A STUDY OF THE DISTORTION OF FINITE AMPLITUDE ULTRASONIC WAVES IN LIQUIDS

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Laszlo Adler

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OF FINITE AMPLITUDE ULTRASONIC
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A Thesis

Submitted to the College of Science and Arts Michigan
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L. Adler

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ABSTRACT

The rate at which the harmonics are developed during the propagation of an initially sinusodial ultrasonic wave depends on the nonlinearity of the liquid. A stainless steel plate was used as a filter to pass only the desired harmonics. The rate of growth of each of these harmonics, as a function of distance and pressure, was measured by means of the diffraction pattern produced by the wave. From the result of these measurements, the parameter B/A (describing nonlinearity) was calculated to be $6.2 \pm .6$ and 9.6 ± 1 for water and m-xylene, respectively.

The rate of growth of the harmonics as measured, deviate somewhat from the values predicted from the theory for a dissapationless medium. An approximate theory was suggested to consider both generation and absorption of the harmonics. Fair agreement was obtained between this approximate theory and the measured values of the harmonics.

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CHAPTER I

INTRODUCTION

In general, a sound beam changes its wave shape as it propagates in a fluid whose pressure density relationship does not follow Hooke's law. This means that harmonics are generated as the wave progresses. In the case of infinitely small amplitude, this change in wave shape is so small that one can neglect it; however, in the case of a finite amplitude sound wave, this effect has to be considered. The distortion of a finite amplitude wave is due to the nonlinear property of the medium in which the wave propagates. The positive and negative increments in pressure are impressed on the mass of medium. The change in the volume of the mass not being equal, the volume change for the positive pressure will be less than the volume change for negative pressure.

Thuras, Jenkins and O'Neil[1] studied finite amplitude effects in gases. They have pointed out that the equation of state for adiabatic processes shows that the linear relationship between pressure and volume does not hold, and consequently, leads to the distortion of the ultrasonic wave. They gave an approximate solution for the generation of harmonics as a function of distance, fundamental pressure and frequency.

Fox and Wallace[2], following the same argument for liquids, assumed that the following power series expresses the relation between pressure and density

$$P = P_0 + A\,\frac{\rho - \rho_0}{\rho_0} + \frac{B}{2}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \dots \qquad (1)$$

where P is the pressure and ρ is the density. P_0 and ρ_0 are the pressure and density of the undisturbed medium. B and A are parameters of the liquid which are functions of temperature.

Fox and Wallace, instead of solving the equation of motion, suggested a graphical analysis of the distorted wave. As they pointed out, the gradually steepening wave front would ultimately form a discontinuity without some stabilizing mechanism. This stabilizing mechanism is the absorption of the ultrasound in the medium. Fox and Wallace evaluated the nonlinearity parameter B/A for liquids using compressibility data.

Fubini-Ghiron[3] used a similar treatment to that of Fox and Wallace, but instead of a graphical method, he used an analytical treatment of the finite amplitude wave distortion. His theory assumes that the medium is dissipationless and the expressions for the growth of harmonics hold up to the discontinuity of the wave front. This analytical method has been redeveloped independently by Keck and Beyer[4], and Hargrove[5]. Hereafter, this method will be referred to as the Fubini-Ghiron method, however, the notation of Hargrove will be used.

Much experimental work was carried out in the last decade to demonstrate and measure the distortion of a finite amplitude wave in liquids. Zarembo, Krasilnikov, and Shklovskaia-Kordi[6] showed the dependence of the ultrasonic abosrption coefficient on sound pressure. They used two different methods: first, they used a thermal probe as a receiver and analyzed the harmonic structure of the disterted wave; secondly, they observed directly the harmonics in a traveling wave. They used an acoustic filter to separate the fundamental and its harmonics.

Breazeale and Hiedemann[7] explained that the appearance of the asymmetry in the refraction of light by an ultrasonic wave is caused by the distortion of the wave form.

Zankel and Hiedemann[8], in their study of light diffraction by an ultrasonic beam, explained the asymmetry of the diffraction pattern by the distortion of finite amplitude waves. Their method was an indirect way of measuring harmonic structure. They assumed a certain percentage of the harmonic content and fitted it into the light intensity measurement of the diffraction pattern.

In another paper[9], they demonstrated the presence of a second harmonic by use of a metal plate which served as an acoustic filter. The second harmonic which passed through the filter plate served as a diffraction grating.

The experimental method of using a filter plate in combination with optical diffraction has now been used to study quantitatively how the harmonic content of an ultrasonic wave varies with distance from a sinusoidally vibrating source. The results of this study, and the non-linearity parameter B/A for water and m-xylene derived from them will be given in this thesis.

CHAPTER II

THEORY

Theory of Nondissipative Liquids

The amount of distortion of a large amplitude sound wave is expressed in terms of the amount of harmonics present. Thuras, Jenkins and O'Neil obtained a solution for the second and third harmonics by solving the equation of motion for the nonlinear wave.

Their solution is an approximation for small pressures not considering any dissipation of the wave. The expression for the amplitude of the second harmonic may be given as

$$P_2 = \frac{\pi(B/A + 2)}{2 \rho_0 C_0^3} X P_{10}^2 f \tag{2}$$

where X is the distance between the transmitter and the receiver. P_{10} is the fundamental pressure at X = 0. ρ_0 is the density of the undistrubed medium. f is the frequency of the ultrasound, C_0 is the sound velocity, and B/A is the nonlinear parameter of liquid.

Experiment shows that the linear relation between the amplitude of the second harmonic and distance holds only for a very small region of distance. At greater distances, the second harmonic levels off. Such a behavior is given by the theory of Fubini-Ghiron[3]. In this theory, instead of solving the equation of motion, an analytical method was carried through between x = 0 and the point of discontinuity; that is, where the wave would reach discontinuity due to the distortion. With the assumption that the crest propagates faster than the trough, Fubini-Ghiron[3] obtained an expression for the amplitude of the n^{th} harmonic in a dissipationless medium which may be given as

$$P_n = \frac{2P_{10}}{nk} J_n(nk) \tag{3}$$

where P_n is the amplitude of the n^{th} harmonic, P_{10} is the fundamental pressure, J_n is the n^{th} order Bessel function, k is the fractional parameter of the discontinuity distance $k = \frac{X}{L}$, and where

$$L = \rho_0 c_0^3 \left\{2\left[\left(B/2A + 1\right) P_{10} f\right]\right\}^{-1} \tag{4}$$

This theory gives the right shape for the harmonic generation, as it levels off the curve, but it still deviates from the experimental values. In the first approximation for small distances and small pressures the two expressions (equations 2 and 3) are identical. The theory of Fubini-Ghiron will be used in a modified form to describe the propagation of sound in a liquid.

