gauss. The sample was suspended from the beam with a Pyrex tube as described in the section, Balance and Optical System. The data from these measurements are listed in Tables VIII, IX, X, and XI and plotted in Figures 11 and 12.

At room temperature bismuth has two principal susceptibilities, one parallel to the trigonal axis of $\chi_{\parallel} = -1.053 \times 10^{-6}$ and the other perpendicular to the trigonal axis of $\chi_{\perp} = -1.482 \times 10^{-6}$.[4]

If the trigonal axis, along OP (see Figure 13), makes an angle ϕ with the vertical and the magnetic field is in the x direction the observed susceptibility is[5]

$$(\chi_{\parallel} \sin^2\phi + \chi_{\perp} \cos^2\phi)\cos^2\theta + \chi_{\perp} \sin^2\theta \ .$$

[4] L. F. Bates, Modern Magnetism (Cambridge Universtiy Press, London, 1961), p. 172.

[5] Ibid., p. 173.

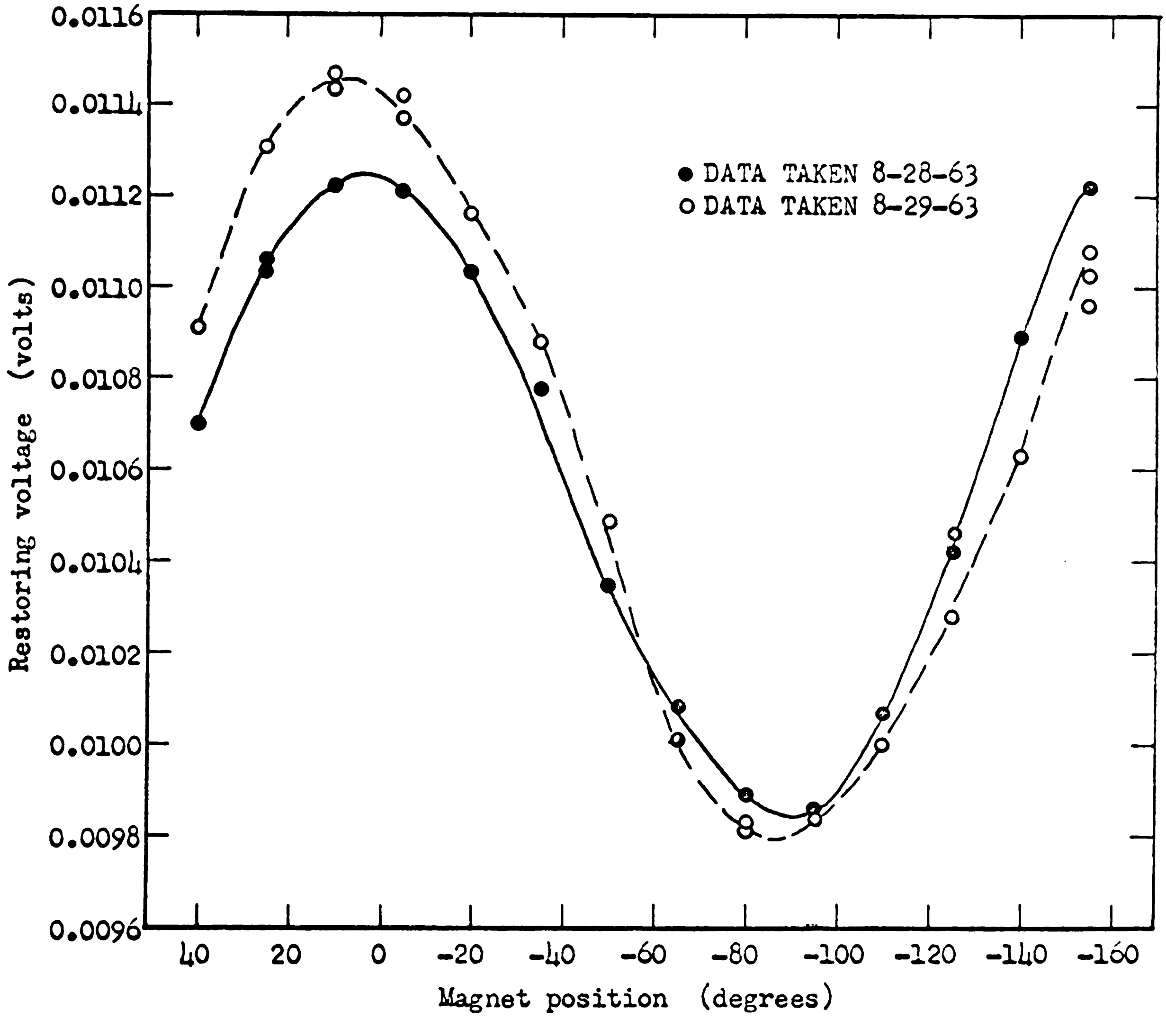

FIGURE 11

DEPENDENCE OF RESTORING VOLTAGE ON MAGNET POSITION,
ROOM TEMPERATURE, 4,200 GAUSS

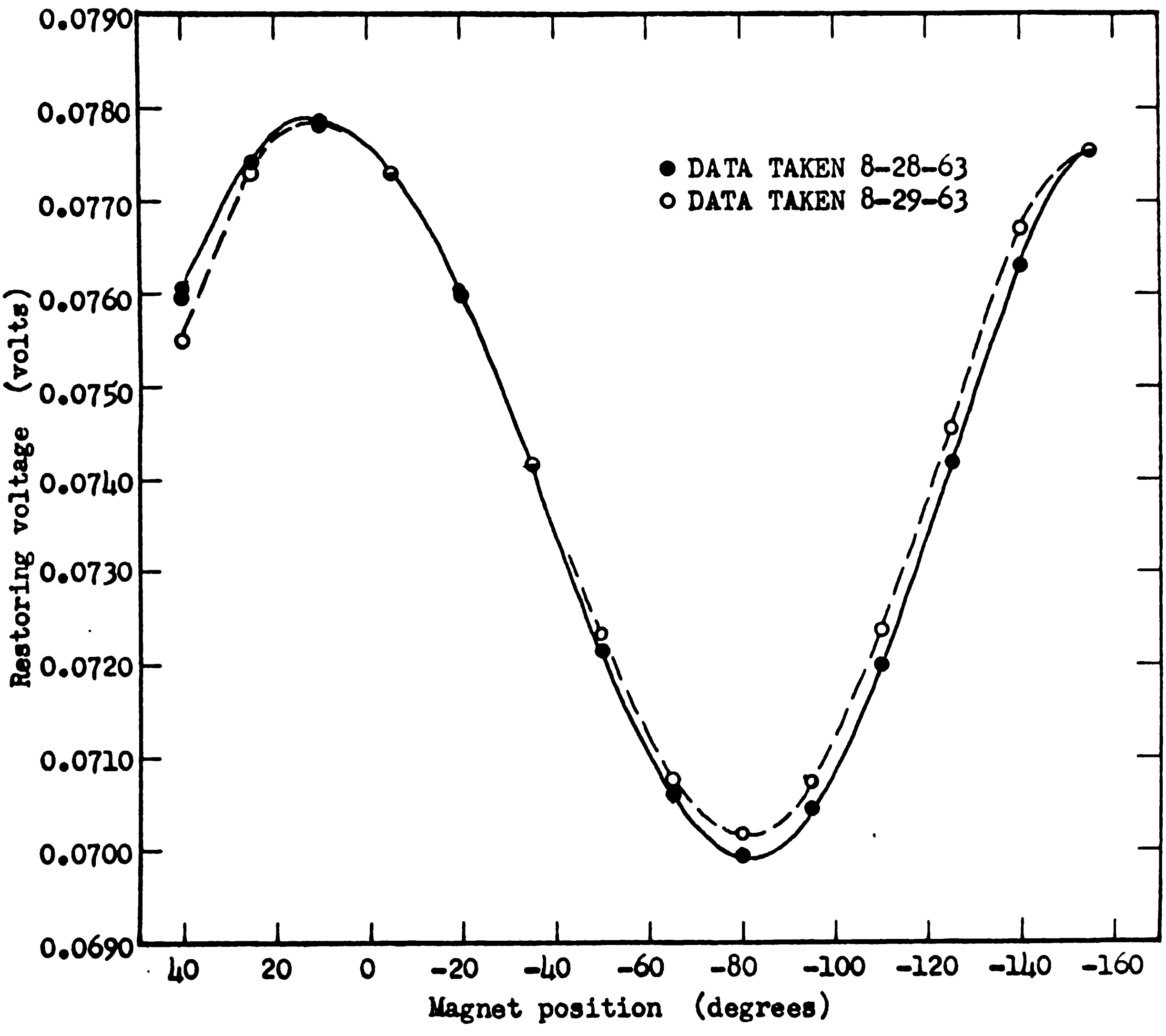

FIGURE 12

DEPENDENCE OF RESTORING VOLTAGE ON MAGNET POSITION,
ROOM TEMPERATURE, 11,270 GAUSS

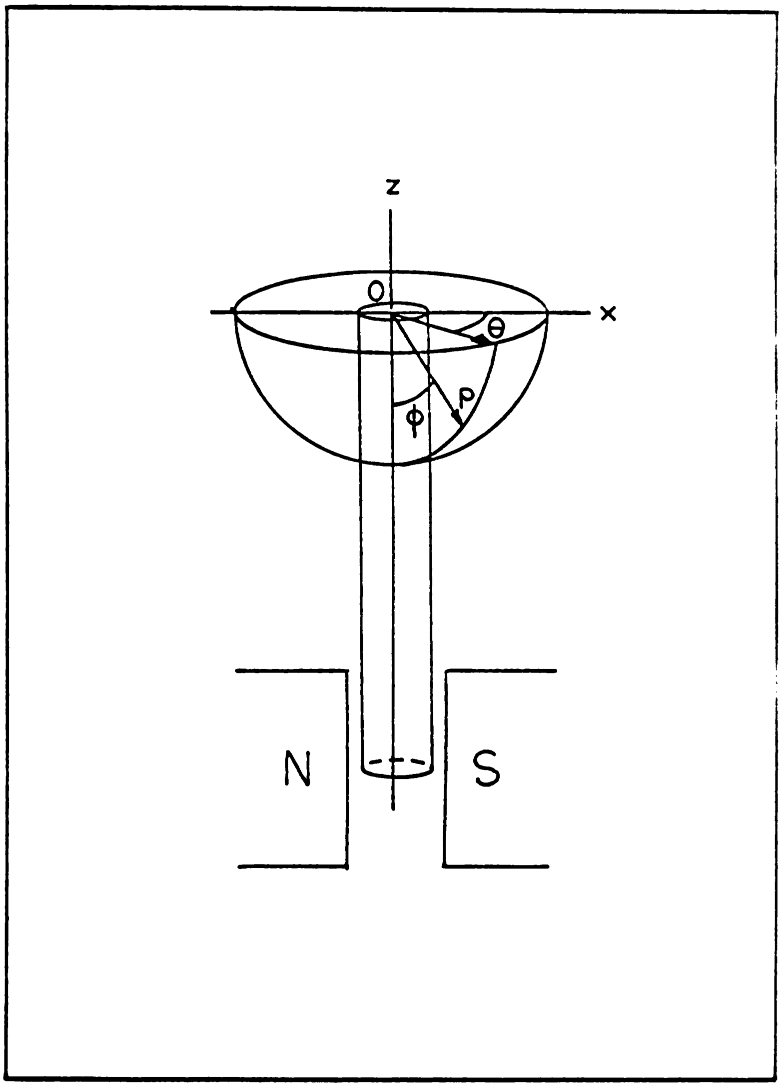

FIGURE 13

CRYSTAL ORIENTATION COORDINATES

TABLE IV

ORIENTATION OF THE TRIGONAL AXIS CALCULATED FROM THE ROTATIONAL DATA

Date	Field (gauss)	Minimum voltage V_{mim} (volts)	Maximum voltage V_{max} (volts)	$\chi_{min}/\chi_{max} = V_{mim}/V_{max}$	Angle[a] ϕ (degrees)
8-28-63	4,200	0.00980	0.01126	0.870	44
8-29-63	4,200	0.00978	0.01146	0.861	45
8-28-63	11,270	0.06903	0.07788	0.887	36
8-29-63	11,270	0.07015	0.07780	0.902	34

[a]The angle between the trigonal axis and the vertical calculated from the data.

