INTERFEROMETER MEASUREMENTS OF THIN FILMS

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Frederick William Kuether

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This is to certify that the

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Thomas H. Osgood.
Major professor

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INTERFEROMETER MEASUREMENTS OF THIN FILMS

by

Frederick William Kuether

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W. C. Luether

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INTRODUCTION

Thin films have been measured by many methods. Some of these are basically similar to the method we have used. Tolansky's[1] method is an example of this. He employs the interference of monochromatic light, multiply reflected in or on the thin film to be measured. Other methods show little or no similarity. Rothen[2] depends on the difference in ellipticity between light reflected at other than normal or grazing incidence from steel plates and similar plates coated with thin protein films whose refractive index is near 1.495. His accuracy of $\pm .5$ A however is exceedingly high. Most of these methods have limitations upon the type and/or thickness of the film which can be measured. Usually they present difficulties in measurement, in calculation or in accuracy. By removing as many of these difficulties as possible, we have derived a method which will accurately determine the thickness of any thin film by a simple measurement.

We have made use of the Edser-Butler[3] bands with surfaces arranged to give a very high resolving power. These bands are the bands observed in a spectroscope when parallel white light which has passed through a thin etalon is examined. The etalon is essentially a parallel-sided air film, preferably bounded by high reflecting surfaces. When white light passes into this system, constructive interference takes place only for those wavelengths which, after multiple reflection, leave the boundary of the air film at the same

phase. The intermediate wavelengths interfere either partially or totally, depending on the phase difference for each reflection and the number of effective reflections. The number of reflections quite naturally depends upon the reflecting power of the surfaces. Thus as the reflecting power increases, the selection of the wavelengths for which constructive interference occurs becomes more critical so that the ability to detect these wavelengths becomes less difficult or, in other words, their resolving power becomes greater.

Usually the transmitted light is observed in a spectroscope where the Edser-Butler[3] pattern of bright lines on a dark field may be used to measure the wavelengths for which constructive interference occurs. These wavelengths may be used to measure the thickness of the film in the following manner.

If n is the order number or the total number of wavelengths in the double thickness of the film, d is the effective thickness of the air film and λ is a wavelength of constructive interference, we can write

$$2d = n\lambda.$$

For successive bands toward the blue or shorter wavelengths, we can also write

$$2d = (n + 1)\lambda_1 \text{ and}$$

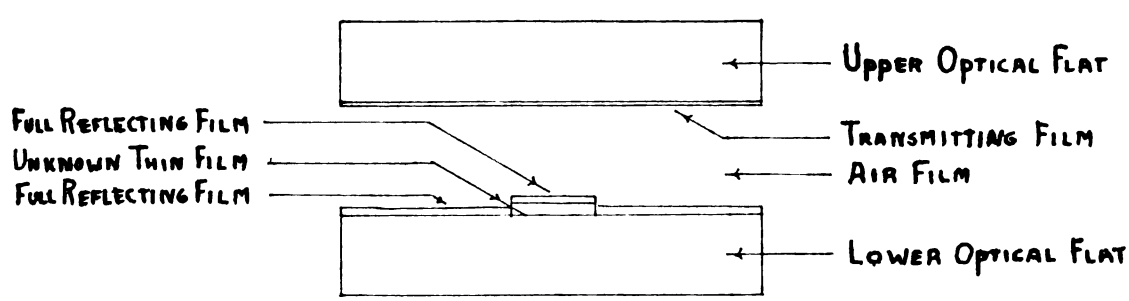
$$2d = (n + 2)\lambda_2 \text{ etc.}$$

Although d could be determined by solution of any two of these linear equations, the type of film which could be measured by this method would depend on its optical proper-

ties for light must pass through the film. The only alternative is to attempt measurements using the light reflected from the air film or etalon.

If energy is to be conserved, the pattern in reflection must be complementary to the Edser-Butler[3] pattern. The dark lines in reflection and the bright lines in transmission would correspond to the same wavelengths so that the equations above would hold for both patterns. The thickness of the etalon as determined by these equations would not be the actual thickness, for phase changes occur upon reflection.[4] Even though these phase changes can be measured and the actual thickness determined, this would complicate the calculations.

Further we have yet to show that the reflected light gives any advantages. To do so, let us construct the etalon like this:



With a full reflecting coating on the lower surface of the air film, light would not enter the film to be measured so that its optical properties need not

be considered. The only limitation is that the surface of the film must be smooth - a condition which is not a product of the method of measurement. The thickness of the air film may be determined on and off the step and the effective thickness of these two positions is the actual thickness of the thin film, for the error due to the phase changes upon reflection are the same for both measurements and therefore cancel when subtracting. It is interesting to point out that as long as the wavelength measurement is made to the same place in the intensity pattern, n need not be an integer but may correspond to the order number of that portion of the pattern.

As mentioned before, the accuracy of this method depends upon the high resolving power resulting from the multiple reflections within the etalon. The Edser-Butler[3] bands which result from this can be shown to have the intensity pattern of the Fabry-Perot Interferometer[5] where the change in order number is due in this case to a change in wavelength, rather than a change in d. The resolving power R of this pattern may be calculated from the chromatic resolving power as given for the Fabry-Perot pattern[6]

$$R = \left|\frac{\lambda}{\Delta\lambda}\right| = \left|\frac{n}{\Delta n}\right|$$

where $\Delta\lambda$ is effectively the uncertainty in the knowledge of λ. Using Rayleigh's[7] condition for determining $\Delta\lambda$ or Δn, it can be shown that[6]

$$R = 2.98\ n r^{\frac{1}{2}} \div (1 - r)$$

where r is the reflection power. If in a particular case the order number is 20 and r equals .89 (which is equivalent to an amplitude reflection coefficient of 94%), then

$$R = 2.98\ (20)\ .94 \div .11 = 509$$

If the wavelength is 5000 A, the thickness of the etalon is

$$d = \tfrac{1}{2}\ n\lambda = \tfrac{1}{2}\ (20)\ 5000 = 50000\ A$$

And the accuracy to which we might expect to measure the wavelength is

$$= 5000 \div 509 = 9.8\ A$$

Meissner[8] in his articles on Interference Spectroscopy gives further physical meaning to the expression for resolving power in this manner

