CONTINUOUS X-RAY INTENSITY FROM ALUMINUM TARGETS AS A FUNCTION OF ELECTRON ENERGY

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CONTINUOUS X-RAY INTENSITY FROM ALUMINUM TARGETS AS A FUNCTION OF ELECTRON ENERGY

by

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A Thesis

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CONTENTS

I. INTRODUCTION

The study of radiation resulting when fast moving electrons strike matter has provided the physicist with invaluable data from which basic laws governing atomic collision phenomena have been deduced. Moreover, from a theoretical standpoint, the underlying phenomena of nature have often been most easily predicted from a consideration of energy relations between impinging electrons and resultant radiation. As measurements became more refined, theories based upon classical macroscopic mechanics were seen to be rough approximations only. The advent of the new mechanics enabled the theorist to give a more exact picture of the workings of nature. However, due to the difficulties of the mathematics involved, approximations had to be employed, the validity of which could be determined only by comparison with experimental results.

The intensity of the continuous x-ray radiation produced by an electron colliding with a nucleus is a function of several factors. These may briefly be stated as that due to azimuthal angle of observation with respect to the direction of the cathode ray beam, the energy of the bombarding cathode rays, the atomic number of the bombarded atom, and the wave length of the emitted radiation. The investigation was undertaken to experimentally determine the absolute intensity of the continuous radiation of wave length 474 XU as a function of bombarding electron energy, and to compare this data with the results predicted by the most recent theory of this problem.

II.

Historical Discussion of Theories and Experiments Concerning the Continuous X-ray Spectrum

As early as 1898, Stokes and Thompson (1) proposed an explanation of the continuous radiation process as irregular electromagnetic pulses due to acceleration of the bombarding cathode rays by the atoms of the target. Sommerfeld,(2) in 1909, using classical electromagnetic theory, presented a more complete explanation of this phenomena. It w as not until 1928 that reliable experimental data on the intensity of emission of the continuous x-rays were made available by Kulenkampff (3). In these results the earlier theories were shown to be in error, principally as regards the spatial distribution of the emitted x-ray energy. In the meantime Kramers, utilizing early quantum mechanics principles, presented a theory which compared favorably with data of Nicholas (5), particularly as regards the short wave length limit. The theory failed to predict Kulenkampff's results of azimuthal distribution however.

With the development of the wave mechanics, Sommerfeld,(6) in 1931 developed a theory in which he represented an electron by its rigorous eigenfunction. The treatment was nonrelativistic. Because of this and other necessary restrictions imposed in order to obtain a solution of the problem, results were obtained which differed considerably with Kulenkampff's experiments.

Scherzer (7) in 1932, using a relativistic treatment of the wave mechanics and employing Dirac wave functions, calculated the intensity distribution at the short wave length limit. Since the short wave length limit restriction reduces the usefulness of the theory and because a more recent work by Sauter (8) is more inclusive, no attempt has been made in this thesis to make a comparison of experimental results with Scherzer's theory.

Sauter's first paper (8), dealing with this subject, employed a wave mechanical treatment, simplified by representing the retarded electron as a modified plane wave. That is, he employed Born approximations which are valid only when the initial kinetic energy of the incident electron is much greater than the energy required for K shell ionization. The wave function is considered in such a manner that the wave is plane at infinity; when actually, the wave front is affected by the nucleus at infinity.

Sauter's second paper (9) is a relativistic treatment of the problem. In this development, he obtains a formula for the resultant continuous x-ray energy, which, to date, furnishes the most complete results of theory with which to make experimental comparisons. The resultant expression for the energy, $J_\nu d\nu$, of a particular frequency in the frequency range $d\nu$, stated as equation 11 in his paper (9), is reproduced in this thesis as equation 1 below. If p_o and p represent the initial and final momentum of the bombarding electron and mc^2 the rest mass energy of the electron, then the non-relativistic approximation is valid when $p_o, p \ll mc^2$. Upon making this approximation, equation 1 simplifies to the earlier expression he obtained in the non-relativistic treatment (8) of this problem.

Sauter's nonrelativistic approximation differs from Sommerfeld's equation by only a frequency dependent, not a direction dependent, factor. This factor is approximately unity except at the short wave length limit where Born's approximation is no longer valid. Now, by multiplying the nonrelativistic approximation by this frequency dependent factor, Sommerfeld's corresponding rigorously valid equation is obtained. Sauter's results also agree with those of Scherzer to within the same frequency dependent factor. In view of these considerations, Sauter assumes that the relativistic equation 1, multiplied by the correction factor, given as formula 13 of his paper (9), and as formula 2 below, results in a valid approximation for the entire spectral range. It is this resulting equation 1, multiplied by the correction factor, formula 2, which is used for theoretical computations in this thesis.

Due to the mathematical difficulties, all of the theories have been developed on the basis of the retardation of one electron by an isolated nucleus, without taking account of the screening action of the external electrons.

Recent experimental data have been obtained by H. R. Kelly (10) for absolute intensities of the continuous x-ray radiation from thin aluminum targets as a function of azimuthal angle, using a bombarding electron energy of 31.7 kev. This data was normalized at the azimuthal angle of 90° to coincide with the relative intensities as plotted by Böhm (11) from Scherzer's equation. The agreement is quite good, the maximum intensity from Kelly's data falling at $\theta = 57^{\circ}$, while the maximum as calculated by Böhm occurs at $\theta = 55^{\circ}$. The agreement is not as good at large and small values of θ where the intensity of radiation is much smaller and the consequent errors of observation greater.

Clark and Kelly (12) report absolute intensity measurements of wave length 474 XU from thin aluminum targets, using the bombarding electron energy of 31.7 kev. Their results are greater than those predicted by Sauter's theory.

