AN EXPERIMENTAL DETERMINATION OF THE DECAY ENERGY OF BA^{131} AND TE^{121}

Thesis for the Degree of M. S.

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ABSTRACT

AN EXPERIMENTAL DETERMINATION OF THE DECAY ENERGY OF BA^{131} AND TE^{121}

The energy for the decay by electron capture of Ba^{131} and Te^{121} was measured. For Ba^{131}, the ground state energy difference between Ba^{131} and Cs^{131} was found to be 1160 ± 8 Kev. This was determined by finding the decay energy to the 1046.5 Kev state by measuring the L to K-capture ratio for the transition to that state. Log ft values are given for the different transitions. A value of $0.22 \pm .03$ was measured for the $\alpha_K / (1+\alpha_T)$ conversion coefficient for the 123.7 Kev transition. The decay energy for the decay to the 575 Kev state of Te^{121} was found to be greater than approximately 400 Kev. Evidence is presented to show that the 1130 Kev transition is in coincidence with a low energy gamma-ray. It is also postulated that the 506 Kev transition is to the ground state.

AN EXPERIMENTAL DETERMINATION OF THE DECAY ENERGY OF BA^{131} AND TE^{121}

By

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I. INTRODUCTION

A. STATEMENT OF PROBLEM

According to nuclear systematics, the ground state energy difference between the two nuclei Ba^{131} and Cs^{131} is about 1.7 Mev. However, on the basis of recent log ft measurements, it is thought that the value for this energy difference is too large (1). The present work has attempted to measure the transition energy for the decay by electron capture of Ba^{131} to Cs^{131}. It was also observed that no experimental determination of the decay energy of Te^{121} had been made. Since the methods used for the Ba^{131} measurements could be easily adapted to Te^{121} the establishment of the Te^{121} to Sb^{121} transition energy was also made a part of this work.

The decay energies determine the forbiddenness of the transitions. Let the comparitive or reduced half-life be defined as $f(\Delta E, Z)t$ where ΔE is the transition energy, Z is the atomic number, and t is the half-life. The function f takes into account the effects of transition energy, and nuclear charge on the half-life. The ft product should be roughly constant for all transitions of the same degree of forbiddenness. For electron capture the function is

$$f(\Delta E, Z) = \frac{1}{4}\pi\left(n_K g_K^2 \Delta E_K^2 + n_{LI} g_{LI}^2 \Delta E_{LI}^2 + \cdots\right)$$

where n is the number of electrons in the shell, g is the Dirac radial wave function, and ΔE_K is the transition energy minus the binding energy of the K electron (2). If capture of L_{II} and higher shell electrons is neglected, f becomes

$$f(\Delta E, Z) = \frac{1}{2}\pi\left(g_K^2 \Delta E_K^2 + g_{LI}^2 \Delta E_{LI}^2\right)$$

For Z<60, the function may be further simplified by assuming that $(\Delta E_{LI} - \Delta E_K) \ll \Delta E_K$ so that

$$ft = \pi/2\, t\,(g_K^2 + g_{LI}^2)\Delta E_K^2$$

or finally, $\log ft = \log \pi/2 + \log g_K^2 + \log(1 + g_{LI}^2/g_K^2) + \log \Delta E_K^2 + \log t$

Thus by knowing ΔE_K, t, g_{LI}, and g_K, it is possible to calculate the log ft. Actually, the last three quantities are available in the literature so that only a knowledge of the transition energy is needed. The log ft values can be divided into the following groups (3):

TRANSITION PROBABILITY	LOG ft	SELECTION RULE	PARITY CHANGE
super-allowed	3.5	$\Delta J=0$	no
allowed	5±1	$\Delta J=0,1; \Delta l=0$	no
l-forbidden	>6	$\Delta J=0,1; \Delta l=2$	no
first forbidden	7±1	$\Delta J=0,1$	yes
second forbidden	13	$\Delta J=2,3$	no
third forbidden	18	$\Delta J=3,4$	yes

Thus by measuring the log ft for an electron capture decay, it will be possible to assign a degree of forbiddenness to its transition, which may in turn give information about the spins and parities of the nuclear states involved.

B. THEORY OF ELECTRON CAPTURE

1. GENERAL

In general, in nuclear reactions it is possible to ignore the electrons existing outside of the nucleus. However, in the process of electron capture, these electrons play a major role. Here a nucleus decays by capturing an atomic electron. This type of nuclear reaction can be characterized by

$$_{Z}P^{A} + _{-1}e^{0} \longrightarrow _{Z-1}D^{A} + \nu$$

where P and D are the parent and daughter nuclei, is a neutrino, and Z and A are the atomic mass and number. In the specific case in which the captured electron is from the K shell, the process is called K-capture.

Electron capture competes with positron emission when the mass of the parent atom is greater than the mass of the daughter plus two electron rest masses. For light nuclei, the energy separation between states of neighboring isobars is usually so large that positron and electron capture decay are almost assured. On the other hand, for heavy nuclides smaller mass differences tend to prohibit positron decay. For very heavy isotopes, alpha decay is sometimes seen to compete with electron capture. In some nuclei, K-capture is energetically impossible. These nuclei can still exhibit L-capture, however.

The capture produces a hole in the electron shell. This is soon filled by higher shell electrons dropping down to fill the vacancy. X-rays are emitted which are characteristic of the isotope. They can be used to show that an electron capture has taken place.

2. ELECTRON CAPTURE ENERGY

a. MASS DIFFERENCE

It is possible to measure the ground state energy difference between the the parent and daughter (ie., the transition energy) in several different ways. Probably the most direct method would be to measure the atomic mass difference in a mass spectrometer. In this method, the transition energy is given by

$$\Delta E = M(Z) - M(Z-1)$$

where $M(Z)$ and $M(Z - 1)$ are the atomic masses in units of m_0c^2, of the parent and daughter, and ΔE is the transition energy.

b. POSITRON END POINT ENERGY

A second method involves the measurement of the end point of the positron spectrum for transitions between known initial and final nuclear states. In this case, the transition energy is given by (2)

$$\Delta E = W_p + E_i + 2$$

where W_p is the end point energy of the positron spectrum in units of m_0c^2, and E_i is the binding energy of the atomic electron that is ejected with the positron.

c. INTERNAL BREMSSTRAHLUNG

The capture process also results in an abrupt change in the charge of the nucleus, decreasing it by one positive unit. This forces the nucleus to adjust its charge distribution. When the atom thus alters its dipole moment it can emit an x-ray photon which is called an internal bremsstrahlung photon. This radiation differs from the radiation produced when a charged particle is accelerated, the latter being known as external bremsstrahlung.

The reaction energy in electron capture is shared between a bremsstrahlung photon and a neutrino. Thus the internal bremsstrahlung radiation produces a continuous spectrum of energies. The upper energy limit, in units of m_0c^2, is given by (4)

$$W_b = \Delta E + (1 - E_K)$$

where W_b is the upper limit of the bremsstrahlung, ΔE is the transition energy, and $(1 - E_K)$ is the energy gained in capturing a K electron (the rest mass energy minus the binding energy of the electron). If the upper limit of the bremsstrahlung can be determined, then it is possible to calculate the transition energy from the above relation.

The energy distribution of the bremsstrahlung is (4)

$$N(W)\,dW = C(W)\,\frac{\alpha}{\pi}\,\frac{W}{W_p^2}\,(W_p - W)^2\,dW$$

where $N(W)dW$ is the number of photons in the energy interval between W and $W + dW$, W_p is the upper energy limit of the photons, α is the fine structure constant, and $C(W)$ is a complicated function of W which, for small values of W, can be considered a constant C. Thus W_p can be found by making a Kurie type plot of $(N/WC)^{\frac{1}{2}}$ vs. W and locating the intersection of the plot with the W axis.

