ANISOTROPY CONSTANTS OF POLYCRYSTALLINE FERROMAGNETIC SUBSTANCES

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ANISOTROPY CONSTANTS OF POLYCRYSTALLINE FERROMAGNETIC SUBSTANCES

BY

Richard Henry Kropschot

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Richard Kropschot

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I. INTRODUCTION

This thesis deals with ferromagnetic materials. Such materials include the ferromagnetic elements iron, nickle and cobalt and ferromagnetic alloys such as supermalloy, permalloy, alnico, deltamax, silectron, and Heusler alloys. Most of the ferromagnetic alloys are composed partially or wholly of the ferromagnetic elements listed above. However, some of the alloys such as Heusler alloys contain entirely paramagnetic elements.

In magnetic media the magnetic field intensity H and the magnetic flux density B are related by

$$\underline{B} = \underline{H} + 4\pi\, \underline{I} \tag{1}$$

where I is the intensity of magnetization or dipole moment per unit volume. In addition in paramagnetic materials

$$\underline{I} = \chi\, \underline{H}\,, \tag{2}$$

where χ is the magnetic susceptibility. For this case

$$\underline{B} = (1 + 4\pi\chi)\, \underline{H}\,, \tag{3}$$

$$\mu = (1 + 4\pi\chi) \tag{4}$$

where μ is the magnetic permeability. In ferromagnetic materials equations (2), (3) and (4) are not valid in general and the relations between B, H and I must be given graphically. Curves relating B and H or I and H are called magnetization curves and serve to define the empirical relationship between the various pairs of variables. The curves plotted with B as the ordinate and H as the abcissa are of great interest in practical calculations. However, from the theoretical point of view the curves relating I and H are usually of greater interest.

The first quantitative theoretical calculations in magnetism are due to Langevin who successfully developed an expression for the susceptibility of a paramagentic solution. [2] Langevin considered a large group of non-interacting dipoles in thermal equilibrium at some temperature T. If we consider one dipole of strength μ in an applied field H, then the energy of the dipole is given by

$$E = -\underline{\mu} \cdot \underline{H} \tag{5}$$

Let the angle between μ and H be θ, then

$$E = -\mu H \cos\theta . \tag{6}$$

By applying the Maxwell Boltzman distribution law, Langevin obtained an expression for the intensity of magnetization in the form

$$I = N \frac{\int_0^\pi \mu \cos\theta \, e^{-\frac{\mu H \cos\theta}{kT}} \sin\theta \, d\theta}{\int_0^\pi e^{-\frac{\mu H \cos\theta}{kT}} \sin\theta \, d\theta} , \tag{7}$$

where N is the total number of dipoles per unit volume. This may be written

$$I = N k T \frac{\partial}{\partial H} \log \int_0^\pi e^{-\frac{\mu H \cos\theta}{kT}} \sin\theta \, d\theta , \tag{8}$$

from which one readily obtains

$$I = N\mu \left\{ \coth \frac{H\mu}{kT} - \frac{1}{\frac{\mu H}{kT}} \right\} \equiv N\mu L(a). \tag{9}$$

In the above L (a) is defined as the Langevin function, that is

$$L(a) = \left(\coth a - \frac{1}{a}\right) \qquad a = \frac{\mu H}{kT} . \tag{10}$$

If $\mu H \ll kT$ one may use the expansion for the hyperbolic cotangent

$$\coth \frac{\mu H}{kT} = \frac{1}{\frac{\mu H}{kT}} \left\{ 1 + \frac{4}{12} \left(\frac{\mu H}{kT}\right)^2 - \cdots \right\} \tag{11}$$

from which we obtain

$$I \simeq \frac{N \mu^2 H}{3kT} . \tag{12}$$

From Eq. (2) we have

$$\chi = \frac{N \mu^2}{3kT} . \tag{13}$$

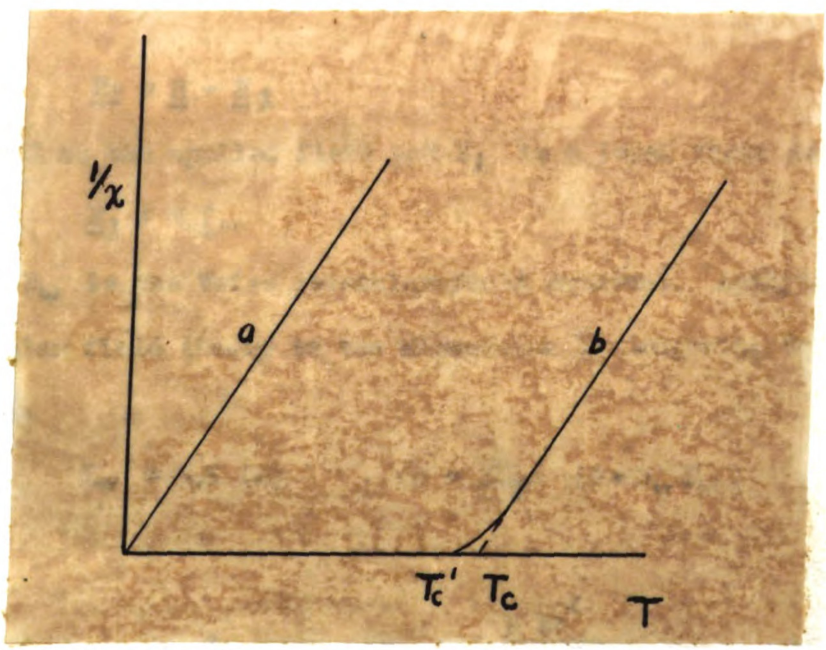

Fig. 1. The reciprocal of the susceptibility as a function of temperature above the Curie point.

If we plot the reciprocal of the susceptibility against the temperature, we obtain a straight line as shown in Fig. 1, curve a. The slope of which is given by $\frac{3k}{N\mu^2}$ and the origin is the intercept. This gives essentially the results found in paramagnetism.

Above the Curie temperature ferromagnetic materials act as paramagnetics. However, it is an experimentally known fact that the curve of $\frac{1}{\chi}$ vs. T for ferromagnetic materials is shifted along the T axis toward higher temperatures, as shown in Fig. 1, curve b. This shift was accounted for by the molecular field theory proposed by Pierre Weiss (1907). The Weiss theory of ferromagnetism postulates that there exists a spontaneously induced molecular field proportional to and in the same direction as the intensity of magnetization.[11] The effective field H_T acting on a magnet can be expressed as

$$\underline{H_T} = \underline{H} + \underline{H}_\lambda \tag{14}$$

where H is the applied field and H_λ is a local field defined by

$$\underline{H}_\lambda = N_\omega \underline{I}_\omega \tag{15}$$

where N_ω is the Weiss molecular field constant. Applying the molecular field theory to the discussion by Langevin, Eq. (9) becomes

$$I_\omega = N\mu L\ (a) \qquad a = \frac{\mu}{kT}\ (H + N_\omega I_\omega) \tag{16}$$

and

$$I \simeq \frac{N\mu^2(H + N_\omega I_\omega)}{3\,k\,T} = \frac{\frac{N\mu^2}{3\,k\,T}}{1 - N_\omega \frac{N\mu^2}{3\,k\,T}}\ H. \tag{17}$$

Therefore

$$\chi = \frac{I_\omega}{H} = \frac{N\mu^2}{3k(T-T_c)} \tag{18}$$

where

$$T_c = \frac{N_\omega N\mu^2}{3k} \ . \tag{19}$$

From Eq. (18) as T approaches T_c the susceptibility is observed to go to infinity, (ie ferromagnetism). T_ℓ is defined as the ferromagnetic Curie temperature and is within 20°K out of approximately 630°K of the paramagnetic or extrapolated Curie temperature T_c.

A comparison of Weiss' theoretical work and experimental evidence may be obtained in the following manner.

Let us consider the case where H = 0, then

$$\frac{I}{\mu N} = \coth \frac{\mu H_\ell}{kT} - \frac{kT}{\mu H_\ell} \ . \tag{20}$$

Set

$$\varsigma = \frac{I}{N\mu} \qquad \tau = \frac{T}{T_c} \ , \tag{21}$$

then

$$\varsigma = \coth \frac{3\varsigma}{\tau} - \frac{\tau}{3\varsigma} \ . \tag{22}$$

The theoretical curve of ς against τ can readily be calculated. The quantities can also be measured by experiment.

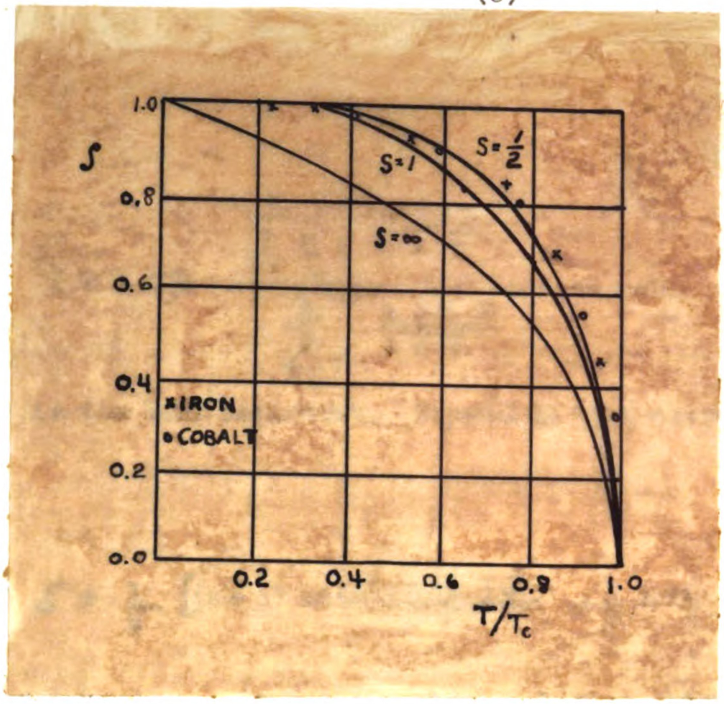

Fig. 2. Saturation intensity of magnetization as a function of temperature, in reduced units. The abscissa is the ratio of the temperature to the Curie temperature, and the ordinate is the ratio of the saturation intensity of magnetization to the value of this intensity at the absolute zero. The curve $S = \infty$ is for the classical theory of Weiss. The other curves are based on the Brillouin rather than the Langevin function, and are the corresponding quantum theory versions appropriated to a spin quantum number S.[11]

Figure 2 shows how remarkably well the two curves agree.

