EFFECT OF AIR DAMPING ON TRANSVERSE VIBRATIONS OF STRETCHED FILAMENTS

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

David Wesley Stauff

1954

This is to certify that the

thesis entitled

EFFECT OF AIR DAMPING
ON TRANSVERSE VIBRATIONS
OF STRETCHED FILAMENTS

presented by

DAVID WESLEY STAUFF

has been accepted towards fulfillment
of the requirements for

M. S. degree in PHYSICS

D. J. Montgomery
Major professor

Date 28 May 1954

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ON
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by

David Wesley Stauff

A Thesis

Submitted to the School of Graduate Studies of
Michigan State College of Agriculture and
Applied Science in partial fulfillment
of the requirements for the
degree of

MASTER OF SCIENCE

Department of Physics

1954

ACKNOWLEDGMENT

The author wishes to express his thanks to Dr. D. J. Montgomery for continued encouragement and advice.

David W. Stauff

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INTRODUCTION

The first experimental determinations of the laws of vibrating strings are credited to Mersenne (1636) and Galileo (1638). A practical application of these laws is made in the vibroscope, an instrument for finding the linear density of fibers and thence their equivalent diameter. The vibroscope consists of a device whereby a filament of known length under tension is induced to vibrate transversely by an applied oscillatory force, and the frequency of mechanical resonance observed. From the values of frequency, length, tension, and other parameters, it is possible to calculate the linear density and hence the equivalent diameter of the fiber, under the assumption of uniform density.

Gonsalves (1), apparently the first to apply the laws of vibrating strings to determination of linear filament density, gave the first-order stiffness correction, an approximation based upon the work of Seebeck (7). Montgomery (4) treated the combined effects of small stiffness and slight nonuniformities, basing his treatment on a generalization of Rayleigh's work (6). In these treatments external damping was neglected, so far as its effect on natural frequencies is concerned. Karrholm and Schroder (3) looked into the effect of air damping in their investigation of the bending modulus of fibers, and found that accurate results could be obtained with a resonance frequency method when the fiber diameter was large and the resonance frequency high.

With diameters substantially less than twenty microns, however, it is possible that corrections will be appreciable at the frequencies used in practical vibroscoping. It is the purpose of the present work to determine the existence of such corrections, and to establish the applicability of some of Stokes' theoretical work.

THEORY

For transverse vibrations of a uniform elastic string or wire of length ℓ, cross-sectional area S, density ρ_0, fixed at both ends, and under a constant tension T, the resonant frequency f_n is given in the n-th mode by

$$f_n = \frac{n}{2\ell}\sqrt{\frac{T}{\rho_0 S}}, \qquad n = 1, 2, 3 \ldots\ldots \tag{1}$$

As seen from the formula, the vibrating string has discrete natural frequencies of vibration, harmonically related, each resonant frequency f_n corresponding to the n-th allowed mode of vibration.

The validity of Eqn. (1) is dependent upon certain assumptions, the first of which is that the string is perfectly flexible. Filaments used in the present work were chosen long and flexible so as to make the effect of stiffness small and under control. According to Voong and Montgomery (9), who have investigated the effect in detail, a stiffness factor can be defined as

$$a \equiv \left(4EI/\ell^2 T\right)^{1/2} \tag{2}$$

where E (dynes/cm.^{2}) is the elastic modulus of the filament material, and I (cm.^{4}) is the moment of inertia of the filament about the neutral axis. In the work reported here, the largest stiffness correction was 2.2% under the assumption of circular cross-section. For our purposes, this effect can be considered negligible.

In the analysis for the stiffness correction, it was assumed that the filament was of uniform cross-section throughout its length. For actual filaments, nonuniformities certainly exist; however, it has been shown (4) that slight nonuniformities do not alter the stiffness correction in the approximation used above.

Amplitudes of vibration were purposely kept small, in conformity with the assumptions of the theory. It was found that a slight change in the resonant frequency did indeed occur if the amplitude became very large and accordingly the transverse displacement of the fiber was kept below 0.1 mm. from equilibrium position at a mid-point distance of five centimeters from the end of the fiber.

The most significant condition on Eqn. (1) as far as this work is concerned is the requirement that energy losses do not disturb the resonant frequency. Such losses may be traced to four sources: energy transferred to supports, internal damping, acoustical radiation, and air damping. The fiber is supported at the top by a large brass clamp which may be considered to have infinite mass as compared with the mass of the fiber. Hence there are no direct energy losses to it. The tension was supplied by the weight of an aluminum foil tab cemented to the bottom of the fiber. Wire-hook weights were hung on the tab when greater tensions were required. Though energy losses to this bottom support were not investigated directly in this work, no movement of the tab could be observed, and it was concluded that energy dissipated in this form was negligible. Internal damping exists in the filament but its effect is small relative to that of air damping, as shown by the greatly increased amplitude in a vacuum over that in air. No investigation of the extent of acoustical radiation has been made, but it seems unlikely that there can be very much energy radiated by a fine filament at small amplitude.

Air damping is the last source of energy loss, and the one with which the present work is primarily concerned. The effect on fibers of not much less than twenty microns in diameter is negligible, but it is perhaps appreciable when smaller fibers are employed. The calculation of air damping on a stretched filament appears not to have been worked out. Stokes (8) has worked on the theory for a related problem, however, in which he considers laminar fluid flow around an infinite cylinder in sinusoidal oscillation normal to its length. A Reynolds number calculation indicated that the flow was laminar, and on the basis of Stokes' results, the theory was worked out for the present case of a vibrating string under tension.