The ratio of the total number of bremsstrahlung photons to the number of K-captures is (4)

$$\int_0^R N(W)/N_c\,dW = \alpha/12\pi \quad W_p^2$$

where N_c is the number of K-captures. A graph of this function vs. the photon end point energy is given in Fig. 1. Since the ratio is proportional to W_p^2, it will be easier to measure experimentally the transition energies for K-captures of high W_p.

d. L/K ELECTRON CAPTURE RATIO

In general, it is also possible for a given nucleus to capture electrons from one of the L sub-shells instead of from the K shell. It can be shown that the ratio of the number of L_I-captures to the number of K-captures can be used to determine the decay energy of an electron capture reaction. According to Rose (5), the ratio of the probabilities of L_I to K-capture is given by

$$a\,\frac{P_{LI}}{P_K} = \left(\frac{q_{LI}}{q_K}\right)^2 \left(\frac{g_{LI}}{g_K}\right)^2$$

where q_i is the energy of the i neutrino, a is the exchange correction, and g_i is the Dirac radial wave function of the i shell, whose value can be determined from the literature (3). The neutrino energy is given by $q_i = \Delta E - E_i$ where E_i is the i shell binding energy. The above relations yield ΔE as

$$\Delta E = \frac{E_{LI} - u\,E_K}{1-u} \qquad \text{where } u = \sqrt{\frac{P_{LI}}{P_K}\left(\frac{q_K}{g_{LI}}\right)^2}$$

A graph of this relation vs. the L_I to K-capture ratio is given in Fig. 2. The variable u is determined from the experiment.

II. EXPERIMENTAL SETUP

The first attempt to obtain the transition energy of

Figure 1. Ratio of the number of bremsstrahlung photons to the number of K-captures vs. the endpoint energy of the photon

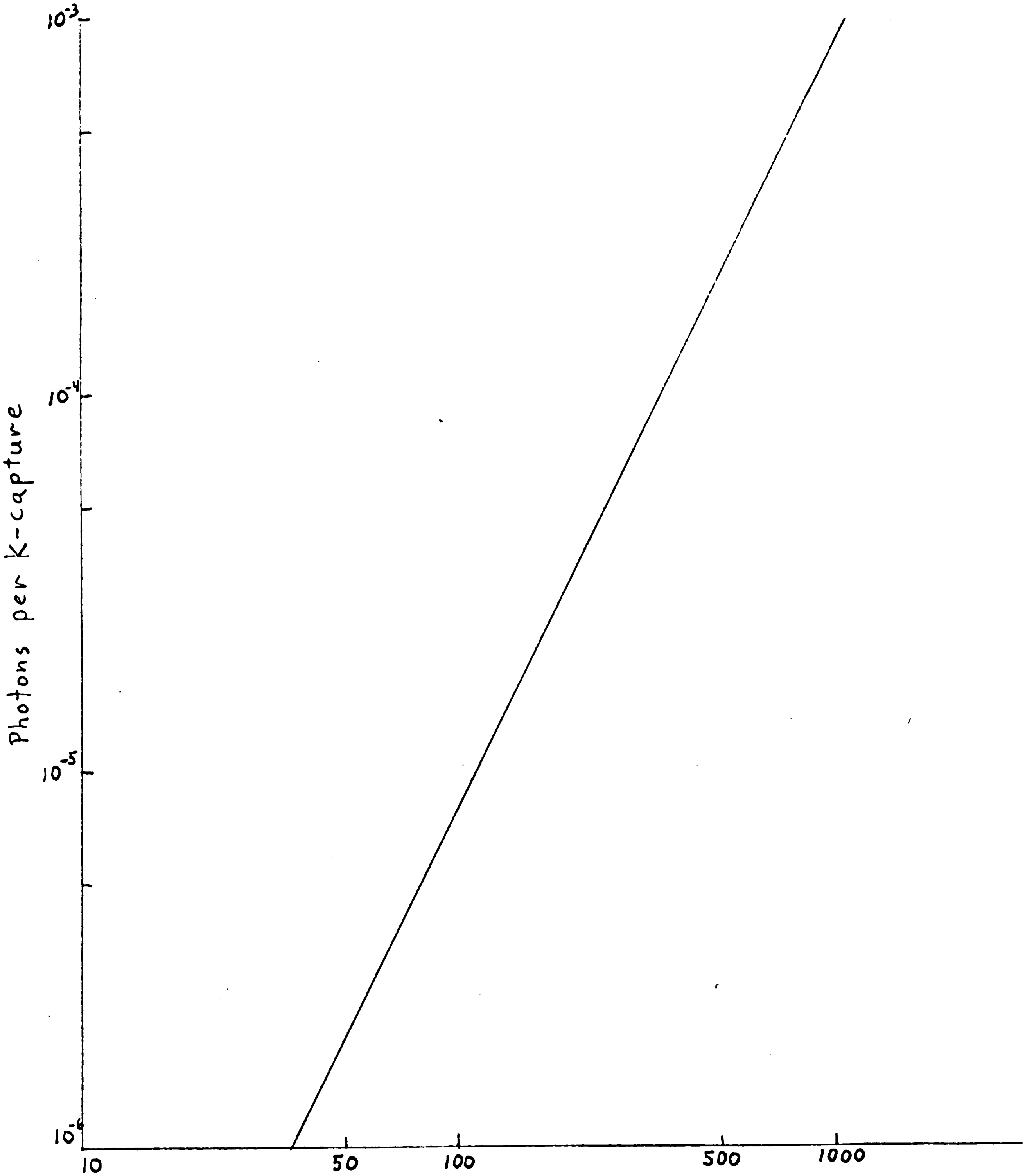
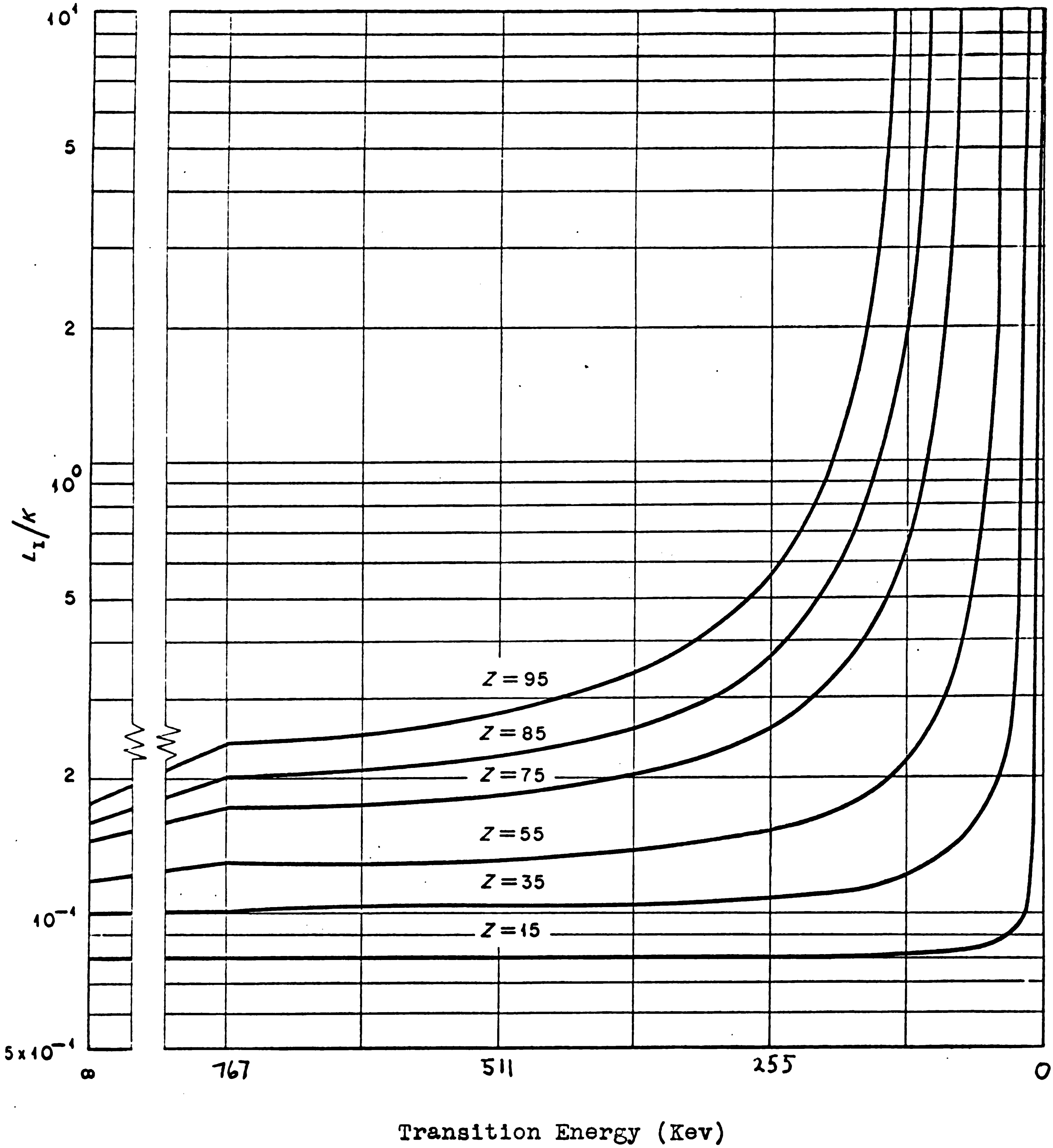


