AN ULTRASONIC IMAGING DEVICE A SIMULATION

Thesis for the Degree of M. S.

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THESIS

ABSTRACT

AN ULTRASONIC IMAGING DEVICE
A SIMULATION

By

Charles Harrison Shubert

This thesis investigates the problem of three dimensional imaging of internal human organs. A new conceptualization of a solution to the imaging problem is presented. Simple, idealized cases are used to describe the concept. An algorithm for handling the imaging of point discontinuities is developed. Results of a computer simulation are presented.

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A SIMULATION

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Charles Harrison Shubert

A THESIS

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To my family
Jan, Sean, Mom, and Oliver

ACKNOWLEDGMENT

I would like to thank Jay Guenon for his help in organizing my thoughts and my computer program.

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INTRODUCTION

THREE-DIMENSIONAL ULTRASONIC DEVICES

This thesis will deal with several of the parameters which are associated with the development of a particular variety of ultrasonic medical diagnostic equipment. The development efforts are directed toward the creation of a device that will be able to capture and display images of internal human organs. The display would probably best be in the form of a computer generated two-dimensional rendering of three-dimensional information. The three-dimensional information is obtained by an echo-ranging technique. The technique is similar in some respects to certain techniques used in seismology, radar, and sonar technology.

CURRENT ULTRASONIC TECHNIQUES

At present one- and two-dimensional techniques are used in medical ultrasonic imaging.

A.) An application of ultrasound to medical diagnosis is the one-dimensional echocardiography.[1] With this technique the transducer is pulsed while in a fixed

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[1]Feigenbaum, H. Echocardiography. Philadelphia: Lea and Febiger, 1972. Chapter 2.

position relative to the patient. The pulses are reflected by tissue interfaces. The time for the pulse to travel from the transducer to the tissue interface and back to the transducer is proportional to the distance between the transducer and the interface. This technique is called echo-ranging. The information available is distance of tissue interfaces along a line. This means that the information is one-dimensional. The major use of this technique is to observe the functioning of the heart's mital valve.

B.) The two-dimensional techniques are primarily cross-sectional views obtained by moving the transducer in a scanning pattern. This is similar to the scanning pattern of an airport-type radar.[2] The display is typically a CRT pattern similar to a radar display. The face of the CRT represents the plane in which the transducer is being scanned. The view is consequently two-dimensional. Three-dimensional views can be constructed by taking multiple cross-sections, but the technique is inherently two-dimensional in nature.[3]

C.) There are other techniques available to visualize internal organs that do not use ultrasonic radiation.

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[2] Uematsu, S.; Walker, A.E. A Manual of Ecoencephalography. Baltimore: The Williams and Wilkins Co., 1971. Pp. 122-130.

[3] Dekker, D; et al. "A System for Ultrasonically Imaging the Human Heart in Three Dimesnions." Computers and Biomedical Research. 7:544-553, 1974.

These fall into two categories.

1.) Non-invasive techniques such as x-ray are primarily those which expose the patient to some sort of radiation.

2.) Invasive techniques such as injection of dye into the blood stream introduce either substances or equipment into the body of the patient.

A NON-INVASIVE THREE-DIMENSIONAL TECHNIQUE

Clearly, there is a need for a non-invasive three-dimensional technique for visualization of internal organs that is low risk. With this in mind a conceptualization for a three-dimensional ultrasonic system has been developed. This thesis will describe that conceptualization. The limitations of this approach to the detection problem will be probed with several simple cases.

1.) The first case is that of one point reflecting an ultrasonic pulse. The question is, what information is necessary to locate that point?

2.) The second case is that of two points. The question is, what information is necessary to resolve these two points when they are separated by some arbitrarily small distance?

3.) The third case is that of multiple points. For this case the multiple number of points is arbitrarily chosen to be eight. The question remains, what information is needed to resolve these points?

METHOD OF DETECTION

Briefly, the method of detection of these points is as follows. A plane wave pulse of ultrasound emanates from the X-Y plane. The pulse travels in the positive Z direction an encounters a discontinuity or discontinuties that reflect(s) the pulse. The reflected pulse or echo is detected by an array of receivers located in the X-Y plane. The echos are converted to signals that have a time of reception and an amplitude associated with them. With this time and amplitude information from the array elements the reflecting points can be reconstructed.

A SIMPLE MODEL

A simple, idealized, model is assumed for the propagation and interaction of the ultrasonic pulse with the medium, the discontinuity, and the receiving elements. After limits on different parameters are established, these parameters will be used in a computer simulation of the device.

DISCUSSION OF THE DEVICE WILL BE PRESENTED IN THREE SECTIONS

I. The first section deals with a description of the operational aspects of a device that uses echo-ranging and time sequencing to locate reflecting points, surfaces, etc. Simple cases will be used to establish the limits on parameters, such as, the time resolution necessary to determine the location of a reflecting point to a specified precision.

II. The second section is the development of an algorithm for dealing with data impinging on the detection device.

III. In the third section results of a computer simulation are presented.

SECTION I: OPERATIONAL DESCRIPTION AND CONSIDERATION OF SOME SIMPLE CASES

ULTRASOUND: WAVE PROPERTIES

The functioning of any device is governed by the physical properties of the phenomena associated with it. In this case the phenomena of ultrasound has associated with it certain wave properties. Ultrasound can be described with quantities, such as, wave velocity, wave-length, frequency, etc. As with any wave, diffraction and interference are present. The speed of acoustic radiation is frequency dependent. Dispersion results and a pulse of ultrasound spreads in time and space.

In this thesis the primary emphasis of conceptualization of the total system precludes an analysis of the dispersion and diffraction-interference properties. It is clear that these properties establish an upper limit to the resolution possible with any ultrasonic device. The current resolution of two-dimensional ultrasonic scanning, as evidenced by photographs published in the media, would seem to be of the order of 1-2 mm. This resolution is sufficient for the initial phases of three-dimensional ultrasonics.

ENERGY LOSS

As ultrasound propagates through any medium it loses energy. The medium absorbs this energy with absorption coefficient as given by Wells[4] of 1 db MHz^{-1} cm^{-1}. This energy loss at certain distances and frequencies becomes a limiting factor. Figure 1 shows energy density as a function of distance traveled by the pulse for several different frequencies.

DETECTING PHASE

With ultrasonic radiation it is possible to detect phase. The ultrasonic transducers used for receiving the echo will respond to pressure wave variations rather than to just a time averaged intensity. This results in the retention to phase information. The net result is improved spatial resultion via improved time resolution without the inherent limitation of dispersion by decreasing the pulse width.

