ABSOLUTE INTENSITY
MEASUREMENTS OF FRESNEL
DIFFRACTION PATTERNS

Thesis for the Degree of M. S.
MICHIGAN STATE COLLEGE
J. Guy Woodward
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INTRODUCTION

Early in the 19th century Fresnel developed and published his theory of diffraction. The theory has been used to predict the distribution of energy in all types of diffraction arising from a cylindrical wave front. In comparing the theoretical diffraction patterns with the observed patterns, Fresnel and other early experimenters were forced to limit their work to visual observations of the positions of the lightest and darkest parts of the patterns because of a lack of instruments to measure light intensity with any degree of accuracy. In recent years, with the accompanying development of photoelectric cells and methods of measuring small electric currents, it has become possible to make an accurate determination of the energy distribution in any diffraction pattern and hence to test Fresnel's theory for the intensity relationships as well as for the linear extent of the patterns.

The results of the first successful attempt at making such a measurement were published by Lyman of Harvard University in 1929[1]. This work consisted in photographing a pattern and determining its contour from a microphotometer trace of the photographic plate. The plates used were carefully calibrated by means of an exposure-density curve made at the same time. By this method Lyman showed the intensity ratios of maxima and minima in a measured straight-edge pattern to check within 4% of the corresponding ratios taken from the theoretical pattern. In the summer of 1937 McClellan at Michigan State College[2] did some preliminary work in which he found an agreement within about 10%, except at very low values of intensity, between the measured and calculated diffraction patterns due to a single narrow slit.

FRESNEL'S THEORY

It is not within the purpose of this discussion to develop Fresnel's theory of diffraction. Such a development may be found in almost any textbook of physical optics[3], and only the final result of the derivation is given here. Fig. 1 represents in cross section the optical system necessary for the diffraction of light in passing through a single slit. L is a line source of monochromatic light which travels out in all directions giving a cylindrical wave front S. If at Q, at a distance a from L, a slit is placed symmetrical with the optical axis of the system, the cylindrical wave front striking it will be diffracted forming a pattern which can be seen in the plane of observation. According to Fresnel the intensity at any point P' in this plane is given by $I \propto x^2 + y^2$ where

$$x = \int_0^v \cos \frac{\pi v^2}{2} dv$$

$$y = \int_0^v \sin \frac{\pi v^2}{2} dv$$

and $$v = s\sqrt{\frac{2(a + b)}{ab\lambda}} ,$$

λ being the wave length of the monochromatic light, s the length of wave front used, and a and b the distances indicated in Fig. 1. Thus we see that the relative intensities of different parts of a pattern can be predicted providing the dimensions used are known. The linear spread of the pattern is determined by the following equation in which d is the distance of P' from the axis:

$$d = v\sqrt{\frac{b\lambda(a + b)}{2a}}$$

These are the equations which will be used later to calculate the theoretical diffraction patterns. Before they can be used, however, the Fresnel Integrals shown above must be evaluated. This work has been done by a number of men using several different methods. The most complete table of these integrals, running from 0 to 8 in steps of .005 for values of v, has been compiled by Sparrow at the University of Virginia[4].

With a single slit the only part of the cylindrical wave front which is effective in sending light to the screen is a length Δs which, for ordinary slit widths, is essentially the same as the width of the slit. Corresponding to Δs there is a Δv given by

$$\Delta v = \Delta s\sqrt{\frac{2(a + b)}{ab\lambda}}$$

The first step in applying the theory to a given case is to determine the value of Δv. This gives the limits for the Fresnel Integrals. For this value of Δv the two values of x at the ends of the interval are read off and subtracted algebraically to give Δx. The same process is followed to obtain Δy. These are squared and added to obtain the value of the intensity for the midpoint v.

METHOD OF MEASUREMENT AND EQUIPMENT USED

The energy distribution in the various diffraction patterns was measured by means of a Visitron type 53-AV photoelectric cell mounted on a carriage which was moved by a lead screw of 3/8 inch diameter and 1 mm. pitch. The cell was moved across a pattern and readings of the photoelectric current produced were taken at frequent intervals. To obtain resolution a narrow slit aperture was always used directly in front of the photocell in the path of the light. The light sensitive device was mounted at one end and the diffracting slit at the other end of a light-tight camera approximately 570 cm. long. The camera consisted of a sheet iron tube 8 inches in diameter terminating in a wooden box 2 meters long at the photocell end. In the end of this box could be placed either a translucent screen for visual observation of a pattern, or a plate holder for use in photographing the pattern, or the metal box containing the photoelectric cell for a direct measurement of the pattern. To eliminate troublesome scattered light in the camera, cardboard baffles were placed at suitable positions inside the tube.

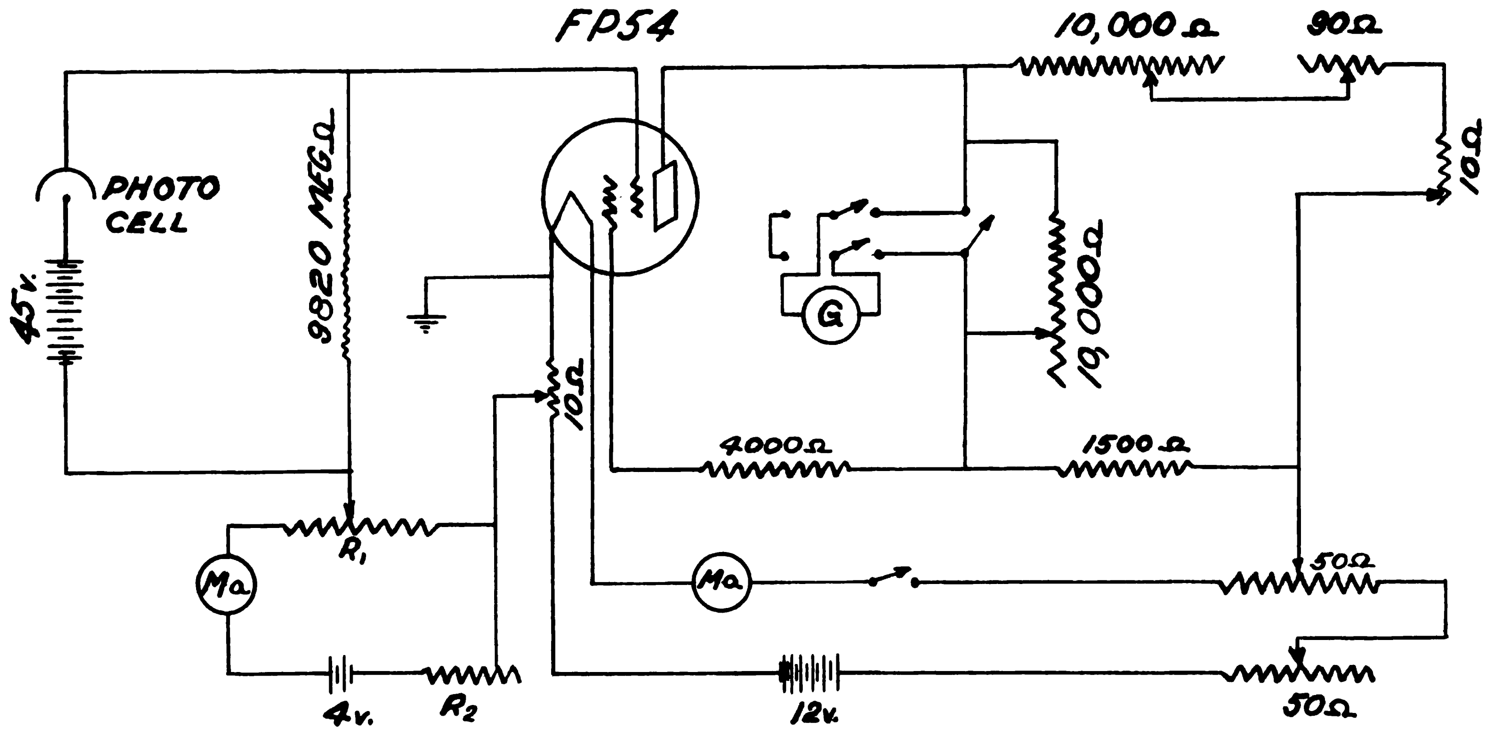
The line source of light postulated by the theory was approximated by using a narrow slit illuminated from behind by a mercury vapor arc. This arc was a General Electric lab arc No. H-3 of the capillary type. The light from it passed through three glass filters to be rendered monochromatic, the filters being Corning G34Y and Cambridge Botanical Supply Nos. 8 and 16. More will be said about this source in a separate section. The source slit was on the optical axis and about 50 cm. from the diffracting slit.

Two thin, steel straight-edges about 8 cm. long, one mounted firmly on the end of the camera and the other held against the first by two rubber band springs, served as the diffracting slit. Different slit widths were obtained by inserting between the two straight-edges small but accurately ground roller bearings whose diameters were known. For each slit width two such separators of the same diameter were used, one at the top and the other at the bottom of the slit. The rubber bands prevented both the separators and the movable straight-edge of the slit from slipping. The source as well as the end of the camera holding the diffracting slit was in one room while the photocell and controls were in another at the opposite end of the camera. During a set of measurements it was necessary to take several readings with the photocell dark. This was made easy by placing a shutter device directly before the diffracting slit, which shutter could be controlled by the operator at the far end of the camera.

The greatest difficulty encountered in this work was the procuring of diffraction patterns wide enough to permit resolving the fine structure and yet of sufficient intensity to produce a measureable photoelectric current. The length of camera used was found by trial to be about the optimum value as a compromise between intensity and resolution. While the mercury vapor arc was an extremely bright source, the light from it passed through three glass filters, three narrow slits, and six meters of air before reaching the sensitive surface of the photoelectric cell. The resulting currents were quite feeble, the greatest current measured in any pattern, with one exception, being less than 5 x 10^{-13} ampere with all measured currents lying between 10^{-15} and 5 x 10^{-13} ampere.

Such currents are, of course, far below the range of any galvanometer. Since a quadrant electrometer would have been very inconvient for this work a vacuum tube amplifying circuit was resorted to. A General Electric FP54 electrometer tube was used in the DuBridge-Brown[5,6] balanced circuit. The schematic diagram together with the constants of the circuit is shown in Fig. 2.