According to Stokes, the damping force (dynes/cm.) on a unit length of a cylinder (cf. Fig. 2) of radius a (cm.), and density ρ_0 (gm./cm.^{3}), vibrating in a fluid of density γ (gm./cm.^{3}) and viscosity μ (poise) is given by

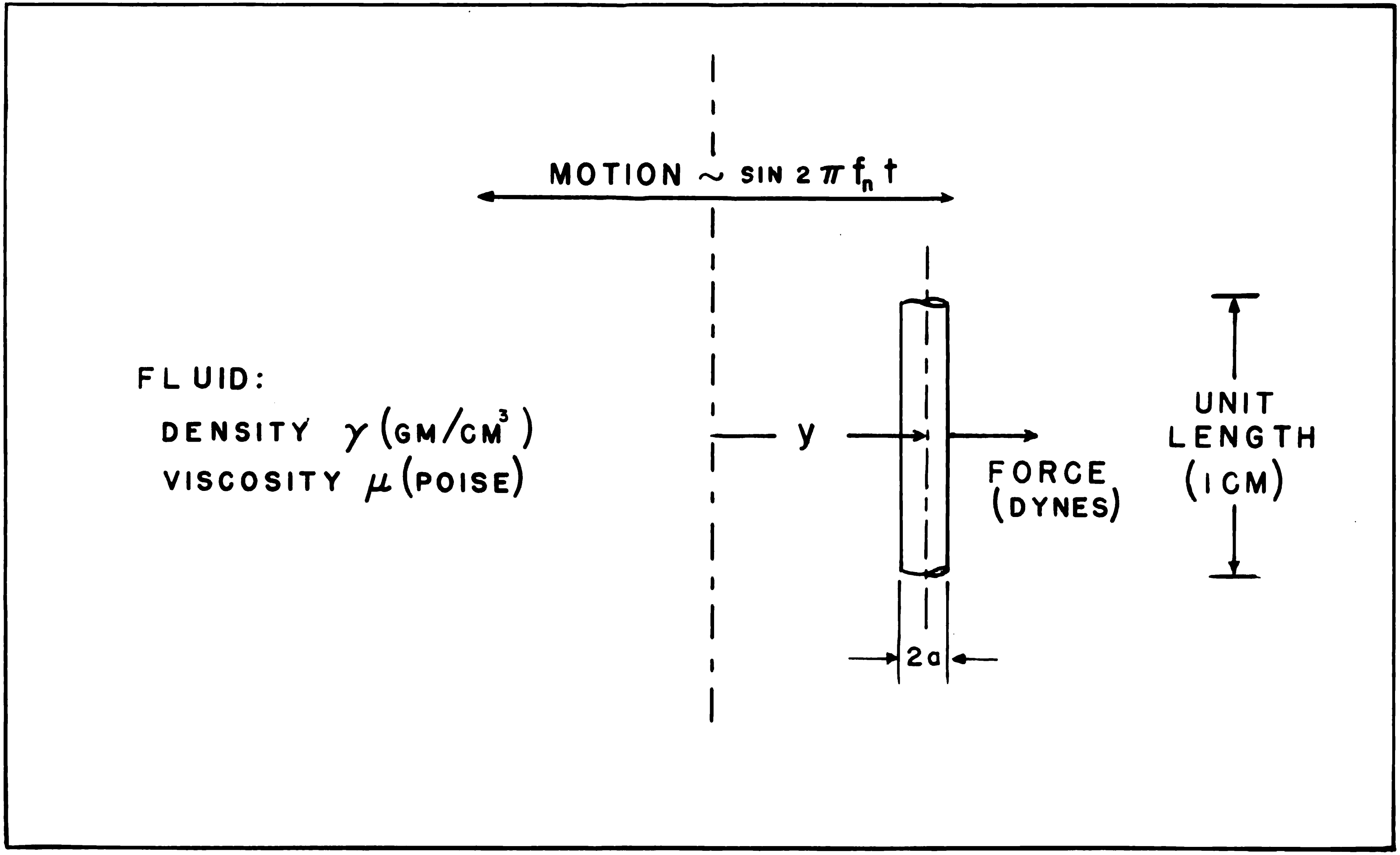

Figure 2. Cylinder vibrating in fluid.

$$F = -K_2 M \frac{d^2y}{dt^2} - K_1 M\omega \frac{dy}{dt}, \qquad (3)$$

where y is the displacement from the position of equilibrium, ω is the angular frequency $= 2\pi f$, M (gm./cm.) is the mass of the gas displaced $= \rho_0 S$, where S (cm.^{2}) is the cross-sectional area of the cylinder, and K_1 and K_2 are complicated dimensionless functions of N, where

$$N \equiv 4\mu / a^2 \gamma \omega. \qquad (4)$$

Karrholm and Schroder (3), following Ising (2), assumed the relative insignificance of the term containing the second deriv. in Eqn. (3). It appears from our work, however, that the term is of importance when very small values of a are considered. With the addition of this added damping force, Newton's Second Law applied to a vibrating string, becomes

$$T \frac{\partial^2 y}{\partial x^2} - R \frac{\partial y}{\partial t} + F_{(x)} e^{i\omega t} = (\rho_0 S + K_2 M) \frac{\partial^2 y}{\partial t^2} \qquad (5)$$

where T = tension (dynes), $R = -K_1 M\omega$, and $F_{(x)}$ is the driving force in dynes/cm. The $K_2 M$ term can be interpreted as an added mass of gas dragged along with the cylinder. Setting $(\rho_0 S + K_2 M) = \rho$, the solution of (5) is

$$y(x,t) = e^{i\omega t} \frac{\sum F_n e^{in\pi x/l}}{T\left(\frac{n\pi}{l}\right)^2 + (Ri\omega - \omega^2 \rho S)} \qquad (6)$$

l being the length of the cylinder (cm.), F_n the nth Fourier component of the driving force.