Figure 2. Transition energy vs. L_I to K-capture ratio



Ba^{131} involved the measurement of the endpoint of the bremsstrahlung spectrum. The spectrum in coincidence with a ground state gamma-ray transition was desired. Fig. 3 is a block diagram of the experimental arrangement used, and Fig. 4 is a simplified decay scheme of Ba^{131} (1). Sodium iodide crystals, three inch by three inch diameter were employed as detectors. The two crystals were mounted on Dumont type 6363 photomultipliers. The pulses from the phototubes were fed to cathode followers which drove coaxial cables connected to the amplifiers. A multichannel pulse height analyzer recorded the results of the experiment. As seen in Fig. 3, the pulses from each detector were also used in a coincidence circuit. The cathode follower outputs were sent to Cosmic Radiation Lab linear amplifiers. One of the signals was then passed through a single channel analyzer to a Cosmic Lab Multiple Coincidence Unit. The other signal went directly to the coincidence unit. The single channel analyzer was adjusted so that it allowed only the desired gamma-ray pulses to pass. Thus the coincidence circuit would gate the multichannel analyzer so that it would record pulses only when a gamma-ray of the desired energy was present.

By introducing an artificial delay in one branch of the circuit, it was possible to record the accidental coincidences. If the counting rates in the two detectors are denoted by N_1 and N_2, the accidental coincidence rate is given by $2TN_1N_2$ where T is the resolving time of the coincidence circuit (7). The individual counting rates

Figure 3. Block diagram of experimental arrangement for measuring internal bremsstrahlung spectrum

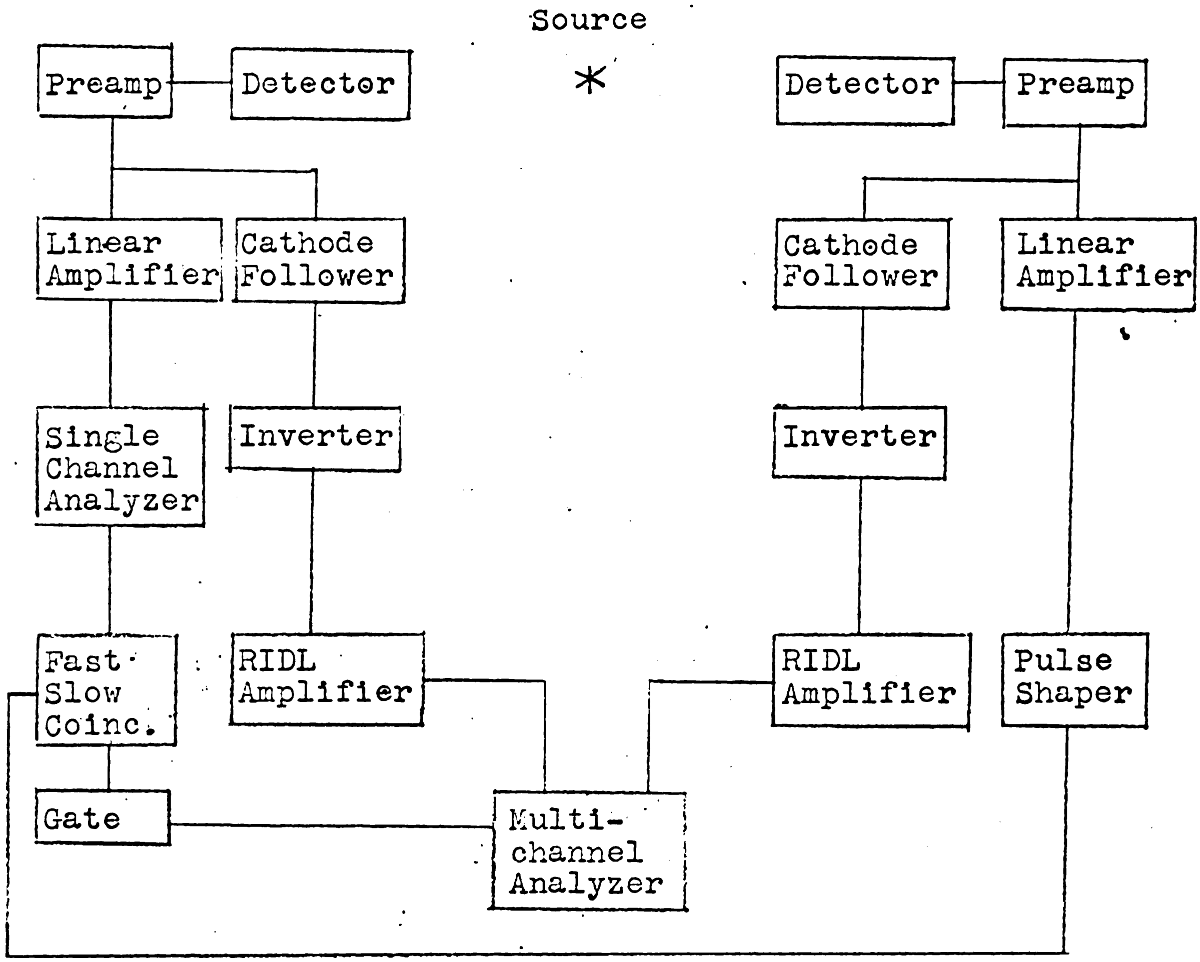
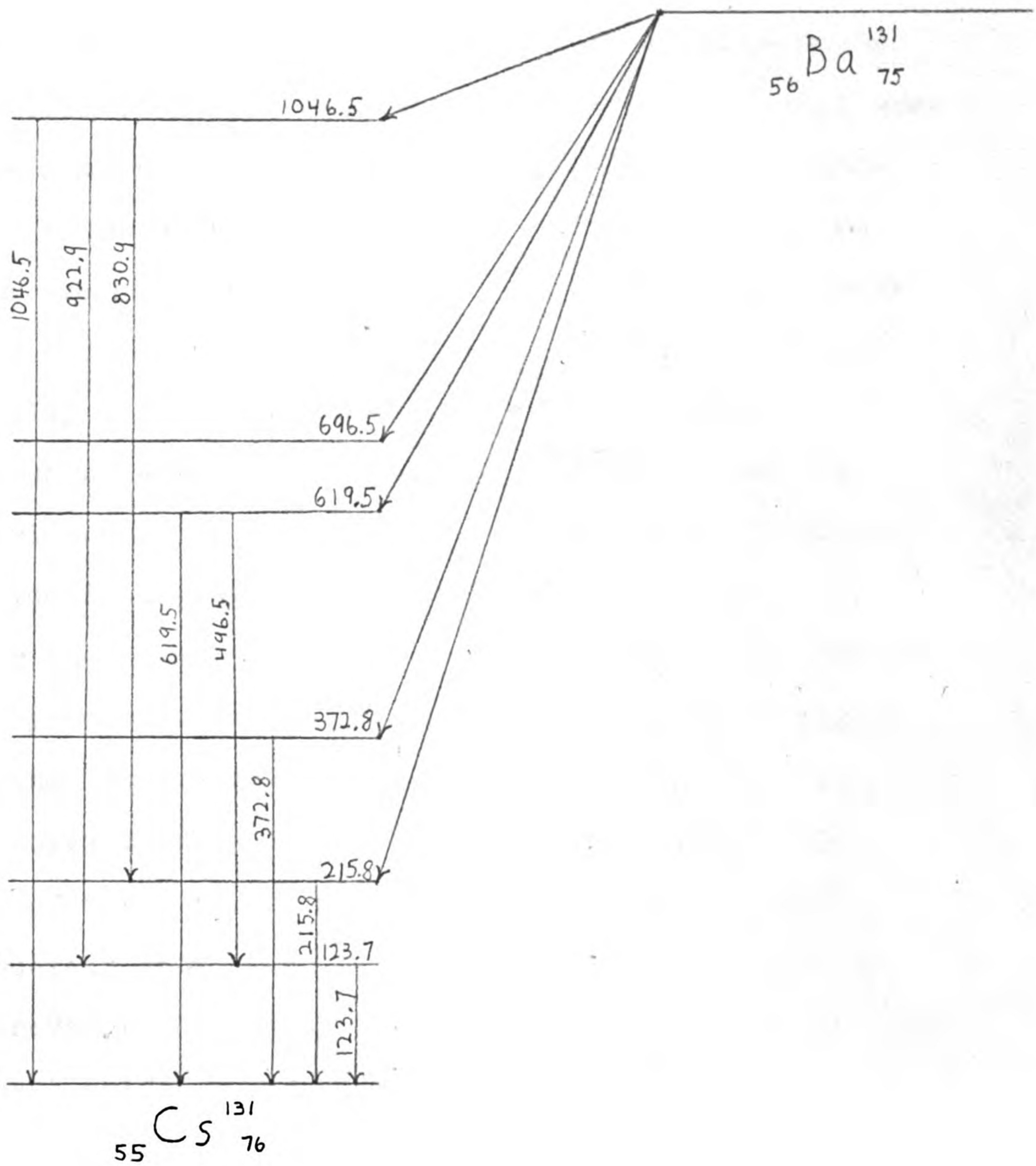