Smick and Kirkpatrick (13) report measurements of absolute intensities from Nickel targets, bombarded with electrons of 15 kev energy, of a wave length 1.431 Å and an azimuthal angle of 88°. Their results are in agreement with Sauter's theory after it has been slightly modified to more fully approximate the conditions of the experiment.

Harworth and Kirkpatrick (14) announce relative intensity measurements of the continuous spectrum from Nickel targets bombarded with electrons of energy 10 to 180 kev. The authors state only partial agreement with theory. The results are given in abstract form; and at this time no discussion can be made of the nature of the discrepancies between data and theory.

III.

Theoretical Equations due to Sauter and their Evaluation for Conditions of this Experiment

A. Theoretical Equations:

The following three equations given by Sauter (9) as formulas 11, 13, and 14 respectively, are those which are evaluated for the conditions of this experiment. Henceforth in this report they will be referred to as formulas 1, 2, and 3.

$$J_\nu \, d\nu = \frac{2 e^6 Z^2}{R^2 c^3} \frac{p}{p_0^3} d\nu \left\{ \frac{(3E_0^2 - c^2 p_0^2) m^2 c^4 \sin^2\theta}{u^4} - \frac{2E_0^2 - c^2 p_0^2}{u^2} - \frac{E c p \cos\theta}{2u^2} \quad \frac{c p_0^2}{2pu} \log \frac{E + cp}{E - cp} \right.$$
$$+ \frac{c p_0^3 (h\nu \cos\theta - c p_0)}{2 P_0^2 u^2} + \left(\frac{m^2 c^4}{2u^2} - \frac{h\nu}{u} + \frac{(h\nu)^2 (h\nu - c p_0 \cos\theta)}{2 c^2 P_0^2 u} \right) \frac{p_0^2}{p P_0} \log \frac{P_0 + p}{P_0 - p}$$
$$+ \log \frac{E_0 E - m^2 c^4 + c^2 p p}{E_0 E - m^2 c^4 - c^2 p_0 p} \left[\frac{(3E_0 h\nu m^2 c^4 - E_0 E c^2 p_0^2) m^2 c^2 \sin^2\theta}{2 p_0 p u^4} \right.$$
$$\left. \left. + \frac{3E_0^2 c^2 p_0^2 - 4E_0 h\nu m^2 c^4 + c^4 p_0^2 p^2}{4 c^2 p_0 p u^2} - \frac{(E_0 E + c^2 p_0^2) h\nu \cos\theta}{4 c p u^2} \right] \right\} \qquad 1$$

The following formula 2 is what Sauter designates as the "Correction Factor" which must be applied to equation 1 to obtain the "Corrected" formula 3, valid in the whole spectral range.

$$\frac{\dfrac{(2\pi\alpha Z)^2}{\beta_0 \beta}}{\left(e^{\frac{2\pi\alpha Z}{\beta_0}} - 1\right)\left(1 - e^{-\frac{2\pi\alpha Z}{\beta}}\right)} \qquad 2$$

The f inal equation used for computations of Sauter's theory is given in the final equation 3, which is

$$J_\nu(\text{corr}) \, d\nu = J_\nu \, d\nu \cdot \frac{\dfrac{2\pi\alpha Z}{\beta_0 \beta}}{\left(e^{\frac{2\pi\alpha Z}{\beta_0}} - 1\right)\left(1 - e^{-\frac{2\pi\alpha Z}{\beta}}\right)} \qquad 3$$

The quantity ($J_\nu d\nu$) corrected gives the energy per **unit area**, per electron, per second, per atom, per unit area of target, where observations are made at a distance R f rom the target and at an azimuthal angle of θ.

The quantities entering these equations are as follows:

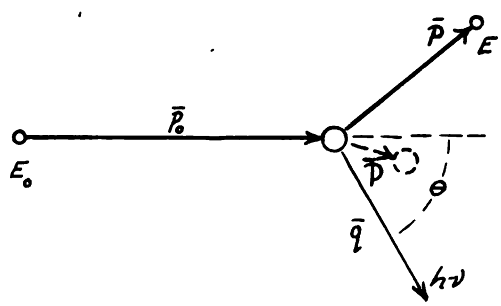
h = Planck's constant
e = Charge on electron
m = rest mass of electron
c = velocity of light
R = distance of defining aperture from the x-ray target.

Z = Atomic number of bombarded nucleus.
E_0 = Initial Total Energy of bombarding electron.
E = Total energy of electron after collision.
P_0 = Initial momentum of bombarding electron.
P = Momentum of electron after collision.
q = Momentum of quantum.
θ = Angle at which quantum is emitted with respect to incident electron direction.
β_0 = Ratio of velocity of incident electron to the velocity of light.
β = Ratio of velocity of electron after collision to that of light.

$$U = E_0(1 - \beta_0 \cos\theta)$$

$$\alpha = \frac{2\pi e^2}{hc} = \text{the fine structure constant.}$$

Referring to the following vector diagram representing the momenta and energies of the type of collision considered here,



it is seen that the momentum $\bar{p}_0$ of the incident electron is

$$\bar{p}_0 = \bar{q} + \bar{p} + \bar{P}$$

Transposing:

$$\bar{p} + \bar{P} = \bar{p}_0 - \bar{q}$$

Sauter defines the quantity P_0 as

$$P_0^2 = (\bar{p} + \bar{P})^2 \equiv (\bar{p}_0 - \bar{q})^2$$

which by the vector diagram is

$$P_o = p_o^2 + q^2 - 2p_oq\cos\theta .$$

Sauter neglects the energy aecuired by the nucleus during the collision process, so that

$$E = E_o - h\nu.$$

The initial energy is obtained from the relation:

$$E_o = (KE)_o + mc^2 .$$

Using the relativistic kinetic energy, total energy is given by

$$E = mc^2\left[(1-\beta^2)^{-\frac{1}{2}} - 1\right] + mc^2.$$

Thus β and β_o may be solved for, using the initial and final total energies.