VELOCITY OF ULTRASOUND

The velocity of sound in human tissue ranges from 1.49 to 1.61 × 10^5 cm/s.[5] Throughout the rest of

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[4]Wells, P.N.T. "The Possibility of Harmful Biological Effects in Ultrasonic Diagnosis." Proceedings of the Symposium on the Cardiovascular Applications of Ultrasound. May, 1973.

[5]El'piner, I. Ultrasound: Physical, Chemical, and Biological Effects. New York: Consultants Bureau, 1964. Pp. 2,332-334.

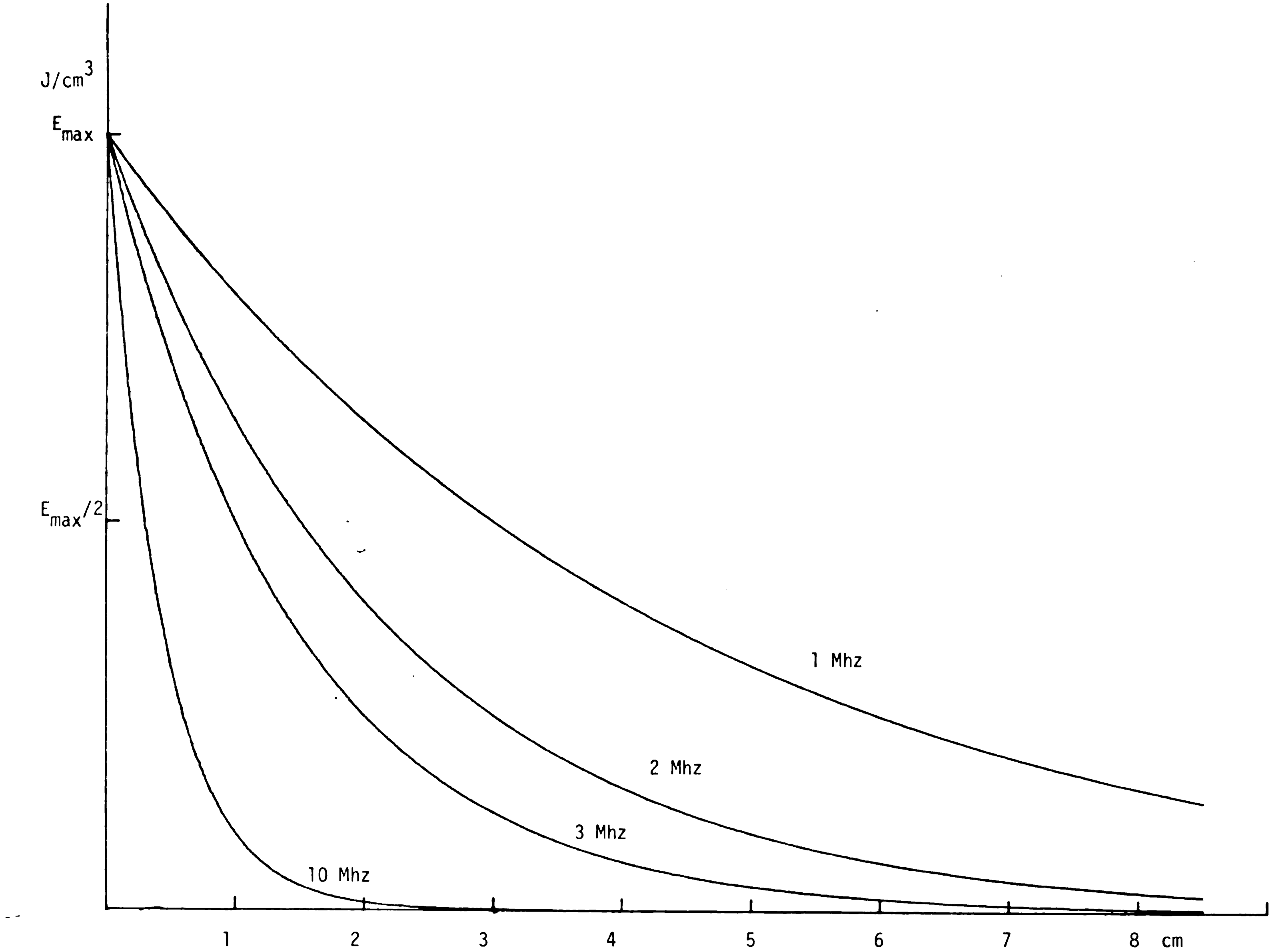


Figure 1 ENERGY DENSITY VS DISTANCE PULSE TRAVELS

this thesis the velocity of sound in human tissue will be assumed constant. This assumption as all others in this thesis are for the purpose of focusing attention on the conceptual nature of the process.

The constant velocity is taken to be the velocity of sound in water (1.5×10^5 cm/s). The effect of this assumption is to further limit consideration of dispersion.

DEVELOPING THE MODEL

Using the physical properties of ultrasound with their ascribed limitations, a step by step development of an imaging model can be undertaken. For this exercise it is assumed that a plane wave radiates in the positive Z direction until it encounters a discontinuity in the medium. At the discontinuity part of the energy is reflected. If it is a point discontinuity, the reflected wave is spherical. By the super-position principle the effect of any discontinuity can be treated as the sum of effects of a group of point discontinuities.

SPATIAL RESOLUTION OF DISCONTINUITIES

The resolution of a discontinuity depends on the resolution of the point discontinuities. If the model can resolve point discontinuities that are very close together, then any other discontinuity can also be well resolved (e.g. a 1 mm resolution is a 1 mm resolution regardless of the shape of the object whose image is being resolved).

THE REFLECTED ULTRASOUND

After the reflection from the point discontinuity the energy is assumed to propagate as a spherical wave whose energy has an r^{-2} dependence. The reflected wave has diminished energy as it returns due to two separate phenomena. First, the abosrption by the medium and, secondly, the spherical spreading of the energy. The incident wave loses energy only by the absorption process.

PULSED ULTRASOUND

At this point it is necessary to add another feature to the plane wave propagating through the medium. It is necessary to pulse the ultrasound to achieve the ranging effect. By assuming a constant velocity for the ultrasound, no dispersive effects are encountered in the model. The pulse is considered arbitrarily narrow in time.

RECEIVING ELEMENT ARRAY

After reflection the pulse propagates away from a point discontinuity as a spherical wave. This reflected pulse eventually encounters a receiving element array located in the X-Y plane. The elements of the array are receiving transducers which respond to the changing energy density. The array elements are in general located at different distances from the point discontinuity.