The FP54 tube, the grid input resistor, and two 22-1/2 volt "C" batteries were all mounted in the same metal box with the photoelectric cell. All other controls except R_1 and R_2 were mounted in a separate copper box which served as a shield. The tube and the control box were connected by a flexible shielded cable. R_1 was a Leeds-Northrup Students' Potentiometer. It was calibrated to read directly in volts by means of R_2 and the milliammeter in series with it, the potentiometer having been previously checked against a Weston Standard Cell to find the correct value of the current. All batteries, with the exception of the two "C" batteries supplying the accelerating potential for the photocell, were heavy duty lead plate storage batteries. The galvanometer, a Leeds-Northrup type H-52290 operating at about 10^{-11} ampere per millimeter, was mounted on a heavy iron block supported by sponge rubber pads



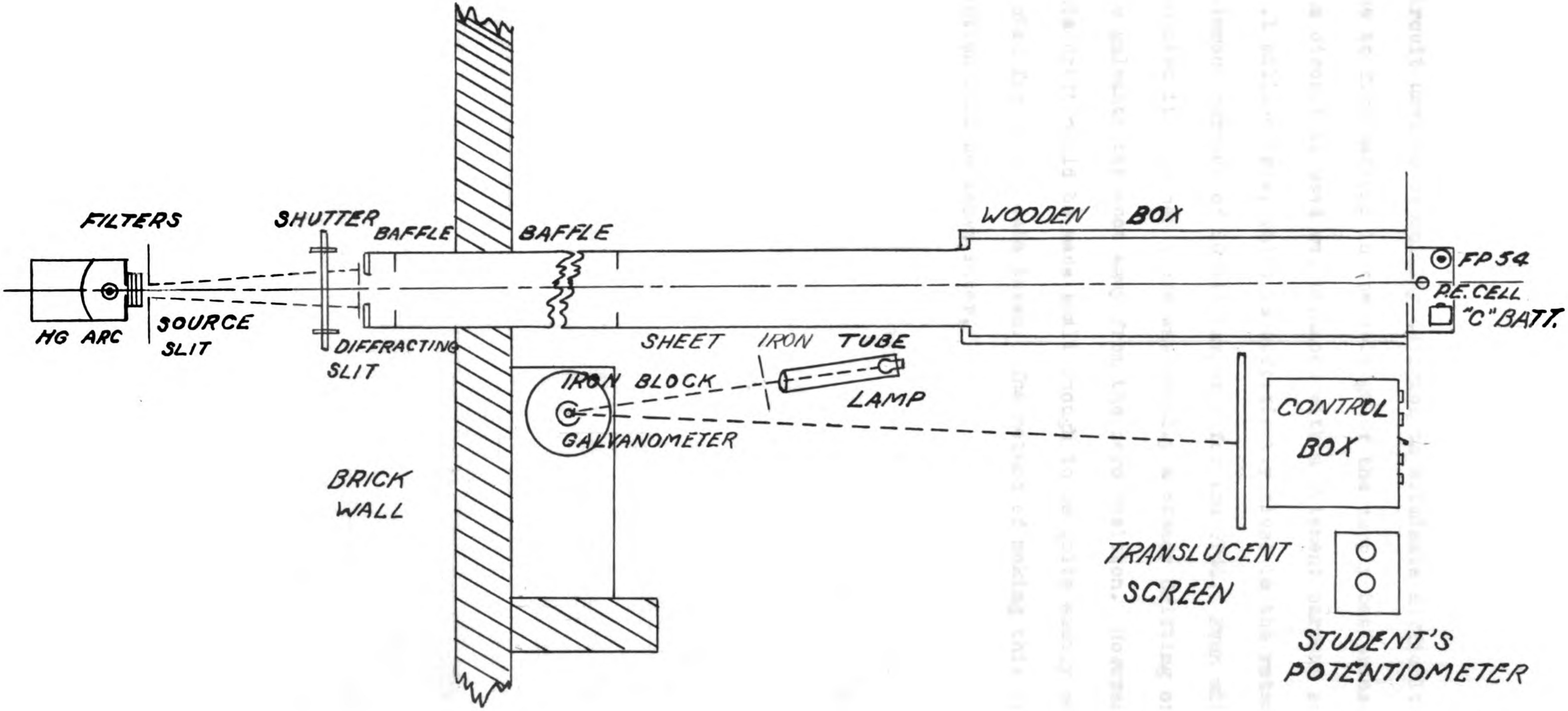
DU BRIDGE ~ BROWN BALANCED CIRCUIT

FIG. 2

on a rigid shelf at one end of the room. It was quite free from disturbing vibrations. A narrow line of light, from a specially constructed lamp incorporating a lens and a 32 C.P. automobile headlamp bulb, was reflected from the galvanometer mirror onto a translucent screen and scale at the operator's desk at the far end of the room, giving a light lever of about 4 meters. Fig. 3 shows the complete set-up of apparatus.

With the set-up and equipment described in the two preceding paragraphs the sensitivity to changes in grid potential of the FP54 was 5.5×10^{-6} volts per millimeter as registered by the galvanometer spot on the translucent screen. With the 10^{10} ohm input resistor this voltage sensitivity corresponded to a current sensitivity of 5.5×10^{-16} ampere per millimeter. The circuit was used as a null instrument. The zero position of the galvanometer was first found, then during a set of measurements the galvanometer spot was kept on the same place on the scale. This was accomplished by adjusting the Students' Potentiometer to compensate for the potential drop across the input resistor with a given photoelectric current. The millivolt range of this potentiometer was used.

A great many troubles, large and small, were met in building up the FP 54 amplifying circuit, but this is not at all an unusual occurrence in working with electrometer tubes. However, if all connections are good, the resistors are dependable, the batteries are fully charged, and the shielding of the circuit is complete this method can be used to great advantage in measuring currents too small for a galvanometer. The DuBridge-Brown



ARRANGEMENT OF APPARATUS

FIG. 3

circuit used is especially designed to eliminate difficulties due to fluctuations in the voltage of the supply batteries. The circuit as used was balanced with a filament current of 86.1 milliamperes, which is sufficiently close to the rated filament current of 90 milliamperes for the FP54. Even with the circuit balanced there was usually a steady drifting of the galvanometer spot away from the zero position. However, this drift could be made small enough to be quite easily corrected for in the data taken. The method of making this correction will be shown later.

LINEARITY OF PHOTOELECTRIC CELL

Although theory tells us that the photoelectric current should be proportional to the intensity of the light falling on the sensitive surface, it was thought wise to test the Visitron 53-AV cell to ascertain the kind of response it gave throughout the range of intensities measured in diffraction patterns. This was done by the inverse square method using the familiar fact that light intensity is inversely proportional to the square of the distance from the source.

The box containing the photocell and its associated equipment was mounted at one end of an optical bench two meters long. The photoelectric currents were measured with a light source placed at different positions on the optical bench. These measurements were made by the same null method described in the last section. The source used here consisted of a 50 watt Mazda lamp in a light tight container which had a small slot (about 1/8 inch by 1/2 inch) opened in one side. This slot was covered by layers of blank newsprint paper. The paper served the double purpose of giving a diffuse source of light and of cutting down the intensity to desired values. When several readings of intensities and distances had been taken over the two meter range of the bench, another layer of paper would be added to give a less intense light and the range covered again with this new source.

In this manner the photocell was checked for values of grid voltage changes from 2.5×10^{-4} to 400×10^{-4} volts. To do this it was necessary to use from one to ten layers of paper. The cell response was found to be strictly linear throughout.

Some of the data taken in this way for the range from 2.5×10^{-4} to 160×10^{-4} volts are tabulated in Table I and shown graphically in Fig. 4. All intensities measured in diffraction patterns lay within this range.

Table I

Linearity of Photoelectric Cell

$I = k/d^2$

I = intensity of light

d = distance from source

k = constant

V = photocell response measured by change of grid potential in volts $\times 10^5$

Subscripts of V indicate the number of layers of paper covering the source.

d	k/d^2	V_5	V_8	V_{10}
46 cm.	472	2390	570	258
56	319	1600	379	172
66	230	1140	280	129
86	135	680	164	74
116	74.3	362	107	53
186	28.9	---	---	24

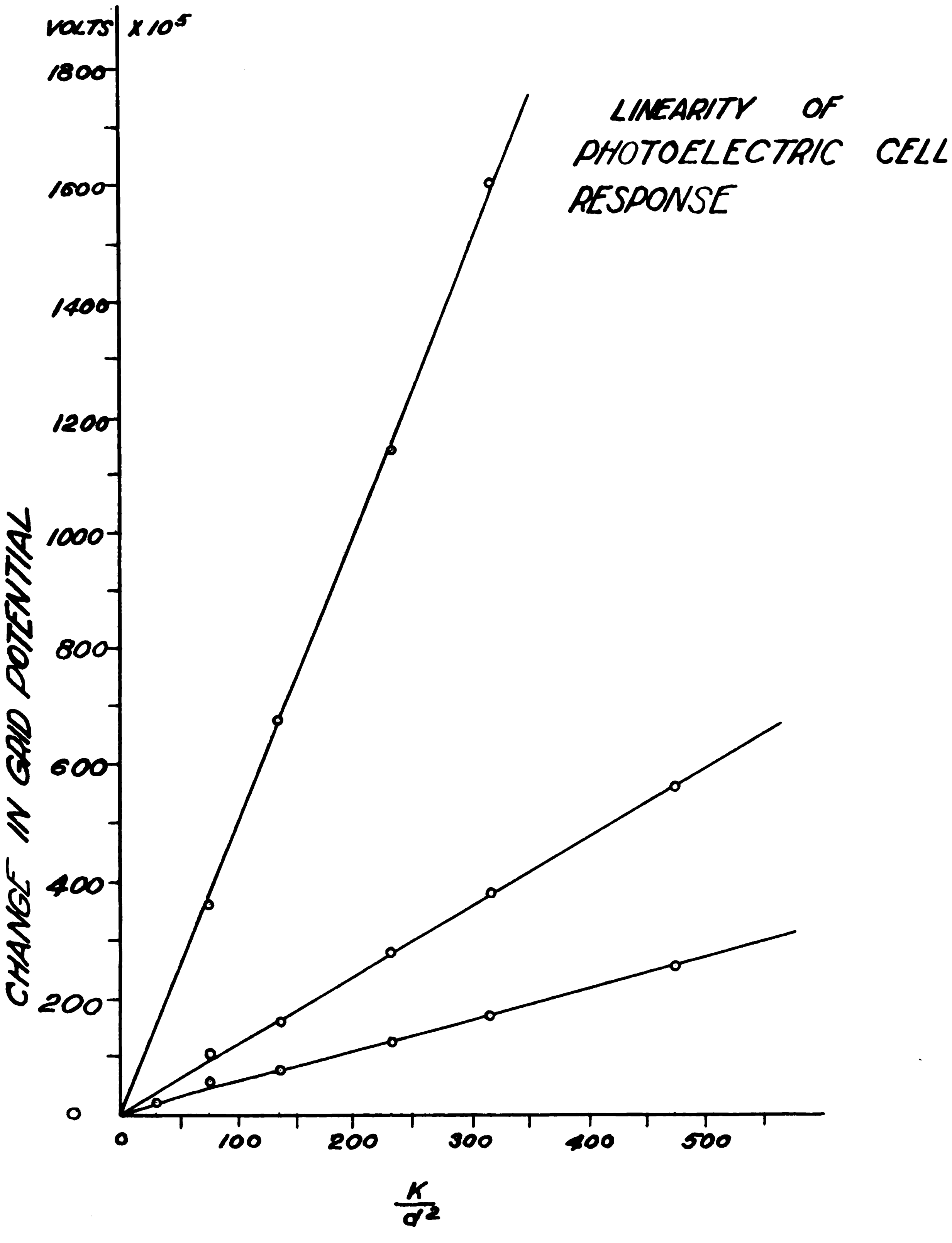

LINEARITY OF PHOTOELECTRIC CELL RESPONSE
VOLTS X 10⁵
1800
1600
1400
1200
1000
800
600
400
200
0
CHANGE IN GRID POTENTIAL
0
100
200
300
400
500
K/d²

