# AN INVESTIGATION OF LIGHT DIFFRACTED BY WIDE, HIGH-FREQUENCY ULTRASONIC BEAMS

Thesis for the Degree of M. S.

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W. Richard Klein

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# AN INVESTIGATION OF LIGHT DIFFRACTED
# BY WIDE, HIGH-FREQUENCY
# ULTRASONIC BEAMS

by

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AN ABSTRACT OF A THESIS

Submitted to
Michigan State University
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Approved: E. A. Hiedemann.

W. Richard Klein

## ABSTRACT

The theory of light diffraction by a sinusoidal, progressive, ultrasonic wave presented by Raman and Nath (1) becomes invalid for high frequency, intense and/or wide sound beams. Mertens (2) has developed a more exact theory for predicting the light diffraction pattern under such conditions. An experimental investigation to determine the range of validity of Mertens' theory is described.

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W. R. K.

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## INTRODUCTION

In 1922, Brillouin (1) predicted that the Debye waves in a liquid should produce effects on light similar to the Bragg reflection. Looking for this effect Debye and Sears (2) found that a liquid striated by an ultrasonic beam acts as an optical grating. Lucas and Biquard (3) discovered the same effect independently at about the same time.

Several authors have treated this subject on a theoretical basis with varying degrees of success. Notable among the early papers are two presenting two different views.

Lucas and Biquard (3) attempted to explain the effect on the basis of refraction of the light due to the periodically varying refractive index, but they could not make any predictions about the intensities of the different diffraction orders.

Bär's (4) experiments revealed the parameters on which the intensity of a diffraction order depends. Bär's results inspired Raman and Nath (5) to a different theoretical approach. The interaction of the ultrasound with the light was considered to have the effect of a pure phase grating whose spacing was the same as the wavelength of the ultrasound. The results of the theory of Raman and Nath show that for a progressive ultrasonic wave with normal incidence the angles at which the diffraction orders should be observed are given by

$$\sin\theta_n = \pm\frac{n\lambda}{\lambda^*} \qquad n = 0, 1, 2, \ldots$$

$\lambda$ and $\lambda^*$ being the wavelengths of the light and the ultrasound respectively, and the intensity of the $n$th diffraction order should be given by

$$I_n = J_n^2(v)$$

where $J_n(v)$ represents the $n$th order Bessel function whose argument $v$ is given by

$$v = \frac{2\pi\mu L}{\lambda} \quad ;$$

$\mu$ represents the maximum variation in the refractive index due to the ultrasound, and $L$ the width of the sound beam.

In a later work, Raman and Nath (6) used Maxwell's equations to describe the propagation of the light through the medium containing ultrasound. An approximate solution of the resulting differential equation shows the predicted intensities to be in agreement with the results of their simplified theory described above.

The validity of these results of Raman and Nath was verified experimentally by Sanders (7).

Intensity predictions obtained using the theory of Raman and Nath appear to be limited in validity to those experimental arrangements where the refraction described by Lucas and Biquard may be neglected. Experimentally, this requires a small gradient of refractive index and/or a narrow sound beam. The first requirement implies a moderate sound intensity and a moderate sound frequency.

Qualitatively, one might think of the observed light diffraction pattern as the resultant of the combined effects of a phase modulation whose magnitude is indicated by the parameter $v$ and an intensity modulation whose magnitude is indicated by a quantity $Hv$. Conditions to be placed on $v$ and $H$ to describe the range of validity of a theory, then, follow as a natural consequence. These conditions for the Raman and Nath theory have been expressed by Extermann and Wannier (8) as

$$H = \frac{2\pi\lambda L}{\mu_0 \lambda^{*2}} << 2$$

and

$$Hv < 2$$

In the above inequalities, it should be noted that the left hand members depend upon the square of the ultrasonic frequency. As a consequence of this dependence, one must use a more complicated model than that of Raman and Nath in describing diffraction by high-frequency ultrasound even for relatively narrow sound beams.

Several authors (8-10) have treated the diffraction problem in the range beyond that where the Raman and Nath theory is valid. Extermann and Wannier, for example, have obtained solutions with what appears to be only a small degree of approximation, but numerical predictions of intensities for a given experimental arrangement are extremely difficult to obtain.

Mertens (10) has made several simplifying assumptions which lead to results in a form from which it is easier to obtain numerical results, but the solutions are less general and the theory appears to be limited in applicability. The purpose of this work is to examine the results of Mertens experimentally and to determine over what range of experimental conditions the theory is useful.

## THE THEORY OF MERTENS

The Raman and Nath theory for the diffraction of light by ultrasonic waves is valid for the limited range of experimental conditions described above. Mertens has extended this range by obtaining a more exact solution of the differential equation of Raman and Nath. The method used is to apply Maxwell's equations to the region of the medium containing the sound beam, along with the initial condition that the light has a plane wave-front at the plane of entrance to the sound beam.

The directions of travel of the light beam and the ultrasound are assumed to be normal. The refractive index is taken to be a linear function of the density of the medium. For a sinusoidally varying ultrasonic beam, then,

$$\mu(x,y,z,t) = \mu_0 + \mu \sin(2\pi\nu^* t - \vec{K}\cdot\vec{r})$$

where $\mu_0$ is the refractive index of the medium, $\mu$ the maximum variation of the refractive index, $\nu^*$ the ultrasonic frequency, $\vec{K}$ the propagation vector of the ultrasound, and $\vec{r}$ the position vector within the medium.

Maxwell's equations as applied to the medium are written:

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}, \qquad \nabla\cdot\vec{H} = 0,$$

$$\nabla\times\vec{H} = \frac{1}{c}\frac{\partial(\mu^2\vec{E})}{\partial t}, \qquad \nabla\cdot(\mu^2\vec{E}) = 0 .$$

$\vec{E}$ represents the electric intensity and $\vec{H}$ the magnetic intensity. Upon elimination of $\vec{H}$

$$\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2(\mu^2\vec{E})}{\partial t^2} + \nabla(\nabla\cdot\vec{E}) \tag{1}$$

$$\nabla\cdot(\mu^2\vec{E}) = 0 \tag{2}$$

Since $\nu^* \ll \nu$, $\nu$ being the frequency of the light, the refractive index is assumed to be time independent, and the time variation is reestablished in the final result. This gives the problem a stationary character.

Equation (1) may be written

$$\nabla^2 \vec{E} = \frac{\mu^2(x,y,z,t)}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \nabla(\nabla \cdot \vec{E}) \qquad (3) .$$

If the plane ultrasonic waves are traveling in the X-direction

$$\mu(x,y,z,t) = \mu(x,t) \quad ,$$

and

$$\mu^2 \nabla \cdot \vec{E} = - E_x \frac{\partial(\mu^2(x,t))}{\partial x} \quad .$$

Substituting this into Eq. (3) gives

$$\nabla^2 \vec{E} = \frac{\mu^2(x,t)}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} - \nabla\left(\frac{1}{\mu^2(x,t)} \frac{\partial \mu^2(x,t)}{\partial x} E_x\right) .$$

The second term on the right hand side is neglected since it is small compared with the other terms (11). Also, if the light is traveling in the Z-direction these further simplifications result in the form

$$\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \frac{\mu^2(x,t)}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \qquad (4)$$

If $\vec{E}$ is written in the form

$$E = e^{2\pi i \nu t} \phi(x,z,t)$$

Eq. (4) becomes

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = - \frac{4\pi^2}{\lambda^2} \mu^2(x,t)\, \phi$$

The solutions of this equation give the values of the amplitudes of the light in the various diffraction orders.