A better approximation to the experimental curve can be obtained by applying quantum mechanics to Langevins' theory. Previously we allowed $\cos\theta$ to take on all values. Now let us assume our system is quantized such that $\cos\theta$ may take on only the values

$$\cos\theta = \frac{M}{S} = \frac{-S, -S+1, \ldots\ldots\ldots\ldots, S}{S} .$$

The integral terms in the expression for the intensity of magnetization are now replaced by sums and the expression for I becomes

$$I = 2N\beta \frac{\sum_{-S}^{S} \frac{m}{s} e^{\frac{m\,2\beta H}{s\,k\,T}}}{\sum_{-S}^{S} e^{\frac{m\,2\beta H}{s\,k\,T}}} \tag{23}$$

where β is the Bohr magnetron. Expanding this expression we obtain

$$\mathcal{S} = \frac{1}{s}\left[(s+\tfrac{1}{2})\coth\frac{(s+\frac{1}{2})\,3\mathcal{S}}{s\,\mathcal{T}} - \frac{1}{2}\coth\frac{3\mathcal{S}}{2\,s\,\mathcal{T}}\right]. \tag{24}$$

This equation reduces to the classical relation obtained previously if we let $S = \infty$. However, the best agreement with experiment is the case where $s = \frac{1}{2}$ as shown in Fig. 2. The value for $\mathcal{S}$ is then given by

$$\mathcal{S} = 2\left[\coth\frac{6\mathcal{S}}{\mathcal{T}} - \frac{1}{2}\coth\frac{3\mathcal{S}}{\mathcal{T}}\right]. \tag{25}$$

Let us investigate the significance of the spontaneously induced magnetic field assumed by Weiss in light of modern day quantum mechanics.[11] The energy terms we are interested in are the exchange energy and the magnetostatic energy. The total energy is related to the Weiss molecular field by

$$\omega_n = 2\beta\, S_z H_T \tag{26}$$

or

$$\omega_n = 2\beta\, S_z(H + N_\omega I_\omega)\ . \tag{27}$$

Then

$$\frac{\omega_n - 2\beta S_z}{I_\omega} = 2\beta\, S_z N_\omega \tag{28}$$

where S_z is the projection of the spin in the direction of H.

We must now obtain an expression for ω_n in order to evaluate N_ω. The exchange energy is given in the form

$$\omega_{ex} = -2\, J_{ij}\, \underline{S}_i \cdot \underline{S}_j \tag{29}$$

where $\underline{S}_i$ and $\underline{S}_j$ are the spin angular momentum vectors of atoms i and j. In order to arrive at this expression let us consider the permitted values of the energy as being

$$W = C \pm J_{ij}$$

where C is a constant and J is the exchange integral given, for the case of two particle systems, by

$$J_{ij} = \int \Psi_i(1)\Psi_j(2)\, H'\, \Psi_j(1)\Psi_i(2)\, d\tau_1 d\tau_2\, . \tag{30}$$

The Ψ's refer to the respective wave functions and H' is the Hamiltonian operator. The positive value of J corresponds to a singlet state whereas the negative value is a triplet. It is a well known fact that in ferromagnetism the lowest state is the condition for which all the spins are lined up. This is true only in the triplet state; therefore we are interested in the conditions on J for the triplet state to be of the lowest energy or in other words J positive.

If we consider a term which is a function of spin in the form

$$f(s) = -\frac{1}{2} - \frac{1}{2}\, S_i \cdot S_j \tag{31}$$

we find that the characteristic values are $\pm$ 1. The plus is for the singlet and the minus for the triplet, as desired. The exchange energy now becomes

$$\omega_{ex} = -2\, J\, S_i \cdot S_j\, . \tag{32}$$

The terms C and $-\frac{J_{ij}}{2}$ drop out because they are not spin dependent.

The exchange energy can also be calculated in a similar manner for the interaction of the i^{th} electron with its Z nearest neighbors.

$$\omega_{ex} = 2\, J\, Z\, S_z \frac{I\omega}{2\beta\, N} \qquad (33)$$

The magnetostatic energy is given by

$$\omega_{mag} = 2\beta\, S_z\, H \quad . \qquad (34)$$

Therefore

$$\omega_h = 2\beta S_z \left\{ H + \frac{J Z\; I_\omega}{2\beta^2 N} \right\} . \qquad (35)$$

Equating this to (27)

$$N_\omega = \frac{Z\; J}{2\beta^2 N} \ , \qquad (36)$$

then from (19)

$$T_c = \frac{J Z\, N\, (2\beta)^2\, S\,(S+1)}{6\,\beta^2 N\, k} = \frac{2\, J Z S\,(S+1)}{3\,k} \ . \qquad (37)$$

We have shown that by using the Weiss molecular field theory or its anâlog in quantum mechanics, the exchange energy, we are able to predict a Curie temperature. However, in order to obtain the desired results we must have a condition with the dipoles lined up at all times. We know from experimental results that this is not always true. It is possible, with no applied field, to have the overall magnetization of the sample be zero and with a small applied field the intensity of magnetization is in general less than the saturation magentization.

In order to account for the fact that in ferromagnetic materials an extremely small applied field causes a relatively large magnetization, Pierre Weiss assumed the existence of domains. Each domain is at saturation magnetization and in absence of an applied field they are oriented randomly. The Weiss theory does not justify the existence of the domain type structure but assumes it in order to account for experimental observations. The domain type structure can, however, be justified in light of modern theories.

The existence of the domain type structure has been verified experimentally by use of powder pattern methods. Fine particles of magnetite in collodial suspension are placed on a prepared surface and observed through a microscope. The pieces of magnetite tend to collect along the domain walls due to the very strong local magnetic fields. The size of the domains seem to be very dependent on the material, the size of the crystal and also on the history of the material.

When an external field is applied the domains with the favored orientation can be observed to grow at the expense of the unfavorably orientated domains. The resultant magnetization can be changed in two manners: first, due to favored domains growing at the expense of unfavored ones or, second, by means of rotation of the magnetization vector in the direction of the applied field.

The domain structure has its origin in the possibility of lowering the energy of a system by going from a saturated configuration such as a single domain structure with high magnetic energy to that of several domains oriented in such a manner that the system has a lower energy.[9]

In addition to the two energy terms already mentioned, exchange and magnetostatic, we must consider two additional energies in order that serious approximations in the domain structure do not have to be made. These two terms are the anisotropy energy, of which we will say more in a following section, and the magnetoelastic energy. The anisotropy energy tends to direct the magnetization vector along certain crystalographic axes. The magnetoelastic energy arises from the fact that the crystal dimensions change slightly when a magnetic field is applied. The change in dimensions is called magnetostriction and is related in a direct manner to the anisotropy energy.

In the region between two adjacent domains, called a Bloch wall, there exists a transition area where the spin direction changes from parallel to an antiparallel allignment. The change does not occur across just one lattice plane but rather across many, depending on the energy. The exchange energy is inversely proportional to the wall thickness and, therefore, tends to make the wall spread throughout the whole crystal. However, in the transition from a parallel to an antiparallel alignment the spin directions are rotated away from the crystalline axis and are, therefore, acted upon by the anisotropy energy. The anisotropy energy is directly proportional to the wall thickness. It is a balance between these two energies that determine the thickness of the wall. For iron the wall thickness is of the order of 1000 Å and the total wall energy per unit area is approximately 1 erg/cm^2.[9]

II. THE ANISOTROPY CONSTANTS

All ferromagnetic substances have a crystalline microstructure. As is characteristic of crystalline materials, ferromagnetic materials are anisotropic in the sense that their physical properties depend on the direction in which they are measured. Therefore, it is to be expected that a portion of the internal energy of a ferromagnetic substance will depend on the orientation of the magnetization vector in respect to the crystalline axes. This portion of the internal energy is referred to as the anisotropy energy. The anisotropy energy of a crystal of cubic symmetry such as iron or nickel may be expressed in a series expansion as

$$E = K_0 + K_1 (\alpha_1^2\alpha_2^2 + \alpha_1^2\alpha_3^2 + \alpha_2^2\alpha_3^2) + K_2 (\alpha_1^2\alpha_2^2\alpha_3^2) \;\ldots\ldots \quad (38)$$

where α_1, α_2 and α_3 are the direction cosines of the magnetization vector with respect to the x, y and z axes, and K_0, K_1, and K_2 are the anisotropy constants. In this expression the odd powers of the direction cosines disappear due to crystalline symmetry and the α squared terms are included in K_0 due to the fact that

$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$$

It is found experimentally that in ferromagnetic materials the constants K_1 and K_2 are quite large while the higher ordered terms can be neglected. This means that the ease with which a given sample can be magnetized is dependent on the choice of axes. It is easier to magnetize an iron crystal along the [100] direction than it is along the [111] direction, while the reverse of this is true in a nickel crystal.

The origin of the anisotropy energy is not immediately obvious. The Heisenburg exchange energy used in explaining ferromagnetism is isotropic since the exchange effect depends only on the relative orientation of adjacent spins and not on the crystal direction. The magnetic dipole interaction between spins does give a term that is dependent on the orientation of the spins with the crystal lattice, but the magnitude of this term is much too small to account for the anisotropy effect. Bloch and Gentile[15] have suggested that the spin orbit coupling combined with an electrostatic coupling of the orbit to the crystal yields the right order of magnitude for the anisotropy constants and at present this is the generally accepted theory.