Fig. 4

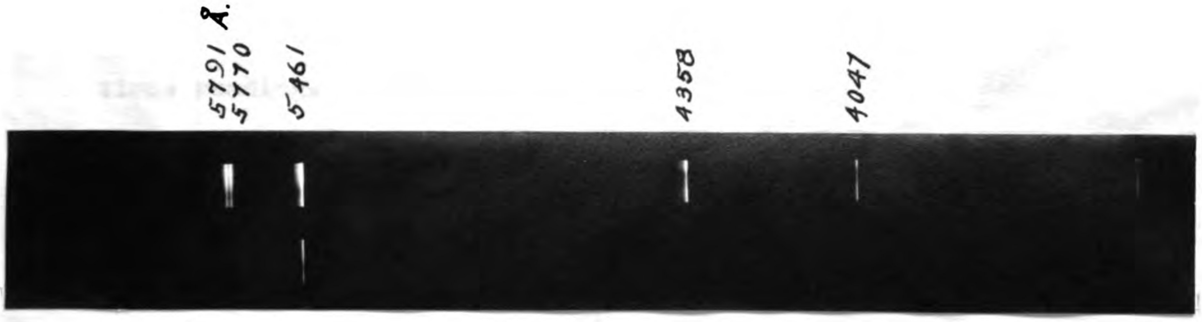
SOURCE

Fresnel's theory postulates a monochromatic source. The mercury vapor arc and filters used in this experimental work come very close to this ideal. The spectrum of mercury shows that there are no strong lines within about 50 Ångstroms of the very strong 5461 Å. line. The system of glass filters used will transmit a narrow band in the neighborhood of this line. Since it is the only strong line in that region, the light passing through the filters will be highly monochromatic with the wavelength 5461 Å. There will, of course, be a little light of other wavelengths coming through, but the amount is so small as to be negligible in comparison with the accuracy of the other measurements.

The three filters work as follows: The Corning G34Y cuts out everything on the blue side of the 5461 Å. line; the Cambridge Botanical filter No. 8 cuts out a band about 200 Å. wide on the red side immediately adjacent to the 5461 Å. line; the No. 16 filter cuts out all the red which is beyond the band cut out by the No. 8. All three will transmit the 5461 Å. line. Two spectrographs, one of the unfiltered mercury arc and the other taken with the three filters in place, are shown in Fig. 5. The exposure times are the same (1 sec.) for both pictures.

The arc was checked with a foot-candle meter to attest the constancy of illumination. It was found that when the line voltage was constant there was no perceptible variation in intensity. However, during the daytime the line voltage varied enough to cause variations of one and two per cent in the source. One particularly troublesome circuit when closed would cause the source to lose as much as five per cent of its intensity. Because of these findings most of the pattern measurements were made late at night and on week-ends when there were no noticeable fluctuations in the line.

Fig. 5



Mercury Vapor Arc

Unfiltered and Filtered

EXPERIMENTAL DATA

With the equipment described above it was possible to measure different types of diffraction patterns including the straight-edge, double-slit, and single-slit patterns. Data are given for diffraction at two single slits of different widths. In measuring a diffraction pattern three variables were recorded for each setting, viz, the distance moved across the pattern by the photoelectric cell, the change in grid potential as read on the Students' Potentiometer, and the time at which the reading was taken with reference to the time at which the first reading was taken. To facilitate making measurements against time the readings were taken with the same time interval (45 seconds) between every two successive readings.

When measurements were being made, every eighth reading was taken with the shutter closed and the photocell darkened and without moving the photocell from the position of the previous reading. These "dark readings" were thus taken only against time and had no dependence on the diffraction pattern. They were used in making the correction for drift. To make the correction, the "dark readings" were plotted with voltages as ordinates and time as abscissas and the points thus fixed were connected by a smooth curve. From this curve the error due to drift of a voltage reading taken with the photocell illuminated by the diffraction pattern could be determined by reading off the ordinate on the drift curve corresponding to the time at which the uncorrected measurement was made. This correction would be added or subtracted from the potentiometer reading depending on whether the drift decreased or increased the readings as time went on.

Since the corrected diffraction pattern readings were to be plotted with voltages as ordinates and distances moved as abscissas, the simplest way of applying the drift correction was to plot the pattern readings and dark readings on the same voltage scale and, using dividers, to add or subtract the correction from the uncorrected pattern reading at the time of plotting the latter. For this reason on the curves representing the patterns measured there will be two curves--the drift curve with voltages as ordinates and times as abscissas, and the corrected pattern curve with voltages as ordinates and distances moved as abscissas.

To obtain a more accurate experimental curve of any pattern an average was taken of three measured curves in each case. The average was obtained by reading the ordinates of corresponding points of the three curves under consideration to obtain a mean value of the ordinate at that point. Since the initial readings of the three measurements of a given pattern were seldom taken at the same point in the pattern, the corresponding points on the curves used in obtaining the average had to be determined from the curves themselves and not from the data.

There now follow tables containing the experimental data. The curves are plotted from the data in these tables. Combined in one tabulation will be the data for all three measurements of a single pattern, but the curves for the measurements will be on separate pages.

Table II

a = 53.5 ± .1 cm. b = 567.9 ± .5 cm. λ= 5461 A.

Δs = 1.586 ± .001 mm. Δv = 4.34 ± .015

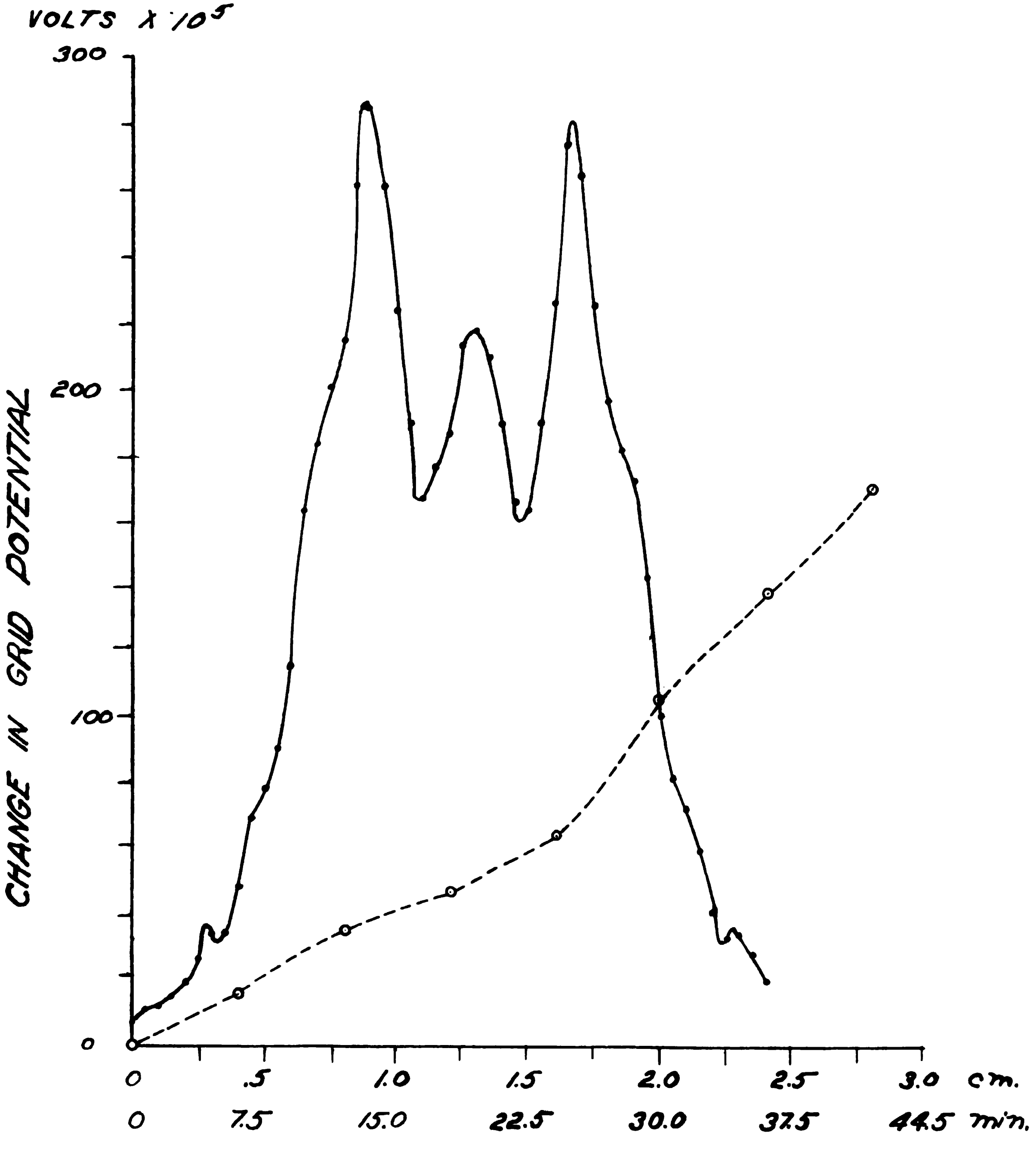
	Dist.	Grid pot. change in volts x 10^5		
Time	Moved	A	B	C
.75 min.	0.0 mm.	10	11	20
1.50	.5	15	13	17
2.25	1.0	18	20	25
3.00	1.5	24	33	40
3.75	2.0	30	33	40
4.50	2.5	39	42	43
5.25	3.0	49	48	51
6.75	3.5	53	65	73
7.50	4.0	70	79	83
8.25	4.5	94	82	88
9.00	5.0	105	100	107
9.75	5.5	120	127	130
10.50	6.0	148	149	151
11.25	6.5	198	162	165
12.75	7.0	222	189	195
13.50	7.5	241	240	245
14.25	8.0	256	284	287
15.00	8.5	300	303	307
15.75	9.0	329	320	325
16.50	9.5	306	356	353
17.25	10.0	270	400	394
18.75	10.5	239	411	403
19.50	11.0	218	372	377

Time	Dist. Moved	Grid pot. change in volts x 10^5 A	B	C
20.25 min.	11.5 mm.	230	322	334
21.00	12.0	242	311	311
21.75	12.5	271	319	312
22.50	13.0	277	337	324
23.25	13.5	271	357	353
24.75	14.0	257	376	374
25.50	14.5	237	373	375
26.25	15.0	239	365	364
27.00	15.5	271	353	357
27.75	16.0	314	354	353
28.50	16.5	368	373	367
29.25	17.0	365	412	412
30.75	17.5	337	468	472
31.50	18.0	311	496	490
32.25	18.5	301	474	468
33.00	19.0	296	438	442
33.75	19.5	270	427	428
34.50	20.0	232	413	416
35.25	20.5	217	397	395
36.75	21.0	214	366	363
37.50	21.5	206	330	334
38.25	22.0	192	322	333
39.00	22.5	186	313	323
39.75	23.0	191	299	303
40.50	23.5	189	283	290

Time	Dist. Moved	Grid pot. change in volts x 10^5 A	B	C
41.25 min.	24.0 mm.	184	285	286