At any given moment the elements are recording an energy density pattern that corresponds to a reflected pulse. The reflected pulse transmits the location of the point discontinuity to the receiving element array. The pulse arrives at each array element at a time determined by the distance traveled by the pulse.

When a pulse strikes a receiving element, the element samples the energy density and converts that information into an electrical signal. The signal has both an amplitude which corresponds to the local energy density and a sampling time associated with it. If the time is measured from some fixed reference such as pulse emission, all receiving elements can be linked together in time. All data from one element can be linked to all data from any other array element. If the receiving elements are located in the X-Y plane and do not all lie on the same straight line, then the location of a reflecting point can be uniquely determined.

TIME AND DISTANCE EQUIVALENCE

The distance from the X-Y plane to the point and hence to the element is the distance traveled by the pulse. The time necessary to travel these distances is is the distance divided by the velocity of the pulse.

DIRECTIONALITY OF ELEMENTS

The receiving elements are not directional except for being pointed in the positive Z direction. The receiving elements will respond to signals from anywhere in the region of positive Z. Variations occur in signal strength due to differences in angles of incidence of reflected pulses with respect to array elements. This model assumes the variations to be insignificant.

Any two points will have reflected pulses arrive simultaneously at a receiving element if the pulses have traveled the same distance after emission. The effect on the signal at the element is additive. In general there is a surface about the element where reflecting points on that surface generate echos that arrive simultaneously at the element. Equivalently, each reflecting point of interest will be on as many surfaces as there are receiving elements. Each surface is unique. The intersection of three unique surfaces defines a point. If the receiving elements are all in the same plane, two points are defined. One point will have a positive Z and the other will have a negative Z. Since there can be no reflecting points with negative Z, the point is well defined. For the system to be unique for both positive and negative Z it would be necessary to locate one receiving element out of the X-Y plane.

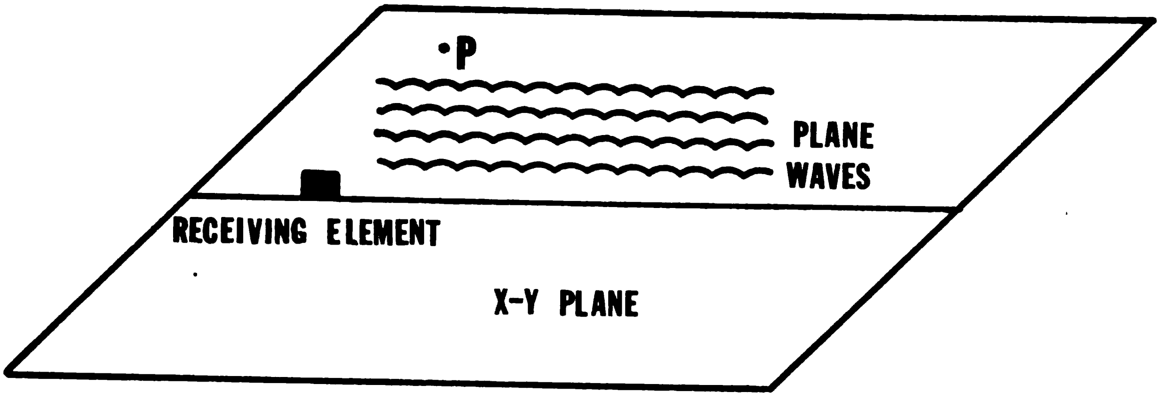
A SINGLE REFLECTING POINT

Consider briefly the one receiving element-one reflecting point case. For one element and one reflecting point the following exist (see Figure 2):

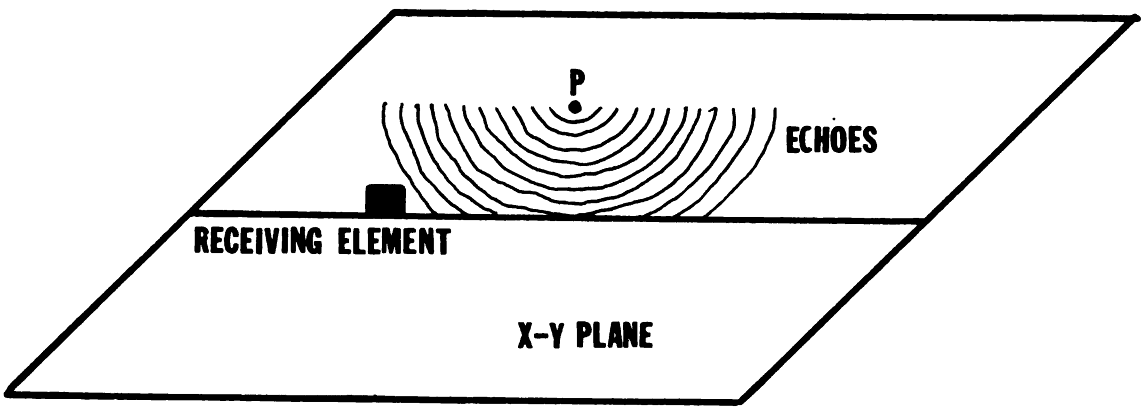
TOTAL DISTANCE TRAVELED BY PULSE = $Z + (X^2 + Y^2 + Z^2)^{1/2}$ for an element located at the origin and a point located at (X,Y,Z). All points located on a surface at the same total distance have echos arriving at the element simultaneously. If the total distance is C, then the surface has the equation:

$$Z = (C^2 - (X^2 + Y^2))/2C$$

In two dimensions this is the equation of a parabola with vertex at C/2. Figure 3 indicates point P on this parabola. It must be remembered that the emitted pulse is a plane wave for this treatment to be valid. Figure 4 shows two elements and how they define the location of point P. Note that P is located at different distances from each element. This means that there are two equations in two unknowns and these can be used to define two of the distances. To expand this to the three dimensional case there are three unknowns and these can be related to three equations only if there are three array elements. In three dimensions the above equation defines a paraboloid of revolution. The only restriction is that Z must be considered greater than zero.



