THE COTTON-MOUTON EFFECT IN LIQUID CRYSTAL AT MICROWAVE FREQUENCIES

Thesis for the Degree of M. S.

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Glen Alan Mann

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This is to certify that the

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The Cotton-Mouton Effect in Liquid Crystals

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Glen Alan Mann

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R. D. Spence
Major professor

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THE COTTON-MOUTON EFFECT IN LIQUID CRYSTAL AT MICROWAVE FREQUENCIES

by

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INTRODUCTION

On the basis of previous experiments[1] it has been suggested that the effect of a magnetic field on the complex dielectric constant of liquid crystals[2] at microwave frequencies should produce a rotation of the plane of polarization of the electric vector in a circular waveguide. This thesis reports measurements of this rotation and suggests an approximate quantitative theory for predicting the magnitude of the rotation which can be expected under a given set of conditions.

THEORY

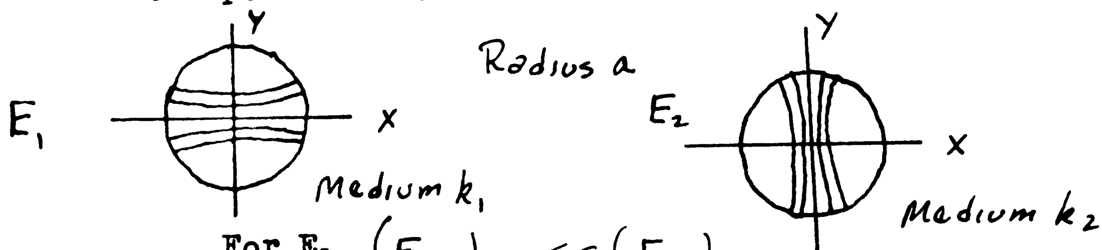
The earlier experiments showed that the substance para-azoxyanisole, which is a liquid crystal between 120°C and 135°C is anisotropic in the presence of the magnetic field. When the magnetic field is applied parallel to the electric vector, the attenuation is markedly decreased, therefore decreasing the imaginary part of the complex dielectric constant. However, when the magnetic field is applied transverse to the electric field, the imaginary part of the dielectric constant is increased, causing an increase in the attenuation. Along with the change in loss there is also a change in the velocity of propagation through the liquid crystal when a magnetic field is applied.

If one uses the above facts to formulate a simple working picture of what will happen to the electric vector as it passes through the sample at some angle besides parallel to the magnetic field, or perpendicular to the field, the electric vector should rotate because of a differential absorption and phase shift of the components parallel and perpendicular to the magnetic field. Using this differential absorption idea to explain the rotation, we can develop the following expression for the angle through which the electric vector will rotate:

Find an approximate solution for waves in a circular wave-guide. Consider two TE_{11} fields E_1 and E_2 which satisfy vector wave equations

$$\nabla^2 E_1 + k_1^2 E_1 = 0$$
$$\nabla^2 E_2 + k_2^2 E_2 = 0$$

and whose polarizations are



For E_1 $(E_{1y})_{AV.} << (E_{1x})_{AV.}$

For E_2 $(E_{2x})_{AV.} << (E_{2y})_{AV.}$

Let $\mathcal{E} = AE_1 + BE_2$

Then

$$\nabla^2 \mathcal{E}_x + k_1^2 A E_{1x} + k_2^2 B E_{2x} = 0$$
$$\nabla^2 \mathcal{E}_y + k_1^2 A E_{1y} + k_2^2 B E_{2y} = 0$$

and

$$\nabla^2 \mathcal{E}_x + k_1^2 \mathcal{E}_x = B(k_1^2 - k_2^2) E_{2x} = B(\delta k)^2 E_{2x}$$
$$\nabla^2 \mathcal{E}_y + k_2^2 \mathcal{E}_y = A(k_2^2 - k_1^2) E_{1y} = -A(\delta k)^2 E_{1y}$$

for small δk, E_{1y} and E_{2x}

$$\nabla^2 \mathcal{E}_x + k_1^2 \mathcal{E}_x \simeq 0$$
$$\nabla^2 \mathcal{E}_y + k_2^2 \mathcal{E}_y \simeq 0$$

The above are approximate solutions since they satisfy the boundary conditions but do not exactly satisfy Maxwell's Equations for this problem. In polar coordinates they become

$$\mathcal{E}_r = 2A\left(\frac{1}{Kr} J_1(Kr) \cos\varphi\, e^{-\gamma_1 z}\right) + 2B\left(\frac{1}{Kr} J_1(Kr) \sin\varphi\, e^{-\gamma_2 z}\right)$$

$$\mathcal{E}_\varphi = -2A\left(\frac{1}{K} \frac{d}{dr} J_1(Kr) \sin\varphi\, e^{-\gamma_1 z}\right) + 2B\left(\frac{1}{K} \frac{d}{dr} J_1(Kr) \cos\varphi\, e^{-\gamma_2 z}\right)$$

Also

$$\mathcal{E}_x = \mathcal{E}_r \cos\varphi - \mathcal{E}_\varphi \sin\varphi$$

$$\mathcal{E}_y = \mathcal{E}_r \sin\varphi + \mathcal{E}_\varphi \cos\varphi$$

The derivative of the Bessel Function is given by

$$\frac{d}{d\rho} J_1(\rho) = \frac{1}{\rho} J_1(\rho) - J_2(\rho)$$

so

$$\mathcal{E}_x = 2A\left[\left(\frac{1}{Kr} J_1(kr)\right)\left(\frac{1+\cos 2\varphi}{2}\right) + \left(\frac{1}{Kr} J_1(kr) - J_2(kr)\right)\left(\frac{1-\cos 2\varphi}{2}\right)\right] e^{-\gamma_1 z}$$

$$+2B\left[\left(\frac{1}{Kr} J_1(kr)\right)\left(\frac{\sin 2\varphi}{2}\right) - \left(\frac{1}{kr} J_1(Kr) - J_2(kr)\right)\left(\frac{\sin 2\varphi}{2}\right)\right] e^{-\gamma_2 z}$$

$$\mathcal{E}_y = 2A\left[\left(\frac{1}{Kr} J_1(kr)\right)\left(\frac{\sin 2\varphi}{2}\right) - \left(\frac{1}{Kr} J_1(kr) - J_2(kr)\right)\left(\frac{\sin 2\varphi}{2}\right)\right] e^{-\gamma_1 z}$$

$$+2B\left[\left(\frac{1}{kr} J_1(kr)\right)\left(\frac{1-\cos 2\varphi}{2}\right) + \left(\frac{1}{Kr} J_1(kr) - J_2(kr)\right)\left(\frac{1+\cos 2\varphi}{2}\right)\right] e^{-\gamma_2 z}$$