Since the purpose of this work is to check the results of Mertens' theory experimentally, the mathematical details will not be discussed here. It will suffice to say that Mertens has found solutions of Eq. (5) and Miller and Hiedemann (12) have shown these solutions to be expressible in the form

$$\phi_n = J_n(v) + iHA_n(v) + H^2 B_n(v)$$

The two parameters $A_n(v)$ and $B_n(v)$ may be expressed as the power series

$$A_n(v) = \sum_{m=0}^{\infty} a_{n,m} v^{2m+n}$$

and

$$B_n(v) = \sum_{m=0}^{\infty}{}' b_{n,m} v^{2m+n-2} \qquad (6)$$

where

$$a_{n,m} = \frac{(-1)^m (2m + n[2n+1])}{6(2^{2m+n+1})(m!)[(m+n)!]} ,$$

and

$$b_{n,m} = \frac{(-1)^m [6m + (n+1)(10n-7)][m + 1/6(2n^2+3n-6)]}{60(2^{2m+n})[(m-1)!][(m+n-1)!]}$$

The prime over the summation sign in Eq. (6) indicates that the summation starts with $m = 2$ when $n = 0$.

The light intensity of the $n$th diffraction order is given by

$$I_n = \phi_n \phi_n^*$$

$$= J_n^2(v) + H^2[A_n^2(v) + 2J_n(v)B_n(v)] + H^4 B_n^2(v)$$

$$= J_n^2(v) + H^2 G_n(v) + H^4 B_n^2(v) \qquad (7)$$

where

$$G_n(v) = A_n^2(v) + 2J_n(v)B_n(v)$$

Table III (appendix) gives values of $G_n(v)$ and $B_n^2(v)$ for the zeroth and first orders. All three terms in Eq. (7) were used in the calculations made in this investigation, but in the theory presented by Mertens the last term is neglected. The experimental results show that the contributions of this term are negligible within the range where the theory is applicable.

## EXPERIMENTAL APPARATUS

The standard optical arrangement for the observation of light diffraction by a progressive ultrasonic wave in a liquid medium was used. (See Fig. 1.) The light source, a 100 watt General Electric AH-4 mercury vapor lamp, was housed in a light tight enclosure. Lens $L_1$, a condenser lens with an adjustable aperture, was used to focus the light on the slit. The aperture was used to control the intensity of the light passing through the system. A filter (not shown) designed to pass only the desired mercury green line was placed between $L_1$ and the slit. This filter was unnecessary for intensity readings since a second filter was contained in the photomultiplier, but it served to give a monochromatic source for visual observations. The lens, $L_2$, was positioned so that the light passing through the tank, T, was collimated. A 4 mm square aperture was placed over the exit window of the tank. This mask covered the entire window, eliminating

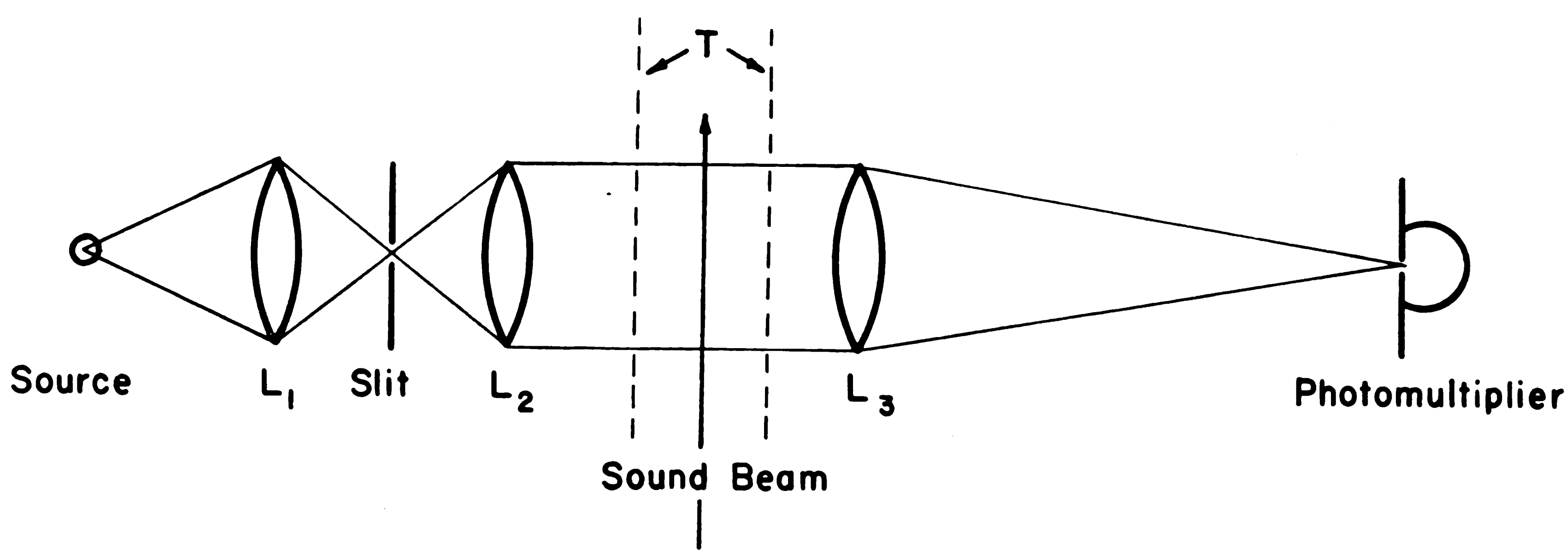



Figure 1. Diagram of the optical arrangement.

stray light. A horizontal slit was placed between the tank and the lens, $L_3$, to further limit the vertical dimension of the light field. Lens $L_3$ was used to project the diffraction pattern onto the plane of the photomultiplier slit. The photomultiplier-microphotometer was type 10-210 manufactured by the American Instrument Company. The tank used had a castor oil absorption well at one end to prevent reflections. The transducer was a 2 inch diameter, x-cut, air backed quartz crystal of fundamental frequency 1.76 Mc which was driven in its third harmonic. The voltage across the transducer was measured with a General Radio vacuum tube voltmeter, type 1800B. The rf power to drive the transducer was supplied by a crystal controlled Johnson-Viking CDC transmitter.

The width of the sound beam was controlled using a pair of hollow reflectors. (See Fig. 2.) The reflectors, R, were designed to present only a plane surface when seen from the transducer side. In order to prevent reflection of sound back onto the face of the transducer, the reflectors were inclined at an angle of $45^{\circ}$ so that the sound was reflected down onto the faces of the two Plexiglas prisms, P. These prisms were oriented so that the sound reflected from their faces would be directed