<u>Theory of Dissipative Liquids</u>

One way to account for dissipation of a finite amplitude ultrasonic wave is the method used by Thuras, Jenkins and O'Neil. They assumed that each harmonic would be absorbed independently of the other.

Here, however, we assume a slightly different model which has led to a better agreement between theory and experiment. We divide the discontunity distance into small intervals and assume that at any distance the generated harmonic can be calculated from the theory of Fubini-Ghiron. The dissipation of the harmonic is introduced by assuming that in the interval dk the average value of the generated harmonic will undergo the absorption. Thus, the actual amount of harmonic present in each interval is the amount generated minus the amount absorbed.

Let us consider the expression for the n^{th} harmonic as expressed by Fubini-Ghiron[3].

$$P_n = \frac{2P_{10}}{nk} J_n(nk) \tag{5}$$

The n^{th} order Bessel function can be expanded into the following infinite series

$$J_n(X) = \sum_{j=0}^{\infty} \frac{(-1)^j \left(\frac{X}{2}\right)^{2j+n}}{j!\,(j+n)!} \tag{6}$$

Expanding equation (6) and neglecting terms higher than the second, we obtain for equation (5)

$$\frac{P_n}{P_{10}} = \frac{(nk)^{n-1}}{2^{n-1}\,n!} - \frac{(nk)^{n+1}}{2^{n+1}(n+1)!} \tag{7}$$

To find the amount of n^{th} harmonic dissipated, we assume an exponential absorption of the average harmonic present, thus

$$\left(\frac{P_n}{P_{10}}\right)_{dissipated} = \frac{\overline{P}_n}{P_{10}}\left(1 - e^{-\alpha_n X}\right) \tag{8}$$

where α_n is the absorption coefficient of the n^{th} harmonic in the medium.

The average value of the relative amount of n^{th} harmonic is given by

$$\frac{\overline{P}_n}{P_{10}} = \frac{\int \frac{P_n}{P_{10}}\,dk}{\int dk} = \frac{(nk)^{n-1}}{2^{n-1}\,n\,n!} - \frac{(nk)^{n+1}}{2^{n+1}(n+2)!} \tag{9}$$

Substituting equation 9 into equation 8, we obtain

$$\left(\frac{P_n}{P_{10}}\right)_{dissipated} = \frac{(nk)^{n-1}}{2^{n-1}\, n\, n!} - \frac{(nk)^{n+1}}{2^{n+1}\,(n+2)!}\left(1 - e^{-\alpha_n X}\right) \quad (10)$$

The actual amount of n^{th} harmonic present is the amount of harmonic generation (equation 7) minus the amount of harmonic dissipated (equation 10).

$$\frac{P_n}{P_{10}} = \frac{(nk)^{n-1}}{2^{n-1}\, n!} - \frac{(nk)^{n+1}}{2^{n+1}(n+1)!} - \left[\frac{(nk)^{n-1}}{2^{n-1}\, n\, n!} - \frac{(nk)^{n+1}}{2^{n+1}(n+2)!}\right]\left(1 - e^{-\alpha_n kL}\right) \quad (11)$$

where x = kl

For the second harmonic, equation 11 becomes

$$\frac{P_2}{P_{10}} = \frac{k}{4} - \frac{k^3}{8} + \left[\frac{k}{4} - \frac{k}{24}\right] e^{-\alpha_2 kL} \quad (12)$$

and for the third harmonic

$$\frac{P_3}{P_{10}} = \frac{k^2}{4} - \frac{81\, k^4}{320} + \left[\frac{k^2}{8} - \frac{27\, k^4}{640}\right] e^{-\alpha_3 kL} \quad (13)$$

This theory is limited to values of α , k, and L, such that the generation of the harmonics is not influenced by their absorption. It is also limited to distances less than the discontinuity distance. By satisfying these conditions experimentally, values of the harmonics as a function of distance which were measured could be compared with this theory. These results will be given after a description of the experiment.

CHAPTER III

EXPERIMENTAL METHODS

Experimental Arrangement

The experimental arrangement used is shown in the schematic illustration in figure 1. Light from the mercury vapor lamp S was condensed by lens L, on the source slip B. Lens L_2 was adjusted by autocollimation to render the light parallel. The light passing through the tank was focused by lens L_3 on the entrance slip B_2 of the photomultiplier microphotometer, a filter F which passes only the 5461A line of mercury was placed between slit B_2 and the photomultiplier Ph.

The ultrasonic transducer Q was an air backed 1 inch square X-cut quartz crystal. This crystal was excited at its fundamental frequency by an RF oscillator which had a maximum output of 50 watts. The RF potential across the quartz was measured using a high-frequency vacuum tube voltmeter.

A specially designed tank T described elsewhere, [10] was used to eliminate reflection of the sound beam, P was a stainless steel plate approximately 1 mm in thickness, used as an acoustic filter to select the desired components of the sound wave.

Pressure Measurement

As is known, the ultrasonic beam acts on the incoming light as an optical phase grating. The angle of the diffracted light in the

n^{th} order is expressed as

$$\sin \Theta = n \frac{\lambda}{\lambda^*} \tag{14}$$

Where λ is the wave length of the light, λ^* is the wave length of the wound, and n can be any integer number.

The intensity of the diffracted light in the n^{th} order for small sound pressure was derived by Raman and Nath[11] to be

$$I_n = J_n^2 (v) \tag{15}$$

where J_n is the n^{th} order Bessel function. The term v is expressed as

$$v = \frac{2 \pi \mu a}{\lambda}$$

where a is the width of the sound field and μ is the variation of the refractive index of the liquid due to the density variation caused by the sound beam. The variation of the index of refraction is caused by the condensations and rarefactions in the medium as a consequence of the sound pressure. This variation is assumed to be proportional to the sound pressure.

