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A NEW METHOD FOR MEASURING  
THE REFLECTING POWER  
OF RETROREFLECTORS

Thesis for the Degree of M. S.  
MICHIGAN STATE COLLEGE

Harry C. Morgan  
1939

THESIS



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A NEW METHOD FOR MEASURING  
THE REFLECTING POWER OF  
RETROREFLECTORS

BY  
HARRY CLARK MORGAN

A THESIS

Submitted in partial fulfilment of the requirements  
for the degree of Master of Science in the  
Graduate School, Michigan State College

Department of Physics  
East Lansing, Michigan  
June 1939

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Acknowledgement

I wish to express my sincere appreciation to Dr. C. W. Chamberlain of the Physics Department of Michigan State College for his invaluable assistance and encouragement. I am also indebted to Mr. Robert L. Rowe for his advice and aid in the photographic phase of this thesis. The ready advice and interest of the Physics Department staff has made my task a pleasant one.

*Harry Clark Morgan*

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### Statement of Problem

The need for lighting aids of some form to increase safety on our highways is recognized by all drivers. Modern headlights when switched to illuminate the road far ahead do fairly well for speeds under fifty miles per hour. The higher speeds, at which a large percentage of us are wont to drive, often bring us upon curves and hazards before our muscles have time to obey the commands of the brain. Approaching cars with their bright lights cut down our visibility to a small fraction of what it was previously. With a constant stream of oncoming traffic we do well to know just where the side of the road is. A good night driver is one who does not worry over the possibility of a man who has become negligent walking on the edge of the pavement.

One safety aid, which is in dispute at the present time, is the illumination of our main highways by overhead electric lights every thousand feet or so. This might be a great aid but for its high cost of installation and nightly consumption of power. Economy dictates that the lights be spaced as far apart as possible. This results in regions of relative darkness succeeded by regions of bright light. At high speeds this rapid fluctuation of intensity strains the eyes, especially the irises which are forced to expand and to contract, and rapidly fatigues the driver.

These facts might well be kept in mind throughout the discussion of the retroreflector which many engineers are enthusiastically investigating.

The retroreflector is a surprising device. It utilizes the light

of the headlights of the oncoming car, and reflects it back to the driver's eyes so that it appears like a small electric light beside the road far ahead. The light may strike the reflector at widely varying angles of incidence and still be reflected back in the direction from which it came.

Devices using this principle have been developed in the past ten years by several commercial companies and have been widely used in road signs to indicate curves, cross roads, railroads, and other hazards of the road. We have come to expect them to warn us of approaching danger. Words are spelled out with letters formed with several "cats eye retroreflectors".

A more recent type which shows great promise has as its fundamental unit three planes which meet at right angles to each other, or might be termed a cube corner. This reflector was devised by Augustin J. Fresnel, a famous French scientist of the early nineteenth century. Incident light is also reflected back upon itself to the driver's eyes by reflectors comprising a plurality of cube corners. This type of reflector has been used to outline the road between Lansing and Detroit as a test installation. The Michigan State Highway department reports that since the installation, night accidents have been reduced forty percent. My personal observations have convinced me that these reflectors have made travel much safer. The road is clearly outlined far ahead, and obstacles, such as parked cars and pedestrians blot out one or more of the reflectors thus showing their presence.

It would be interesting at this point to discuss some of the properties of a good retroreflector. Obviously it must return light

which falls upon it at different angles of incidence with the axis of the reflector. The button must not return the light directly back to its source, but in a cone of about one degree so that part of the reflected light enters the driver's eyes. This angle represents a practical compromise and gives the driver a brightly illuminated reflector at distances ranging from two or three thousand feet to about a hundred feet from the car to the reflector.\* When an angle of about forty degrees is made by the incident light and the axis of the reflector, the car is close to the retroreflector. The reflector need not return light at greater angles.\*

The reflecting surface of the retroreflector should not deteriorate with time. Deterioration of the reflecting powers of a reflector would require frequent replacement and thus added cost and an increased road hazard if the poor units are not immediately replaced.

Very little work has been published on methods of measurement of the optical properties of retroreflectors. Accurate measurements of the efficiency of retroreflectors at different angles of incidence as well as quality of workmanship is very desirable. Such measurements enable the highway engineer to determine the merit of various retroreflectors and to select those retroreflectors which will best serve the driving public. The information derived from these measurements will indicate what further research and improvement is necessary.

\* R. Kingslake An apparatus for testing highway sign reflector units. Journal of the Optical Society of America Vol. 28, Sept. 1938 page 323.

### Theory of the cats eye retroreflector

Figure 1 diagrammatically represents the simple theory of the cats eye retroreflector. A converging lens brings parallel light to a focus. Light ray 1 is bent so as to approach the normal line to the lens. When it reaches the focus at point B, ray 1 undergoes reflection at an angle which is twice the angle between the incident ray at B and the normal to B and the lens. The reflected ray enters the lens at N and emerges from the lens parallel to the entrant path, but displaced. Ray 2 makes an angle with the normal to the lens, is refracted by the lens and is brought to the lens focus at point A where it undergoes reflection. The reflected ray makes an angle with the incident ray of twice the angle that the incident ray makes with the normal to A. This reflected ray is refracted by the lens at point L so that it emerges parallel to but displaced from the entrant path. Surface AB must therefore coincide with the focal plane of the lens and also have the normal at every point in its surface pass through the center O of the lens.

Figure 2 shows an early type. The system consists of the major portion of a glass sphere and a metal reflector a short distance behind the glass.

Figure 3 shows a more recent type incorporating several improvements. The focus of the lens is at the rear surface of the lens. Silver is used to make this surface reflect the light back through the lens. This silver coat is protected by a heavy coat of lacquer or paint. This type secures retroreflection at much greater angles of incidence than the type shown in figure 2.

The maximum angle of incidence with the lens axis is determined by the physical limitations imposed by the design. Theoretically a cats eye could be made which would retroreflect at ninety degrees incidence with the lens. However as stated on page three retroreflecion is not necessary at such a great angle. As has been shown, the reflected light must not return perfectly upon its incident path but in a small cone which includes the driver's eyes. It is difficult to design a reflector which will accomplish this. The lens is cast and as a result does not have optical surfaces. Light incident at angles with the lens axis is distorted and the returned light forms different kinds of patterns.

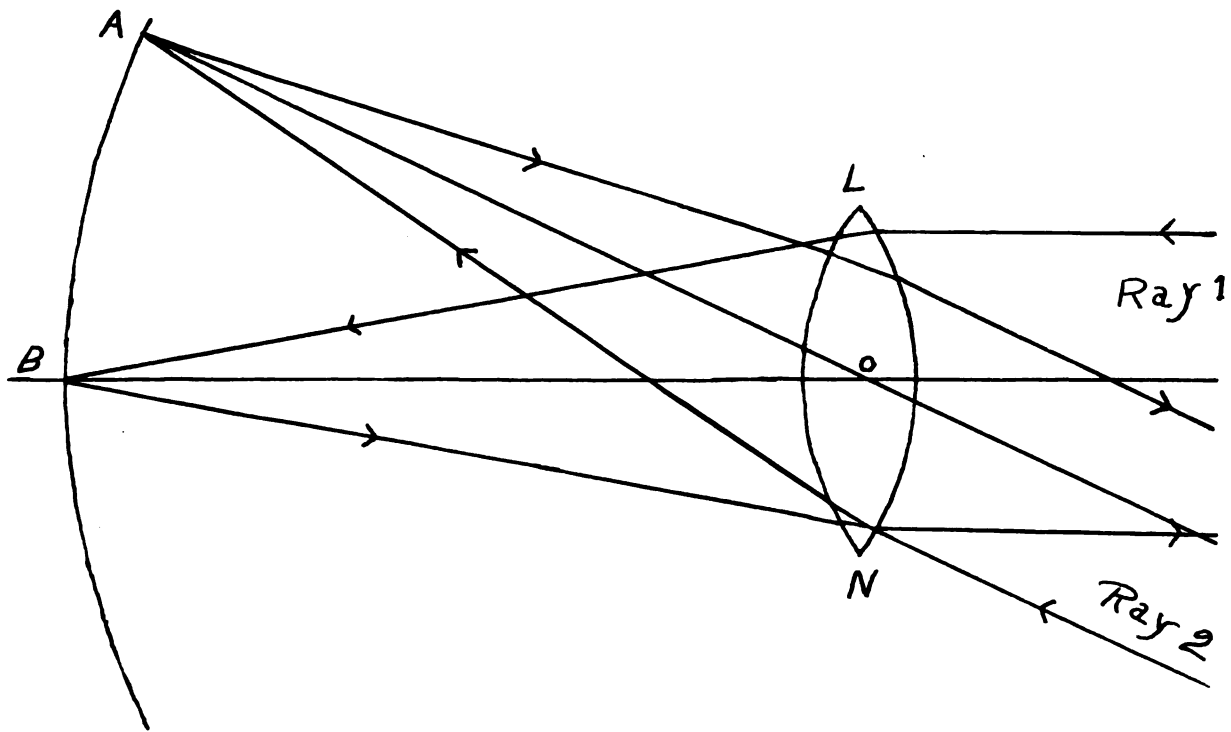


Figure 1

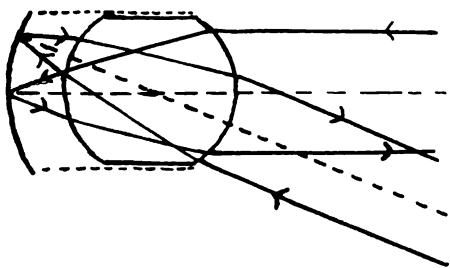


Figure 2

Early type of  
retroreflector

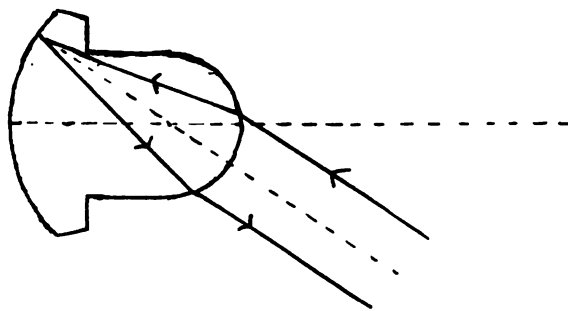


Figure 3

Improved retroreflector

## Theory of central triple retroreflectors

The theory of the Fresnel facet may be more easily understood if we consider the simpler two dimensional case before the three dimensional case.

Figure four shows two reflectors, AB and BC at right angles to each other. A light ray I is incident at F making an angle  $x$  with the normal EF and a complementary angle  $y$  with BC. By the law of reflection the ray DF makes the same angle  $x$  with the normal EF and the same complementary angle  $y$  with BF. Ray DF is incident at D, making an angle  $y'$  with the normal ED and a complementary angle  $x'$  with AB. The reflected ray R by the law of reflection makes the same angle  $y'$  with the normal ED and the same complementary angle  $x'$  with AB. Since the sum of the inside angles of a triangle equals 180 degrees, it is obvious that  $x' + y + 90 = 180$  degrees or that  $x' + y = 90$  degrees and  $x' + y' = 90$  degrees. ED is perpendicular to AB, and EF is perpendicular to BC. AB was constructed perpendicular to BC. DE is thereby parallel to BC. Since  $y' = y$ , the reflected ray R is parallel to the incident ray I, but displaced, for any angle of incidence  $x$ .

