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THE IMPEDANCE METHOD AND ITS APPLICATION TO THE DETERMINATION OF THE ACOUSTIC IMPEDANCE OF LIQUID MEDIA

Thesis for the Degree of M. S.

MICHIGAN STATE UNIVERSITY

Robert Richard Slocum

1956

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THE IMPEDANCE METHOD AND ITS APPLICATION TO THE DETERMINATION OF THE ACOUSTIC IMPEDANCE OF LIQUID MEDIA

By

Robert Richard Slocum

AN ABSTRACT

A THESIS

Submitted to the College of Science and Arts of Michigan State University of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Department of Physics

1956

Approved E. A. Hiedemann

THE IMPEDANCE METHOD AND ITS APPLICATION TO THE DETERMINATION OF THE ACOUSTIC IMPEDANCE OF LIQUID MEDIA.

The theory of electroacoustics has provided methods whereby analysis of transducer performance may be carried out by measurements of the reactive and resistive components of the electrical impedance offered to the terminals of the transducer. From plots of the impedance components vs. frequency, quantities proportional to the acoustic impedance of the medium into which the transducer is operating can be derived. After empirically determining the appropriate constants of the transducer, the acoustic impedance of the medium may be derived.

Examples of the kind of information that may be obtained about the transducer performance characteristics are given, and the results of measurements on liquids are presented. The underlying theory is outlined, and a discussion of the general utility of the impedance method is included.

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I wish to thank Dr. E. A. Hiedemann, who suggested the development of this method, for his guidance and many suggestions during the development of the equipment, but more especially for his understanding and encouragement.

R. Slocum

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I. INTRODUCTION

The impedance method is a familiar one in the field of acoustics. The term 'motional impedance' was first introduced by A. E. Kennelly and G. W. Pierce in 1912, when they were studying the variation of impedance with frequency for a telephone receiver and discovered that the electric impedance could be influenced by the motion of the coupled mechanical system[1].

The impedance concept was introduced into mechanics and acoustics as a result of the similarity in form of the mathematical equations for acoustical and mechanical vibrations with those of oscillating electric circuits. Its introduction proved invaluable to the analysis of transducer performance and facilitated improvements in transducers by leading to the development of a comprehensive theory of electroacoustics.

The impedance concept carried over very directly into ultrasonics, where it continued to be of vital importance in transducer design and theory. Radiation resistance, motional impedance, and impedance matching are important considerations for anyone working with ultrasonic transducers.

Because the motional impedance of a transducer is affected by the medium in which it is operating as well as the manner in which the crystal is mounted, and since a change in the motional impedance in turn causes a variation in the electrical impedance offered to the generator at the input terminals of the transducer, observations of the electrical impedance lead to a knowledge of some of the important properties of the media being irradiated. This study utilizes the impedance method for the measurement of the characteristic impedance of various liquids.

The theory will be outlined in Section II, and its specific application to this problem and the apparatus, and some of the problems involved, including its limitations, will be considered subsequently.

II. THEORY

As has been pointed out, the study of equivalent circuits and the analogy between electrical and mechanical oscillatory phenomena has led to the development of the theory of electro-acoustics. By means of this theory it is possible to represent electro-mechanical transducers by a single form of equivalent circuit. Although such a representation by a single equivalent network allows maximum generality for application of theory to different cases, it has become common practice to use different types of circuits for different transducers, on the assumption that some networks are physically more meaningful than others. Here the discussion of transducers will follow the more general method, in which the transducer is considered as a four-terminal network.

Following the theory given by Hunt[2], we consider a transducer as a four-terminal network, with two terminals representing the electrical input coupled through a "black box" element, which represents the general conversion of electrical to mechanical energy, to a mechanical circuit of a single degree of freedom (figure 1).

On the electrical side, a current I flows through an electrical impedance Z_e, with a voltage E across the input terminals; on the mechanical side, there is a velocity v, a mechanical impedance Z_m, and a force F across the output terminals.

Two equations are needed to describe the behavior of this system: one in terms of the electrical quantities, including the reactions caused by the motion of the mechanical system; the other in terms of the mechanical variables, as well as any mechanical reactions due to the currents and voltages in the electrical network. The symbols T_{em} and T_{me} represent "transduction coefficients", denoting electromechanical coupling. Assuming a steady state, with time appearing in the form $e^{i\omega t}$, the equations are

$$E = Z_e \cdot I + T_{em} \cdot v$$
$$F = T_{me} \cdot I + Z_m \cdot v \qquad (1)$$

Some important properties of the electromechanical interaction can be displayed by observing the driving-point impedance appearing at either pair of terminals. The electric driving-point impedance at a terminal pair is defined as the complex ratio of voltage across the terminals to the current in the terminal pair, when all other electromotive forces and current sources are suppressed. Putting F=0 in Eqs.(1) and solving for I in terms of E, we get

$$Z_{ee} = \left[\frac{E}{I}\right]_{F=0} = Z_e + \frac{(T_{em} T_{me})}{Z_m} \qquad (2)$$

The mechanical driving-point impedance is similarly found to be

$$Z_{mm} = \left[\frac{F}{V}\right]_{E=0} = Z_m + \frac{(-T_{em}T_{me})}{Z_e} \qquad (3)$$

Equations (2) and (3) show the usual electrical and mechanical impedance with an additive term containing the transduction coefficients. These extra terms represent a modification of the impedance caused by the presence of a bilateral electromechanical coupling. The additive term in Eq. (2) indicates the modification of the electrical impedance by the motion in the mechanical system; this term, $\left(-\frac{T_{me}T_{em}}{Z_m}\right) = Z_{mot}$, has been given the name of "motional impedance".

