THE THEORY OF THE USE OF THE SUPER-REGENERATIVE RECEIVER FOR THE DETECTION OF THE NUCLEAR MAGNETIC RESONANCE

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Salah Izzat Tahsin

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by

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Salah I. Tahsin

CONTENTS

I. INTRODUCTION

In 1924, W. Pauli[1] suggested that the hyperfine structure of atomic spectra could be explained by the association of a magnetic moment to the nucleus. Uhlenbeck and Goudsmit introduced the electronic spin (1925) to explain the fine structure in spectral lines. The success with which the electronic spin explained these details encouraged physicists to associate mechanical spin with the nuclear magnets. At present the evidence for the existance of nuclear spin and nuclear magnetic moment is numerous and both are considered as characteristic of the nucleus.

The nuclear spin system can be made to absorb energy under certain conditions which will be discussed later. A knowledge of this energy provides a large amount of information about the nucleus itself and may also lead to a further knowledge of the constituents of the media in which the nucleus is imbedded.

Many methods have been used to detect this energy. Gorter[2] in 1936 made an unsuccessful attempt to detect heating of the crystalline lattice as a consequence of the energy absorbed by the spin system from the radio-frequency field and transferred to the lattice by the relaxation mechanism.

1. N. Bloembergen. "Nuclear magnetic relaxation" p. 9, The Hague Martinus Nijhoff, from W. Pauli, Naturwissenshafters 12, 741, (1924)

2. C. J. Gorter. Physica 3, 995 (1935)

The first successful method was devised and used by Purcell, Torrey and Pound in 1946[3]. This method consisted of measuring the losses arrising from nuclear absorption from a sample placed in a resonant cavity. The poor sensitivity of the method for increased power levels led Bloembergen, Purcell and Pound[4] to the use of a radio-frequency bridge. The principles involved in these two methods will be made clear when we discuss the experimental details.

In 1947, Roberts[5] suggested the use of the super-regenerative receiver for the detection of nuclear radio-frequency resonance absorption. The full description and the theory of this method is the subject of this thesis.

In Part I, a brief review of the theory of nuclear magnetic resonance is presented. In Part II, the main subject is discussed.

3. E. M. Purcell, H. C. Torrey and R. V. Pound, Phys. Rev. 69, 37 (1946)

4. N. Bloembergen, E. M. Purcell and R. V. Pound, Phys. Rev. 73,679 (1948)

5. A. Roberts, Rev. Se. inst. 18,845 (1947)

PART I

THEORY OF NUCLEAR MAGNETIC RESONANCE

1. Nuclear spins and magnetic moments

Seeking an explanation of the hyperfine structure in line spectra and the intensity alteration in band spectra, it was necessary to assume that the atomic nucleus as well as the electron possess an intrinsic angular momentum or nuclear spin designated by (I).

When one computes the magnetic moment due to a spinning sphere of mass (m) and a uniformly distributed surface charge (e), one finds:

$$\bar{\mu} = \frac{e}{2mc}\bar{P} \tag{1}$$

where ($\bar{P}$) is the angular momentum of the spinning shell and (c) is a constant numerically equal to the velocity of light. The nucleus does not conform to this model or any other proposed model. In fact, the resonance absorption phenomena provides one of the experimental methods for finding the relation between the nuclear magnetic moment and the vector $\bar{P}$. A variety of experiments have shown that

$$\bar{\mu} = g\frac{e}{2Mc}\bar{P} \tag{2}$$

Where (g) is a constant of the nucleus called the "gyromagnetic ratio", a number characteristic of a given nucleus in a given nuclear energy state. (M) is the proton mass and (e) is the proton charge.

If (I) is the nuclear spin quantum number, the angular momentum of the nucleus ($\bar{P}$) due to its spin is

$$|\bar{P}| = I\frac{h}{2\pi}$$

where (h) is Planck's constant. The expression for the magnetic moment becomes:

$$|\mu| = g\, I \frac{e\hbar}{4\pi Mc} \qquad (3)$$

Whenever a magnetic moment ($\bar{\mu}$) is subjected to a magnetic field or whenever interaction between the field and the magnetic moment exists, a precession of ($\bar{\mu}$) takes place at a certain angular velocity about the direction of the external magnetic field, similar to the precession of a mechanical top in the gravitational field. This precession is called the "Larmor precession" and the frequency of precession is the "Larmor frequency".

Classical electrodynamics give the Larmor frequency as

$$\nu_L = g \frac{e}{4\pi Mc} H \qquad (4)$$

Where (H) is the applied external field.

Substituting for $g \frac{e}{4\pi Mc}$ from (3) into (4) we get

$$\nu_L = \frac{\mu H}{I \hbar}$$

Thus a determination of the Larmor frequency for a certain field intensity leads to a determination of the nuclear magnetic moment.

2. <u>The principle of nuclear resonance</u>

On quantum mechanical basis, the components of the magnetic moment in the direction of the field can take only certain assigned values. A magnetic quantum number (m) is associated with these components, which may take any one of the values ($\pm m$) where (m) is related to (I), the nuclear spin quantum number, by

$$m = I,\ I-1,\ I-2,\ \cdots\cdots,\ -I$$

Thus there are $(2I+1)$ possible components in the field direction or, in other words, there are $(2I+1)$ permitted orientations of spin relative to the direction of the field.

When a magnet is placed in a magnetic field, its tendency is to orient itself in the direction of the field where it possesses minimum potential energy. The same is true of the nuclear magnets, the smaller the angle they make with the direction of H, the smaller is their potential energy and vice versa. Thus, when the field is applied, the nuclear magnets will be distributed on $(2I+1)$ energy levels. To effect a transition from one energy level to a higher one an amount of energy $h\nu = (\frac{e\hbar}{2Mc} H)$ is required, the method of inducing such a transition is the main object of nuclear magnetic resonance experiments. Gorter (2) remarked that, just as in Na-vapor an anomalous electric dispersion occurs at the position of the yellow resonance line, there must be an anomaly in the nuclear paramagnetic susceptibility in the radio-frequency range, if the substance is placed in a large magnetic field H, the anomalous dispersion will be accompanied by absorption at the "Larmor frequency".

If one subjects a sample containing nuclear magnets in a magnetic field to radiation at the Larmor frequency, a nucleus in a lower energy level may absorb a quantum of energy and jump to a higher energy level. The allowed transitions are only those with $\Delta m = \pm 1$ which is the selection rule for this phenomenon.

Consider the magnetic moment ($\bar{\mu}$) in the constant field ($\bar{H}_0$), Fig. (1), which is precessing about ($\bar{H}_0$) as explained before.

A small rotating magnetic field is somehow imposed at right angles to ($\bar{H}_0$), this small field will exert a torque $\bar{L} = \bar{\mu} \times \bar{H}_1$ which is also rotating with the same frequency as ($\bar{H}_1$). If the angular velocity of ($\bar{H}_1$) is not the same as the Larmor angular velocity of precession of ($\bar{\mu}$), the field ($\bar{H}_1$) and ($\bar{\mu}$) will fall in and out of phase ($\omega - \omega_L$) times a second, where (ω) is the angular velocity of ($\bar{H}_1$), and consequently the torque ($\bar{L}$) will tend to increase the angle (θ) when in phase and decrease it when out of phase, thus the time average of the torque action on ($\bar{\mu}$) will be zero. On the other hand, if $\omega = \omega_L$, the torque will keep its tendency to either increase (θ) over the whole period or decrease it over the whole period, resulting in our absorption of energy in the first case and emission in the second case. This is what is called the <u>resonance phenomenon</u>.