A much simpler proof which will be used in the three dimensional case is as follows. The lines R'D, D'D, I'F, F'F represent extensions of the original lines shown. Since the included angles formed by two intersecting lines are equal,  $x' = x'' = x''' = x$  and  $y' = y'' = y''' = y$ . It can be seen that the incident ray I upon reflection at F is rotated through an angle of  $y + y$  or  $2y$  degrees. Ray FD is again reflected and rotated through an angle of  $x + x$  or  $2x$  degrees from its incident path. The ray R has thus been rotated through a total angle of

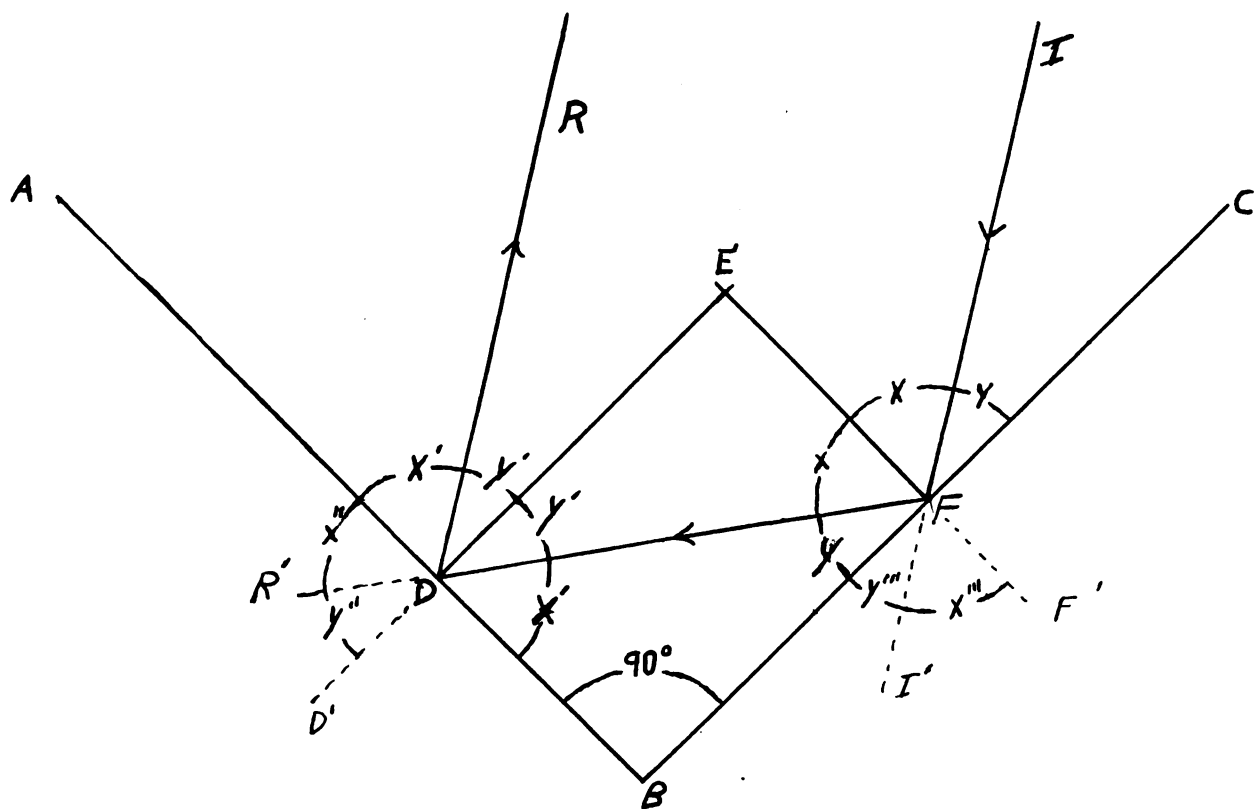


Figure 4

$2x + 2y$  or 180 degrees (since  $x + y = 90$  degrees) upon the double reflection. The ray R is thus parallel to ray I but reversed in direction.

Figure five illustrates the three dimensional case of retroreflection. It is plainly seen that the addition of one more dimension complicates the geometry of the problem. FC, JB, AN, are normals to the points of incidence of the light ray. Projections of the light ray onto yz planes are: BC onto BG, AB onto BN. Projections of light rays onto xz planes are: IC onto CD, CB onto EC, CB onto EF, AB onto MA, AB onto BK, AB projected down onto EH, RA onto PA, RA onto GH. It is not necessary to prove that angles are equal as on page 7, it will be assumed that their equality is obvious. The method used just preceeding this discussion will now be resorted to.

The incident ray I is rotated through an angle  $2w$  in the plane determined by BECDI, but zero degrees in the plane xz. Ray BC incident at B is reflected along AB. By virtue of the projection of AB onto BN and of BC onto BG ray BC is rotated through an angle of  $2o$ . By virtue of the projection of BC onto EC and of AB onto EH ray BC is rotated through an angle of  $2w$ . Ray BA incident on point A is reflected along RA. The ray BA is rotated zero degrees in the y direction in which it is measured. Ray BA is rotated through an angle of  $2u$  as shown by the projections. Summing the angular rotations we find:

$w + w + 2o = 180$  degrees by the fact that  $w + o = 90$  degrees and that

$2u + v + v = 180$  degrees by the fact that  $u + v = 90$  degrees. The reflected ray RA is thus parallel to the incident ray IC but reversed in direction.

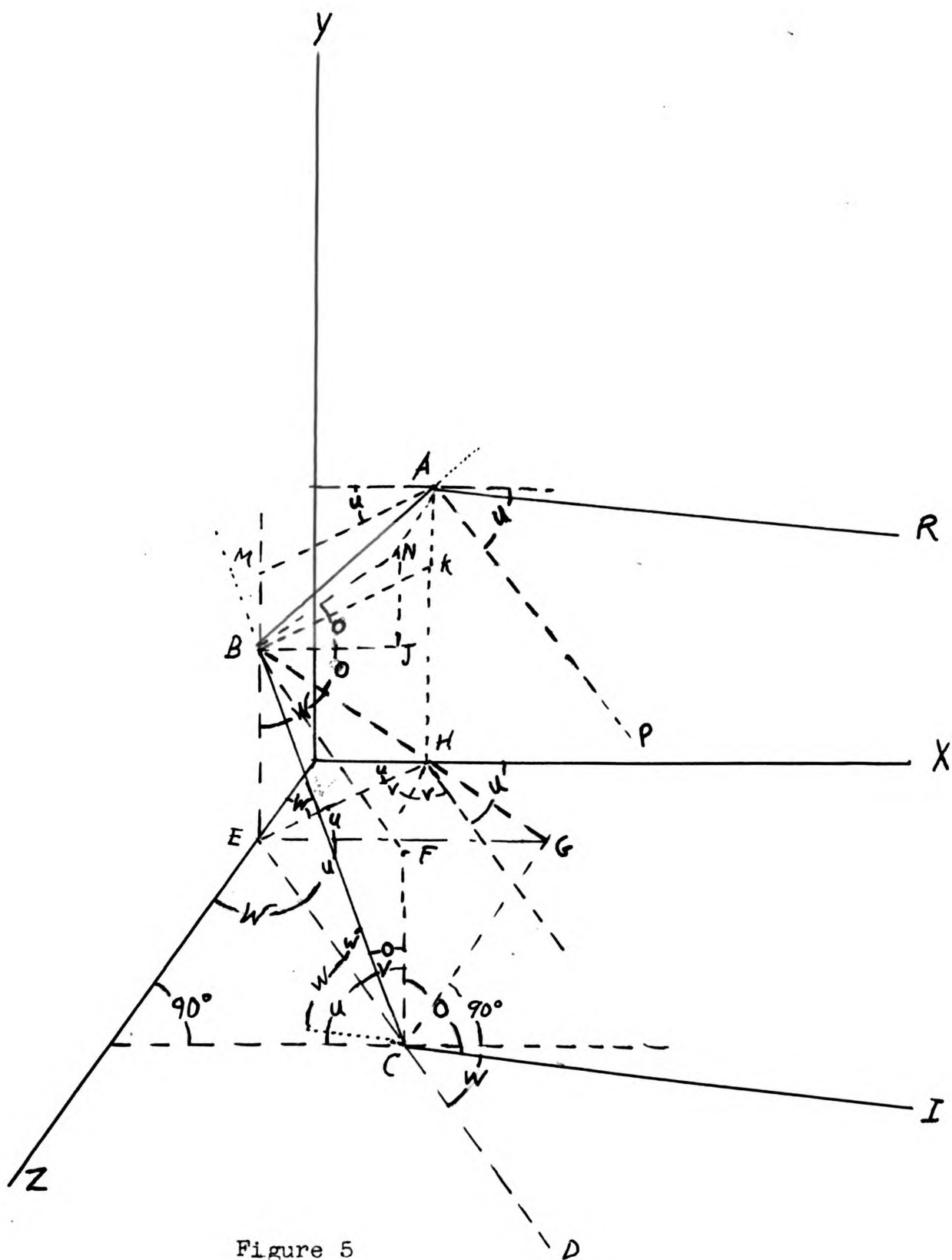


Figure 5

One of the first central triple retroreflector<sup>S</sup><sub>λ</sub> was made up of triangular faces forming the cube corners as shown in figures six, seven and eight. This reflector however is not very efficient. By observing the reflection of an enlarged facet with parallel light incident upon it one finds the corners as shown by figure 7 with the lines ABC, DEF, GHI dark. This means that some of the light which falls upon the facet leaks out the side after two reflections and is lost. This amounts to about thirty per cent loss. At twenty five degrees incidence with the axes of the retroreflector, only the region JOMK is illuminated. Thus nearly two thirds of the incident light is lost. This is a very interesting experiment and should be carried further. The commercial reflector<sup>S</sup><sub>λ</sub> made up of these facets have about one hundred units joined edge to edge in the form of a disk.

An improved design is a facet composed of three square surfaces meeting at right angles. This concave facet is shown in figure 9. At axial incidence the reflector turns all the light which falls upon it. This does not take into consideration the light which is lost because of imperfect internal reflection. At about forty degrees incidence, as shown by figure ten, the point B is turned outwardly, and the bottom in, relative to the paper. The only portion of the mirror which is filled with reflected light is the area ADGB, making about twenty five percent reflection. The rest of the incident light is lost out the sides after one or two reflections. Figure eleven shows the same reflector rotated thirty degrees in the opposite direction. The region ABCDEF, about thirty five percent of the surface, is filled with reflected light. These results were

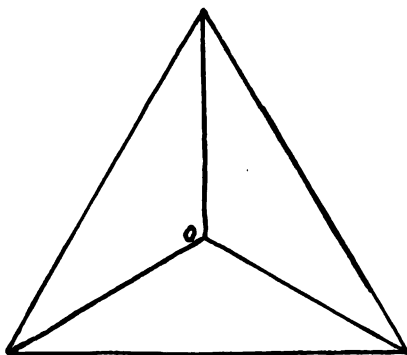


Figure 6

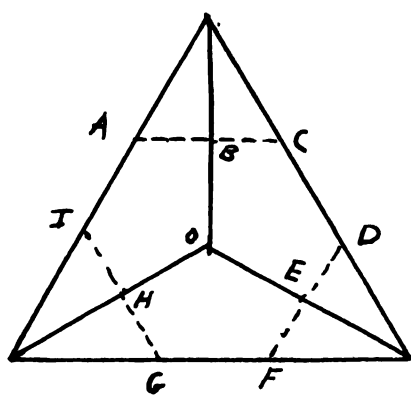


Figure 7

Normal Incidence

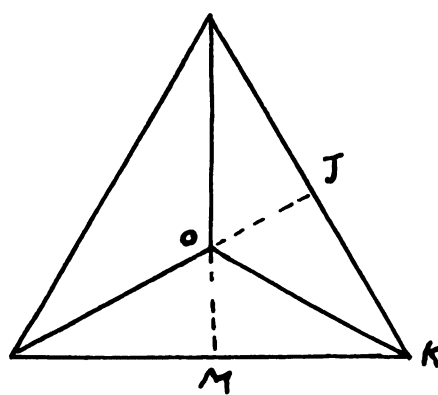


Figure 8

25° Incidence

obtained with facets made with silvered mirrors viewed through a half silvered mirror held at forty five degrees between facet and the observer. The parallel light enters the system from the side and is reflected from the half silvered mirror.