$$\beta^2 = 1 - \frac{m^2c^4}{E^2}$$

The momenta of the electron before and after collision may be evaluated from the expressions involving their total energies.

$$p = mv\,(1-\beta^2)^{-\frac{1}{2}} \quad \text{where } v = \beta c.$$

Utilizing the value of β^2 above,

$$p^2 = E^2/c^2 - m^2c^2$$

Likewise

$$p_o^2 = E_o^2/c^2 - m^2c^2$$

B. <u>Quantities Used in the Computations</u>

To evaluate the expressions 1 and 2, the values of the physical constants e, m, and h used were those given by DuMond (15), and are:

$m = 9.11780 \times 10^{-28}$ gm

$e = 4.80650 \times 10^{-10}$ esu

$h = 6.63428 \times 10^{-27}$ erg-sec.

$c = 2.99776$ cm/sec.

Other quantities entering into the calculations are as follows:

$Z = 13$ for Aluminum

$R = 34.0$ cm = distance of defining aperture from target.

$\theta = 60^{\circ}$ = Azimuthal angle of observation.

The wave lengths of the K absorption limits of Cd and of Ag which were used were taken from the tables given by Compton and Allison (16). These are

λ_{Cd}(K limit) = 463.13 XU

λ_{Ag}(K limit) = 484.48 XU.

From these we get

ν_{Cd}(K limit) = 6.4728×10^{18} sec^{-1}

ν_{Ag}(K limit) = 6.1876×10^{18} sec^{-1}.

The constant factors which enter equations 1 and 2 have been calculated on the basis of the above constants. They are as follow s:

Mean frequency between absorption limits of the Ross balanced foils.

$$\nu = \frac{\nu_{Cd} - \nu_{Ag}}{2} = 6.3302 \times 10^{18} \text{ sec}^{-1}$$

$$\Delta\nu = \nu_{Cd} - \nu_{Ag} = 2.85244 \times 10^{17} \text{ sec}^{-1}$$

$$h\nu = 4.19963 \times 10^{-8} \text{ ergs}$$

$$1ev = 1.60336 \times 10^{-12} \text{ ergs}$$

$$q = \frac{h\nu}{c} = 1.40092 \times 10^{-18} \text{ gm } \frac{cm}{sec}$$

and the expression which we arbitrarily place equal to A,

$$A = \frac{2e^6 Z^2}{R^2 C^3} = 3.81752 \times 10^{-71}$$

TABLE I. QUANTITIES ENTERING $J_\nu \, d\nu$ COMPUTATIONS

		30 KEV	40 KEV	50 KEV	60 KEV	80 KEV
$(KE)_0 = 1.60336 \cdot 10^{-12}$ kev	$\times 10^{8}$	4.8101	6.4135	8.0168	9.6202	1.2827
$E_0 = (KE)_0 + mc^2$	$\times 10^{7}$	8.6748	8.8351	8.9955	9.1558	9.4765
$E = E_0 - h\nu$	$\times 10^{7}$	8.2548	8.4152	8.5755	8.7358	9.0565
$\beta_0 = \left[1 - \frac{m^2c^4}{E_0^2}\right]^{1/2}$		0.32836	0.37405	0.41267	0.44621	0.50238
$u = E_0(1 - \beta_0 \cos\theta)$	$\times 10^{7}$	7.2505	7.1627	7.1394	7.1131	7.0960
$p_0 = \left[\frac{E_0^2}{c^2} - m^2c^2\right]^{\frac{1}{2}}$	$\times 10^{17}$	0.95021	1.1024	1.2383	1.3628	1.5881
$p = \left[\frac{E^2}{c^2} - m^2c^2\right]^{\frac{1}{2}}$	$\times 10^{18}$	3.3427	6.3965	8.4398	10.105	12.869
$P_0 = [p_0^2 + q^2 - 2p_0 q \cos\theta]^{\frac{1}{2}}$	$\times 10^{17}$	.8885	1.0395	1.1746	1.2985	1.5229
$A \frac{p}{p_0^3}$	$\times 10^{37}$	1.4873	1.8225	1.6967	1.5240	1.2264
$\frac{(3E_0^2 - c^2p_0^2)\, m^2c^4 \sin^2\theta}{u^4}$		3.9654	4.2235	4.4379	4.6183	4.9002
$\frac{2E_0^2 - c^2p_0^2}{u^2}$		2.7086	2.8143	2.9047	2.9838	3.1168
$\frac{E c p_0 \cos\theta}{2u^2}$		0.11182	0.13476	0.15614	0.17635	0.21407
$\frac{c p_0^2}{2up} \log \frac{E + cp}{E - cp}$		0.13625	0.18392	0.23196	0.28021	0.37668
$\frac{c p_0^3 (h\nu \cos\theta - c p_0)}{2u^2 P_0^2}$		-0.08176	-0.11486	-0.14176	-0.17236	-0.23395
$\frac{m^2c^2}{2u^2}$		0.63855	0.65067	0.65860	0.66347	0.66666
$\frac{h\nu}{u}$		0.05792	0.05847	0.05882	0.05904	0.05918