3. <u>Relaxation time</u>

Any method of achieving nuclear resonance cannot be restricted to absorption alone or emission alone as is evident from the discussion of the previous section. At resonance two types of transition are equally probable, those magnetic moments in a high energy state may emit energy and go to the next lower state and those in a low energy state may absorb energy and go to the next higher energy level. To simplify the argument consider the proton with $I = \frac{1}{2}$, then $m = +\frac{1}{2}, -\frac{1}{2}$,

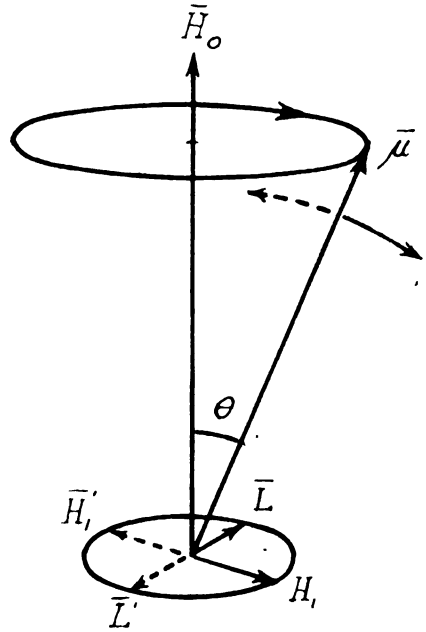


Figure 1

thus two energy states are available for the proton, if the population of the two states is the same, no net absorption or emission takes place, but fortunately at thermal equilibrium a Boltzmann distribution of energy levels exists.

Pake[6] calculated from the Boltzmann factor the excess number of protons in the lower state at room temperature in a field of about 20000 gauss and found that for every million nuclei in the upper energy state there are a million and fourteen nuclei in the lower energy state, it is these fourteen nuclei in each two million which are responsible for the net nuclear magnetization of the sample and for the absorption of energy at resonance. Lowering the temperature of the sample increases the population of the lower energy states and thus more energy is absorbed at resonance.

Rabi[7] computed the transition probability for an isolated magnetic moment with $I=\frac{1}{2}$ in a constant strong magnetic field and a weak precessing field and came to the conclusion that there is an equal probability of transition in one direction as in the other. Therefore, if a net absorption is to occur there must be, at all times, an excess number of nuclear magnets in the lower state. This calls for a relaxation mechanism to bring the excited nuclear magnets to the

6. G. E. Pake. "Fundamentals of nuclear magnetic resonance absorption" Typewritten notes at Washington University, Saint Louis, Mo. p. 9.

7. I. I. Rabi. Phys. Rev. 51,652 (1937)

lower state. The time required for all but 1/e of the equilibrium excess number to reach the lower energy state is called the "relaxation time".

The equal probability of transition found by Rabi does not apply completely to practical cases. Rabi considered an isolated magnetic moment while in practice we deal with a very large number of such moments very close to each other. The interaction of these moments is responsible for the relaxation process. The important interactions are, the spin-Lattice and the spin-spin interactions. In the first, the spin system gives up its energy to the vibrating atoms in the lattice and thus "cool down" to a lower energy state. The second interaction considers the additional field set up at the position of a magnet by a neighboring nuclear magnet. If the two magnets are anti-parallel, the "local" field set by one at the position of the other has two unique effects. The component of the local field along the external field will change the Larmor frequency, thus causes a dispersion in the resonance frequency and increases the width of the resonance line. The component perpendicular to the external field will be precessing with the frequency of precession of the source magnet. If the nuclear magnets are identical, their frequency of precession will be the same and nuclear resonance will occur flipping over both nuclei resulting with their exchanging orientations. This "spin-spin collision" phenomenon does not change the population of the levels but it shortens the half life of a certain level thus accelerating the relaxation process.

The relaxation mechanism was the subject of extensive research by many pioneers, Bloembergen, Purcell and Pound[5] have studied relaxation in fluids and found that Brownian motion at the Larmor frequency provides the relaxation mechanism. Spin-Lattice relaxation times thus far measured range from 10^{-4} seconds or less in certain solutions containing paramagnetic ~~ons~~ ions to several hours for very pure ice crystals at liquid nitrogen temperature[6].

4. Nuclear Susceptibility

The magnetic flux density or magnetic induction in a magnetic substance is given as

$$\bar{B} = \bar{H} + 4\pi \bar{M}$$

where $\bar{M}$ is the magnetic moment per unit volume of the sample. In isotropic substances $\bar{M}$ is proportional to the magnetic field H.. The proportionality factor (χ) is defined as

$$\chi = \frac{\partial M}{\partial H}$$

and at ordinary room temperature it is usually written

$$\chi = \frac{M}{H}$$

(χ) is called the magnetic susceptibility, its relation to the absolute temperature is called the "Curie susceptibility" and is given as

$$\chi = \frac{N}{3kT} g^2 \mu_0^2 \, I(I+1)$$

where (N) is the number of nuclei per unit volume, (g) is the gyromagnetic ratio (μ_0) is the nuclear magneton and (I) is the nuclear spin quantum number.

The magnetic susceptibility (χ) is important in this treatment since it is the source of the nuclear energy absorption. It is considered a complex quantity

$$\chi = \chi' - j\chi''$$

F. Bloch[8] in seeking a description of the nuclear induction effect set up and solved the equations of motion involving the nuclear magnetization $\bar{M}$ and the total magnetic field. From these equations he obtained theoretical expressions for (χ).

The expressions for (χ') and (χ'') as found by Bloch are called the "Bloch susceptibilities" and are

$$\chi' = \frac{1}{2}\chi_0 \omega_0 T_2 \frac{T_2(\omega_0 - \omega)}{1 + T_2^2(\omega_0 - \omega)^2}$$

and

$$\chi'' = \frac{1}{2}\chi_0 \omega_0 T_2 \frac{1}{1 + T_2^2(\omega_0 - \omega)^2} \qquad (6)$$

Where (ω_0) is the resonance frequency, $\omega(t)$ is the Larmor frequency and (T_2) is the time required for the components of the magnetic moments to die out to $1/e$ of their initial value.

χ' versus time is plotted in figure 2(a), χ'' versus time is plotted in figure 2(b).

5. <u>Radio-frequency signal at resonance</u>

In nearly all the methods used in measuring nuclear signals the sample is inserted in an inductance coil. The energy absorbed at resonance may be considered as due to the introduction of a dissipative

8. F. Bloch. Phys. Rev. 70,460 (1946)

resistance $r(t)$ in series with the inductance. We find first the potential drop at resonance and then $r(t)$.

At resonance, the p.d. across the coil is

$$\tilde{V} = \mu L_o \frac{d\hat{I}}{dt} = j\omega\mu L_o \tilde{I}$$

where L_o is the inductance of the air filled coil and $\tilde{I}$ is the complex current flowing through the coil.

But $$\mu = 1 + 4\pi\chi = 1 + 4\pi\chi' - j\,4\pi\chi''$$

therefore $$\hat{V} = j\omega L_o \hat{I}\,(1 + 4\pi\chi' - j\,4\pi\chi'')$$

or $$\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = j\omega L_o(1 + 4\pi\chi') + 4\pi\omega L_o\chi''$$

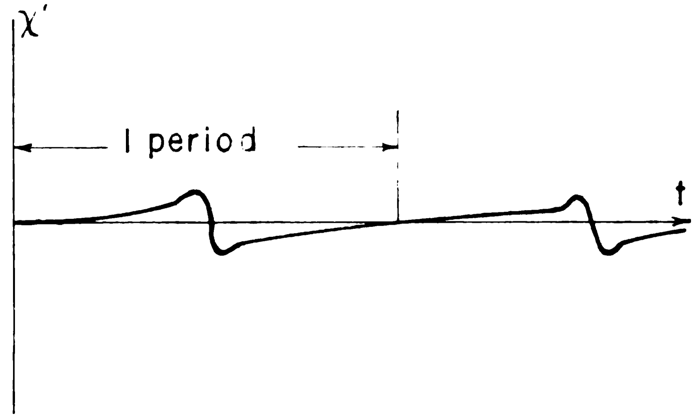
where $\tilde{Z}$ is the impedence of the coil. We are interested in the dissipative or real part of this impedance, •,i.e.