a) OUTGOING WAVE



b) REFLECTED WAVES

c) DISTANCE PULSE TRAVELS BETWEEN
OMISSION AND RECEPTION

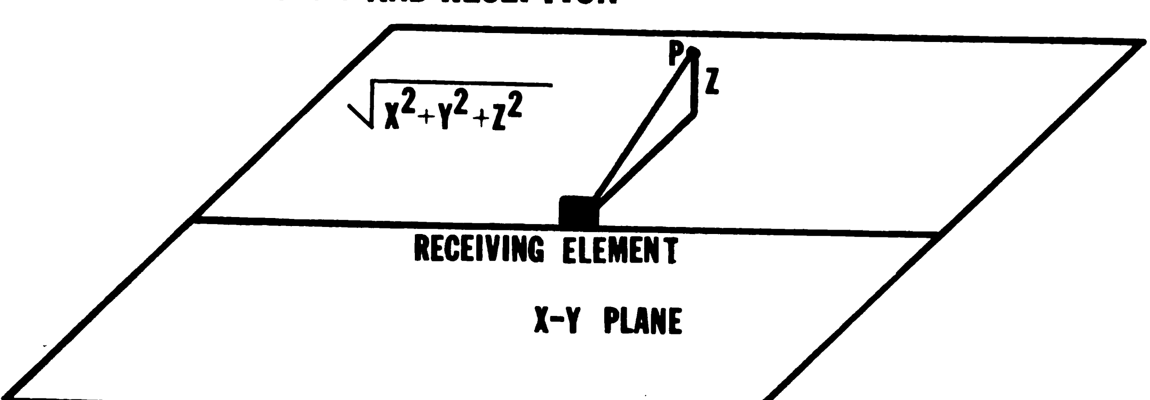


FIGURE 2

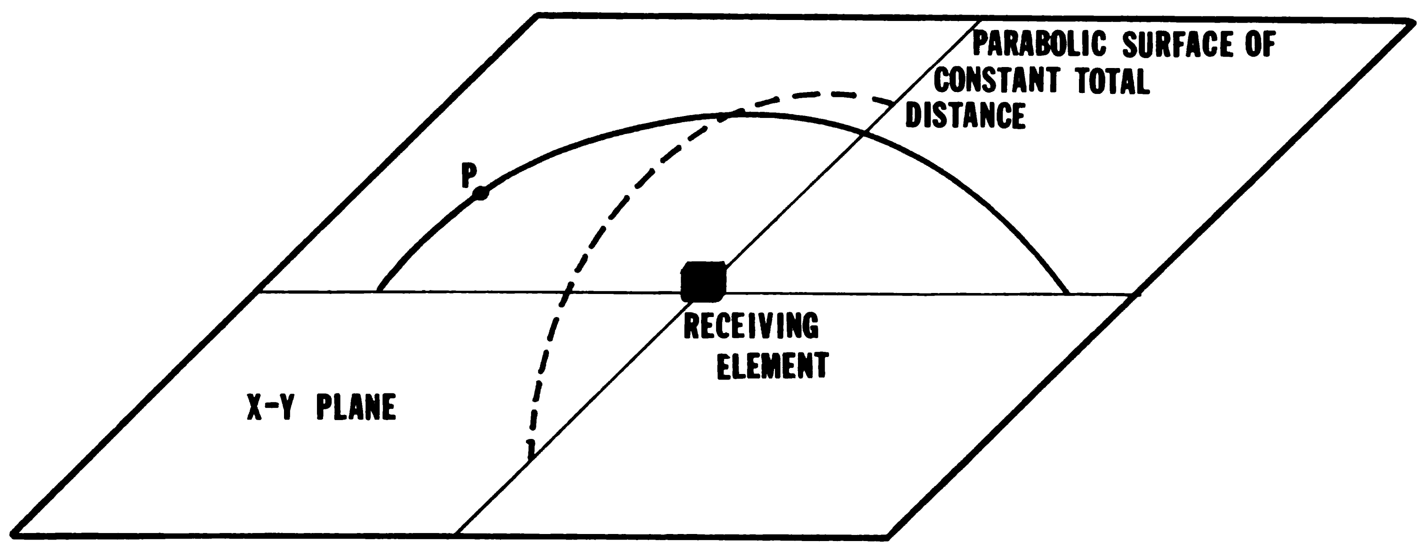


fig. 3

REFLECTING POINT ON ONE SURFACE

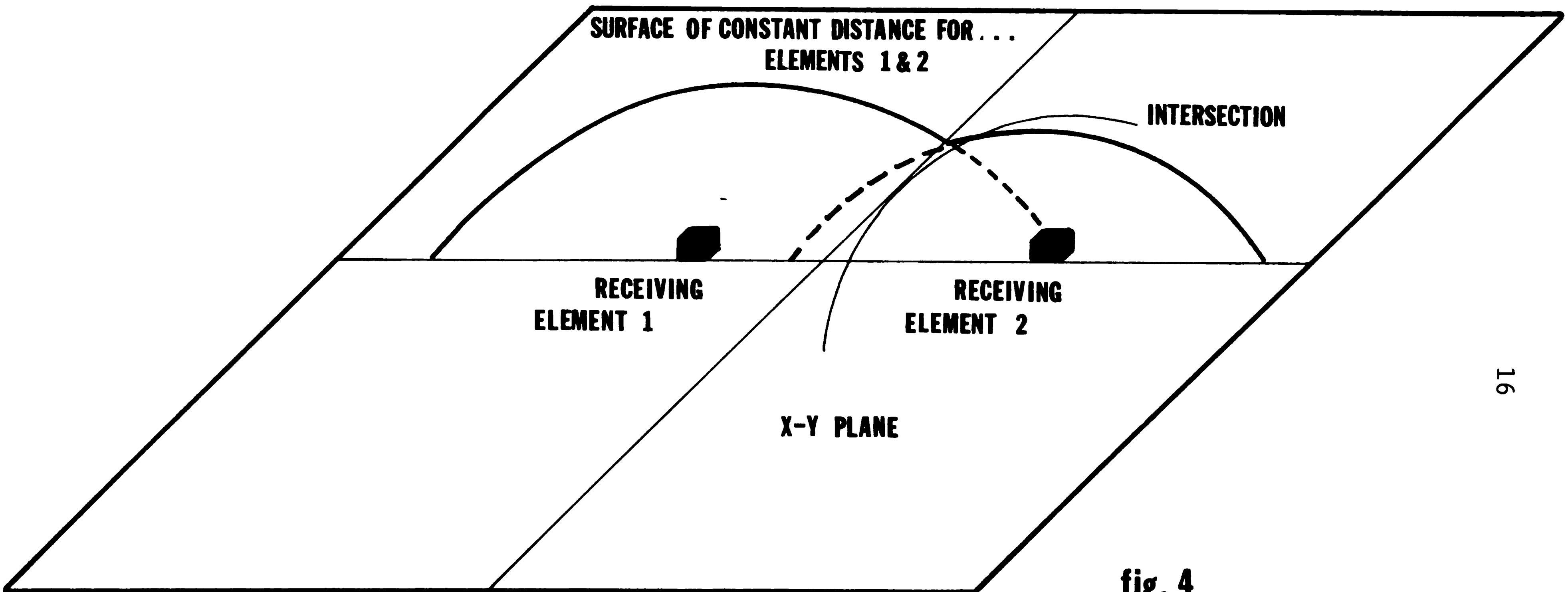


fig. 4

REFLECTING POINT ON TWO SURFACES

From the above discussion it can be concluded that an imaging device must have the following characteristics:

1) there must be at least three elements .
2) all elements must not lie on the same straight line
3) plane wave pulses are emitted and propagate in the positive Z direction
4) a method must be provided for recording information from each element as a function of time
5) an algorithm must be available for converting recorded information back to point information
6) a mechanism for displaying the information must be available.