Then Eq.(2) may be rewritten to include this definition in the form $Z_{ee} = Z_e + Z_{mot}$, where Z_e is the "clamped" or "blocked" impedance, measured when the mechanical system has in some manner been prevented from vibrating. Since the motional impedance, Z_{mot}, is directly proportional to the negative product of the transduction coefficients, its nature will depend on the size of these coefficients and whether they are real or complex. However, the general behavior of the motional impedance, especially its variation with frequency, may be studied by considering the behavior of the mechanical admittance y_m, since $Z_{mot} = (-T_{em}T_{me})/y_m$, where $y_m = 1/Z_m$.

If the mechanical impedance Z_m is written in the generalized form $Z_m = r_m + j\omega l_m + \frac{1}{j w c m} = r_m + jx_m$, where the quantities r_m, l_m, and c_m refer to the mechanical resistance, mass, and compliance, respectively, and x_m denotes the mechanical reactance, we see that the variation of this impedance with frequency may be exhibited by expressing Z_m as a vector from the origin in the complex x_m-r_m plane. In general both the length and phase of this vector will vary as the frequency changes, and the tip of the vector will trace out a curve which is called an "impedance locus". If r_m is not a function of frequency the impedance locus will be a vertical straight line at a distance r_m from the origin. The phase angle is zero at the angular frequency of mechanical resonance, $\omega_0^2 = 1/l_m c_m$.

If a similar plot is made of the mechanical admittance, y_m, the admittance locus is a circle, which is just the geometrical inversion of the straight-line impedance locus. This admittance circle has a diameter of length $1/r_m$. Since Z_m is real at mechanical resonance, y_m is also real when $\omega_0^2 = 1/l_m c_m$ (figure 2).

If we call the frequencies for phase angles of $\pi/2$ and $-\pi/2$ radians ω' and ω'' (the quadrantal frequencies) respectively, we may write $r_m = -\omega' l_m + \frac{1}{\omega' c_m} = -\omega' l_m \left[\frac{1-\omega_0^2}{\omega'^2}\right]$

and $$r_m = +\omega'' l_m - \frac{1}{\omega'' c_m} = \omega'' l_m \left[\frac{1-\omega_0^2}{\omega''^2}\right]$$

From these relations it can be shown that $\omega' \omega'' = \omega_0^2$ Further, the damping of the mechanical system, $r_m/2\, l_m$, is found to be $\frac{r_m}{2 l_m} = \frac{\omega'' - \omega'}{2}$, and the mechanical quality factor Q_m,

$$\frac{\omega_0 l_m}{r_m} \equiv Q_m = \frac{\omega_0}{\omega'' - \omega'}$$

Thus, measurement of the resonance frequencies and both quadrantal frequencies gives enough information to determine uniquely the relative values of r_m, l_m, and c_m. Only relative values are so obtained because measurement at the electrical terminals gives the admittance locus multiplied by the scaling factor $(-T_{em}T_{me})$. This product of the transduction coefficients may be considered as a vector operator which both alters the scale and rotates the diameter of the admittance circle about the origin by an angle denoted by 2β. It also serves as a units operator, converting the locus of mechanical admittance into a locus of electrical impedance.

The elect4ical impedance vector, Z_e, may also be represented on the complex plane. Since $Z_{ee} = Z_e + Z_{mot}$, the plot of the total driving-point impedance, Z_{ee}, will be a combination of the motional impedance circle with the tip of the blocked impedance vector, Z_e, as its moving origin, resulting in an impedance loop of the form shown in figure 3.

Such loops occur in the vicinity of the resonance frequency, and smaller loops appear at the harmonics, with the possibility of parasitic modes giving rise to loops also.

When the mechanical system is blocked the electrical impedance, Z_e, and its frequency-variation can be measured directly. After making similar measurements when the crystal is allowed to vibrate freely, the motional impedance may then be found by subtracting Z_e from Z_{ee} at each frequency. Since it is usually impossible to block the motion of the crystal completely, it is necessary to infer the shape of the blocked-impedance locus by smoothly joining the measured values of Z_e made at frequencies far above and far below resonance, being careful to avoid the region of a harmonic mode. This interpolation may be done most easily if the values of R_e and X_e are plotted in rectangular coordinates with frequency as the abscissa. Then direct subtraction gives the coordinates of r_m and x_m for each point of the motional impedance locus:

$$r_m = R_{ee} - R_e$$

$$x_m = x_{ee} - x_e$$

In this way the motional-impedance circle may be drawn and its diameter and resonance and quadrantal frequencies found directly. In addition to being able to calculate the values of r_m, l_m, and c_m from these diagrams, it is also possible to find such quantities as the mechanical quality factor, Q_m, the mechanical power delivered to an external load, the effective efficiency, η, of conversion of electrical power into useful mechanical power, and the maximum efficiency of power conversion.