As the crystal is rotated about its vertical axis a maximum susceptibility, $\chi_{max} = \chi_{\perp}$ will be observed at $\theta = 90^{o}$ and a minimum susceptibility, $\chi_{min} = \chi_{\parallel} \sin^2\phi + \chi_{\perp}\cos^2\phi$, at $\theta = 0^{o}$. The ratio of these observed susceptibilities is

$$\frac{\chi_{min}}{\chi_{max}} = \frac{\chi_{\parallel}}{\chi_{\perp}} \sin^2\phi + \cos^2\phi \ .$$

By using the above values for $\chi_{\parallel}$ and $\chi_{\perp}$ and the experimental values of $\chi_{min}/\chi_{max} = V_{min}/V_{max}$ listed in Table IV, ϕ can be found from

$$\frac{\chi_{min}}{\chi_{max}} = 0.724 \sin^2\phi + \cos^2\phi = 1 - 0.276 \sin^2\phi \ .$$

The value of ϕ was calculated to be 45^{o} at 4,200 gauss and 35^{o} at 11,270 gauss (see Table IV). This calculation may be unreliable for the following reason. In the process of cutting the sample from the larger crystal a small piece broke off the top of the sample. Crude measurements showed that the angle between the normal to the surface of the break and the vertical axis of the sample was $16^{o} \pm 2^{o}$. If it is assumed that the sample cleaved along the basal plane,[6] then the trigonal axis would make an angle of 16^{o} with the vertical axis of the sample. This assumption is borne out moreover by results described in the next section.

Ferromagnetic contamination of the sample would make the measured minimum susceptibility less (in absolute value) and would also decrease the measured rotational values of χ_{min}/χ_{max}, the low field values more than the high field values, thereby accounting qualitatively for this descrepancy.

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[6] *Ibid.*, p. 174.

De Haas-Van Alphen Effect

The crystal was suspended by the Pyrex tube as in the rotational measurements and the magnet was rotated to the direction of maximum room temperature susceptibility. In this orientation the magnetic field is perpendicular to the trigonal axis of the crystal, i.e., $\chi = \chi_{max} = \chi_{\perp}$. The measurements were made at 4.2°K.

The data from these measurements are listed in Table XII. The low field at the top of the sample has been neglected as this decreases the restoring voltage at 17,685 gauss by only 2% and less at lower fields.

The restoring voltage, V, versus magnetic field curve (see Figure 14) clearly shows the quadratic rise in the restoring voltage. The oscillations can be seen more clearly in the plot of V/H^2 versus H (see Figure 15) and the plot of V/H^2 versus 1/H illustrates the periodic variation in 1/H (see Figure 16). The double peak at 13,000 gauss is attributed to spin splitting of the Landau levels by Kunzler at al..[7]

Comparison of the position of the susceptibility peaks and the period of the oscillations are compared with magnetothermal oscillations[8] with the field along the binary axis of the crystal in Table V. The close agreement shown in Table V indicates that the field was nearly parallel to the binary axis of the crystal when the susceptibility measurements referred to in the preceding section were made.

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[7]J. E. Kunzler, F. S. L. Hsu, and W. S. Boyle, Phys. Rev. III, 128, 1094 (1962).

[8]*Ibid.*, p. 1090.