TABLE I. CONCLUDED

		30 KEV	40 KEV	50 KEV	60 KEV	80 KEV
$\frac{(h\nu)^2(h\nu - cp_0 \cos\theta)}{2c^2 p_0^2 u}$		-0.00172	-0.00156	-0.00143	-0.00133	-0.00117
$B = \frac{m^2c^4}{2u^2} - \frac{h\nu}{u} + \frac{(h\nu)^2(h\nu - cp_0 \cos\theta)}{2c^2 p_0^2 u}$		0.57891	0.59064	0.59834	0.60310	0.60631
$C = \frac{p_0^2}{p p_0} \log \frac{p_0 + p}{p_0 - p}$		2.40536	2.67790	2.7988	2.9469	3.1876
BC		1.3925	1.5492	1.6746	1.7773	1.9327
$D = \log \frac{E_0E - m^2c^4 + c^2p_0p}{E_0E - m^2c^4 - c^2p_0p}$		1.5110	2.741	3.458	3.9839	4.7531
$E = \frac{(3E_0 h\nu m^2c^4 - E_0Ec^2p_0^2)m^2c^2\sin^2\theta}{2p_0pu^4}$		0.48749	-0.00965	-0.31174	-0.44542	-0.61831
$F = \frac{3E_0^2c^2p_0^2 - 4E_0h\nu m^2c^4 + c^4p_0^2p^2}{4uc^2p_0p}$		1.4354	1.2245	1.2632	1.3247	1.4530
$G = \frac{(E_0E + c^2p_0^2)h\nu\cos\theta}{4u^2cp}$		0.07945	0.04525	0.03701	0.03311	0.02932
$D(E+F-G)$		2.78545	3.2057	3.1623	3.3711	3.8278
$J_\nu\, d\nu$ (ergs, etc)	$\times 10^{37}$	7.5926	10.4503	9.9089	9.3788	8.2406
β		0.12139	2.2787	2.9503	3.4677	4.2597
$\frac{(2\pi\alpha Z)^2}{\beta_0\beta}$		8.8807	4.1532	2.9074	2.2877	1.6542
$\frac{\frac{(2\pi\alpha Z)^2}{\beta_0\beta}}{\left(e^{\frac{2\pi\alpha Z}{\beta_0}} - 1\right)\left(1 - e^{-\frac{2\pi\alpha Z}{\beta}}\right)}$		1.7468	1.1474	1.0390	0.99837	0.96891
$J_\nu(\text{corr.})\, d\nu$	$\times 10^{36}$	1.3263	1.1990	10 2953	0.9.3635	0.7 9844

IV. Experimental Apparatus

A. General Discussion of Experimental Conditions to be Satisfied:

The theory with which the experimental results of this work are to be compared concerns the collision process between a bombarding electron of known total energy and an isolated atom; which process may result in the emission of a quantum of continuous x-ray energy of specified frequency at a definite azimuthal angle. The energy of the bombarding electron must be controlled, and the number of these electrons colliding with a nucleus per unit target area should be kept as small as possible and yet be accurately determined. These two requirements are fulfilled by controlling the x-ray tube voltage and current closely. The next important item is to have as few atoms per unit area of the target as feasible and still determine their number. This involves using a very thin target (of Aluminum in this case) and knowing its thickness. As implied above, it is also necessary to make measurements on a known frequency range of quanta emitted at a known azimuthal angle; both of which are fulfilled in this experiment. Finally, the number of quanta produced per unit of time under the given conditions must be measured. The procedure involved in determining all of these factors is treated below.

B. Production of X-rays.

1. The High Voltage Source:

The power supply used in this work has been described by Pettitt (17). It consists of a voltage doubler circuit employing two "Kenotron" rectifier tubes, an electrostatic voltmeter, and a special circuit and equipment used for controlling and measuring the emission of electrons in the x-ray tube. The voltage doubler circuit uses a 500 cycle source of power, and the electrical constants of the circuit are such that the ripple voltage is approximately 20 volts per milli-ampere. The calibration of the electrostatic voltmeter is accomplished by short wave length limit measurements made with the Bragg spectrometer. A schematic overall diagram of the apparatus is shown in figure 1.

In order to read the very small current used in the x-ray tube (of the order of 0.1 microampere), it is necessary that the entire cathode system, from the point where the current is measured, be electrostatically shielded so that no corona currents interfere with the measurement. The microammeter is connected between the shield and cathode current source and must read the actual space current in the tube. The instrument used for measuring the current is a General Electric reflecting type galvanometer, number 32C, 236G1. It has a current sensitivity of .008 μa. per division, and is critically damped with an external resistance of 46,000 ohms. The instrument was shunted so as to have a sensitivity of 0.01 microampere per division.