Van Vleck[12] has calculated a theoretical value for the anisotropy constant K_1, that agrees with experiment as far as sign, approximate magnitude and temperature dependence. He gives K_1 to be of the order of $\frac{A^4}{10\ k\ T_c h^2 \nu^2}$ per atom if due to dipole-dipole interactions, and of the order of $\frac{A^4}{h^3 \nu^3}$ if due to the quadrupole effect. A is the spin-orbit constant, T_c is the Curie temperature, and $h\nu$ is a quantity dependent on the energy levels. The calculated value of K_1 is the order of 10^5 ergs/cc. The value for K_1 in the expression is also temperature dependent, which agrees with experimental evidence. In fact it is found experimentally that the constants K_1 and K_2 may change sign between high and low temperatures.[9]

The anisotropy constants for various Fe - Co, Fe - Ni, Co - Ni, and Fe - Co-Ni alloys are summarized in a paper by Bozorth (1937).[5] I will list briefly some of the results obtained.

Table 1. Values of K_1 and K_2 for various ferromagnetic elements and alloys

Composition Fe	Co	Ni	Temp °C	$K_1 \times 10^3$ ergs/cc	$K_2 \times 10^3$ ergs/cc
100			20	421	150
50	50		20	-68	-390
50		50	20	33	-180
	50	50	20	-108	-40
		100	20	-34	50

It was the purpose of this research to obtain experimental values for the anisotropy constants K_1 and K_2 in polycrystalline ferromagnetic substances. The anisotropy energy and the magnetostatic energy are the only portions of the internal energy that are dependent on the orientation of the magnetization vector. The other two terms, exchange and magnetoelastic, are independent of angle. The torque per unit volume on a disk or oblate spheroid sample due to an applied magnetic field is given by

$$T = \frac{\partial E}{\partial \phi} \tag{39}$$

where ϕ is the angle between the magnetization vector and an arbitrary axis in the plane of the sample in which the torque is measured. In a high magnetic field the magnetiostatic energy becomes a constant along with the exchange and magnetoelastic energy. The torque on a sample is then only a function of the anisotropy energy.

For the rotation of the magnetization vector I in the(100) plane the torque on a given sample can be computed by letting $\alpha_1 = \cos\beta$, $\alpha_2 = \sin\beta$ and $\alpha_3 = 0$. Where β is the angle between I and the x axis. The torque is then given by

$$T = \frac{K_1}{2} \sin 4\beta \tag{40}$$

In a similar manner the torque on a sample cut from the (110) plane is of the form

$$T = \sin 2\phi\left(1 - \frac{3}{2}\sin^2\phi\right)\left(K_1 + \frac{K_2}{2}\sin^2\phi\right) \qquad (41)$$

where ϕ is the polar angle.

For a sample cut in an arbitrary direction Bitter and Tarasov[4] have given the following results:

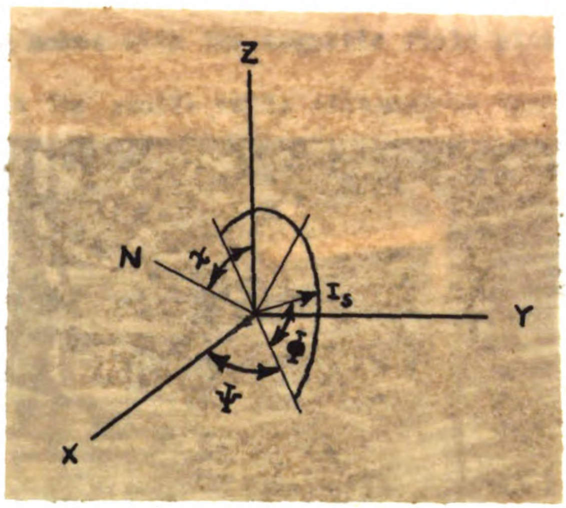

Fig. 3. Angles defining orientation of disk X, Y, Z are cubic axes of the crystal; N is the normal to the disk; I_s is the direction of the magnetization in the plane of the disk; ϕ is the angle measured in the plane of the disk.

In terms of the angles defined in Fig. 3

$$\begin{aligned} \alpha_1 &= \cos\phi\cos\psi - \sin\phi\sin\psi\cos\chi\,, \\ \alpha_2 &= \cos\phi\sin\psi + \sin\phi\cos\psi\cos\chi\,, \qquad (42) \\ \alpha_3 &= \sin\phi\sin\chi\,, \end{aligned}$$

the torque on a given sample, for the case of small K_2 (assuming $K_2 = 0$), is given by

$$\frac{T}{K_1} = A_1 \sin 2\phi + A_2 \sin 4\phi + B_1 \cos 2 \quad + B_2 \cos 4\phi, \qquad (43)$$

where

$$A_1 = \tfrac{1}{4}\sin^2\chi\ \left(1 - 7\cos^2\chi\right) - \left(1 + \cos^2\chi\right)\cos 4\psi\ ,$$

$$A_2 = -\frac{7}{8}\sin^4\chi - \left[\left(\frac{1}{8}\right)\sin^4\chi + \cos^2\chi\right]\cos 4\psi \quad ,$$

$$B_1 = -\frac{1}{2}\sin^2\chi \cos\chi \quad \sin 4\psi, \qquad (44)$$

$$B_2 = -\frac{1}{2}(1+\cos^2\chi \quad)\cos\chi \sin 4\psi .$$

III. THE TORQUE MAGNETOMETER

In order to investigate the anisotropy energy torque curves were used. Torque curves are curves relating the angle the direction of rolling makes with the magnetic field plotted against the torque produced on the sample while alligned in that direction.

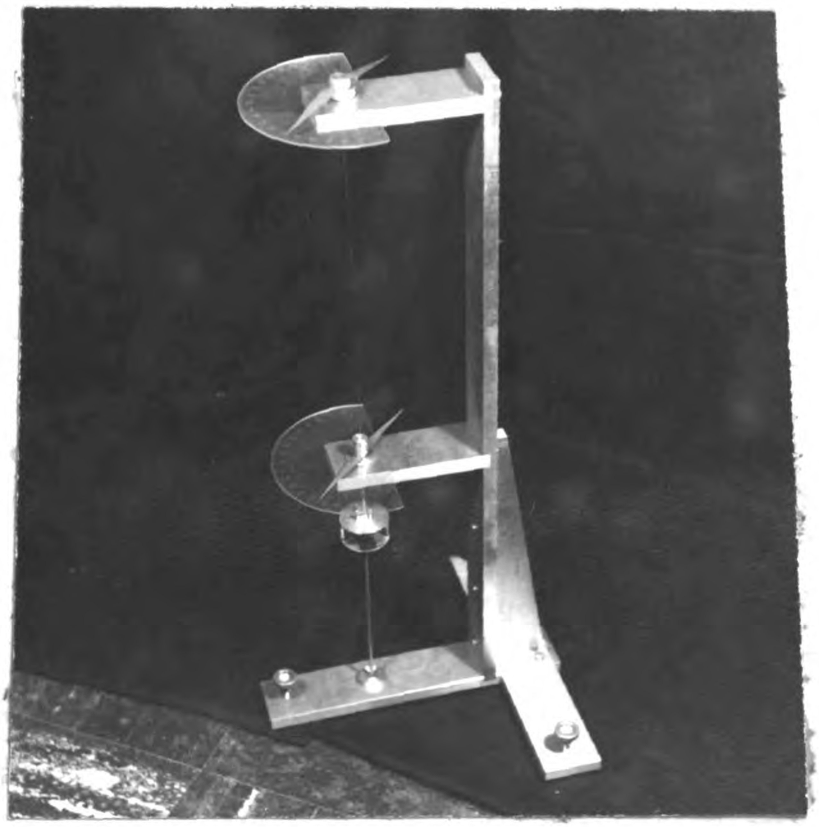

Fig. 4. The Torque Magnetometer.

The torque magnetometer shown in Fig. 4 was used to measure the torque on disk and oblate spheroid samples. A sample was clamped in the sample holder which is supported by a 1/8 inch brass shaft running in glass bearings made from short lengths of capillary tubing.

A torsion fiber with a brass indicator at either end was used to suspend the sample holder. The lower indicator reading gives the sample orientation while the difference in the two readings represents the retarding torque on a sample in opposition to the torque produced by an applied magnetic field.

The constants of the suspension were evaluated by use of the torsion pendulum. The quantity determined experimentally was the product

$$\mu a^4 = \frac{8 \pi l I}{P^2} \tag{45}$$

where μ is the modulus of rigidity of the wire, a is its radius, and l is its length. I is the moment of inertia of the pendulum and P is its period. To evaluate μa^4 a length of wire l = 123.6 cm was used and a cylindrical block of aluminum of mass m = 475.070 grams and radius r = 3.8137 cm was used as a pendulum. The results of the torsion pendulum measurements are shown below.

Trial	Number of Oscillations	Time Sec.
1	50	440
2	50	440
3	25	220
4	25	220

From these data it was found that

$$\mu a^4 = 138,600 \text{ dyne cm}^2.$$

The retarding torque produced by the fiber is then given by

$$T = \frac{\pi}{2} \frac{\mu a^4}{l} \theta \tag{46}$$

where θ is the angular separation of the indicators and l is the length of the wire used in the instrument. The instrument wire length used during the course of the experiment was 30.493 cm as measured by use of a height guage. The torque can then be expressed as

$$T \text{ (dyne cm)} = 7140\, \theta \text{ (rad.)}$$

A uniform magnetic field was applied to the sample by means of flat cylindrical pole pieces of a divided electromagnet. The magnitude of the field was determined from a calibration curve obtained by taking a large number of readings with a flux meter.

The magnetometer was designed so that a sample about the size of a quarter or smaller, depending on its crystal allignment, would give a sizeable deflection. The samples investigated were either transformer laminations or a silicon steel called "silectron".