$$Re\left\{\tilde{Z}\right\} = r(t) = 4\pi\omega L_o\chi''$$

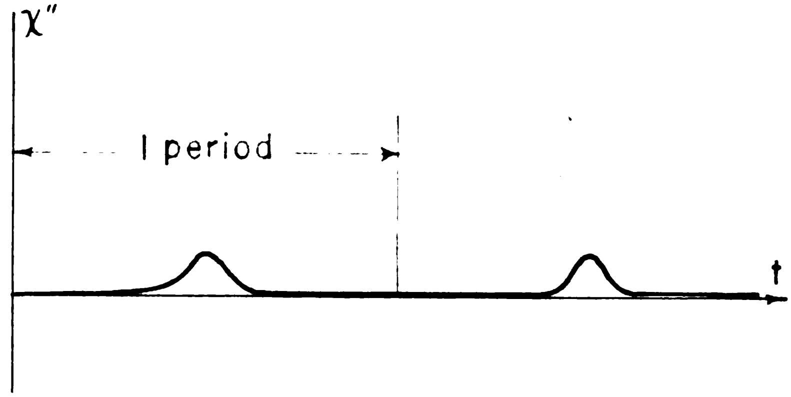
In terms of the circuit Q - factor $\frac{\omega L_o}{R}$, this becomes

$$r(t) = 4\pi QR\chi''(t)$$

where R is the series ohmic resistance of the circuit. Since the only time variable in the expression for $r(t)$ is $\chi''(t)$, the plot for $r(t)$ is the same as that for $\chi''(t)$ shown in figure 2(b) with a simple change of scale.



(a)



(b)

Figure 2

PART II

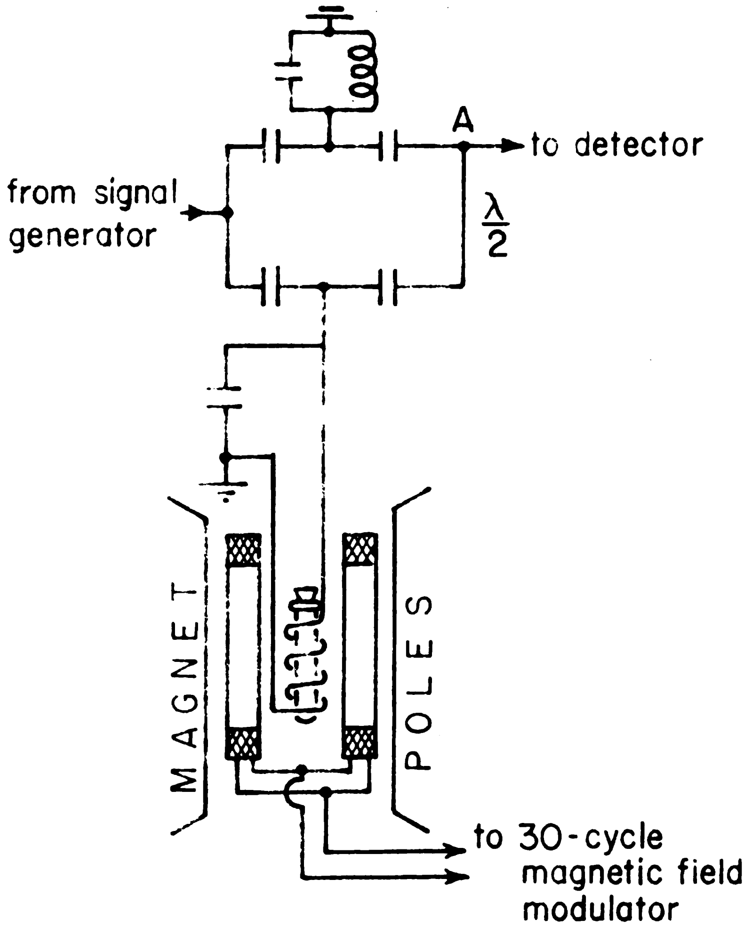
USE OF SUPER-REGENERATIVE RECEIVERS IN DETECTING NUCLEAR RESONANCE ABSORPTION

Although our main concern is the theory of super-regenerative receivers as used in nuclear resonance experiments, an account of the bridge method seems to be of a great help in pointing out the advantages of the super-regenerative receiver. Furthermore, both methods have the same basic arrangement for producing nuclear resonance absorption.

The essential features of the bridge method is the balancing of a signal flactuations. In figure (3), the important parts of the circuit arrangement is shown. The signal is fed to the two branches of the bridge. Two identical oscillating circuits are connected to each branch of the bridge. One of the oscillatory circuits has the sample as the core of its coil. The branches are so arranged that when the signal components from the two branches meet at point (A), they are 180 degrees out of phase. Their amplitudes cancel giving a null effect to the detector.

The sample coil is put at right angles to the direction of a strong uniform magnetic field. The field is modulated by a 60 cycle current carrying coils.

The operation starts by the application of the signal. This sets the oscillatory circuits on the branches oscillating. The frequency of oscillation is determined by other experiments and is kept constant during operation. Next, the magnetic field is turned on and increased gradually until equation (3) is satisfied. This means that



A
to detector
from signal
generator
$\frac{\lambda}{2}$
MAGNET
POLES
to 30-cycle
magnetic field
modulator

Figure 3

nuclear resonance is taking place. The modulating coils serve to swing the magnetic field back and forth through resonance 120 times a second so that the signal may be easily detected and amplified.

The energy absorbed at resonance in the tank circuit is taken out of the energy flowing through the branch of the bridge to which it is connected. This destroys the null effect at point A since the component of the signal flowing through the other branch is still the same. The slight difference in energy is sent to a detector and observed on the oscilloscope screen. It is to be noticed that at least three parts of this arrangement should be designed very carefully and present many parameters to be taken care of. These are, the signal generator, the oscillatory circuit and the detector arrangement.

It will be shown next that all of these parts are taken care of in one simple super-regenerative circuit.

<u>The principle of super-regeneration</u>

The main feature of the super-regenerative receiver is the periodic building up and decay of free oscillations.

<u>Regeneration</u>

An easy approach to the principle of super-regeneration is through a discussion of the regenerative receiver. Figure (4) represents a simple regenerative receiver circuit.

The regenerative receiver consists of two distinct circuits, a diode detector circuit (the tank circuit) with the cathode and the grid forming the diode and a suitable grid leak combination for detection. The detected current is fed to a one stage amplification circuit (the plate circuit) which uses all three electrodes as its tube.

When the circuit is connected to the power supply a direct current flows through the plate circuit, the coupling between the plate circuit and the tank circuit is so arranged that the energy fed to the tank circuit is not enough to set up free oscillations.

When a signal is received, a varying voltage is impressed on the detector circuit and is amplified in the plate circuit. A part of the energy from the tickler coil (L_2) is fed back to the tank circuit through the coupling between L_1 and L_2. The energy fed back adds further amplification to the signal. This process of amplification proceeds until the signal amplitude reaches an equilibrium value. Two important points must be emphasized.

a) The energy fed back should always be kept under a certain critical value at which free oscillations are started in the tank circuit. These free oscillations, if started, block the reception of any signal.

b) The simplicity of the circuit, its combined detection and amplification is very desirable.

<u>Super-regeneration</u>

Keeping the same elements in the regenerative receiver, we can turn it into a super-regenerative receiver as follows:

a) The coupling between the tickler and the tank coils is increased until free oscillations are set up in the tank circuit.

b) The voltage supplied to the plate of the tube is periodically turned on and off. This results in a periodic build up and decay of oscillations. The frequency of these bursts of oscillations is the same as the frequency with which the plate supply voltage is turned on and off. Making the circuit alternately oscillatory and non-oscillatory is called the "quenching action".