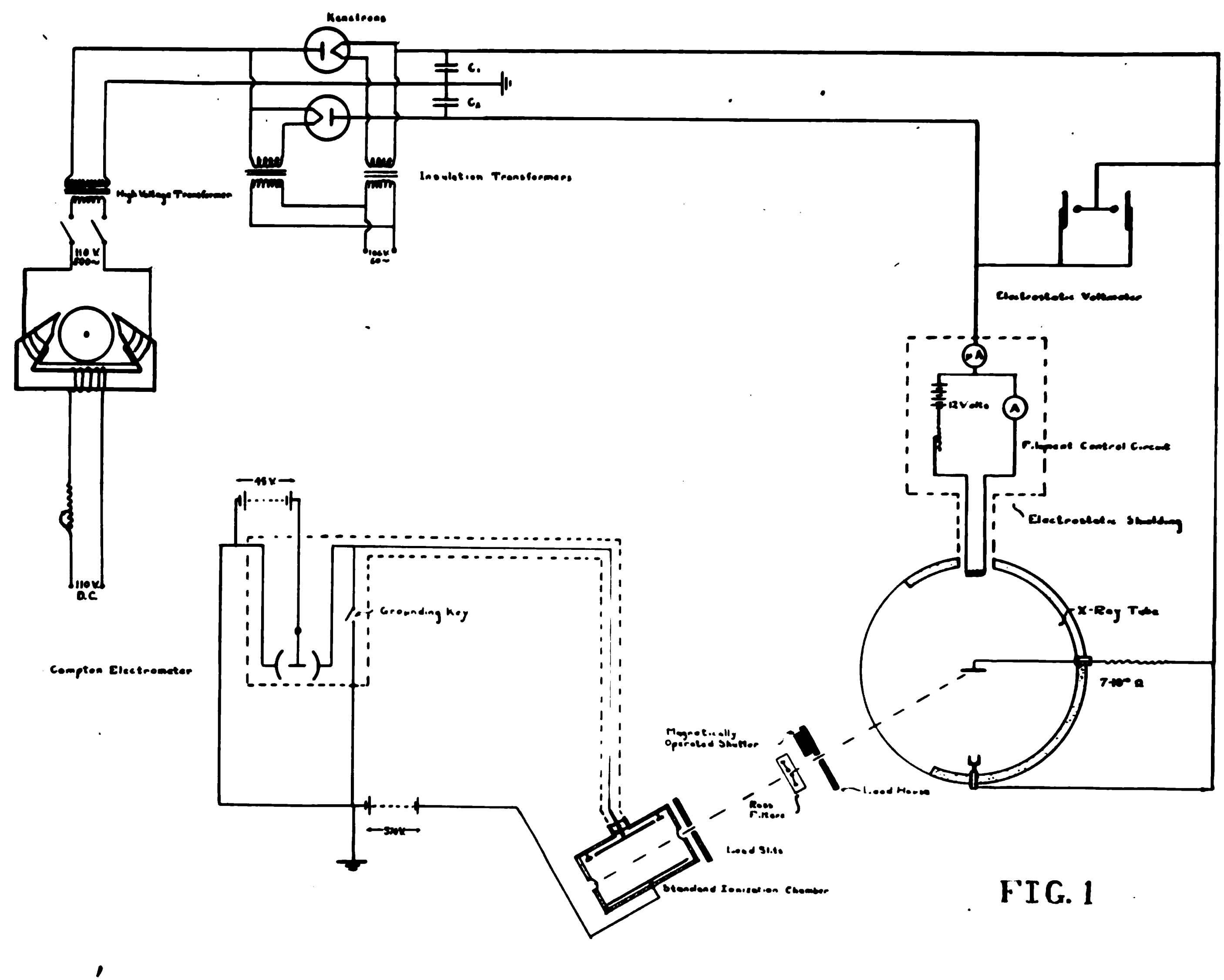

Kenotrons
C1
C2
High Voltage Transformer
Insulation Transformers
110 V
110 V D.C.
Electrostatic Voltmeter
12 Volts
Filament Control Circuit
Electrostatic Shielding
X-Ray Tube
Grounding Key
Compton Electrometer
45 V
Magnetically Operated Shutter
Lead House
Ross Filters
Lead Slits
Standard Ionization Chamber

FIG. 1

2. The X-Ray Tube

The tube shown diagramatically in figure 2 is made of a steel cylinder $2\frac{1}{2}$ inches deep, 8 inch diameter. The top plate is removable. A two inch diameter pyrex glass tube leads from the center of the bottom plate to the oil diffusion pump. A spiral tungsten cathode is mounted by the wall of the tube. At the opposite end of the diameter passing through the cathode is mounted the aluminum anode cup of $1\frac{1}{2}$ inch diameter. Focusing of the electron beam is accomplished by a small steel disk placed behind the cathode. The aluminum anode is mounted on a brass rod which extends upward in a pyrex tube of $2\frac{1}{4}$ inch diameter and is fastened to a brass cap at the tube's top. The targets are placed one inch off center in the direction of the cathode. The thin film targets are mounted on a rectangular steel wire frame, 1 inch by $1\frac{1}{4}$ inches. This frame fastens in a brass rod which extends upward in a pyrex tube of $1\frac{3}{4}$ inch diameter and fastens to a removable brass plate at the top of the pyrex tube. The anode and target are thus well insulated from the metal tank of the tube which is at the same potential as the cathode. The only positive potentials in the tube are on the target and anode, assuring that all space current is actually collected at these points. The tube must then be insulated from ground for one-half the tube voltage. A horizontal slot three-fourths inch wide is cut in the tube wall and covered with an aluminum window of .0193 cm. thickness. The entire tube may be rotated