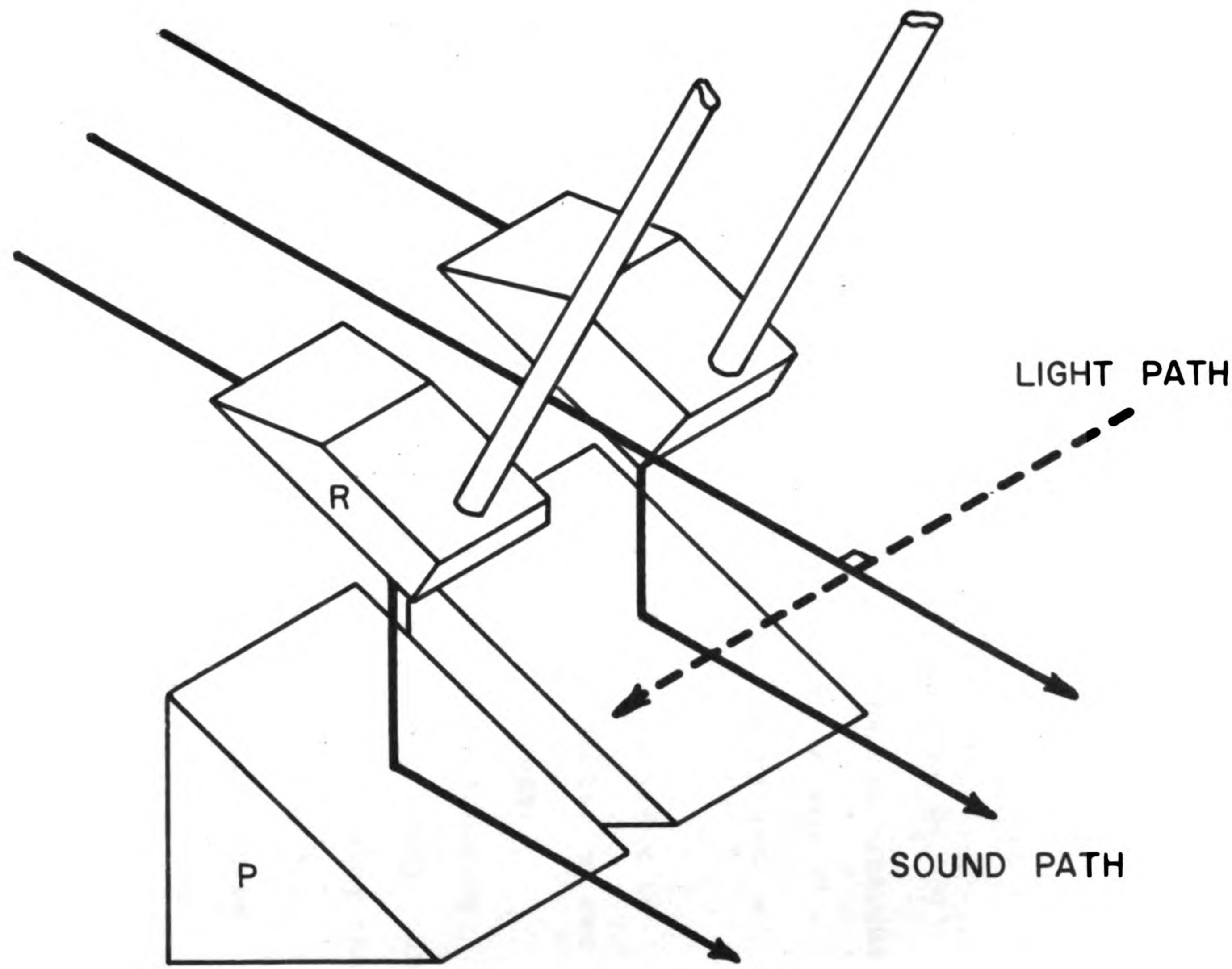



Figure 2. Diagram of the sound aperture.

into the absorption well at the end of the tank.

The dimensions of the sound aperture assembly made it necessary to work with the transducer at a minimum distance of 4 cm from the light beam.

## EXPERIMENTAL PROCEDURE

The results of Mertens expressed by Eq. (7) show that it is possible to describe the experimental conditions using the two parameters $v$ and $H$ . It is necessary then to examine the theory over a range of these two parameters.

Since the variation of refractive index is linear with quartz voltage, the parameter $v$ can be varied by changing the quartz voltage.

The parameter $H$ can be varied by changing either the sound beam width or the frequency. In this investigation the width was varied using the sound aperture described previously. One disadvantage is that the range of $v$ which can be attained experimentally decreases as the beam width is reduced.

To insure that the directions of travel of the sound and the light were mutually perpendicular the quartz voltage was set so that about fifty percent of the light was in the zeroth order. The quartz was then rotated about a vertical axis and simultaneously the light intensity was noted on the photomultiplier. The angular setting was taken to be correct when the light intensity was at a minimum.

The sound aperture was positioned by projecting its shadow image onto a viewing screen.

Using the schlieren technique, the optical aperture was located so that the light passed through the most uniform part of the sound field. Also, since the greater variation was in the vertical direction, a horizontal slit was used to limit the light field to about 0.4 mm in the vertical direction.

When working at high sound intensities dissipated energy creates temperature differences and hence optical inhomogeneity within the medium resulting in undesirable optical refraction. Also, as the temperature of the system changes the acoustical and electrical impedances change giving unstable conditions. For this reason the system was allowed to come to thermal equilibrium between readings.

In taking the readings the quartz voltage was varied in 10 volt intervals from zero to the maximum output voltage attainable.

First the intensity of the zeroth diffraction order was observed. Data were taken for sound beam widths from 1.6 cm to 4.7 cm in 0.2 cm intervals (the odd interval being that from 4.4 cm to 4.7 cm). Three sets of readings were then taken for the first order.

## PRESENTATION OF DATA

### ZEROTH ORDER

Measured zero-order light intensities vs $\nu$ are shown on the following pages. Four curves, representative of the 16 taken are shown in Figs. 3-6.

To normalize the $\nu$ scale it is necessary to assume that some characteristic point on the experimental curve corresponds to the same characteristic point on the theoretical curve since the values of $\nu$ cannot be measured directly. When the second minimum on a curve could be reached experimentally this point was used. Otherwise the first minimum was used.

Figure 7 shows the intensity of the light at the first and second minima of the zeroth order vs the width of the ultrasonic beam.

Figure 8 shows the intensity of the zero-order maxima vs sound beam width.

In determining $H$ the following values were used:

| | |
|---|---|
| sound velocity | 1500 m/sec |
| refractive index of water | 1.33 |
| wavelength of light | 5461 Å |
| frequency of ultrasound | 5235 kc |