Figure 4. Simplified decay scheme for Ba^{131}.



N_i are given by $N_i = Neps$ where N is the actual singles count rate, e is the efficiency of the detector for the radiation of interest, p is the peak to total ratio, and s is the solid angle subtended by the detector. By subtracting the spectrum of accidentals from the coincidence spectrum, it is possible to obtain the spectrum of true coincidences. This corrected spectrum should contain the internal bremsstrahlung spectrum. The results of these experiments are presented in the next section.

The second attempt to measure the transition energy involved the determination of the K to L capture ratio. Essentially the same experimental setup was used as in Fig. 3, which was explained above. Since the spectrum in coincidence with an x-ray was desired, a 6 millimeter sodium iodide crystal with a thin beryllium window was used for the x-ray detector. This thin window was considerably more efficient in transmitting the low energy x-rays than the aluminum window of the three inch crystal employed in the bremsstrahlung measurements. The results of the K to L ratio measurements were recorded by a Nuclear Data Transistorized Multichannel Analyzer, which proved to be highly stable with respect to gain and zero level shifts. This property was used to advantage on the long runs necessary to obtain reasonable statistics in the coincidence spectrum.

The Ba^{131} was obtained in the form of $Ba(NO_3)_2$ from Oak Ridge National Laboratory. It was reactor produced from a sample of 23% enriched Ba^{130}. By standard radiochemical procedures, the Ba^{131} was separated from Cesium contaminants. The separated material was made into a disk source of diameter 0.6 cm., which was mounted in the center of an aluminum holder of dimentions 6.5 by 9.0 cm.

The Te^{121} was obtained from Brookhaven National Laboratory where it was cyclotron produced. It was also purified by radiochemical procedures. The Te^{121} was deposited on a 3.6 cm. long strip of gold foil, which was then mounted on an aluminum holder as described above. This source was allowed to age for about 12 half-lives to reduce the amount of the 17 day activity which was initially present.

III. EXPERIMENTAL RESULTS

A. BA^{131} RESULTS

1. INTERNAL BREMSSTRAHLUNG EXPERIMENTS

The first attempt to measure the decay energy of Ba^{131} involved the determination of the endpoint of the internal bremsstrahlung spectrum in coincidence with a ground state transition. With reference to the decay scheme in Fig. 4, it was possible to use the 1046.5, 619.5, or the 372.8 Kev ground state transitions. If the transition energy to the 1046.5 Kev state is assumed to be of the order of 200 Kev, then the graph in Fig. 1 shows that the ratio of the number of bremsstrahlung photons to the number of K-captures is of the order of 10^{-5} for this transition. For the other transitions of interest the ratio is of the order of 10^{-4}. This extremely low production rate, coupled with the fact that the photons would have an energy distribution, made the determination of the endpoint impossible. Data were taken for each of the ground state transitions mentioned above, but the low energy portion of the bremsstrahlung spectrum was completely masked by second order effects such as gamma-ray summing in the detectors. Absence of any high energy bremsstrahlung would tend to support the assumption of a decay energy considerably less than the 1.7 Mev postulated by systematics.

2. L/K CAPTURE RATIOS

An attempt was made to obtain the decay energy by finding the K to L capture ratio. If P_K is the fraction of electron captures that proceed by K-capture, w_K is the

K fluorescence yield, s_K is the detector solid angle, e_K is the detector efficiency for K x-rays, N_{g-K} is the number of gamma-ray x-ray coincidences, and N_g is the number of gamma-rays detected, then

$$P_K \omega_K e_K S_K = \frac{N_{g-K}}{N_g}$$

where P_K is defined by the relation

$$\frac{1}{P_K} = \frac{N_K + N_L + N_M + \cdots}{N_K} \approx 1 + A \frac{P_{LI}}{P_K}$$

Here N_i is the number of captures to the i shell, a is the exchange effect correction (8), and the factor A is to account for capture by L_{II} and higher shells. Solving for P_{LI}/P_K in terms of experimentally measurable quantities

$$a \frac{P_{LI}}{P_K} = \left(\frac{N_g e_K \omega_K S_K}{N_{g-K}} - 1 \right) \frac{1}{A} .$$

According to Brysk and Rose (6)

$$a \frac{P_{LI}}{P_K} = \left(\frac{q_{LI}}{q_K} \right)^2 \left(\frac{g_{LI}}{g_K} \right)^2$$

where g_{LI} and g_K are the Dirac radial wave functions, and q_i is the energy of the neutrino ($\Delta E - E_i$), where E_i is the i shell binding energy. Thus the transition energy can be obtained in terms of tabulated constants and experimentally determined numbers. The results for Ba^{131} follow.

Fig. 5 is the spectrum in coincidence with the Cesium 31.64 Kev K x-ray (with accidental coincidences removed) and Fig. 6 is a singles spectrum (with background removed) corrected to the same time as the coincidence run. Both spectra were analyzed by using standard "stripping" techniques, beginning at the high energy end. The following experimental numbers were obtained.