A point on the envleope of the curve y (x,t) at fixed x will have its maximum displacement, as a function of ω, when the absolute value of the denominator is a minimum. It is convenient to work with the square of the absolute value, and this quantity is

$$|D|^2 = \left[T\left(\frac{n\pi}{\ell}\right)^2 - \omega^2\rho_o S - \omega^2 K_2\gamma S\right]^2 + \left(K_1\gamma S\omega^2\right)^2. \tag{7}$$

The equation resulting from setting the derivative of $|D|^2$ to zero is intractable, but a good approximation may be obtained by writing

$$K_1(N) \equiv N\,\alpha(N)\,\phi(N) \quad \text{and} \quad K_2(N) \equiv N\,\phi(N), \tag{8a,b}$$

where ϕ and α are slowly varying functions of N, and hence of ω, and may be taken as constant in the differentiation. Figure 1 shows the dependence of ϕ and α on N. If we introduce ω_o, the fundamental resonant frequency in the absence of damping,

$$\omega_o^2 \equiv \left(\frac{\pi}{\ell}\right)^2 (T/\rho_o S), \tag{9}$$

the expression for $|D|^2$ becomes

$$|D|^2 = (\rho_o S)^2\left(\omega^2 - n^2\omega_o^2 + 4\pi\mu\phi\omega\right)^2 + \left(4\pi\mu\alpha\phi\omega\right)^2. \tag{10}$$

Differentiation of this expression with respect to ω, ϕ and α being taken as constant, leads to a cubic in ω. Instead of solving the resulting equation directly for ω_n, the n^{th} resonant frequency in the presence of damping, we write

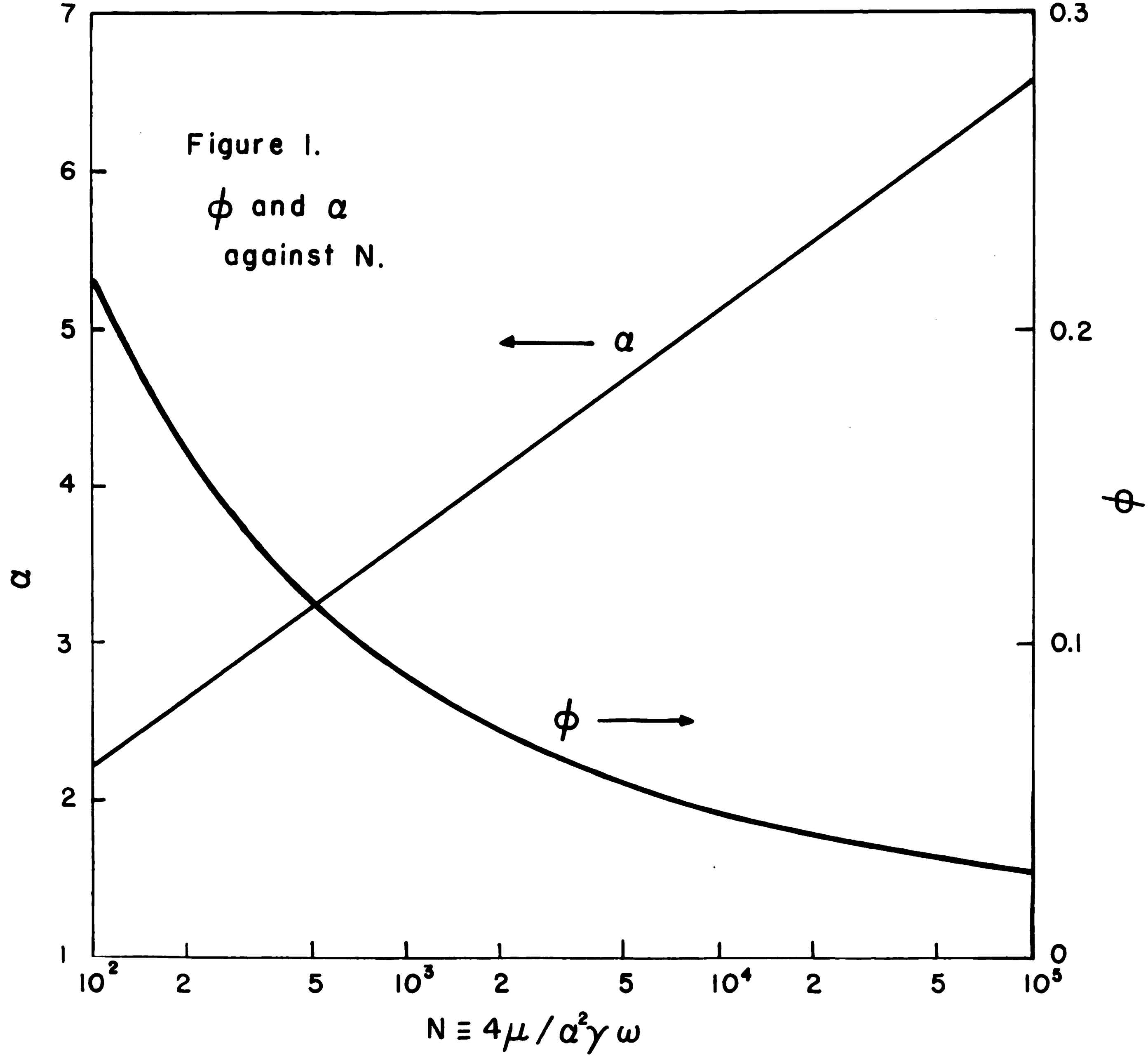

Figure 1.
ϕ and α against N.