The measurement of the intensity of the diffracted light leads to the value of the variation of the index of refraction and indirectly to the sound pressure. However, the difficulty is that the relationship between pressure and the variation index of refraction is not very accurately known. A theoretical relation was derived by Lorentz-Lorenz 12, 13 for the variation of the refractive index, with the assumption that the molecules are optically isotropic. It is known, however, this does not hold true for many liquids.

Raman and Krishnan 14 tried to use some correction term considering the anisotropic property of the molecules, but to apply their result in actual calculation is too complicated for the present purposes.

Several empirical formulas were worked out for the variation of the refractive index with pressure, of which Eykman's 15 formula seems to fit best with the experimental measurements for work on water and m-xylene, which are the liquids used in this work. From the Eykman formula the variation of refractive index is found to be

$$\mu = \frac{(\mu_o - 1)(\mu_o + 1.4\mu_o + .4)}{\mu_o^2 + .8\mu_o + 1} \qquad (16)$$

where μ_o is the refractive index of the undisturbed liquids. From this formula, the relationship between pressure amplitude and the Raman-Nath parameter v is

$$P = .56\frac{v}{L}\ \text{atm.} \qquad (17)$$

for water and

$$P = .25 \frac{V}{L} \text{ atm.} \tag{18}$$

for m-xylene.

With these expressions, it is possible to determine the pressure amplitude from the light intensity in any diffraction order. The procedure used was as follows:

The relative intensity of the diffracted light in the first order of the diffraction spectra was measured with the photomultiplier microphotometer. From the measured intensity values, the Raman-Nath parameter v was calculated using equation 15.

From the evaluated v-values, the pressure amplitude, was determined by means of equations 17 and 18.

Demonstration of the Higher Harmonics

As a demonstration that the higher harmonics of the fundamental are present as a result of the distortion of the wave, the following experiment was made: the filter plate at the front of the sound beam was rotated through a 90° angle and the light intensity of the second order was recorded. Because the spacing between diffraction orders is proportional to the frequency (equation 11), any second harmonics present would contribute to the second order in the diffraction pattern.