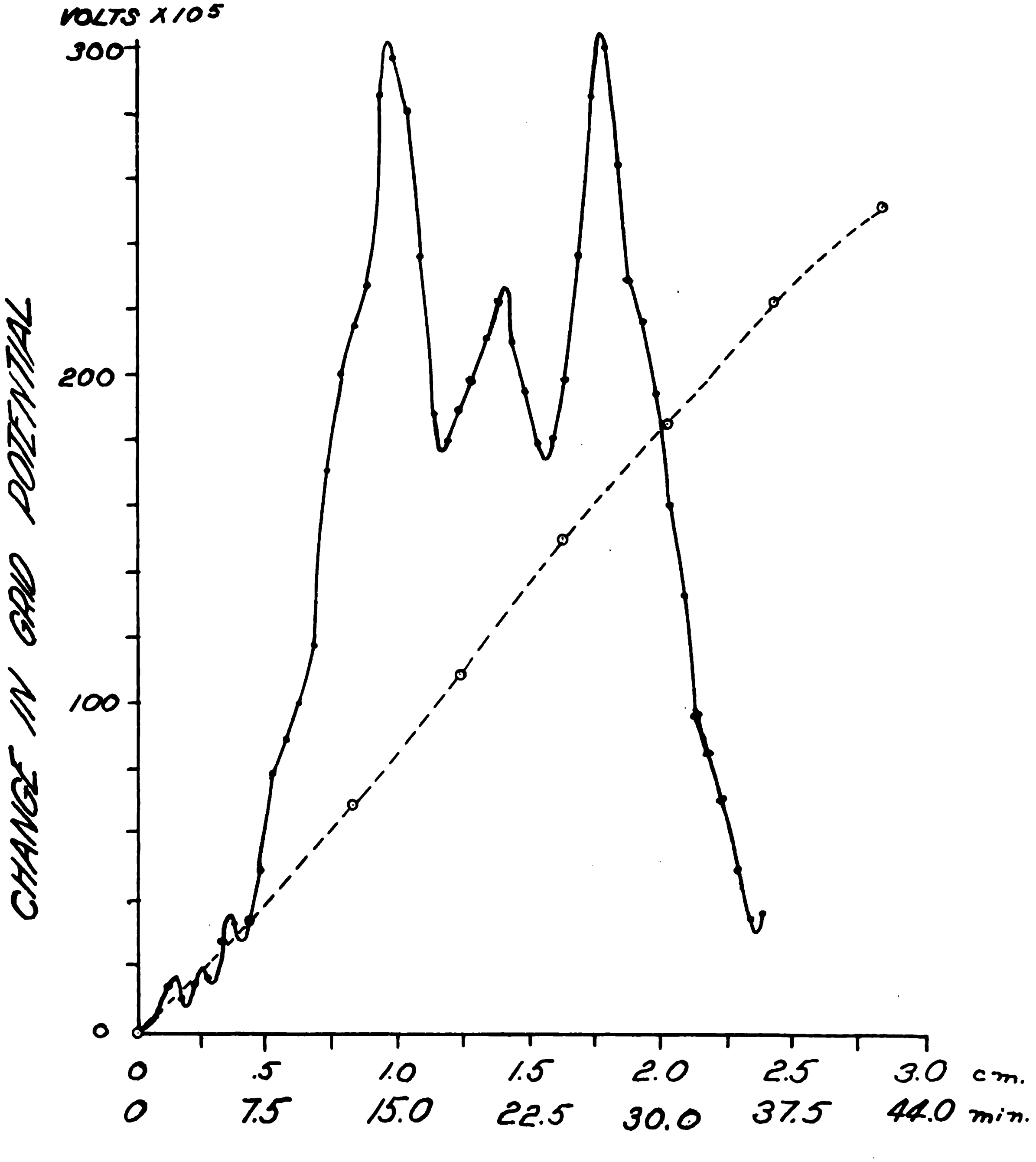
Dark Readings

0.00		0	0	0
6.00		16	35	41
12.00		36	70	86
18.00		47	110	122
24.00		64	151	163
30.00		105	186	198
36.00		138	223	234
42.00		169	252	260



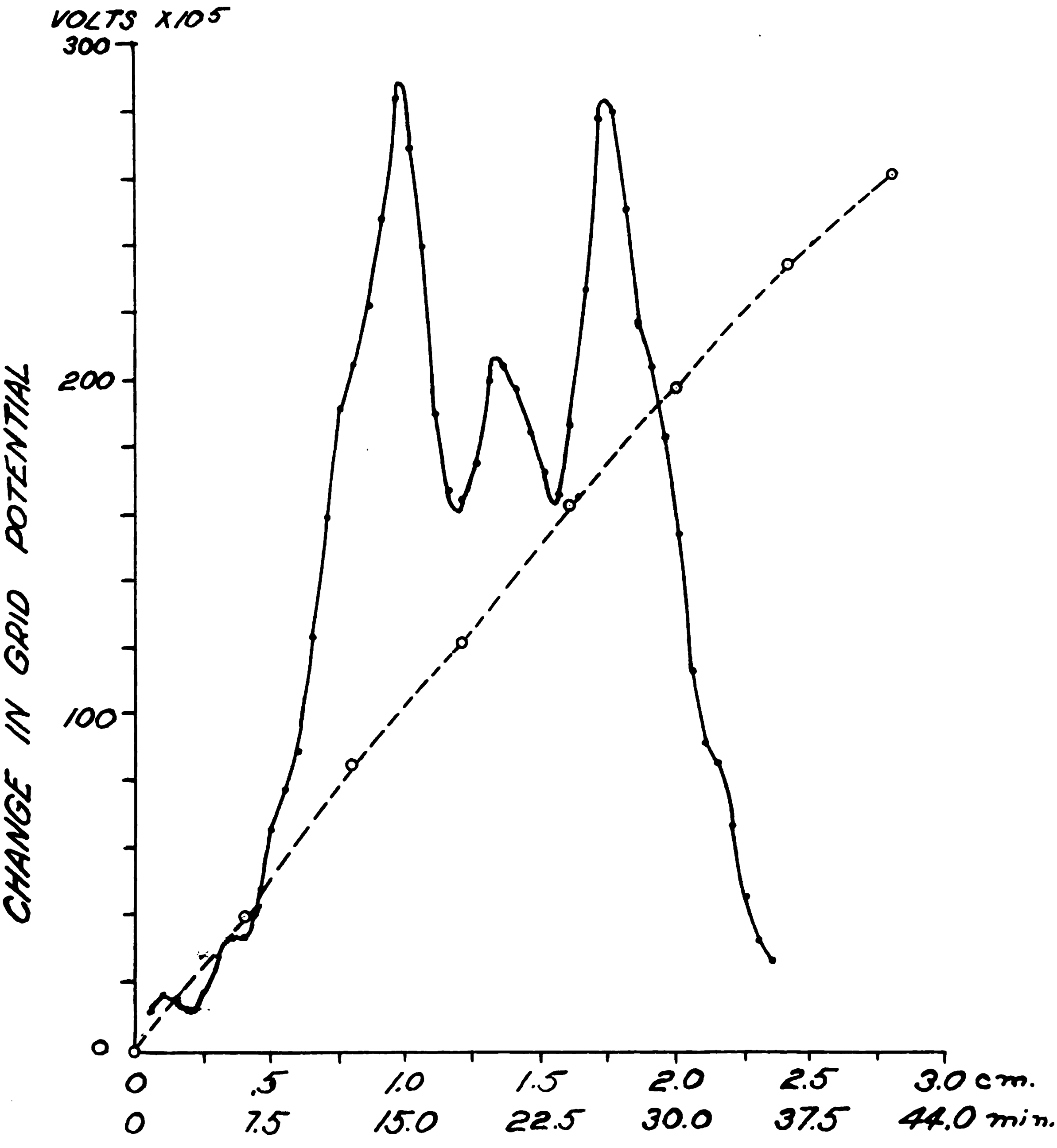
FROM TABLE II-A

FIG. 6



FROM TABLE II-B

FIG. 7



FROM TABLE II-C

FIG. 8

Table III

From curves of Figs. 6, 7, and 8.

Dist.	volts x 10^5 A	Dist.	volts x 10^5 B	C	Dist.	volts x 10^5 Average
0.00 cm.	8	.11 cm.	6	13	0.00 cm.	9
.05	12	.16	15	18	.05	15
.10	13	.21	11	16	.10	14
.15	16	.26	16	14	.15	15
.20	20	.31	18	19	.20	19
.25	27	.36	29	30	.25	29
.30	35	.41	34	35	.30	35
.35	35	.46	35	35	.35	35
.40	49	.51	50	49	.40	49
.45	70	.56	80	67	.45	72
.50	79	.61	90	79	.50	83
.55	91	.66	101	90	.55	94
.60	116	.71	118	124	.60	119
.65	164	.76	171	159	.65	165
.70	184	.81	201	192	.70	192
.75	201	.86	215	205	.75	207
.80	215	.91	228	222	.80	222
.85	262	.96	286	248	.85	265
.90	286	1.01	297	283	.90	289
.95	262	1.06	281	269	.95	271
1.00	224	1.11	237	240	1.00	234
1.05	190	1.16	189	191	1.05	190

Dist.	volts x 10^5 A	Dist.	volts x 10^5 B	C	Dist.	volts x 10^5 Average
1.10 cm.	167	1.21cm.	181	168	1.10 cm.	172
1.15	178	1.26	190	165	1.15	178
1.20	187	1.31	199	176	1.20	187
1.25	214	1.36	212	200	1.25	209
1.30	218	1.41	223	205	1.30	215
1.35	210	1.46	211	198	1.35	206
1.40	190	1.51	196	185	1.40	190
1.45	166	1.56	181	173	1.45	175
1.50	163	1.61	182	167	1.50	171
1.55	190	1.66	200	187	1.55	193
1.60	226	1.71	237	227	1.60	230
1.65	274	1.76	285	278	1.65	279
1.70	265	1.81	300	280	1.70	282
1.75	226	1.86	265	251	1.75	247
1.80	197	1.91	230	218	1.80	215
1.85	182	1.96	218	205	1.85	202
1.90	172	2.01	195	184	1.90	184
1.95	143	2.06	161	155	1.95	153
2.00	101	2.11	134	115	2.00	117
2.05	82	2.16	97	93	2.05	91
2.10	72	2.21	86	87	2.10	82
2.15	60	2.26	73	68	2.15	67
2.20	42	2.31	51	47	2.20	47
2.25	33	2.36	36	35	2.25	35
2.30	34	2.41	37	29	2.30	33
2.35	28				2.35	28