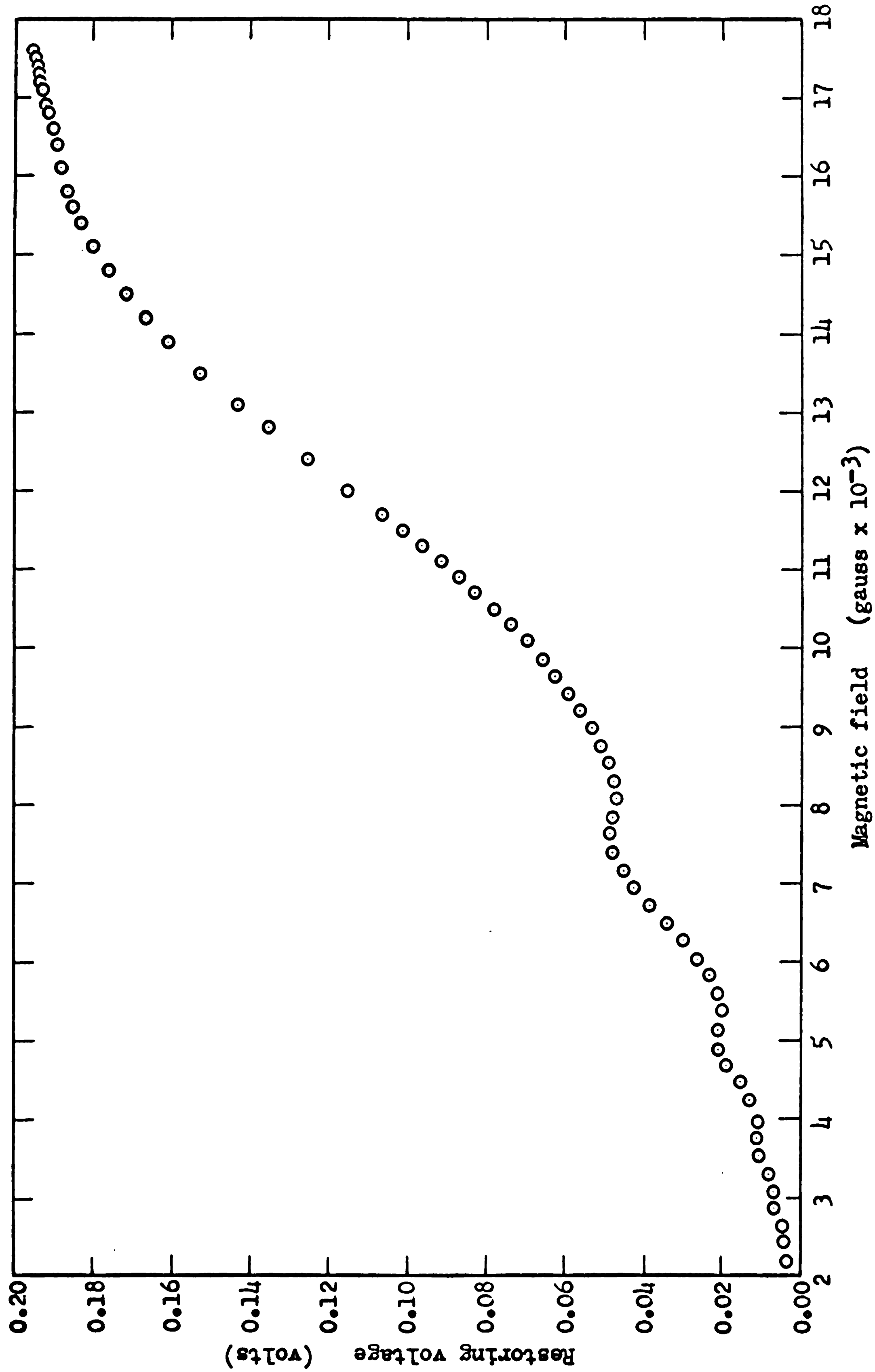


FIGURE 14

DEPENDENCE OF RESTORING VOLTAGE ON THE MAGNETIC FIELD, 4.2°K

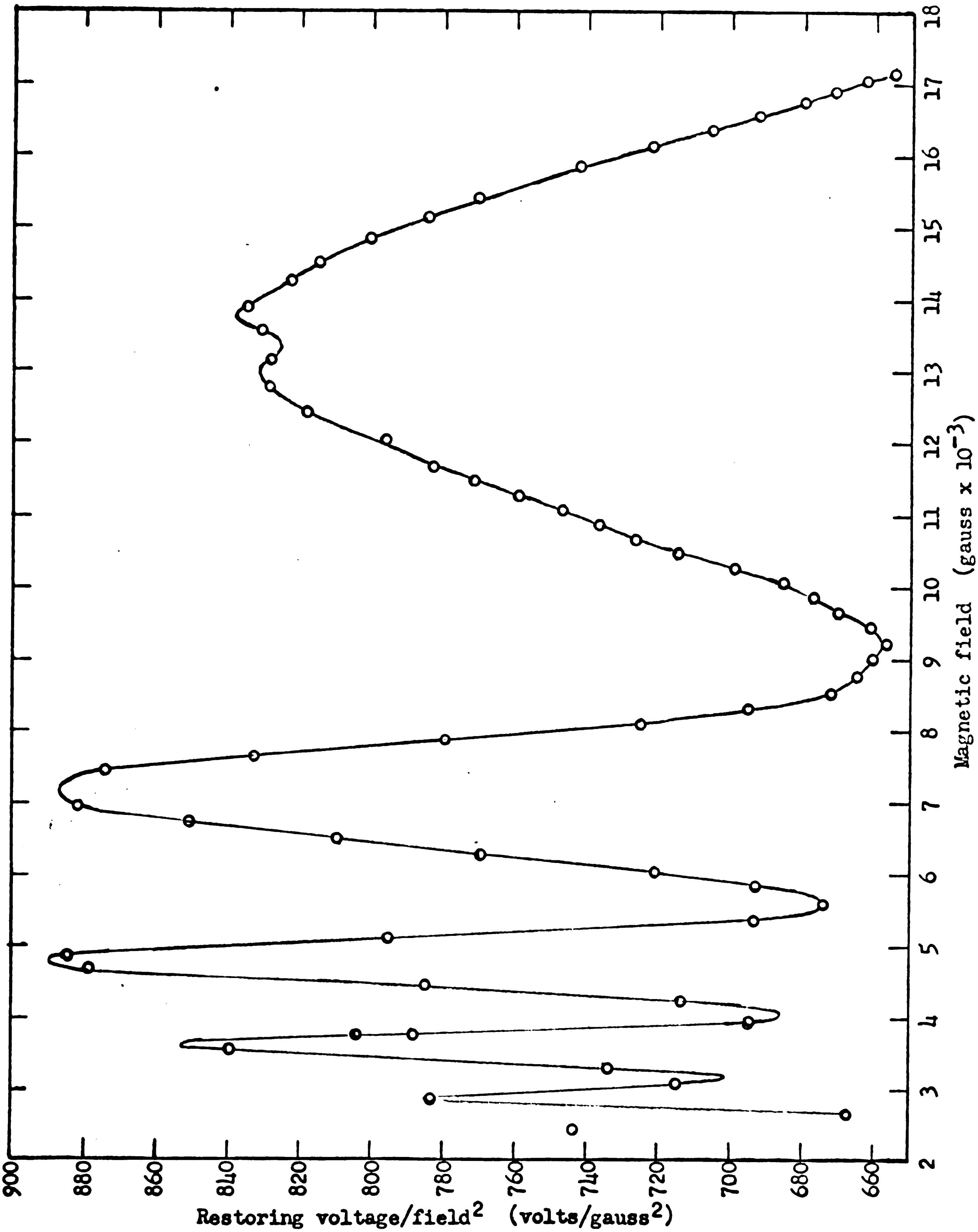
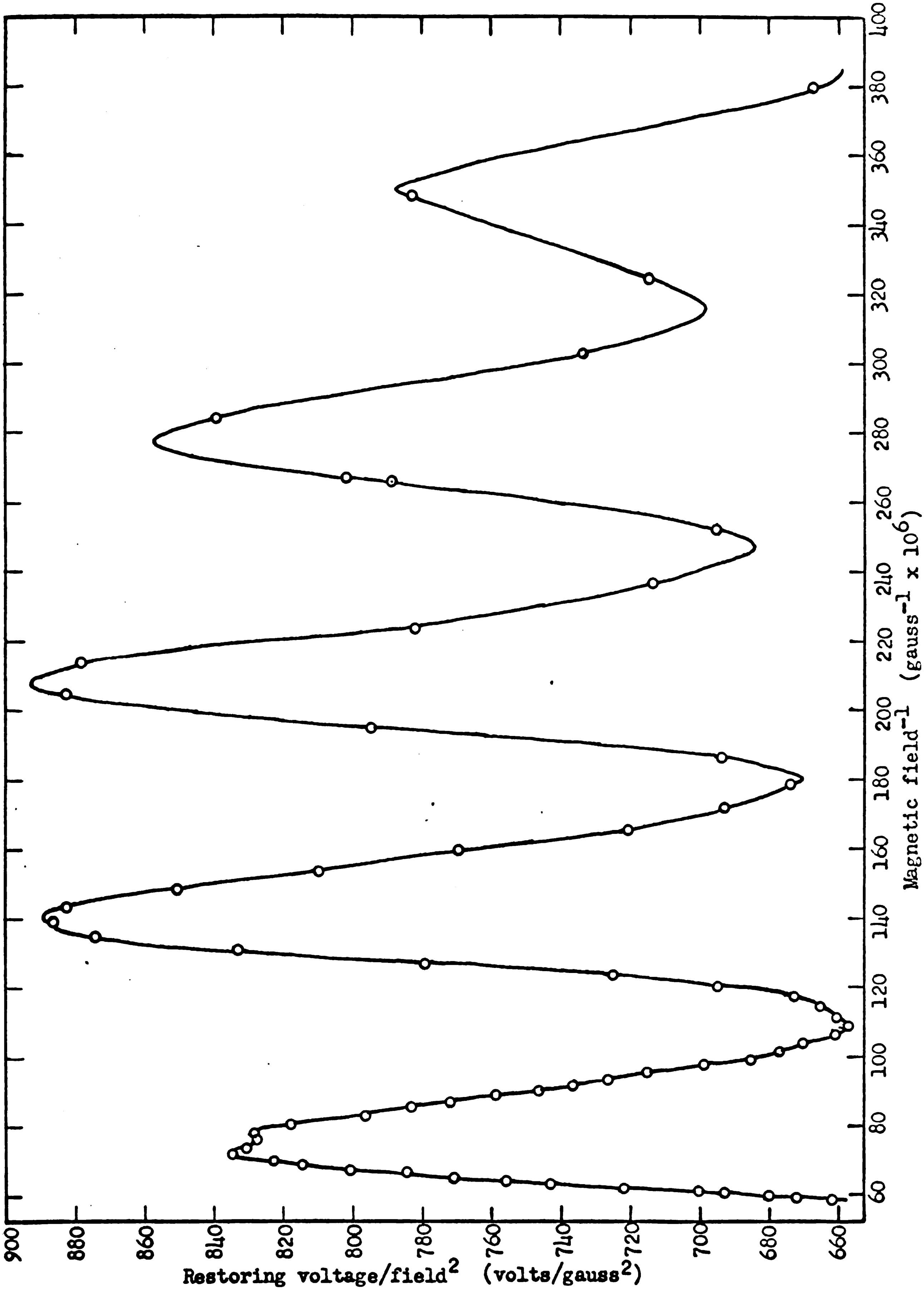