Now let us consider a specific equivalent circuit for a piezoelectric crystal. An equivalent circuit is a combination of R, C, and L elements, which, when substituted for the element in an electrical circuit, will exhibit the same electrical properties as the element itself. The equivalent circuit which is now universally accepted for a piezoelectric transducer[3] is the one shown in figure 4, where the series arm R, C, L is related to the mechanical elements resistance, compliance, and mass, respectively, and the parallel condenser, C_0, expresses the property of the crystal as a capacitance. This equivalent circuit is treated in detail by Cady[3], Mason[4], Vigoureux[5], et.al.

At mechanical resonance, L and C have equal magnitudes but opposite phase, thus cancelling out, and the equivalent circuit reduces to a simple R-C network. It is important to note that the circuit elements in the equivalent network are considered to be strictly constant over a narrow range in the vicinity of a resonance, and that similar circuits apply near each of the harmonic modes, R, L, and C, taking different values for each harmonic.

If a coil whose reactance at the resonant frequency is of the same magnitude but of opposite sign as the reactance, $1/\omega C_0$, of the element C_0, is inserted in parallel with the crystal, then the resulting network will show only the pure resistance. A measurement of this resistance offered by the transducer and coil at the electrical terminals at the frequency of mechanical resonance will permit the evaluation of R of the equivalent network. However, this resistance, which corresponds to the mechanical losses of the crystal, depends on the method of mounting and the internal losses in the transducer as well as on the loading medium. If the internal losses and those due to mounting were negligible compared to the radiation losses into the load, measurement of the resistance at resonance would give directly the characteristic impedance of the liquid being irradiated. An arrangement where these approximations are quite good can be practically realized for a free crystal operating in a highly absorbing liquid. A more accurate way of eliminating the effect of the internal and mounting losses is to make a resistance measurement at resonance when the crystal is operating into air, and another measurement when it is operating in the liquid under study. This corresponds to having two equations from which the mounting and internal losses may be eliminated.

The common liquids behave like pure resistance, but some of the more viscous ones exhibit a reactance component in addition to the resistance. In such a case one may evaluate the reactance component by measuring the change in resonant frequency of the crystal from the unloaded to the loaded condition. Actually it appears in practice that one may consider the unloaded frequency as that measured for the crystal in air, provided the liquids under study are relatively viscous.

Thus it is clear that the characteristic impedance, $Z_0 = R_m + jX_m$, of a liquid may be found by measuring the change of electrical resistance, $\Delta R_e = K_1 R_m$, at resonance, and the change of resonant frequency, $\Delta f = K_2 X_m$, between the unloaded and the loaded conditions[6]. The constants K_1 and K_2 may be evaluated theoretically from the geometry of the crystal and the type of mounting[4,6]. In practice these constants are determined experimentally by carefully measuring ΔR_e and Δf for a liquid whose characteristic impedance is accurately known.

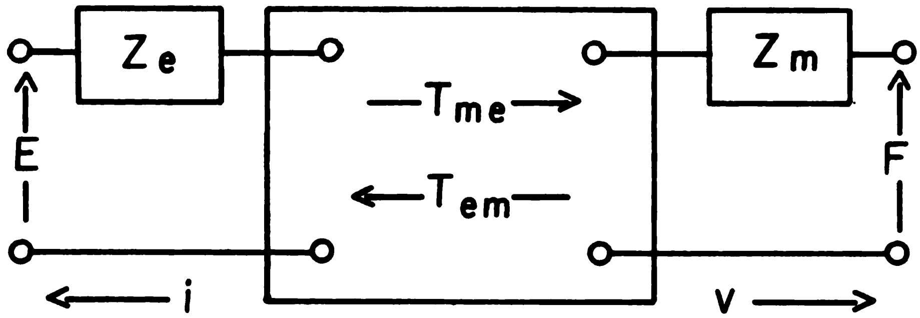

Figure 1. General four-terminal network, valid for all transducer types.

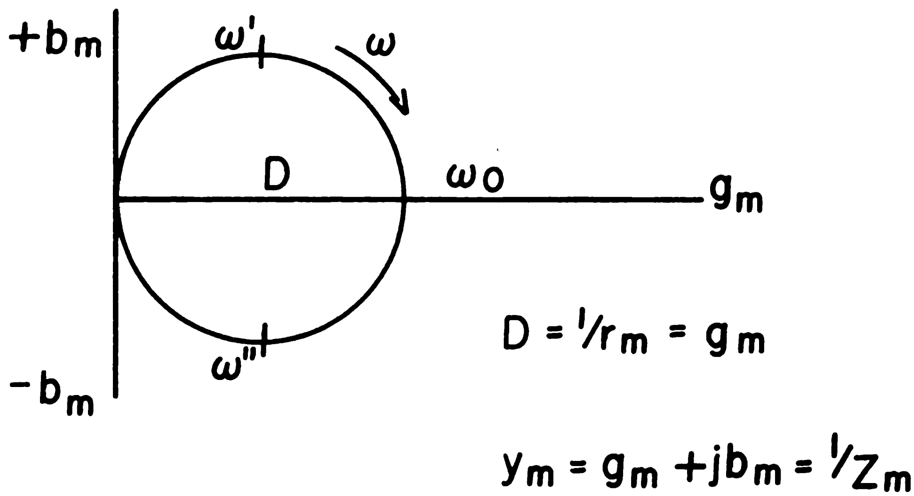

Figure 2. Motional admittance circle for r_m independent of frequency (vertical straight-line impedance locus).