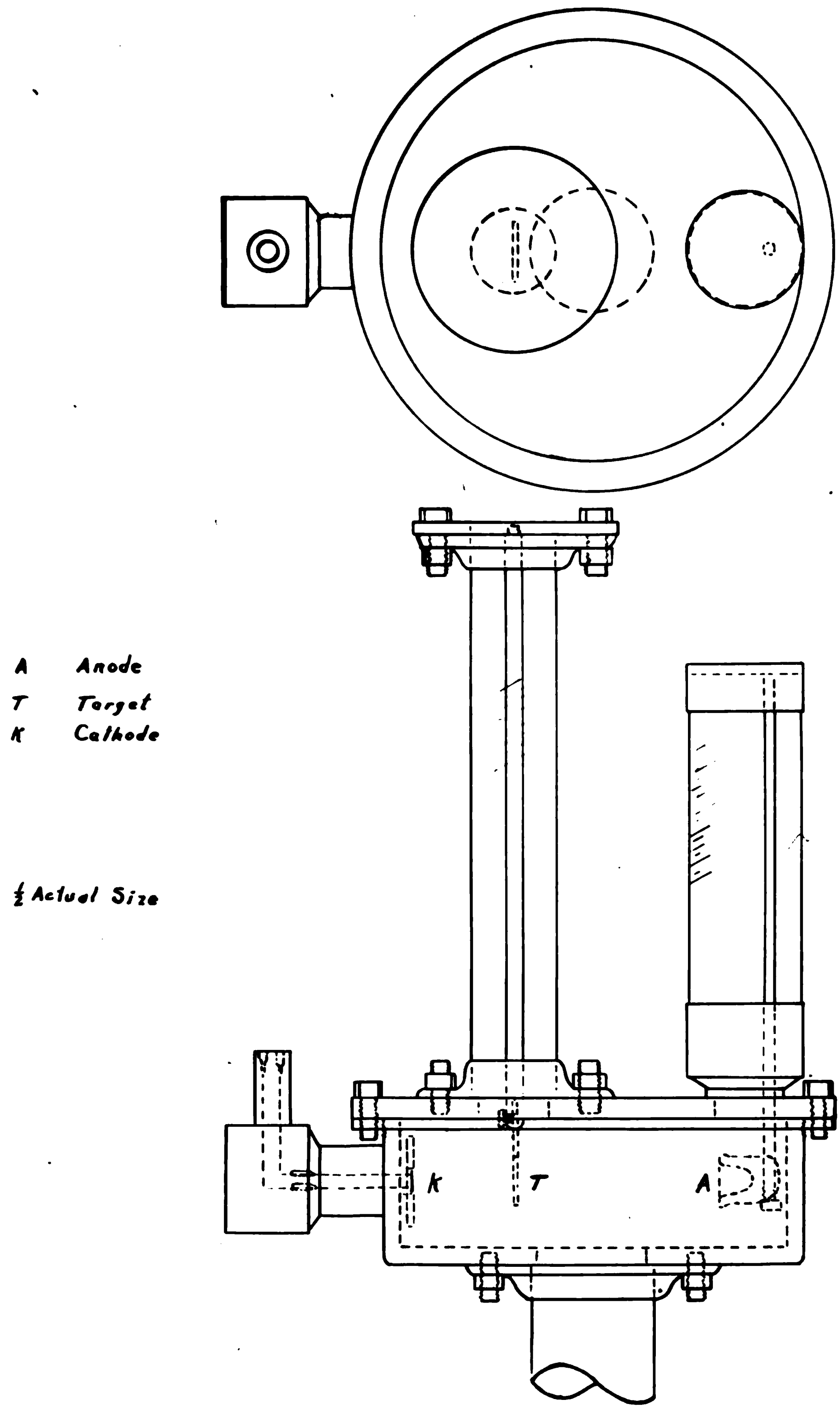


Fig 2

about an axis through the target, thus enabling azimuthal determinations of intensity to be made.

3. <u>Thin Film Targets</u>

The theory deals with radiation resulting from the retardation of a single electron by an isolated nucleus. This, of course, is not possible to obtain experimentally; nor would the intensity be perceptible were the condition obtainable. In order to fulfill this condition as well as to eliminate the possibility of the electron losing energy by minor collisions before the major retardation occurs, it is requisite that the target be as thin as possible concomitant with a reasonable intensity. A typical target is given in figure 3, showing the well defined focal spot. Not only does the small cathode current of 0.15 microampere increase the life of a target; but also, the theoretical conditions are, thereby, more closely approximated.

The target backing is commercial cellophane, cleaned with alcohol. Measurements with cellophane targets alone in the tube showed no measurable intensity from them. Thus no correction has been applied to the backing material. The aluminum is evaporated on the targets in a specially constructed evaporation chamber. Four targets, along with a Michelson interferometer mirror, were mounted in the evaporation chamber and coated simultaneously. The mirror was then used to measure the target thickness, a subject treated separately below.

Fig 3

Aluminum Target

4. Ross' Balanced Filters

To isolate a narrow region dν of the continuous x-ray spectrum, a method devised by Ross and discussed by Kirkpatrick (18) is employed. This method uses two adjacent elements in the atomic table. The filter thicknesses are such that their absorptions are the same for the x-ray radiation, except in the narrow band between their K absorption limits. By interposing first one filter and then the other in the path of the X-ray beam, the intensity of radiation within this band that is measured, is proportional to the difference in magnitudes of the galvanometer deflections for the respective filters. The narrower the band between the K absorption limits, the more strictly monochromatic will be the measurements. However, the band width must be sufficient to produce measurable differences in intensity.

The discussion of Ross filters by Kirkpatrick (18) shows that there is a preferred thickness for which the maximum intensity difference will be obtained from the filters. The filters used are of Cd and Ag and the values of thickness are worked out by H. R. Kelly (10). This allows a band 21.35 XU wide at the mean wave length of 473.80 XU.

5. Standard Ionization Chamber

The x-rays pass through a defining lead aperture of 0.912 cm^2 cross section into the standard ionization chamber. The cylindrical chamber is of brass with mica windows at either end. The collector plates are of aluminum. Guard rings assure a uniform electric field over the collector plate where ions are collected for measurement. The chamber is filled with CH_3Br gas at 68 cm pressure. The absorption of x-rays within the chamber takes place principally by photoelectric absorption.

In the photoelectric absorption process, the quanta interact with the bromine atom, ejecting an electron from one of its shells. The energy used in ejecting an electron from a carbon atom is negligibly small compared with the energy required for the bromine atom. The ions formed in the electric field are then drawn over to the collector electrodes, which charge is transmitted to the electrometer.

C. <u>Electrostatic Capacity of Electrometer Circuit</u>

In order to convert electrometer deflections to the corresponding voltage on the collector plates of the ionization chamber and thence to units of charge, it is necessary to know the capacitance of the electrometer system. The circuit for determining this capacitance is given in figure 4 below.