SOME SIMPLE, IDEALIZED CASES

At this point it is useful to consider in some depth several simple, idealized cases. These cases will allow the definition of some working parameters.

CASE ONE

The first case is that of one reflecting point location in a large, homogeneous, isotropic medium. The velocity of sound in the medium is 1.5×10^3 m/s. An ultrasonic pulse is emitted from the X-Y plane and propagates in the positive Z direction. The pulse has an arbitrarily short duration. The receiving array consists of three elements. These elements are located in the X-Y plane. A point, P, is arbitrarily located at (X,Y,Z). The reflected pulse from that point can be used to determine its location.

DETERMINING THE DISTANCES

The distance traveled by the pulse from the X-Y plane to the point is Z. The distance traveled by the echo from point P to each element is:

for element #1 located at $(X_1, Y_1, 0)$

$$d_1 = ((X_1 - X)^2 + (Y_1 - Y)^2 + Z^2)^{1/2}$$

for element #2 located at $(X_2, Y_2, 0)$

$$d_2 = ((X_2 - X)^2 + (Y_2 - Y)^2 + Z^2)^{1/2}$$

for element #3 located at $(X_3, Y_3, 0)$

$$d_3 = ((X_3 - X)^2 + (Y_3 - Y)^2 + Z^2)^{1/2}$$

DETERMINING THE TIMES

The time of arrival of the echo at each element is:

$$t_1 = (d_1 + Z)/v, \text{ where } v = 1.5 \times 10^3 \text{ m/s}$$
$$t_2 = (d_2 + Z)/v$$
$$t_3 = (d_3 + Z)/v$$

DETERMINING THE LOCATION OF P

These times of arrival determine the location of point P. Upon reception of the echos, the t's are determined. If the velocity, v, and the positions of the receiving elements are known, and the equations are independent, the position of the point can be determined. Note that there is a two-fold degeneracy in Z, but, since only positive Z is considered this is not a problem. The equations would not be independent if the receiving elements were all located on the same straight line. This can be seen by setting Y_1,Y_2, and Y_3 equal to zero. Then there would be a two-fold degeneracy in Y. This can be generalized to any line in the X-Y plane merely by transforming that line into the X-axis and noting the degeneracy in Y. This clearly shows that to locate a point the minimum number of elements necessary is three. Also, the three elements must not all lie on the same straight line.

CASE TWO

The second case is that of two points. In this case the location of the receiving element #1 will be taken to be (1,0,0) where the units are given in cm. Element #2 is located at (-0.5, 0.87, 0) and the third at (-0.5, -0.87, 0). The location of the second point with respect to that of the first is varied so that a clear idea of the relationship between the geometrical resolution of two points and the time resolution of echos from them can be established. The times of arrival, t, will be calculated in the manner used in the single point case. The velocity is assumed to be 1.5×10^5 cm/s and the distances are in centimeters. Geometrical resolution will be demonstrated by varying the distance between the two points from 1 cm to 0.01 cm. Two differing situations are discussed. These involve X-Y (i.e. lateral) resolution and purely Z (i.e. depth) resolution. The results of these calculations are tabulated in Table 1. It is clear from Table 1 that the most difficult points to resolve for a given separation distance are those that lie in a plane parallel to the X-Y plane. The least difficult points to resolve are those that lie on a line perpendicular to the X-Y plane. The implication to be drawn from that fact is that receiving elements should be separated as far as is possible (within the physical limits of the device) so that time resolutions can be maximized.

<u>1 CM POINT SEPARATION</u>

For a point separation of 1 cm and using three elements, it appears that a time resolution of 10^{-6} seconds is adequate to resolve the two points. For a point separation of 0.1 cm the time resolution must improve by a factor of 10 to 10^{-7} seconds for the resolution of two points. As the geometrical resolution improves the time resolution must improve accordingly.

TABLE 1 - RESOLUTION OF TWO POINTS

POINT SEPARATION	POINT LOCATION	ELEMEMT NUMBER	ELEMENT LOCATION	DISTANCE	TRAVEL TIME FOR PULSE
1 cm	(0,0,5)	1	(1,0,0)	5.099 cm	6.732×10^{-5}s
	(0,1,5)			5.196	6.797
	(0,0,6)			6.082	8.055
	(0,0,5)	2	(-.5,.87,0)	5.099	6.732
	(0,1,5)			5.026	6.684
	(0,0,6)			6.082	8.055
	(0,0,5)	3	(-.5,-.87,0)	5.099	6.732
	(0,1,5)			5.360	6.906
	(0,0,6)			6.082	8.055
0.1 cm	(0,0,5)	1	(1,0,0)	5.099	6.732
	(0,.1,5)			5.100	6.733
	(0,0,5.1)			5.197	6.798
	(0,0,5)	2	(-.5,.87,0)	5.099	6.732
	(0,.1,5)			5.082	6.721
	(0,0,5.1)			5.197	6.798
	(0,0,5)	3	(-.5,-.87,0)	5.099	6.732
	(0,.1,5)			5.116	6.744
	(0,0,5.1)			5.197	6.798
0.01 cm	(0,0,5)	1	(1,0,0)	5.099	6.732
	(0,.01,5)			5.099	6.732
	(0,0,5.01)			5.108	6.745
	(0,0,5)	2	(-.5,.87,0)	5.099	6.732
	(0,.01,5)			5.097	6.731
	(0,0,5.01)			5.108	6.745
	(0,0,5)	3	(-.5,-.87,0)	5.099	6.732
	(0,.01,5)			5.100	6.733
	(0,0,5.01)			5.108	6.745

LIMITATIONS ON TIME RESOLUTION

The limitations on time resolution become more severe as the demands for geometrical resolution increase. For instance, for a time resolution of 10^{-8} seconds it is conceivable that a pulse of that time duration would be necessary. To form such a pulse would require frequencies above 100 Mhz. From Figure 1 it is clear that such frequencies would be readily absorbed. This absorption would be limiting to the point that spreading of the pulse due to dispersion would be a secondary consideration.