It was found that samples with a readily observable anisotropy energy could be selected by bending a sample of the material until it broke. If the sample was soft and flexible its anisotropy constants were small, however, if it was brittle and showed large crystallites on the broken edge the sample displayed an anisotropy energy large enough to be measured readily by the torque magnetometer.

The surface of the metal strips were prepared by polishing them lightly with an abrasive after which an indelible ink was applied to the surface so a line could be scribed parallel to the direction of rolling. A compass was used to outline a disk. The disk was cut out as close to the line as possible by use of tin snips and then ground to the line on an external grinder. Samples of silectron were made in a form which approximated an oblate spheroid by first cutting out a rough sample and soldering to it a brass shaft which was inserted in a collet on a lathe and turned to proper shape. The sample was finished by filing it while it revolved.

The procedure for obtaining the torque curves was as follows: The sample was clamped firmly in the center of the sample holder and a strong magnetic field applied. The pointer on the lower indicator was set at predetermined angular positions spaced 5 degrees apart and extending over a range of 180 degrees. At each setting of the lower pointer the upper pointer was rotated to a position at which the torque on the sample was just balanced by that produced by twisting the torsion fiber. The difference between the pointer readings gave the angle through which the fiber had been twisted. The torque on the sample was then obtained from Eq. (46). In plotting the torque data the orientation of the sample in respect to the magnetic field was represented by the angle between the scribed line (along the rolling direction) and the direction of the magnetic field.

There are two regions on any one of the curves, from 0 to 180 degrees, that the readings are found to be unstable. They are located on either side of zero torque where the slope is positive. The regions extend to a point about half way from zero to the maximum or minimum point. It is believed that the instability is a characteristic of the sample rather than the instrument since the same instability was observed by Bitter and Tarasov[4] using an instrument of quite different design.

The errors in the torque measurements arise primarily from the frictional torques in the instrument and errors in reading the

indicator scales. The error due to frictional torques appears to amount to less than 0.5 of a degree while the error in reading the indicator scales is estimated at 1.0 degree. These errors amount to about 3.0 percent of the maximum deflection for most samples and since the first order anisotropy constant K_1 is directly proportional to the maximum deflection, they constitute a most serious limitation in the accuracy of the instrument. The second order anisotropy constant K_2 is much more sensitive than K_1 to small errors in the torque curve and, therefore, the torque magnetometer used in this experiment cannot be used to determine K_2.

While most of the torque curves were obtained with the sample at room temperature it was thought desirable to measure the torque on a few samples at the same temperature as liquid nitrogen in order to obtain an indication of the temperature dependence of the anisotropy constants in polycrystalline materials. This was accomplished by placing the sample in a small cup made of poly-foam which was placed in the sample holder and kept filled with liquid nitrogen while the torque curve was measured. The sample was then allowed to warm up and the torque curve was measured again at room temperature.

IV. DATA

The torque curves are shown on the following pages followed by their interpretation.

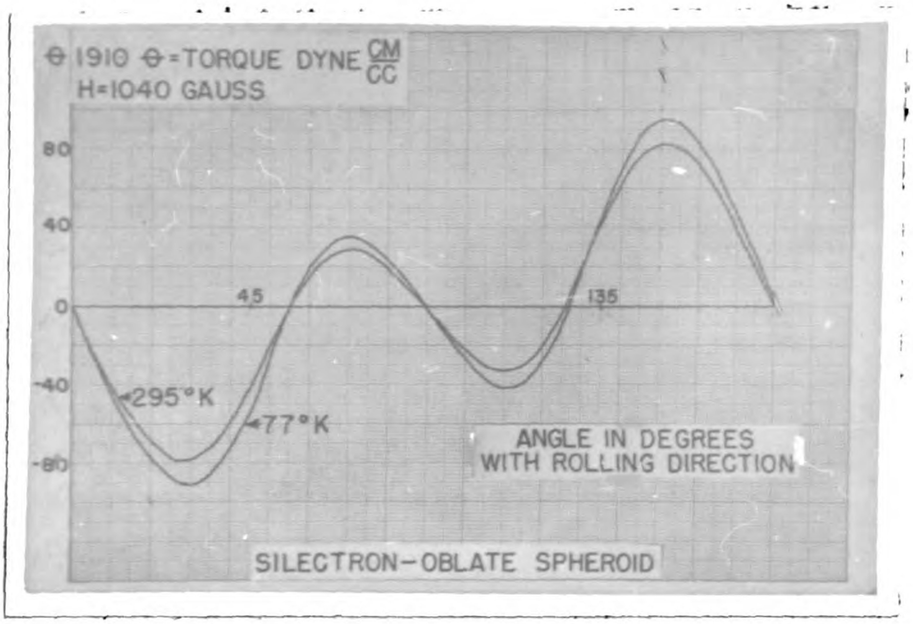

Fig. 5. Sample 1. Torque curve of a polycrystalline oblate spheroid of iron silicon, in (110) plane.

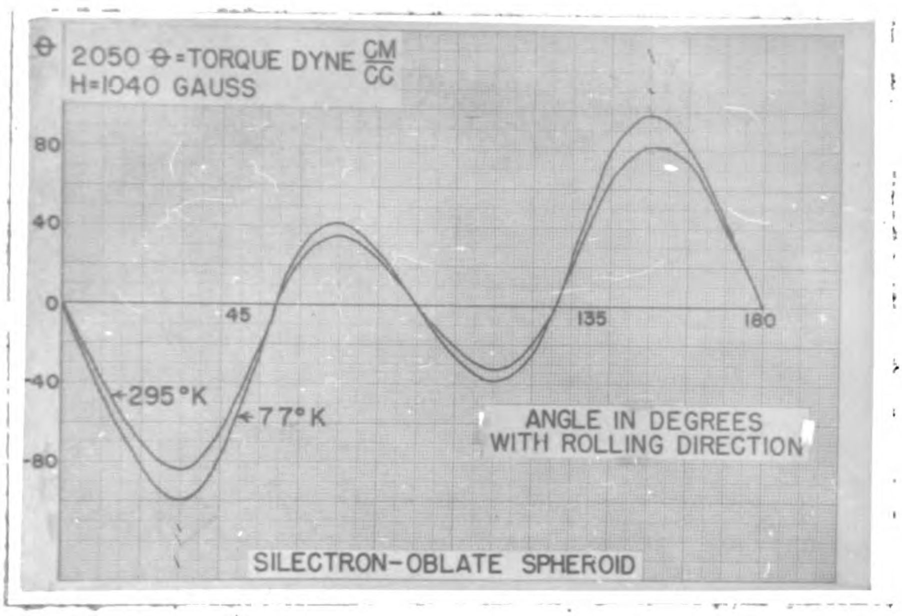

Fig. 6. Sample 2. Torque curve of a polycrystalline oblate spheroid of iron silicon, in (110) plane.

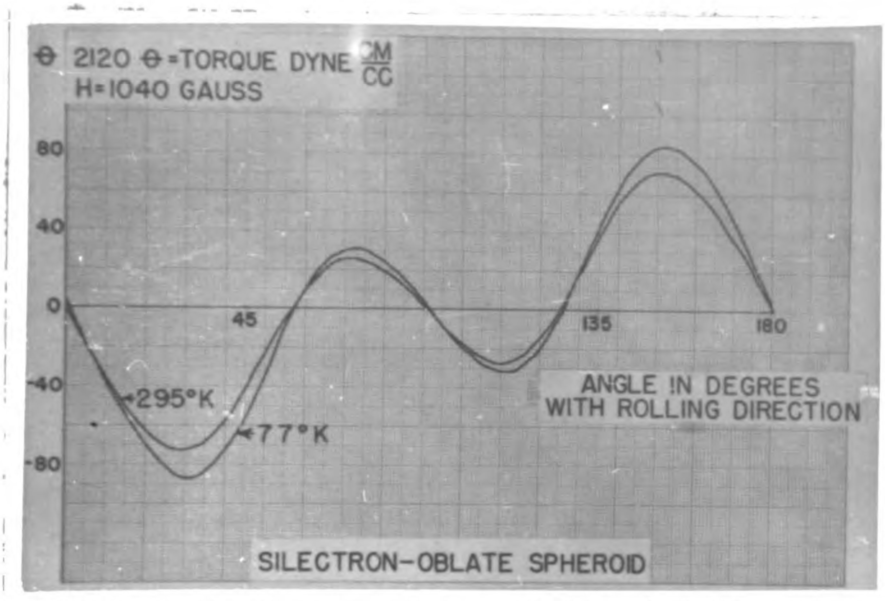

Fig. 7. Sample 3. Torque curve of a polycrystalline oblate spheroid of iron silicon, in (110) plane.

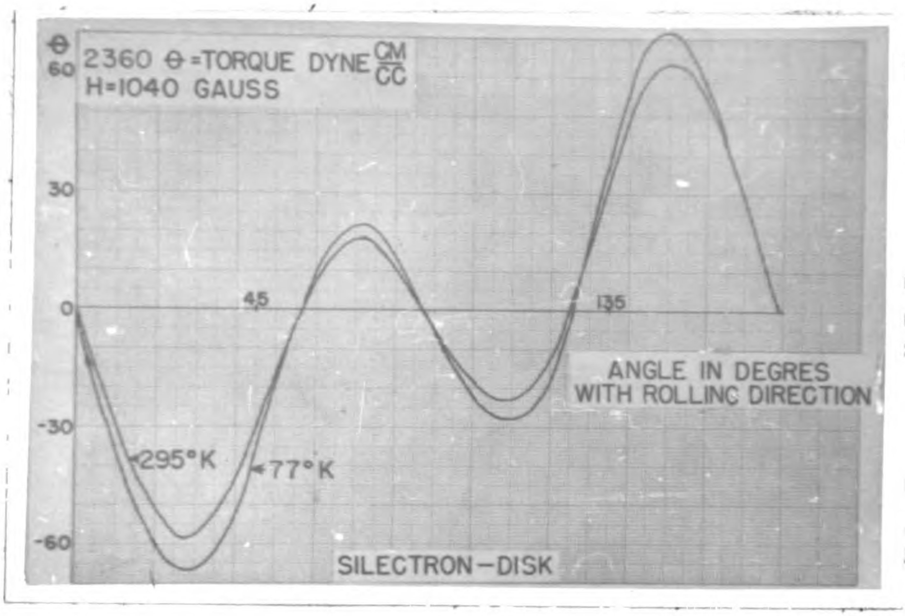

Fig. 8. Sample 4. Torque curve of a polycrystalline disk of iron silicon, in (110) plane.