$N_{K-1046.5} = 6079 \pm 80$ $\qquad N_{1046.5} = 360056 \pm 600$

$e_K s_K = 0.026 \pm .002$ $\qquad \omega_K = 0.876 \pm .005$

Figure 5. Ba^{131} coincidence spectrum with cesium x-ray

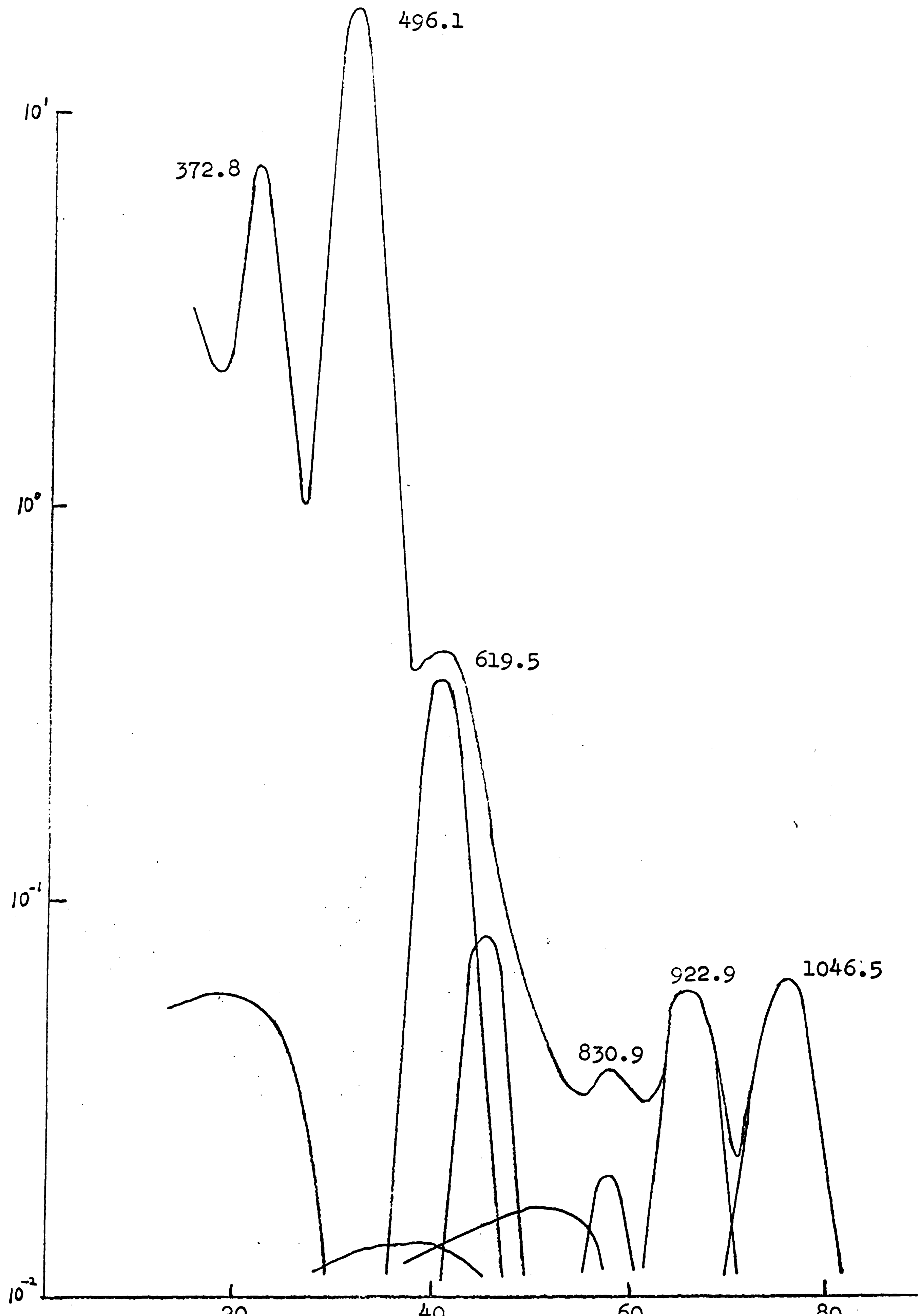
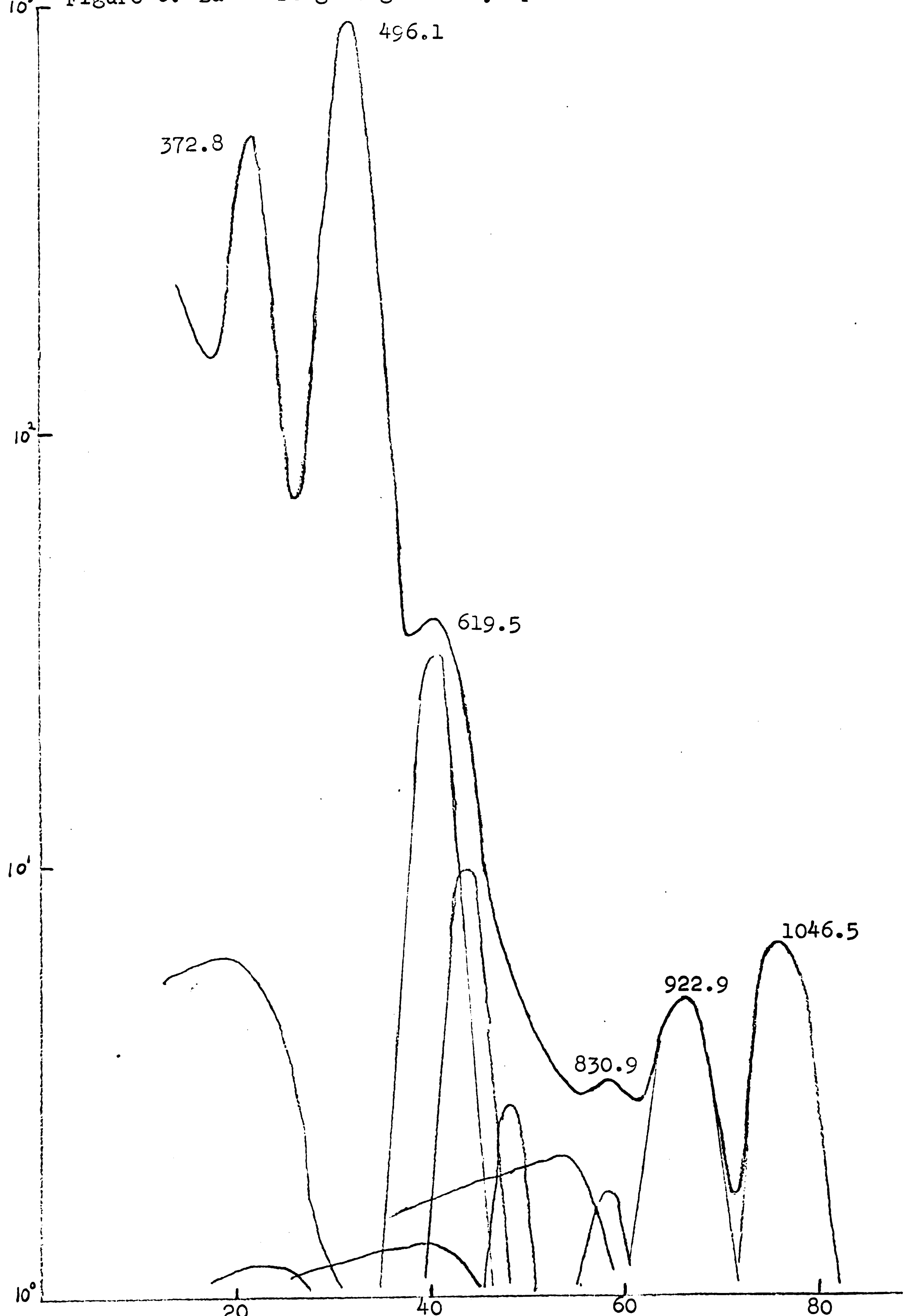


Figure 6. Ba^{131} singles gamma-ray spectrum



The constant a is given by the relation*

$$a = 1 + 4/Z$$

A value of 1.07 was used. The decay energy to the 1046.5 Kev level by this method was found to be 97±25 Kev. Thus the ground state energy difference between Ba^{131} and Cs^{131} was found to be 1143±25 Kev.

It is apparent that certain errors will be introduced because of the uncertainty in the fluorescence yield, detector efficiency, and solid angle measurements. In many cases these errors can be eliminated. If the isotope has two or more levels which decay to the ground state, it may be possible to find P_{LI}/P_K ratios for both transitions. According to Fig. 2, valid results may be obtained for levels to which the transition energy is of the order of 500 Kev or less. By taking the ratio of these ratios, the efficiencies, solid angle, and fluorescence yield cancel, giving a more accurate experimental result. In this case

$$\frac{P_{K1}}{P_{K2}} = \frac{N_{K-1}}{N_{K-2}} \quad \frac{N_2}{N_1}$$

where P_{K1} is the probability of K-capture to the first state, and P_{K2} is the K-capture probability to the second state. This can be solved to give

$$\left(\frac{q_{LI}}{q_K}\right)_1^2 = \left[\frac{1 + aA\,(g_{LI}/g_K)^2 (q_{LI}/q_K)_2^2}{P_{K1}/P_{K2}} - 1\right] \frac{1}{aA\,(g_{LI}/g_K)^2}$$

from which the decay energy can be found by the relation

$$\left(\frac{q_{LI}}{q_K}\right)_1 = \frac{\Delta E - E_{LI}}{\Delta E - E_K}$$

*This relation is, strictly, only for $Z < 20$. The value used was obtained by extrapolation.

where the shell binding energies are tabulated (3). The 1046.5 Kev state was used as state 1 and the 619.5 Kev state was used as state 2. The experimental numbers for the 619.5 state are

$$N_{K-619.5} = 21488 \pm 150 \qquad N_{619.5} = 1148789 \pm 1050$$

The value of $(q_{LI}/q_K)_2$ for capture to the 619.5 Kev state was found to be 1.060 by successive approximations. This gave a value of 1.390 for $(q_{LI}/q_K)_1$ for capture to the 1046.5 Kev state. The decay energy to the 1046.5 level by this method was found to be 113 ± 8 Kev. Thus the ground state energy difference between Ba^{131} and Cs^{131} was found to be 1159 ± 8 Kev.