$$\omega_n \equiv 2\pi f_n \equiv n\,\omega_o(1-\varepsilon). \tag{11}$$

Introduction of this expression into the cubic gives the following result for ε , the fractional frequency shift due to damping, to first-order terms:

$$\varepsilon = \left(\frac{\phi}{f_n}\right)^2\left(\frac{\mu}{\rho_o S}\right)^2\left[\alpha^2 + 1 + \frac{1}{\left(\frac{\phi}{f_n}\right)\left(\frac{\mu}{\rho_o S}\right)}\right]. \tag{12}$$

We write

$$\frac{\phi}{f_n}\cdot\frac{\mu}{\rho_o S} \equiv \lambda\ ; \tag{13}$$

then

$$\varepsilon = \lambda^2\left[\alpha^2 + 1 + \frac{1}{\lambda}\right]. \tag{14}$$

Now α^2 runs from 10 to 50 in the range of interest, and it is usually possible to neglect unity in comparison with it with small error in the correction; then

$$\varepsilon \simeq \lambda\left[1 + \alpha^2\lambda\right]. \tag{15}$$

It turns out that $\alpha^2\lambda$ is usually less than unity, and an upper limit on the relative frequency shift ε is then given by Eqn. (15) with $\alpha^2\lambda = 1$:

$$\varepsilon \sim 2\lambda = \frac{2\phi}{f_n}\ \frac{\mu}{\rho_o S}. \tag{16}$$

For vibroscoping under ordinary conditions in air at normal temperature and pressure, for a frequency shift of 1 cycle/sec, the linear density $\rho_o S$ is 10 to 20 μ gm/cm. Hence to get a readily measurable shift, it is necessary to work with filaments finer than 10 μ gm/cm, corresponding to diameters less than 0.0010" for glass or aluminum, 0.0005" for copper, and 0.0003" for gold or tungsten.

Then the relation (15), or its approximate form, Eqn. (16), can be checked by varying μ , in going from air to hydrogen, say, at atmospheric pressure, or from air to vacuum; by varying ρ_0 , in working with different materials; and by varying the cross-sectional area of the filaments. All of these methods were used.

APPARATUS AND PROCEDURE

The electrostatic form of vibroscope used is essentially a "string electrometer," and as such is similar to that described by Montgomery and Milloway (5), (see Fig.3). The fiber was suspended inside of a glass tube to facilitate observations in different gases at varying pressures. Since the fiber was observed with the vacuum pump in operation, a plenum chamber and capillary were included in the system (Fig.4) to isolate line fluctuations from the fiber. Three pairs of curved brass electrodes, diametrically opposed on the sides of the tube, were connected to an audio oscillator through the circuit shown in Figure 3. A switchboard made it possible to apply the output of the oscillator to any pair of plates in a phase relationship suitable to excite desired modes of vibration. The length of the fiber was measured with a cathetometer, and the amplitude of vibration was measured on a reticle of the cathetometer telescope. Figure 5 is a photograph of the apparatus.

Filaments used were of tungsten, glass, copper, and gold, ranging from 2 to 12 microns in diameter and from 2.5 to 19.3 gm./cm.^{3} in density. The filaments were vibroscoped in air at normal pressure and at reduced pressures ranging from 29 to 50 μ Hg.