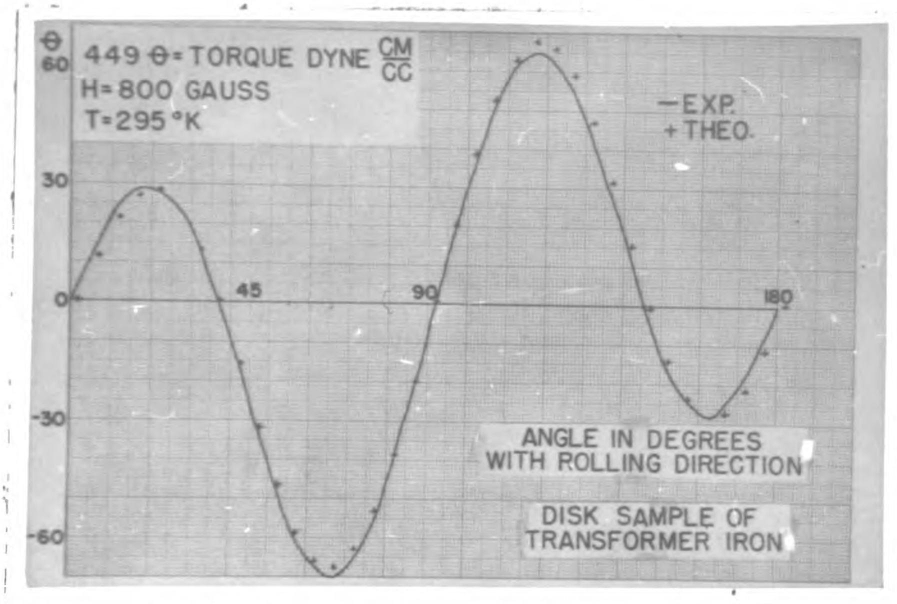

Fig.9. Sample 5. Torque curve of disk sample of transformer iron.

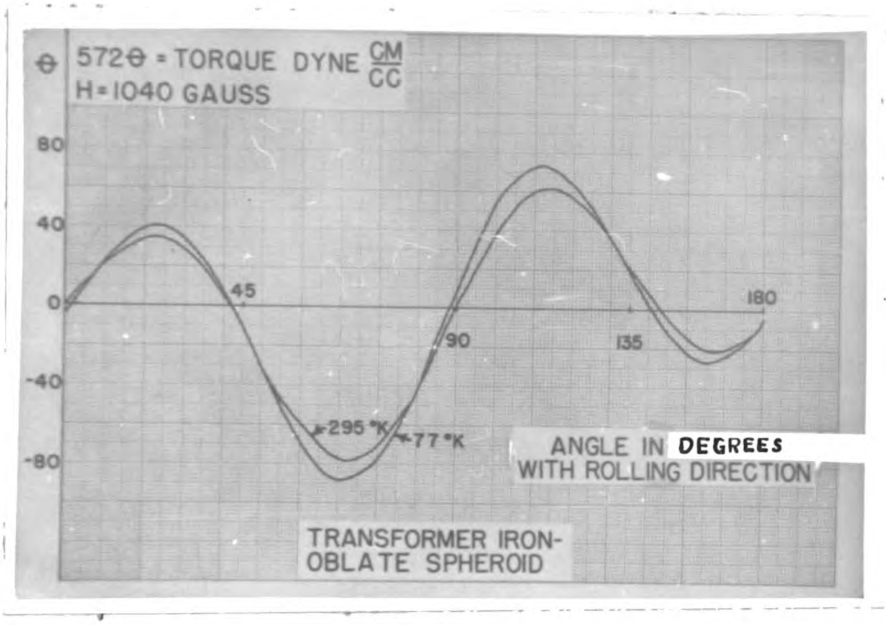

Fig. 10. Sample 6. Torque curve of oblate spheroid sample of transformer iron.

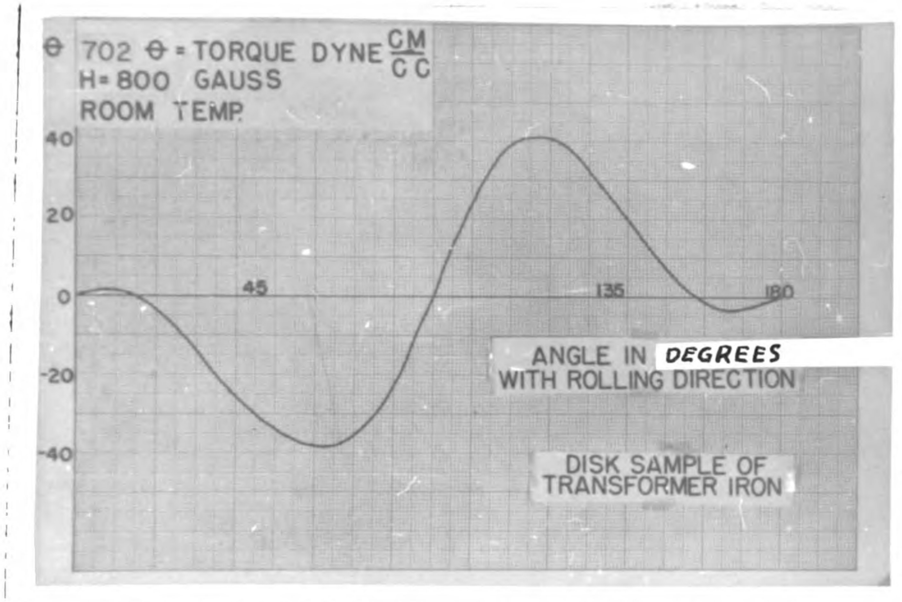

Fig. 11. Sample 7. Torque curve of disk sample of transformer iron.

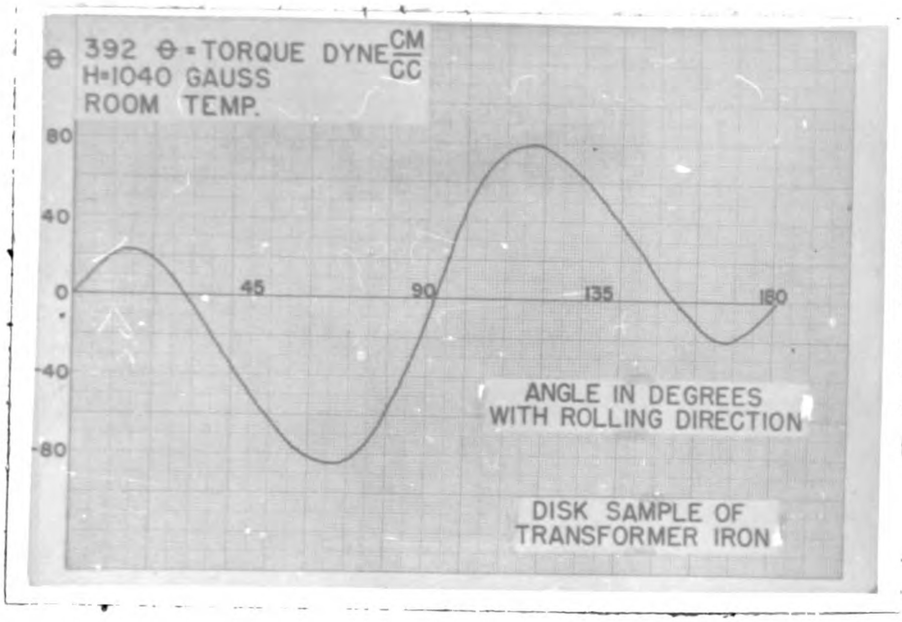

Fig. 12. Sample 8. Torque curve of disk sample of transformer iron.

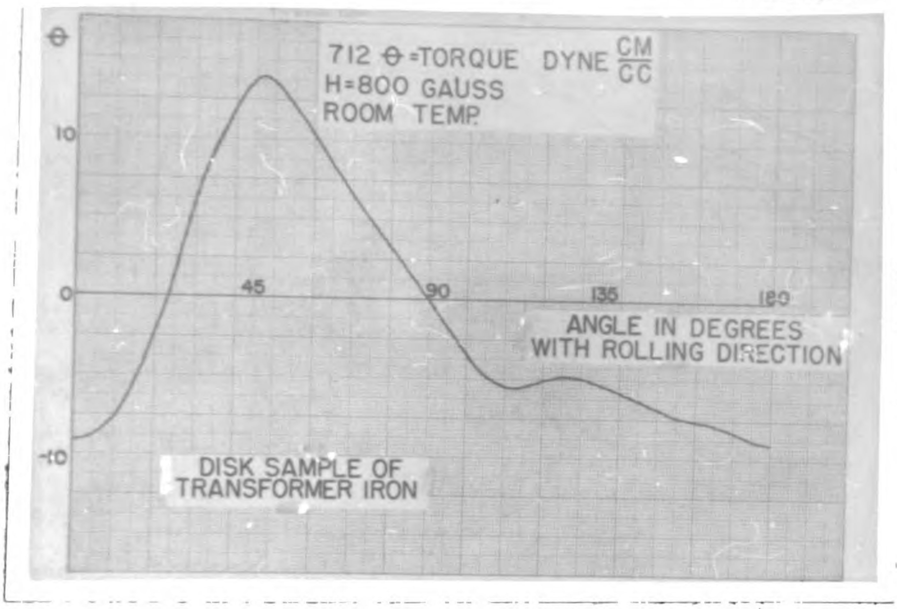

Fig. 13. Sample 9. Torque curve of disk sample of transformer iron.