FIGURE 15

FIELD DEPENDENCE OF DH-VA OSCILLATIONS, 4.2°K



Restoring voltage/field^2 (volts/gauss^2)
900 880 860 840 820 800 780 760 740 720 700 680 660
Magnetic field^{-1} (gauss^{-1} x 10^6)
60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400

FIGURE 16

TABLE V

COMPARISON OF SUSCEPTIBILITY PEAKS AND MAGNETOTHERMAL PEAKS

Peak number p	Susceptibility peaks Field H (gauss)		$10^7/pH$	Magnetothermal peaks H // binary axis (gauss)	$10^7/pH$
1	13,750 12,940	13,330	745	15,500 14,200	673
2	7,170		703	7,150	700
3	4,800		693	4,720	703
4	3,610		693	3,750	700
5	2,850		701	-----	---

V DISCUSSION

This work has successfully shown that the Gouy method is useful for detecting De Haas-Van Alphen oscillations. Moreover, the method has the advantage that the oscillations in free energy are measured rather than those in susceptibility. Oscillations in metals should readily be observable providing the sensitivity of the apparatus is increased by an order of magnitude.

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VII APPENDIX

TABLE VI

ABSOLUTE SUSCEPTIBILITY DATA, TAKEN AT ROOM TEMPERATURE (22°C) WITH THE SAMPLE IN A VACUUM

Low field[a] H_2 (gauss)	High field[b] H_1 (gauss)	$H_1^2 - H_2^2$ (gauss^2 x 10^{-6})	Voltage[c] (volts)	Restoring voltage[d] (volts)
0	0	0	0.07446	-------
0	0	0	0.07438	-------
0	390	0.152	0.07430	0.00008
0	0	0	0.07437	-------
0	951	0.904	0.07385	0.00054
0	0	0	0.07441	-------
0	1,175	1.380	0.07357	0.00084
0	0	0	0.07441	-------
0	2,410	5.808	0.07109	0.00333
0	0	0	0.07443	-------
0	3,196	10.21	0.06852	0.00591
0	0	0	0.07443	-------
0	3,860	14.90	0.06574	0.00867
0	0	0	0.07441	-------
0	4,197	17.62	0.06416	0.01024
0	0	0	0.07439	-------
20	5,550	30.80	0.05657	0.01784
0	0	0	0.07443	-------
90	7,559	57.13	0.04140	0.03302
0	0	0	0.07442	-------
200	8,881	78.83	0.02875	0.04567
0	0	0	0.07441	-------
860	12,230	148.8	-0.01131	0.08574
0	0	0	0.07443	-------
1,320	14,076	196.4	-0.03624	0.11066
0	0	0	0.07441	-------
1,740	16,061	255.0	-0.06533	0.13972
0	0	0	0.07437	-------
2,220	17,685	307.8	-0.08924	0.16362
2,220	17,685	307.8	-0.08921	0.16359
0	0	0	0.07438	-------
0	0	0	0.07437	-------

[a]The magnetic field at the low field end of the sample.