R = (effective number of rays)(order number)

where the effective number is a simple function of the reflection power

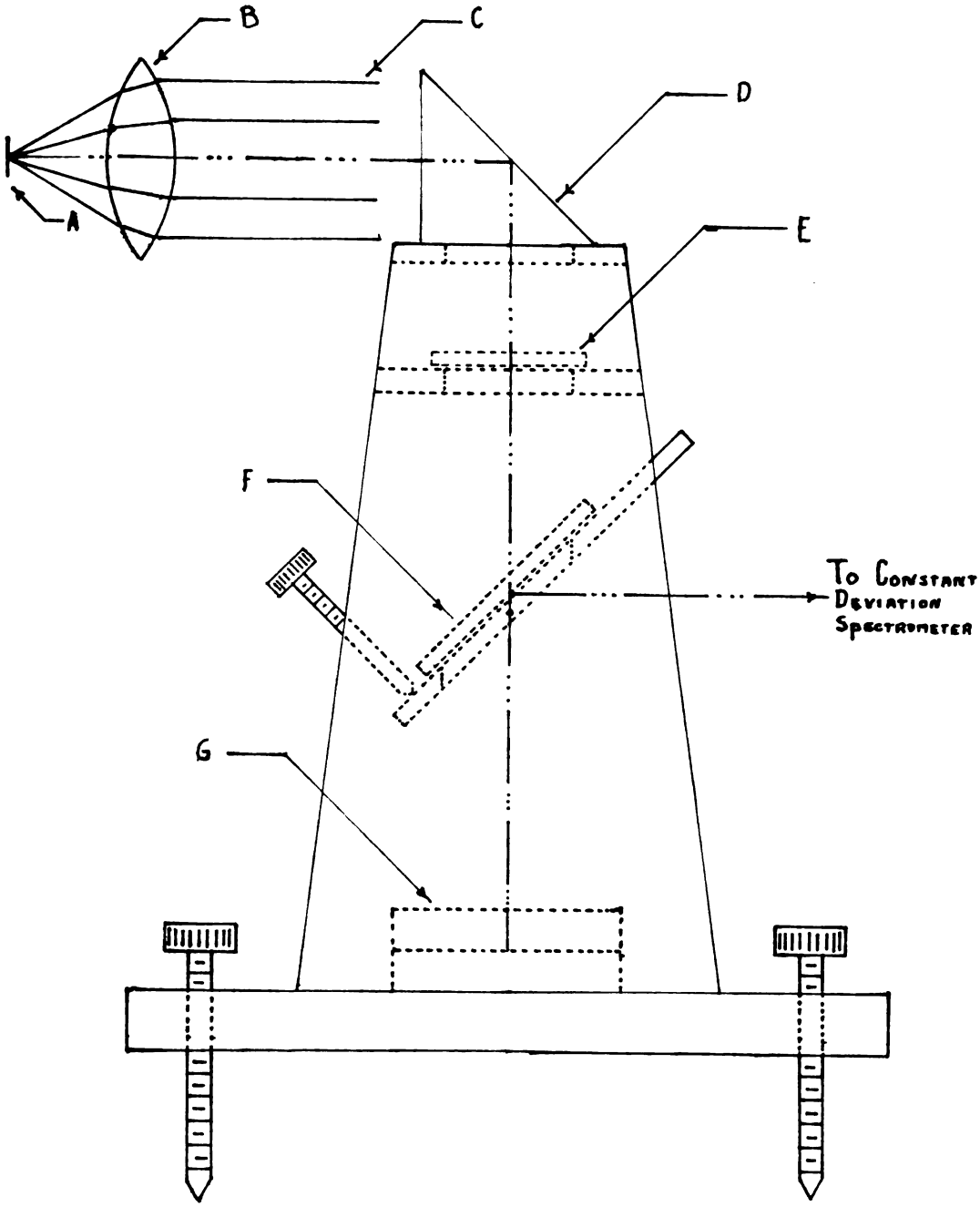
$$N_{eff} = 2.98\ r^{\frac{1}{2}} \div (1 - r)$$

Hence the multiple reflection in the etalon acts much as a grating where the number of rulings is controlled by the reflecting power of the surfaces. This method of obtaining resolving power does not have the mechanical difficulties encountered in the production of a grating.

The uncertainty in the knowledge of the wavelength for which constructive interference occurs when the transmission pattern is observed agrees well with our measurements. However in reflection this uncertainty is much less. This is due to the asymmetrical pattern and other factors which we will justify later.

METHOD OF MEASUREMENT

The optical path used was



where A is a ribbon filament source of light

B is an achromatic collimating lens

C is a group of parallel rays from a point on the filament

D is a totally reflecting 45° prism

E is a heat absorbent glass which reduces the thermal effects at the etalon

F is a half silvered mirror inclined at 45° to the light rays

G is the etalon with the full reflecting lower plate.

The mount for the prism was supported by three pointed screws passing through the base plate on which rested the etalon. This allowed the front surface of the prism to be adjusted perpendicular to the light at the proper height. Since the table for the prism and the base plate for the etalon were parallel to within .0005 inches, no adjustment was needed between these two elements. To direct the reflected light toward the spectrometer, a fifty percent reflecting mirror was mounted on a swivel table just above the etalon. A screw whose direction of motion was perpendicular to the surface of the table was used to adjust the mirror to a 45° angle so that the reflected light was horizontally directed toward the spectrometer. As a great deal of heat was radiated by the source of light, a filter of heat-absorbent glass was placed on the table just below the prism. This materially reduced the amount of heat absorbed by the etalon. It was convenient when studying the transmission pattern of an etalon to place the etalon on the filter table

and complete the optical system with a full mirror placed on the base plate.