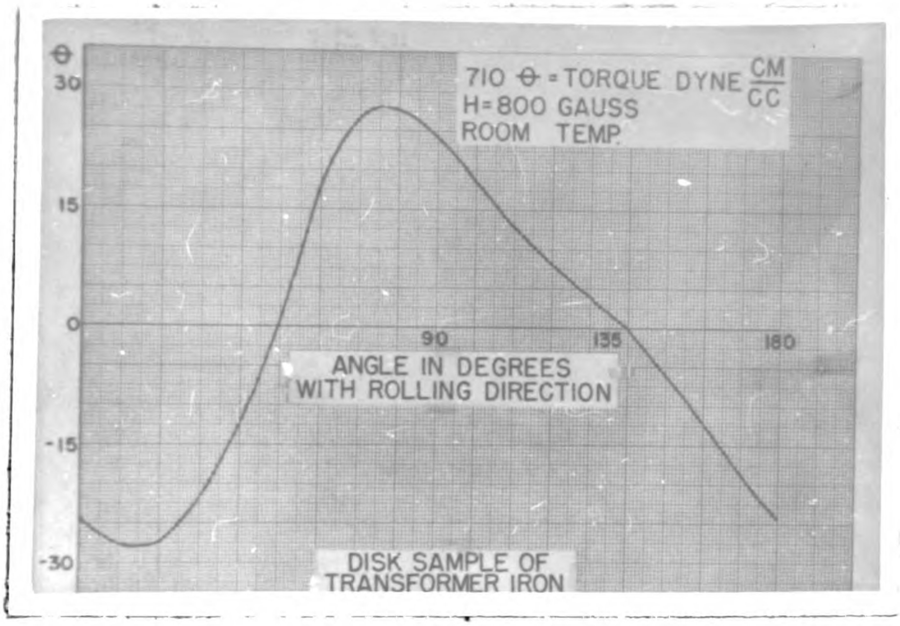

Fig. 14. Sample 10. Torque curve of disk sample of transformer iron.

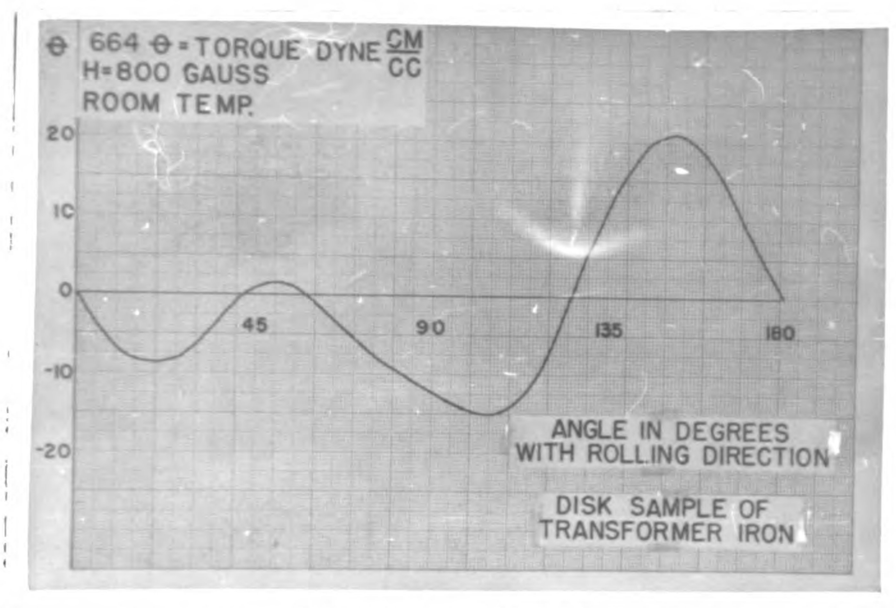

Fig. 15. Sample 11. Torque curve of disk sample of transformer iron.

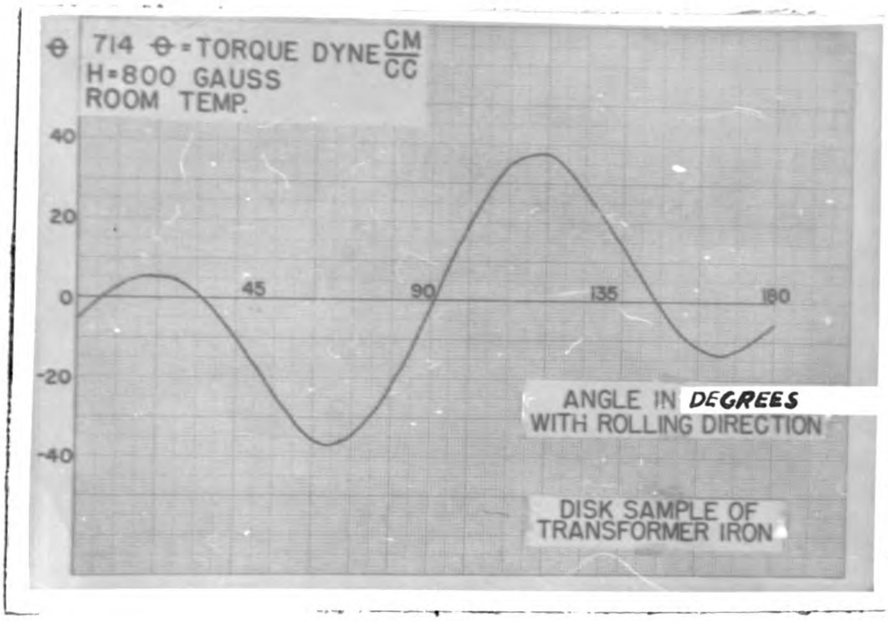

Fig. 16. Sample 12. Torque curve of disk sample of transformer iron.

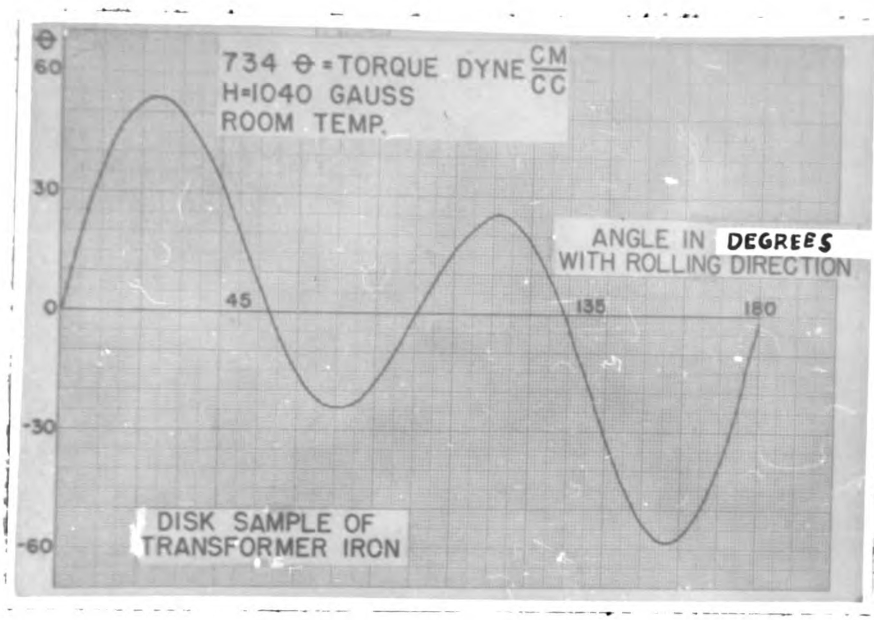

Fig. 17. Sample 13. Torque curve of disk sample of transformer iron.

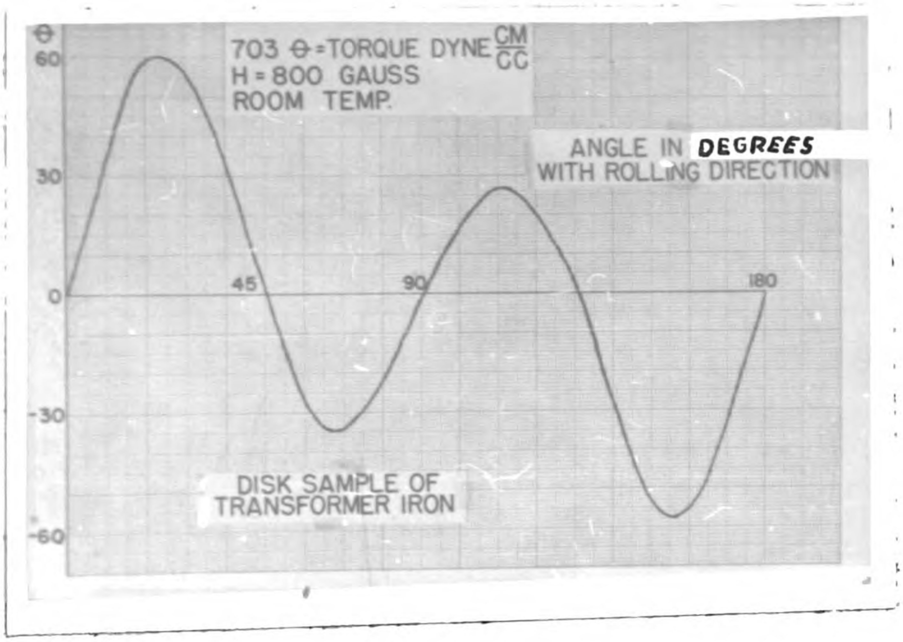

Fig. 18. Sample 14. Torque curve of disk sample of transformer iron.