Commercial retroreflectors are composed of about a hundred cube corners as illustrated in figure eleven. The front face of the reflector is somewhat convex, and the back face is composed of the cube corners. The reflection of light is dependent upon total reflection from the cube faces. More specifically, the light is incident upon the cube faces in the transparent media of the reflector at angles greater than the critical angle. If light is incident upon a cube face at less than the critical angle, that light is lost.

The preceding theory is of necessity brief. In the near future, Dr. C. W. Chamberlain, of this department, will publish a complete theory of the central triple retroreflector. He will also discuss mathematically the efficiency of the reflectors.

The following is a list of the names of the persons who have been  
 named in the report of the Committee on the subject of the  
 proposed amendment to the Constitution of the State of New York,  
 and the names of the persons who have been named in the report  
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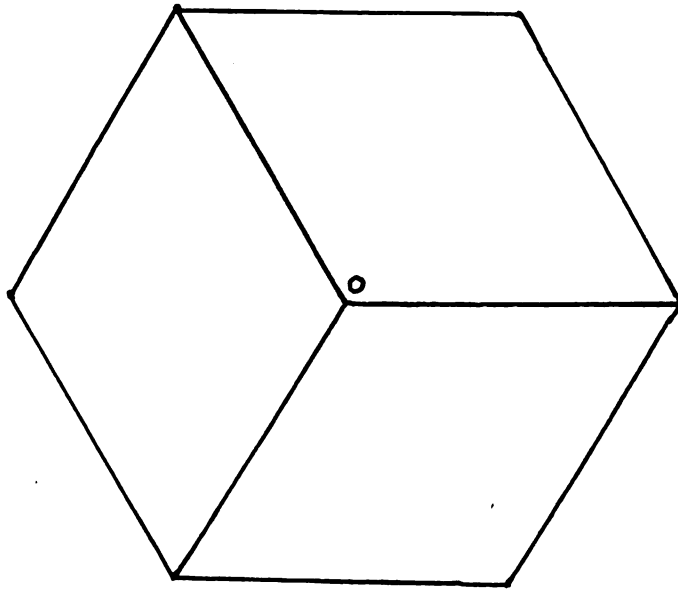


Figure 9

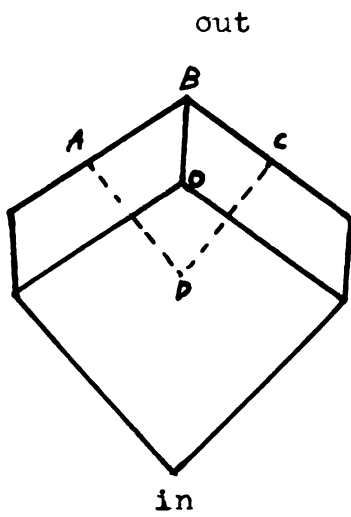


Figure 10  
40° Incidence

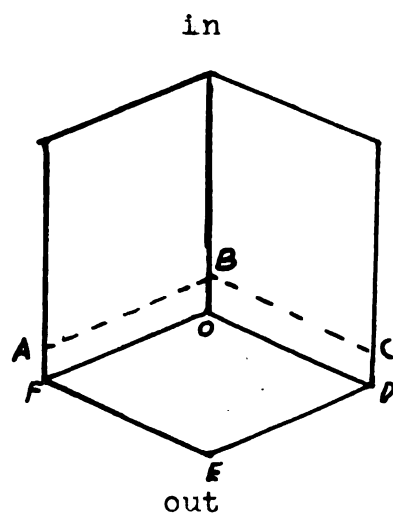


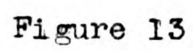
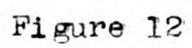
Figure 11  
30° Incidence

### Methods of Measurement

There have been several laboratory methods devised to test retroreflectors. The standard test adopted in 1932 in the joint I.E.S-S.A.E. Standard Specification for Laboratory Tests of Reflex Reflectors for Motor Vehicles is a visual one. The reflector under test is placed one hundred feet from a ten thousand candle power headlight thus furnishing an illumination of one foot candle at the reflector. The apparent candle power of the reflector is compared visually by an observer who is looking at the reflection with his eyes seven inches above the head lamp. With an illuminated opal glass screen placed beside the button the intensity of the light placed behind the screen can be varied at will.\*

A more accurate measuring device was made by R. Kingslake, University of Rochester, Rochester, New York. Figure twelve is a simple sketch of his method. L is the lamp house, S asbestos screens, C the condensing lens, R the photovoltaic cell, D the diaphragm, M the microammeter, T the objective lens, and B the reflector under test.

The principle of Mr. Kingslake's apparatus is as follows. The hole in the photovoltaic cell permits light to emerge via the condensing lens as a point source. The lens T refracts the light so that it is parallel when incident upon the button being tested. The button reflects back through the lens T to the point source in the center of the photovoltaic cell (the hole) and causes no meter deflection. If the reflector returns the light in a small cone, the light in the cone causes a meter deflection. The diaphragm limits the size of the cone to any size desired. The meter deflections are assumed to give

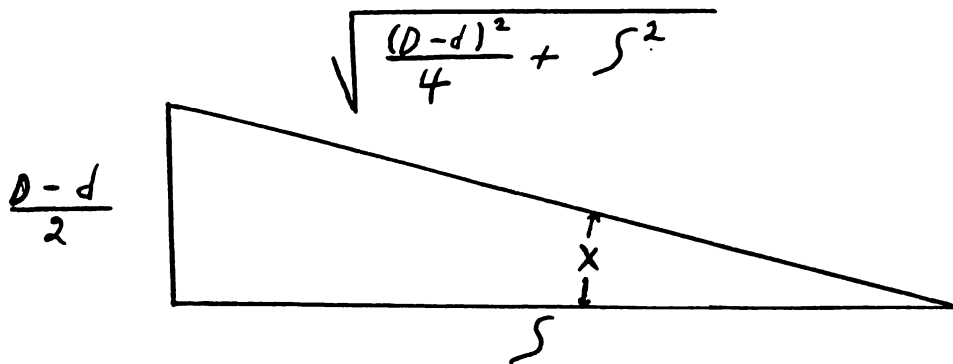


an indication of the efficiency of the button as a retroreflector for highway use.\*

\* Apparatus for Testing Highway Sign Reflectors by R. Kingslake,  
Journal of the Optical Society of America Vol 28 Sept. 1938

## Method and Measurement of Reflected Cone

I am indebted to Dr. C. W. Chamberlain for this excellent method. Figure thirteen illustrates the arrangement of apparatus. P is parallel light from a heliostat arranged so as to bring parallel sunlight into the laboratory. H is a half silvered mirror at forty five degrees incidence with the sunlight, R is the retroreflector under examination, and A is a white paper screen. Two times the angle  $x$ , shown in figure 13, is equal to the angle of the apex of the cone.



The above figure shows known quantities from which it is obvious that

$$\sin x = \frac{\frac{D-d}{2}}{\sqrt{\frac{(D-d)^2}{4} + S^2}}$$

and that

$$2x = 2 \arcsin \frac{\frac{D-d}{2}}{\sqrt{\frac{(D-d)^2}{4} + S^2}}$$

Table I

S was found to be 21 feet,  $5\frac{1}{4}$  inches. Light was normal to reflector.

Central Triple Retroreflectors

Reflector	d	D	2x
green	1 $\frac{3}{8}$ in	4.5 in	42'
white	1 $\frac{3}{8}$ in	7 in	1°18'
amber (same as green reflector)			

Cats eye retroreflectors

Reflector	d	D	2x
green A	13/16 in.	3 in	30'
green B	(too faint to be measured)		
green C	13/16 in	2 in	16'

The green and amber central triple retroreflectors are designed for the rear of trucks. Therefore the small cone would probably be satisfactory. The white one, which is like the reflectors between Lansing and Detroit appears to be correct. The cats eye cones seem rather small.

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### Description and Theory of Apparatus

Figure fourteen illustrates the apparatus used. L is a six to eight volt thirty-two candle power auto headlamp bulb. M is a collimating lens of about seven inches focal length. C is the rotating block on which the button being tested is fastened. P is a scale calibrated in degrees, and O the pointer. H , H are half aluminized mirrors. E is a Weston Photronic photovoltaic cell. K is a triple pole, double throw knife switch. G is a very sensitive galvanometer. A is a zero to ten ammeter. V is a zero to fifteen volts voltmeter. R is a variable resistor. S is a single pole single throw knife switch. D is a diaphragm with the same aperture as the reflector.

Assume that the mirrors have fifty percent films. Fifty percent of the light is transmitted by the first mirror, and fifty percent of the transmitted light is reflected by the second mirror, twenty-five percent is incident upon the photoelectric cell. The fifty percent of the light incident upon the first mirror is reflected to the button being tested. If the button reflects one hundred percent of the light, the fifty percent is reflected back to the first mirror where half of the fifty percent or twenty-five percent is transmitted to the photoelectric cell which is moved to receive it. If the retroreflector being tested does not reflect all of the incident light, the photoelectric cell will have a decreased current output which when compared with the current generated by the light reflected from mirror number two is a measure of the reflecting power of the retroreflector. The