$J_0(\rho)$ is related to $J_1(\rho)$ and $J_2(\rho)$ by

$$J_0(\rho) = \frac{2}{\rho} J_1(\rho) - J_2(\rho)$$

So $\mathcal{E}_x$ and $\mathcal{E}_y$ become

$$\mathcal{E}_x = A\left[J_0(Kr) + J_2(Kr)\cos 2\varphi\right]e^{-\gamma_1 z} + B\,J_2(Kr)\sin 2\varphi\, e^{-\gamma_2 z}$$

$$\mathcal{E}_y = B\left[J_0(Kr) - J_2(Kr)\cos 2\varphi\right]e^{-\gamma_2 z} + A\,J_2(Kr)\sin 2\varphi\, e^{-\gamma_1 z}$$

which shows that in the central portion of the waveguide we have approximately two perpendicularly polarized plane waves with propagation constants γ_1 and γ_2

$$\gamma_1^2 = K^2 - k_1^2 = (\alpha_1 + i\beta_1)^2 = K^2 - \omega^2 \epsilon_1 \mu$$

$$\gamma_2^2 = K^2 - k_2^2 = (\alpha_2 + i\beta_2)^2 = K^2 - \omega^2 \epsilon_2 \mu$$

From earlier data values for ϵ_1', ϵ_1'', ϵ_2' and ϵ_2'' are

$$\epsilon_1' = 3.80\,\epsilon_0 \qquad \epsilon_2' = 3.45\,\epsilon_0$$

$$\epsilon_1'' = .4\,\epsilon_0 \qquad \epsilon_2'' = .8\,\epsilon_0$$

other known values are

$$\lambda = 3.2 \text{ cm}$$

$$a = 1.68 \text{ cm}$$

$$K^2 = \left(\frac{1.84}{1.68}\right)^2 = 1.20$$

$$k_1^2 = \left(\frac{2\pi}{\lambda}\right)^2 (3.80 - .4i) = 14.6 - 1.54i$$

$$k_2^2 = \left(\frac{2\pi}{\lambda}\right)^2 (3.45 - .8i) = 13.3 - 3.07i$$

Therefore

$$\gamma_1^2 = -13.4 + 1.54i$$

$$\gamma_2^2 = -12.1 + 3.07i$$

And α and β become

$\alpha_1 = .21 \qquad \alpha_2 = .44$

$\beta_1 = 3.68 \qquad \beta_2 = 3.51$

From consideration of plane waves we can determine the

of the angle of rotation to be

$$\tan 2\psi = \frac{2\tan\psi_0 \, e^{-(\alpha_2-\alpha_1)z}}{1-\left(\tan\psi_0 \, e^{-(\alpha_2-\alpha_1)z}\right)^2} \cos\left[(\beta_1-\beta_2)z\right]$$

where ψ_0 is the angle the incident wave makes with the magnetic field and z is the length of sample.

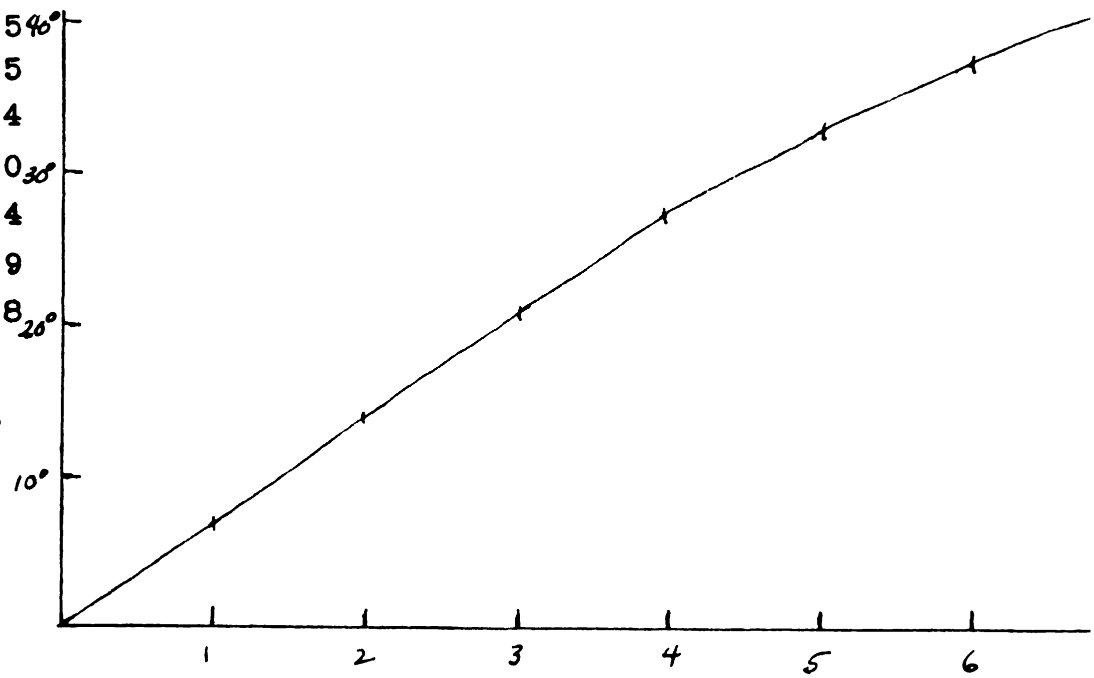
Using the computed values

$$\tan 2\psi = \frac{2\tan\psi_0 \, e^{-.23z}}{1-\left(\tan\psi_0 \, e^{-.23z}\right)^2} \cos(.17z)$$

For $\psi_0 = 45°$ $\tan\psi_0 = 1$

and $\tan 2\psi = \dfrac{\cos(.17z)}{\sinh(.23z)}$

z	$\psi-\psi_0$
0	0
1	6.5
2	13.5
3	20.4
4	27.0
5	32.4
6	36.9
7	40.8



$\psi-\psi_0$

EXPERIMENTAL APPARATUS

In order to produce incident waves other than parallel and perpendicular to the magnetic field, circular waveguide was used. One inch tubing was used for the waveguide. This will propagate the dominant TE_{11} mode at a frequency of 9,375 megacycles. The TE_{11} mode was used because it is the only mode in circular waveguide which has the electric vector in any degree of linear polarization. All of the other modes in circular waveguide have electric field configurations which are so complicated as to render them unsuitable for polarization rotation study.[3]

To provide a magnetic field, a small electromagnet was constructed. The pole pieces of the magnet were made 2 3/4 inches in diameter to allow for a sample length of 7 cm. Eight coils were wound with 2,200 turns on each coil. This was obtained with 1 Amp. Max. The wire that was used to wind the coils was flat, ribbon-like copper wire with a Form X coating. Because it was so thin, a current of 1 Amp. through each coil was the maximum for this wire. Between each field coil was placed a water cooling coil. The theoretical field for the magnet is 3,000 gauss.