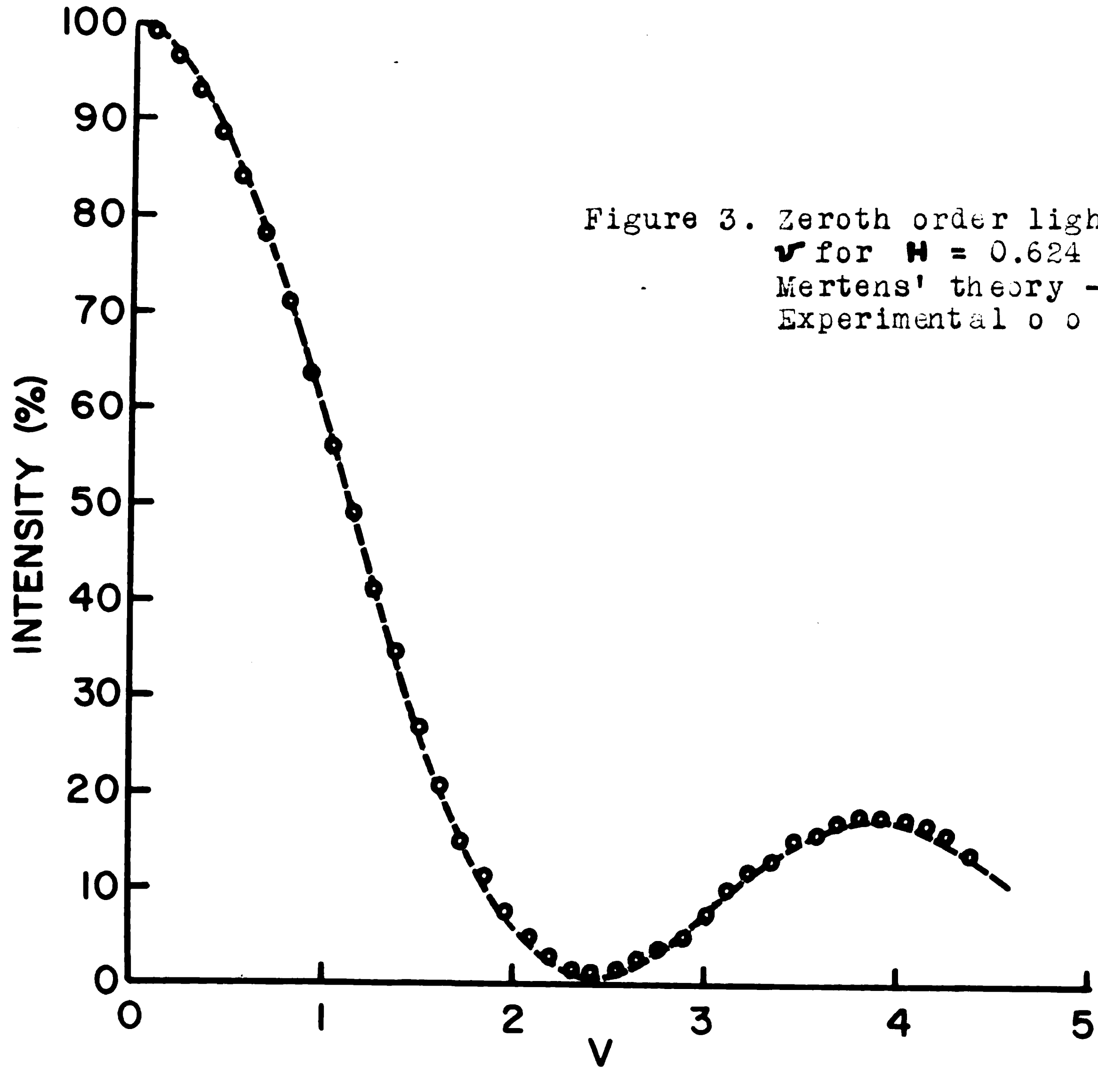



Figure 3. Zeroth order light intensity vs
$v$ for $H$ = 0.624 ($L$ = 2.0 cm).
Mertens' theory - - - - -
Experimental o o o o o

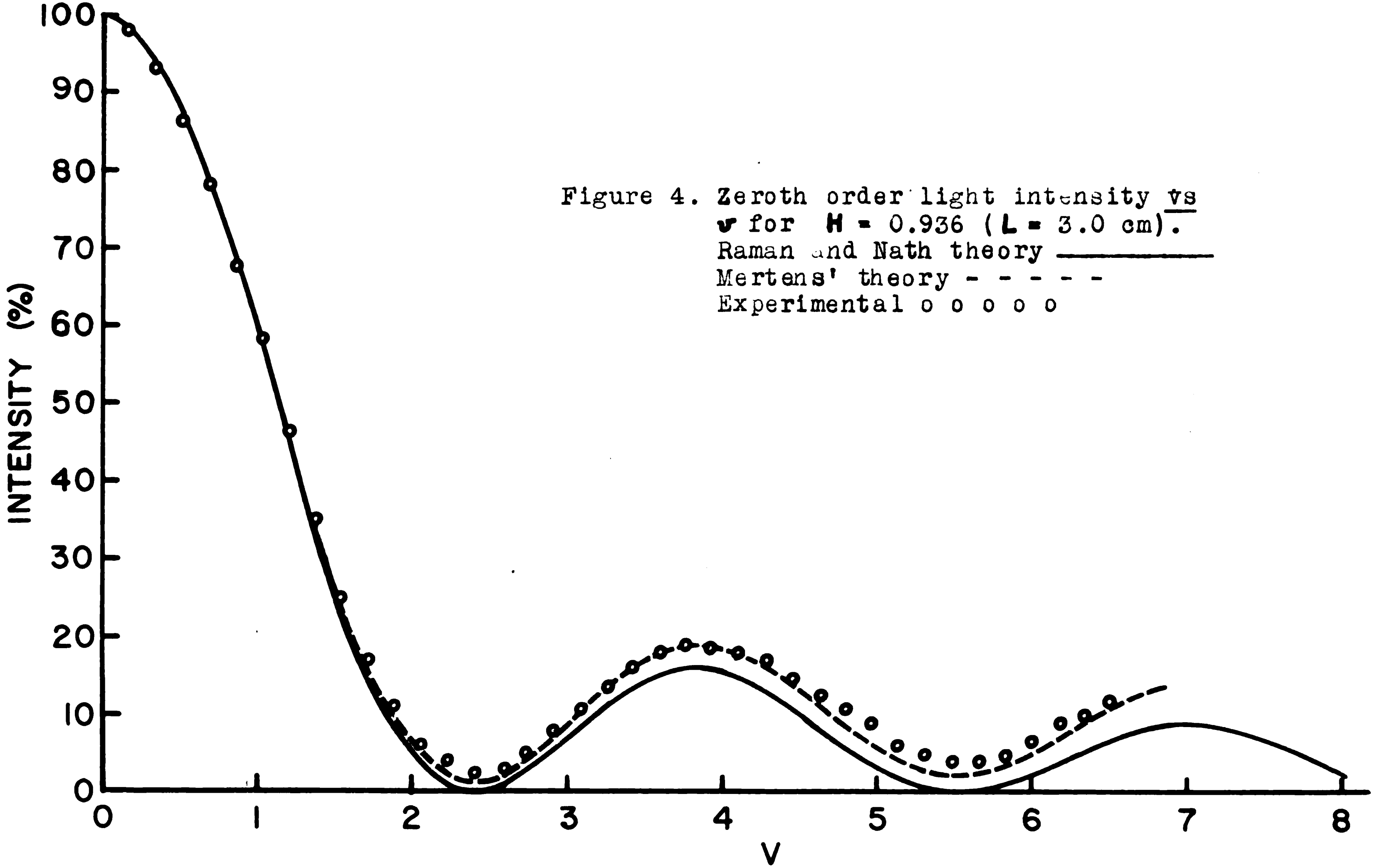



Figure 4. Zeroth order light intensity *vs*
$v$ for $H = 0.936$ ($L = 3.0$ cm).
Raman and Nath theory ———
Mertens' theory - - - - -
Experimental o o o o o