Our circuit now is a super-regenerative receiver in a rather primitive sense. In the coming sections we will discuss the two types of quench and the different modes of operation.

1. Separate quench super-regenerative receivers

In separately quenched super-regenerative receivers, the quenching voltage.from a separate oscillator is applied to one of the electrodes of the tube. The point of application of the quench voltage is customarily the plate of the tube.

To understand the basic principles of the operation of the separately quenched super-regenerative receiver, we shall take the previous circuit as a prototype of all separately quenched receivers and analyze its operation in detail. For the sake of definiteness, we shall assume that the quenching action is accomplished by a plate supply voltage whose wave form is rectangular as shown in figure (5).

During the interval (τ) the supply voltage rises to the value E_{bb} which is sufficient to allow self-sustained oscillations to

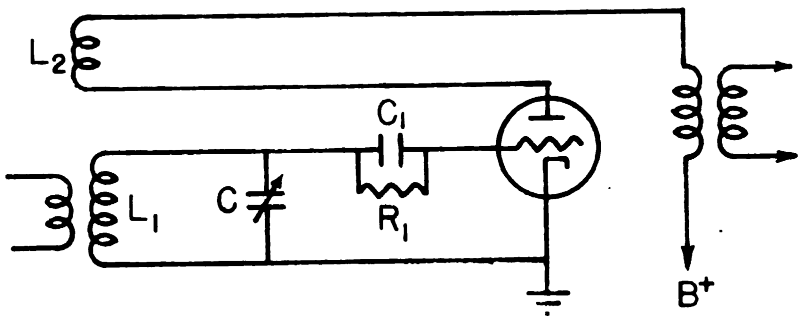


Figure 4

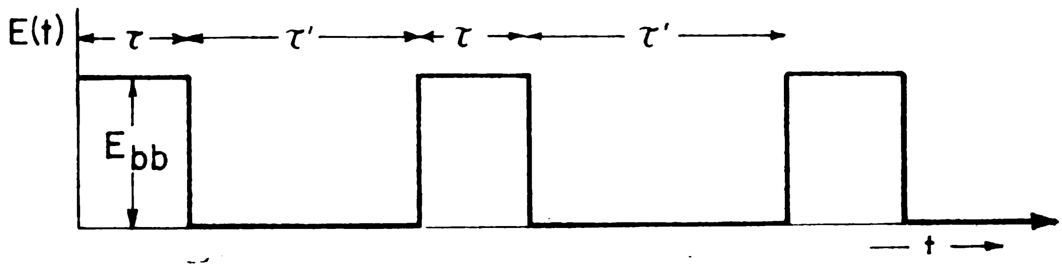


Figure 5

(a) (b)

Figure 6

build up in the tank circuit. The interval (τ) is followed by the interval (τ'), which may or may not be equal to (τ) in length and during which the plate voltage falls to zero. During the interval (τ') the amplitude of the oscillations in the tank circuit decays because of energy losses in the tank circuit.

To understand the process of build up and decay of oscillations in the tank circuit, we introduce the equivalent circuit shown in figure 6(a).

The L-C combination is that of the tank circuit. The constant resistance R is responsible for the ohmic and radiation losses of the tank circuit. $r(t)$ represents the time dependent resistance which is added to the circuit by inserting the sample in the inductance coil. We have previously shown that

$$r(t) = 4\pi Q R \chi''(t)$$

The time dependent negative resistance $R^{-}(t)$ allows oscillations to build up in the circuit during the period τ.

We may form an estimate of the factors which determine $R^{-}(t)$ by the following analysis.

The current (i) in the tank circuit is given as

$$i = j\omega c \cdot e_g \quad ,$$

where ($j\omega c$) is the capacitive susceptance and (e_g) is the grid voltage. The plate voltage is

$$e_p = -\mu e_g \quad ,$$

where (μ) is the amplification factor of the tube.

From the equivalent plate circuit of figure 6(b), it is clear that the plate current is $\displaystyle i_p = -\frac{\mu e_g}{r_p + j\omega L_t}$

The e.m.f. induced in L_1 due to the voltage drop in L_2 is

$$e = j\omega M i_p = -j\omega M \frac{\mu e_g}{r_p + j\omega L_2}$$

$$= -\frac{M\mu}{C} \cdot \frac{i}{r_p + j\omega L_2}$$

We may simplify this term by assuming that $j\omega L_2 << r_p$ and replacing M by its absolute value so that

$$e = \frac{|M|\, i}{C} \cdot \frac{\mu}{r_p} = \frac{|M|\, i}{C} g_m \quad ,$$

where (g_m) is the transconductance of the tube and is defined as

$$g_m = \frac{\delta i_p}{\delta e_g} \quad ,$$

which is the slope of the tube characteristic curve.

Since
$$e = R^{-}(t)\, i$$

we get the expression for $R^{-}(t)$ by dividing the last expression for (e) by (i)

$$R^{-}(t) = \frac{|M|}{C} g_m \tag{7}$$

For the last relation to hold, the slope of the characteristic curve should be a constant, at least at the operating part of the curve because we assumed linearity when we used the relation

$$e_p = -\mu e_g$$

We now return to the equivalent circuit of figure 6(a). The differential equation for the current is

$$L\frac{di}{dt} + R(t)\, i + \frac{1}{C}\int i\,dt = 0$$

where

$$R(t) = R - R^{-}(t) + r(t) \tag{8}$$

Differentiating and collecting terms we get

$$\frac{d^2 i}{dt^2} + \frac{R(t)}{L}\frac{di}{dt} + \left[\frac{1}{LC} + \frac{1}{L}\frac{dR(t)}{dt}\right] i = 0 \tag{9}$$

The differential equation as it stands has variable coefficients which arise from the time dependence of $r(t)$ and $R^-(t)$. It is of the Hill type since $r(t)$ and $R^-(t)$ are periodic in time. Its exact solution is rather complicated. However, we may make some approximations based on the fact that $r(t)$ and $R^-(t)$ vary very slowly compared to the natural frequency of the circuit.

Let us put the differential equation (9) in the simpler form

$$I''(t) + f(t)I'(t) + g(t)I(t) = 0$$

where $$f(t) = \frac{R(t)}{L} \quad and \quad g(t) = \frac{1}{LC} + \frac{dR(t)}{Ldt}$$

assume now a solution

$$I = v(t)\, e^{-\frac{1}{2}\int f(t)dt}$$

when substituted in the differential equation we get

$$v''(t) + \left[g(t) - \frac{1}{4}f^2(t) - \frac{1}{2}f'(t)\right]v(t) = 0$$

making the substitution

$$K^2(t) = g(t) - \frac{1}{4}f^2(t) - \frac{1}{2}f'(t)$$

the equation takes the form

$$v''(t) + K^2(t)v(t) = 0$$

Before we go further in the solution, let us examine the time dependence of our variables.

$v(t)$ varies with the natural frequency of the circuit, which is of the order of 10^7 c./sec. $K(t)$ depends on $R(t)$ which in turn depends on $r(t)$ and $R^-(t)$. In normal operation the fundamental frequency of $R^-(t)$ and $r(t)$ are the order of 10^5 c./sec. and 10^2 c./sec., respectively.

With $k(t)$ a very slowly varying function of time, we may assume an exponential solution

$$v(t) = e^{\pm j\int_0^t k(t)\,dt + \gamma}$$

or

$$v(t) = A e^{j\int_0^t y(t)\,dt} \qquad wher \quad y(t) = f[k(t)]$$

Substituting this solution in the differential equation for v, we get

$$y^2 - j y' = k^2(t)$$

With $y(t)$ a very slow variable of time we can assume $y' \ll y^2$.