Since the sample is a liquid crystal between 120°C and 135°C, a temperature bath had to be made. This was a copper tank 7 inches long, 3 inches wide and 7 inches high. This tank was inserted between the pole pieces of the magnet.

With a pole piece separation of 3 inches, the field obtained was 1,400 gauss as shown in Fig. 1. To study the rotation of the plane of polarization, two rotatable joints were made for the waveguide. One was placed so that the klystron could be rotated to change the incident angle of the electric vector. The other joint was placed on the detector so that it could measure the rotation for a constant incident angle but at different field strengths. A large circular protractor was mounted beneath the rotating detector. To operate the klystron, a power supply and square wave generator was built from the circuit diagram of a Browning TVN7.

The para-azoxyanisole was prepared in the following manner. Para-nitro-anisole was mixed with nine normal sodium hydroxide. This solution was placed in a flask and heated to 80°C, while constantly being stirred. Then glucose was added to the solution. This is an exothermic reaction. The temperature of 80°C had to be maintained while adding the glucose to get the maximum yield of para-azoxyanisole. The resulting mixture was dilluted to 2 liters with water and suction filtered. About 200 ml. of water was added to the precipitate and steam bubbled through the mixture for three or four hours until the unreacted para-nitro-anisole was removed. The resulting product was filtered again and the precipitate was dissolved in hot benzene and ethyl alcohol and filtered once more. The last process or re-crystallizing from the hot benzene and ethyl alcohol was repeated three or four times until the precipitate was a light yellow and the filtrate was also a light yellow. Fifty gm. of the para-azoxyanisole was made.

Three pictures are included of the apparatus. The first shows the set up that was used for obtaining the results. The second is a picture showing the face of the magnet and the meter that was used to find the maximum and minimum points. The third is a picture of the waveguide and temperature bath tank.