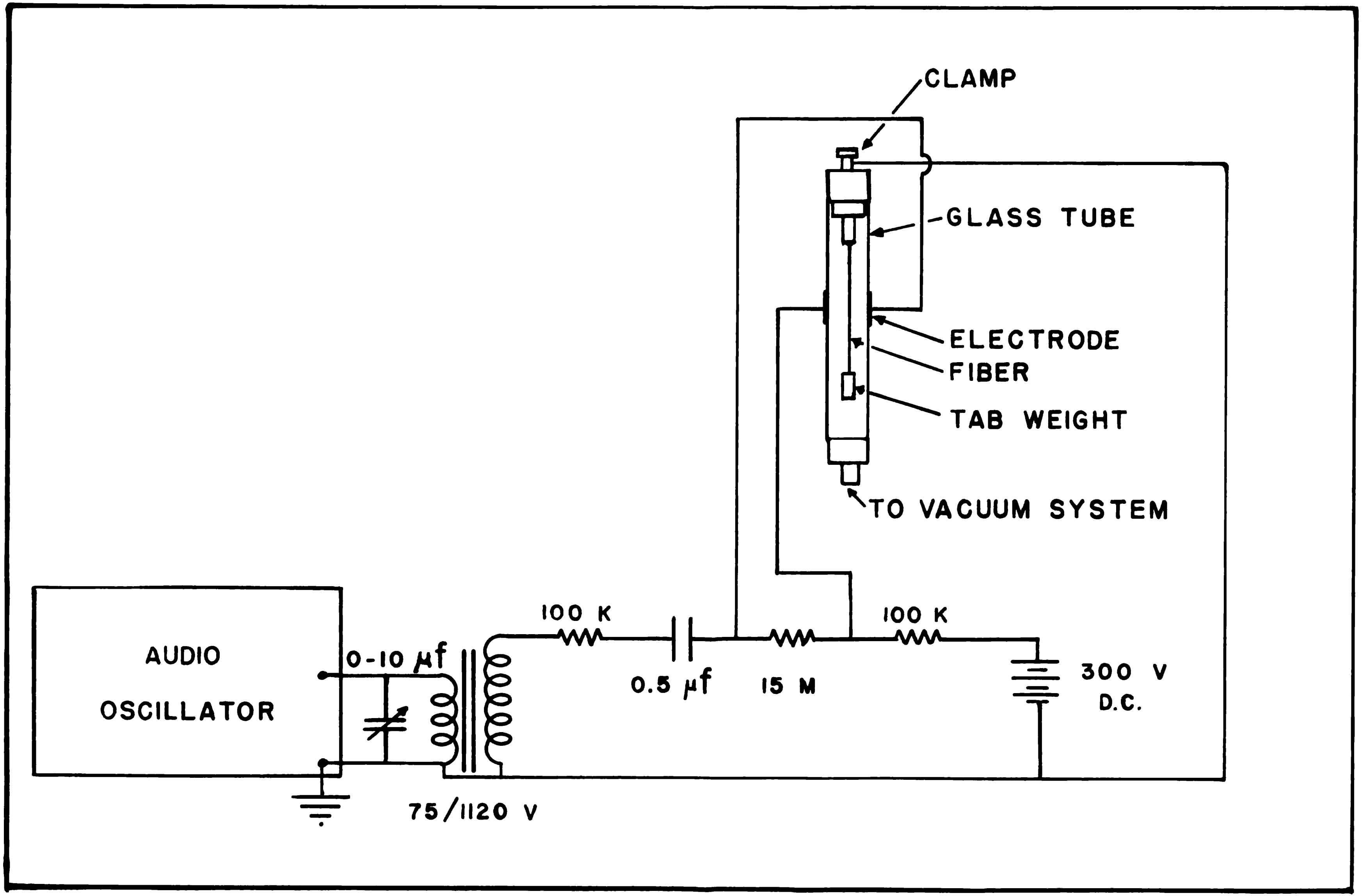

Figure 3. Schematic diagram of electrostatic vibroscope.

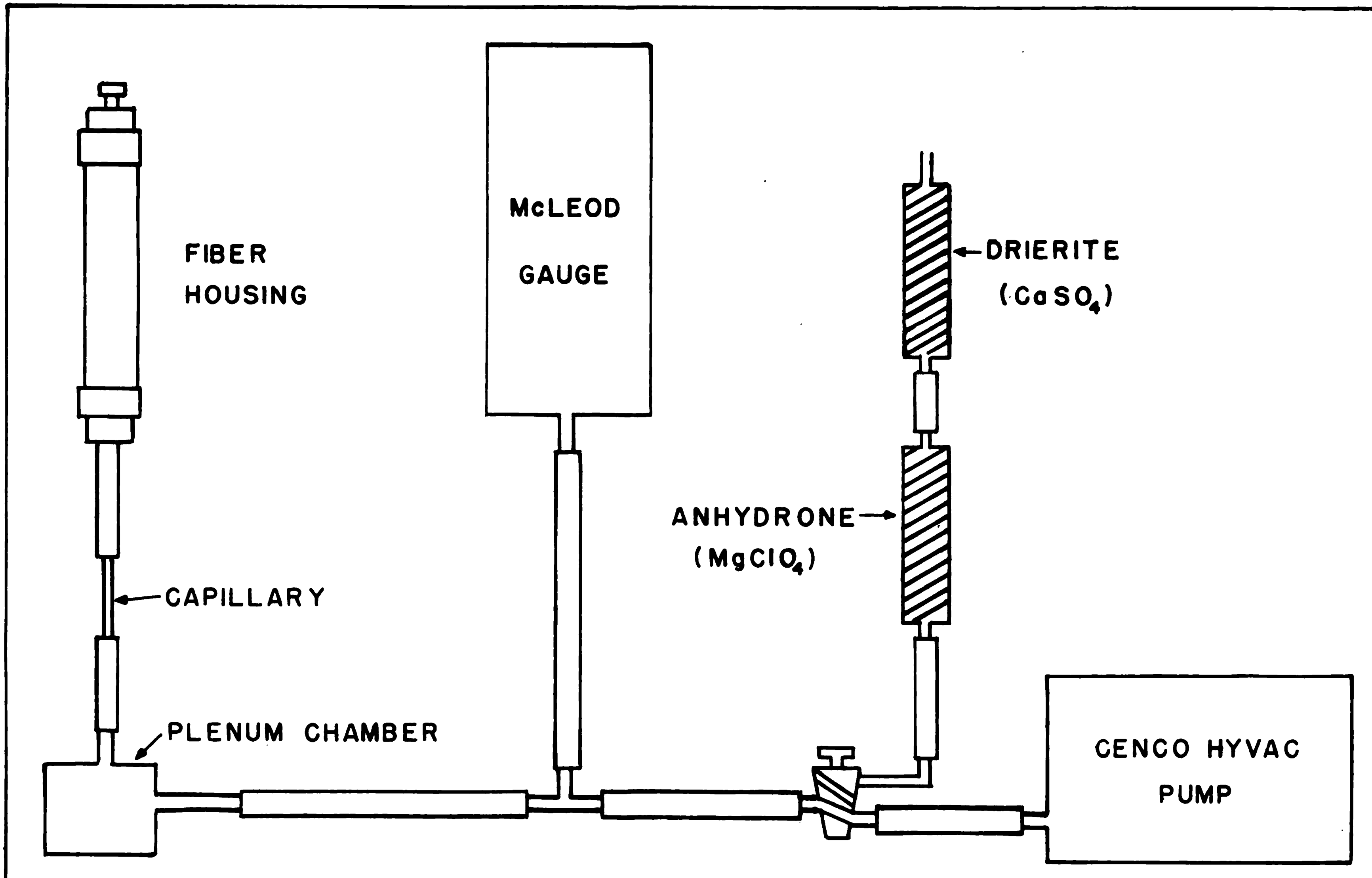

Figure 4. Block diagram of vacuum system.