Therefore to a first approximation

$$y = \pm k$$

And to a second approximation

$$y = \pm k - \frac{j}{2}\frac{k'}{k}$$

The corresponding second approximation for (v) becomes

$$v(t) = A e^{\pm j\int_0^t k(t)\,dt - \frac{1}{2}\int_0^t \frac{k'}{k}\,dt} = \frac{A}{\sqrt{k}}\, e^{\pm j\int_0^t k(t)\,dt} \quad .$$

The general solution for (I) becomes

$$I = A e^{-\frac{1}{2}\int_0^t f(t)\,dt} \cdot \frac{e^{\pm j\int_0^t k(t)\,dt}}{\sqrt{k}} \quad .$$

Giving $f(t)$ and $g(t)$ their values, the complete solution becomes

$$I = A e^{-\frac{R(t)}{2L}t} \cdot \frac{e^{\pm j\sqrt{\frac{1}{Lc} - \left(\frac{R}{2L}\right)^2}\,t}}{\left[\frac{1}{Lc} - \left(\frac{R}{2L}\right)^2\right]^{1/4}} \quad ,$$

where we have dropped a term $\frac{R'(t)}{2L}$ because of its smallness and considered the average value of R (t) for the evaluation of the exponents.

The quantity under the radical is easily recognized as (ω^2), the natural frequency of the circuit, thus

$$I = Ae^{-\frac{R(t)}{2L}t}\,\frac{e^{\pm j\omega t}}{\sqrt{\omega}}$$

The first term in the solution, namely

$$Ae^{-\frac{R(t)}{2L}t}$$

represents the amplitude of the oscillations. To discuss the building up and decay of the oscillations envelope we write the above expression as

$$Ae^{\frac{R^-(t)-[R+r(t)]}{2L}t} \tag{10}$$

Here we have to consider two time intervals.

1. During the interval (τ), fig. (5), $R^-(t)$ is greater than $R+r(t)$ the oscillations build up.
2. During the interval (τ'), $R^-(t)$ is less than $R+r(t)$, the oscillations decay.

All super-regenerative receivers can be classified on the basis of expression (10), such a classification proceeds as follows:

1. Separately quenched receivers - $R^-(t)$ has a constant repetition period. The receiver operates in one of two modes.
   a) The <u>linear</u> mode of operation - The exponent in expression (10) has a very small value.
   b) The <u>logarithmic</u> mode of operation - The exponent has a large value.
2. Self-quenched receivers - $R^-(t)$ has a variable repetition period.

Although in these receivers the amplitude of oscillations normally does not build up to the point where it is limited by the equilibrium value set by the tube characteristics, the amplitude does reach a value sufficiently high that the build up curve is no longer linear. It is, therefore, logarithmic.

Now, we proceed to discuss the modes of operation in detail.

a. The linear mode -

If the exponent

$$\frac{R^{-}(t) - [R + r(t)]}{2L} \tau$$

is small, the amplitude of the oscillations at the end of the period (τ) may be found by taking the first two terms of the exponential expression (10). The amplitude then becomes

$$A \left\{ 1 + \left[\frac{R^{-}(t) - [R + r(t)]}{2L} \right] \tau \right\}$$

Noting that $R^{-}(t)$ and R are constant during the interval (τ), it is clear that the amplitude is linearly proportional to $r(t)$, i.e. to the resistance introduced by nuclear absorption.

Therefore, when the sample is off resonance, all the envelopes will have practically the same amplitude and enclose the same area. At resonance the amplitude is less and consequently the enclosed areas are less.

Fig. 7(b) shows two types of envelopes at and off resonance.

b. The logarithmic mode -

In this mode the oscillations are allowed to build up to their equilibrium value, determined by the characteristics of the tube, before they are quenched. Going back to expression (10) we see that this happens when the exponent has a large value during (τ). When the sample is off resonance, all the envelopes enclose practically the same area, except for a slight variation due to the unstability of the noise voltage. At resonance, the oscillations take longer time to build up and a shorter time to decay. This results with a smaller area under the oscillation envelopes.

Fig. 7(a) shows the change in this area when the sample is at resonance.

2. Self-quenched super-regenerative receivers

As the name indicates, this type of receiver quenches its own oscillations. This is achieved by a simple alteration in the circuit of figure (4). The values of the components in the grid circuit C_1R_1 are so chosen that the oscillations are interrupted periodically in a manner often known as "squegging".

In the separately quenched oscillator C_1R_1 have such values that C_1 partially charges and discharges at the frequency of the oscillator. If the values of C_1 and R_1 are increased, C_1 will not discharge when the polarity changes. It will keep on charging at the positive peaks of oscillations until the grid is driven so negative that it cuts off the plate current. The condenser C_1 will then discharge through the resistance R_1 until the grid voltage has the right value for the plate current to flow and for the oscillations to start again. The process is repeated periodically with a frequency determined by the circuit parameters.

The quench frequency is not a constant as is the case with separately

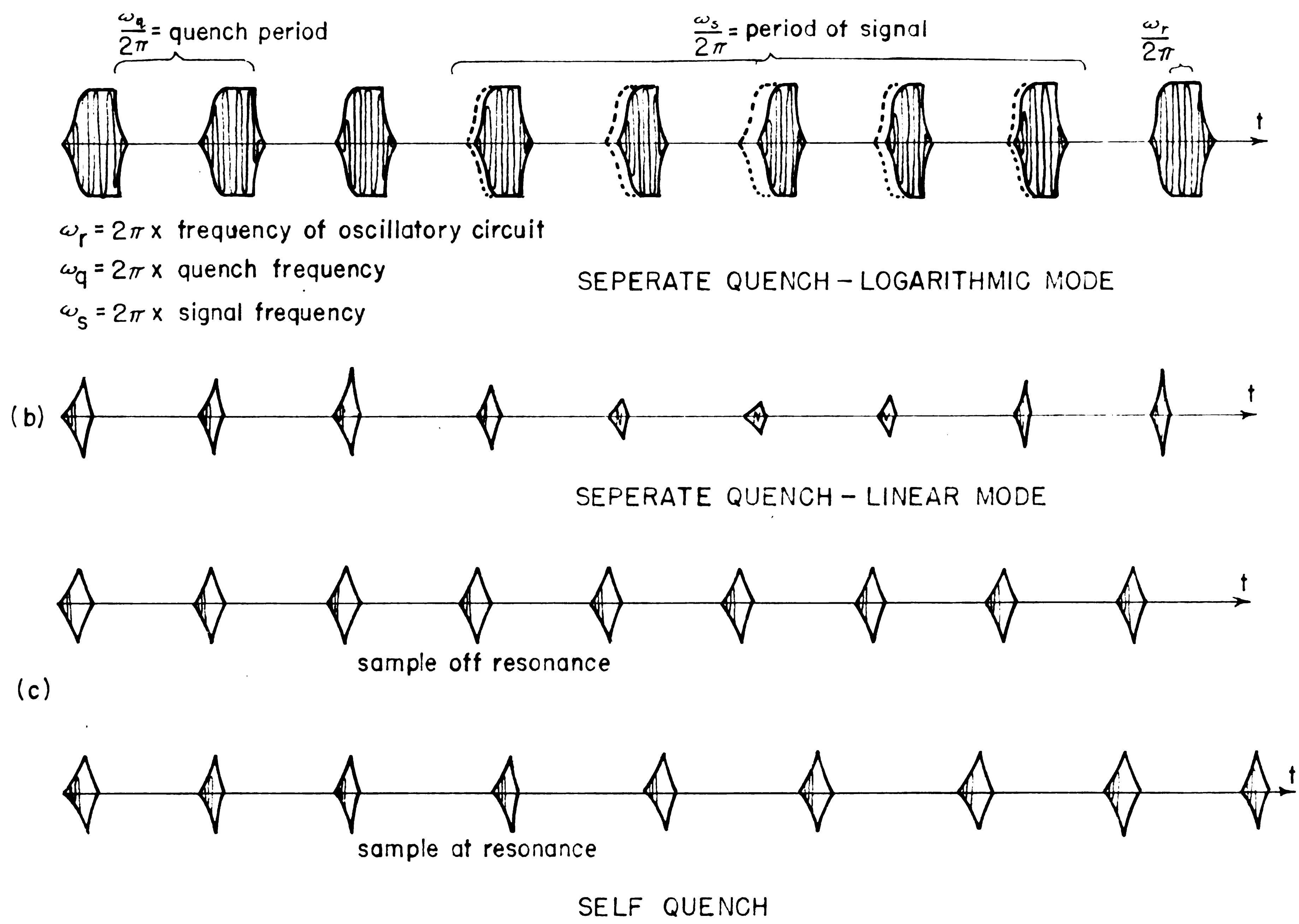

$\frac{\omega_q}{2\pi}$ = quench period
$\frac{\omega_s}{2\pi}$ = period of signal
$\frac{\omega_r}{2\pi}$
t
$\omega_r = 2\pi$ x frequency of oscillatory circuit
$\omega_q = 2\pi$ x quench frequency
$\omega_s = 2\pi$ x signal frequency
SEPERATE QUENCH – LOGARITHMIC MODE
(b)
SEPERATE QUENCH – LINEAR MODE
sample off resonance
(c)
sample at resonance
SELF QUENCH