3. CONVERSION COEFFICIENT MEASUREMENTS

It is possible for an excited nucleus to decay by not emitting a gamma-ray photon. One such process is known as internal conversion. Here, the energy of the excited nucleus is transferred internally to an atomic electron, which is ejected with the transferred energy minus the electron binding energy. The fraction of K electrons which are converted, to the number of emitted gamma-rays is called the K conversion coefficient and is designated α_K. The fraction of total transitions going by K conversion is $\alpha_K/(1+\alpha_T)$ where α_T is the total conversion coefficient $(\alpha_T = \alpha_K + \alpha_L + \cdots)$. Following the emission of a K conversion electron, a K x-ray may be emitted. These x-rays can produce coincidences with those gamma-rays which are in cascade with the transition which is internally converted.

Thus certain gamma-ray peaks may be enhanced in the coincidence spectrum. The number of coincidences will now be

$$N_{K-\gamma} = \left(N_\gamma e_K s_K \omega_K P_K\right) + \left(\frac{\alpha_K}{1+\alpha_T}\right)\left(N_\gamma e_K s_K \omega_K\right)$$

where the second term arises as a result of the conversion x-rays. The above relation can be solved to yield

$$\frac{\alpha_K}{1+\alpha_T} = \frac{N_{K-\gamma}}{N_\gamma e_K s_K \omega_K} - P_K$$

The data taken in the previous section can be used to give $\alpha_K/(1+\alpha_T)$ for the 123.7 Kev transition by finding the enhancement of the 922.9 Kev gamma-ray peak which is in coincidence with the 123.7 Kev transition. Thus

$$\left(\frac{\alpha_K}{1+\alpha_T}\right)_{123.7} = \frac{N_{922.9-K}}{N_{922.9} e_K s_K \omega_K} - P_{K(1046.5)}$$

The following experimental data were used.

$N_{K-922.9} = 2827 \pm 55$ $\qquad N_{922.7} = 128675 \pm 360$

$e_K s_K = 0.021 \pm .002$ $\qquad P_K(1046.5) = 0.74 \pm 0.15$

From these data, the fraction of K conversion electrons is 0.29 for the 123.7 Kev transition.

The number of 1046.5 coincidences is given by

$$N_{K-1046.5} = N_{1046.5}\, e_K \omega_K s_K P_{K(1046.5)}$$

By taking the quotient of this expression and the expression for the number of 922.9 Kev coincidences, the relation for the 123.7 Kev transition becomes

$$\left(\frac{\alpha_K}{1+\alpha_T}\right)_{123.7} = \left(\frac{N_{K-922.9}}{N_{K-1046.5}} \; \frac{N_{1046.5}}{N_{922.9}}\right)\left(P_{K(1046.5)}\right) - P_{K(1046.5)}$$

Thus the uncertainties in the efficiencies, solid angle and fluorescence yield have been eliminated. The result for this method gives the fraction of K conversion electrons for the 123.7 Kev transition as 0.22±.03. This value is in good agreement with the value of 0.18±.04 which was found by Kelly and Horen (1) another way.

B. TE^{121} RESULTS

The method of capture ratios to determine the energy of decay by electron capture was applied to Te^{121}. The corrected singles and coincidence spectra are given in Fig. 7, and a tentative decay scheme in Fig. 8 (9). According to this decay scheme, the 575 and 1130 Kev transitions are to the ground state. Measurements were made using both of these transitions. These data were obtained fot the 575 transition.

$N_{K-575} = 169101 \pm 500 \quad N_{575} = 4269150 \pm 2100 \quad e_K s_K = 0.053$

From these data, a value of $0.12 \pm .02$ was obtained for P_{LI}/P_K. As can be seen in Fig. 2, this value falls on the flat portion of the decay energy curve, hence the uncertainty in the decay energy to the 575 Kev level is very large. At best, a lower limit (corresponding to $P_{LI}/P_K = 0.14$) can be established at about 400 Kev.

Examination of the spectra in Fig. 7 indicated certain discrepancies in the tentative decay scheme. It can be seen that the 506 Kev transition was not enhanced in K x-ray-gamma-ray coincidences relative to the 575 Kev transition, as would be expected from the tentative decay scheme. Since the 70 Kev transition would be expected to be highly converted, it can be concluded that the 506 Kev transition is to the ground state.

Measurements with the 1130 Kev transition showed that it is enhanced in coincidence. Thus it must be in coincidence with a low energy (>30.5 Kev) highly converted transition. All these conclusions are incorporated in the proposed decay scheme, Fig. 9.

Figure 7. Te^{121} singles and coincidence gamma spectra

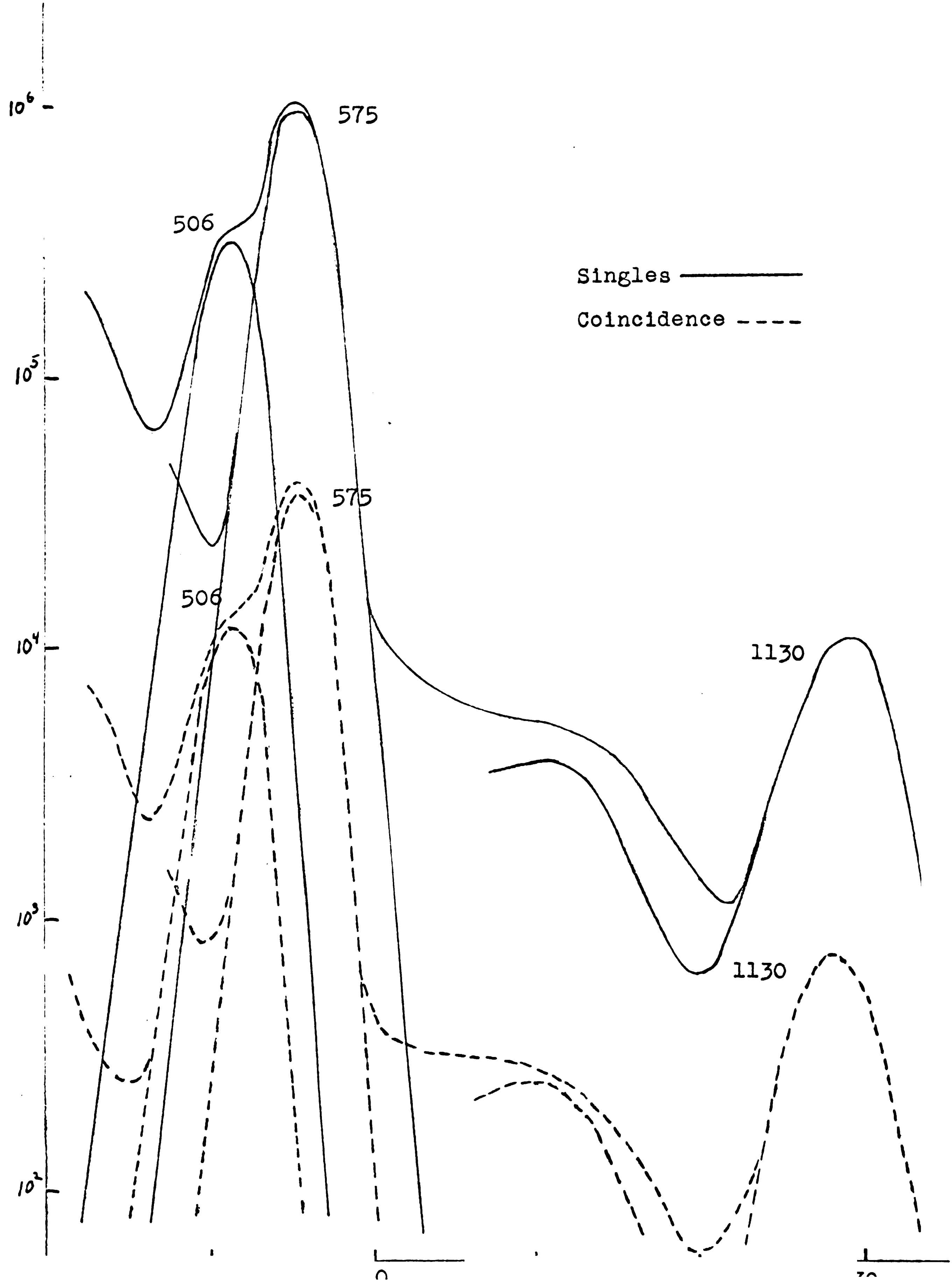


Figure 8. Tentative decay scheme for Te^{121}.