RECEIVING ARRAY CONFIGURATION

In Table 1 it can be seen that the receiving elements are located at the corners of an equilateral triangle of side 1.73 cm. This configuration offers the least redundancy of information. This is due to the fact that this is a configuration that allows the maximum separation of the array elements and separation of the elements is necessary for resolution. The triangular configuration will be the basic unit for testing the number of elements needed to resolve reflecting points in the computer simulation.

CASE THREE

The last simple case to be considered is the eight point case. The receiving element array is the same as in the previous case. The eight points are located at the vertices of a cube. The dimensions of the cube are 1 cm × 1 cm × 1 cm. The eight vertices and their distances from the elements are listed in Table 2 along with the pulse travel times.

TABLE 2 - RESOLUTION OF EIGHT POINTS

POINT SEPARATION	POINT LOCATION	ELEMENT NUMBER	ELEMENT LOCATION	DISTANCE	TRAVEL TIME FOR PULSE
1 cm	(4,4,4)	1	(1,0,0)	6.403 cm	6.935×10^{-5} s
	(5,4,4)			6.928	7.285
	(4,5,4)			7.071	7.380
	(5,5,4)			7.549	7.699
	(4,4,5)			7.071	8.047
	(5.4,5)			7.549	8.366
	(4,5,5)			7.681	8.454
	(5.5,5)			8.124	8.749
	(4,4,4)	2	(-.5,.87,0)	6.787	7.191
	(5,4,4)			7,488	7.658
	(4,5,4)			7.303	7.535
	(5.5,4)			7.958	7.972
	(4,4,5)			7.421	8.280
	(4,5,5)			7.895	8.597
	(5,4,5)			8.066	8.711
	(5,5,5)			8.505	9.003
	(4,4,4)	3	(-.5,-.87,0)	7.741	7.827
	(5,4,4)			8.362	8.241
	(4,5,4)			8.405	8.270
	(5,5,4)			8.981	8.654
	(4,4,5)			8.302	8.868
	(5,4,5)			8.841	9.256
	(4,5,5)			8.925	9.283
	(5,5,5)			9.468	9.645

SECTION II: ALGORITHM FOR SIMULATION

The simulation is of a device that pulses ultrasonic radiation into a uniform medium. At arbitrary points in the medium there are reflecting points. The ultrasonic pulse is reflected by these reflecting points and directed back toward an array of receiving elements. The device then converts the reflected pulse into an electrical signal and processes that information to determine the location of the reflecting points.

The algorithm consists of several parts. The parts are:

A.) simulation of reflected pulse data from arbitrarily selected reflecting points

B.) simulation of data processing to determine reflecting point location

C.) simulated device output.

A) <u>SIMULATION OF REFLECTED PULSE DATA FROM ARBITRARILY SELECTED REFLECTING POINTS</u>

ASSUMPTIONS

A major assumption concerns the pulse construction. The pulse is considered to be non-dispersive. This implies the pulse length is sufficient to contain many cycles of the ultrasonic radiation. The pulse is assumed to propagate as a plane wave pulse parallel to the plane of the receiving elements. The reflecting points are assumed to reflect the pulse so that the reflected pulse is spherical.

The reflecting points are assumed to intercept very little of the incoming energy. The pulse wavefront is assumed broad enough with respect to the reflecting point that the reflecting points that lie behind (have greater Z) it are not in its shadow. The reflecting points may be chosen to reflect part or all of the energy incident upon them.

The velocity of the pulse through the medium is assumed to be constant and within the range of velocities of sound in human tissue. As a result of this assumption time becomes a direct measure of distance. Only one outgoing pulse is considered. The cumulative effect on processing the information from many pulses is not considered.

These assumptions are designed to simplify the mathematical treatment so that the emphasis may be placed on the conceptual aspects of the simulation. By showing the concept to be valid for the simplistic case, the groundwork is laid for dealing with the more complex case.

The simulation of the reflected pulse data is divided into the following sub-sections:

I) generation of the outgoing pulse

II) generation of the reflected pulse

III) representation of the received pulse

IV) one reflecting point - many receiving elements

V) many reflecting points - many receiving elements

VI) matrix representation of information

The simulation will proceed from the single reflecting point and single receiving element case to the many reflecting points and many receiving elements case.

I) GENERATION OF AN OUTGOING PULSE

The outgoing pulse is a plane wave pulse that originates in the X-Y plane and propagates in the positive Z direction. The receiving elements also lie in the X-Y plane. The choice is arbitrary. Its major function is to simplify the mathematical treatment of the problem.

As has been stated earlier, time can be used to represent the distance the pulse has traveled. This follows from the assumption of uniform velocity in the medium. For the generation of data representing an outgoing pulse striking a reflecting point, the pulse travel time is calculated as:

t_{rp} = (constant) Z, where

Z is the perpendicular distance to the X-Y plane and

(constant) = (velocity of sound)$^{-1}$

The energy density of the pulse at the reflecting point is taken to be:

$E_{rp} = E_\circ e^{-0.23\ Z\ (freq)}$, where

$E_\circ$ = initial energy density of the pulse (taken to be 1)

E_{rp} = energy density of the pulse at reflecting point

(freq) = frequency of the ultrasound in Mhz

This is equivalent to a 1 dB decrease in power of the pulse per centimeter per Mhz.[6]

The outgoing pulse arrives at the reflector at some time, t_{rp}, with some energy density, E_{rp}.

II) GENERATION OF THE REFLECTED PULSE

At the reflecting point the pulse is reflected in such a manner that the reflected pulse is assumed to be spherical. The reflecting point may have a value of reflectivity ranging between zero and one.

The energy density of the pulse at the receiving element (X',Y',Z') after reflection from the reflecting point located at (X,Y,Z) is:

$$E_R = \frac{R\ E_{rp} e^{-0.23[(X'-X)^2 + (Y'-Y)^2 + (Z'-Z)^2]^{\frac{1}{2}}(freq)}}{(X'-X)^2 + (Y'-Y)^2 + (Z'-Z)^2},$$

where R is the REFLECTIVITY of the point times the cross-sectional area of the point times geometrical constants, and E_{rp} is the energy density of the pulse as it arrives at the reflecting point.