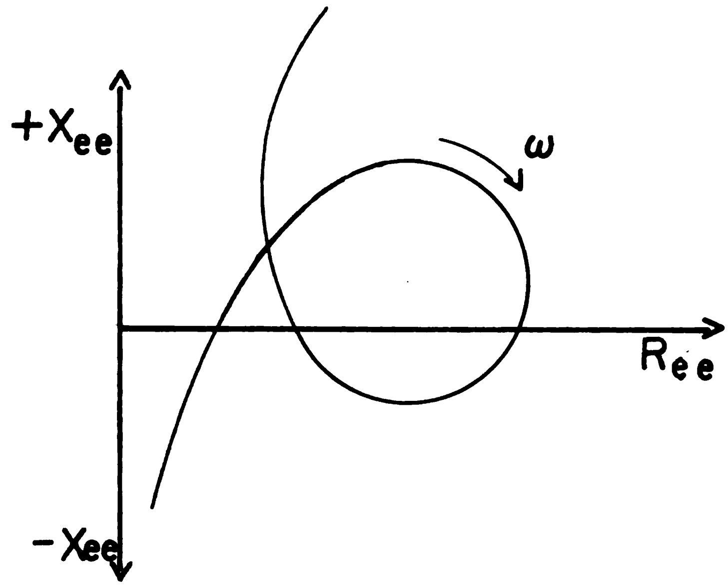

Figure 3. Typical electrical impedance loop (generally non-circular)for frequencies very near resonance.

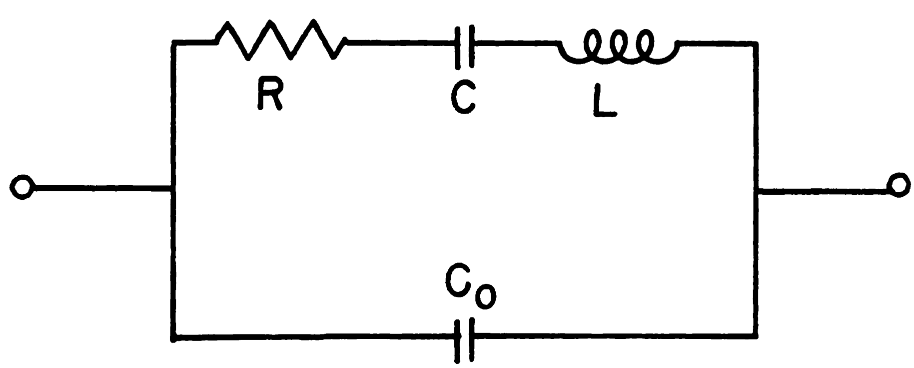

Figure 4. Universally accepted equivalent circuit for piezoelectric transducer.

III. OBJECT OF EXPERIMENT

Because of the rather general utility of the impedance method it was considered desirable to build up sufficient apparatus so that the technique would be available whenever it might be needed for studies in the ultrasonics laboratory, in conjunction with both optical and pulse methods.

As an example of the knowledge that may be gained by this method, the decision was made to undertake an investigation of several liquids to determine their acoustic impedances, from which the sound velocities may be derived, if desired. This was also done with an eye toward determining whether there might be a sufficient time-variation in the acoustic impedance of egg albumin to make feasible the development of a method for checking the freshness of eggs by ultrasonic radiation.

In order to illustrate the usefulness of this method a variety of applications are displayed. Toward this end two impedance loops (a plot of reactance vs. resistance with frequency as a parameter) have been included. An impedance loop is useful because it renders immediately visible many important performance characteristics of transducers.

Another interesting practical problem is the effect of the method of mounting on the transducer performance characteristics, for which the impedance method can give valuable information. Due to time limitations only one crystal was considered from this particular point of view.

IV. EXPERIMENTAL SET-UP AND APPARATUS

For all investigations in the vicinity of one megacycle a Brush Hypersonic Generator Model BU-214 was used as a source of alternating current. This oscillator has an output frequency range from 300 to 1,200 kilocycles. This frequency range is covered in five stages, involving altering the coil in the frequency-control tank-circuit. Within the five ranges the frequency is continuously variable by means of an auxillary coil located inside the main coil of the control tank circuit, causing small variations of inductance of the L-C tank circuit.

The measuring instrument per se was a General Radio Company R-F Bridge, Type 1606-A, in conjunction with a Hallicrafters General Communications Radio Receiver, Model SX-62 A, which functioned as a null detector for determination of bridge balance conditions. The type 1606-A Radio Frequency Bridge is a null instrument for use in measuring impedance at frequencies from 400 kc to 60 Mc. The bridge is used with a series substitution method for measuring an unknown impedance, Z_x, in terms of its series resistance component, R_x, and series reactance component, X_x. The resistance is read from a variable-condenser dial directly calibrated in reactance in ohms at a frequency of 1 Mc.

The resistance dial reading is independent of frequency, and reads from 0 to 1000 ohms; the reactance dial from 0 to 5000 ohms at 1 Mc.

The important characteristics required of the measuring apparatus are the following:

(1) The oscillator must be as stable as possible in frequency, the output frequency and voltage being relatively independent of the load. It would be desirable to have one oscillator to cover the entire frequency range of interest to the experimenter, but it may be more convenient to use several oscillators of different frequency ranges, as was done in this case. The generator must also be adequately shielded so as to avoid stray coupling between it and the detector.