[b]The magnetic field at the high field end of the sample.

[c]The voltage drop across R_{12} (see Figure 9) with the beam restored to null position.

[d]The voltage drop across R_{12} required to balance the magnetic force on the sample.

TABLE VII

ABSOLUTE SUSCEPTIBILITY DATA, TAKEN AT ROOM TEMPERATURE (22^oC) WITH THE SAMPLE IN ONE ATMOSPHERE OF OXYGEN

Low field H_2 (gauss)	High field H_1 (gauss)	$H_1^2 - H_2^2$ (gauss^2 x 10^{-6})	Voltage (volts)	Restoring voltage (volts)
0	0	0	0.07146	-------
0	390	0.152	0.07135	0.00011
0	0	0	0.07144	-------
0	951	0.904	0.07092	0.00052
0	0	0	0.07144	-------
0	1,175	1.380	0.07066	0.00081
0	0	0	0.07149	-------
0	0	0	0.07147	-------
0	2,410	6.808	0.06807	0.00343
0	0	0	0.07153	-------
0	0	0	0.07151	-------
0	3,196	10.21	0.06550	0.00601
0	0	0	0.07150	-------
0	3,860	14.90	0.06268	0.00883
0	0	0	0.07150	-------
0	4,197	17.62	0.06116	0.01034
0	4,197	17.62	0.06113	0.01038
0	0	0	0.07150	-------
20	5,550	30.80	0.05344	0.01807
0	0	0	0.07152	-------
90	7,559	57.13	0.03802	0.03349
0	0	0	0.07151	-------
0	0	0	0.07146	-------
200	8,881	78.83	0.02514	0.04631
200	8,881	78.83	0.02512	0.04633
0	0	0	0.07144	-------
0	0	0	0.07148	-------
860	12,230	148.8	-0.01570	0.08715
860	12,230	148.8	-0.01570	0.08715
0	0	0	0.07143	-------
0	0	0	0.07146	-------
1,320	14,076	196.4	-0.04085	0.11231
0	0	0	0.07146	-------
0	0	0	0.07143	-------
1,740	16,061	255.0	-0.07024	0.14167
0	0	0	0.07143	-------
0	0	0	0.07144	-------
2,220	17,685	307.8	-0.09448	0.16592
2,220	17,685	307.8	-0.09448	0.16592
0	0	0	0.07144	-------

NOTE: See Table VI footnotes for an explanation of the headings.

TABLE VIII

ROTATIONAL DATA, AT ROOM TEMPERATURE (22°C) AND AT A FIELD OF 4,200 GAUSS, TAKEN AUGUST 28, 1963

Time[a]	Magnet position[b] (degrees)	Field[c] (gauss)	Field-on voltage[d] V_1 (volts)	Field-off voltage[e] V_2 (volts)	Restoring voltage[f] $V_2 - V_1$ (volts)
7:42	40	0	-------	0.16158	-------
7:47	40	0	-------	0.16165	-------
7:49	40	0	-------	0.16168	-------
7:56	40	0	-------	0.16161	-------
8:00	40	4,200	0.15096	0.16166	0.01070
----	25	4,200	0.15061	0.16167	0.01106
8:04	25	4,200	0.15065	0.16168	0.01103
8:08	10	4,200	0.15047	0.16169	0.01122
8:10	10	4,200	0.15046	0.16169	0.01123
----	-5	4,200	0.15049	0.16170	0.01121
8:18	-20	4,200	0.15068	0.16171	0.01103
8:25	-35	4,200	0.15104	0.16172	0.01068
----	-50	4,200	0.15138	0.16173	0.01035
8:34	-50	4,200	0.15140	0.16174	0.01034
8:37	-65	4,200	0.15167	0.16175	0.01008
8:54	-80	4,200	0.15190	0.16179	0.00989
8:57	-95	4,200	0.15195	0.16180	0.00985
8:59	-95	4,200	0.15194	0.16180	0.00986
9:00	-110	4,200	0.15173	0,16180	0.01007
9:02	-110	4,200	0.15174	0,16181	0.01007
9:05	-125	4,200	0.15135	0.16181	0.01046
9:07	-125	4,200	0.15139	0.16182	0.01042
9:11	-140	4,200	0.15094	0.16183	0.01089
----	-155	4,200	0.15061	0.16183	0.01122
----	-155	4,200	0.15060	0.16183	0.01123
----	-155	0	-------	0.16183	-------

[a]The time at which the reading of V_1 or V_2 was taken.

[b]The position of the magnet relative to an arbitrarily picked position.

[c]The magnetic field at the high field end of the sample.

[d]The voltage drop across R_{12} (see figure 9) with the field on and the beam restored to null position.

[e]The voltage drop across R_{12} with the field off and the beam restored to null position. As the "field-off" readings are taken only at the beginning and the end of a run; the values listed when the field is on are obtained by assuming a linear drift with time.

[f]The voltage drop across R_{12} required to balance the magnetic force on the sample.

TABLE IX

ROTATIONAL DATA, AT ROOM TEMPERATURE (23°C) AND AT A FIELD OF 4,200 GAUSS, TAKEN AUGUST 29, 1963

Time	Magnet position (degrees)	Field (gauss)	Field-on voltage V_1 (volts)	Field-off voltage V_2 (volts)	Restoring voltage $V_2 - V_1$ (volts)
6:01	-155	0	-------	0.16339	-------
6:06	-155	4,200	0.15231	0.16339	0.01108
6:07	-155	4,200	0.15236	0.16339	0.01103
6:09	-155	4,200	0.15238	0.16340	0.01102
6:11	-155	4,200	0.15244	0.16340	0.01096
6:15	-140	4,200	0.15277	0.16340	0.01063
6:19	-125	4,200	0.15313	0.16341	0.01028
----	-110	4,200	0.15341	0.16341	0.01000
----	-95	4,200	0.15357	0.16341	0.00984
----	-80	4,200	0.15360	0.16341	0.00981
6:27	-80	4,200	0.15358	0.16342	0.00984
----	-80	4,200	0.15359	0.16342	0.00983
6:29	-65	4,200	0.15341	0.16342	0.01001
----	-50	4,200	0.15293	0.16342	0.01049
----	-35	4,200	0.15255	0.16343	0.01088
6:41	-20	4,200	0.15227	0.16343	0.01116
7:00	-5	4,200	0.15210	0.16347	0.01137
7:01	-5	4,200	0.15205	0.16347	0.01142
7:03	-5	4,200	0.15205	0.16347	0.01142
7:05	10	4,200	0.15201	0.16348	0.01147
7:07	10	4,200	0.15205	0.16348	0.01143
7:09	10	4,200	0.15204	0.16348	0.01144
7:11	25	4,200	0.15218	0.16348	0.01130
----	25	4,200	0.15218	0.16349	0.01131
7:16	40	4,200	0.15258	0.16349	0.01091
----	40	0	-------	0.16349	-------
7:21	40	0	-------	0.16451	-------