Figure 7

quenched receivers. When nuclear absorption is taking place, the oscillations start from the new level. The building up is slower as was explained before. Consequently, the condenser C_1 takes a longer time to charge and the cut off point is reached later. This delay in time of operations results in a rarefaction of oscillation bursts or, in other words, in a quench frequency modulation. Figure 7(c) shows two sequences of bursts, one with the sample off resonance, and the other with the sample at resonance.

Experimental observations show that absorption does not result with pure frequency modulation. A certain percentage of amplitude modulation is always present. This may be explained as follows:

The decay of oscillations does not start unless the grid voltage reaches a certain fixed value for cut off. This means that the condenser C_1 should accumulate a certain charge (Q). At resonance, it takes longer to accumulate the required charge than it takes when the sample is off resonance. Therefore, whatever the period for build up may be, the area under the build up envelope is the same. However, the area under the decay envelope is not the same. At resonance the decay is faster and will include a smaller area. In other words, for the build up envelope, there is a restriction by a definite charge which makes the areas under those envelopes the same. The decay envelope, on the other hand, is restricted by the exponent during (τ') which is larger at absorption.

It will be shown later that the oscillation envelopes which occur at the quench frequency can be expanded in a Fourier series. The amplitude modulation occurs only in the zeroth harmonic or the constant term representing the d.c. component.

The effect of the amplitude modulation of the d.c. component on the signal is shown in Figure (8). The d.c. level is depressed at resonance. The super-imposed quench oscillations follow the same outline. The resultant pattern is always observed on the oscilloscope for the undetected absorption line.

3. Types of circuits

The super-regenerative circuits may take a variety of forms. However, all of them can be represented by the symbolic circuit shown in Figure (9). The (Z's) are impedances, some of which may be omitted. These symbolic circuits represent r-f paths in the actual circuits only. The subscripts g, c, j and t stand for the grid, cathod, plate and tank circuits respectively. Figure 10(a) is the symbolic representation of the circuit shown in Figure 10(b). The lettering on both figures represent the same parts to simplify the comparison. C_{pc} is the plate-cathode capacitance which represents an r-f linkage between the plate and cathode. Figure 11(a) represents another possible alteration in the symbolic circuit of Figure (9). It represents the actual circuit shown in Figure 11(b). This form of the circuit is more practical than that of Figure 10(b). It eliminates the difficulty of putting two coils around the sample which causes serious troubles. The energy feed back is direct. This circuit, with some alterations, is widely used for nuclear resonance experiments.

The last discussion was concerned with self-quenched super-regenerative receivers. The separately quenched receivers can be represented symbolically by the same circuit as in Figure (9). Another unit is to be added to represent the quenching oscillator. Figure 12(a) is the symbolic representation of the circuit in Figure 12(b).

4. Types of signals observed

The patterns observed on the oscilloscope assume many shapes which depend on the same controllable variables in the circuit. We will discuss some of these patterns and their sources assuming that our signal always has the shape of a Gaussian curve.

Side bands - Due to the uncommon shape of the oscillations envelope, one should expect a Fourier series analysis of these envelopes to reveal a large number of high order harmonics.

Let the envelope of these bursts be defined by

$$F(t) = f(t)\, e^{j\omega t}$$

Remembering that the building up takes place during the period (τ), Figure (4), and the decay during (τ'), we can define

$$\begin{aligned} f(t) &= e^{R_1 t} && 0 \leq t \leq \tau \\ &= e^{-R_2 t} && \tau \leq t \leq \tau + \tau' \end{aligned}$$

The function is sectionally continuous, periodic and possesses derivatives within the internal $(\tau+\tau')$, therefore, it can be expanded in a Fourier series of the form

$$F(t) = e^{j\omega t}\sum \alpha_n e^{jn\omega_q t}$$

Where (ω_q) is the quench frequency and α_n is defined as

$$\alpha_n = \frac{1}{\tau+\tau'}\int_0^{\tau+\tau'} f(t)\, e^{-jn\omega_q t}\, dt \quad .$$

Substituting for $f(t)$ we get

$$\alpha_n = \frac{1}{\tau+\tau'}\left[\int_0^{\tau} e^{R_1 t} e^{-jn\omega_q t}\, dt + \int_{\tau}^{\tau+\tau'} e^{-R_2 t} e^{-jn\omega_q t}\, dt\right]$$

Carrying out the integration and evaluating, we get the following result

$$\alpha_n = \frac{1}{\tau+\tau'}\left[\frac{e^{(R_1-jn\omega_q)\tau}-1}{R_1-jn\omega_q} - \frac{e^{-(R_2+jn\omega_q)(\tau+\tau')}-e^{-(R_2+jn\omega_q)\tau}}{R_2+jn\omega_q}\right]$$

We are not interested in the actual values of (α_n). We are merely interested in its behaviour. This we can do without going through the evaluation of the above expression. It is apparant that $\alpha_n \to 0$ as $n \to \infty$. This insures the convergency of (α_n). Let (n) now get large enough to justify dropping the resistances R_1 and R_2, we get the following expression

$$\alpha_n \sim \frac{1-e^{-jn\omega_q(\tau+\tau')}}{jn\,\omega_q}$$

This expression is enough to show us some of the properties of (α_n). First, one notices that it contains an oscillatory term which oscillates more and more rapidly as (n) gets larger. The (n) in the denominator gives a rough estimate of the rate of decrease of (α_n) with respect to (n). Remembering the approximations we made, we notice that (α_n) decreases slower than $\frac{1}{n}$ as $n \to \infty$. This indicates that a large number of the harmonics show up on the oscilloscope screen. These are the side bands we started to discuss. If ν_0 is the frequency of the central band we find the side bands at frequencies ($\nu_0 \pm n\nu_q$). Where (ν_q) is the quench frequency. These side bands are not totally useless, since they may be used as frequency marks to measure line widths. The quench frequency is known to a good approximation. One can compare the side band separation with the unknown band width to get a rough estimate of the unknown width (42 kc./sec. = 10 gauss).