(2) The bridge must be a radio frequency type, capable of a high degree of accuracy over the entire frequency range of interest, and well shielded against stray pick-up from either the generator or detector. It is strongly advisable that co-axial connectors be used for connections to both the source and the receiver. Obviously the reactance-resistance ranges must be commensurate with the corresponding impedance components being measured.

(3) The detector must be of high sensitivity in order to enable very precise location of the balance point, since this is the quantity of major concern in this method. It must be well shielded, for the same

reasons mentioned in connection with the signal generator and the R-F bridge. It must possess a local oscillator capable of putting out a strong, clear signal, and should have an automatic volume-control switch.

Frequently measurements were made with a U.S. Army Signal Corps Frequency Meter BC-221 J, with a range from 125 to 20,000 kc. The prime requisite of the frequency measuring device in this application is accuracy. The meter used here was accurate to 1000 cycles per second. (see figure 1 for diagram of apparatus)

The procedure for making measurements was the following:

> The signal generator was set somewhere in the vicinity of the resonant frequency of the crystal, which was operating into air. This frequency was measured on the frequency meter, and then recorded. Then the bridge was balanced with the crystal short-circuited. After connecting the crystal into the unknown arm of the bridge, a balance was again obtained using the reactance and resistance dials. The reactance and resistance were then recorded. This procedure was then repeated in its entirety for several different frequencies, until it was certain that the resonant peak in a plot of resistance vs. frequency had been located.

Next the crystal was immersed in one of the liquids under consideration and the procedure outlined above was repeated, step by step. An example of these plots is shown for a general case (figure 2.)

Measurements were made for several different crystals in each of the liquids investigated. Also some measurements at higher frequencies were made, using another oscillator that was available. In general reactance measurements were not made, since only the resistance and the resonant frequency were required for obtaining the acoustic impedance of the liquids. However, in order to obtain impedance loops, reactance data were measured in a few cases.

As a rough indication of the effect of the crystal mounting, measurements were made on an unmounted barium titanite disc, and then again after the same crystal had been glued onto a phenolic resin composition board backing on one side.

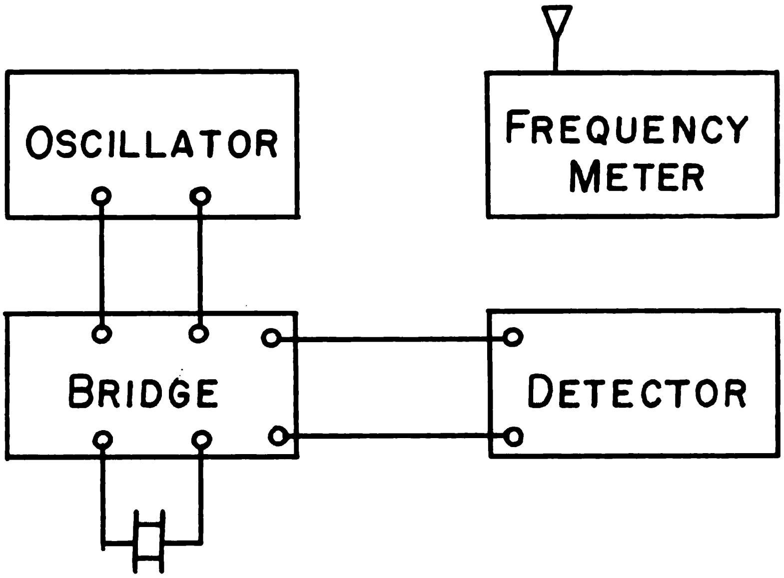

Figure 1. Block diagram of experimental apparatus.

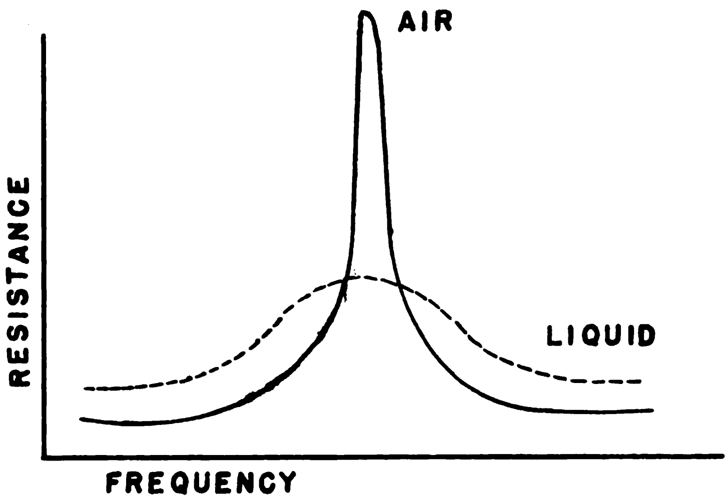

Figure 2. Typical curves of resistance vs. frequency in air, and with liquid load.

V. DATA

Figure (1) is a plot of measured electrical resistance vs. frequency for a freely vibrating barium titanate crystal with a resonant frequency of 1.0911 megacycles per second. The broken curve is the one which applies to the unmounted crystal. The solid curve is the same plot for the crystal when mounted with air-backing in a brass crystal-holder of the type commonly used for ultrasonic investigations of liquids.