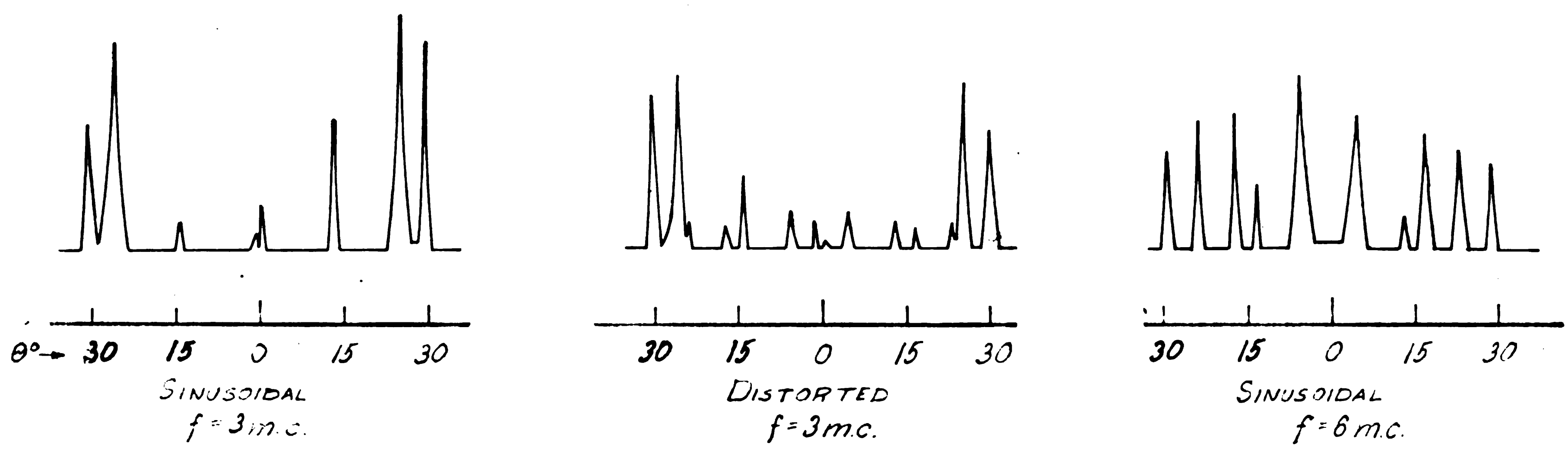

Figure II

DEMONSTRATION OF SECOND HARMONIC

SOUND AMPLITUDE TRANSMISSION VS. INCIDENCE ANGLE TO PLATE

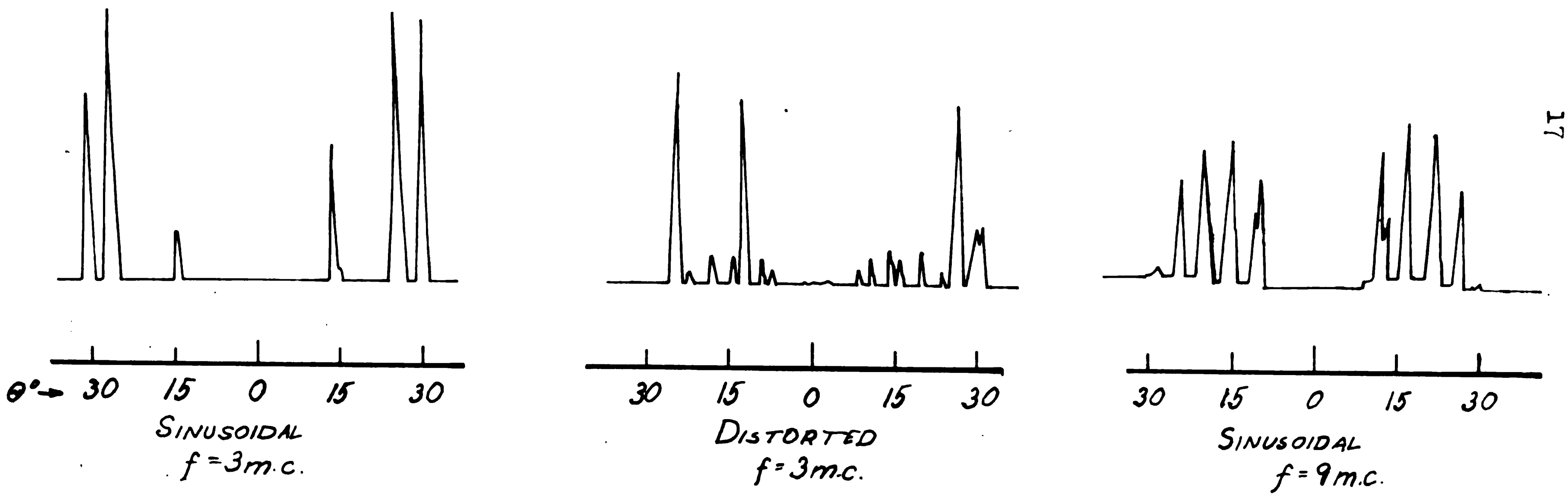

Figure III

DEMONSTRATION OF THIRD HARMONIC

SOUND AMPLITUDE TRANSMISSION VS. INCIDENCE ANGLE TO PLATE

At first, a 3 mc quartz was used and placed close to the plate and a very low sound pressure was applied. Under that condition the finite amplitude effect is negligible. The transmission curve is shown in figure 2a. Since the light intensity is a function of the sound pressure, the peaks show the transmission of the fundamental pressure. Then the transducer was moved back to 50 cm and the ultrasonic pressure was increased to 1 atm. Under this condition, the experiment was repeated. As the result shows, in figure 2b, the extra peaks are indicating the presence of second harmonic since they coincide with the transmission peaks of the 6 mc sound wave shown in figure 2c. A similar measurement was made to demonstrate the presence of the third harmonics as shown in figure 3. This method gives the possibility to select the angle of maximum transmission for the different harmonics. It is also demonstrated that the harmonics present are not generated in the transmission plate, but in the liquid.

Calibration of the Plate

One cannot expect that the energy of the incoming sound beam to the plate will be transmitted into the outgoing beam one hundred percent, for due to the multiple reflection inside the plate and the generation of shear waves inside the solid medium, there will be a certain amount of loss.

To detect the sound energy loss, or more correctly, the amount of sound transmitted, measurements of the sound amplitude were made before and after the plate using the optical method. To obtain a calibration for the second harmonic of a 3 mc wave, a 6 mc source was used.

The ratio of the outgoing sound amplitude to the incoming sound amplitude gave the transmission coefficient of the plate. The transmission coefficients obtained were .92 for water and .74 for m-xylene. For the measurement of the second harmonic these coefficients were taken into consideration.

Measurement of the Second Harmonic in Water

In the actual measurement of the second harmonic, a 3 mc quartz was used to generate the fundamental frequency. The plate was placed near to the light beam with a selected angle to the sound wave, at which angle only the second harmonic was transmitted (assuming no fourth harmonic is present). The fundamental and the third harmonics were reflected and the second harmonic was measured, using the method described, at various distances and at various fundamental pressures. The fundamental pressure was measured by measuring the quartz voltage. The quartz voltage was calibrated to the acoustical pressure by an optical method using equation 12.

The distance was varied from zero to the discontinuity distance and the fundamental pressure was varied from .1 atm to 1.1 atm.

In figure 4 the relative percentage of the second harmonics was plotted vs. the fractional parameter of the discontinuity distance <u>k</u> for 3 different fundamental pressures: 1.1 atm, .88 atm and .44 atm, with the corresponding discontinuity distances: 39 cm, 48 cm and 97 cm. The solid line represents the theoretical curve for a dissipationless medium. As one can see, the experimental points are in good agreement with the dissipationless theory for low k values. As it was expected, the medium can be considered dissipationless if the sound wave propagates a small distance. However, with increasing k values, the experimental points deviate more and more drastically from the theory, indicating that the medium cannot be considered dissipationless when the propagation distance is too great.

It is significant to point out that the deviation from the theory is greater for smaller pressures than for higher ones. Since the discontinuity distance is inversely proportional to the fundamental pressure, the absorption becomes more dominating, as the sound wave has to travel farther to reach the discontinuity distance.

Figure 5 shows the theoretical curves considering dissipation using equation 10 with the experimental points. As we can see, there is a much better agreement between theory and experiment, even if one considers only this simple model for dissipation. For small pressures the agreement is much better than for higher pressures. There is a possibility that at higher pressures some fourth harmonics are gener-