CONSTRUCTION OF THE ETALON AND ITS SURFACES

To clean the optical surfaces of the etalon we used acetone, concentrated HNO_3, and then rinsed with water. It was found that if the surface was washed with a wetting agent which was then removed by rinsing with conductance water, a surface ideal for coating was obtained. Aluminum coatings were evaporated onto these plates from heated aluminum beads in the usual manner. During the evaporation, the thickness of the coating was controlled by comparison with a series of Wratten Neutral Filters which varied by factors of two from 3.12 to 50% transmission.

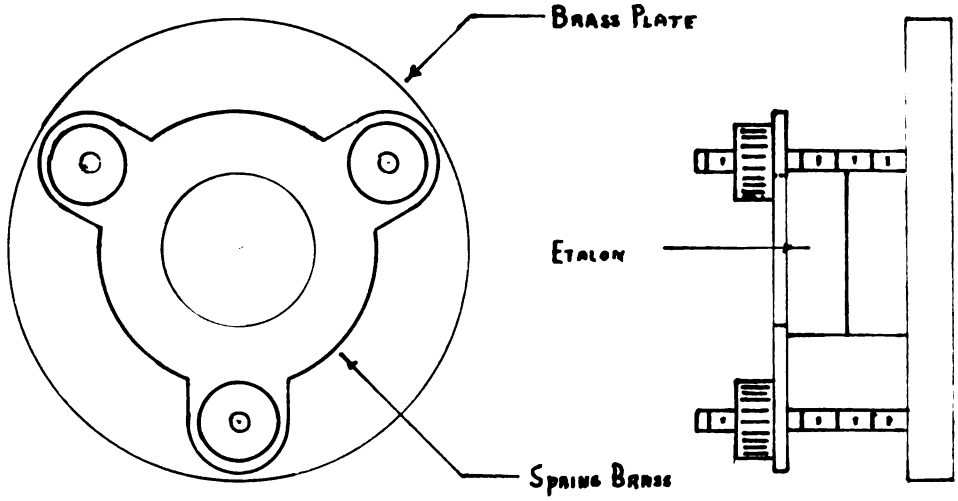
The step we measured was aluminum. It was placed on a portion of a full reflecting surface by shielding other portions of the surface with the microscope slides on which the surface was allowed to rest. The first millimeter of a step formed in this manner shows a gentle downward slope. We have no explanation to offer for this effect, but it does not appear to extend to the major portion of the step.

The etalon is formed using three pin points of soft wax to separate the high reflecting surfaces. Although such surfaces brought in contact usually have an air film between them of 100,000 A or less depending on the pressure applied, the system is very unstable and useless for our purposes. When the two reflecting surfaces are brought together upon the wax, there is always a wedge present between the surfaces.

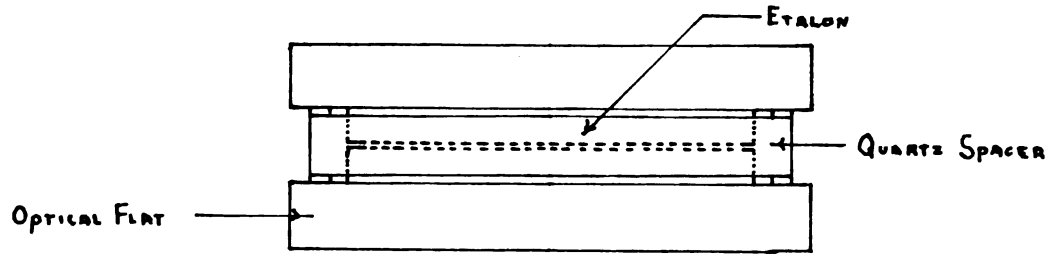
In monochromatic light, this results in a dark line pattern with the lines parallel to the axis of the wedge. Pressure must be applied to deform the wax and obtain an etalon whose sides are parallel. The adjustment for this condition is much more critical if the apparent thickness of the etalon is a multiple of the wavelength being used. The reason for this will follow from the method of adjustment. To adjust to this separation, one of the dark bands in the field must be chosen to remain in the field. The remaining bands are removed by pressing down the thicker portion of the etalon and increasing the narrower portion of the etalon by lifting that portion of the upper plate. As the etalon becomes parallel, the band in the field will gradually curve to form a closed circle; for only the rays reflected directly under the eye will pass through the etalon normal to its boundary; the others will have their angle of passage increased as they are reflected further from the normal to the eye. This is the familiar Fabry-Perot pattern which may be observed visually when the eye is completely relaxed (fringe pattern in focus at infinity).

It was found necessary that the etalon thickness be a near multiple of the wavelength of the monochromatic light used for adjustment because at an etalon thickness of 50000 A, as is commonly used in this method, a change in order of one due to the obliqueness of the reflected light would form a narrow dark ring 7.5 cm in radius. We could not hope to have one of these rings on an inch square unless an effort were made to adjust the thickness of the etalon to the proper value. The

Fabry-Perot etalon has many such rings because the order number and the effective thickness of the etalon is thousands of times greater, and hence a unit change in order corresponds to a much smaller angle of obliqueness. However with this ring in the etalon, very critical adjustments may then be made so that in the absence of wedge, the same diameter ring pattern follows the eye as it traverses the etalon. The step film offers a hindrance to this method, especially for small plates where the step has been placed at one edge, so that reasonably large plates with the step in the center give greater satisfaction. At best this adjustment is by no means simple, for if the etalon is to be stable, the strain at the three points of support must be equal. Further thermal problems in etalon adjustment were the effects due to hands, room and absorption of light. These effects were decreased with the aid of a mount which added to the thermal capacity of the system and allowed pressure to be applied to the etalon without direct human contact.



Were it essential to remove these errors more exactly, a quartz spacer such as used in the Fabry-Perot etalon could be employed. The spacer could be of a reasonable size if this construction were used.



This would be a specialized piece of equipment but its construction is quite possible. A further refinement of the equipment should employ a method by which the adjustment for a parallel etalon could be made on the etalon table.

We were limited by having only plates an inch or less in width. Greater stability and ease of adjustment could be obtained using two-inch plates, preferably circular, for they give a more uniform stress when used in the holder described above.