NOTE: See Table VIII footnotes for an explanation of the headings.

TABLE X

ROTATIONAL DATA, AT ROOM TEMPERATURE (22°C) AND AT A FIELD OF 11,270 GAUSS, TAKEN AUGUST 28, 1963

Time	Magnet position (degrees)	Field (gauss)	Field-on voltage V_1 (volts)	Field-off voltage V_2 (volts)	Restoring voltage $V_2 - V_1$ (volts)
11:01	-155	0	-------	0.16199	------
11:03	-155	0	-------	0.16201	------
11:07	-155	11,270	0.08448	0.16201	0.07754
11:12	-140	11,270	0.08570	0.16202	0.07632
11:13	-140	11,270	0.08572	0.16202	0.07630
-----	-125	11,270	0.08783	0.16202	0.07419
11:19	-110	11,270	0.09003	0.16203	0.07200
11:22	-95	11,270	0.09159	0.16203	0.07044
11:24	-80	11,270	0.09210	0.16204	0.06994
11:27	-65	11,270	0.09143	0.16204	0.07061
11:31	-50	11,270	0.08990	0.16205	0.07215
-----	-35	11,270	0.08789	0.16205	0.07416
-----	-20	11,270	0.08608	0.16206	0.07598
-----	-5	11,270	0.08474	0.16206	0.07732
11:43	10	11,270	0.08421	0.16207	0.07786
11:44	10	11,270	0.08420	0.16207	0.07787
11:47	25	11,270	0.08466	0.16208	0.07742
11:53	40	11,270	0.08600	0.16209	0.07609
12:00	40	11,270	0.08616	0.16210	0.07594
12:04	40	0	-------	0.16213	------
12:06	40	0	-------	0.16209	------

NOTE: See Table VII footnotes for an explanation of the headings.

TABLE XI

ROTATIONAL DATA, AT ROOM TEMPERATURE (23°C) AND AT A FIELD OF 11,270 GAUSS, TAKEN AUGUST 29, 1963

Time	Magnet position (degrees)	Field (gauss)	Field-on voltage V_1 (volts)	Field-off voltage V_2 (volts)	Restoring voltage $V_2 - V_1$ (volts)
4:45	40	0	-------	0.16358	-------
4:50	40	0	-------	0.16353	-------
4:56	40	11,270	0.08810	0.16360	0.07550
4:59	40	11,270	0.08804	0.16361	0.07557
5:01	25	11,270	0.08632	0.16362	0.07730
5:04	10	11,270	0.08583	0.16363	0.07780
5:07	-50	11,270	0.08632	0.16365	0.07733
5:10	-20	11,270	0.08762	0.16366	0.07604
----	-20	11,270	0.08758	0.16367	0.07609
5:15	-35	11,270	0.08951	0.16368	0.07417
5:19	-50	11,270	0.09136	0.16369	0.07233
5:21	-65	11,270	0.09292	0.16370	0.07078
5:23	-80	11,270	0.09351	0.16371	0.07020
5:25	-95	11,270	0.09298	0.16371	0.07073
5:27	-110	11,270	0.09134	0.16372	0.07238
5:31	-125	11,270	0.08918	0.16374	0.07456
5:34	-140	11,270	0.08703	0.16375	0.07672
----	-155	11,270	0.08625	0.16376	0.07751
5:43	-155	0	-------	0.16378	-------

NOTE: See Table VIII footnotes for an explanation of the headings.

TABLE XII

DE HAAS-VAN ALPHEN EFFECT DATA TAKEN AT 4.2°K

Time[a]	Field-on voltage[b] V_1 (volts)	Field-off voltage[c] V_2 (volts)	Restoring voltage[d] $V_2 - V_1$ (volts)	Field[e] H (gauss x 10^{-3})	Field^2 H^2 (gauss x 10^{-3})^2	$(V_2 - V_1)/H^2$ (volts/gauss^2)	$1/H$ (gauss^{-1} x 10^6)
5:37	-------	0.28894	-------	0	0	-----	-----
5:40	-------	0.28897	-------	0	0	-----	-----
5:43	-------	0.28922	-------	0	0	-----	-----
5:45	-------	0.28920	-------	0	0	-----	-----
5:47	-------	0.28914	-------	0	0	-----	-----
5:49	0.28824	0.28922	0.00098	0.395	0.156	6280	2530
5:51	0.28960	0.28923	-0.00037	0.624	0.389	-951	1600
5:52	0.28906	0.28923	0.00017	0.624	0.389	437	1600
5:54	0.28888	0.28924	0.00036	0.624	0.389	925	1600
5:56	0.28859	0.28925	0.00066	0.847	0.717	921	1180
5:58	0.28834	0.28926	0.00092	1.08	1.16	796	930
6:00	0.28797	0.28927	0.00130	1.30	1.68	773	771
6:02	0.28764	0.28928	0.00164	1.52	2.31	710	658
6:04	0.28725	0.28929	0.00204	1.76	3.08	662	563
6:06	0.28650	0.28930	0.00280	1.98	3.92	714	505
6:09	0.28635	0.28931	0.00296	2.21	4.87	608	453
6:11	0.28492	0.28931	0.00439	2.43	5.90	744	412
6:13	0.28467	0.28932	0.00465	2.64	6.97	667	379
6:15	0.28288	0.28933	0.00645	2.87	8.24	783	348
6:17	0.28256	0.28934	0.00678	3.08	9.49	714	325
6:19	0.28126	0.28925	0.00799	3.30	10.9	734	303
6:21	0.27896	0.28936	0.01040	3.52	12.4	839	284