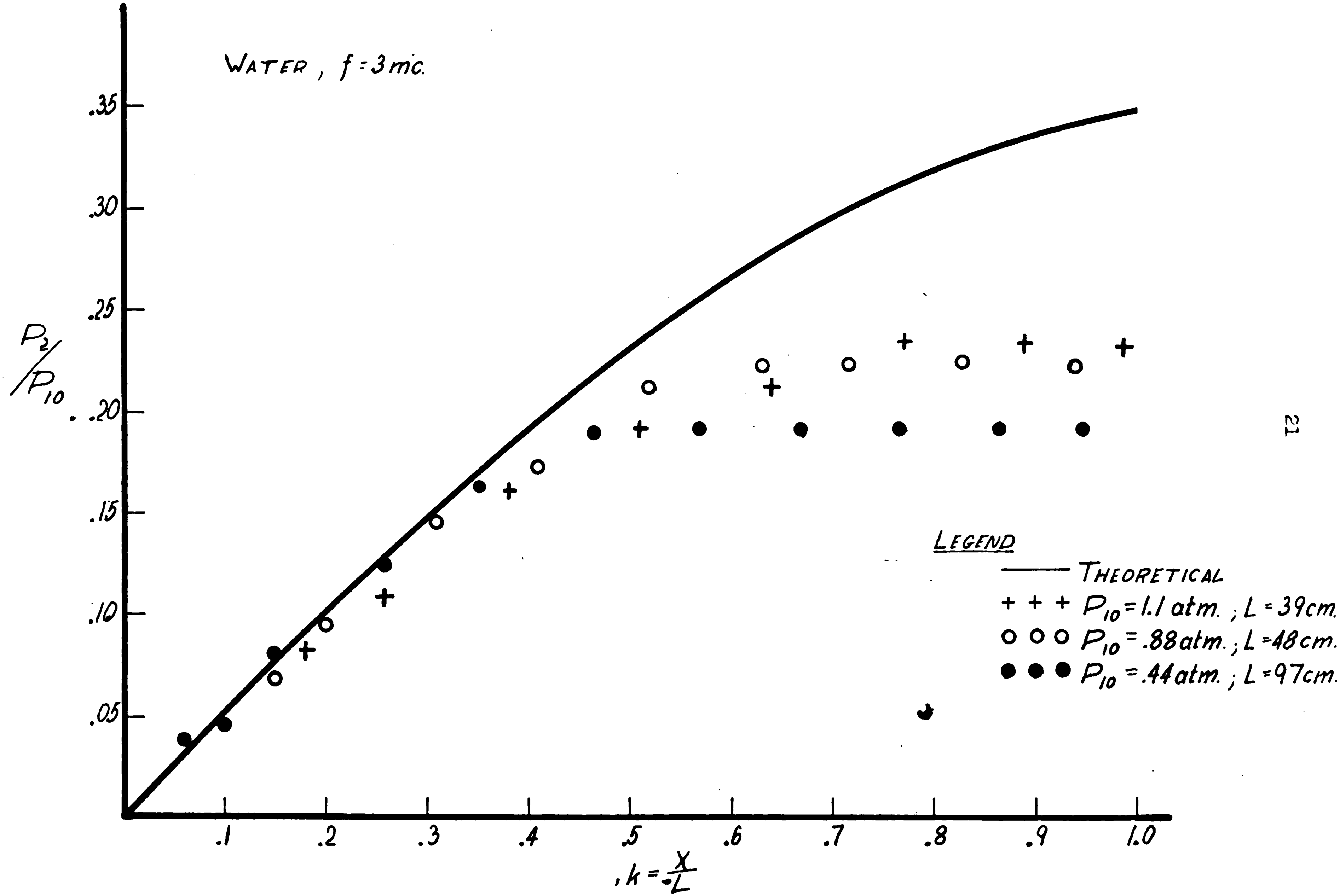

Figure IV

SECOND HARMONIC VS. k

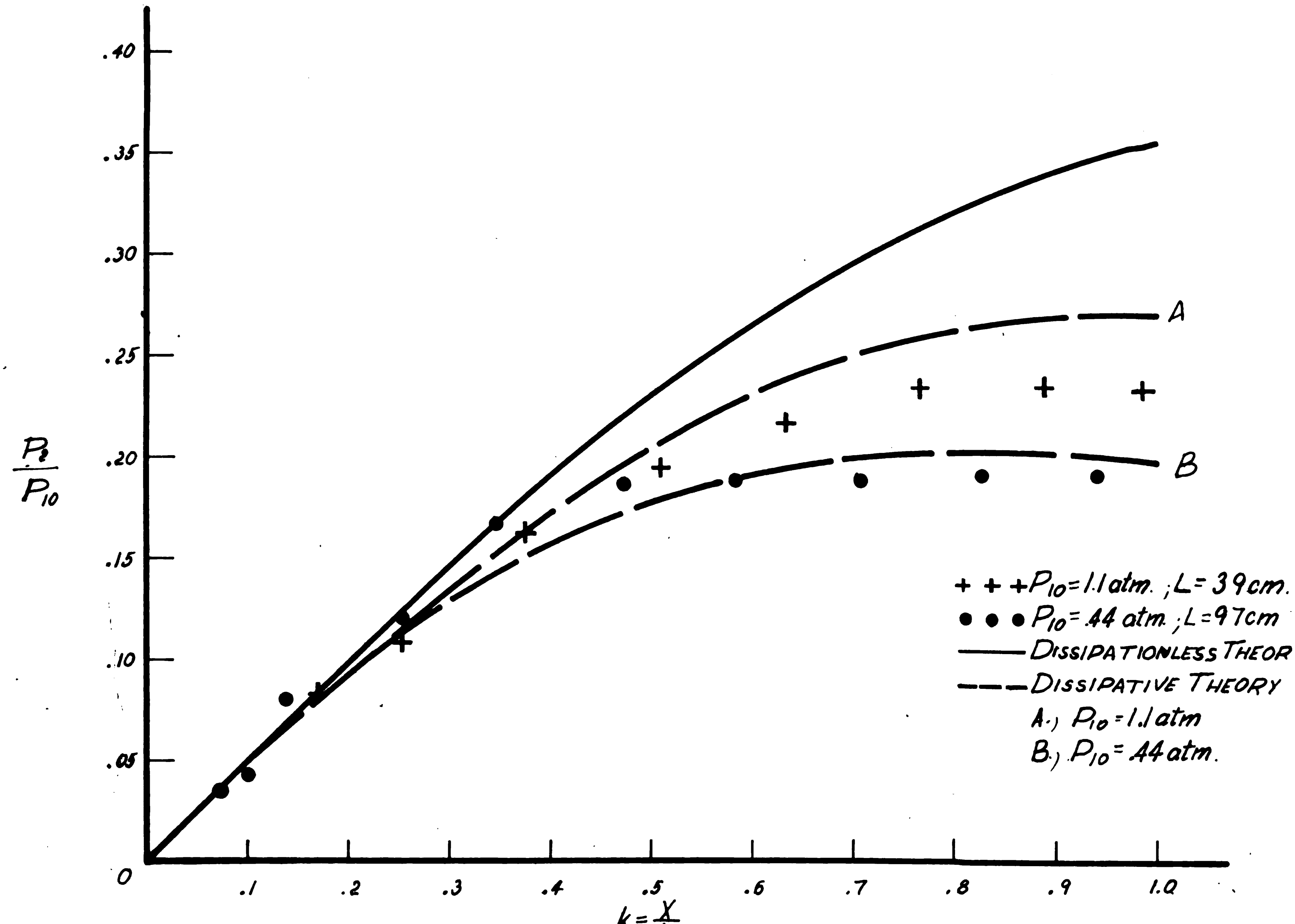

Figure V

SECOND HARMONIC VS. k

Water - f = 3 mc

ated which also pass through the plate. But probably some more terms should be included into the dissipationless theory. The fact that absorption affects the generation of harmonics could cause some of these discrepancies.