Table IV

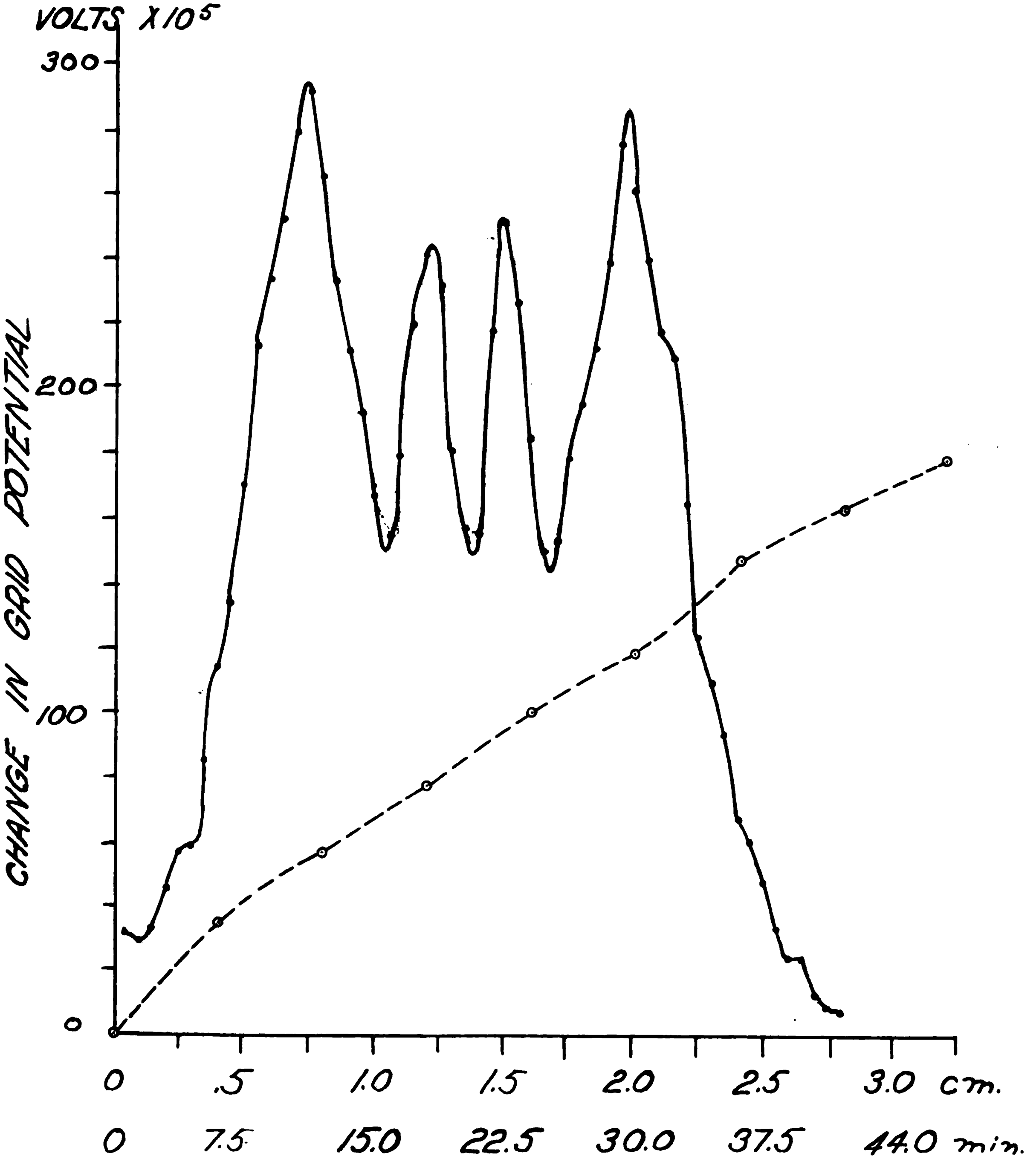
a = 53.5 ± .1 cm. b = 567.9 ± .5 cm. λ = 5461 A.

Δs = 1.979 ± .001 mm. Δv = 5.42 ± .015

Time	Dist. Moved	Grid pot. change in volts x 10^5 A	B	C
.75 min.	0.00 mm.	38	23	23
1.50	.5	40	24	31
2.25	1.0	48	32	42
3.00	1.5	65	47	62
3.75	2.0	81	56	78
4.50	2.5	90	67	90
5.25	3.0	117	85	119
6.75	3.5	152	115	145
7.50	4.0	175	137	166
8.25	4.5	214	162	208
9.00	5.0	259	204	257
9.75	5.5	283	231	284
10.50	6.0	304	246	301
11.25	6.5	333	281	352
12.75	7.0	350	307	363
13.50	7.5	327	285	333
14.25	8.0	298	260	293
15.00	8.5	280	246	280
15.75	9.0	263	240	268
16.50	9.5	241	217	241
17.25	10.0	230	200	235
18.75	10.5	260	214	270

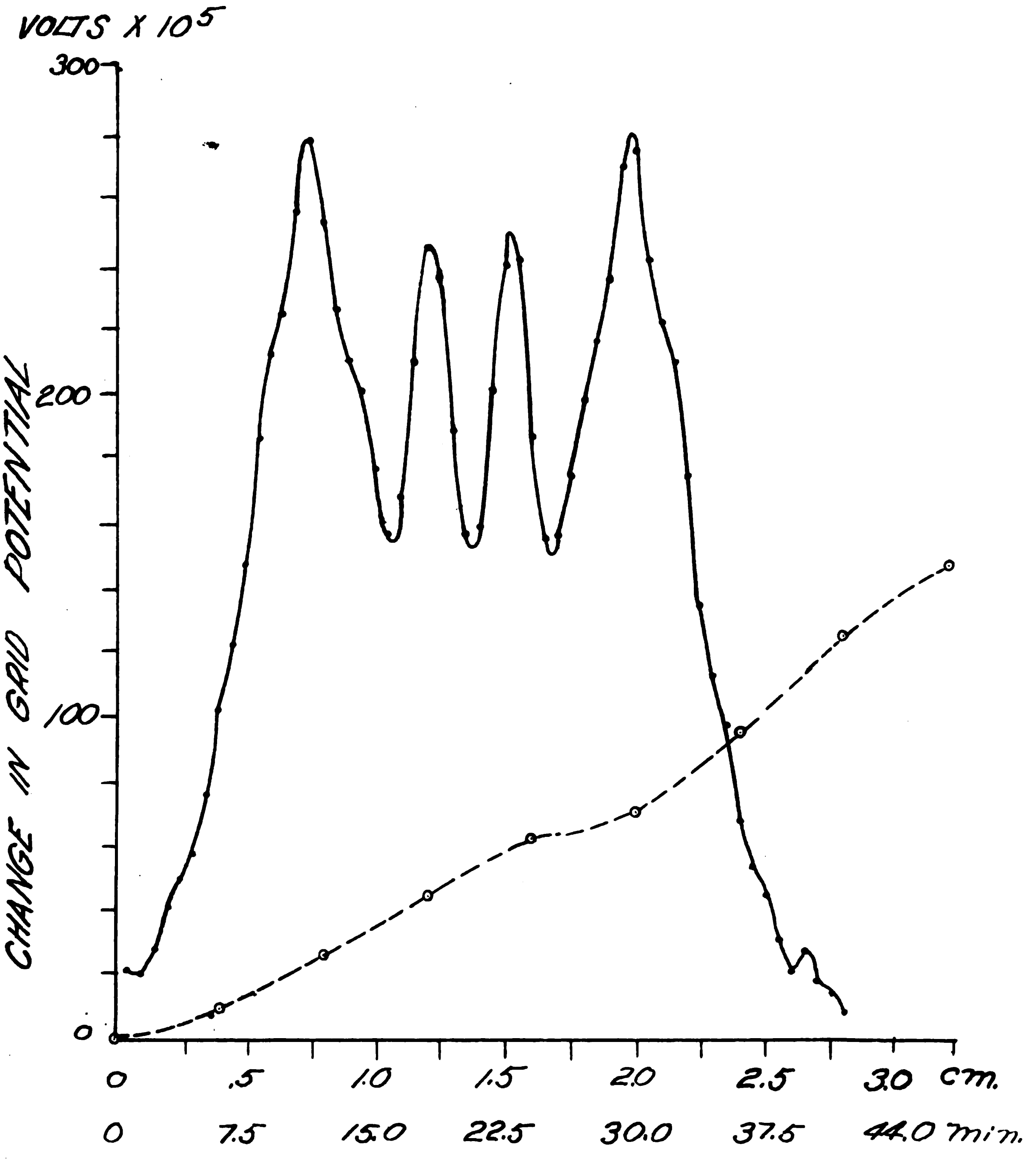
	Dist.	Grid pot. change in volts x 10^5		
Time	Moved	A	B	C
19.50 min.	11.0 mm.	303	259	325
20.25	11.5	328	297	343
21.00	12.0	321	290	316
21.75	12.5	274	246	272
22.50	13.0	252	216	253
23.25	13.5	264	220	283
24.75	14.0	320	264	354
25.50	14.5	355	303	371
26.25	15.0	333	305	346
27.00	15.5	294	262	296
27.75	16.0	260	221	268
28.50	16.5	266	224	280
29.25	17.0	295	244	312
30.75	17.5	318	270	338
31.50	18.0	339	290	357
32.25	18.5	367	311	388
33.00	19.0	407	348	423
33.75	19.5	398	358	408
34.50	20.0	379	327	385
35.25	20.5	360	311	372
36.75	21.0	360	307	355
37.50	21.5	317	276	316
38.25	22.0	279	241	287
39.00	22.5	265	222	274

	Dist.	Grid pot. change in volts x 10^5		
Time	Moved	A	B	C
39.75 min.	23.0 mm.	250	210	258
40.50	23.5	227	184	228
41.25	24.0	222	173	218
42.75	24.5	211	170	219
43.50	25.0	200	161	211
44.25	25.5	193	154	209
45.00	26.0	195	163	203
45.75	26.5	186	156	202
46.50	27.0	184	155	202
47.25	27.5	185	153	202
	Dark Readings			
0.00		0	0	0
6.00		35	10	22
12.00		56	26	70
18.00		77	44	84
24.00		100	62	100
30.00		118	70	125
36.00		147	94	145
42.00		163	124	164
48.00		178	146	192