Figure (2) is a plot of the measured electrical resistance vs. the measured electrical reactance for the unmounted one megacycle barium titanate crystal, exhibiting the typical impedance loop form.

Figure (3) is a plot of the motional resistance vs. the motional reactance. The plotted values are the difference between the measured electrical impedance components and the blocked impedance components, the latter being obtained by extrapolation, as discussed in section II.

The diameters of the motional impedance circles are inversely proportional to the losses of the crystal. Since the loading medium was air for both cases, the decrease in diameters indicates clearly the losses caused by the mounting. Since the quality factor, Q, is related to the half-width of the resonant peaks, it is obvious that the mounting considerably lowers the Q. By measuring the impedance of the crystal unmounted, then again when in the crystal holder, and then when operating into a liquid of known impedance, it would be possible to determine the losses of the holder.

Figure (4) is a graph of the measured electrical series resistance at the anti-resonant frequency vs. the known values of the acoustic impedance of the four liquids in which experiments were made. The crystal used for these data was a two-inch square barium titanate of nominal frequency of one megacycle per second.

Figure (5) is a plot similar to Figure (4) for measurements performed with a one megacycle per second quartz crystal.

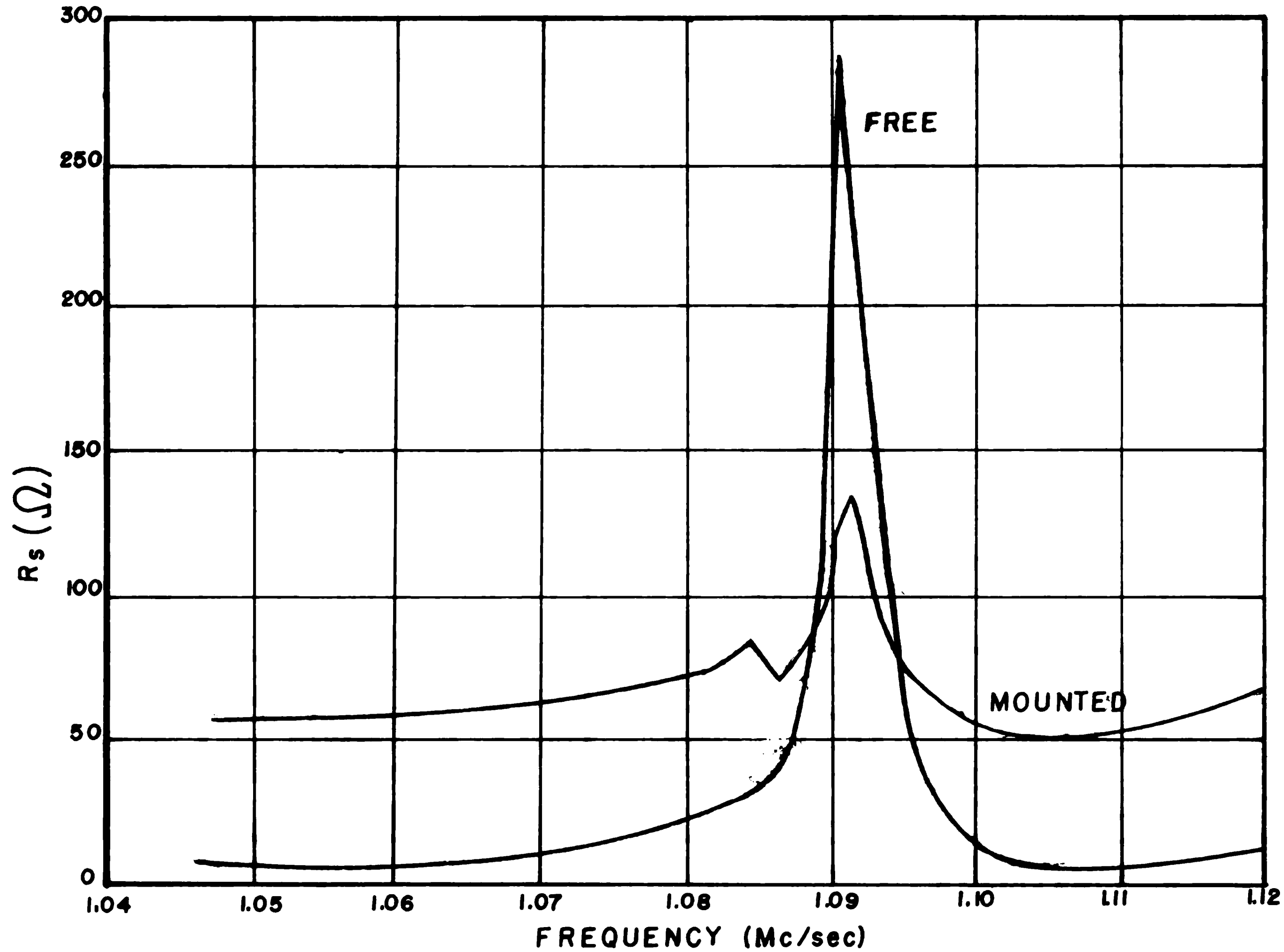

Figure 1. Plot of R_s vs. frequency for unmounted barium titanate disc, and also for air-backed mounting.