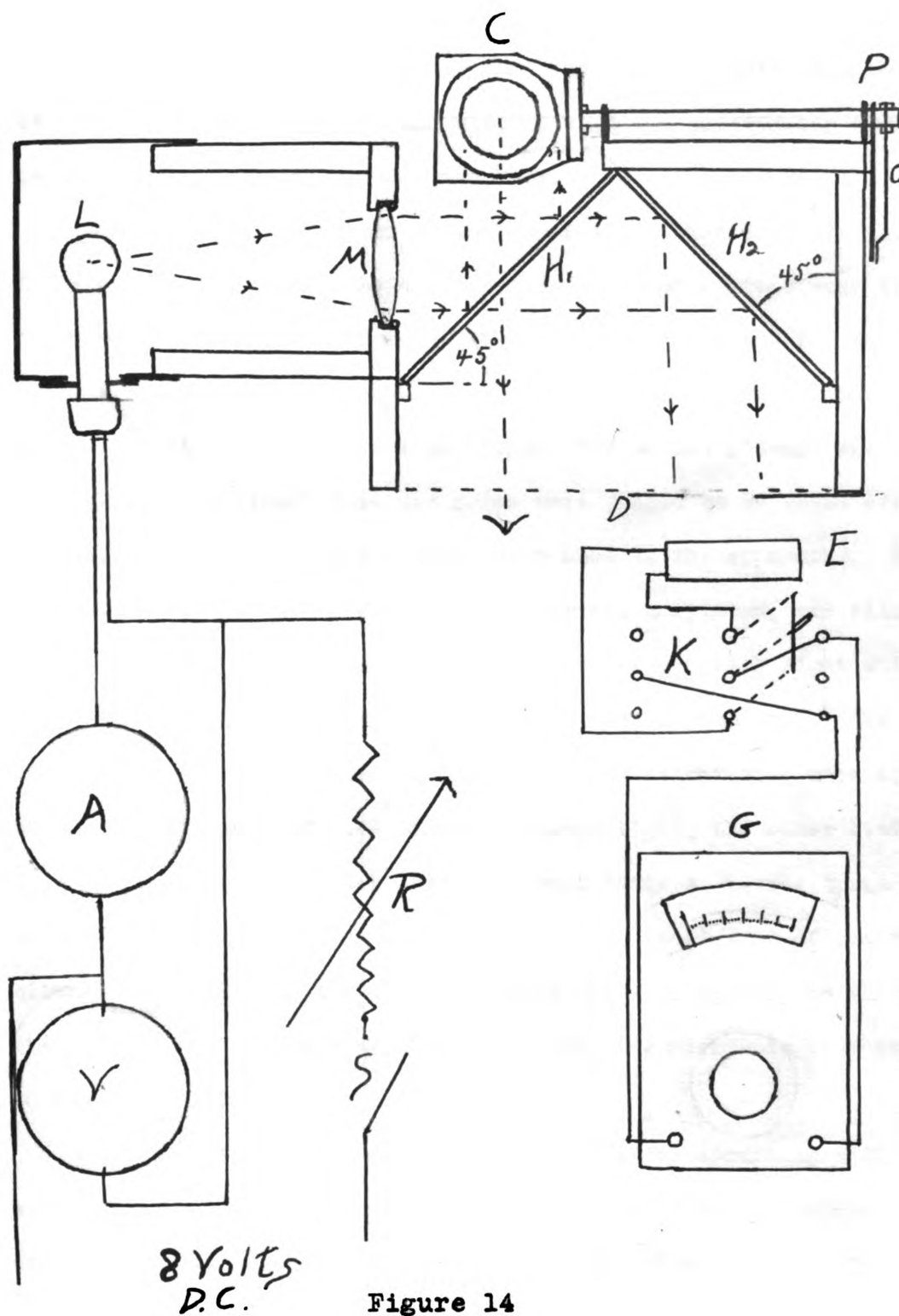


Figure 14

mirrors need not be fifty percent reflectors and transmitters, nor be alike, just so long as the actual reflecting and transmitting power is known for each film.

The switch K was inserted so that the photoelectric cell could be disconnected and at the same time so that the <sup>1</sup>galvanometer could be short-circuited to make it dead beat.

The most difficult part of the apparatus to construct is the fifty percent mirrors. The mirrors were silvered a great many times, but a satisfactory fifty percent film which was the same for each mirror could not be produced. The evaporation of aluminum in a vacuum onto the mirrors was then tried. The second attempt was successful. By visual test the films were judged to be about eighty percent reflecting. These mirrors were used in the apparatus. After the data, which <sup>are</sup> recorded in this paper <sup>were</sup> completed, the film thickness was tested by interrupting a beam of parallel light which fell upon the photoelectric cell with the mirrors and noting the difference in galvanometer reading. The results obtained were these: one mirror transmitted 2.4% of the incident light, the other 1.69%. This indicates the unreliability of visual tests on mirror transmitting and reflecting powers. The standard visual test of standing between two lights of the same intensity and viewing the one through the mirror and the other by reflection from the mirror is thus shown to be unreliable.

A photovoltaic cell was selected because the apparatus will eventually be taken out on the road to make tests on reflectors which are in service. It also eliminates the need for a high voltage direct current battery for the photocell. Since the manufacturer did

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The Secretary of the Interior  
J. M. McKIM

not furnish curves which showed the relationship between the intensity of incident light and the current generated by the photo-electric cell, data <sup>were</sup> ~~was~~ taken to determine this relationship. The current changes as the cell is placed at different distances from a standard electric light bulb on an optical bench. This data is listed in table II and is plotted on graph I.

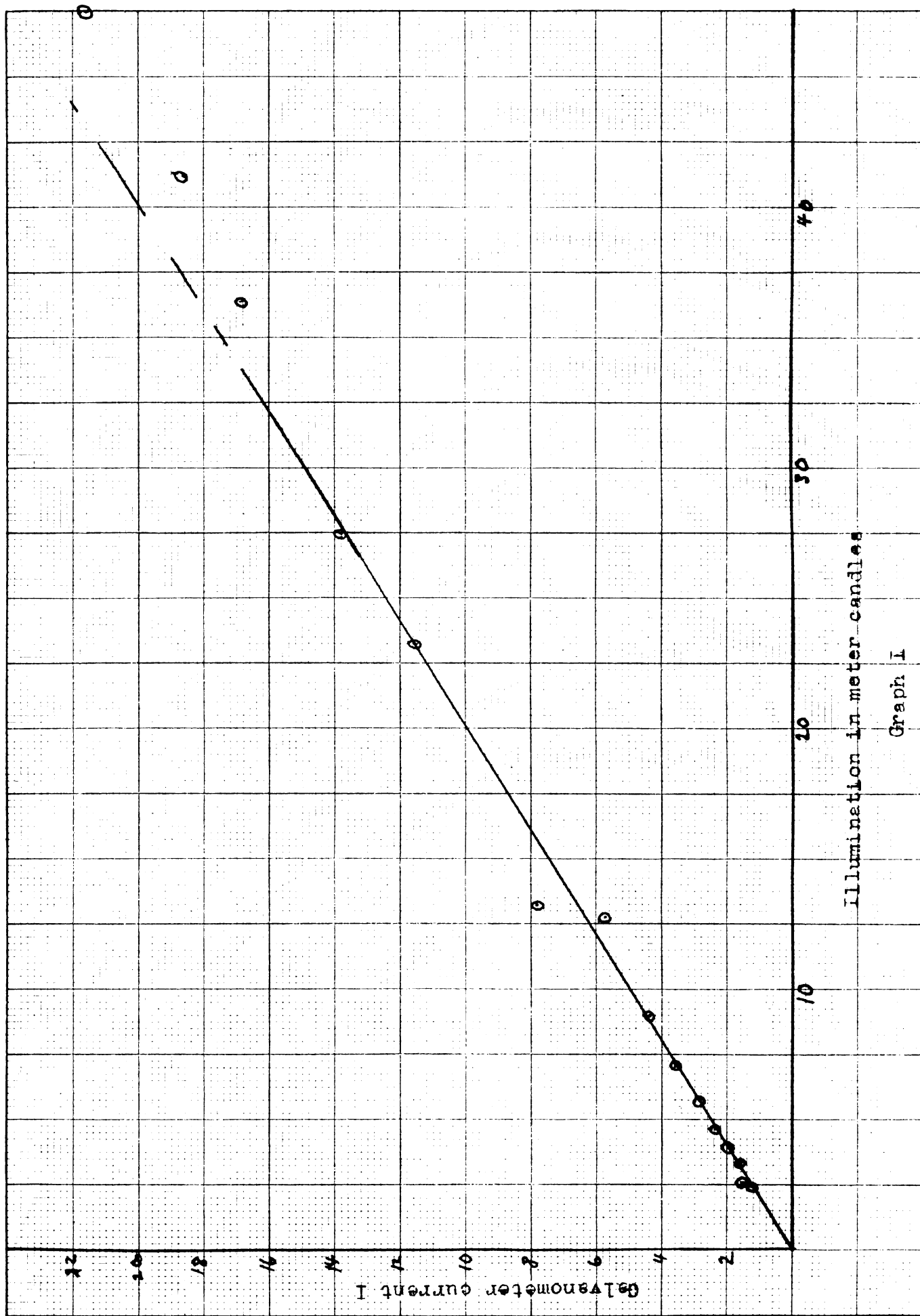
Table II

Source 6-8 volt auto bulb standardized as 5.82 Candle power

current I of Galv.	d in meters from light	$1/d^2$	light intensity in meter candles
1.3	1.5	.444	2.58
1.5	1.4	.458	2.67
1.7	1.3	.592	3.44
1.98	1.2	.694	4.04
2.38	1.1	.827	4.81
2.90	1.0	1.000	5.82
3.62	.9	1.234	7.18
4.40	.8	1.56	9.08
5.78	.7	2.20	12.80
7.85	.6	2.776	13.24
11.45	.5	4.00	23.25
13.70	.45	4.94	28.75
16.80	.4	6.25	36.35
18.57	.375	7.11	41.30
21.68	.35	8.16	47.45

Theoretically the output current of a photovoltaic cell doubles when the illumination is twice as great. For low light intensities and a low external resistance in the cell circuit this is true. Graph one shows that this is true to within a few percent. When the illumination exceeded 35 meter candles the distance between the light source and the cell was too small for the inverse square law to be applied. The variation of cell output for the same illumination will be discussed later.





After a few experiments, it became evident that alternating current could not be used to power the lamps in the apparatus. The line voltage fluctuations made consistent readings impossible. Storage batteries were then resorted to with excellent results. All measurements have been made with this source of direct current energizing the light bulbs.

# Measurement of the Efficiency of Retroreflectors at Different Angles of Incidence

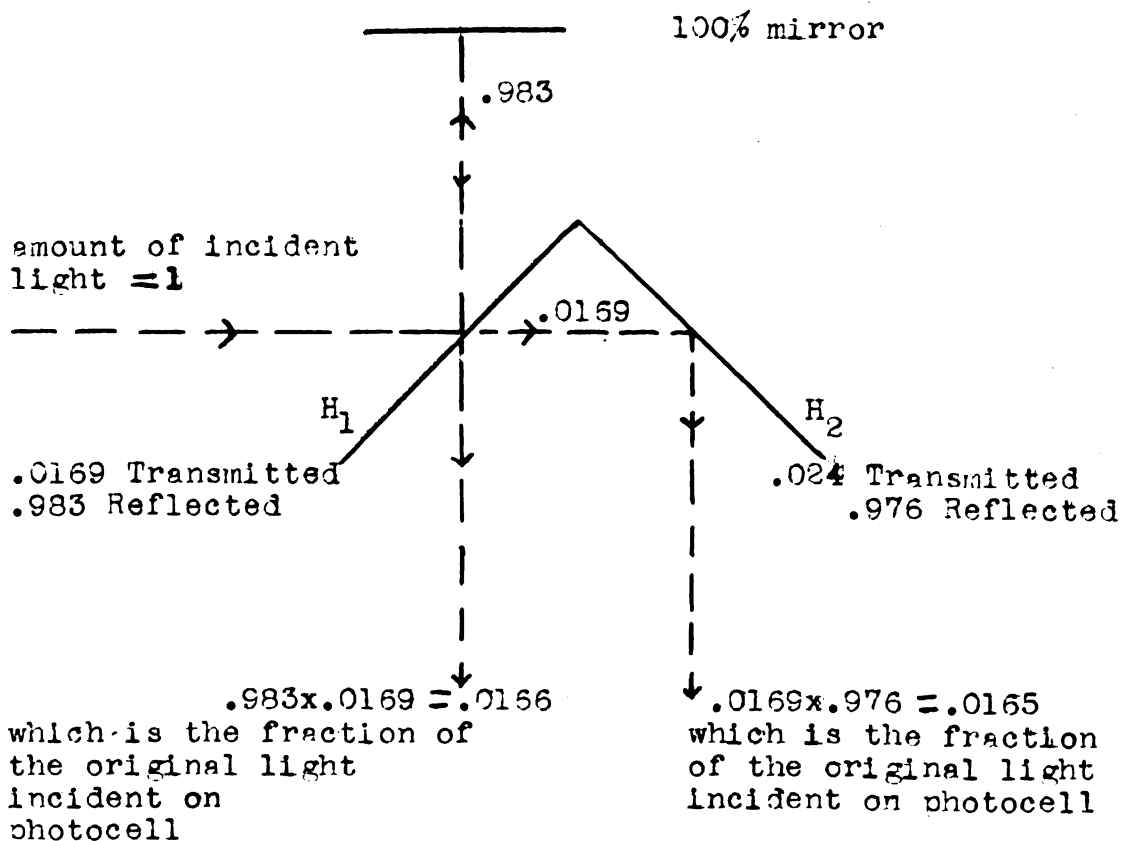
The procedure has already been outlined in brief. A more detailed explanation is in order at this point. The light is kept at constant brilliancy by proper manipulation of the variable register R in figure fourteen. The reflector is secured to block C. The photocell is then illuminated with the light passing through diaphragm D. The galvanometer deflection is noted. The photo-electric cell is then moved over so that the reflected light from the retroreflector is incident upon its surface. The galvanometer deflection is noted. The angle of incidence of the light on the reflector is now changed and the galvanometer deflection is noted. After the retroreflector is explored at all desired angles of incidence, a check is made upon the light emerging from the diaphragm D. The galvanometer was usually shorted and the photocell disconnected by throwing switch K over to the left. This protected the galvanometer from damage by large currents when the lights were turned on to change the angle of incidence. It often appeared to make the data more consistent since the cell had time to recover between readings.