Absorption line patterns - When super regenerative receivers are used for nuclear resonance detection, experimenters usually notice that the absorption line they get on the oscilloscope screen takes many shapes. We attempt now to explain some of these observable patterns.

a) Modulating field's effects

The procedure to get nuclear resonance is to set the external magnetic field at a certain high value ($\sim 10^4$ gauss) and then increase the frequency of the r-f transverse field until the Larmor frequency is reached and equation (4) is satisfied. It was explained before

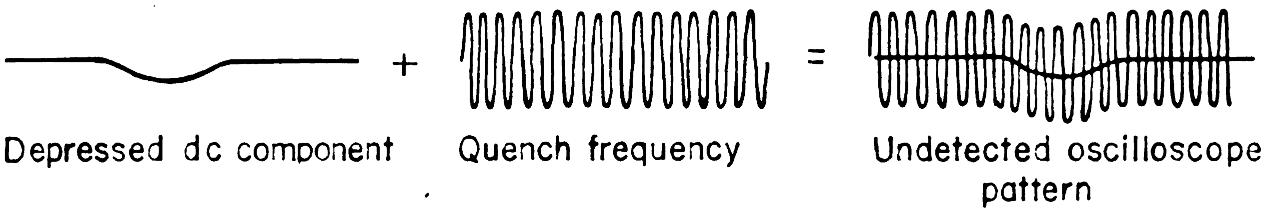


Figure 8

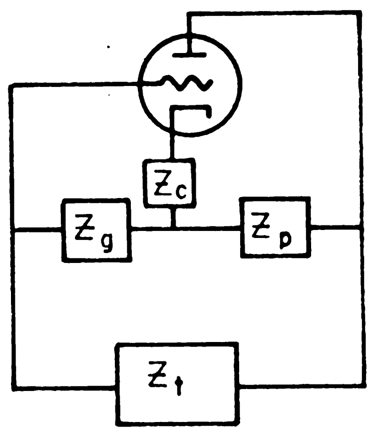
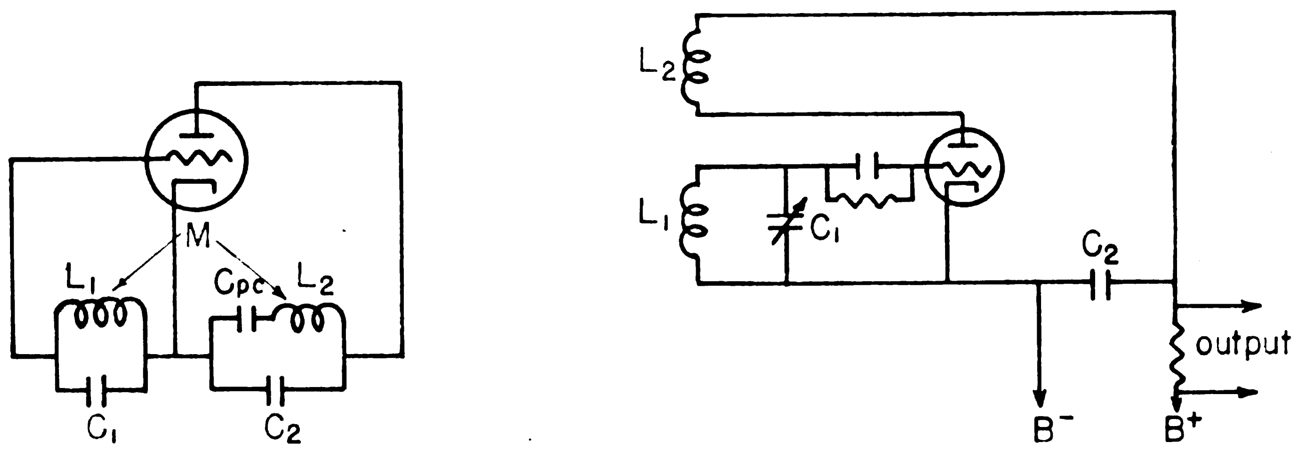
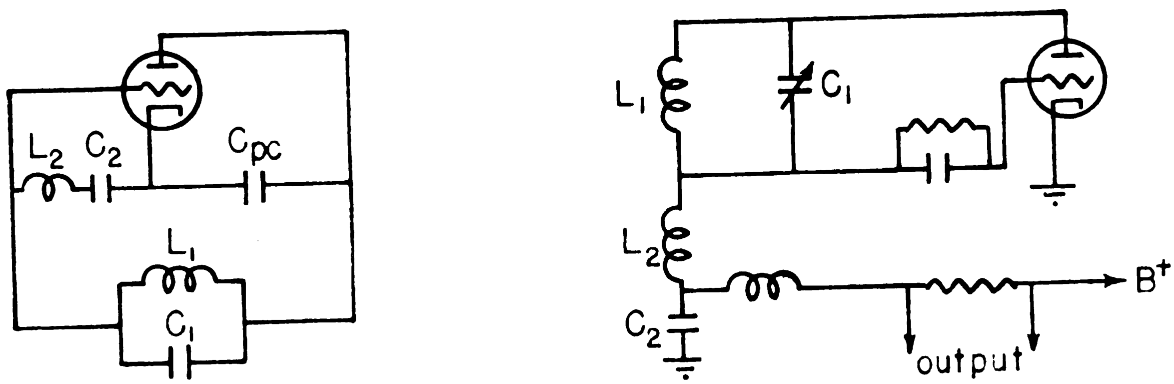


Figure 9



(a) (b)

Figure 10



(a) (b)

Figure 11

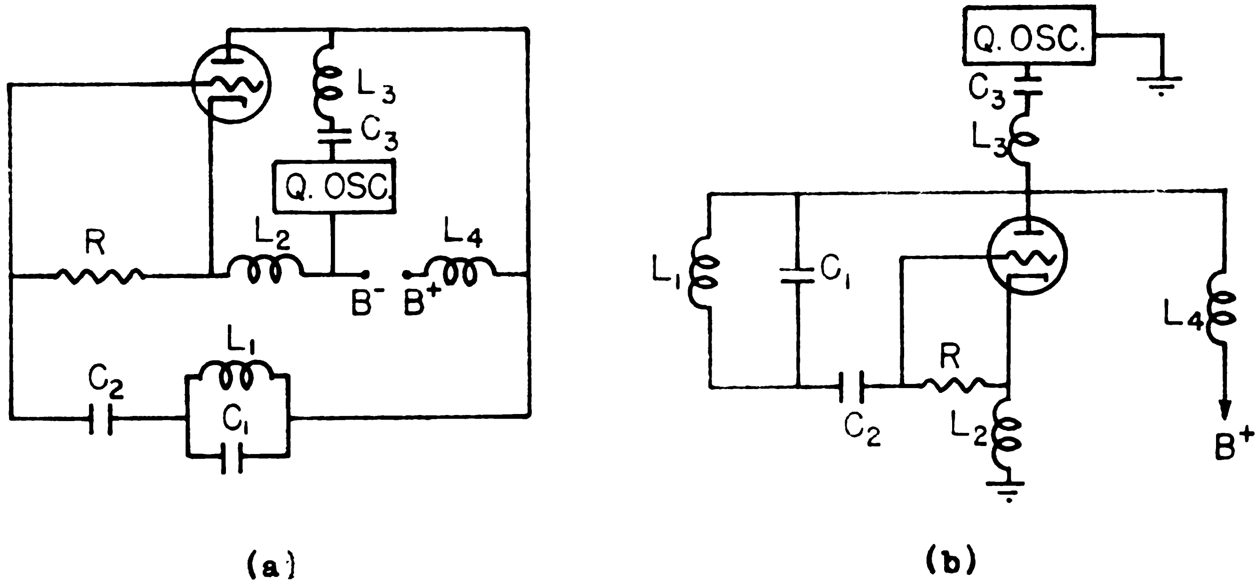


Figure 12

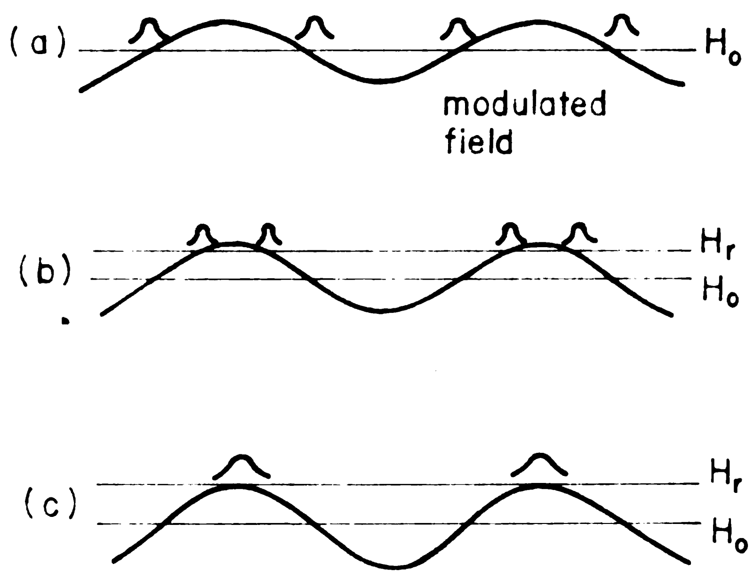


Figure 13

that to get a persistent signal we should make the field oscillate about the resonance value. This was done by a modulating field parallel to the external field (see Figure 3.)