Wedge errors may be greatly reduced by taking measurements just on and off the step film. Since for short periods of time it was found that wedge errors could be less than 20 A over the whole plate, a measurement over a short portion of the plate should not exceed 5 A.

The heaviest reflecting surface used on the upper plate consisted of an aluminized coating showing 3% transmission or approximately 90% reflection, for which the resolving power

appeared to continue to increase.

MEASUREMENTS AND METHODS OF CALCULATION

To obtain the desired accuracy, various methods of calculation were employed, each of which had advantages of simplicity or accuracy in the determination of n or d. In the section following, we will enumerate their advantages. Consider the following data which was obtained with a Constant Deviation Spectroscope, for the characteristic points in the reflection pattern of a 15% transmission top coating for an etalon thickness of 66,000 A.

Order number	Wavelength	Order number	Wavelength
n	6665 A	n + 5	5327 A
n + 1	6349 A	n + 6	5123 A
n + 2	6058 A	n + 7	4933.5 A
n + 3	5791.5 A	n + 8	4758.5 A
n + 4	5552 A		

METHOD A. The equations which hold are

$$n\lambda = 2d, \quad (n + 1)\lambda_1 = 2d, \quad (n + 2)\lambda_2 = 2d, \text{ etc.}$$

These equations are of the form $xy = c$ where x is the order number and y is the wavelength. Since by the notation above, x is a constant, this equation may be considered linear and the method of Least Squares can apply to improve the accuracy of the results.

Let c_1, c_2, c_3, —— be the observed thicknesses of the etalon for each wavelength measurement. Then the re-

sultant errors or residuals, if c is the apparent double thickness, can be written

$$(c_1 - c),\ (c_2 - c),\ (c_3 - c),\ \ldots\ldots = \rho, \rho_1, \rho_2\ \ldots\ldots$$

The first condition of Least Squares is that $\Sigma\rho = 0$ or that

$$\rho + \rho_1 + \rho_2 + \ldots\ldots = 0$$

The second condition is that the sum of the squares of the residuals shall be a minimum, or what is mathematically equivalent in this case, the rate of change of the sum of the squares of the residuals is zero. As n is known to be a constant, we write

$$\frac{\partial}{\partial n}\left[\Sigma\rho^2\right] = 0$$

Hence

$$\frac{\partial}{\partial n}\left[(n\lambda - c)^2 + ((n+1)\lambda_1 - c)^2 + \ldots\ldots\right] = 0$$

and we can write

$$\lambda\rho + \lambda_1\rho_1 + \lambda_2\rho_2 + \ldots\ldots = 0$$

From the data above the residuals become

$$\rho = 6665\ n - c$$

$$\rho_1 = 6349\ n + 6349 - c$$

$$\rho_2 = 6058\ n + 12116 - c$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

The summation of the residuals becomes

$$50557.5\ n + 188023 - c = 0$$

For the second condition we write

$$\lambda\rho = (6665)^2\ n - 6665\ c$$

$$\lambda_1\rho_1 = (6349)^2\ n + (6349)^2 - 6349\ c$$

$$\lambda_2\rho_2 = (6058)^2\ n + 2(6058)^2 - 6058\ c$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

The summation of the weighted residuals then becomes

$$287{,}402{,}368\ n + 988{,}509{,}710 - 50557.5\ c = 0$$

Solving these simultaneous equations for n and c, we obtain

$$n = 19.940298$$

$$c = 2\ d = 132{,}905.07\ A$$

$$d = 66{,}453.04\ A$$

Assuming that these values are certainly more accurate than the determination of any one wavelength, it is of interest to divide c by the appropriate order numbers, n, n plus 1, n plus 2, etc., to estimate the uncertainty of our measurement of wavelength. For this set of data, errors range from - 2 A to + 1.9 A so that the probable uncertainty of our measurements is ±2 A. Subsequent sets of data treated in the same manner show errors not exceeding ±.9 A. Although the etalons were smaller for these sets of data, higher reflecting power and particularly smaller wedges brought about the increase in resolving power.

The probable numerical errors of the values of n and c, as obtained by this method, with equal weights assumed for each pair of values, may be shown to be[8]

$$a_n = .6745 \left[\frac{n' \Sigma \rho^2}{(n'-2)(n' \Sigma \lambda^2 - \Sigma \lambda \cdot \Sigma \lambda)} \right]^{\frac{1}{2}}$$

$$a_c = .6745 \left[\frac{\Sigma \rho^2 \cdot \lambda^2}{(n'-2)(n' \Sigma \lambda^2 - \Sigma \lambda \cdot \Sigma \lambda)} \right]^{\frac{1}{2}}$$

where n' is the number of independent readings.

Upon calculation for this set of data, they become

$$a_n = 6.659 \times 10^{-3}$$

$$a_o = 15 \text{ A}$$

When this is applied to the effective thickness, the probable error is ± 8 A. If two such calculations were made to determine the thickness of a step, one would be safe in assuming the thickness of the step to be known to 25 A. This calculation is cumbersome and its importance lies in an accurate determination of the order number.

METHOD B. As mentioned before, the order number and separation can be determined between any two of the linear equations

$$n\lambda = 2\,d$$

.....

$$(n + a)\lambda_a = 2\,d$$

where a is any integer.

Solving between these for d

$$d = \tfrac{1}{2}\, a\, \frac{\lambda\lambda_a}{\lambda - \lambda_a}$$

If there are six bands between $\lambda = 6300 \pm 1$ A and $\lambda_a = 4500 \pm 1$ A,

$$d = \frac{6}{2}\, \frac{(6300 \pm 1)(4500 \pm 1)}{(1500 \pm 2)} = 60540 \text{ A}$$

with a maximum possible error of

$$.0159 - .208 - .1334 = .1701\%$$

Using the value for d to determine the order number

$$n_\lambda = 2\,d \div \lambda = \frac{2(60540)}{6300} = 19.22$$

with a maximum possible error of

$$.1701 + .0159 = .1884$$

This method although simple does not give the accuracy hoped for. Its value lies in a rapid determination of the order number to a reasonable error of .2%.