It was now necessary to calculate the correction factor needed to compensate for the differences in transmission and reflection percentages of the two mirrors.

The mirror which transmits 1.69% of the incident light was placed in the position of  $H_1$  and the mirror which transmits 2.4% was placed in the position of  $H_2$  in figure fourteen. One way to determine

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \int_0^x f(t) dt$ . It is shown that  $f(x)$  is a constant function, and its value is determined by the initial condition  $f(0) = 1$ . The second part of the paper is devoted to the study of the properties of the function  $g(x)$  defined by the equation  $g(x) = \int_0^x g(t) dt$ . It is shown that  $g(x)$  is a constant function, and its value is determined by the initial condition  $g(0) = 1$ . The third part of the paper is devoted to the study of the properties of the function  $h(x)$  defined by the equation  $h(x) = \int_0^x h(t) dt$ . It is shown that  $h(x)$  is a constant function, and its value is determined by the initial condition  $h(0) = 1$ . The fourth part of the paper is devoted to the study of the properties of the function  $k(x)$  defined by the equation  $k(x) = \int_0^x k(t) dt$ . It is shown that  $k(x)$  is a constant function, and its value is determined by the initial condition  $k(0) = 1$ . The fifth part of the paper is devoted to the study of the properties of the function  $l(x)$  defined by the equation  $l(x) = \int_0^x l(t) dt$ . It is shown that  $l(x)$  is a constant function, and its value is determined by the initial condition  $l(0) = 1$ . The sixth part of the paper is devoted to the study of the properties of the function  $m(x)$  defined by the equation  $m(x) = \int_0^x m(t) dt$ . It is shown that  $m(x)$  is a constant function, and its value is determined by the initial condition  $m(0) = 1$ . The seventh part of the paper is devoted to the study of the properties of the function  $n(x)$  defined by the equation  $n(x) = \int_0^x n(t) dt$ . It is shown that  $n(x)$  is a constant function, and its value is determined by the initial condition  $n(0) = 1$ . The eighth part of the paper is devoted to the study of the properties of the function  $o(x)$  defined by the equation  $o(x) = \int_0^x o(t) dt$ . It is shown that  $o(x)$  is a constant function, and its value is determined by the initial condition  $o(0) = 1$ . The ninth part of the paper is devoted to the study of the properties of the function  $p(x)$  defined by the equation  $p(x) = \int_0^x p(t) dt$ . It is shown that  $p(x)$  is a constant function, and its value is determined by the initial condition  $p(0) = 1$ . The tenth part of the paper is devoted to the study of the properties of the function  $q(x)$  defined by the equation  $q(x) = \int_0^x q(t) dt$ . It is shown that  $q(x)$  is a constant function, and its value is determined by the initial condition  $q(0) = 1$ .

the correction is illustrated as follows:



$$\frac{.0166}{.0165} = 1.007$$

Hence the calculated % for a 100% mirror is too high by .7%.

Therefore all calculated percentages of reflection will be high by .7%. To correct a calculated percentage, multiply the calculated percentage by .7% and divide by 100. Then add this result to the calculated percentage.

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## Efficiency of cats eye reflectors for different angles of incidence.

Auto bulb operating at 8 volts, 3.86 amps.

Table III

green reflector A

Angle of incidence in degrees	Galv. deflection	Galv. deflection if 100% efficient	Amount reflected in %
90 right	.1	5.4	0
40	.1		0
35	.3		3.7
30	.5		7.1
25	1.4		23.9
20	1.8		31.3
15	1.9		33.1
10	2.0		35.0
5	2.1		37.8
0	2.1		37.8
5 left	2.05		35.8
10	1.9		33.1
15	1.8		31.3
20	1.6		27.6
25	1.3		22.1
30	1.1		18.4
35	1.0		16.5
38	.7		11.0
40	.15		.9
90	.1	5.4	0

red reflector A, same type

Angle of incidence, zero degrees. Galvanometer deflection .7

divisions. Galvanometer deflection for 100% reflection 5.4 divisions.

Amount reflected in percent is 13%. It should be noted that the .1 division deflection for ninety degree incidence is due to scattered light, and that this amount must be subtracted from the other deflections in order that the actual deflection may be found. This data is plotted on graph II.



Table IV

Green reflector B (cats eye type)

Angle of incidence in degrees	Galv. deflection	Galv. deflection for 100% reflection	% of light reflected
90 right	.1	5.45	0
40	.15		.7
35	.7		11.1
30	.9		14.8
25	.9		14.8
20	.9		14.8
15	.9		14.8
10	.9		14.8
5	.8		13.0
0	.8		13.0
5 left	.8		13.0
10	.8		13.0
15	.85		13.9
20	.9		14.8
25	.85		13.9
30	.8		13.0
35	.7		11.1
40	.25		2.7
41	.2		1.8
90	.1	5.2	0
		5.4	
		mean 5.35	

Above data plotted on graph II

Table V

## Correction factors

Calculated %	Subtract from calculated %
50	.35
45	.32
40	.28
35	.24
30	.21
25	.18
20	.14
15	.10
10	.07
5	.04



Table VI

Green reflector C (cats eye type)

Angle of incidence in degrees	Galv. deflection	Galv. deflection for 100% reflection	% of light reflected
90 right	.1	5.4	0
37	.3		3.6
36	.5		9.0
35	.7		11.0
30	.9		14.7
25	1.1		18.4
20	1.3		22.2
15	1.4		23.0
10	1.6		27.6
5	1.8		31.3
0	1.8		31.3
5 left	1.7		29.4
10	1.6		27.6
15	1.5		25.7
20	1.3		22.2
25	1.1		18.4
30	1.0		18.4
35	.85		13.8
40	.4		5.5
41	.2		1.8
42	.1		0
90	.1	5.4	0

This data is plotted on graphs II and III

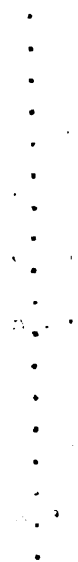
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Table VII

Green reflector C (cats eye type) rotated  $90^{\circ}$  from data in Table VI  
to check on reflector symmetry.

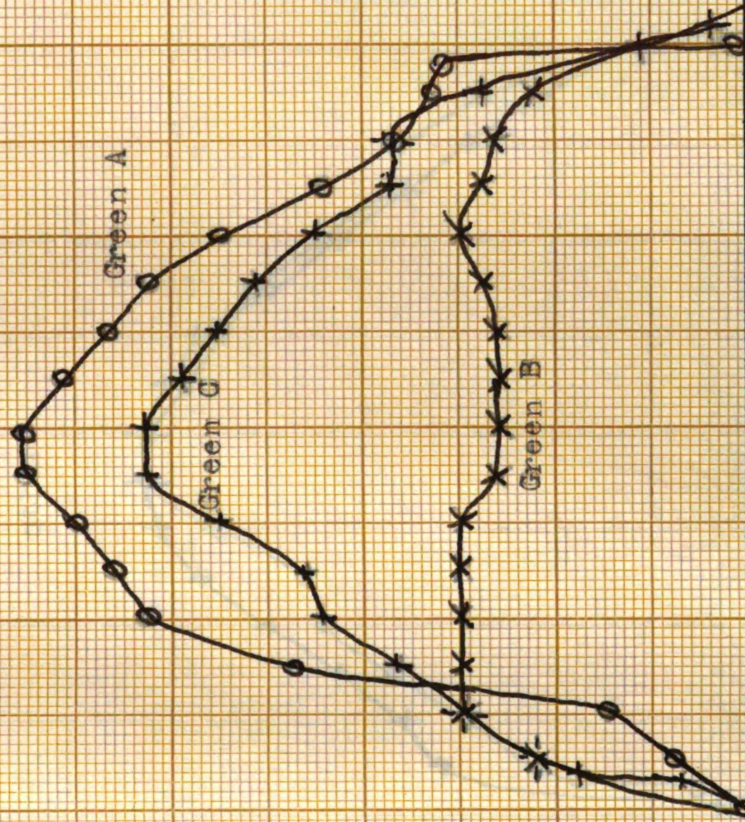
Angle of incidence in degrees	Galv. deflection	Galv. deflec- tion for 100% reflection	% of light reflected
90 right	.1	5.5	0
39	.2		1.8
35	1.0		16.1
30	1.1		17.9
25	1.3		21.4
20	1.5		25.0
15	1.7		28.6
10	1.8		30.4
5	1.85		31.3
0	1.85		31.3
5 left	1.8		30.4
10	1.7		28.6
15	1.5		25.0
20	1.3		21.4
25	1.1		17.9
30	.9		14.6
35	.7		10.7
40	.4		5.4
41	.2		1.8
42	.1		
90	.1	5.8	
		5.4	
		mean 5.56	

This data is plotted on graph III.