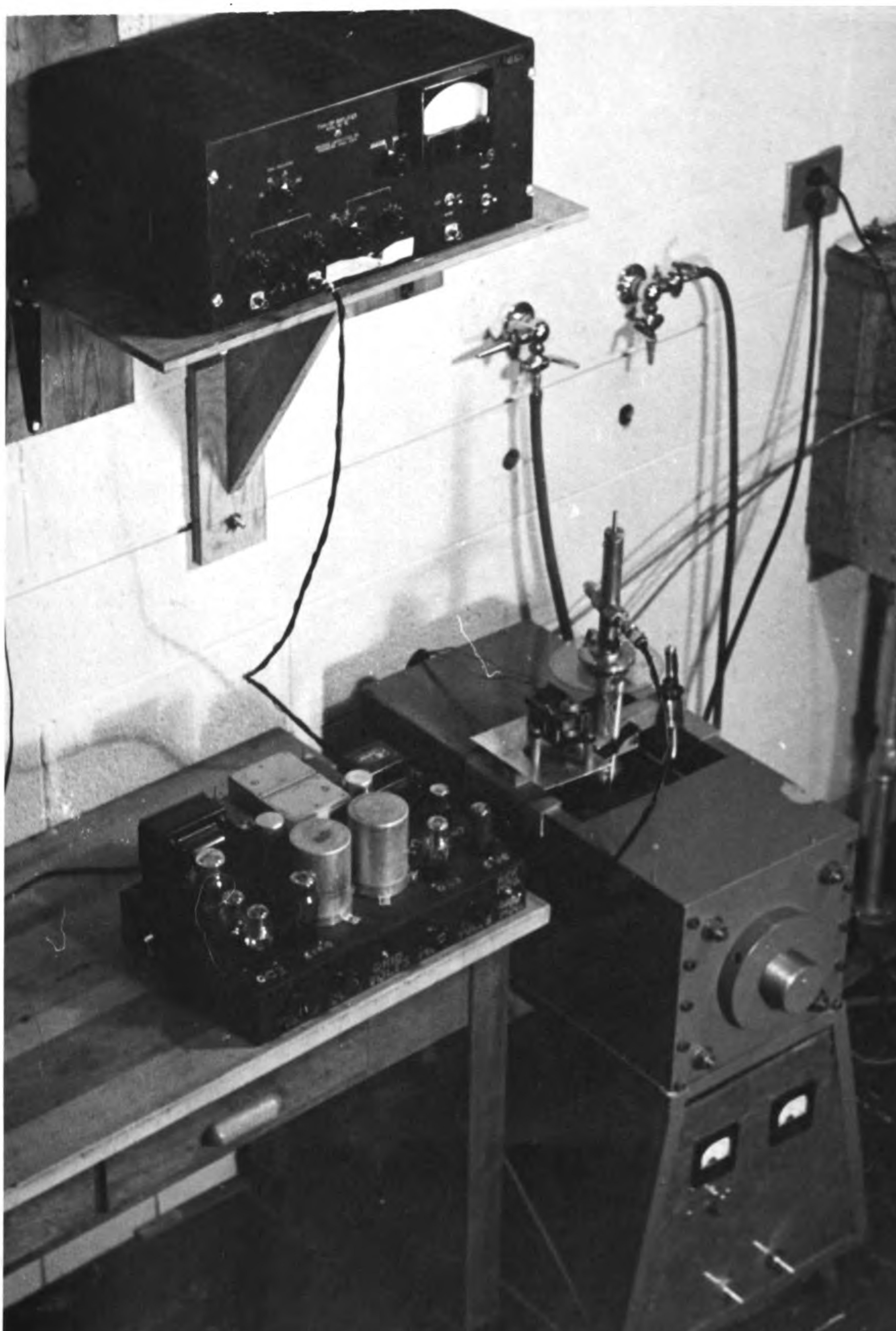


Fig. 2a - Apparatus Set Up

Fig. 2b - Magnet and Meter

Detector

Temperature Bath

Klystron

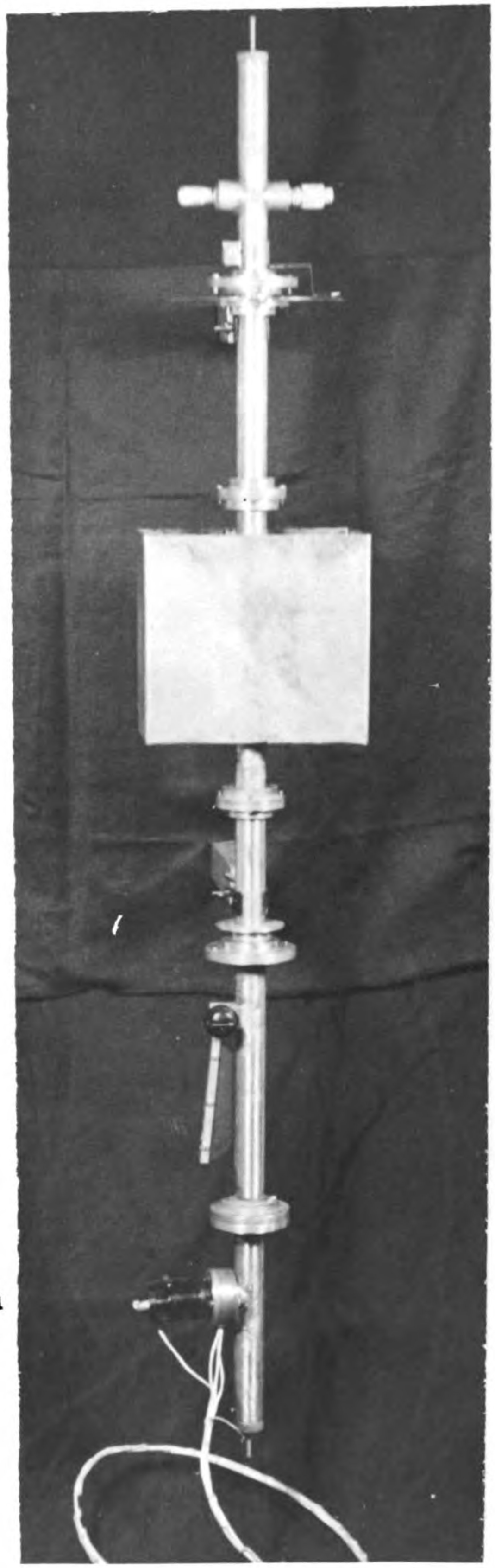


Fig. 2c

EXPERIMENTAL PROCEDURE

To determine the direction of the incident wave, the apparatus was put together with the sample in the cell and a point was found for which the plane of polarization did not rotate when the field was turned on. If the simple picture we have is correct, then when the electric field is parallel to the magnetic field there are no components perpendicular to the field. Therefore, the electric vector will not rotate. When this point was found, the protractor was fixed so that the indicator on the detector read 0° for a maximum output.

To determine the angle of the incident wave, the detector was set at the angle desired and the klystron was rotated until a maximum output was obtained. The incident wave was taken for 0°, 30°, 45° and 60° to the magnetic field for one given length of sample. The two maxima and the two minima were observed for no field, 500 gauss and 1,300 gauss. The length of the column was then changed and the readings at the various angles were again taken with no field, 500 gauss and 1,300 gauss.

Cell Empty

Max. -	66 at 6°	67 at 187°	No field
Min. -	2.6 at 89°	2.5 at 269°	
Max. -	64 at 185°	64 at 6°	1,100 gauss
Min. -	2.5 at 270°	2.6 at 89°	

Cell with 7.87 cm. Crystal Temp. Below Liquid Phase

Max. -	97 at 213°	96 at 30°	No field
Min. -	.2 at 113°	.2 at 292°	
Max. -	94 at 211°	92 at 31°	1,100 gauss
Min. -	.2 at 111°	.2 at 293°	

z = 2.29 cm.

	Max.	Min.	
No field	240° - 76.0 60° - 80.0	150° - 0.1 330° - 0.1	
500 gauss	228° - 96.0 47° - 96.0	139° - 0.1 319° - 0.1	E 60° H Temp = 127°C
1,300 gauss	217° - 100.0 41° - 112.0	136° - 0.1 315° - 0.1	

No field	225° - 64.0 45° - 66.0	135° - 0.1 315° - 0.1	
500 gauss	211° - 102.0 34° - 104.0	124° - 0.1 305° - 0.1	E 45° H Temp = 127°C
1,300 gauss	27° - 120.0 211° - 110.0	122° - 0.1 302° - 0.1	

No field	210° - 60.0 30° - 64.0	120° - 0.1 300° - 0.1	
500 gauss	18° - 140. 199° - 136.	110° - 0.1 290° - 0.1	E 30° H Temp = 127°C
1,300 gauss	197° - 176. 17° - 180.	109° - 0.1 289° - 0.1	

No field	180° - 86.0 0° - 88.0	90° - 0.1 270° - 0.1	
500 gauss	0° - 200.0 181° - 200.0	271° - 0.1 89° - 0.1	E 0° H Temp = 127°C
1,300 gauss	179° - 220.0 0° - 240.0	91° - 0.1 270° - 0.1	

z = 3.60 cm.

	Max.	Min.	
No field	180° - 72.0 0° - 72.0	90° - 0.1 270° - 0.1	
500 gauss	172° - 360. 354° - 360.	267° - 0.1 87° - 0.1	E 0° H Temp = 128°C
1,300 gauss	172° - 400. 354° - 400.	267° - 0.1 88° - 0.1	
No field	210° - 96 30° - 96	120° - .1 300° - .1	
500 gauss	13° - 248 192° - 240	103° - 2.4 284° - 2.4	E 30° H Temp = 128°C
1,300 gauss	190° - 240 11° - 254	281° - 2.4 102° - 2.4	
No field	225° - 36 45° - 36	135° - .1 315° - .1	
500 gauss	24° - 132 213° - 128	114° - 2.4 294° - 2.4	E 45° H Temp = 128°C
1,300 gauss	209° - 144 24° - 152	292° - 3.2 112° - 3.2	
No field	250° - 68 60° - 68	150° - .1 330° - .1	
500 gauss	231° - 72 44° - 80	133° - 1.2 313° - 1.6	E 60° H Temp = 128°C
1,300 gauss	227° - 80 41° - 80	129° - 2.4 310° - 2.4	

ℓ = 4.89 cm.