METHOD C. Consider two bands with the same order number, having wavelengths measured on and off the step. Let s be the thickness of the step.

$$n\lambda_{on} = 2d_{on}$$

$$n\lambda_{off} = 2d_{off}$$

Subtracting the first from the second

$$n(\lambda_{off} - \lambda_{on}) = 2(d_{off} - d_{on}) = 2s$$

$$n\Delta\lambda = 2s$$

The accuracy of this equation may be greatly improved by applying the order number as calculated by either of the methods given above. The choice depends upon the percentage accuracy of the knowledge of the shift in wavelength $\Delta\lambda$.

The data of a step of 260 A with a separation of 76,000 A with a 15% transmission top is as follows: (The order number was calculated by the second method above).

Order number	Wavelength on step	Wavelength off step	Shift in wavelength	2s	s
23.9299	6508.5	6530.5	22	526	263
24.9299	6249.0	6270.0	21	524	262
25.9299	6008.0	6029.0	21	544	272
26.9299	5783.5	5803.0	19.5	525	262
27.9299	5578.5	5597.5	19	530	265
28.9299	5385.0	5403.0	18	522	261
29.9299	5207.0	5223.5	16.5	494	247
30.9299	5038.0	5055.0	17	526	263
31.9299	4877.5	4894.0	16.5	527	264

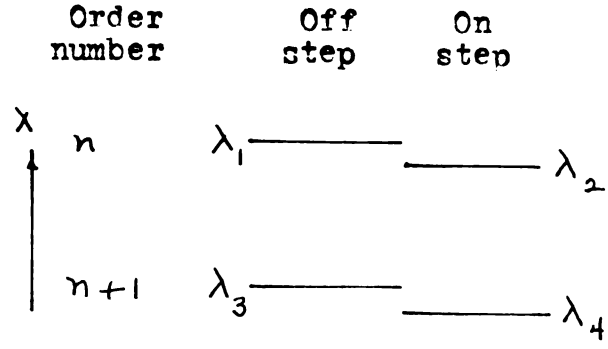
The average value of s, the thickness of the step, is 262.0 A. Since by comparison the error in $\Delta\lambda$ is the only effective error, we may use the Gaussian Distribution Curve to determine the probable numerical error in s. The expression for this is

$$.8453\left(\frac{\text{summation of deviations from the mean}}{n'(n'-1)^{\frac{1}{2}}}\right) = 1.1 \text{ A}$$

where n' is nine, the number of independent readings. This method is simple in calculation and yet very accurate. It is interesting to point out that if we divide $2s$ by the appropriate order number to determine the values for $\Delta\lambda$, the greatest disagreement between measured and calculated values is .9 A. This accuracy is obtained because the errors due to drum calibration and calibration curve of the spectrometer are negligible over the short range of $\Delta\lambda$. In fact if the spectrometer is in reasonable adjustment, a calibration curve is not needed to determine the corrected wavelengths in that for a small value of $\Delta\lambda$, the differences remain the same.

Although the above data indicates an accuracy greater than we expected, no calculation by this method has shown a probable error greater than 5 A. Slightly greater accuracy can be obtained by weighting the determinations of s by the corresponding value of $\Delta\lambda$. This is effectively the method of Least Squares with an assumed value for n which is more accurate.

METHOD D. The above method of calculation shows its physical significance if we consider any two orders just on and off the step.



For these lines we can write

$$n\lambda_2 = 2d_{on}$$

$$(n+1)\lambda_3 = 2d_{off}$$

$$(n+1)\lambda_4 = 2d_{on}$$

then between the first and last equation

$$n\lambda_2 = (n+1)\lambda_4$$

which can be written

$$\frac{1}{\lambda_2} = \frac{n}{\lambda_4} \cdot \frac{1}{n+1}$$

Multiplying by 2s, we have

$$\frac{2s}{\lambda_2} = \frac{n}{\lambda_4} \cdot \frac{2s}{n+1}$$

By subtraction between the first and last equation, we have

$$\frac{n}{\lambda_4} = \frac{1}{\lambda_2 - \lambda_4}$$

And likewise between the middle and last equation

$$\frac{2s}{n+1} = \lambda_3 - \lambda_4$$

Substituting and simplifying, we have

$$S = \left(\frac{\lambda_2}{2}\right)\left(\frac{\lambda_3-\lambda_4}{\lambda_2-\lambda_4}\right)$$

Now $\frac{\lambda_3-\lambda_4}{\lambda_2-\lambda_4}$ is just the fraction shift of order n + 1 between λ_2 and λ_4 when the step is removed. λ_2 is the effective wavelength of the radiation and 2 is the usual factor for the double distance light must travel. This indicates that just as in Tolansky's[1] work the fractional shift of the pattern due to the step must be measured, but that in our case a wavelength must be measured to determine the effective radiation. The accuracy of this measurement does not rely on these measurements, but on an accurate determination of the order number and a measurement of the shift, $\Delta\lambda$. Further it has the advantage that many measurements may be made with comparative ease and the thickness averaged between these.

It is readily seen that a Least Squares determination could be obtained from the series of equations $n\Delta\lambda = 2s$, $(n + 1)\Delta\lambda_1 = 2s$, $(n + 2)\Delta\lambda_2 = 2s$ .... The inaccuracy in $\Delta\lambda$ is so large that the method is virtually useless.

ANALYSIS OF MEASUREMENT

The actual thickness of the step excluding the wedge error has been measured to certainly less than 5 A. The wedge error has been shown to be 5 A or less, so that the actual error in the measurement of the step does not exceed 10 A. With this accuracy, variations in the surfaces of the optical plates become detectable. All of the errors mentioned can be lowered, but with the proper equipment the

last two can be virtually removed with no great loss in the utility in the method of measurement.

From our observation, the accuracy of these measurements in numerical error decreases very slowly for larger steps. A 5000 A step then can be measured to virtually the same accuracy as a 260 A step.