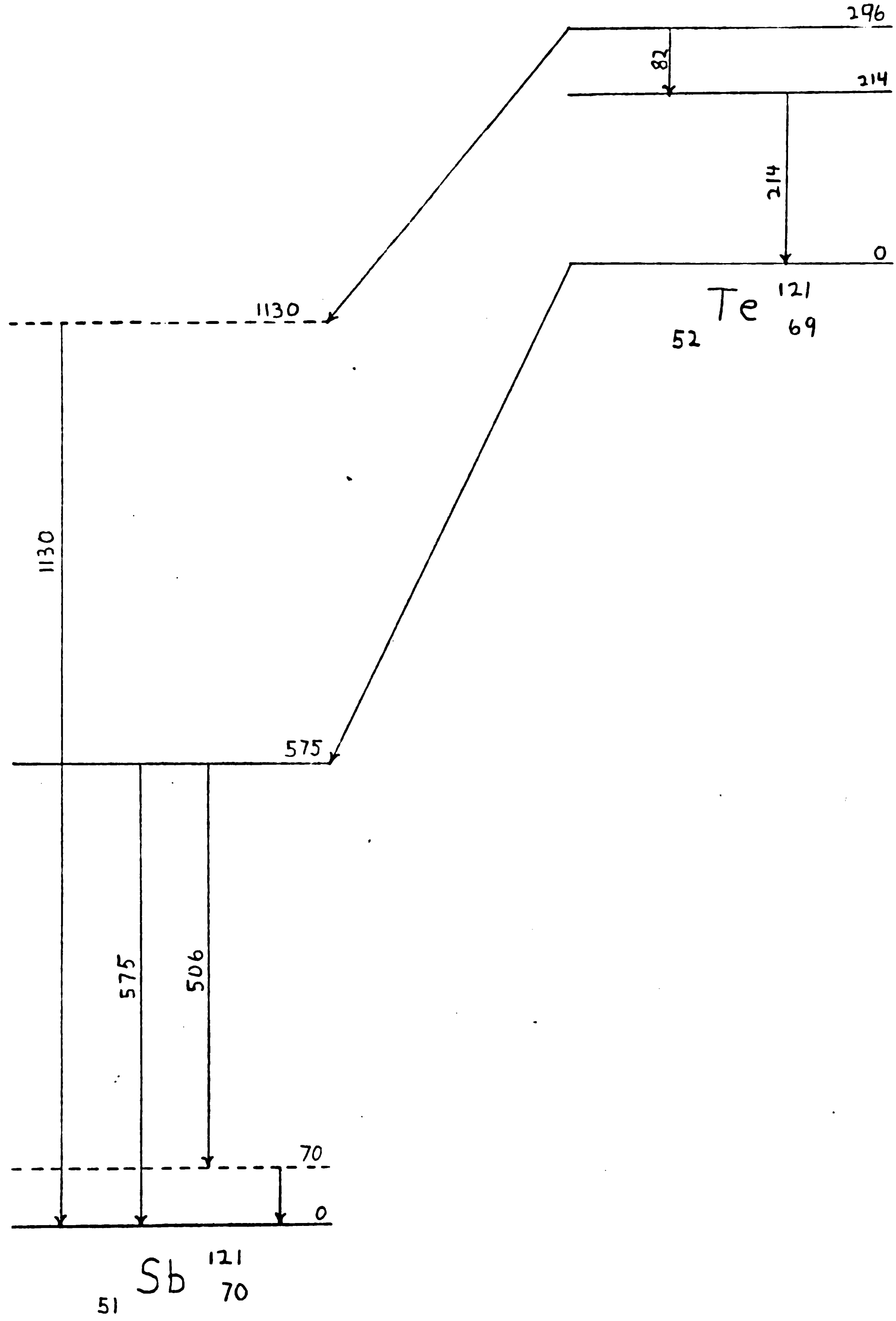
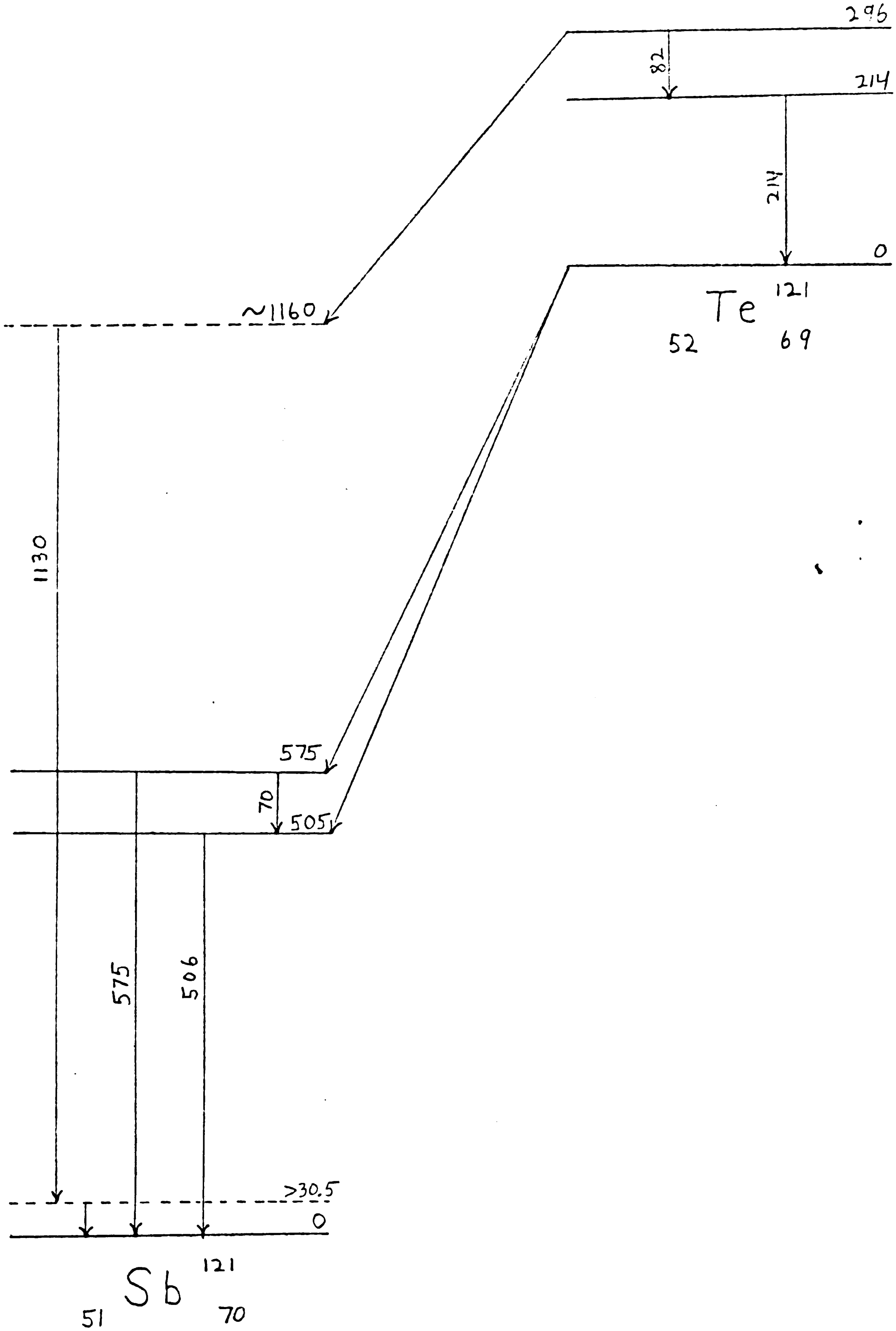


Figure 9. Proposed decay scheme for Te^{121}.