	Max.	Min.	
No field	240° - 310.	150° - 0.1	
	60° - 360.	330° - 0.2	
500 gauss	210° - 500.	308° - 9.8	E 60° H
	32° - 590.	126° - 11.0	Temp = 130°C
1,300 gauss	26° - 720.	118° - 17.0	
	212° - 700.	300° - 18.0	
No field	225° - 300.	135° - 0.1	
	45° - 320.	315° - 0.1	
500 gauss	200° - 1200.	109° - 14.0	E 45° H
	19° - 1360.	289° - 16.0	Temp = 129° C
1,300 gauss	15° - 1880.	107° - 18.4	
	195° - 1700.	286° - 19.6	
No field	210° - 480.	120° - 0.1	
	30° - 480.	300° - 0.1	
500 gauss	10° - 2320.	98° - 5.6	E 30° H
	189° - 2080.	278° - 5.6	Temp = 128°C
1,300 gauss	7° - 2400.	279° - 6.4	
	188° - 2400.	98° - 6.4	
No field	180° - 84.0	90° - 0.1	
	0° - 88.0	270° - 0.1	
500 gauss	178° - 720.0	268° - 0.1	E 0° H
	358° - 720.0	88° - 0.1	Temp = 127°C
1,300 gauss	358° - 800.0	87° - 0.1	
	178° - 840.0	268° - 0.1	

z = 6.40 cm.

	Max.	Min.	
No field	180° - 12. 0° - 12.	90° - .1 270° - .1	
500 gauss	356° - 110. 174° - 120.	85° - .2 264° - .2	E 0° H Temp = 129°C
1,300 gauss	171° - 160. 353° - 160.	83° - .2 264° - .2	

No field	210° - 13. 30° - 13.	120° - .1 300° - .1	
500 gauss	40° - 71. 186° - 73.	95° - .7 275° - .8	E 30° H Temp = 129°C
1,300 gauss	177° - 88. 1° - 90.	92° - 1.0 271° - .9	

No field	225° - 11. 45° - 12.	135° - .1 315° - .1	
500 gauss	10° - 38. 188° - 44.	100° - 1.0 280° - 1.1	E 45° H Temp = 129°C
1,300 gauss	184° - 52. 0° - 55.	97° - 1.4 277° - 1.4	

No field	240° - 12. 60° - 13.	150° - .1 330° - .1	
500 gauss	30° - 16. 210° - 17.	120° - 1.2 297° - 1.3	E 60° H Temp = 129°C
1,300 gauss	198° - 21. 21° - 21.	291° - 1.8 111° - 1.8	

z = 7.87 cm.

	Max.	Min.	
No field	240° - 9.6 60° - 8.8	150° - .1 330° - .1	
500 gauss	21° - 27. 202° - 30.	110° - .6 291° - .7	E 60° H Temp = 128°C
1,300 gauss	193° - 52. 14° - 58.	101° - 1. 283° - 1.	

	Max.	Min.	
No field	225° - 9.6 45° - 9.8	135° - .1 314° - .1	
500 gauss	7° - 54. 186° - 53.	99° - .3 278° - .3	E 45° H Temp = 129°C
1,300 gauss	183° - 100. 3° - 110.	273° - .5 93° - .5	

	Max.	Min.	
No field	110° - 9.6 30° - 9.8	120° - .1 300° - .1	
500 gauss	7° - 77. 189° - 78.	92° - .2 273° - .2	E 30° H Temp = 129°C
1,300 gauss	180° - 120. 2° - 130.	271° - .2 91° - .2	

	Max.	Min.	
No field	180° - 9.2 0° - 9.0	90° - .1 270° - .1	
500 gauss	176° - 96. 358° - 100.	267° - .1 87° - .1	E 0° H Temp = 129°C
1,300 gauss	178° - 200. 2° - 230.	87° - .1 266° - .2	

RESULTS

The first page of data shows that when the cell is empty, or if the cell is filled with sample below the temperature for the liquid crystal phase, there is no rotation when the field is applied. The rotation seen for 0° incidence gives a measure of the reading error. These points show rotations $\psi - \psi_o$ of 1° to 5°. The average is 3° which would seem to indicate that the error in adjusting the angle of the incident wave is of this order of magnitude.

Fig. 3 shows the rotation $\psi - \psi_o$ for an incident wave of 30° to the magnetic field vs. the length of the sample in the cell. The two curves shown are for different field strengths as indicated.

Fig. 4 and Fig. 5 show the same thing as Fig. 3, except that the angle of the incident wave is 45° and 60° respectively.

Fig. 6 illustrates the way in which the rotation $\psi - \psi_o$ varies with the length of the sample with a maximum field and at three different angles of incidence. It is interesting to note that Fig. 6 shows the rate of rotation the electric vector is decreasing, even though the sample length is increasing and the magnetic field is strong enough to saturate the sample. This is in accordance with the theory of differential absorption as being the cause of the rotation of the plane of polarization.