These measurements are average thicknesses over very small areas of the etalon. The area may be approximated from the dimensions of the spectrometer slit for we have attempted to use parallel light in this experiment. The width of the slit was .020 mm and the effective length should not exceed .5 mm as all measurements were made at the tip of the indicator in the field of view of the spectrometer.

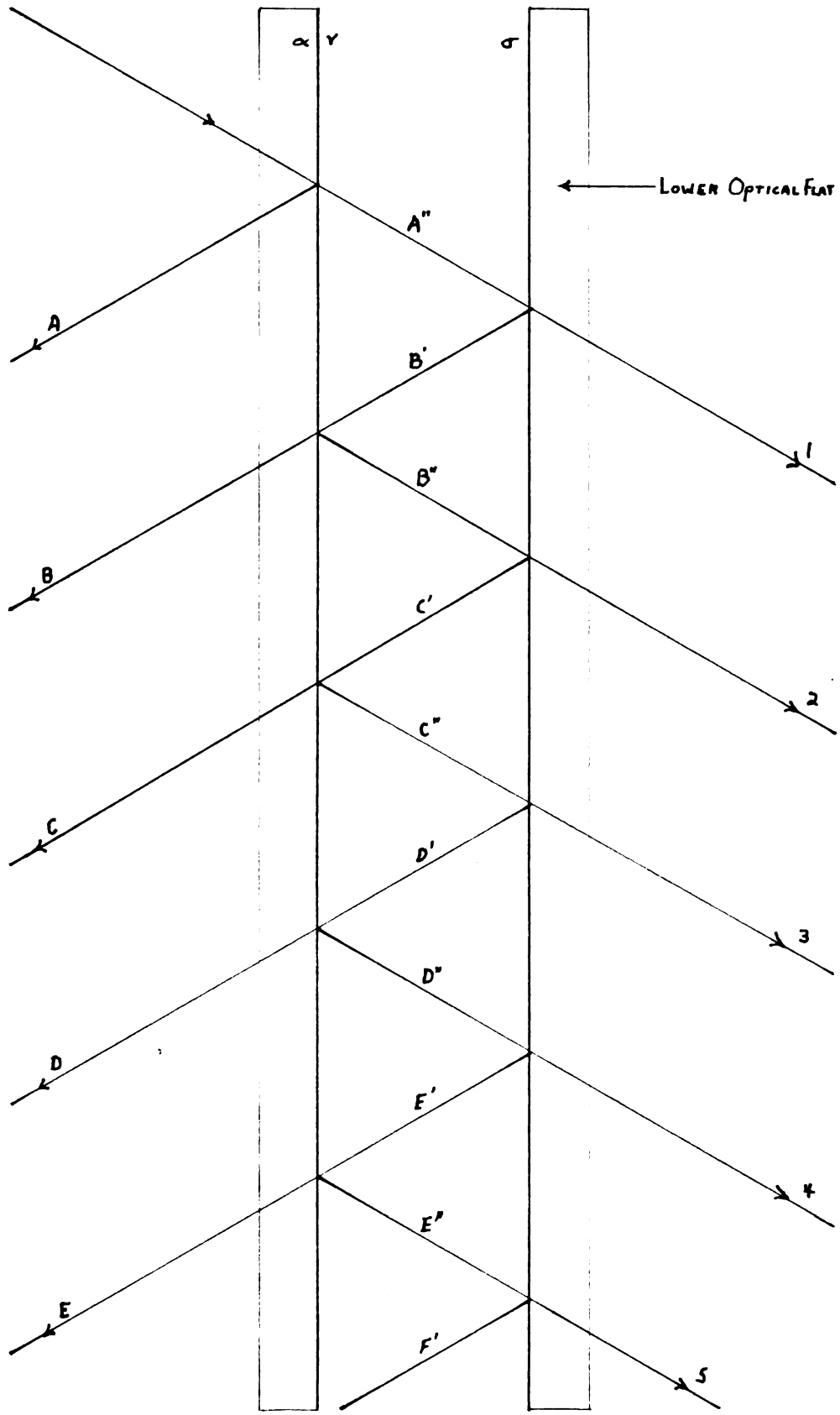
If a wedge of 3000 A per inch is deliberately placed upon the etalon, the pattern as formed in the spectrometer has a markedly lower resolving power when the axis of the wedge is adjusted parallel to the length of the slit. Over the .020 mm width of the slit this corresponds to a change in separation of 23 A. Now this change is doubly effective as the light must travel this distance twice so that the resolving power corresponds to 12 A surface variations in the surface of the Aluminum films. Because of the imperfection of our source of parallel light, the effective width of the slit at the surface of the etalon may be 100% in error. However it is quite reasonable to claim that surface variations on the Aluminum films cannot exceed 25 A for films 1000 A or less in thickness.

It has been found that surfaces in thick commercial films of the same type show irregularities much greater than this. Although the thickness of the coat may be a factor, more probably it is due to the method of deposition. Even on aged films of 1000 A or less, surface irregularities have been found which may indicate migration.

CONSIDERATION OF THE ASYMMETRICAL PATTERN

The resolving power of the reflection pattern has been shown to be markedly greater than would be expected. This is in part due to the asymmetrical pattern formed by the etalon when observed in reflected light; an asymmetry which has been easily detectable in monochromatic and white light. Its production is not entirely due to the full reflecting lower surface since the asymmetry does not disappear for a partially transmitting lower film.. For an explanation of these facts, let us consider the drawing on the following page. Though light was normally incident, it is convenient to construct it unrefracted and at a large angle.

It follows that if the second reflecting surface were to transmit rays 1, 2, 3, 4, ..., the intensities of B', C', D', E', ... will be correspondingly less. Hence as the second surface becomes full reflecting, the intensities of B', C', D', E', ... become correspondingly greater. If the analogy to the grating is continued, the resolving power in reflection becomes correspondingly greater. Fur-


α
γ
σ
LOWER OPTICAL FLAT
A
A"
B'
B
B"
C'
C
C"
D'
D
D"
E'
E
E"
F'
1
2
3
4
5

ther, any alterations in the phase change due to higher reflecting surfaces will be the same for all rays and may be considered part of the effective thickness of the etalon. It appears then that the lower surface is not responsible for the asymmetry noted in the reflection pattern.