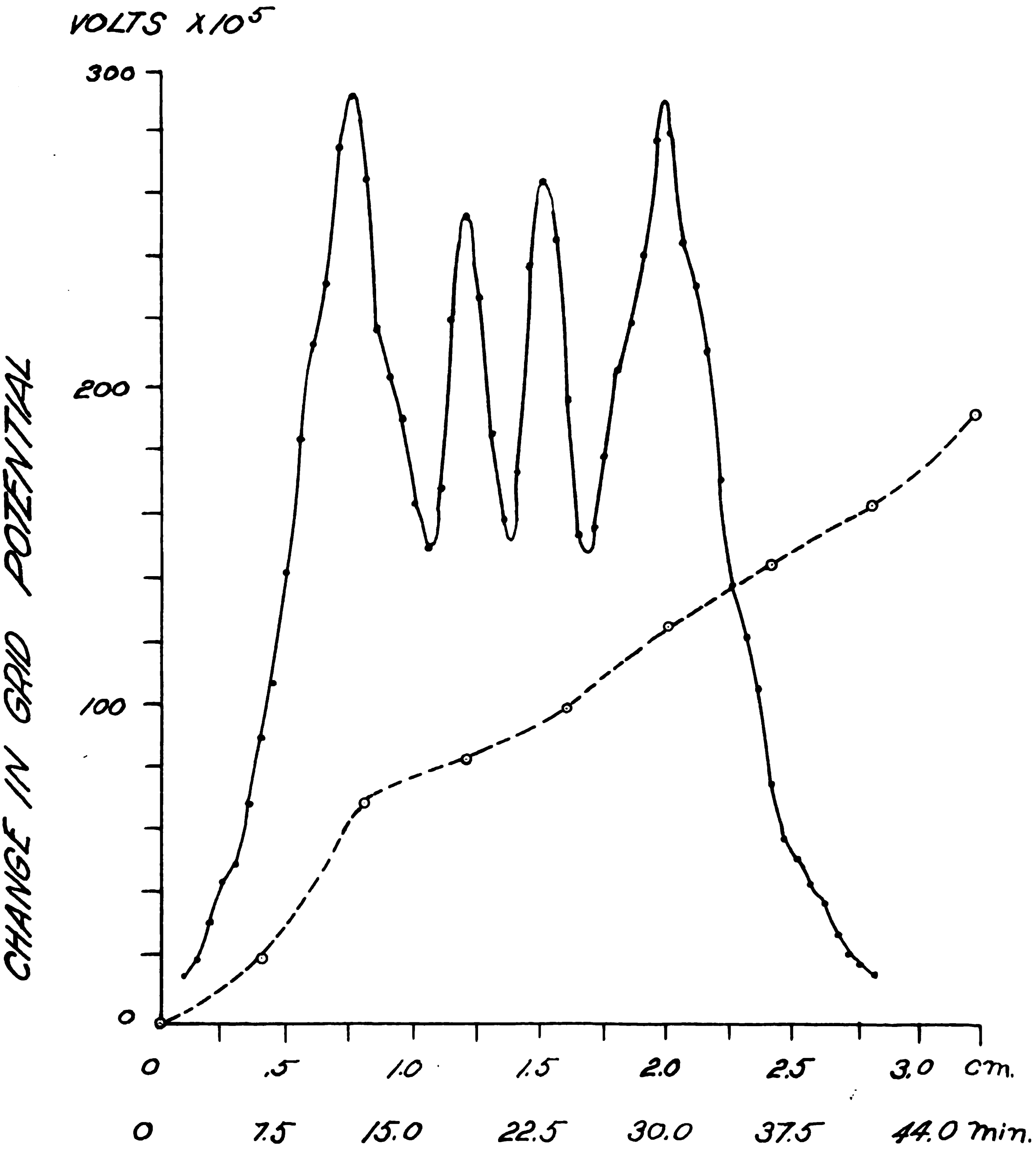
FROM TABLE IV-A

FIG. 9



FROM TABLE IV-B

FIG. 10



FROM TABLE IV-C

FIG. 11

Table V

From curves of Figs. 9, 10, and 11

	volts x 10^5			volts x 10^5		volts x 10^5
Dist.	A	B	Dist.	C	Dist.	Average
.05 cm.	32	22	.04 cm.	--	.05 cm.	27
.10	39	21	.09	16	.10	22
.15	33	28	.14	21	.15	27
.20	46	41	.19	33	.20	40
.25	57	50	.24	45	.25	51
.30	59	58	.29	51	.30	56
.35	85	76	.34	70	.35	77
.40	114	102	.39	91	.40	102
.45	134	122	.44	108	.45	121
.50	170	147	.49	143	.50	153
.55	213	186	.54	143	.55	195
.60	234	212	.59	215	.60	220
.65	252	224	.64	235	.65	234
.70	279	256	.69	277	.70	271
.75	291	278	.74	293	.75	287
.80	265	253	.79	267	.80	262
.85	233	226	.84	220	.85	226
.90	212	210	.89	205	.90	207
.95	193	201	.94	192	.95	195
1.00	168	177	.99	165	1.00	170
1.05	155	157	1.04	151	1.05	154
1.10	180	168	1.09	170	1.10	173
1.15	220	210	1.14	223	1.15	218

Dist.	volts x 10^5 A	B	Dist.	volts x 10^5 C	Dist.	volts x 10^5 Average
1.20 cm.	242	245	1.19cm.	255	1.20 cm.	247
1.25	232	236	1.24	230	1.25	233
1.30	182	189	1.29	187	1.30	186
1.35	157	157	1.34	160	1.35	158
1.40	156	159	1.39	175	1.40	163
1.45	218	201	1.44	240	1.45	216
1.50	252	240	1.49	266	1.50	253
1.55	227	241	1.54	248	1.55	239
1.60	185	187	1.59	198	1.60	190
1.65	150	155	1.64	155	1.65	153
1.70	153	156	1.69	157	1.70	155
1.75	179	175	1.74	180	1.75	178
1.80	196	198	1.79	207	1.80	200
1.85	213	216	1.84	222	1.85	217
1.90	239	235	1.89	243	1.90	239
1.95	276	270	1.94	279	1.95	275
2.00	262	275	1.99	281	2.00	273
2.05	240	241	2.04	247	2.05	243
2.10	218	222	2.09	233	2.10	224
2.15	210	210	2.14	213	2.15	211
2.20	165	175	2.19	173	2.20	171
2.25	124	135	2.24	140	2.25	133
2.30	109	113	2.29	123	2.30	115
2.35	93	98	2.34	107	2.35	99

Dist.	volts x 10^5 A	B	Dist.	volts x 10^5 C	Dist.	volts x 10^5 Average
2.40 cm.	68	68	2.39 cm.	76	2.40 cm.	71
2.45	61	54	2.44	59	2.45	58
2.50	48	45	2.49	53	2.50	49
2.55	34	32	2.54	45	2.55	37
2.60	25	22	2.59	38	2.60	28
2.65	25	28	2.64	29	2.65	27
2.70	14	19	2.69	23	2.70	19
2.75	10	15	2.74	20	2.75	15
2.80	8	10	2.79	16	2.80	11

COMPARISON WITH THEORETICAL CURVES

Using the formulae given above, the values of Δv for the two cases discussed were determined. From a table of Fresnel Integrals the intensities at various points in the patterns were found. These are tabulated in Tables VI and VII and plotted in Figs. 12 and 13. Before these curves can be compared with the measured ones they must be corrected for the width of the photoelectric cell aperture. The effect of this aperture would be to lower the high points, to raise the low points, and to average out the very fine structure.

This aperture correction can be made on a theoretical curve by using the fact that the total quantity of light falling on the photo-sensitive surface in a given time would be proportional to the area under the theoretical curve bounded by the limits giving the width of the aperture. This statement is illustrated in Fig. 12 where the line mn represents the aperture width and the area under the curve within these limits is efgh. This area is taken to be proportional to the average intensity at the midpoint of the aperture. A planimeter was used to measure these areas at points .50 mm. apart on the scale of the curve and these readings used to construct patterns which could be compared with the experimental curves. These data are shown in Tables VIII and IX. The width of the aperture was measured with a travelling microscope and found to be .49 mm.

To construct the theoretical and experimental curves on the same graph both must be adapted to the same scale. This can easily be done by comparing the corresponding readings for the exact center of the pattern. For example in the $\Delta v = 4.34$

pattern d = 0.00 cm. and I = 87 at the center of the theoretican pattern and d = 1.37 cm. and I = 2.5 at the center of the observed pattern drawn from the average values of Table III. Hence to put these two patterns on the same scale the 1.37 cm. must be added to each value of d on the theoretical pattern and each value to I on the theoretical pattern must be multiplied by 215/87 = 2.47.

If the centers of the calculated and observed patterns for $\Delta v = 5.42$ (the latter drawn from average values given in Table V) are compared, it is found that a quantity 1.29 cm. must be added to d to give the corrected distances, d_0, and the areas must be multiplied by a factor of 2.78. However, when the factor 2.78 is used and the resulting curve plotted on the same graph as the experimental curve it is very evident that the center point of the latter curve is in error and cannot be used in finding the multiplying factor. For this reason the first maximum to the right of the center has been used to obtain the multiplying factor of 2.53.

Since the theoretical curves are symmetrical with the center of the pattern, readings will repeat themselves on either side of the center. The data for the theoretical curves are given in Tables VIII and IX. In Figs. 14 and 15 the corresponding experimental and calculated durves are drawn together, the former from data in Tables III and V and the latter from Tables VIII and IX.

Table VI

a = 53.5 ± .1 cm.   b = 567.9 ± .5 cm. λ=5461 A.

Δs = 1.586 ± .001 mm. Δv = 4.34 ± .015

v	$\overline{\Delta x}^2$	$\overline{\Delta y}^2$	I	d
0.00	1.56	.731	2.29	0.00 mm.
.10	1.42	.781	2.20	.42
.20	1.08	.906	1.99	.85
.30	.736	1.04	1.78	1.27
.40	.530	1.12	1.80	1.70
.45	.497	1.12	1.62	1.90
.50	.510	1.12	1.63	2.12
.60	.682	1.12	1.80	2.54
.70	1.02	1.21	2.23	2.97
.80	1.37	1.39	2.76	3.39
.85	1.49	1.46	2.95	3.60
.90	1.61	1.51	3.12	3.82
.95	1.56	1.51	3.07	4.02
1.00	1.51	1.44	2.95	4.24
1.10	1.42	1.12	2.54	4.66
1.20	1.44	.740	2.18	5.09
1.25	1.51	.605	2.12	5.30
1.30	1.59	.518	2.11	5.51
1.35	1.64	.477	2.12	5.72
1.40	1.66	.473	2.13	5.94
1.45	1.59	.476	2.07	6.15
1.50	1.44	.502	1.94	6.36

v	$\overline{\Delta x}^{2}$	$\overline{\Delta y}^{2}$	I	d
1.60	1.06	.456	1.52	6.78 mm.
1.70	.778	.312	1.09	7.21
1.75	.719	.246	.97	7.42
1.80	.701	.205	.91	7.63
1.85	.701	.194	.90	7.84
1.90	.687	.212	.90	8.06
1.95	.635	.251	.89	8.28
2.00	.537	.297	.83	8.48
2.10	.297	.325	.62	8.90
2.20	.158	.240	.40	9.33
2.25	.138	.200	.34	9.54
2.30	.135	.183	.32	9.75
2.35	.075	.194	.27	9.96
2.40	.114	.230	.34	10.18
2.45	.078	.283	.36	10.39
2.50	.037	.298	.34	10.60
2.60	.000	.235	.24	11.02
2.70	.002	.133	.14	11.45
2.80	.002	.118	.12	11.87
2.90	.023	.131	.15	12.30
3.00	.086	.063	.15	12.72
3.10	.086	.006	.09	13.14
3.20	.049	.000	.05	13.57
3.30	.060	.000	.06	13.99
3.40	.063	.041	.10	14.42