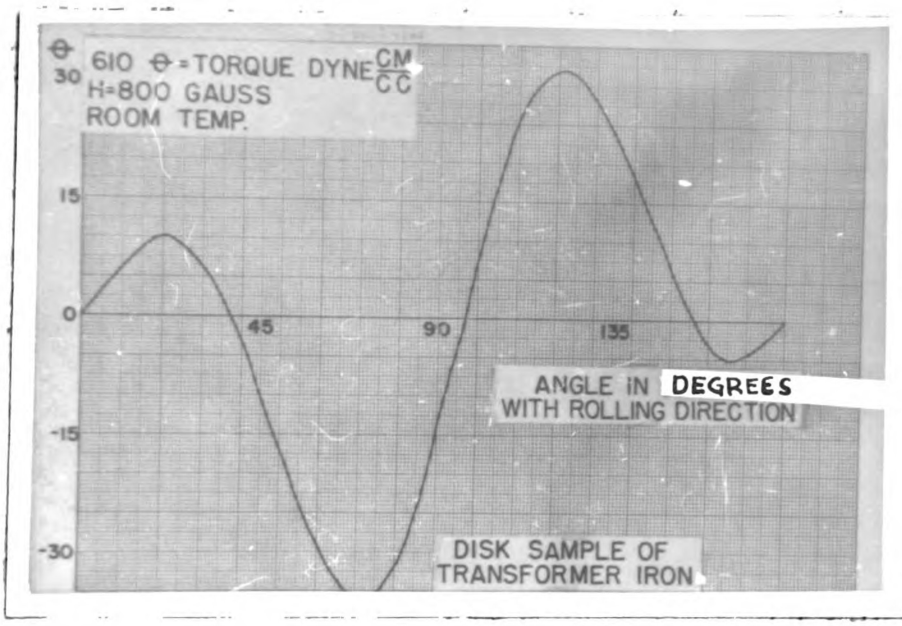

Fig. 19. Sample 15. Torque curve of disk sample of transformer iron.

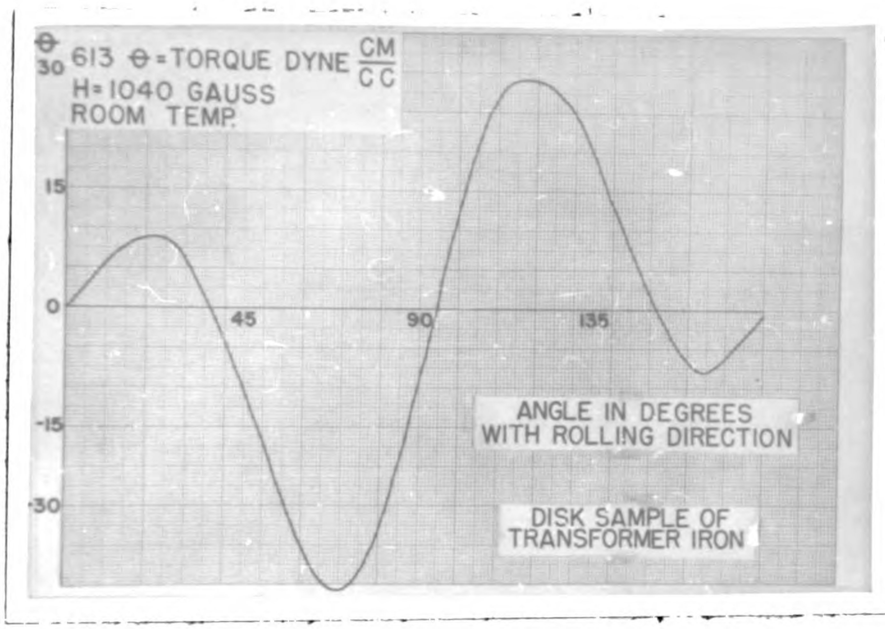

Fig. 20. Sample 16. Torque curve of disk sample of transformer iron.

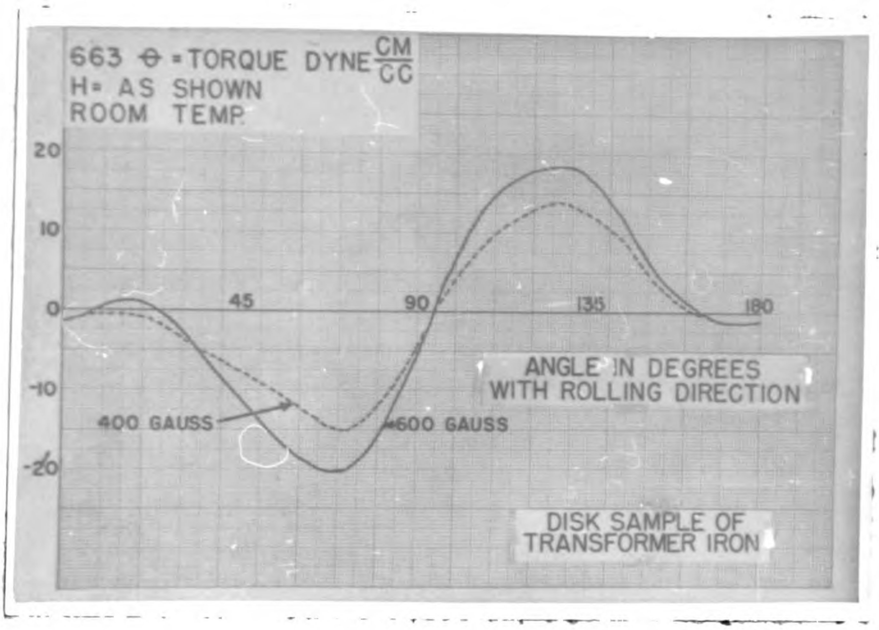

Fig. 21. Sample 17. Torque curve of disk sample of transformer iron.

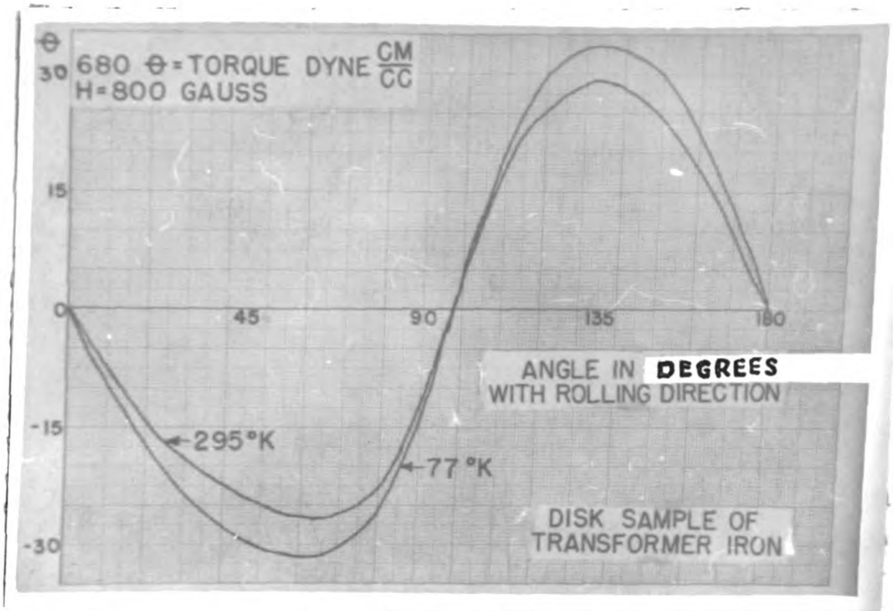

Fig. 22. Sample 18. Torque curve of disk sample of transformer iron.

V. Interpretation of Data

The torque on a sample cut from the (110) plane of a single crystal is given by Eq. (41) to be

$$T = \sin 2\phi\,(1 - 3/2 \sin^2\phi)\,(K_1 + K_2/2 \sin^2\phi).$$

Let us define

$$\begin{aligned} \beta_1 &\equiv \sin 2\phi_1 (1 - 3/2 \sin^2\phi_1) \\ \beta_2 &\equiv \sin 2\phi_2 (1 - 3/2 \sin^2\phi_2) \end{aligned} \tag{47}$$

where ϕ_1 and ϕ_2 satisfy the conditions

$$\begin{aligned} &\left.\frac{\partial T}{\partial \phi}\right|_{\phi_1} = 0 \\ &\left.\frac{\partial T}{\partial \phi}\right|_{\phi_2} = 0 \\ &\phi_2 > \phi_1 \\ &\frac{\partial T}{\partial \phi} \neq 0 \quad \text{for} \quad \phi_1 < \phi < \phi_2 \quad . \end{aligned} \tag{48}$$

Solving for ϕ_1 and ϕ_2 and inserting the results in the above one obtains

$$\begin{aligned} \beta_1 &= .56109 \\ \beta_2 &= .21002. \end{aligned}$$

We can then write

$$\begin{aligned} K_1 + \frac{\sin^2\phi_1}{2} K_2 &= \frac{T_1}{\beta_1} \\ K_1 + \frac{\sin^2\phi_2}{2} K_2 &= \frac{T_2}{\beta_2} \end{aligned} \tag{49}$$

where T_1 and T_2 are the values of the torque at ϕ_1 and ϕ_2 respectively. Solving these equations for K_1 and K_2 one obtains

$$\begin{aligned} K_1 &= 2.2449 T_1 - 1.236\, T_2 \\ K_2 &= 13.338\, T_2 - 4.9923\, T_1 \end{aligned} \tag{50}$$

Eqns. (50) are strictly applicable only for samples cut from the (110) plane of a <u>single</u> crystal. However, if one is dealing with a polycrystalline sheet in which most of the crystallites have been so oriented that their (110) planes lie in the plane of the sheet one may reasonably expect that the torque curve will be of the form given by Eq. (41). If an experimental torque curve follows this form one may use Eqns. (50) to obtain two constants K_1 and K_2. These constants will not be true anisotropy constants since they do not apply to a single crystal but rather to an aggregate of single crystals. In fact, the departure of K_1 and K_2 from the values which characterize a single crystal will serve to indicate the percent of the crystallites which are oriented in the assumed manner. If we designate the first order anisotropy constant by K_{1s} and the corresponding quantity for the polycrystalline sample by K_{1p} then the fraction of oriented crystallites is K_{1p}/K_{1s}.