The asymmetry must be explainable then as the result of conditions at the first surface. Theoretical considerations by Stratton[9] and Jenkins & White[10] indicate that for an integral order number, ray A will not be in phase with rays B, C, D, ... The phase difference will be shown to be dependent upon phase changes due to reflection at the glass-Aluminum boundary, reflection at the air-Aluminum boundary, and changes in the order number N. With optical constants for $\lambda = 5893$ taken from the International Critical Tables, the phase changes upon reflection become respectively $21^{\circ}\ 40'$ and $33^{\circ}\ 50'$. These are true only for full reflecting films, but we will apply them as they approximate the phase changes for the more complex case of transmitting films.

To relate the factors above to the phase difference between rays A and B, let the phase of the incident light be $e^{i\omega t}$ for any frequency which allows rays B, C, D, .. to interfere constructively. For clarity, let a, b' and b indicate in radians an advance in phase reflection at the glass-transmitting Aluminum, air-transmitting Aluminum and air-Aluminum reflections. Then we can write for ray A

$$e^{i(\omega t - a)}$$

If n is the order number corresponding to the effective thickness of the etalon, b and b' absorbed as part of the thickness, we can write for ray B, since its angular distance in the etalon is b' less than $2\pi n$,

$$e^{i(\omega t + b' - 2\pi n)}$$

Consecutive rays will then differ in angular distance by the factor $2\pi n$. They can be written

$$e^{i(\omega t - 4\pi n + b')}$$

$$e^{i(\omega t - 6\pi n + b')}, \text{ etc.}$$

If now the incident phase is taken as

$$e^{i(\omega t - b')}$$

Rays A, B, C, ... have simplified phase conditions given by

$$e^{i(\omega t - [a + b'])}$$

$$e^{i(\omega t - 2\pi n)}$$

$$e^{i(\omega t - 4\pi n)}$$

It follows then that the phase difference between A and B is $2\pi n - (a + b')$. Within the accuracy of our knowledge of a and b', this phase difference remains independent of wavelength. Since phase changes a and b' can be shown to be of the same nature, they are addative.

The Vibration Curve[11] method of determining the intensity pattern formed by rays A, B, C, D, ..., although not adequate for this situation, does give insight into the physical nature of the etalon. Its graphical solution of

The problem allows one to watch the effect of the phase as it determines the intensity pattern.

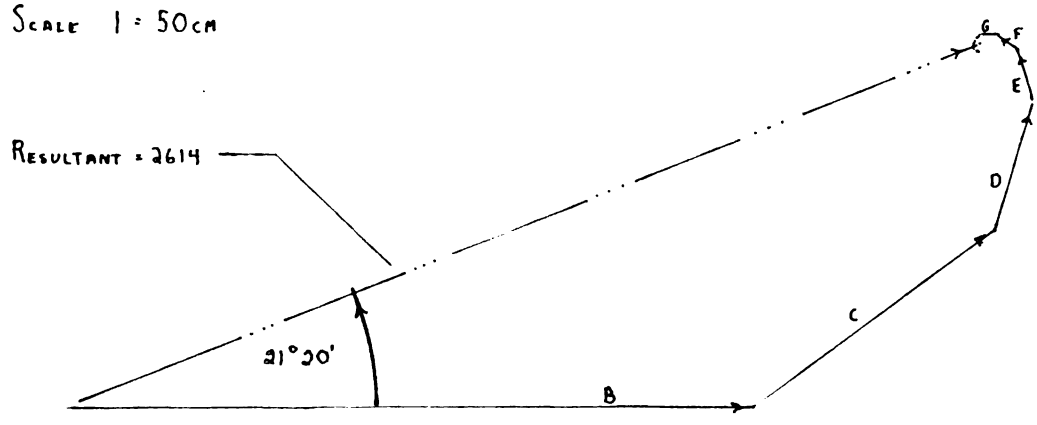
Let the fractional amplitude coefficients of the first surface be α for reflection and γ for transmission. Similarly let σ be the fractional reflection coefficient for amplitude at the second surface. Assuming incident light of unit amplitude, the amplitudes of rays A, B, C, D, ... then become α, $\sigma\gamma^2$, $\alpha\sigma^2\gamma^2$, $\alpha^2\sigma^3\gamma^2$, ... respectively. Rays B, C, D, ... differ in amplitude by the factor $\sigma\gamma$, the only term of which we are free to change is α, the reflection coefficient of the first surface.

Let us assume values for these coefficients which may be suitable for a 50% reflecting top coating; $\alpha = .45$, $\gamma = .45$, $\sigma = .90$. These values, although appropriate on the basis of amplitude considerations, are questionable when the energies of A and A" are added and compared with the incident energy. Conversely, when energy considerations are employed to determine these amplitude coefficients, a similar lack of agreement results for amplitude consideration. Though the vibration method is not adequate to the problem, we will continue because of the physical significance illustrated by this method. The amplitudes for the rays A, B, C, D, ... then become respectively .45, .1821, .0787, .0340, .0147, .0063, .0027, .0012, .0005, .0002, .0001, ...

Because of the symmetry in phase which exists between rays B, C, D, ..., we will evaluate the amplitude for these separately and add their resultant vectorily to

ray A, using ray B as a reference. To determine the amplitude for fractional order numbers, we will use order numbers varying by steps of .1.

For n = 1 ...., rays B, C, D, ... will be in phase and the resultant amplitude is merely the sum of the individual amplitudes, or .3205. For n = 1.1 ...., the rays B, C, D, ... differ in phase by 360° ÷ 10, or 36°. Constructing graphically to find the resultant amplitude



The amplitude then is .2614 at a positive 21° 20' with respect to the direction of ray B.