# Cats eye retrorreflector

% of light reflected by retrorreflector



Right

Left

Angle of incidence in degrees

Graph II

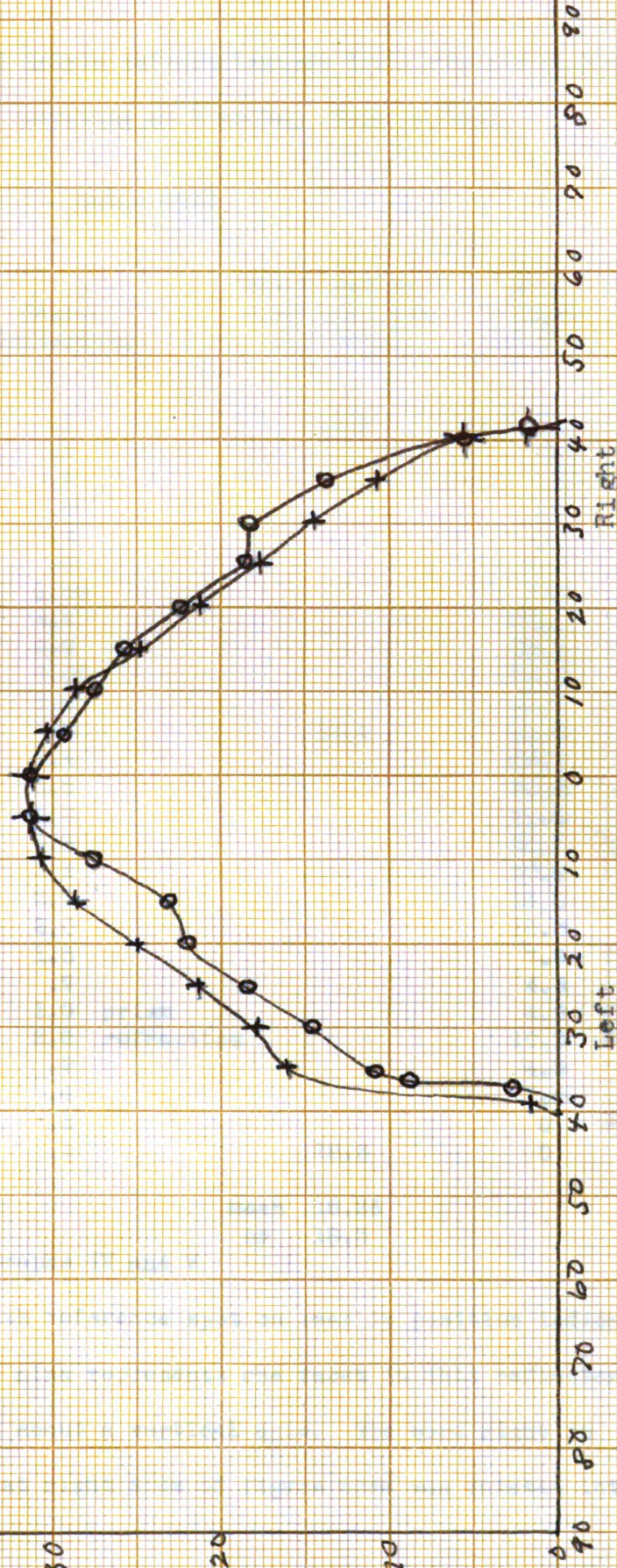
% of light reflected by retroreflector

Cats eye retroreflector

Symmetry test

Two sets of data taken at 90° with each

other



The efficiency of central triple retroreflectors for different angles of incidence. Auto bulb operating at 8 volts, 3.86 amperes.

Table VIII

## White reflector

Angle of incidence in degrees	Galv. deflection	Galv. Deflection for 100% reflector	% of light reflected
90 right	.1	18.3	0
50	.1		0
45	.2		.5
40	.2		.5
35	.3		1.1
30	.6		2.7
25	1.05		5.2
20	4.2		22.3
15	6.9		32.1
10	7.45		39.9
5	7.8		41.9
0	7.8	18.2	41.9
5 left	7.4		39.7
10	6.5		34.8
15	5.8		31.0
20	4.8		25.5
25	3.9		20.6
30	2.9		15.2
35	2.0		10.3
40	1.5		7.6
45	.9		4.3
50	1.0 prism		4.9
55	2.0 refraction		10.3
60	.9		4.3
65	.4		1.6
70	.1		0
90	.1	18.3	0
		mean 18.26	
		or 18.3	

This data is plotted on Graphs IV and V

Reflector was oriented with reference spot on rear in position indicated for installation. Figure nine represents one facet in this reflector.

The reflector was rotated about a vertical axis. The word right in the first column means that right side of figure nine was rotated into

the paper about the axis, and the word left means that the left side of the figure was rotated into the paper.

The reflector was now rotated in the holder ninety degrees, and the data following was taken for different angles of incidence.

Table IX

## White reflector

Angle of incidence in degrees	Galv. deflection	Galv. deflection for 100% reflection.	% of light reflected
90 right	.1	18.2	0
50	.1		0
45	.2		.5
40	.2		.5
35	.5		2.2
30	.9		4.3
25	3.0		15.8
20	5.6		29.9
15	6.9		36.9
10	7.5		40.2
5	7.9		42.3
0	7.9	18.2	42.3
5 left	7.4		39.7
10	6.8		36.4
15	5.9		31.5
20	4.4		23.4
25	2.3		12.0
30	.8		3.8
35	.3		1.1
40	.2		.5
45	.15		.3
50	.1		0
90	.1	18.4	
		18.3	
		mean 18.3	

This data is plotted on graph V.

Table X

## Green central triple retroreflector

Angle of incidence in degrees	Galv. deflection	Galv. deflection for 100% reflection	% of light reflected
90 right	.1	17.7	0
65	.1		0
60	.2		.6
55	.2		.6
50	.3		1.1
45	.4		1.7
40	.6		2.8
35	.8		4.0
30	1.0		5.1
25	1.2		6.2
20	1.5		7.9
15	2.1		11.4
10	2.4		13.1
5	2.5		13.6
0	2.5		13.6
5 left	2.3		12.5
10	2.0		10.8
15	1.8		9.6
20	1.2		6.2
25	.9		4.5
30	.7		3.4
35	.5		2.3
40	.4		1.7
45	.2		.6
50	.2		.6
55	.1		0
90	.1	17.4	0
		17.5	
		mean 17.5	

This data is plotted on Graphs IV and VI.

This reflector has a line through the center of the reflector, dividing the reflector into two equal halves. Figure nine shows a facet. The cube face on the left side of this figure in the reflector is nearest the center line. This same face of the facets on the other side of the center line also is nearest the center line.

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The green reflector in table X was rotated in its holder ninety degrees.

Table XI

Angle of incidence in degrees	Galv. deflection	Galv. deflection for 100% reflection	% of light reflected
90 right	.1	18.0	0
40	.1		0
35	.2		.5
30	.7		3.3
25	1.4		7.2
20	1.9		10.0
15	2.1		11.1
10	2.4		12.7
5	2.5		13.2
0	2.5	17.8	13.2
5 left	2.5		13.2
10	2.2		11.6
15	1.9		10.0
20	1.7		8.8
25	1.1		5.5
30	.6		2.7
35	.2		.5
40	.1		0
90	.1	18.1	0
		mean 18.0	

This data is plotted on graph VI

Yellow central triple retroreflector is identical in design with retroreflector measured above.