The extent to which the torque measured for a given sample agrees with the torque function given in Eq. (41) can be checked by calculating K_{1p} and K_{2p} and plotting the curve obtained from these values on the experimental curve.

Figures 5,6,7 and 8 show curves of a 3% silicon steel called silectron.* It is known that the direction of rolling is the [100] axis and that the plane of the sheet coincides with the (110) plane. Subject to the assumptions mentioned above it is possible to calculate K_{1p} by use of Eq. (50).

Fig. 5 shows the torque curves obtained from a sample of silectron cut in the form of an oblate spheroid at temperatures of 295

* Silectron samples obtained from Allegheny Ludlum Steel Corporation, Brackenridge, Pa.

and 77 degrees K. The torque on the sample was zero along the rolling direction, (i.e. [100]). The torque was also zero at 55 and 90 degrees corresponding to the [111] and [110] directions. This corresponds to the fact that the energy takes either a maximum or a minimum value along the axes. (The [111] direction should be at 54.7 degrees from the [100] direction but the measured values are within the limits of accuracy of the instrument.) The anisotropy constant for this sample at 295 degrees K is 2.75×10^5 as given by Eq. (50). Bozorth[6] gives the value of $K_{1s} = 2.87 \times 10^5$ for a single crystal of 3% silicon steel therefore one would expect this sample to be 95% alligned.

The value of K_{1p} increased about 12% when the temperature of the sample was lowered to 77 degrees K. ($K_{1p} = 3.08 \times 10^5$) It was observed that lowering the temperature shifted the points of zero torque. This may be due to the instruments reaction to the low temperature or it may be caused from a change in internal structure of the sample.

Figures 6 and 7 show two more oblate spheroid samples cut from the same strip of silectron as sample 1. For sample 2

$$K_{1p} = 2.94 \times 10^5 \text{ at } 295^\circ K$$

and $$= 3.53 \times 10^5 \text{ at } 77^\circ K,$$

and for sample 3

$$K_1 = 2.74 \times 10^5 \text{ at } 295^\circ K$$

and $$= 3.27 \times 10^5 \text{ at } 77^\circ K.$$

The curves also have zero torque at 0 and 90 degrees and at approximately 54.7 degrees and can be interpreted in the same manner as sample 1.

The variation in the values of the K's for the three samples appears to be due either to large localized crystals in the stock the sample was cut from or to a change in crystal structure caused from soldering the brass shaft to the samples.

The curve of a disk sample of silectron is shown in Fig. 8. In this sample the points of zero torque are also slightly shifted with a reduction in temperature and the anisotropy constants are lower than those of the oblate spheroid samples. For the disk

$K_{1p} = 2.61 \times 10^5$ at $295°$ K

and $= 2.95 \times 10^5$ at $77°$ K.

These results agree with the results of Bitter and Tarasov[4] who who found that when oblate spheroid samples were used the curve shifted to a more symmetrical configuration and the anisotropy constants were larger than those of comparable disk samples.

Fig. 9 shows the torque curve of a disk sample cut from a transformer lamination. Examination of the torque curve leads one to believe that this sample lies in the (110) plane since the torque is zero at 0, 37 and 92 degrees, corresponding to the [110], [111] and [100] directions. The constants are

$K_{1p} = 5.23 \times 10^4$

and $K_{2p} = 1.94 \times 10^4$ at $295°$ K.

Using these values for K_1 and K_2 the theoretical curve was found to be in good agreement with experiment. Comparing K_{1p} with K_{1s} for a single crystal indicates that the sample is approximately 12% alligned with the remaining 88% distributed randomly.

The above sample was filed into an approximate oblate spheroid

and the resultant curve is shown in Fig. 10. The constants are

$$K_{1p} = 6.81 \times 10^4 \text{ at } 295^\circ K$$

and $$= 7.98 \times 10^4 \text{ at } 77^\circ K.$$

As was observed in the silectron samples K_{1p} increases for an oblate spheroid sample and also increases with decrease in temperature. A shifting of the points of zero torque with change in temperature was also observed in this sample.

For the majority of the curves obtained in this experiment the simple type of interpretation previously employed was not found appropriate. However, Akulov and Brüchatov[14] have suggested a method by which one can infer the structure of a sample even when the crystallites have several different states of orientation. They assume that an experimentally measured torque curve may be represented by the expression

$$T = A \sin 2\phi + B \sin 4\phi \qquad (51)$$

They next assume that the possible orientation of the crystallites in the sample fall into five distinct groups which are shown in Table 2.

TABLE 2.

The Group	1	2	3	4	5
Plane containing the magnetization vector and the rolling direction	(100)	(100)	(110)	(110)	Randomly orientated crystals
Rolling direction	[100]	[110]	[100]	[110]	
Relative volume	W_1	W_2	W_3	W_4	W_5

In this table the W's indicate the relative volume of the sample occupied by each group and are therefore subject to the restriction

$$\sum W_i = 1 \tag{52}$$

The total energy is given by

$$E = \sum_{i=1}^{5} W_i E_i \tag{53}$$

Where E_i represents the energy of the i^{th} group. On differentiating the last expression with respect to ϕ one finds

$$T = \sum_{i=1}^{4} W_i T_i \tag{54}$$

On inserting in the formula the appropriate formulas for the T_i one obtains an expression for the total torque

$$T = \frac{K_1 s}{2}\left\{(W_1 - W_2)\sin 4\phi + \tfrac{3}{4}(W_3 - W_4)\sin 4\phi + \tfrac{1}{2}(W_3 - W_4)\sin 2\phi\right\} \tag{55}$$

Comparing the coefficients in this expression with those of Eq.(51) one obtains

$$\begin{aligned} \frac{K_1}{2}\left[4(W_1 - W_2) + 3(W_3 + W_4)\right] &= 4B \\ \frac{K_1}{2}\left[(W_3 - W_4)\right] &= 2A \end{aligned} \tag{56}$$

This set of equations has no unique solution. However, if one assumes that any two of the groups are of negligible importance in a given sample one may solve for the relative volumes of the remaining two. Such an approximation allows four possible combinations of the groups

$$\begin{array}{lll} \text{I} & W_2 \neq 0 & W_4 \neq 0 \\ \text{II} & W_1 \neq 0 & W_4 \neq 0 \\ \text{III} & W_2 \neq 0 & W_3 \neq 0 \\ \text{IV} & W_1 \neq 0 & W_3 \neq 0 \end{array} \tag{57}$$

From these result

$$
\begin{aligned}
-4\,W_2 + 3\,W_4 &= \frac{8\,B}{K_{1s}} \quad , \qquad -W_4 = \frac{4\,A}{K_{1s}} \\
4\,W_1 + 3\,W_4 &= \frac{8\,B}{K_{1s}} \quad , \qquad -W_4 = \frac{4\,A}{K_{1s}} \\
-4\,W_2 + 3\,W_3 &= \frac{8\,B}{K_{1s}} \quad , \qquad W_3 = \frac{4\,A}{K_{1s}} \\
4\,W_1 + 3\,W_3 &= \frac{8\,B}{K_{1s}} \quad , \qquad W_3 = \frac{4\,A}{K_{1s}} \quad .
\end{aligned}
\tag{58}
$$

W_5 is determined from Eq. (52).

In order to pick the proper set of the above equations one invokes the requirement that the W's shall be all of the same sign. The possibility of negative values for the W's arises from the fact that the direction of the positive torque, and therefore the sign of A and B, is arbitrary although not independent. This requirement plus the known sign on the value of the first anisotropy constant generally yields a unique choice of the above equations. The method indicated above was applied to samples 7 and 8. For sample 7

$$A = -\,30$$

$$B = 14$$

Of the four Eqns. (58) one must choose equation III.

Thus

$$-4\,W_2 + 3\,W_3 = \frac{8\,B}{K_{1s}}$$

$$+W_3 = \frac{4\,A}{K_{1s}}$$

or

$$W_2 = \frac{3\,A - 2\,B}{K_{1s}}$$

$$W_3 = \frac{4\,A}{K_{1s}} \quad .$$

Substituting the constants A and B into the above equations one has

$W_2 = 15\%$

$W_3 = 20\%$

$W_5 = 65\%$

In a similar manner for curve 8

$A = -50$

$B = 47.4$

Inserting these values into equation III as before one obtains

$W_2 = 23\%$

$W_3 = 19\%$

$W_5 = 58\%$

Even with this type of an analysis there are serious limitations imposed. For the curves shown in Figures 13, 14, 15 and 16 an analysis of this type would be nearly, if not entirely, impossible. About the only information which can be obtained from a curve of this type is that the sample is anisotropic.

By use of higher order terms of Eq. (51) one may readily obtain an expression which is valid for curves 13 and 14. For curve 13 it was necessary to use only one extra term, $C \sin 6\phi$, to obtain an empirical curve that agrees essentially with experiment. The constants are

$A = 19.0$

$B = 45.2$

$C = 2.0$

For curve 14 even higher order terms are necessary.

In order to analyse curves 15 and 16 in the manner above it is necessary to use an expansion in both the sine and cosine.

Curve 17 was used to study the effect of using fields of strength insufficient to produce saturation. Above a field of approximately 800 gauss the sample was found to be nearly at saturation and an increase in field did not appreciably change the shape of the curve.

Curve 18 may be interpretated if one assumes that the sample had an impurity of alligned material such as cobalt giving rise to the sin 2 ϕ form of the curve. The curve also shows an increase in the anisotropy constant with decrease in temperature as was observed in previous samples.

The results of this work may be summarized as follows:

1. The torque magnetometer furnishes a simple rapid method of obtaining the torque curves of polycrystalline samples.

2. In situations in which the polycrystalline sample shows a high degree of orientation it is possible to infer such information as the effective first order anisotropy constants, the relative abundance of particular orientations and the temperature dependence of the effective first order anisotropy constant from the torque curves.

ANISOTROPY CONSTANTS OF POLYCRYSTALLINE FERROMAGNETIC SUBSTANCES

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