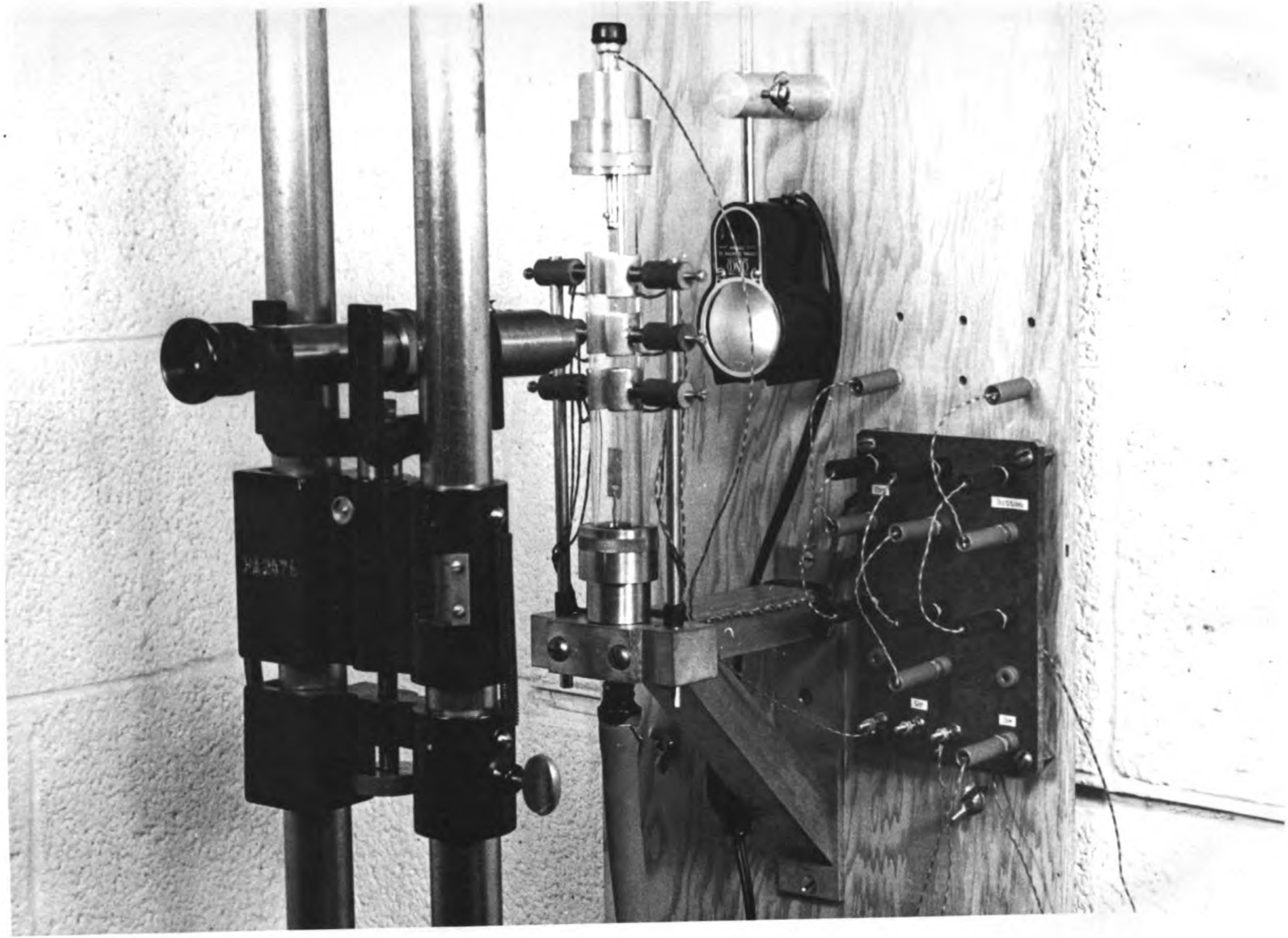

Figure 5. Electrostatic vibroscope.

Since the viscosity of a gas is nearly independent of pressure so long as the mean free path is very small compared with the relevant dimensions of a body in motion relative to the gas, it is necessary to go to quite low pressures to insure the absence of viscosity effects. At 30 μ Hg, the mean free path for air molecules is greater than a millimeter, and hence viscosity effects should be completely negligible for filaments of the diameters used. To establish this conclusion experimentally, the pressure was increased to 50 μ Hg. There was no shift in resonant frequency, and it appears safe to take the observed shift from atmosphere to 30 μ Hg as the same as the shift to a complete vacuum. Some of the fibers were also vibrated in hydrogen at one atmosphere, the viscosity of this gas being about half that of air. Columns of Drierite ($CaSO_4$) and Anhydrone ($MgClO_4$) were incorporated in the system to insure the dryness of the gas admitted.