Angle of incidence was zero degrees

Galvanometer deflection was 3.9 divisions

Galvanometer deflection for 100% reflection was 15.4, 15.7, 15.3,  
mean 15.5

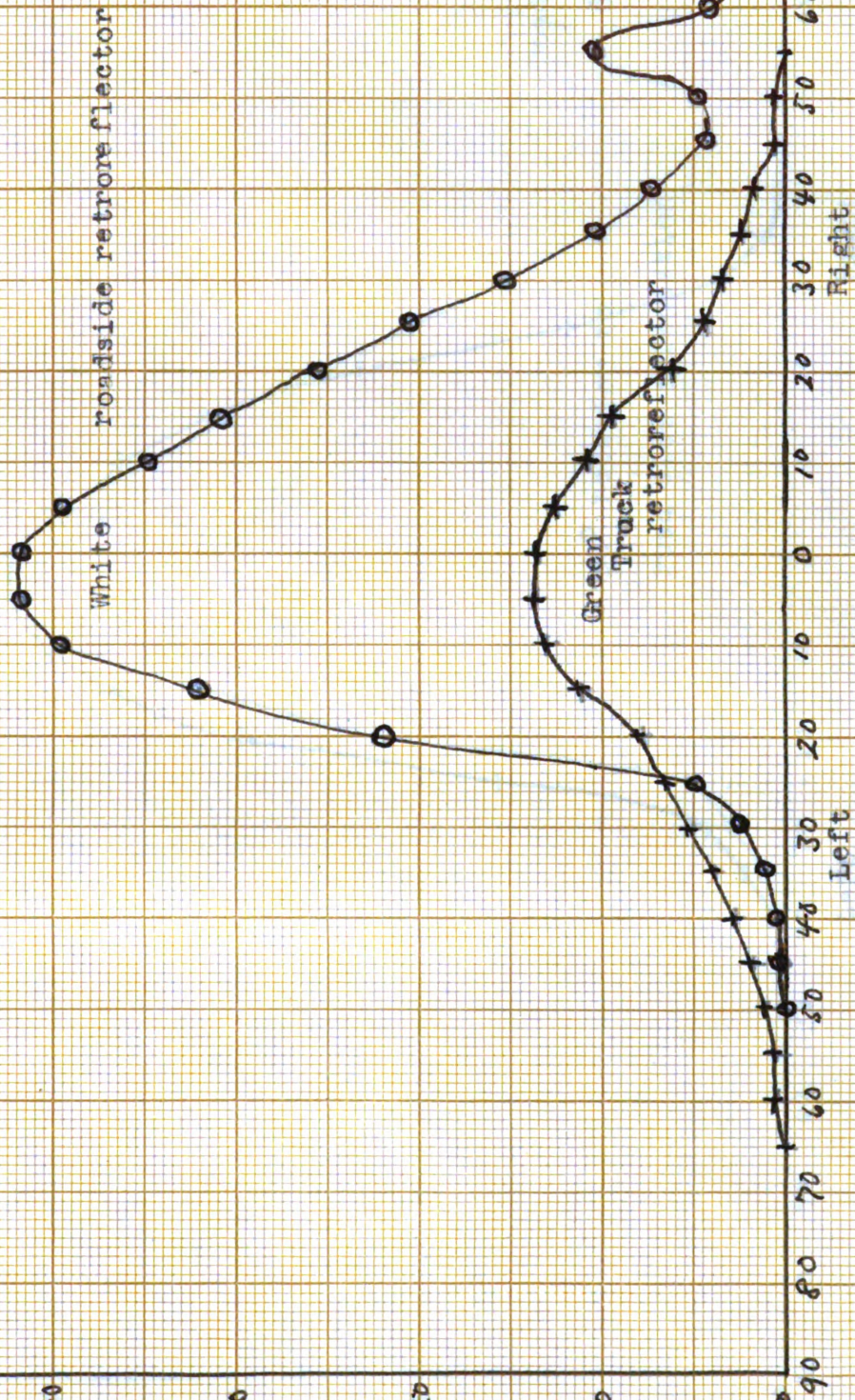
Actual % of light reflected is 25.2%



# Central triple retroreflector

Oriented as they would be in service - dot up

% of light reflected by retroreflector



Angle of incidence in degrees  
Graph IV

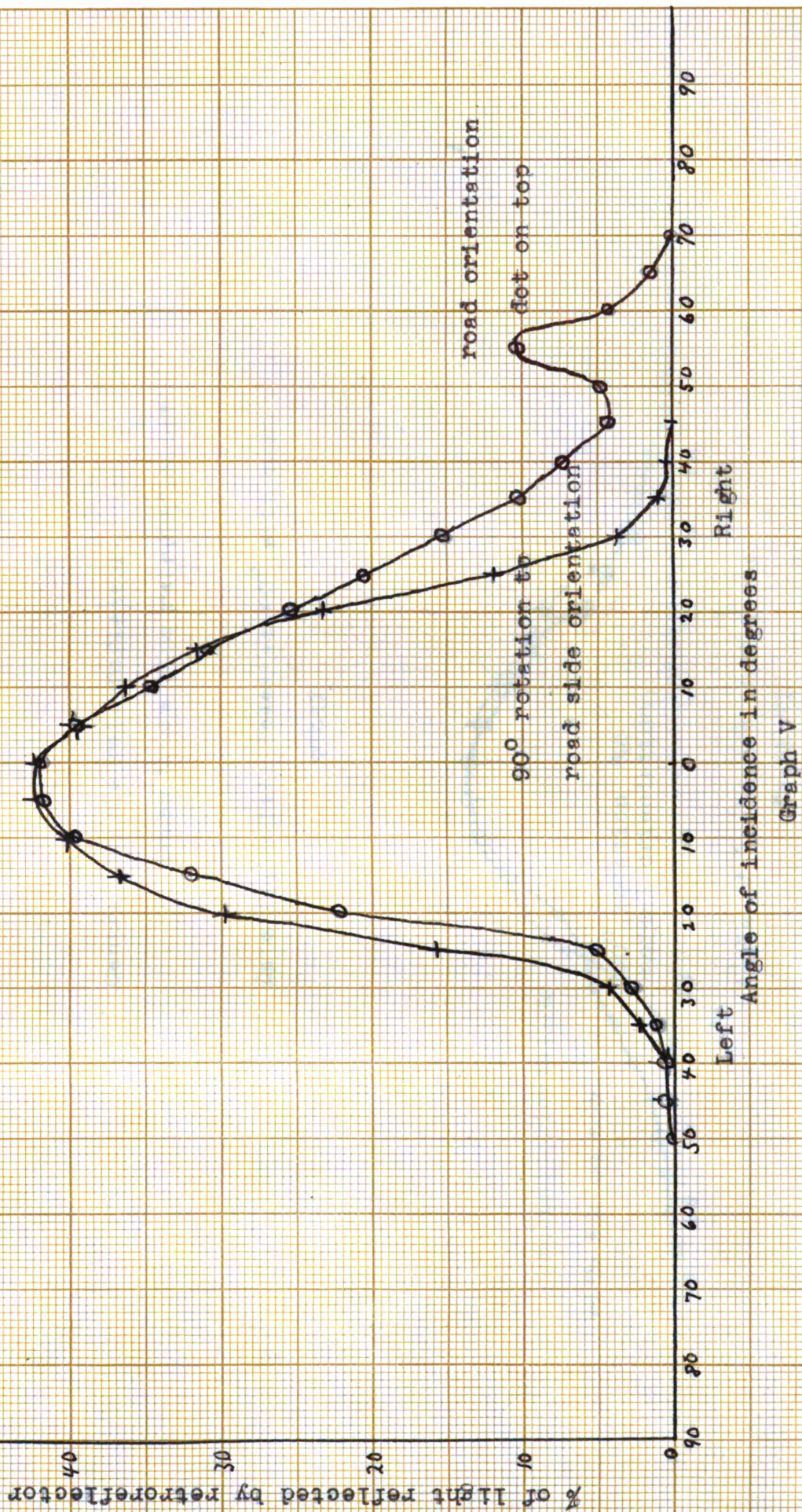
Right

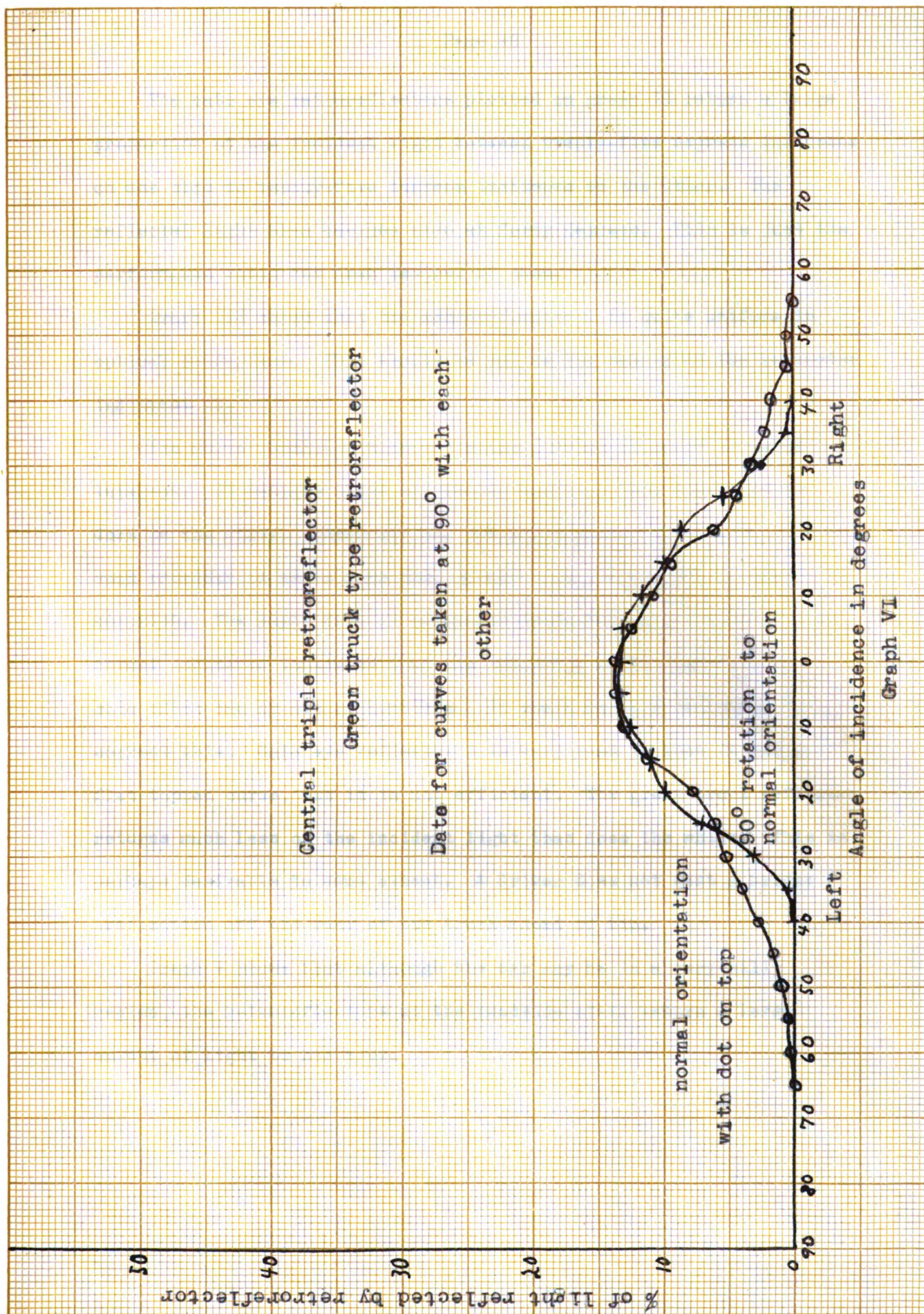
Left

# Central triple retroreflector

White roadside type

Data for curves taken at 90° with each other





The cats eye retroreflectors plotted on graph II return a large proportion of the incident light between twenty-five degrees incidence on one side to twenty-five degrees incidence on the other. The reflected light then becomes zero at forty degrees. This is just the performance which has been desired of them.

Graph III shows that the retroreflector C is quite uniform in optical properties. This should be so, as the theory of the reflector indicates it.

The white central triple reflector plotted on graph IV is of interest. It returns light at greater angles on one side than it does on the other. Therefore the reflector should be mounted with such orientation as to make this greater angle available to the motorist. The rapid drop of reflected intensity on each side of normal incidence indicates that the reflector leaks light out the side after one or two reflections. It would be very desirable to correct this. Cats eye reflector B is very good in that it has a flat topped curve, but it is not efficient. The green truck reflector returns much less of the incident light than does the white. This is perhaps desirable to some extent. A driver does not want a bright reflection off the rear of the truck ahead of him.

Graph V shows that although the car may be in a very hilly region, the retroreflectors at the roadside still return a large amount of light to his eyes.



Photographs of Reflector Patterns at Different Angles of Incidence.

At first it was planned to use a densitometer to determine the intensity of the reflected light as recorded upon a photographic film placed where the photoelectric cell is placed on the apparatus diagrammed in figure fourteen. However after the photographs were examined, the extreme irregularity of the patterns made such a procedure inadvisable. Therefore the photographs were taken directly on the paper and as shown in this paper are negatives. Eastman kodabrom F 2 smooth single weight paper was used for all photographs.

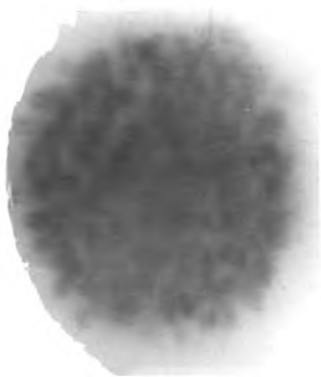
The next five pages display the patterns obtained at the indicated angles of incidence. In the photographs of the central triple reflector white, it is interesting to note how the reflected pattern cuts off with a definite shadow. Also note the rotation of the cut off pattern when the reflector has been rotated ninety degrees. The black disk in all photographs indicates approximately the intensity and diameter of one hundred percent reflection. The workmanship on the die which made the reflectors is clearly indicated, as well as the altering of the angles between the cube faces to produce a one degree cone of light. The large faint disk shown on some photographs is due to the circular aperture of the paper holder which could not have glass between the paper and the incident light beams.

The reflected patterns of the cats eye reflectors are instructive. These pictures show clearly that at angles of incidence other than axial, a very distorted cone is reflected. This may well cause the reflected cone to miss the eye altogether.