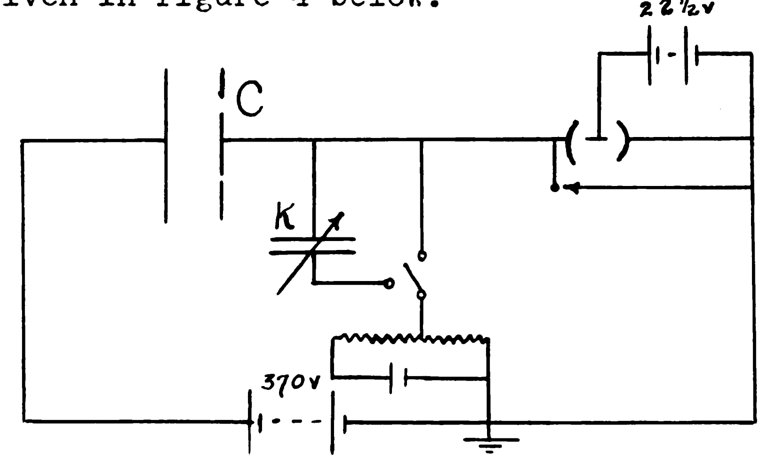


Fig 4

C = Capacity of Electrometer System.

K = Capacity of Standard Condenser.

v = Volts applied directly on quadrants of electrometer.

V = Volts applied to standard condenser.

When the charge collected in the ionization chamber is Q, the potential v, transmitted to the electrometer quadrants is given by

$$Q = Cv$$

This potential gives the electrometer a definite deflection δ. By applying known voltages direct to the electrometer circuit, and noting the corresponding readings of the instrument, curves relating electrometer deflections to applied voltage may be drawn. Then all that is required to determine the charge collected, causing a given electrometer deflection, is the capacity C of the electrometer circuit. For the particular instrument used, the relationship of voltage to electrometer deflection was a straight line. In determining the capacitance of the system, known voltages were applied to a standard condenser (General Radio Co. type 222, serial 660) placed in series with the electrometer circuit, and the deflection of the instrument recorded. This data gave a straight line relating electrometer deflection to applied voltage with a known capacitance in series with the electrometer circuit's unknown capacitance. If now a voltage, v, applied directly to the electrometer circuit gives a deflection δ, then, as we noted, the charge on the electrometer is given by

$$Q = Cv \qquad 4$$

And if a voltage, V, applied to the standard condenser of capacitance K, gives the same deflection of the electrometer, then the charge on the electrometer is again Q. This time Q is given by

$$Q = \frac{1}{\frac{1}{K} + \frac{1}{C}} \cdot V \quad . \qquad 5$$

Equating 4 and 5 and solving for C gives

$$C = \left(\frac{V}{v} - 1\right) K \quad . \qquad 6$$

Now, let "a" be the slope of the straight line obtained from a plot of v against δ, and "b" the slope of the straight line obtained from a plot of V against δ.

$$a = \frac{\delta}{v} \qquad\qquad b = \frac{\delta}{V}$$

Equation 6 becomes

$$C = \left(\frac{a}{b} - 1\right) K \quad , \qquad 7$$

which is the expression used for determining the capacitance of the system. However, there exists the difficulty that the slope b, and consequently the experimentally determined value of C, changes materially with a change in K. The proper value of K to use for determining C must be chosen.

For any given experimental value of C, consider the error dC as determined from equation 7. This is

$$(dC)^2 = (a/b - 1)^2 (dK)^2 + (K/b)^2 (da)^2 + (aK/b^2)^2 (db)^2 \qquad 8$$

The values of da and db are calculated from their respective equations.

$$\delta = av \quad \text{and} \quad \delta = bV .$$

Thus:

$$da = \frac{1}{v} d\delta - \frac{\delta}{V^2} dv .$$

Sinc e the voltage applied with and without K in the circuit is the same, and since the error dv and dV in determining the applied voltage is the same in both instances.

$$da = db$$

Knowing from many determinations of electrometer deflections that the calibration curve was linear, the mean of many deflections for an applied voltage of 0.09 was used to obtain the curve of figure 5. From this data

$$a = 98.7/.09 = 1096.7 \text{ mm per volt.}$$

The data below was used for the determination of capacitance, using a value of K = 581 $\mu\mu$f.

volts	time during which voltage is applied	δ mm
0.90	15 sec.	88.8
		92.3
		92.4
		91.9
		88.1
		94.2
		92.4
	mean	91.4

The mean deflection was 91.4mm. The number, n, of readings was 7; and the sum of the squares of the mean deviations from the mean equals 28.55. The expression for probable error is

$$0.6745 \sqrt{\frac{\sum (\text{mean deviations from the mean})^2}{n(n-1)}}$$

$$d\delta = 0.6745 \sqrt{28.55/7.6} = 0.55 \text{ mm}$$

The voltage could be determined to within .0001 volt.

Then

$$(da)^2 = (db)^2 = (1/.09)^2 (0.55)^2 + (91.4/(.09)^2 (.0001)^2$$

$$= 38.67$$

$$da = db = 6.2 \text{ mm per volt.}$$

Now, the error, dC, from expression 8 may be calculated. The probable error dK in the capacitance of the standard condenser was taken as $1\mu\mu f$. Data taken using a value of $K = 58.0\mu\mu f.$, yielded a mean electrometer deflection of 56.8 mm. Then

$$b = \delta/V = 56.8/.09 = 631.1 \text{ mm per volt}$$

Voltage applied direct to electrometer gave a = 1097 mm per volt. Using the value found before for da and db, we get

$$(dC)^2 = (98.7/56.8 - 1)^2 + (58/631)^2 38.67$$

$$+ (1097.58/(631)^2)^2 38.67$$

$$dC = 1.35 \text{ f.}$$

Several values of dC for corresponding values of the standard capacity K were found in this manner and plotted in figure 6. This curve shows that a minimum error will be made in determining the unknown capacitance by using a value of K between 55$\mu\mu$f and 100$\mu\mu$f. The mean capacity found using several values of K within this range gave C = 39.64$\mu\mu$f.

V.

The Working Equation Used to Reduce the Experimental Data

A. Fraction of x-rays measured in the ionization chamber to those produced at target.

For this analysis, reference will be made to the following figure 7 which shows the arrangement of apparatus in the path of the measured x-ray beam.