IV. CONCLUSIONS

For Ba^{131}, two values were obtained for the electron capture decay energy to the 1046.5 Kev state by slightly different methods. The results were 97±25 Kev and 113±8 Kev. These results agree quite well with the value of 117±10 Kev which was recently reported for this transition by Robinson (10). Using my value of 113 Kev, the following results were calculated for transitions to the other states, and their respective log ft values. Energies are in Kev.

LEVEL	DECAY ENERGY	LOG ft
1046.5	113	6.1
696.5	463	7.7
619.5	540	6.3
372.8	787	6.9
215.8	944	7.2

On the basis of systematics, no parity change is expected for these transitions. Thus the log ft values found show that all these transitions are ℓ forbidden. This is in agreement with what would be predicted on the basis of available shell model states. Measurement of the $\alpha_K/(1+\alpha_T)$ conversion coefficient for the 123.7 Kev transition gave 0.22±.03. This result agrees closely with the result of Kelly and Horen (1) who obtained a value of 0.18±.04 from conversion electron measurements. This tends to support the results of the decay energy measurements of the present work.

The decay energy for the decay by electron capture of Te^{121} to the 575 Kev state was shown to be greater than approximately 400 Kev. It was concluded that the 1130 Kev transition is in coincidence with a highly converted low energy gamma-ray of energy greater than 30.5 Kev. It was also concluded that, since the 506 Kev transition was not enhanced in coincidence relative to the 575 Kev transition, the 506 Kev transition is to the ground state. The 70 Kev transition is then from the 575 Kev level. These conclusions are presented in a modified decay scheme.

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APPENDIX: ERROR ANALYSIS

A summary of the formulae used in evaluating the errors of the various quantities measured in this work is included in this section. The numerical values of these errors are also included.

By standard rules for combining independent errors in products and quotients (11), the error in P_{LI}/P_K is found to be

$$\left(\Delta \frac{P_{LI}}{P_K}\right)^2 = (\Delta P)^2 + \left(\frac{\Delta A}{A^2}\right)$$

where

$$\left(\frac{\Delta P}{P}\right)^2 = \left(\frac{\Delta N_\gamma}{N_\gamma}\right)^2 + \left(\frac{\Delta N_{\gamma-K}}{N_{\gamma-K}}\right)^2 + \left(\frac{\Delta \epsilon_K S_K}{\epsilon_K S_K}\right)^2 + \left(\frac{\Delta \omega_K}{\omega_K}\right)^2 + \left(\frac{\Delta A}{A}\right)^2$$

The error in the efficiency included error in measurement as well as quoted error in the tabulated efficiencies for NaI crystals. Errors in w_k and A are as listed in the literature (3). The error in the singles and coincidence totals include statistical counting error, background subtraction error and a correction for uncertainty in the peak sums (area under the curves). By alternating true and accidental coincidence runs of equal length, errors due to the uncertainty of the half-life were minimized. The error in the decay energy is obtained from

$$\Delta E = \frac{E_{LI} - \mu E_K}{1-\mu} \qquad \text{where} \quad \mu = \sqrt{\frac{P_{LI}}{P_K}\left(\frac{g_K}{g_{LI}}\right)^2}$$

The errors in the numerator and denominator of ΔE are not independent (11) so,

$$\Delta(\Delta E) = \frac{\Delta(E_{LI} - \mu E_K)}{1-\mu} + \frac{\Delta \mu}{(1-\mu)^2}$$

Here $\left[\Delta(E_{LI} - \mu E_K)\right]^2 = (\Delta E_{LI})^2 + \left[\Delta(\mu E_K)\right]^2$ where $\left[\frac{\Delta(\mu E_K)}{\mu E_K}\right]^2 = \left(\frac{\Delta E_K}{E_K}\right)^2 + \left(\frac{\Delta \mu}{\mu}\right)^2$

and

$$\Delta(1-\mu) = -\Delta\mu = -\mu\left(\left[\frac{\Delta(g_K/g_{LI})}{g_K/g_{LI}}\right]^2 + \frac{1}{4}\left[\frac{\Delta(P_{LI}/P_K)}{P_{LI}/P_K}\right]^2\right)^{1/2}$$

In the second method of determining the decay energy, ΔE is given by the relation

$$\Delta E = \frac{E_{LI} - (q_{LI}/q_K)\,E_K}{1-(q_{LI}/q_K)}$$

Hence the error in ΔE is

$$\Delta(\Delta E) = \frac{\Delta\left[E_{LI} - (q_{LI}/q_K)_1 E_K\right]}{1-(q_{LI}/q_K)_1} + \frac{\Delta(q_{LI}/q_K)_1}{\left[1-(q_{LI}/q_K)_1\right]^2}$$

where

$$\left[\Delta\left(\frac{q_{LI}}{q_K}\right)_1^2\right]^2 = (\Delta\beta)^2 + (\Delta\lambda)^2 + (\Delta\gamma)^2$$

and

$$\left(\frac{\Delta\beta}{\beta}\right)^2 = \left[\frac{\Delta\left(\frac{K_1}{K_2}\right)}{K_1/K_2}\right]^2 + \left[\frac{\Delta A}{A}\right]^2 + \left[\frac{\Delta\left(\frac{g_{LI}}{g_K}\right)^2}{(g_{LI}/g_K)^2}\right]^2$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)^2 = \left[\frac{\Delta A}{A}\right]^2 + \left[\frac{\Delta\left(\frac{g_{LI}}{g_K}\right)^2}{(g_{LI}/g_K)^2}\right]^2$$

$$\frac{\Delta\gamma}{\gamma} = \left[\frac{\Delta\frac{K_1}{K_2}}{K_1/K_2}\right]^2 + \left[\frac{\Delta\left(\frac{g_{LI}}{g_K}\right)^2}{(g_{LI}/g_K)^2}\right]^2 + \left[\frac{\Delta\left(\frac{q_{LI}}{q_K}\right)_2^2}{(q_{LI}/q_K)_2^2}\right]^2$$

$$K_1 = \frac{N_{1046-K}}{N_{1046}} \qquad K_2 = \frac{N_{620-K}}{N_{620}}$$

The numerical results for all these error terms are summarized in the following tables:

RESULTS FOR CAPTURE RATIO DETERMINATION

Transition	$\frac{\Delta N_\gamma}{N_\gamma}$	$\frac{\Delta N_{\gamma-K}}{N_{\gamma-K}}$	$\frac{\Delta \epsilon_K S_K}{\epsilon_K S_K}$	$\frac{\Delta \omega_K}{\omega_K}$	$\frac{\Delta A}{A}$	$\frac{\Delta E_{LI}}{E_{LI}}$	$\frac{\Delta(g_K/g_{LI})}{g_K/g_{LI}}$	$\frac{\Delta(P_{LI}/P_K)}{P_{LI}/P_K}$	$\frac{\Delta(\Delta E)}{\Delta E}$
1046	.03	.03	.05	.0057	.05	.0001	.02	.40	.235
620	.05	.06							

RESULTS FOR RATIO OF CAPTURE RATIOS DETERMINATION

$\frac{\Delta(q_{LI}/q_K)_1}{(q_{LI}/q_K)_1}$	$\frac{\Delta(K_1/K_2)}{(K_1/K_2)}$	$\frac{\Delta A}{A}$	$\frac{\Delta(g_{LI}/g_K)}{g_{LI}/g_K}$	$\frac{\Delta(q_{LI}/q_K)_2}{(q_{LI}/q_K)_2}$	$\frac{\Delta E_{LI}}{E_{LI}}$	$\frac{\Delta(\Delta E)}{\Delta E}$
.28	.088	.05	.02	.02	.0001	.07