# White Central Triple Retroreflector

## Reflected Pattern

Dot on reflector next to 100% Spot axis of rotation  $\rightarrow$



Reflection at  $\perp$  incidence 100% Reflection  
exposure 10 seconds



rotated in  
Reflection at  $40^\circ$  incidence 100% Reflection  
exposure 10 seconds

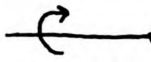
rotated in



Reflection at  $20^\circ$  incidence 100% Reflection  
exposure 10 seconds

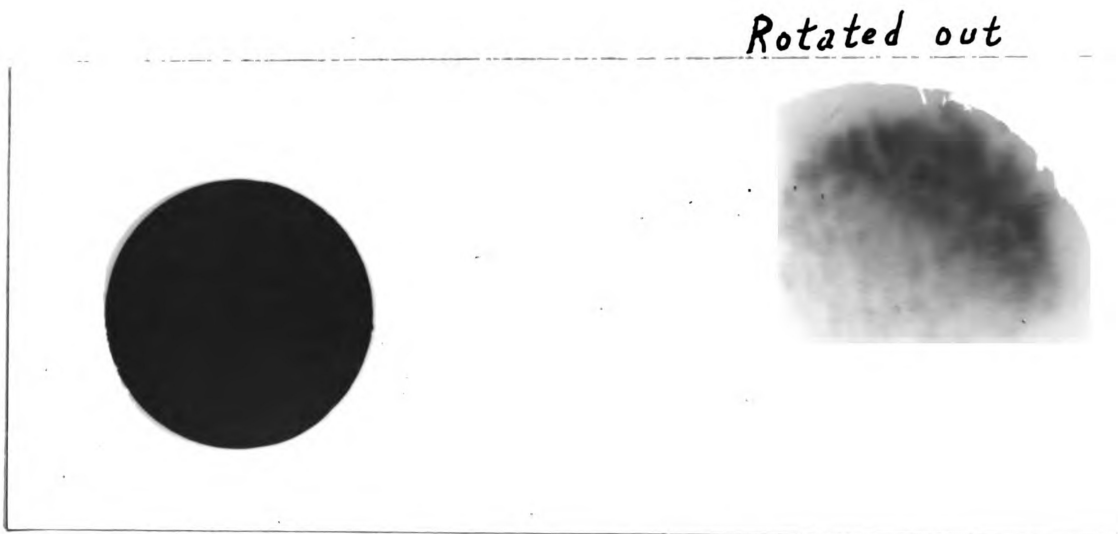
White Central Triple Retro reflector  
Reflected Pattern

Reflector rotated  $90^\circ$  with  
Respect to preceding patterns

Rotation axes 




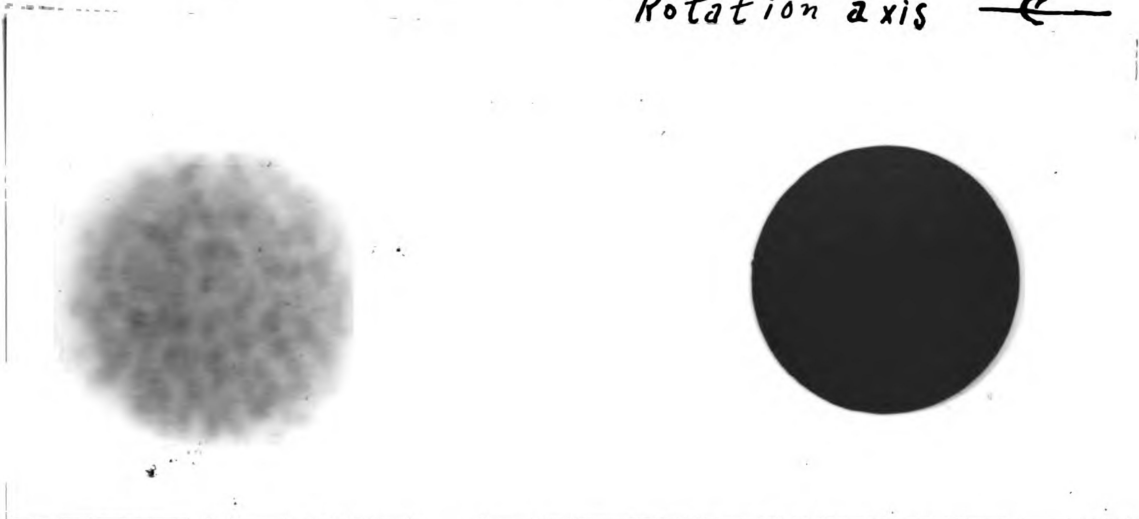
100% Reflection      Reflection at  $25^\circ$  incidence  
exposure 10 seconds



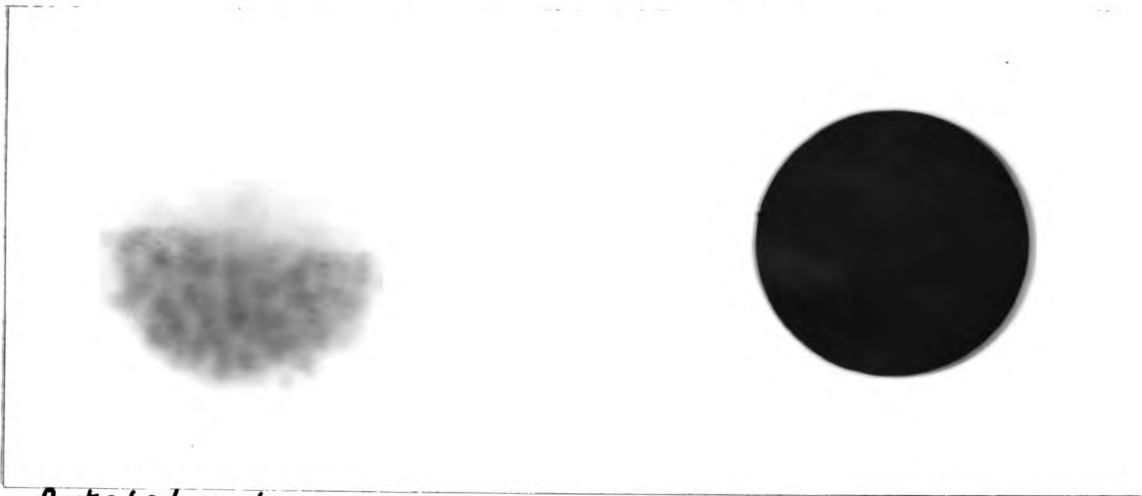
100 % Reflection      Reflection at  $25^\circ$  incidence  
exposure 10 seconds

Green Central Triple Retroreflector  
Reflected Pattern

Rotation axis 



Reflection at  $\perp$  incidence 100% Reflection  
exposure 10 seconds



Rotated out  
Reflection at  $26^\circ$  incidence 100% Reflection  
exposure 10 seconds  
Rotated out

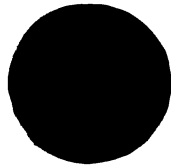


Reflection at  $21^\circ$  incidence 100% Reflection  
exposure 10 seconds

# Green Cats Eye Retro reflector A

## Reflected Pattern

rotation axis



100% Reflection

exposure - 10 seconds

Reflection at  $\perp$  incidence

- 20 seconds



Reflection at  $20^\circ$  incidence  
exposure 20 seconds

Reflection at  $30^\circ$  incidence  
exposure 20 seconds

Reflection  
at  $40^\circ$   
incidence  
exposure  
20 seconds

# Green Cats Eye Retroreflector B

## Reflected Pattern



100% Reflection  
exposure 10 seconds

Reflection at  $\perp$  (perpendicular)  
incidence  
50 seconds

Amount of scattered light with  
reflector removed

exposure 50 seconds

The last photograph of the reflected pattern of reflector B indicates how small and distorted the cone is even at axial incidence. The darkening of the paper generally on one-half of the paper recording the reflected beam is due to scattered light. The apparatus, inside and out, is rendered as light absorbing as possible. A large percentage of the scattered light is caused by the diffusing of light by the aluminum films on the plates  $H_1$  and  $H_2$ , Figure 14.

### Design of Improved Apparatus

To improve the portability several changes suggest themselves. A cell is needed which has a greater current output which will permit the use of a zero to fifty microampere meter. If the mirrors could be made to be about fifty percent transmitting the photocell would have more light incident upon it.

The limitations of the photovoltaic cell limits the accuracy of the data. The photovoltaic cell is quite sensitive to infra red light, which would make the output current change with changing room temperature. The surface of the cell does not have uniform sensitivity as the following data shows. The 13/16 inch diameter spot was incident upon the four quarters of the cell surface successively. The regions covered overlapped somewhat.

Quarter A Galvanometer deflection 5.7 divisions

Quarter B Galvanometer deflection 5.2 divisions

Quarter C Galvanometer deflection 5.3 divisions

Quarter D Galvanometer deflection 5.9 divisions

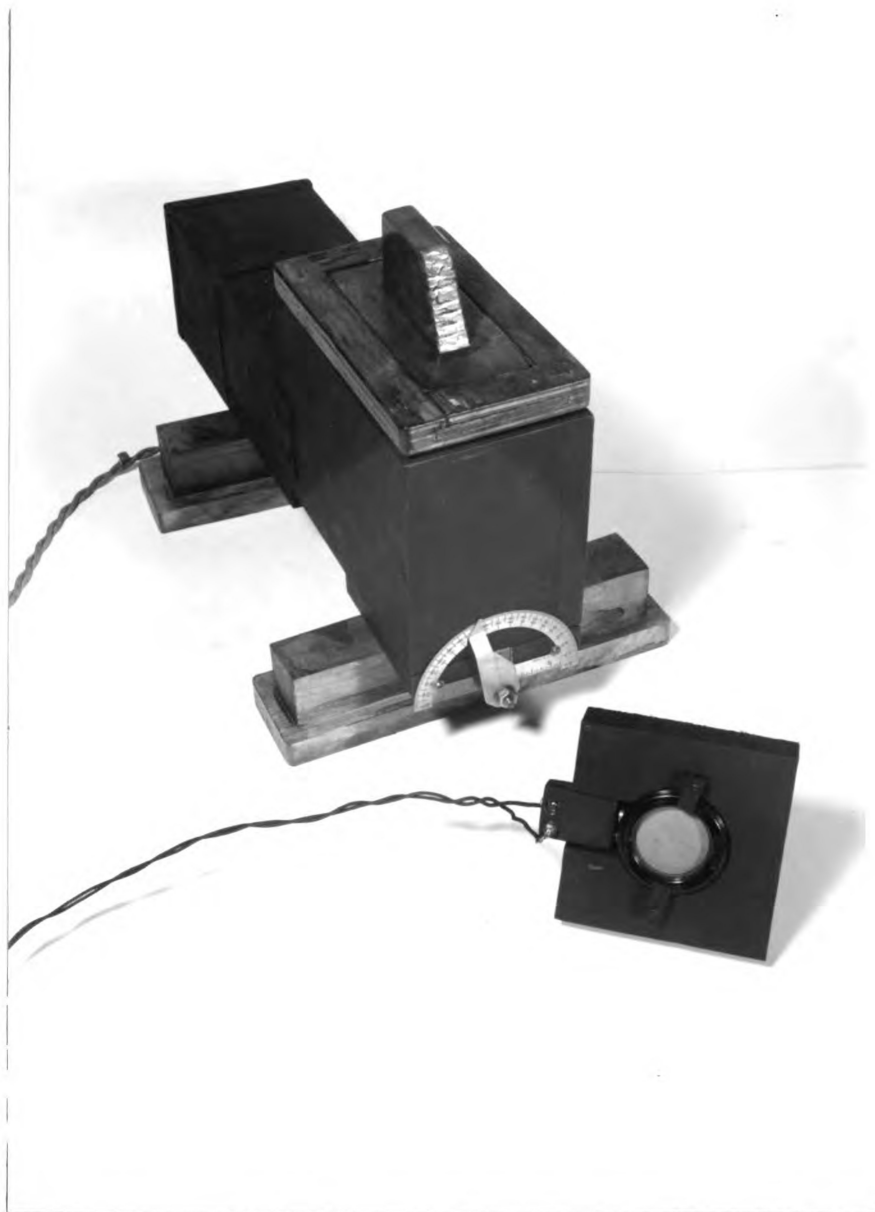
This shows a difference of about ten percent in deflections which should be the same. Although extreme care was taken to have the light in the reflector efficiency experiment incident upon the same spot each time, errors undoubtedly were made.

Probably if a vacuum emission type photoelectric cell were to be used in place of the photovoltaic cell, improved laboratory results could be obtained.

The new photocells being made at the present time have a visual correction filter which will make the response curve of the photo-



cell very similar to that of the eye. This improved type cell with correction filter would give results that would compare with the eye. The data obtained would represent the reflection efficiency which is effective on the eye which is what is desired.



Reflector Box and  
Photovoltaic cell



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Green Central Triple Retroreflector  
Truck Type - Above  
Cats Eye Retroreflector A - below

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