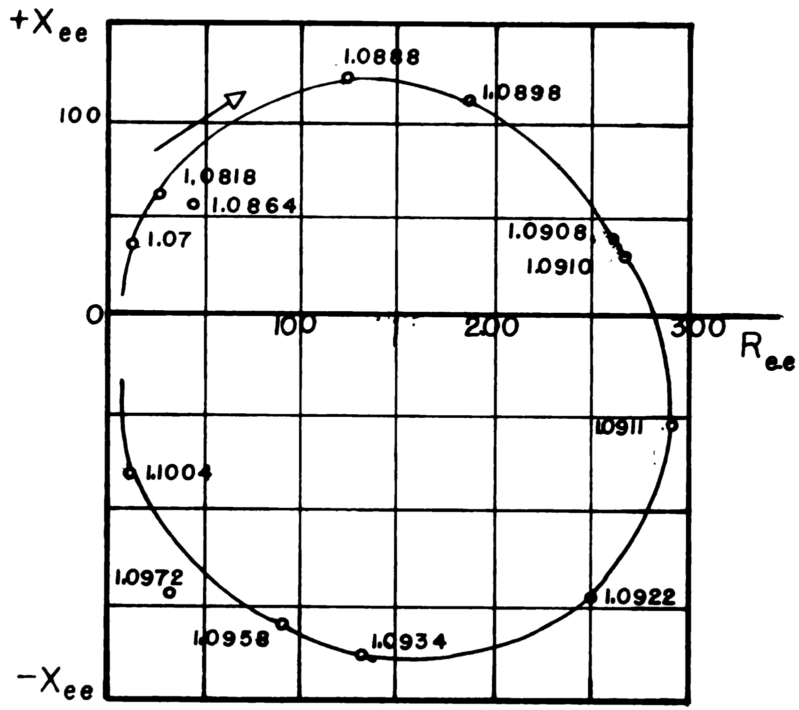

Figure 2. Electrical resistance vs. electrical reactance for barium titanate in air.

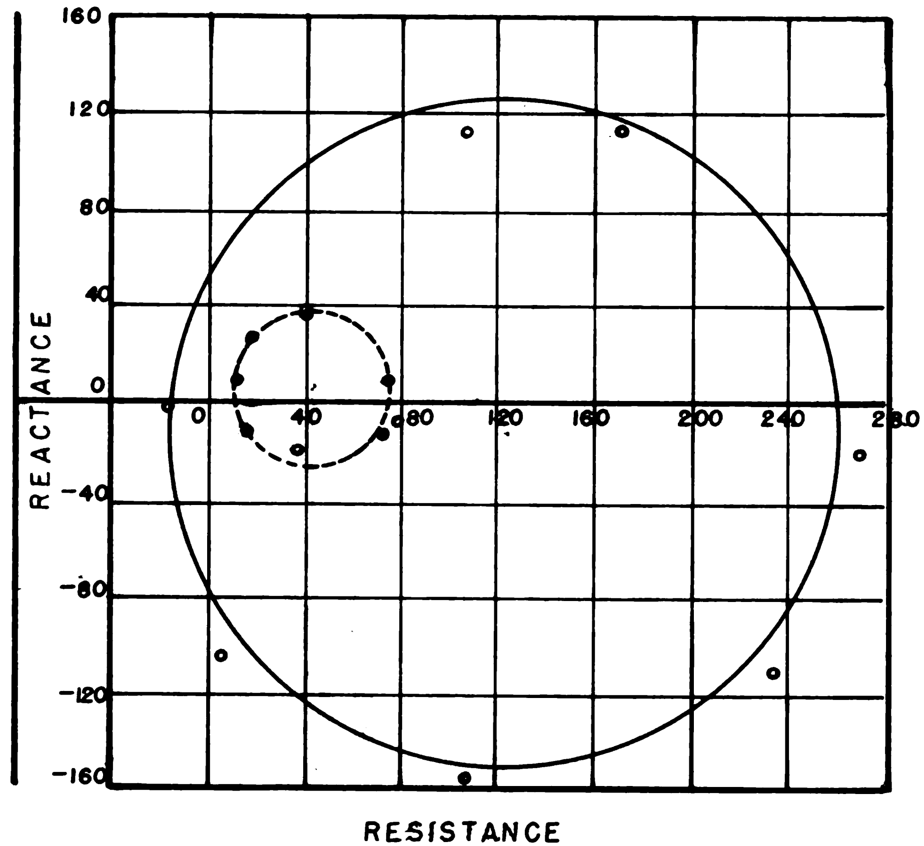

Figure 3. Motional impedance circles derived from the loops of Figure 2.