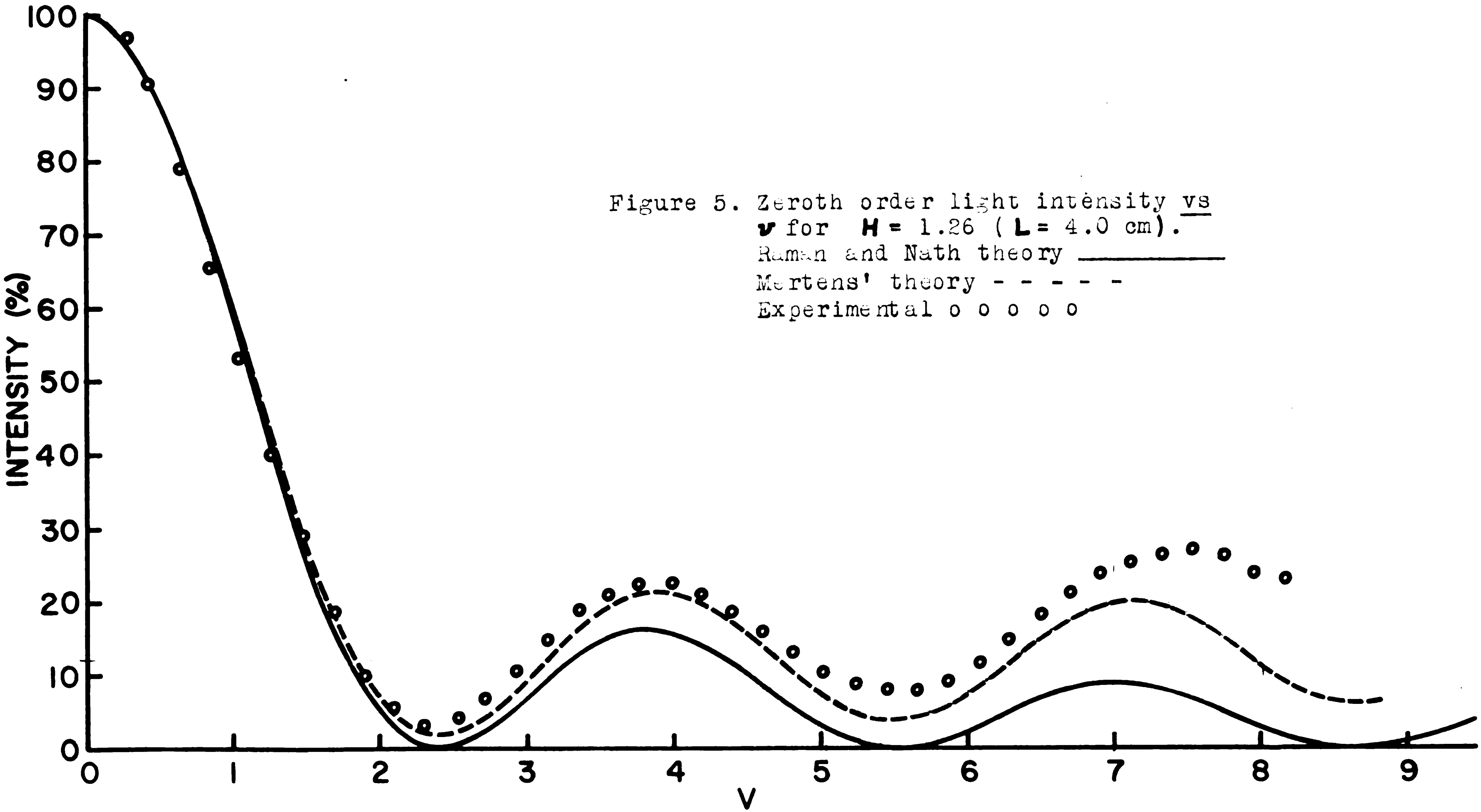



Figure 5. Zeroth order light intensity vs $\nu$ for $H = 1.26$ ( $L = 4.0$ cm).
Raman and Nath theory ———
Mertens' theory - - - - -
Experimental o o o o o

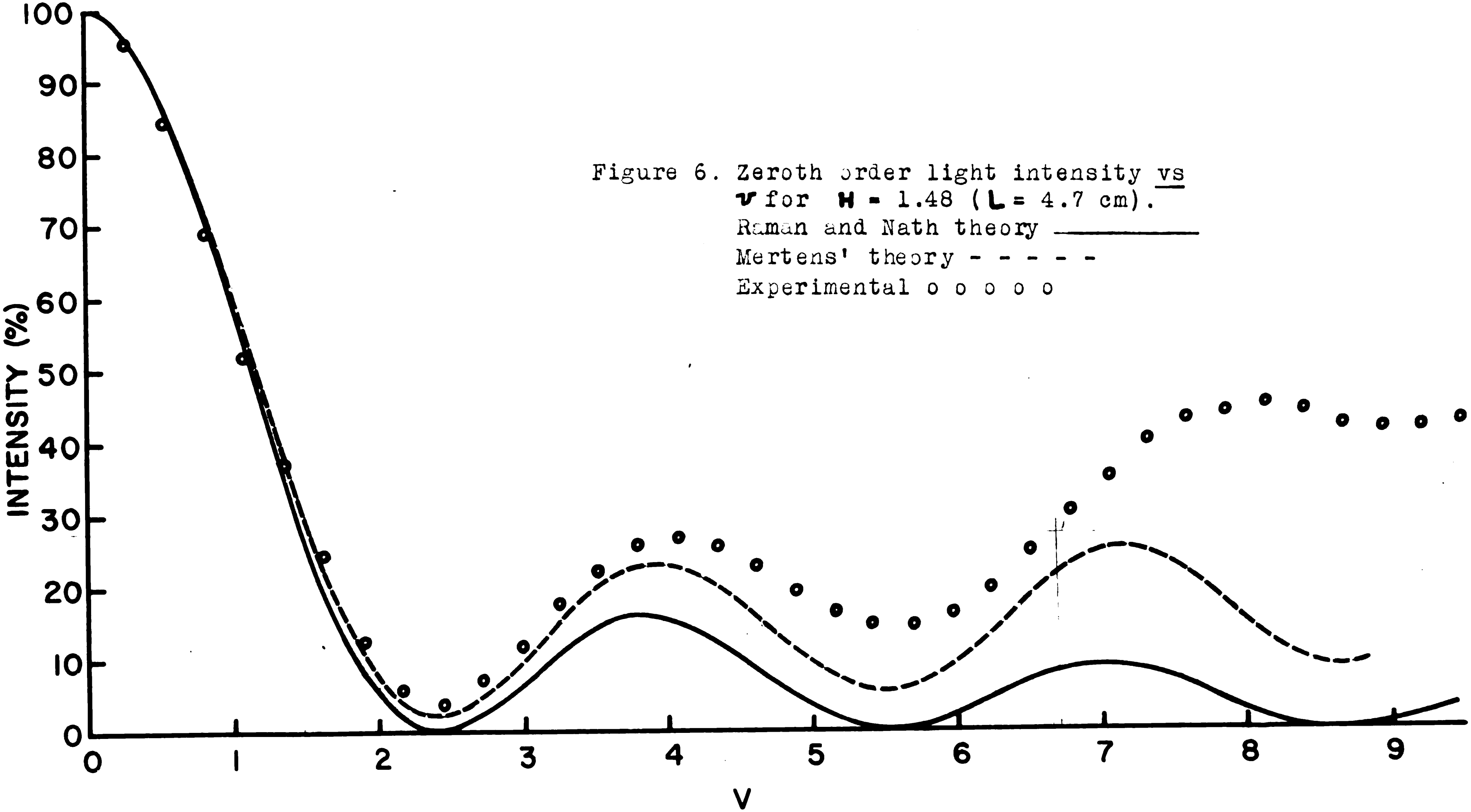



Figure 6. Zeroth order light intensity <u>vs</u> $\nu$ for $H = 1.48$ ($L = 4.7$ cm).
Raman and Nath theory ————
Mertens' theory - - - - -
Experimental o o o o o