For n = 1.2 etc., the rays B, C, D, ... differ in phase by 72° and a similar construction gives .1882 at 25° 40' with B. Similarly

Order number	Resultant amplitude	Phase with B
n = 1.3	.1504	20°
n = 1.4	.1338	11° 20'
n = 1.5	.1271	0°
n = 1.6	.1338	-11° 20'
n = 1.7	.1504	-20°
n = 1.8	.1882	-25° 40'
n = 1.9	.2614	-21° 20'
n = 2.0	.3205	0°

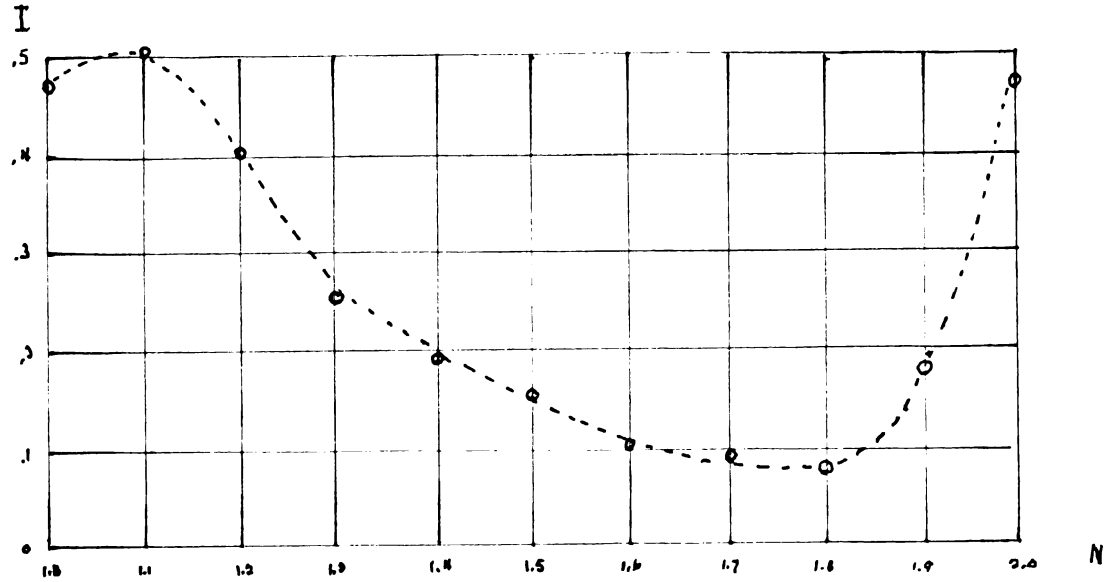
These indicate a symmetrical bright line pattern.

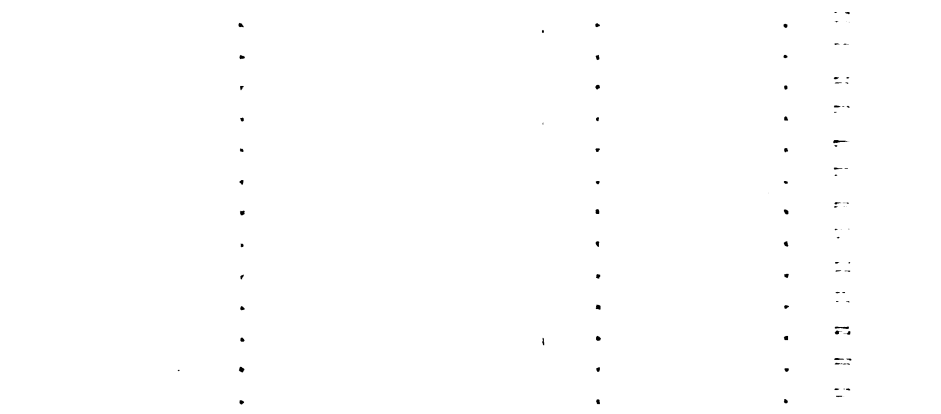
When the phase difference $2\pi n - (a + b')$ between A and B is used to determine the amplitude of A, B, C, D, E, ..., we have

Order number	Amplitude	Intensity (Amplitude)^2
n = 1.0	.687	.472
n = 1.1	.718	.506
n = 1.2	.640	.410
n = 1.3	.517	.268
n = 1.4	.441	.195
n = 1.5	.382	.146
n = 1.6	.342	.117
n = 1.7	.306	.094
n = 1.8	.293	.086
n = 1.9	.426	.182
n = 2.0	.687	.472
n = 2.1	.718	.506
n = 2.2	.640	.410

As can be seen the pattern repeats itself as we go to higher orders. This is another condition for which the vibration curve breaks down, for the resolving power was observed to increase with the order number.

If the values above are tabulated in a graph, the result is





This intensity distribution agrees very well with the observed pattern. The sharp change in the pattern corresponding to an order number of 1.92 is the characteristic point from which all of our measurements were made and agrees with the order numbers calculated earlier in the paper. However the intensity pattern more closely resembles that of 25% transmission coating. The discrepancy evidently lies in our inability to assign appropriate values for the amplitude coefficients. The presence of a black minimum, even for a 15% transmission top coating, shows that the effects taking place within the etalon are not as simple as the Vibration Curve might indicate, for according to it, the first reflection should mask the higher order reflections.

We have justified the higher resolving power in reflected light as caused by two effects, a larger number of effective rays caused by a full reflecting lower surface, and an asymmetrical pattern which presents extremely high resolving power at the portion corresponding to .92 of an order. The asymmetry has been shown due to the phase difference between rays A and B, C, D, ... From the Vibration Curve, it follows that if A were in phase with B, C, D, ..., a symmetrical bright line pattern would result; if A were 180° out of phase, a symmetrical dark line pattern would result; and at values between these, the pattern becomes asymmetrical. It is difficult to see from the first condition above how any pattern could exist if the

surfaces of the etalon were perfect reflectors and transmitters unless as a result of the finite width of the effective area of the slit and/or the interference within the etalon, conditions are more analogous to a grating than we suspect.

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