Measurement of Second Harmonics in m-xylene

The same method was carried out to obtain measurement in m-xylene. The measurement was more difficult for higher k values than in water. When high ultrasonic pressure was used, the disturbance in the liquid was very high. This disturbance is attributable to the presence of a quartz wind. Plate vibration could also be observed in xylene more than in water. However, for small k values the experimental points are in good agreement with the theory (figure 6).

Measurement of Third Harmonics in Water

To show that the method is applicable for measuring the value of harmonics higher than the second, the plate was calibrated for the third harmonic in the same way as was described for the second harmonic, and one set of measurements was carried out.

As was expected, figure 7 shows that the experimental points deviate more rapidly from the dissipationless theory than in the case of the second harmonics. Since the absorption is proportional to the square of the frequency, this can be explained. The approximate dissipationless theoretical curve shows good agreement with the experi-

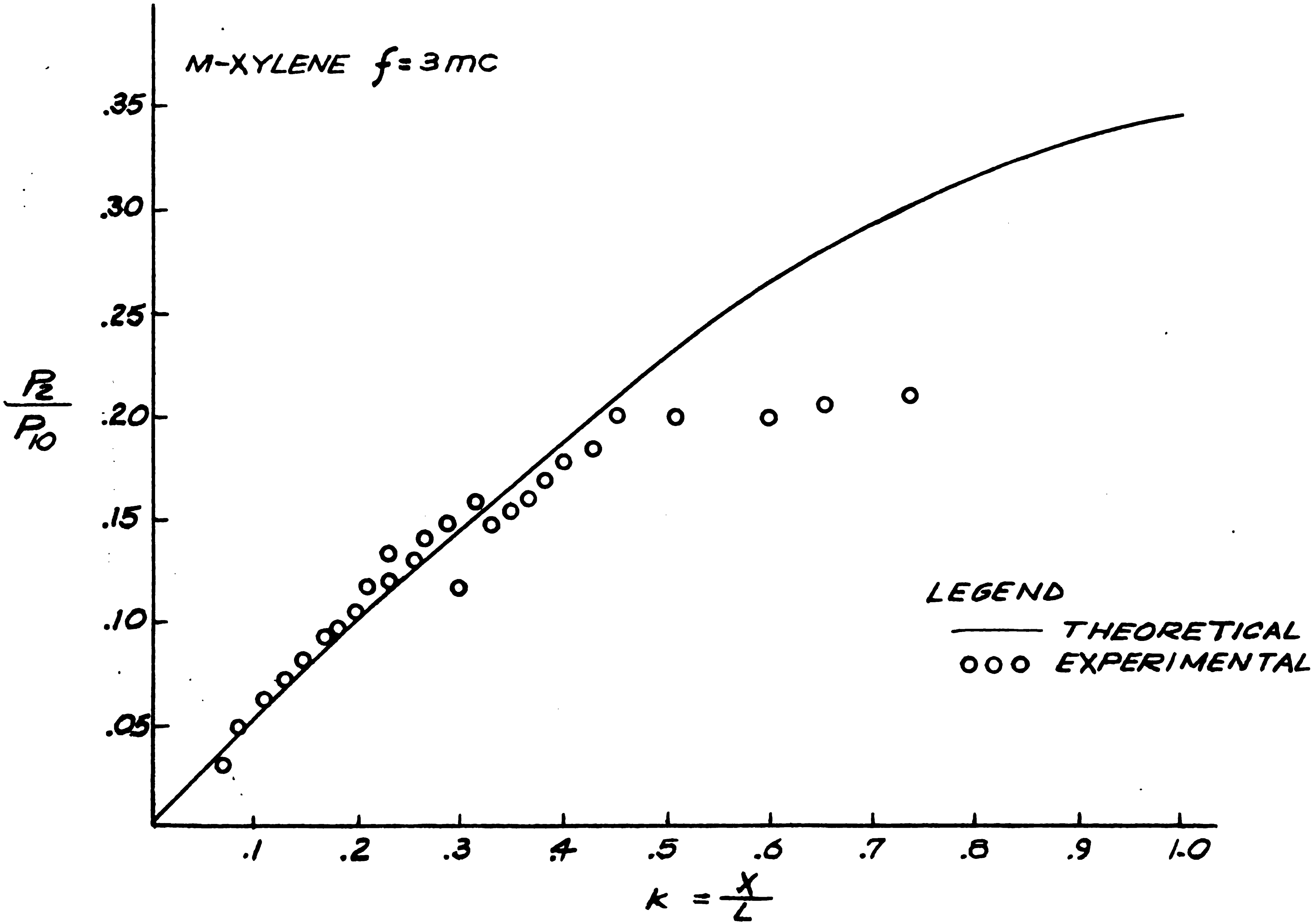

Figure VI

SECOND HARMONIC VS. k

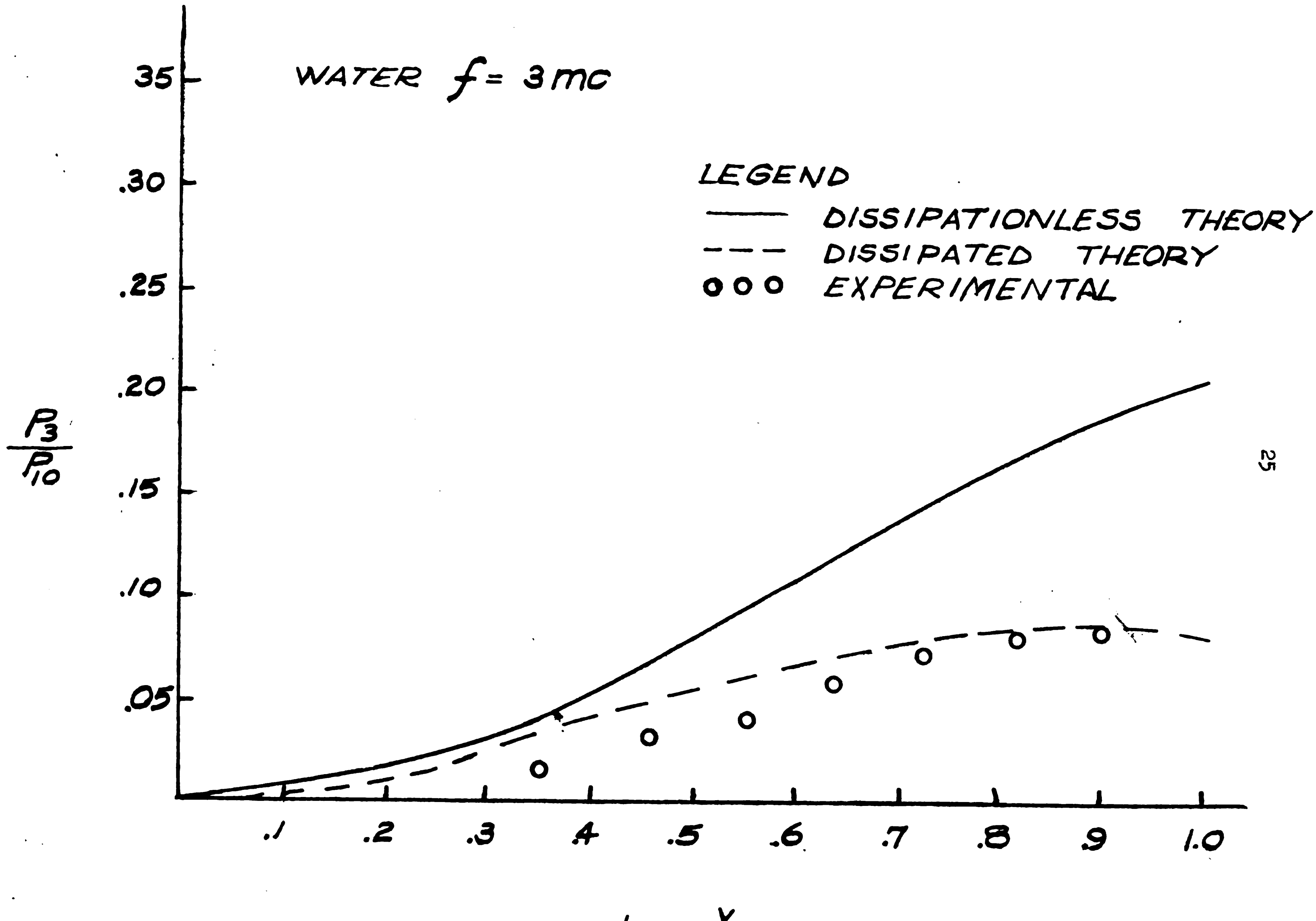

Figure VII

THIRD HARMONIC VS. k

mental points, indicating the correctness of the assumption for dissipation.

Evaluation of B/A Values for Water and m-xylene

The equation of state for liquids expressed by Fox and Wallace[2] introduces the nonlinearity parameter B/A. The growth of the second harmonic of finite amplitude sound waves is related to the B/A values, thus measurements of the second harmonic can be used to determine the value of B/A.

A quick way to determine the nonlinearity parameter is to use the dissipationless theory for harmonics as developed by Fubini - Ghiron. Considering the expression for second harmonics and neglecting higher order terms, one obtains

$$B/A = \text{const.} \frac{P_2}{P_{10}^2 X} - 2 \qquad (14)$$

Thus, measuring second harmonics at various distances and various pressures leads to the value of B/A. Figures 8 and 9 show $P_2/P_{10}^2 X$ vs. P_{10} for water and m-xylene.

The extrapolation of these curves give the B/A value. As one can see, the evaluation of B/A occurs at $XP_{10} = 0$, obviously satisfying the condition for the dissipationless case. Although at higher XP_{10} values, the curves deviate from the theoretical curve, these points are not used for the determination of the B/A value because the extrapolation