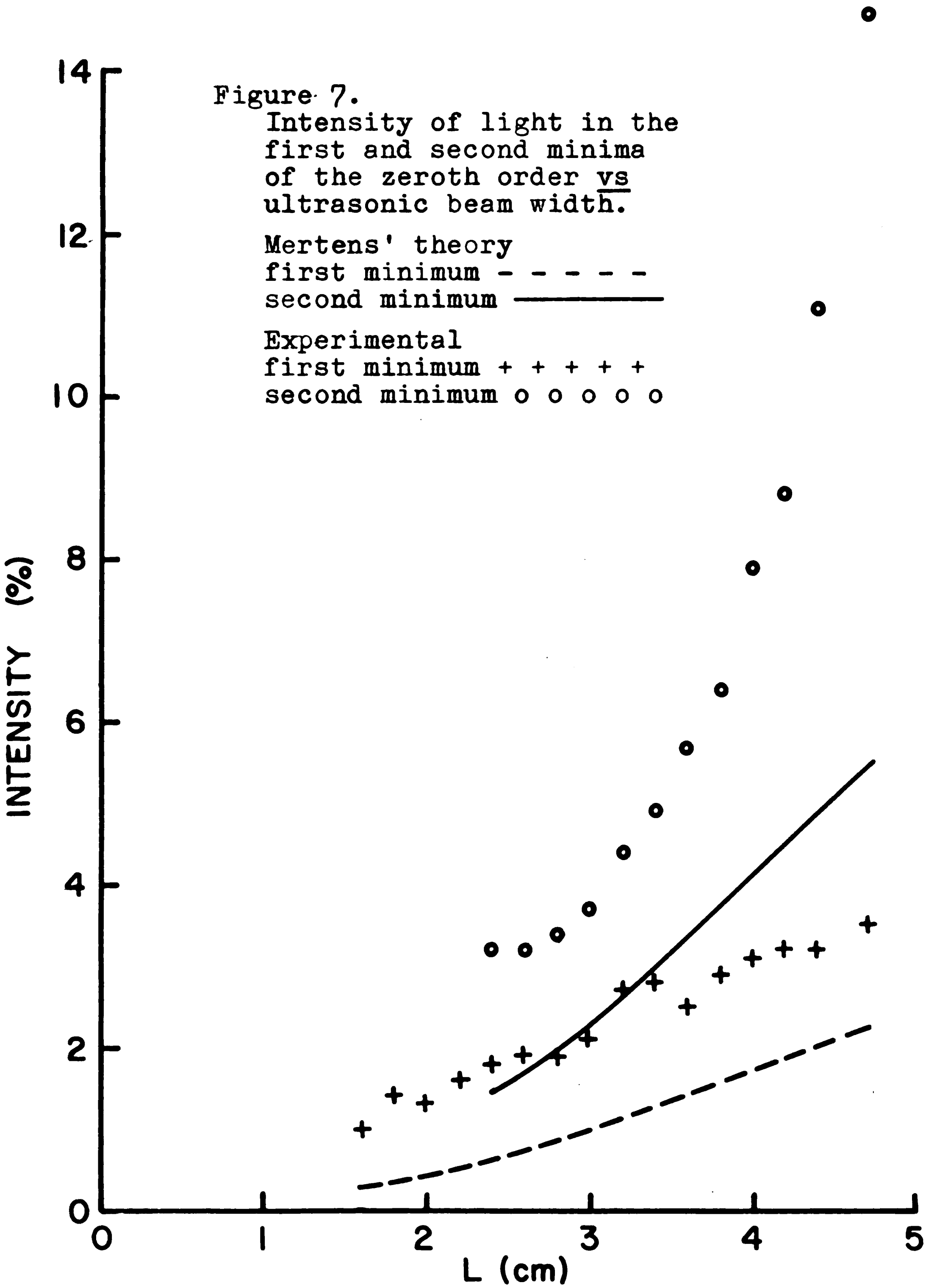



Figure 7.
Intensity of light in the first and second minima of the zeroth order *vs* ultrasonic beam width.

## DATA (continued)

### FIRST ORDERS

Three sets of measurements of first order light intensities were made. These data are shown in Figs. 9-11. The average of the measured positive and negative first order light intensities is shown since an asymmetry was observed. However, in no case for points shown did these measured intensities differ by more than 5 percent (absolute).

The $v$ scale was normalized as described for the zeroth order. The rf voltage to $v$ ratios for the zeroth and first orders were compared and found to agree within experimental error.

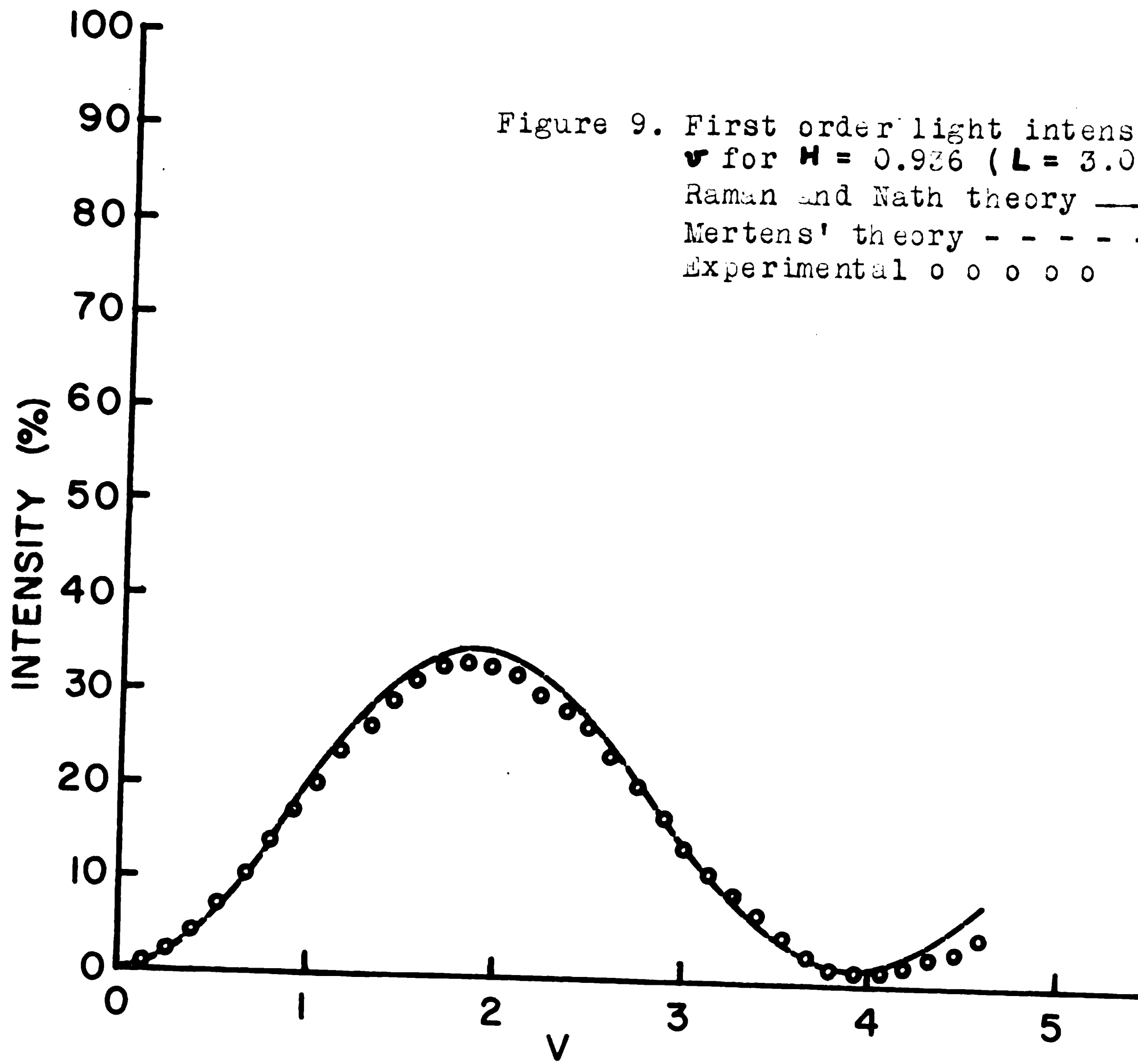



Figure 9. First order light intensity $\underline{vs}$ $v$ for $H$ = 0.936 ( $L$ = 3.0 cm).
Raman and Nath theory ————
Mertens' theory - - - - -
Experimental o o o o o