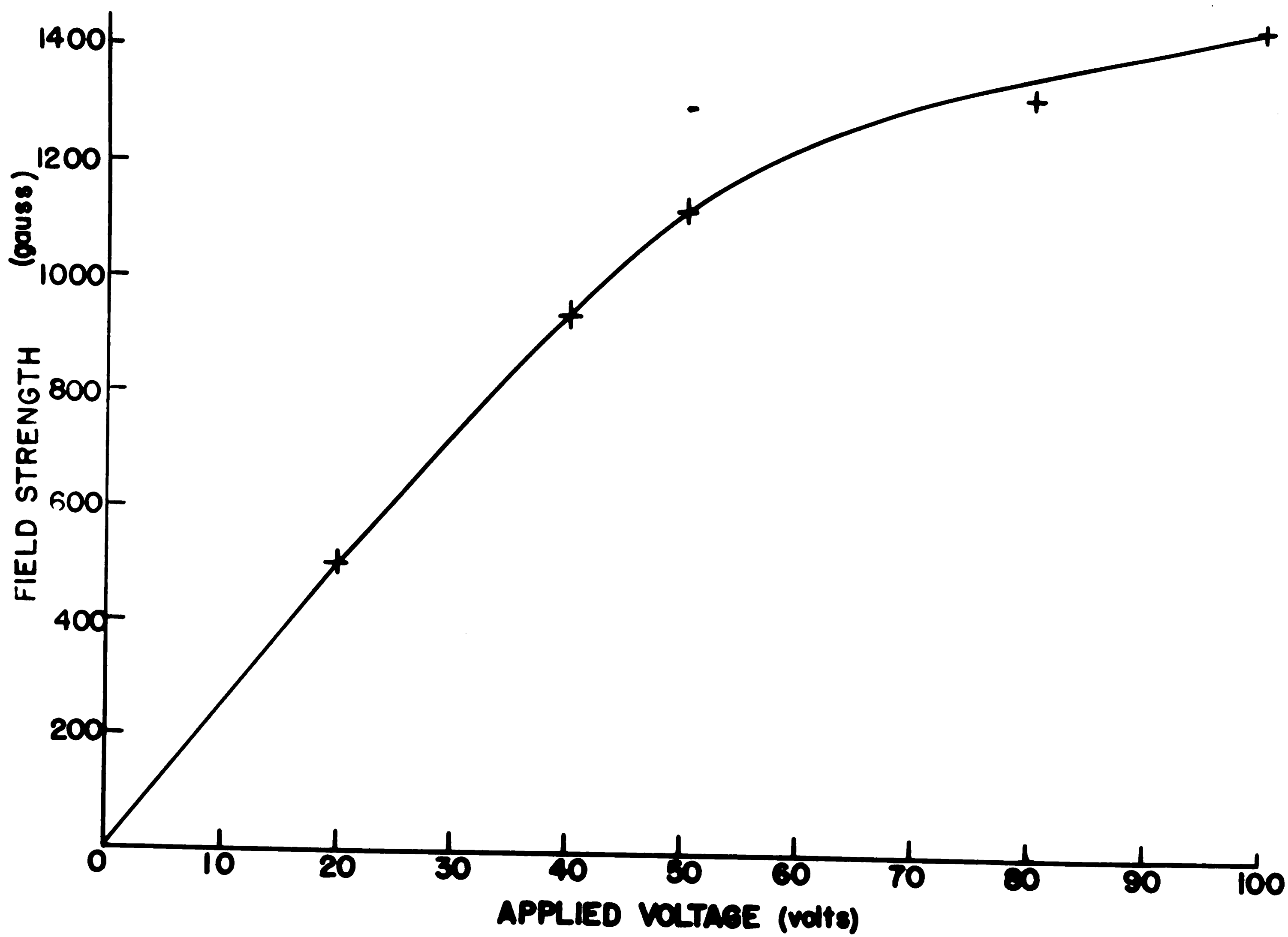


Fig. 1 - Field Strength of Magnet vs. Applied Voltage

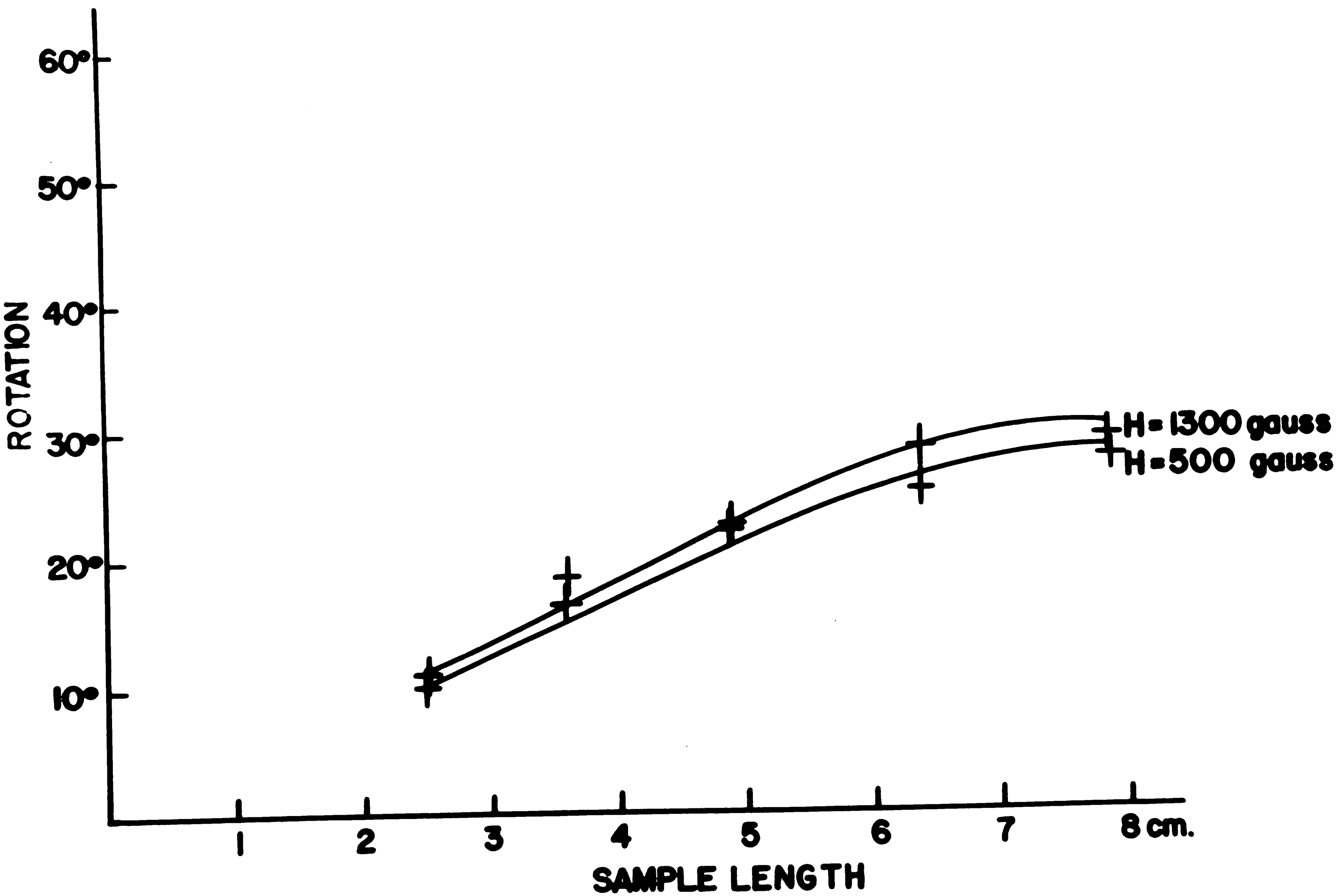


Fig. 3 - Rotation vs. Sample Length for Incident Wave 30° to Field

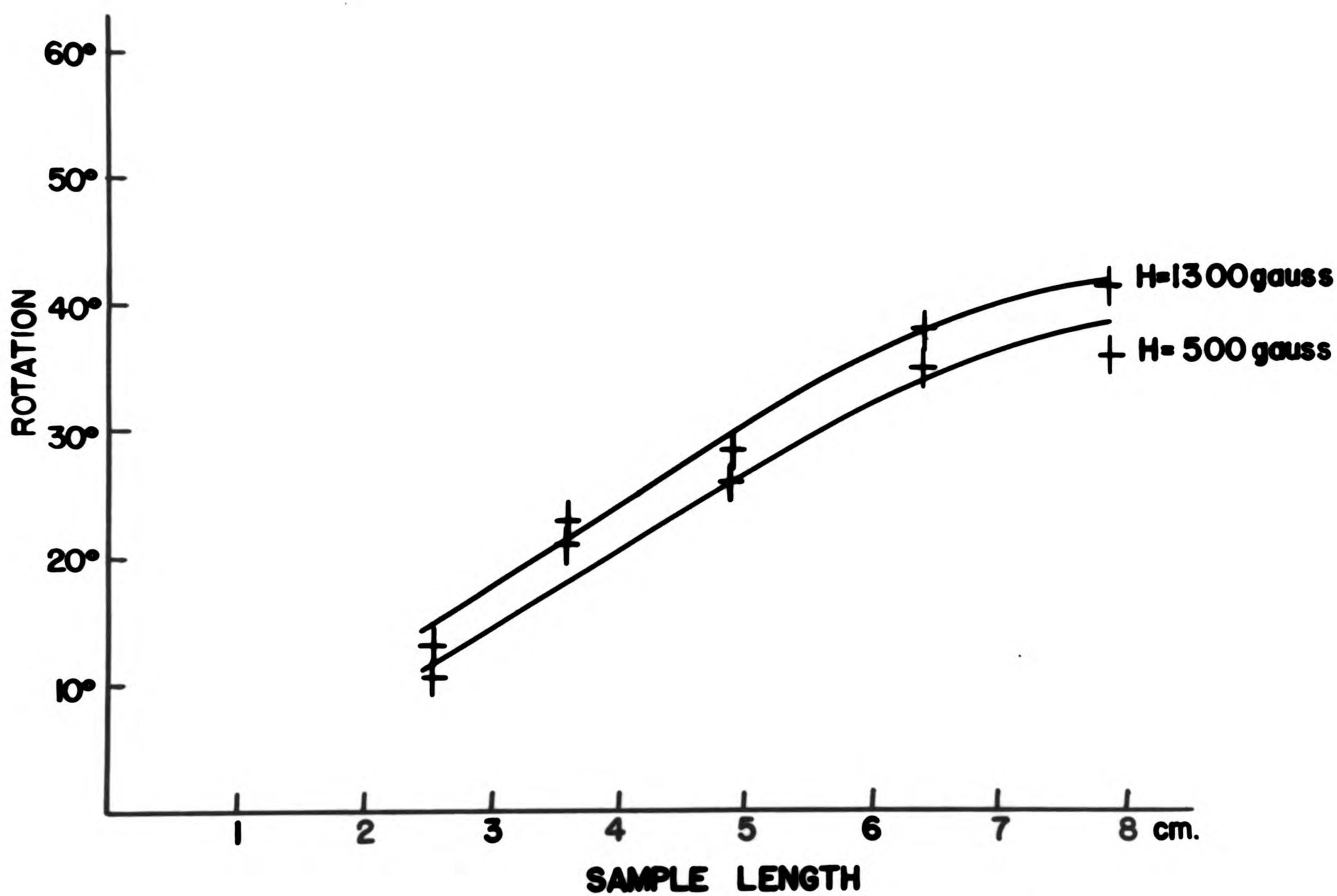


Fig. 4 - Rotation vs. Sample Length for Incident Wave 45° to Field

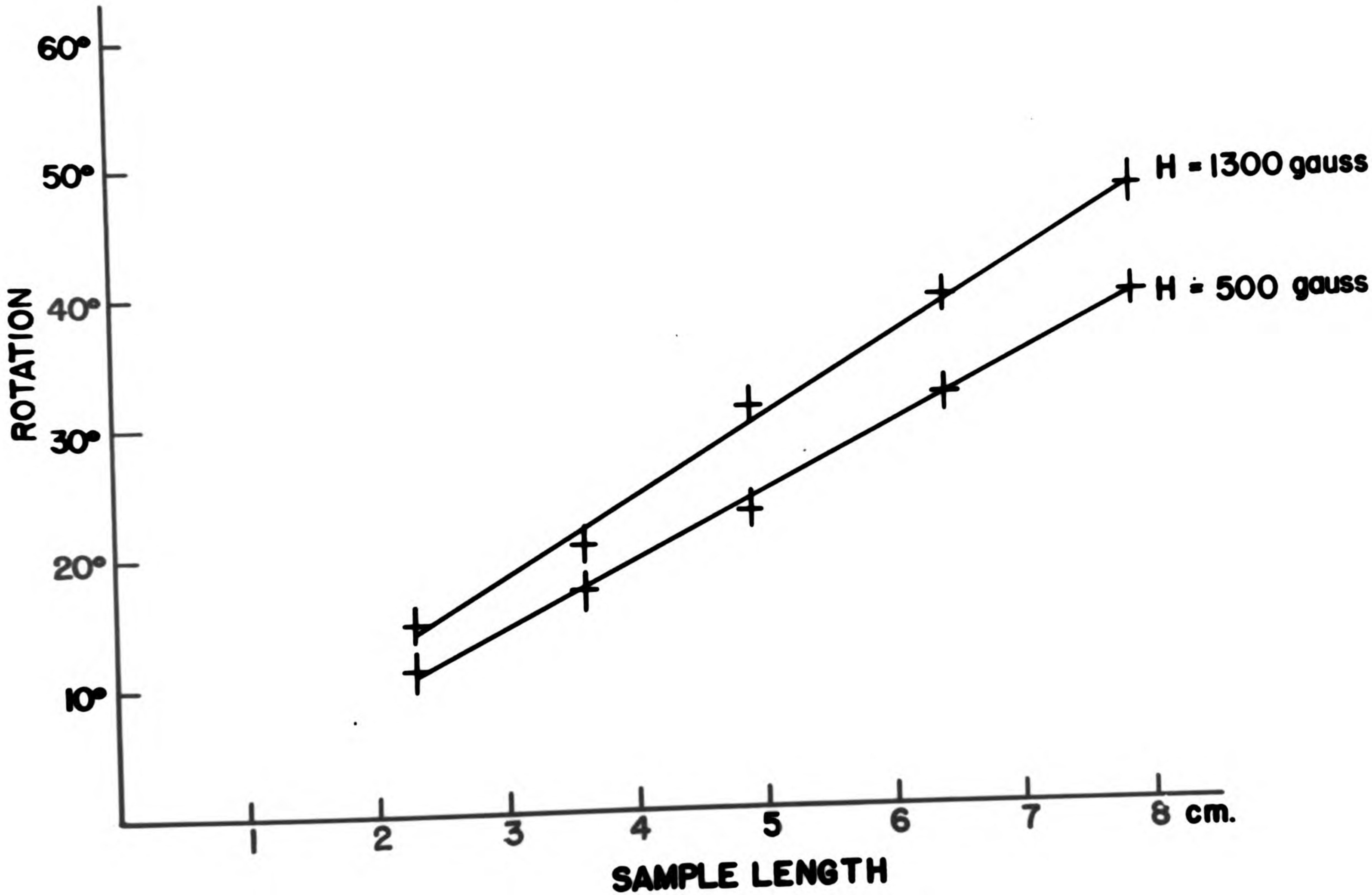


Fig. 5 - Rotation vs. Sample Length for Incident Wave 60° to Field

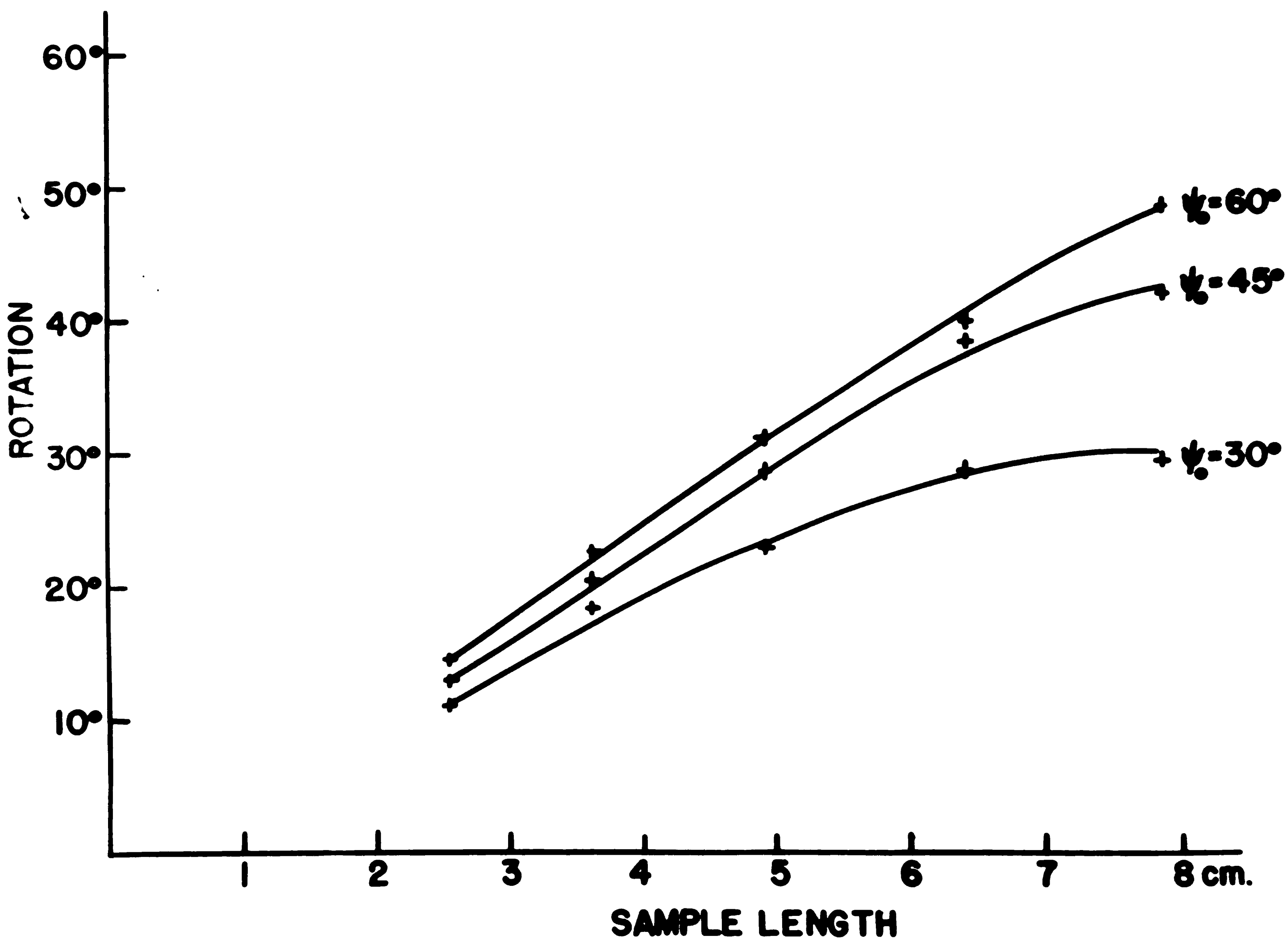


Fig. 6 - Rotation vs. Sample Length for Three Different Incident Waves at 1300 gauss

CONCLUSION

The simple picture that we proposed for the rotation of the plane of polarization due to differential absorption seems to agree fairly well with the experimental results. There is some difference that does occur.

From the curve taken with electric vector parallel to the magnetic field, it is seen that the average error in determining the position of the maximum or minimum is 3°. With an error of this amount, the rotation obtained at a field of 1,300 gauss may not be greater than the angle that the electric vector makes with the magnetic field as it impinges upon the sample. To answer this question, the magnet should somehow be made to give a field of about 3,000 gauss. Even with our assumption that the rotation is caused by a differential absorption, the field picture of the TE_{11} mode shows that the electric vector is not straight but is curved slightly so there is always a small component of the electric field that is perpendicular to the magnetic field.

Another possible reason for the experimental curve being different from our theoretical assumptions is that the molecules of the fluid tend to be oriented by a temperature gradient[4] and also the molecules near the wall of the cell tend to be aligned parallel to the wall.

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