Table VII

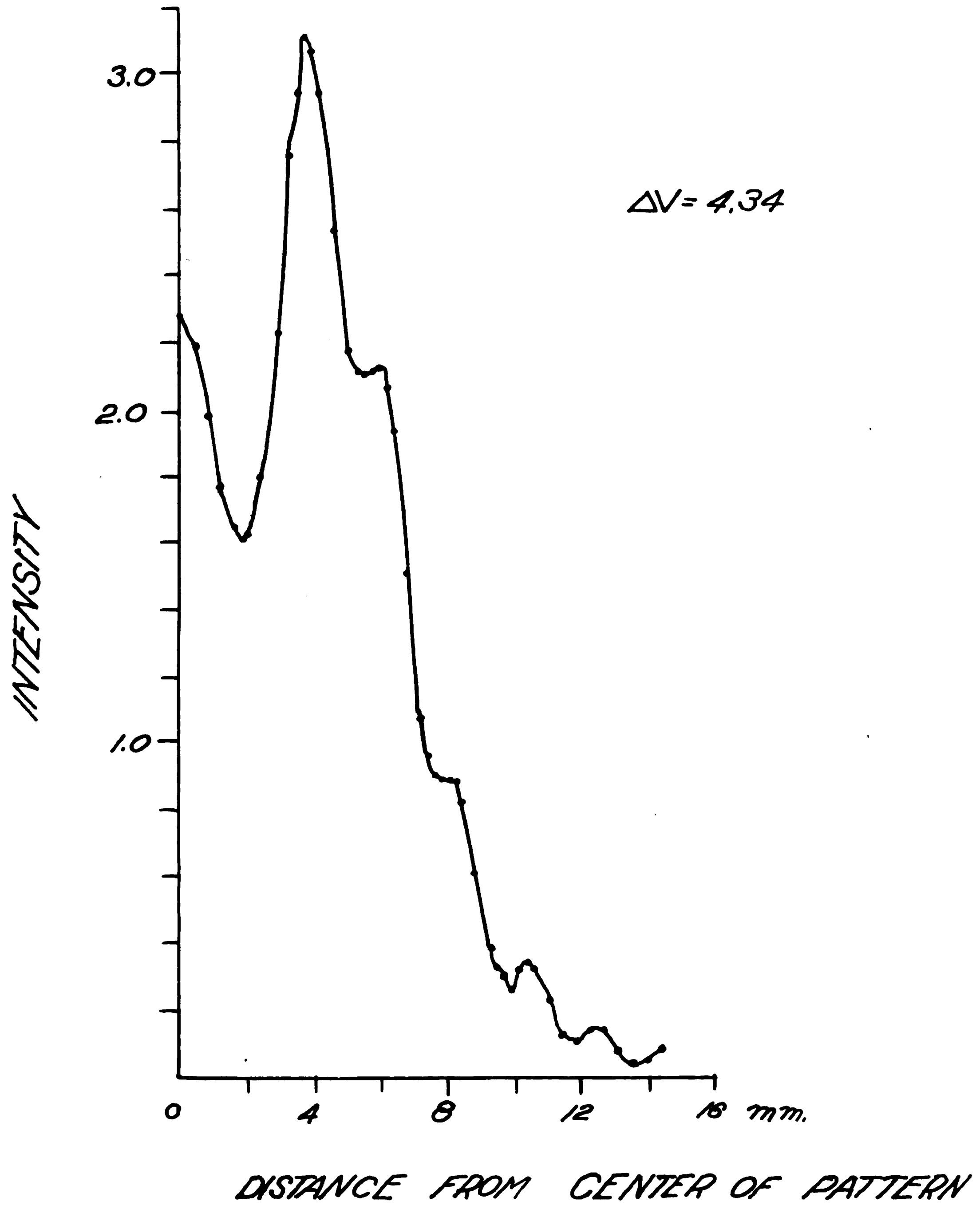
a = 53.5 ± .1 cm. b = 567.9 ± .5 cm. λ= 5461 A.

Δs = 1.979 ± .001 mm. Δv = 5.42 ± .02

v	$\overline{\Delta x}^2$	$\overline{\Delta y}^2$	I	d
0.00	.632	.789	1.42	0.00 mm.
.10	.745	.865	1.61	.42
.20	1.04	1.06	2.10	.85
.30	1.32	1.28	2.60	1.27
.35	1.39	1.32	2.71	1.48
.40	1.39	1.32	2.71	1.70
.45	1.30	1.25	2.55	1.91
.50	1.19	1.10	2.29	2.12
.55	1.08	.951	2.03	2.33
.60	.988	.792	1.78	2.54
.65	.924	.666	1.59	2.75
.70	.893	.588	1.48	2.97
.75	.887	.573	1.46	3.18
.80	.891	.669	1.56	3.39
.90	.857	.891	1.75	3.82
1.00	.731	1.25	1.98	4.24
1.05	.670	1.37	2.04	4.45
1.10	.640	1.42	2.06	4.66
1.15	.660	1.39	2.05	4.87
1.20	.743	1.35	2.09	5.09
1.25	.891	1.30	2.19	5.30
1.30	1.08	1.30	2.38	5.51

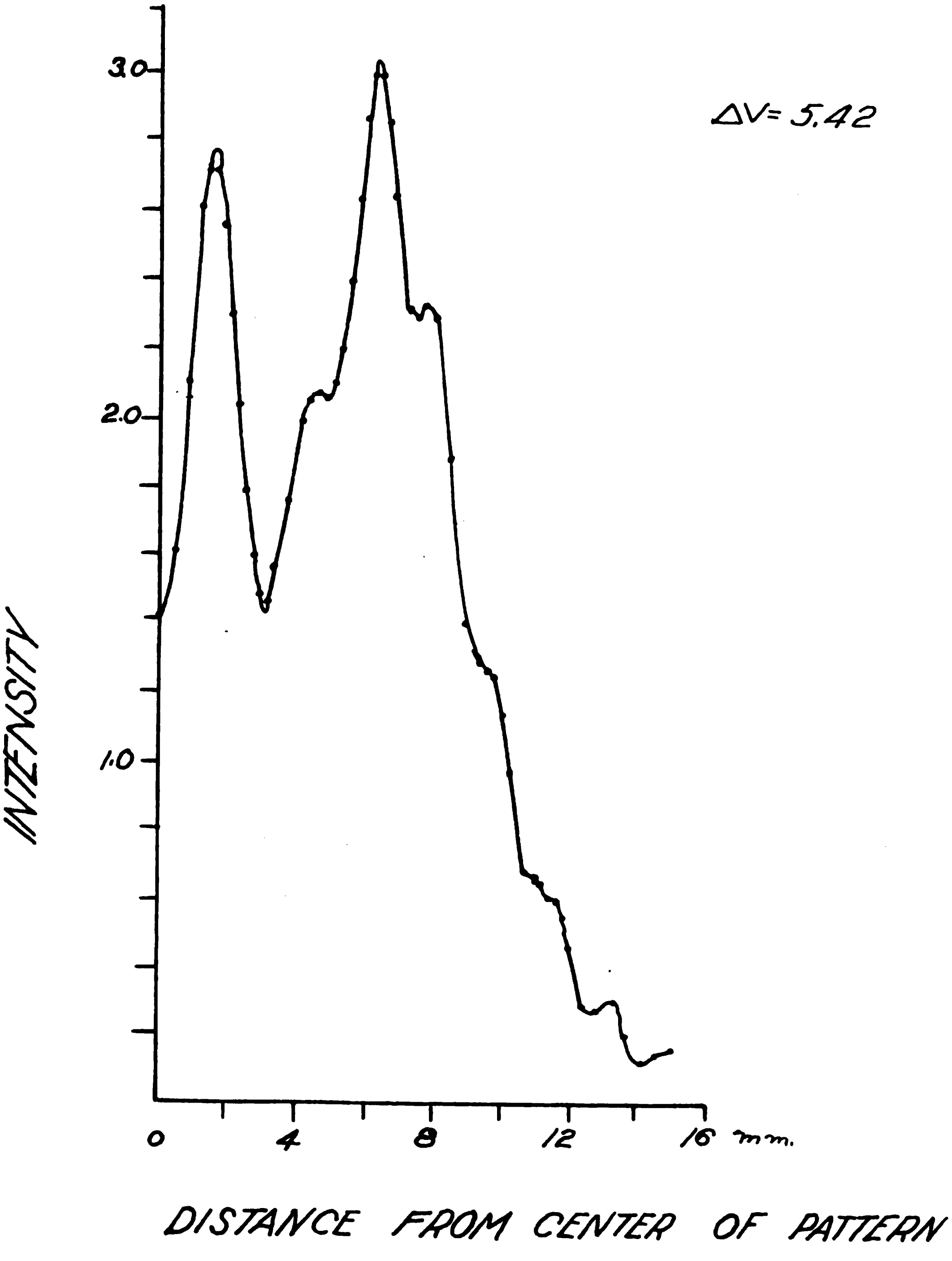
v	$\overline{\Delta x^2}$	$\overline{\Delta y^2}$	I	d
1.35	1.30	1.32	2.62	5.72 mm.
1.40	1.46	1.39	2.85	5.94
1.45	1.54	1.44	2.98	6.15
1.50	1.54	1.44	2.98	6.36
1.55	1.49	1.35	2.84	6.57
1.60	1.44	1.19	2.63	6.78
1.70	1.49	.815	2.30	7.21
1.75	1.59	.686	2.28	7.42
1.80	1.69	.621	2.31	7.63
1.90	1.66	.615	2.28	8.06
2.00	1.32	.554	1.87	8.48
2.10	1.04	.364	1.40	8.90
2.20	1.02	.255	1.28	9.33
2.25	1.02	.262	1.26	9.54
2.30	.941	.297	1.24	9.75
2.35	.803	.331	1.13	9.96
2.40	.638	.333	.97	10.18
2.50	.423	.242	.67	10.60
2.55	.371	.288	.66	10.81
2.60	.304	.196	.64	11.02
2.65	.377	.223	.60	11.23
2.70	.318	.275	.59	11.35
2.75	.228	.309	.54	11.66
2.80	.144	.301	.45	11.87
2.90	.072	.211	.28	12.30
3.00	.066	.206	.27	12.72

v	$\overline{\Delta x}^2$	$\overline{\Delta y}^2$	I	d
3.10	.018	.270	.29	13.14 mm.
3.20	.001	.199	.20	13.57
3.30	.004	.118	.12	13.99
3.40	.011	.130	.14	14.42
3.50	.066	.085	.15	14.84



FROM TABLE VI

FIG. 12



FROM TABLE VIII

FIG. 13

Table VIII

$\Delta v = 4.34$

d	area	d_c		area x 2.47
0.00 cm.	87	1.29 cm.		215
.05	84	1.34	1.24	207
.10	73	1.39	1.19	180
.15	67	1.44	1.14	165
.20	63	1.49	1.09	156
.25	69	1.54	1.04	170
.30	87	1.59	.99	215
.35	107	1.64	.94	264
.40	114	1.69	.89	282
.45	98	1.74	.84	242
.50	82	1.79	.79	203
.55	80	1.84	.74	198
.60	78	1.89	.69	193
.65	68	1.94	.64	168
.70	49	1.99	.59	121
.75	35	2.04	.54	86
.80	31	2.09	.49	77
.85	30	2.14	.44	74
.90	22	2.19	.39	54
.95	15	2.24	.34	37
1.00	13	2.29	.29	32
1.05	15	2.34	.24	37
1.10	10	2.39	.19	25
1.15	6	2.44	.14	15