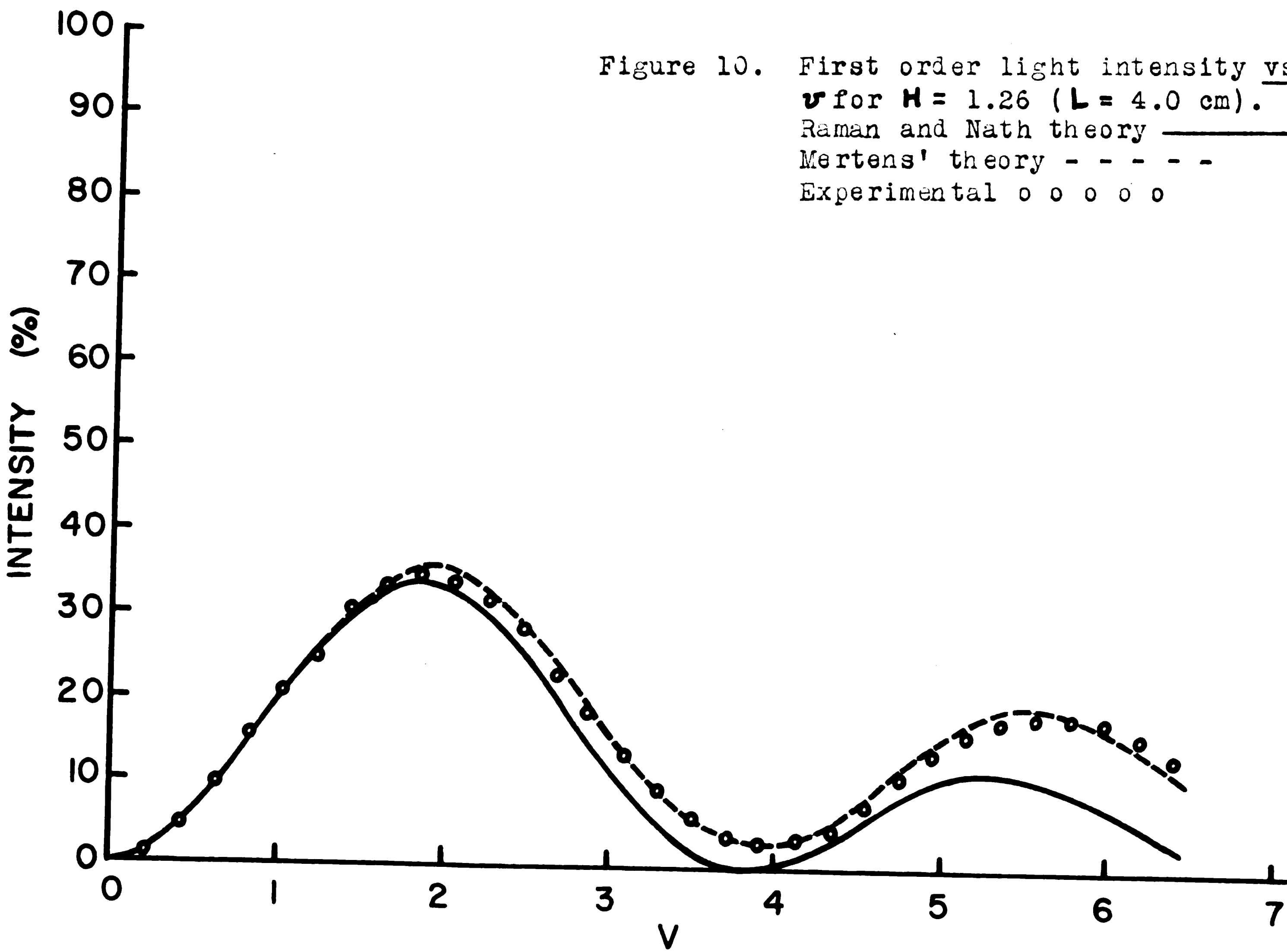
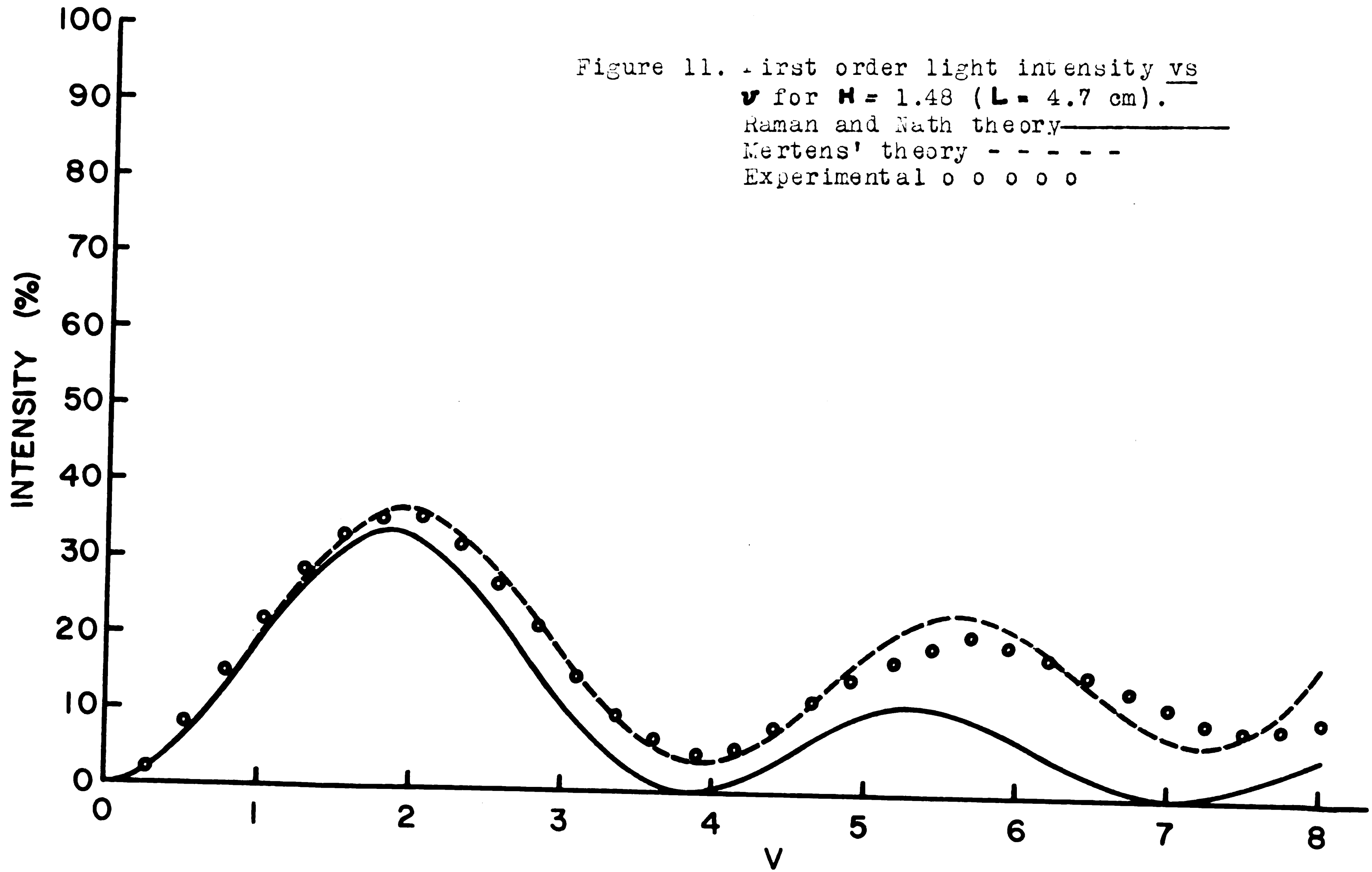



Figure 10. First order light intensity vs $v$ for $H = 1.26$ ($L = 4.0$ cm).
Raman and Nath theory ————
Mertens' theory - - - - -
Experimental o o o o o


INTENSITY (%)
100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
V
Figure 11. First order light intensity vs
v for H = 1.48 (L = 4.7 cm).
Raman and Nath theory
Mertens' theory - - - - -
Experimental o o o o o

## DISCUSSION OF RESULTS

Several discrepancies between the model upon which the theory of Mertens is based and the actual experimental situation should be noted. First, in spite of the precautions taken, it is probable that some non-uniformity existed in the sound field. The observed intensity of a given diffraction order will then be some "average" of the intensities corresponding to the local values of $v$ over the light aperture. As a consequence, the extrema of the light intensity _vs_ $v$ curves will be less sharp, or less light will be observed in the maxima than expected and more light will be observed in the minima than expected. As the sound intensity increases the absolute spread in $v$ would also increase.