III) REPRESENTATION OF RECEIVED PULSE

The received pulse has two characteristics which are of interest. The first is the energy density of the pulse and the second is the time the pulse arrives at the receiving element. A clock is assumed to be started upon emission of the pulse. When the reflected pulse is received, the time on the clock is noted.

The simulation of the energy density of the pulse has been described above. The length of time needed for the pulse to travel from the X-Y plane and be reflected to a receiver is just a constant times the distance the pulse has traveled. This equivalency of time and distance was noted earlier.

The simulation of the received information is the calculation of received energy density at a receiving element located at (X',Y',Z'), E_{RE}, and the calculation of the time the pulse arrived at (X',Y',Z'), t.

$$t = (\text{constant})[Z + [(X'-X)^2 + (Y'-Y)^2 + (Z'-Z)^2]^{\frac{1}{2}}],$$

where $(\text{constant}) = (\text{velocity of sound})^{-1}$.

It should be noted that the received information contains only one piece of information that relates to the location of the reflecting point. That is the information carried in the time part of the simulation. This is the case for one reflecting point-one receiving element.

IV) ONE REFLECTING POINT - MANY RECEIVING ELEMENTS CASE

For the more complicated case of many receiving elements the representation of the received pulse is generalized to i receiving elements.

$$E_{REi} = \frac{R\ E\ e^{-0.23[Z + [(X_i' - X)^2 + (y_i' - Y)^2 + (Z_i' - Z)^2]^{\frac{1}{2}}](freq)}}{(X_i' - X)^2 + (Y_i' - Y)^2 + (Z_i' - Z)^2}$$

$$t_i = (constant)[Z + [(X_i' - X)^2 + (Y_i' - Y)^2 + (Z_i' - Z)^2]^{\frac{1}{2}}]$$

V) MANY REFLECTING POINTS - MANY RECEIVING ELEMENTS CASE

This case has i receiving elements and j reflecting points. The received information has the following form:

$$E_{REij} = \frac{R_j E\ e^{-0.23[Z_j + [(X_i' - X_j)^2 + (Y_i' - Y_j)^2 + (Z_i' - Z_j)^2]^{\frac{1}{2}}](freq)}}{(X_i' - X_j)^2 + (Y_i' - Y_j)^2 + (Z_i' - Z_j)^2},$$

$$t_{ij} = (constant)[Z_j + [(X_i' - X_j)^2 + (Y_i' - Y_j)^2 + (Z_i' - Z_j)^2]^{\frac{1}{2}}]$$

VI) MATRIX REPRESENTATION OF INFORMATION FROM A SET OF RECEIVING ELEMENTS AND REFLECTING POINTS

The above information can be represented by two-dimensional matrices. The form of the information is:

$$I = \begin{pmatrix} E_{RE11}, t_{11} & E_{RE12}, t_{12} & \cdots & E_{RE1j}, t_{1j} \\ E_{RE21}, t_{21} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ E_{REi1}, t_{i1} & \cdots & \cdots & E_{REij}, t_{ij} \end{pmatrix}$$

This matrix may be modified to represent a more typical data arrangement. This is done by creating the following matrix:

$$T' = \begin{pmatrix} t'_{11} & t'_{12} & \cdots & t'_{1i} \\ t'_{21} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ t'_{k1} & \cdots & \cdots & t'_{ki} \end{pmatrix}$$

where k is greater than j.

t'_{ki} represents a window of time where a signal may be acquired. The i indicates which receiving element is being considered. As k is incremented, t'_{ki} is also being incremented. All t_{ij}'s are truncated such that any given t_{ij} is equal to some t'_{ki}. Another matrix I', a $k \times i$ matrix, has its elements I'_{ki} set equal to E_{REij}. This is done for all E_{REij}'s. Any element I'_{ki} not set equal to some E_{REij} is set equal to zero. The matrix I' could look something like this:

$$I' = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & E_{RE22} & \cdots & \cdots \\ E_{RE13} & 0 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & E_{REij}(I'_{ki}) \end{pmatrix}$$

Each column of I' represents the received information at receiving element i as time progresses. The progression of time is equivalent to a progression down the rows of I'. I' can be viewed as being a sequential record of events occurring at a set of receiving elements.

The simulation of reflection pulse data is then in the form of an $i \times k$ matrix. The location of the receiving elements is also in matrix form. Both of these matrices are now ready to be processed to determine the locations of the reflecting points.

B) <u>SIMULATION OF DATA PROCESSING TO DETERMINE REFLECTING POINT LOCATION</u>

The same assumptions used in generating the data are also used in this section.

The processing section will be divided into three sub-sections:

I) available information;

II) processing sequence; and

III) output

I) AVAILABLE INFORMATION

Information available to be processed comes from simulation of reflected pulse data. These data are essentially a recording of the energy density at each receiving element. For the recorded information to be processed it is necessary for the location of the receiving elements to be specified. The locations of the receiving elements are specified by the transfer of information from the reflected pulse simulation. This allows a generality to be inherent in the processing simulation. The generality is with respect to the locations and numbers of receiving elements. The locations and positions of the receiving elements determine the form and content of the simulated reflected pulse data for a given set of reflecting points.

Another important piece of information is the size of the space within which the reflecting points are assumed to be located. This dimensioning of the processing simulation can be either internal or external to the processing simulation. The dimensioning or sizing of this reflecting point space has essentially two parameters. One is the overall physical dimension of the reflecting point space. This determines the total length of time for the reflected pulse data. The second parameter is the physical resolution of the points within the reflecting point space. This determines the number of partitions each time record has. In this processing simulation the overall dimension of the reflecting point space is internal to the processing unit. The partitioning of the space is determined in the reflected pulse data simulation unit.

II) PROCESSING SEQUENCE

The following sequence outlines the functional aspects of the processing unit. The processing unit is basically concerned with determining whether a point in the reflecting point space is a reflecting point. Secondarily, the processing unit determines the reflecting characteristics of the reflecting point. These characteristics being: size of the reflecting point combined with shape and reflectivity of the point. These things are all combined under the somewhat inaccurate term reflectivity.

The processing sequence consists of the following parts:

1.) An arbitrary point in the reflecting point space and a receiving element are chosen. The coordinates of the point and the element are determined.

2.) The distance of the point from the X-Y plane is determined. This is the distance a pulse would have traveled from its emission to the point under consideration. It is clear that this is equal to the Z-component of the point's coordinates. The distance of the point from the receiving element is calculated. This is the distance a reflected pulse would have traveled. The sum of these two distances is the total distance a pulse would need to travel if the selected point was reflecting the pulse to the chosen receiving element.