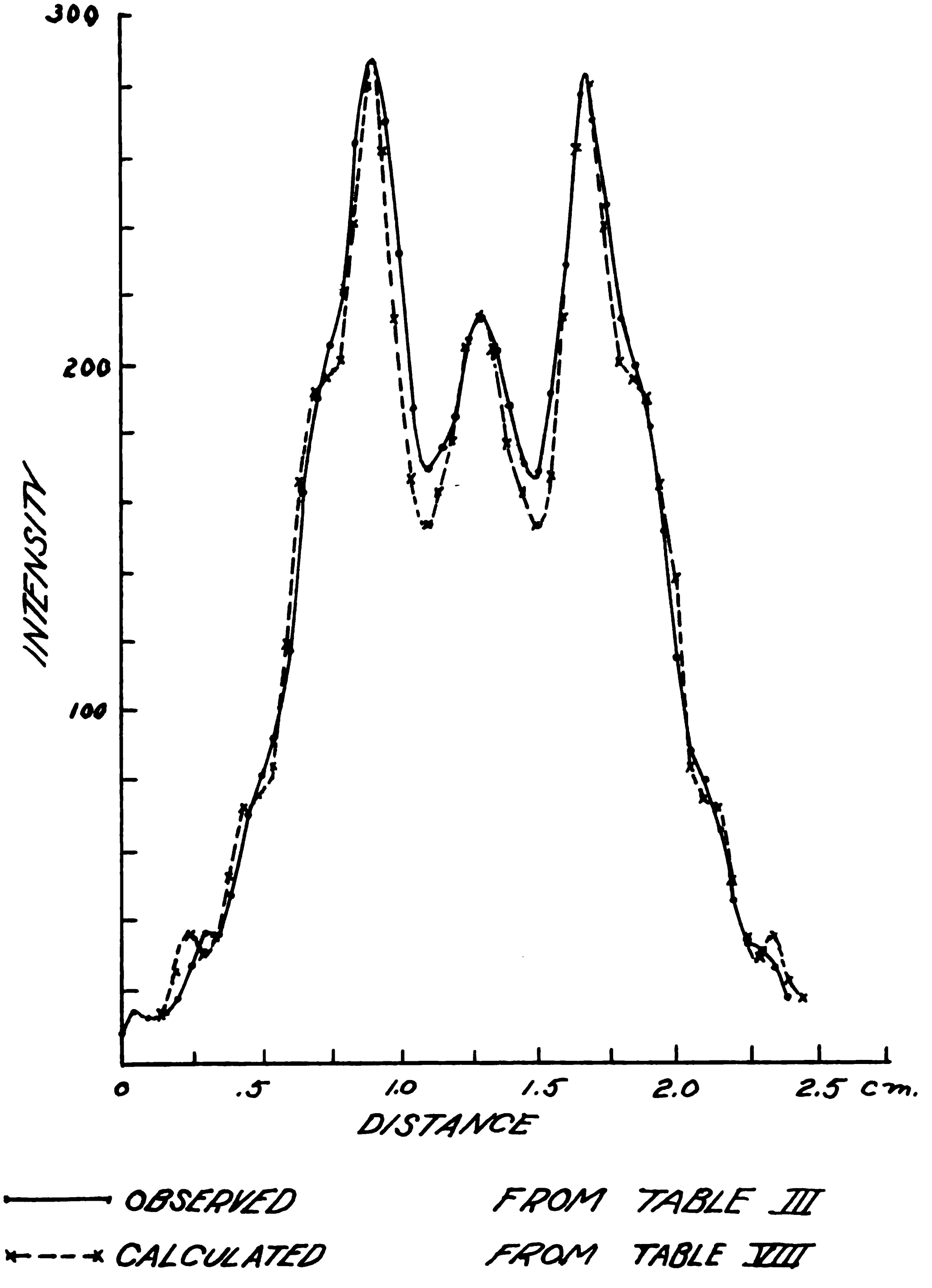

300
200
100
INTENSITY
0
.5
1.0
1.5
2.0
2.5 cm.
DISTANCE
OBSERVED FROM TABLE III
CALCULATED FROM TABLE VIII

FIG. 14

Table IX

$\Delta v = 5.42$

d	area	d_c		area x 2.53
0.00 cm.	54	1.37 cm.		137
.05	65	1.42	1.32	164
.10	81	1.47	1.27	205
.15	100	1.52	1.22	253
.20	91	1.57	1.17	230
.25	70	1.62	1.12	177
.30	55	1.67	1.07	139
.35	58	1.72	1.02	147
.40	68	1.77	.97	173
.45	75	1.82	.92	190
.50	76	1.87	.87	192
.55	88	1.92	.82	223
.60	110	1.97	.77	278
.65	108	2.02	.72	273
.70	91	2.07	.67	225
.75	87	2.12	.62	220
.80	85	2.17	.57	215
.85	72	2.22	.52	182
.90	55	2.27	.47	139
.95	49	2.32	.42	124
1.00	42	2.37	.37	106
1.05	27	2.42	.32	68
1.10	24	2.47	.27	61

d	area	d_c		area x 2.53
1.15 cm.	22	2.52 cm.	.22 cm.	56
1.20	16	2.57	.17	40
1.25	10	2.62	.12	25
1.30	11	2.67	.07	28
1.35	9	2.72	.02	23
1.40	6	2.77		15
1.45	6	2.82		15

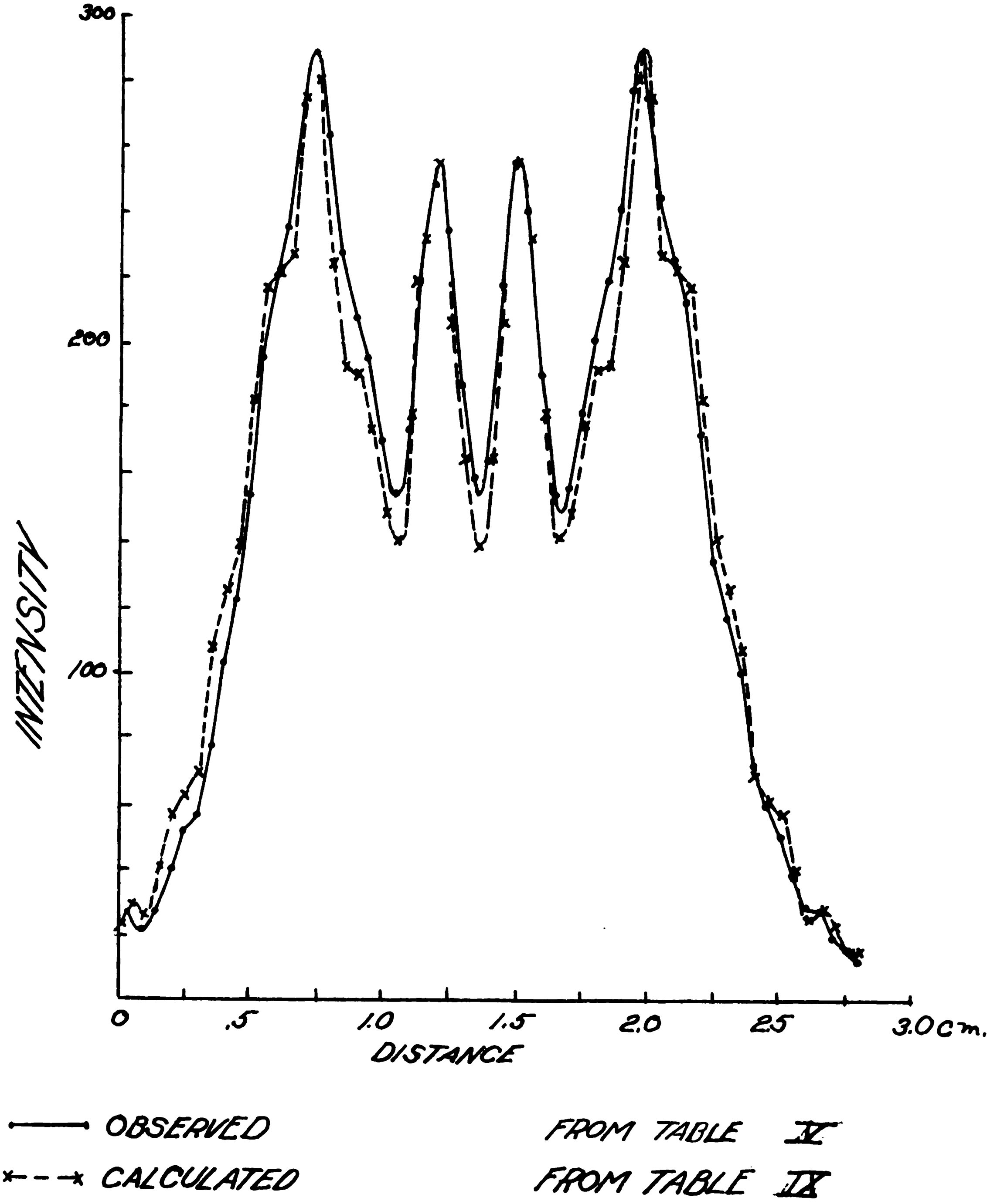

300
200
100
INTENSITY
0
.5
1.0
1.5
2.0
2.5
3.0 cm.
DISTANCE
OBSERVED
FROM TABLE V
CALCULATED
FROM TABLE IX

FIG. 15

DISCUSSION OF ERRORS

Most of the measurements given in the above experimental data could be made with a fair degree of accuracy. Any error in Δv would be due to errors in a, b, and Δs. The values of Δv used could not be more than 0.4% from the correct value. Potentiometer settings could be made to 2×10^{-5} volts, or about 1% on higher values and about 6 or 7% at lower values. However, by taking the average of three measured patterns the error due to the potentiometer readings would be cut down somewhat, probably to about 0.5% at higher values on the curve. Another source of error was found in measuring the aperture width. Due to irregularities of and diffraction effects at the edges of the slit aperture, the cross hairs of the travelling microscope could not be set as exactly as might be desired and, although an average of several readings was used, the value .49 mm. for the aperture width is probably still in error by 1%.

An investigation of the error introduced by the error in Δv showed that at maxima and minima a change of 0.2% in the value of Δv could change the intensity of the theoretical curve by as much as 4% while on the more linear parts of the curve the the change caused by changing Δv was quite small. This, it seems, might explain a considerable part of the observed disagreement between theoretical and experimental intensities. Agreement is seen to be quite good where the slope of the I vs. d curves is rather constant, but at the points of abrupt change the disagreement is more marked.

Except far down on the curves the difference between theoretical and observed values is less than 8% and at most points is much better. Taking into account all sources of error, 0.5% for potentiometer readings, 1% for aperture width, and 4% due to the error in Δv, the maximum possible error is estimated to be about 5.5%. The remaining 2.5% is probably due to the finite width of the source slit. It can easily be shown that the effect of the source slit width is to lower the high points, raise the low points, and leave unchanged the linear parts of the pattern curves. This effect would be quite noticeable with a camera having the dimensions given above and with the source slit at a width any greater than about .01 mm.

CONCLUSION

Within the limits of experimental error in measuring a diffraction pattern photoelectrically, the single slit diffraction patterns as observed and as calculated by Fresnel's theory are identical.

ACKNOWLEDGMENT

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