A second effect not considered in the theory of Mertens is the scattering of light within the medium and elsewhere in the optical system.

Finally, Mertens' model is based upon diffraction by a sinusoidal ultrasonic wave. The presence of finite-amplitude effects will cause some deviation from intensities predicted for a sinusoidal waveform. Intensity measurements taken with the transducer at various distances from the optical aperture (giving various degrees of finite amplitude distortion) indicated that within experimental error the zeroth order was unaffected by finite-amplitude effects. However, a finite-amplitude distorted sound wave does produce an asymmetric diffraction pattern. For this reason the agreement between theoretical predictions and experimental results is expected to be poorer for higher orders.

A choice of ±3 % (absolute) as a criterion for satisfactory agreement between experiment and theory was made to allow for the above effects.

As in the case of the Raman and Nath theory, the limits of validity of the theory of Mertens can be expressed in terms of the parameters $v$ and $H$. Table I shows the values of $v_{max}$, the maximum value of $v$ for which the deviation between theory and experiment does not exceed ±3 %.

From Table I, it is seen that a reasonable upper limit for the validity of the theory of Mertens is

$$Hv < 8$$

Also, it is seen that an upper limit of usefulness (allowing a reasonable range of $v$) of the theory is

$$H < 1.25$$

Figures 9-12 show reasonable agreement between theory and experimental results for the first orders, over approximately the same range as for the zeroth order.

The upper limit of validity of the theory of Raman and Nath may also be obtained from the results. For this purpose, a deviation of ±2 % (absolute) between experiment and theory was allowed. The slightly more rigid tolerance was used in this case since at the lower sound intensities encountered the effects of inhomogeniety of the sound field should be smaller, and also the more stable experimental conditions allow greater accuracy in measuring light intensities. Table II shows the results of this comparison. It is seen in Table II that the theoretical limit of

$$H v < 2$$

is in excellent agreement with the experimental results. It will be recalled that the other condition was written

$$H << 2$$

In this table it is seen that an upper limit of usefulness is given by

$$H < 0.6 \quad .$$

Table I: Maximum values of the parameter $v$ and the quantity $Hv$ for a deviation of less than ±3 % (absolute) between the values of light intensity predicted by the theory of Mertens and experimental values for the zeroth order.

| L (cm) | $v_{max}$ | H | $Hv_{max}$ |
|---|---|---|---|
| 3.4 | 7.6* | 1.07 | 8.1* |
| 3.6 | 7.5 | 1.13 | 8.5 |
| 3.8 | 7.2 | 1.19 | 8.6 |
| 4.0 | 7.1 | 1.26 | 9.0 |
| 4.2 | 5.1 | 1.32 | 6.7 |
| 4.4 | 4.6 | 1.38 | 6.4 |
| 4.7 | 4.1 | 1.48 | 6.1 |

Table II: Maximum values of the parameter $v$ and the quantity $Hv$ for a deviation of less than ±2 % (absolute) between the values of light intensity predicted by the Raman and Nath theory and experimental values for the zeroth order.

| L (cm) | $v_{max}$ | H | $Hv_{max}$ |
|---|---|---|---|
| 1.6 | 4.0* | .50 | 2.0* |
| 1.8 | 4.0* | .57 | 2.3* |
| 2.0 | 4.0 | .62 | 2.5 |
| 2.2 | 3.2 | .69 | 2.2 |
| 2.4 | 3.4 | .76 | 2.6 |
| 2.6 | 3.2 | .82 | 2.6 |
| 2.8 | 3.0 | .88 | 2.6 |
| 3.0 | 2.8 | .94 | 2.6 |
| 3.2 | 2.0 | 1.01 | 2.0 |

* indicates that the maximum value could not be reached experimentally, the actual value being larger than that shown.

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## APPENDIX

Table III: Tabulation of $G_n(v)$ and $B_n^2(v)$ for the zeroth and first diffraction orders

The intensity of the $n$th diffraction order may be obtained from

$$I_n = J_n^2(v) + H^2 G_n(v) + H^4 B_n^2(v)$$

Following is a tabulation of the constants $G_n(v)$ and $B_n^2(v)$ for the zeroth and first diffraction orders*. The results of this investigation show that the terms in $B_n^2(v)$ are negligible over the range where the theory is valid, but they are tabulated for completeness.

| $v$ | $G_0(v)$ | $B_0^2(v)$ | $G_1(v)$ | $B_1^2(v)$ |
|---|---|---|---|---|
| 0.0 | .000000 | .000000 | .000000 | .000000 |
| 0.2 | .000411 | .000000 | -.000201 | .000017 |
| 0.4 | .001574 | .000000 | -.000717 | .000061 |
| 0.6 | .003295 | .000003 | -.001306 | .000114 |
| 0.8 | .005894 | .000008 | -.001262 | .000156 |
| 1.0 | .007260 | .000015 | -.001311 | .000168 |
| 1.2 | .008918 | .000023 | -.000053 | .000142 |
| 1.4 | .010078 | .000029 | .002318 | .000086 |
| 1.6 | .010680 | .000029 | .005777 | .000027 |
| 1.8 | .010807 | .000022 | .010095 | .000000 |
| 2.0 | .010669 | .000010 | .014855 | .000043 |
| 2.2 | .010653 | .000000 | .019520 | .000184 |
| 2.4 | .010697 | .000006 | .023517 | .000431 |
| 2.6 | .011691 | .000044 | .026343 | .000763 |
| 2.8 | .013378 | .000131 | .027660 | .001133 |
| 3.0 | .015895 | .000280 | .027366 | .001472 |
| 3.2 | .019095 | .000494 | .025630 | .001705 |
| 3.4 | .022680 | .000761 | .022877 | .001771 |
| 3.6 | .026245 | .001053 | .019730 | .001636 |
| 3.8 | .029348 | .001327 | .016909 | .001315 |
| 4.0 | .031593 | .001534 | .015113 | .000872 |

* These calculations were made by Richard C. Jennings and Bill D. Cook.

| | | | | |
|---|---|---|---|---|
| 4.2 | .032707 | .001624 | .014898 | .000415 |
| 4.4 | .032603 | .001564 | .016573 | .000083 |
| 4.6 | .031410 | .001347 | .020137 | .000014 |
| 4.8 | .029465 | .001000 | .025267 | .000316 |
| 5.0 | .027269 | .000592 | .031354 | .001038 |
| 5.2 | .025404 | .000222 | .037605 | .002142 |
| 5.4 | .024443 | .000013 | .043166 | .003511 |
| 5.6 | .024846 | .000087 | .047262 | .004947 |
| 5.8 | .026880 | .000545 | .049339 | .006205 |
| 6.0 | .030572 | .001440 | .049161 | .007040 |
| 6.2 | .035253 | .002650 | .047068 | .007326 |
| 6.4 | .040874 | .004187 | .043270 | .006879 |
| 6.6 | .046559 | .005826 | .038682 | .005775 |
| 6.8 | .051561 | .007332 | .034261 | .004198 |
| 7.0 | .055202 | .008458 | .030973 | .002459 |
| 7.2 | .057001 | .008985 | .029614 | .000952 |
| 7.4 | .056756 | .008772 | .030706 | .000086 |
| 7.6 | .054604 | .007798 | .034356 | .000205 |
| 7.8 | .051008 | .006184 | .040250 | .001513 |
| 8.0 | .046697 | .004193 | .047687 | .004015 |