3.) The time between emission of the pulse and the reception of the reflected pulse, if the point is a reflecting point, is a constant times the total distance. This reception time is part of the information available to the processing section.

4.) The reflected pulse data are checked to determine if the receiving element recorded a pulse at the time calculated above. A decision is made at this point and criterion must be established so that the decision may be made.

The criterion is simple for an ideal case. In the ideal case the presence of anything other than zero energy density in that time slot is indication of the presence of a pulse. In cases other than the ideal, the decision making criterion is more difficult to establish.

The criterion will then be either empirical or couched in statistical theory. The ideal case is assumed to be sufficient to deal with the conceptual aspect of the simulation. A realization of the concept in terms of physical application would lead to an in depth study of decision making criterion.

An indication of the presence of a pulse would cause the chosen point to be recorded as being a possible reflecting point. It is only a possible reflecting point because there are other points in the reflecting point space whose total distance is the same as the point under consideration.

5.) The rest of the receiving elements go through the above sequence. When the indication of a pulse is present the record for the chosen point is incremented. The record for the point is of the number of times the point is considered a possible reflecting point. Each point in the reflecting point space goes through this sequence.

6.) Each point in the reflecting point space has a record of indication of pulse presence at a receiving element being kept. After all points in the reflecting point space have been considered the record of indication of pulse presence at receiving elements is examined. Some points may have records that show no pulses were reflected to the receiving elements from these points. Other points may have some indication of pulse presence and some points may have many indications of pulse presence. Criteria must be established to determine whether there is enough indication of pulse presence to justify calling a point one of the reflecting points. For the ideal case the criterion would be indication of pulse presence at all receiving elements. As the simulation progresses to a more sophisticated level, this criterion would need to be eased to encompass a larger degree of uncertainty.

7.) The points that are selected as possible reflecting points then go through the last stage or processing. This stage is the calculation of the reflectivity of the points. The reflectivity as calculated from the energy density of the pulse at each receiving element is averaged in this simulation to determine the reflectivity of the point. This is viewed as adequate for a small number of reflecting points, but would need to be refined as the model is made more sophisticated. This simple criterion for determination of reflectivity works exceedingly well for a relatively small number of reflecting points.

III) OUTPUT

The form of output is chosen for easy comparison with input information. The input information is chosen in terms of location x,y,z-components and reflectivity in decimal form with values ranging between 0.000 and 1.000. The output is also in this form.

A computer program that executes this simulation was written and the results follow in the next section.

SECTION III: USING THE ALGORITHM

In this section the results from using a program based on the algorithm presented in SECTION II are presented and discussed.

The program considered in this section deals with a space consisting of eight reflecting points located at the vertices of a cube. The choice was made for two reasons. First, multiple points were needed to give an indication of the applicability of the algorithm to relatively complicated situations. Secondly, the cubic orientation of the points allows the algorithm to demonstrate the capacity to deal with three-dimensional point configurations.

The program is divided into two parts. The first part simulated data as would be available at an array of receiving elements. The input to this part was number of receiving elements in the array, number of reflecting points, the reflectivity of the points, velocity of ultrasound, frequency of the ultrasound, and an index relating to the spatial resolution. The output of Part 1 was the number of receiving elements, their location, and a matrix of elements, I'_{ki}. I'_{ki} represents the signal strength at receiving element i at time interval k.

Part 2 of the program used the output of Part 1 as its input. The output of Part 2 was directed toward determining the location of the reflecting points and their reflectivity.

The only variable for this simulation was the number of receiving elements. For comparison purposes the inputs to Part 1 are listed beside the output of Part 2.

RESULTS OF COMPUTER SIMULATION:

Three receiving elements located at (1,0,0), (-1,1,0), (-1,-1,0)

INPUT POINT AND REFLECTIVITY		OUTPUT POINT AND REFLECTIVITY	
(4,4,4)	0.1234	(4,4,4)	0.1234
(4,4,5)	0.2345	(4,4,5)	0.2345
(4,5,4)	0.3456	(4,5,4)	0.3456
(4,5,5)	0.4567	(4,5,5)	0.4567
(5,4,4)	0.5678	(5,4,4)	0.5678
(5,4,5)	0.6789	(5,4,5)	0.6789
(5,5,4)	0.7891	(5,5,4)	0.7891
(5,5,5)	0.8912	(5,5,5)	0.8912
		(5,3,5)	0.2408

Five receiving elements located at (1,0,0), (-1,1,0), (-1,-1,0), (1,2,0), (1,-2,0).

INPUT POINT AND REFLECTIVITY	OUTPUT POINT AND REFLECTIVITY
input is same as above	output is same as above except for the last value which is: (3,2,5) 0.1080

Six receiving elements located at (1,0,0), (-1,1,0), (-1,-1,0), (1,2,0), (1,-2,0), (3,1,0)

INPUT POINT AND REFLECTIVITY	OUTPUT POINT AND REFLECTIVITY
input is same as above	output is same as input

DISCUSSION OF RESULTS

The criterion used to determine the existence of a reflecting point was somewhat less rigorous for the five and six receiving element cases than it was for the three element case. With the three element case the criterion for the point under consideration to be considered a reflecting point was for all elements to agree that the point was a reflecting point. For the other two cases all but one element had to be in agreement.

These criterion lead to some interesting observations. The first concerns the three element case. In this case all elements had to be in agreement that a point was a reflecting point before it could be considered a reflecting point. With this rather restrictive criterion the three elements were in error. A point which was not a reflecting point was determined to be a reflecting point. The reflectivity ascribed to the point was rather large in comparison to some "real" points. Clearly, three elements are not capable of resolving the eight points unambiguously.

With the five element case there is a less rigorous criterion. Only four of the elements must agree to determine that a point is a reflecting point. Here, too, a point is identified as a reflecting point when it is not. The reflectivity of the misidentified point has diminished in magnitude relative to the "real" points.

In the six element case the points are identified unambiguously. The criterion here is that five out of the six elements must identify the point as a reflecting point for the point to be considered a reflecting point.

From this small sample the following may be inferred. First, it takes more than the minimum of three receiving elements to identify the locations of reflecting points if there are very many. In general the greater the number of reflecting points in the space under consideration, the greater the minimum number of elements needed to locate them. Secondly, as the criterion for identifying reflecting points is made less rigorous, the number of receiving elements must increase to maintain a given level of discrimination between points and non-points.

The conceptualization is clearly capable of accomplishing the discrimination between point discontinuities. If this approach is in fact practical in a medical diagnostic sense is still not clear.

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