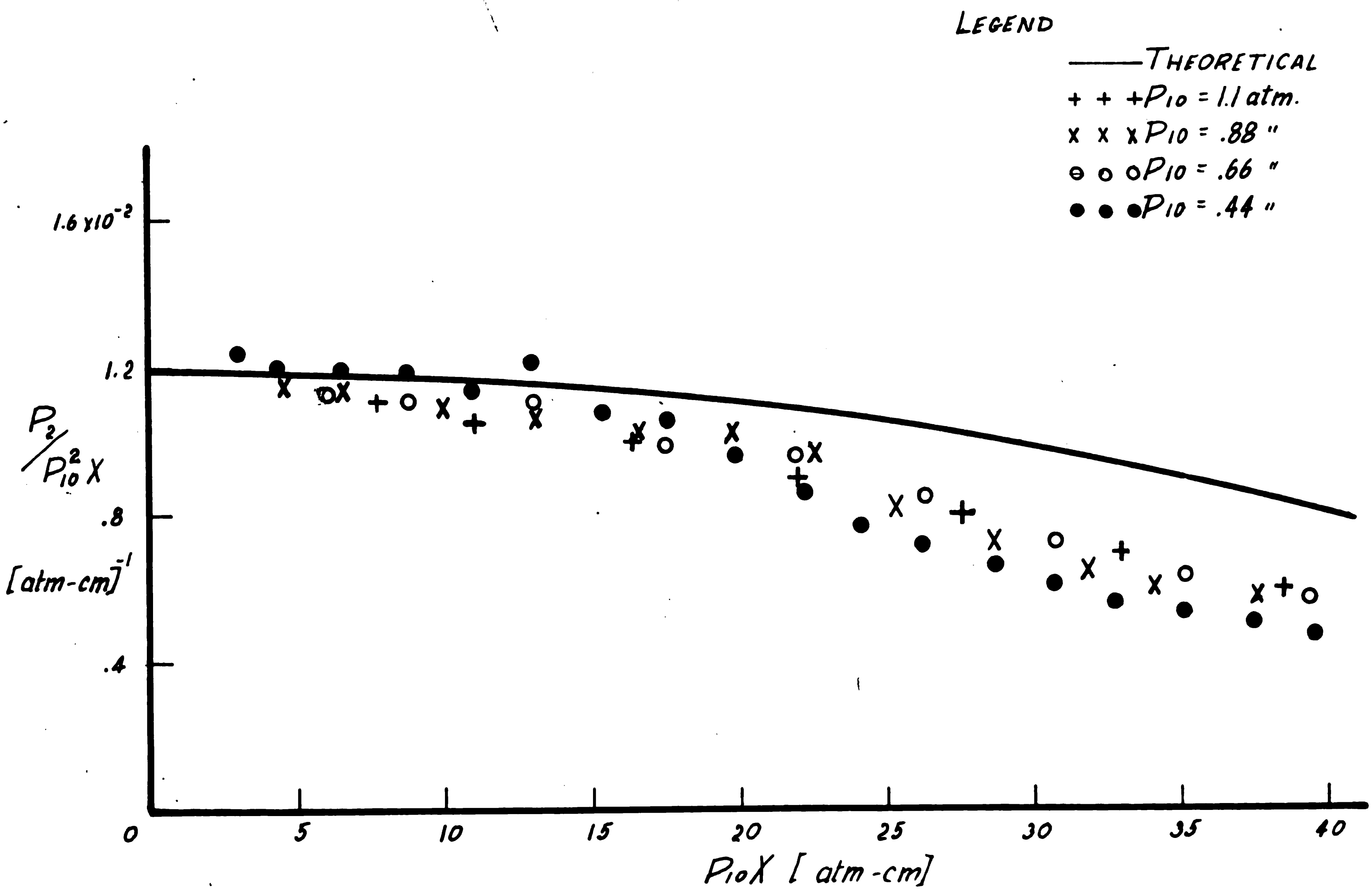

Figure VIII

DETERMINATION OF BIA

of Water - f = 3 mc

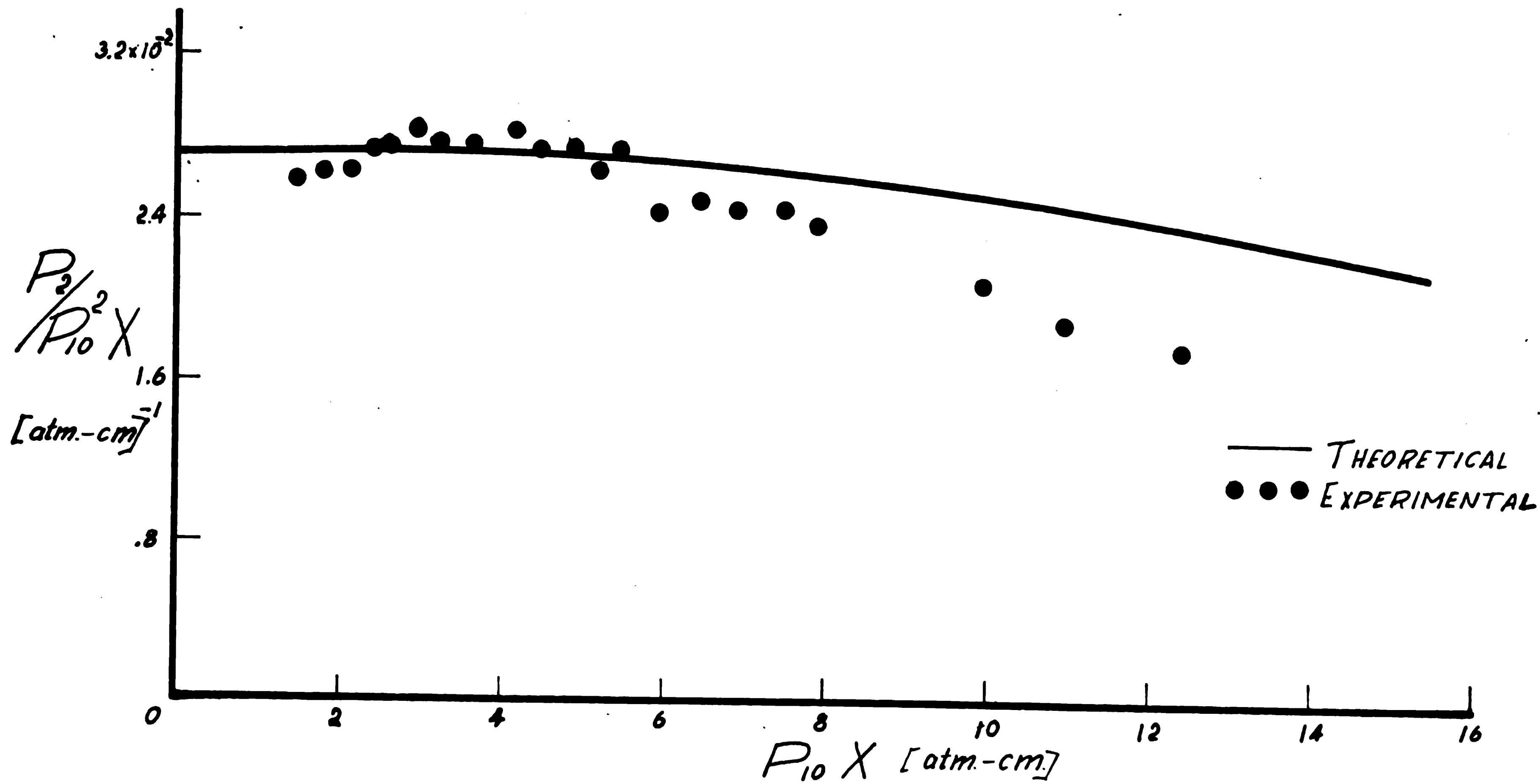

Figure IX

DETERMINATION OF B/A

of m-xylene - f = 3 mc

neglects the effect of dissipation.

Using this approach, the value of B/A for water was found to be $6.2 \pm .6$ and for m-xylene 9.6 ± 1.

CHAPTER IV

SUMMARY OF RESULTS

A method for quantitative measurements of harmonics as a result of the distortion of a finite amplitude sound wave was developed. This method used an acoustic filter and the optical diffraction method of measuring ultrasonic pressure. Measurements were taken for the relative percentage of the second and third harmonic growth in water and for the second harmonic growth in m-xylene at various distances and various fundamental pressures. As was expected, there was a drastic deviation between the dissipationless theory and the experiment, indicating absorption of harmonics.

An approximate theory was suggested to consider the dissipation of the harmonics. A reasonable agreement was obtained between this theory and the experiment for small pressures.

It has been shown that one can evaluate B/A from distortion measurement even though the exact theory is not known. From these measurements the nonlinearity parameter B/A is evaluated as $6.2 \pm .6$ for water and 9.6 ± 1.0 for m-xylene.

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