Some difficulty was experienced in getting a bias charge on the glass fibers, and 600 volts D.C. had to be applied. Three hundred volts was sufficient bias for the other fibers. It was decided to work with fine metal fibers since they can be obtained in series of more or less known diameters of a high degree of fineness. For ease in handling, wires of large diameter and hence of low density were desired. Aluminum came first to mind, and since it does not draw too well, it had to be used in the form of Wollaston wire. Great difficulty was experienced in removing the copper jacket without the aluminum core breaking up into small unsuitable pieces. Finally it became necessary to abandon the aluminum wire and concentrate on gold, from which the silver jacket could be removed quite easily. The copper wire used was bare drawn, 0.0004 in. diameter being the smallest size available.

Tungsten filaments were best observed directly in front of the source of illumination, in this case a microscope lamp. Glass filaments were seen more readily in transmitted light and the light from the lamp was therefore directed at the fiber obliquely to the line of sight. Gold and copper showed up best under reflected light, a small desk lamp alongside the cathetometer being used in this case.

All fibers were cemented to an aluminum foil tab at the bottom and hung from the brass clamp at the top. Increased tension was obtained by hanging copper wire hooks on the tab. Measurements of the mass of the hooks and tabs (with fibers attached) were made on an analytical balance.

RESULTS

Figure 6 is an example of the observations. Normalized amplitudes of vibration of a glass filament vibrated in air at one atmosphere, hydrogen at one atmosphere, and vacuum (29 μHg), are plotted as a function of frequency. A shift to higher frequency and a corresponding sharpening of the resonant peak are noted as the viscosity of the surrounding fluid is decreased. This shift was quite well defined in the case of glass, and the resonant frequencies here are at 252, 263, and 268 cps for air, hydrogen, and vacuum respectively. The normalization of the amplitude to unity at resonance conceals the great increase in amplitude in vacuum over that in air at normal pressure. From air at normal pressure to vacuum, the calculated shift is 11 cycles/sec., whereas the observed value is 16 as seen from Fig. 6. For the shift from air to hydrogen, the calculated shift is 9, whereas the observed value is 11. It appears that the theory gives roughly the value of the shift, but that the predicted value is perhaps a third low.

Table I gives values of calculated and observed frequency shifts for various fibers vibroscoped in air at normal pressure and in a vacuum. Similar data for fibers vibrated in hydrogen at atmospheric pressure and in a vacuum is recorded in Table II. Observed shifts are of relatively low precision: points on

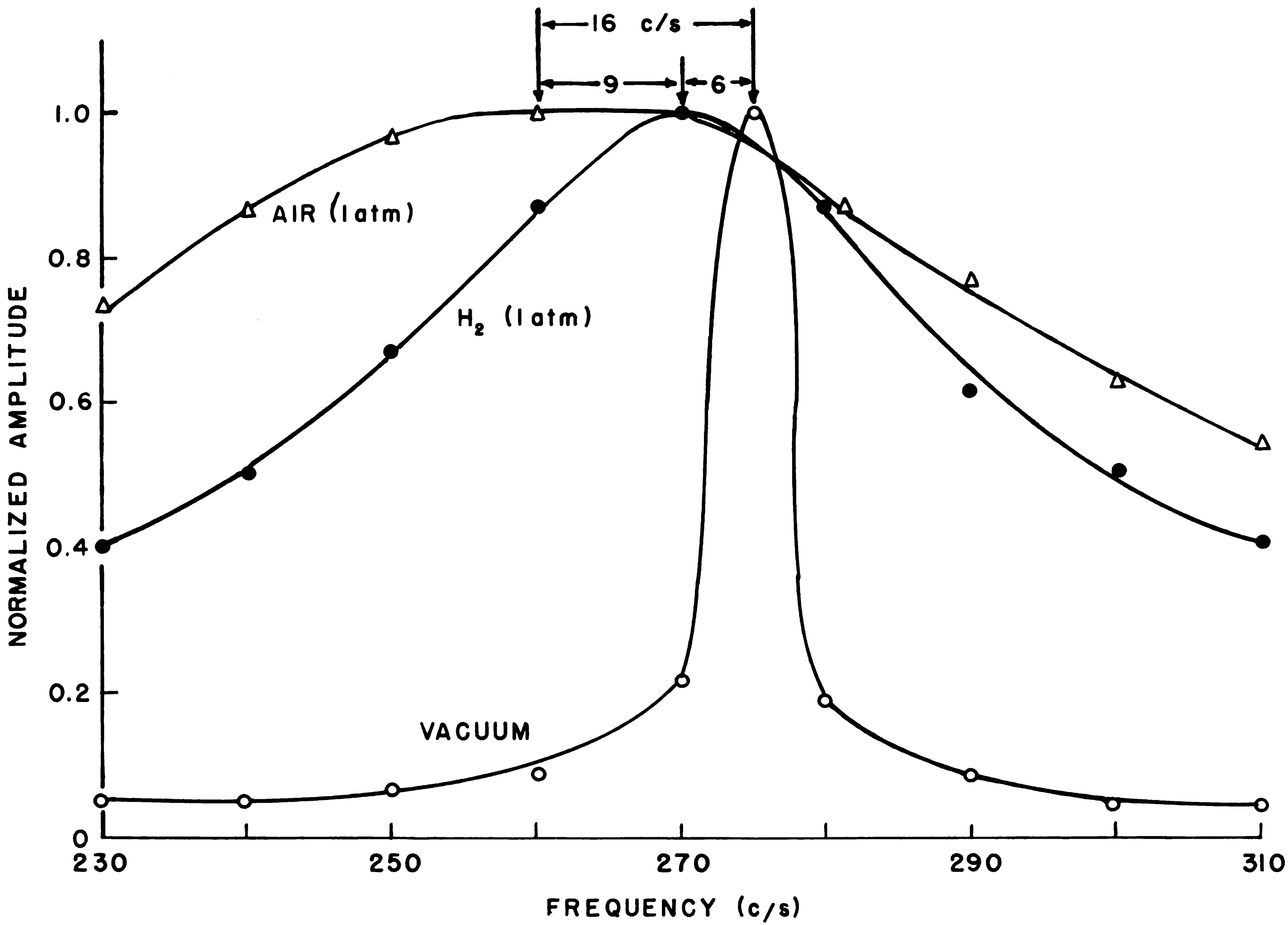

Figure 6. Response curves for glass filament.