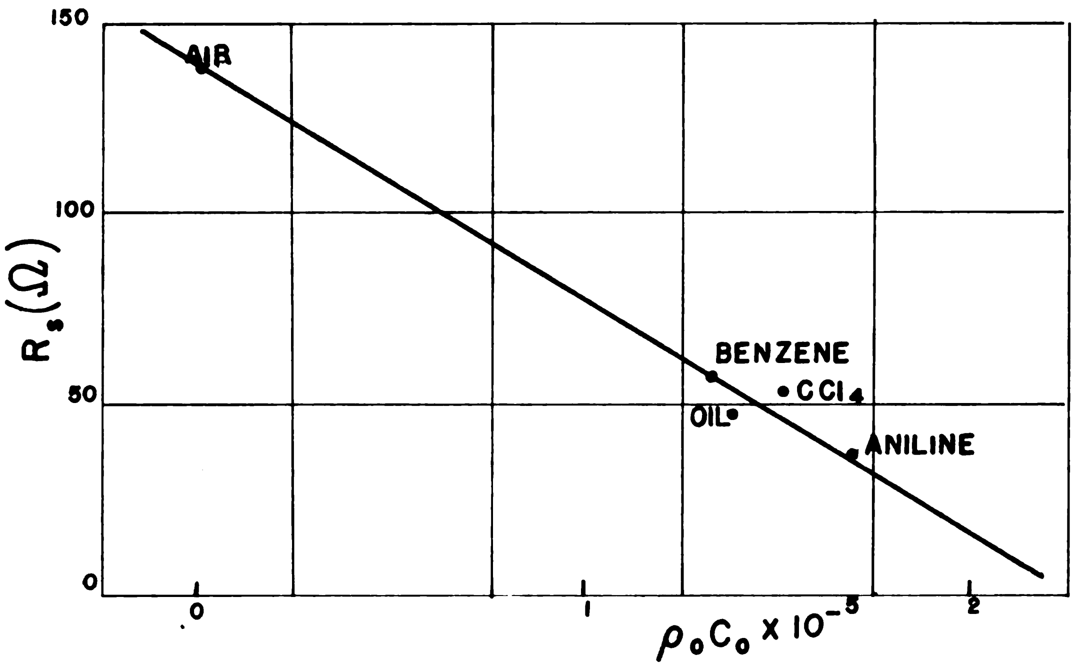

Figure 4. Comparison of measured values of the acoustic impedance of four liquids with their known values, taken with barium titanate crystal.

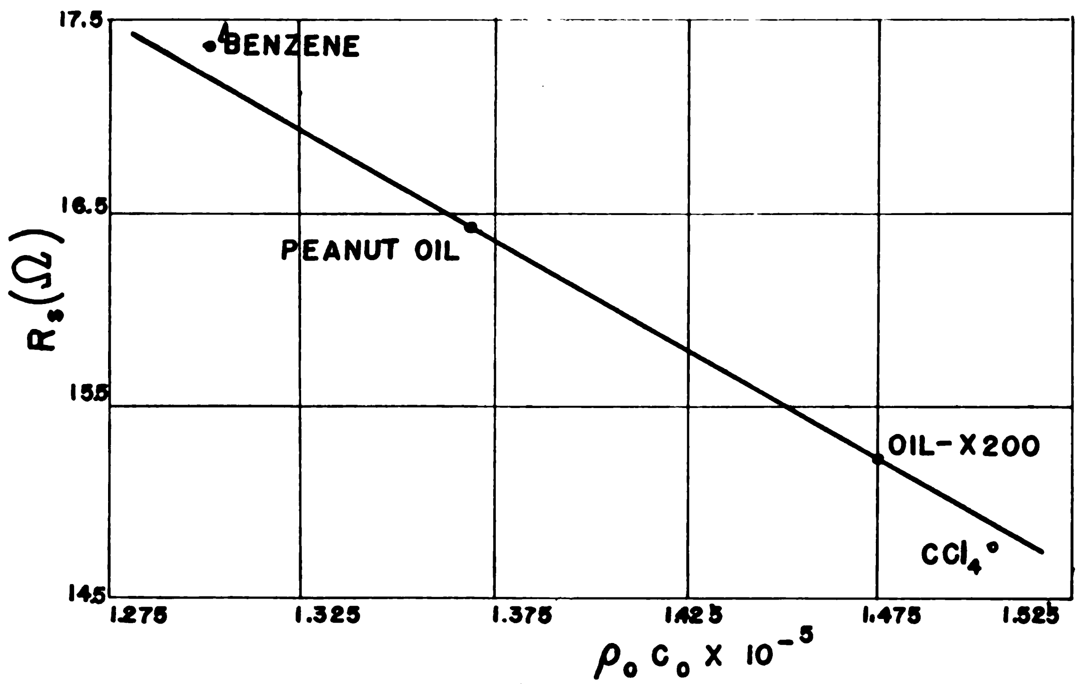

Figure 5. Same as Figure 4, using quartz

VI. CONCLUSIONS AND SUGGESTIONS

The data shown in Section V, and the experience gained in assembling and using the apparatus, clearly point out several important conclusions. Firstly, the percentage errors in the measurement of the acoustic impedance of liquids at a frequency of one megacycle with the present apparatus are as high as ten percent, rendering it impossible to determine the time-variation of egg albumin to better than ten percent. This clearly seems insufficiently accurate. Secondly, the performance of the measurements with the present equipment is very slow and troublesome.

It appears, however, that it will be possible in a comparatively short time, to increase the accuracy to within one or two percent. Cady, in a paper for the Office of Naval Research[9], points out that a similar method, but operating at the much greater frequency of fifteen megacycles per second, is capable of accuracies as high as one percent. Practically no changes are necessary for extension of the equipment used for this study to the fifteen megacycle region.

For studies in highly viscous liquids which show a marked difference between the resistive and reactive components of the acoustic impedance it will be necessary to construct a more sensitive frequency measuring device than is now available in the ultrasonics laboratory, because the reactive component is proportional to the change in resonant frequency from air to liquid, and this frequency change is extremely small (on the order of a few hundred cycles per second).

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