Suppose now that the external field H_0 was set at a value a little below that required for resonance. Then resonance will not occur until the modulating field brings H_0 to the required value. We are assuming here that the radio frequency is kept constant.

Figure 13(a) shows the ideal case where H_0 has the right value for resonance. The modulated field goes through resonance at equal intervals. The signal is drawn where it occurs. Figure 13(b) shows the case where H_0 is a little below resonance value and the modulated field hits resonance value at a quick succession. This shows how the two signals are brought nearer to each other. Figure 13(c) gives the extreme case where the modulated field hits resonance value at its peak. This gives a little larger signal since the field stays at that value for a comparitively longer time.

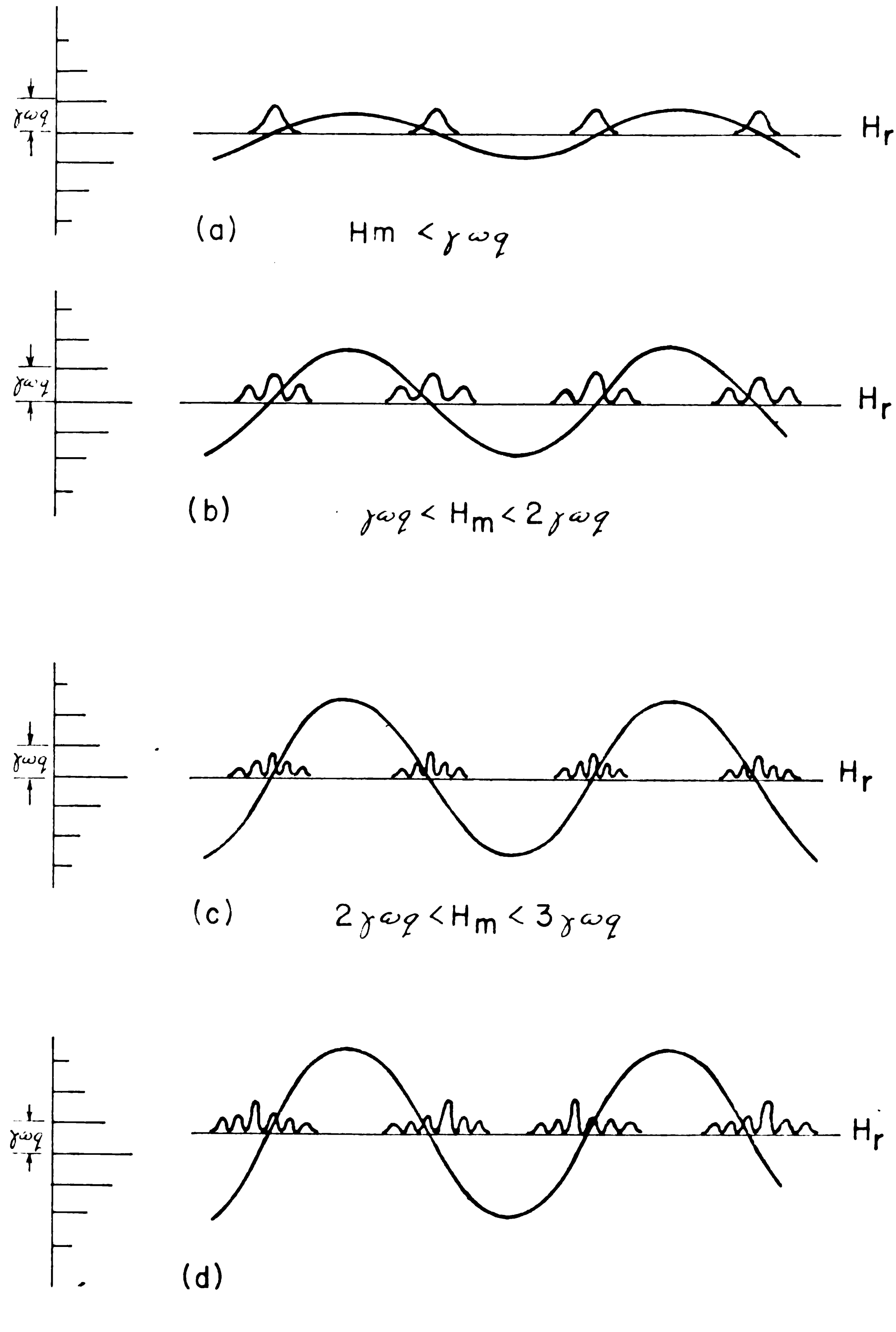
b) <u>Signal spectrum</u>

We have previously shown that the quenching of the oscillations in a super-regenerative receiver introduces many frequency components which differ from the fundamental frequency of the oscillatory circuit of the receiver. The frequencies of these side bands, as they are often called, differ from the fundamental frequency by multiples of the quench frequency. The super-regenerative receiver is, therefore, capable of detecting nuclear resonance at many frequencies which are centered about the fundamental frequency. The amplitudes of the side bands get smaller the further they are from the central band so that the absorption lines correspondingly get weaker.

Using a modulating field of a large amplitude makes the sample sweep through resonance more than twice each cycle of modulation. This phenomenon is made clearer by the help of Figure 14.

In Figure 14(a), the modulating field H_m is smaller than ($\gamma\omega_\ell$) the field to be added to H in equation (4) due to the increase is the Larmor frequency. This is the regular case we have been discussing so far. The absorption takes place twice each cycle at equal intervals. Figure 14(b) shows the case where H_m is made larger than ($\gamma\omega_\ell$). Resonance takes place six times a second. Figure 14(c) shows the effect of a further increase in H_m.

Figure 14(d) is a combined effect of a large H_m and a shift of H_0 from the field H_r required for resonance at the given Larmor frequency. The result is an unsymmetrical set of resonance lines. H_m has the same amplitude as in Figure 14(c) so that on its positive peaks it sweeps through resonance six times and only four times on its negative value. One can imagine any combination of the last two principles which give as many patterns as one wishes.



$\gamma\omega q$
H_r
(a)
$Hm < \gamma\omega q$
(b)
$\gamma\omega q < H_m < 2\gamma\omega q$
(c)
$2\gamma\omega q < H_m < 3\gamma\omega q$
(d)

Figure 14

SUMMARY

The use of super-regenerative receivers for the detection of nuclear magnetic resonance absorption has many advantages over other methods used for this purpose. Of these advantages we may list the following:

1. Sensitivity - Signals as weak as 20 microvolts are detectable.
2. Simplicity -
    a) It acts as an oscillator and a detector at the same time.
    b) Only one tube, a triode, need be used. Only tuning is by varying the capacity of the tank coil condenser.
    c) Simple circuit arrangements, a limited number of circuit parameters.
3. Side bands may be used as frequency marks to which other frequencies may be compared for frequency measurement,

However, the super-regenerative receivers have some disadvantages. Among these we emphasize:

1. Side bands confuse the signal patterns. Sometimes it is hard to tell the signal from side bands.
2. Measures absorption only, no dispersion effects are detected.
3. It takes very high precision and careful design to build up a satisfactory super-regenerative receiver.

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