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TABLE XII (continued)

6:23	0.27829	0.28937	0.01108	3.75	14.1	788	267
6:24	0.27809	0.28937	0.01128	3.75	14.1	802	267
6:27	0.27842	0.28938	0.01096	3.97	15.8	695	252
6:28	0.27662	0.28939	0.01277	4.23	17.9	714	236
6:31	0.27377	0.28940	0.01563	4.47	20.0	782	224
6:33	0.27016	0.28941	0.01925	4.68	21.9	879	214
6:34	0.26828	0.28941	0.02113	4.89	23.9	883	205
6:36	0.26850	0.28942	0.02092	5.13	26.3	795	195
6:38	0.26942	0.28943	0.02001	5.37	28.8	694	186
6:42	0.26831	0.28945	0.02114	5.60	31.4	674	179
6:45	0.26598	0.28946	0.02348	5.82	33.9	693	172
6:46	0.26317	0.28947	0.02630	6.04	36.5	721	166
6:48	0.25911	0.28947	0.03036	6.28	39.4	770	159
6:51	0.25527	0.28949	0.03422	6.50	42.3	810	154
6:53	0.25094	0.28950	0.03856	6.73	45.3	851	149
6:57	0.24688	0.28951	0.04263	6.95	48.3	883	144
6:59	0.24383	0.28952	0.04569	7.18	51.6	886	139
7:01	0.24165	0.28953	0.04788	7.40	54.8	874	135
7:03	0.24100	0.28954	0.04854	7.63	58.2	834	131
7:05	0.24152	0.28955	0.04803	7.85	61.6	780	127
7:07	0.24221	0.28956	0.04735	8.08	65.3	725	124
7:09	0.24170	0.28957	0.04787	8.30	68.9	695	121
7:12	0.24066	0.28958	0.04892	8.53	72.8	672	117
7:14	0.23852	0.28959	0.05107	8.76	76.7	666	114
7:17	0.23631	0.28960	0.05329	8.98	80.6	661	111
7:21	0.23399	0.28962	0.05563	9.20	84.6	657	109
7:23	0.23097	0.28962	0.05865	9.42	88.7	661	106
7:24	0.23090	0.28963	0.05873	9.42	88.7	662	106

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TABLE XII (continued)

7:26	0.22736	0.28965	0.06228	9.64	92.9	670	104
7:28	0.22381	0.28965	0.06584	9.86	97.2	677	101
7:31	0.22012	0.28966	0.06954	10.1	101	686	99.3
7:33	0.21596	0.28967	0.07371	10.3	105	699	97.4
7:35	0.21123	0.28968	0.07845	10.5	110	716	95.5
7:37	0.20695	0.28969	0.08274	10.7	113	727	93.7
7:39	0.20261	0.28969	0.08708	10.9	118	737	92.0
7:41	0.19818	0.28970	0.09152	11.1	123	747	90.3
7:44	0.19330	0.28971	0.09641	11.3	127	759	88.7
7:46	0.18814	0.28972	0.10158	11.5	132	772	87.2
7:49	0.18322	0.28974	0.10652	11.7	136	784	85.8
7:51	0.17430	0.28975	0.11545	12.0	145	796	83.1
7:54	0.16395	0.28976	0.12581	12.4	154	818	80.6
7:56	0.15469	0.28977	0.13508	12.8	163	828	78.3
8:01	0.14683	0.28979	0.14296	13.1	173	828	76.1
8:04	0.13774	0.28980	0.15206	13.5	183	831	73.9
8:06	0.12924	0.28981	0.16057	13.9	192	835	72.1
8:09	0.12323	0.28982	0.16659	14.2	202	823	70.3
8:13	0.11824	0.28984	0.17160	14.5	211	815	68.9
8:16	0.11387	0.28985	0.17598	14.8	220	801	67.5
8:18	0.11027	0.28986	0.17959	15.1	229	785	66.1
8:21	0.10741	0.28987	0.18246	15.4	237	770	65.0
8:24	0.10520	0.28989	0.18469	15.6	244	756	64.0
8:28	0.10354	0.28990	0.18636	15.8	251	743	63.1
8:31	0.10200	0.28992	0.18792	16.1	260	722	62.0
8:33	0.10077	0.28993	0.18916	16.4	268	706	61.1
8:36	0.09972	0.28994	0.19022	16.6	275	693	60.4
8:38	0.09890	0.28995	0.19105	16.8	280	680	59.7

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TABLE XII (continued)

8:41	0.09805	0.28996	0.19191	16.9	286	672	59.2
8:45	0.09741	0.28998	0.19257	17.1	291	662	58.7
8:47	0.09638	0.28998	0.19360	17.2	295	655	58.3
8:50	0.09637	0.29000	0.19363	17.3	299	647	57.8
8:52	0.09605	0.29001	0.19396	17.4	303	639	57.4
8:55	0.09546	0.29002	0.19456	17.5	307	634	57.1
8:58	0.09491	0.29003	0.19512	17.6	309	631	56.9
9:03	-------	0.29050	-------	0	0	---	----
9:05	-------	0.29000	-------	0	0	---	----
9:06	-------	0.29007	-------	0	0	---	----
9:08	-------	0.29010	-------	0	0	---	----
9:15	-------	0.28972	-------	0	0	---	----

[a] The time at which the reading of V_1 or V_2 was taken.

[b] The voltage drop across R_{12} (see figure 9) with the field on and the beam restored to null position.

[c] The voltage drop across R_{12} with the field off and the beam restored to null position. As the "field-off" readings are taken only at the beginning and the end of the run; the values listed when the field is on are obtained by assuming a linear drift with time.

[d] The voltage drop across R_{12} required to balance the magnetic force on the sample.

[e] The magnetic field at the high field end of the sample.