the frequency curves were repeatedly observed to be reproducible to about one cycle/sec., except with very fine fibers. The shift is small in most cases, and a difference is being observed. The oscillator, a Hewlett-Packard Model 200-D, may have drifted slightly during the course of the experiment. Observations of the Lissajous figures of the oscillator output against a line frequency on a cathode ray oscilloscope served as a method of calibration. The line frequency, checked against a tuned circuit frequency meter was not found to drift appreciably. Eqn. (1) was used to check the diameter of each fiber (column "a," Tables I and II), which in most cases varied slightly from manufacturer's specifications. Length measurements made with the cathetometer were probably accurate to 0.5%. The cathetometer was capable of much higher precision, but the point of attachment of the fiber to the tab cannot be precisely determined. Error due to the values of tension and density, as applied in (1), were probably not more than 0.3% and 1.0% respectively.

Some of the discrepancy may perhaps be attributed to innacuracy in frequency readings; but the observed values are consistently high, and we must conclude that there exists some effect not considered in the theory. It is unlikely, however, that the error in predicted values will be more than one or two cycles/sec. low in the case of small shifts or more than 20 or 30% low in the case of large shifts.

TABLE I

RESONANT FREQUENCY SHIFTS FROM AIR (1 atm.) TO VACUUM

Nom. Dia.	T (mg)	f (c/s)	ℓ (cm)	a (μ)	ε (%)	Δf calc. (c/s)	Δf obs. (c/s)
Gold (19.3 g/cm^3)							
0.0003"	72.9	99	10.53	5.18	0.7	0.7	2
"	25.6	58	10.53	5.24	1.1	0.7	1
"	25.6	62.5	9.80	5.24	0.9	0.6	1
0.0002"	74.1	261	8.00	2.61	1.0	2.6	7
"	25.8	119	10.00	2.72	2.0	2.4	3
0.0001"	6.4	239	5.88	1.14	6.9	16.5	20
Tungsten (19.0 g/cm^3)							
0.0005"	36.6	56	10.13	6.77	0.8	0.4	0
Copper (8.89 g/cm^3)							
0.0004"	238.4	227	9.45	6.73	0.6	1.3	2
"	147.5	179	9.45	6.70	0.7	1.3	2
"	50.9	106	9.45	6.65	1.1	1.1	2
Glass (2.5 g/cm^3)							
0.0004"	51.0	268	10.11	4.62	4.2	11.1	16

TABLE II

RESONANT FREQUENCY SHIFTS FROM H_2 (1 atm.) TO VACUUM

Nom. Dia.	T (mg)	f (c/s)	l (cm)	a (μ)	ε (%)	Δf calc. (c/s)	Δf obs. (c/s)
Gold (19.3 g/cm^3) 0.0003"	25.6	61.5	9.80	5.33	0.4	0.2	1
Tungsten (19.0 g/cm^3) 0.0005"	36.6	56	10.13	6.77	0.8	0.4	0
Glass (2.5 g/cm^3) 0.0004"	51.0	263	10.11	4.62	0.9	2.4	5

CONCLUSIONS

Experimental observations of the effect of air damping of transverse vibrations of stretched filaments of glass, tungsten, copper, and gold by the vibroscope method lead to the following conclusions:

1. Air damping will have an effect on the natural frequency and amplitude of vibration of fine fibers. Under ordinary conditions the frequency shift is less than one cycle/sec. for fibers of linear density greater than 10 micro-gm/cm, but for finer fibers it becomes of increasing importance.

2. Stokes' theory works well enough to allow rough calculations to be made for the effect of air damping in measurements of the resonant frequency made with a vibroscope; but the observed shift seems to be about one or two cycles/sec. higher than calculated in the case of small shifts or about 20 or 30% higher in the case of large shifts.

3. A rigorous evaluation of the applicability of Stokes' theory will require further investigation.

LITERATURE CITED

1. Gonsalves, V. E., Textile Research Journal 17, 369 (1947).

2. Ising, G., "Undersokning rorande elektrometrar," Lund, Sweden, Glecrupska Universitets bokhandeln, Part II, 281-3 (1919).

3. Karrholm, E. M., and Schroder, B., Textile Research Journal 23, 208-14 (1953).

4. Montgomery, D. J., J. Applied Physics 24, 1092 (1953).

5. Montgomery, D. J., and Milloway, W. T., Textile Research Journal 22, 729 (1952).

6. Lord Rayleigh, "The Theory of Sound," vol. I, 2nd ed., London, Macmillan, 1894, p. 113ff; reprinted by Macmillan in 1926 and 1929, and by Dover Publications, Inc., New York, in 1945.

7. Seebeck, A., Abhandl. math.-phys. Klasse sachs. Akad. Wiss. (Leipzig) 1, 131 (1852).

8. Stokes, G. G., "Mathematical and Physical Papers," vol iii, Cambridge, University Press, 1901.

9. Voong, E. T. L., and Montgomery, D. J. Textile Research Journal 23, 821-30 (1953).