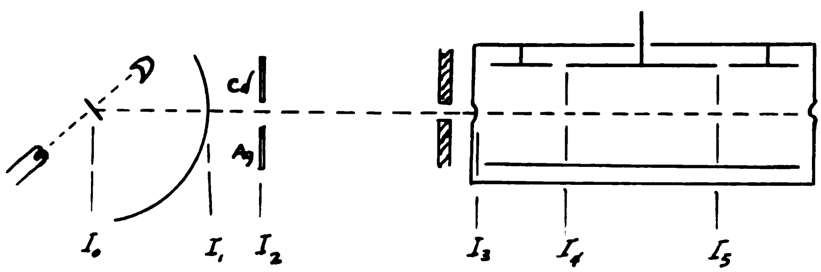


Fig 7

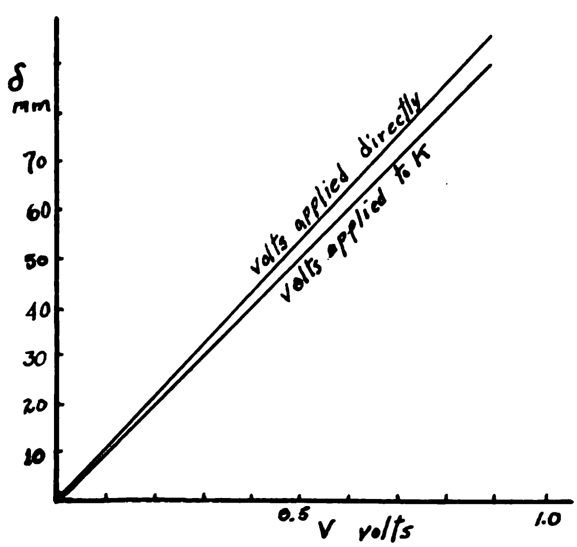


Fig 5

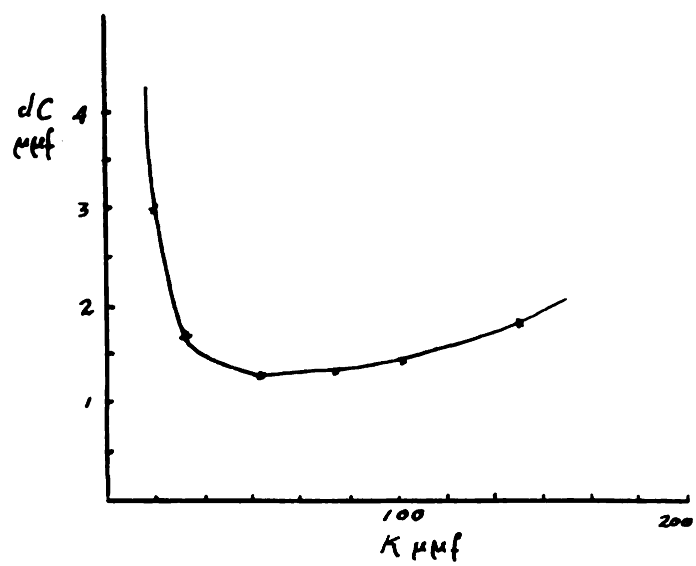


Fig 6

Here,

I_o = Original intensity of x-rays produced at target.

I_1 = Intensity of x-rays passing the x-ray tube window.

I_2 = Intensity of x-rays passing the Ross filter.

I_3 = Intensity of x-rays passing the mica window of ionization chamber.

I_4 = Intensity of rays reaching front end of collecting electrode of ionization chamber.

I_5 = Intensity of rays leaving rear end of collecting electrode.

t_1 = Thickness of Al window of x-ray tube.

t_2 = Thickness of Ag or Cd filter.

t_3 = Thickness of mica window.

t_4 = Distance from mica window to front end of collector electrode.

t_5 = Length of collecting electrode.

μ_1 = Linear absorption coefficient of Al window.

μ_2 = Linear absorption coefficient for Ag or Cd. (depending on which is being considered)

μ_3 = Linear absorption coefficient of mica.

μ_4 = Linear absorption coefficient of methyl bromide in chamber.

The fraction of the original intensity of radiation produced that is absorbed in the useful part of the chamber is then given by

$$f = \frac{I_4 - I_5}{I_o}$$

With the Cd filter in the path, a deflection of the electrometer is obtained which is proportional to f.

Likewise, for the Ag filter, and we may represent this as:

$$\delta_{Cd} = \left(\frac{I_4 - I_5}{I_o}\right)_{Cd} \quad , \delta_{Ag} = \left(\frac{I_4 - I_5}{I_o}\right)_{Ag}$$

The various intensities are related to each other as follows:

$$I_1 = I_o \exp(-\mu_1 t_1)$$

$$I_2 = I_1 \exp(-\mu_2 t_2) = I_o \exp(-\mu_1 t_1 - \mu_2 t_2)$$

and similarly; so that finally

$$I_5 = I_o \exp(-\sum_{i=1}^{5} \mu_i t_i)$$

and $\left(\frac{I_4 - I_5}{I_o}\right)_{Cd} = \left[1 - \exp(-\mu_5 t_5)\right] \exp(-\sum_{i=1}^{4} \mu_i t_i)$

Then $\delta_{Cd} - \delta_{Ag} \sim S = 0.90 \left[1 - \exp(\mu_5 t_5)\right]\left[\exp(-\sum_{i=1}^{4} \mu_i t_i)_{Cd} - \exp(\sum_{i=1}^{4} \mu_i t_i)_{Ag}\right]$ 9

The constant 0.90 which appears above is now discussed. The assumption has been made in this discussion that all the ions formed were collected before recombining. To test whether this were actually true or not, a continuous source of x-rays were produced at least three times as intense as those to be measured. Electrometer deflections were then plotted against voltage applied to the collector electrodes from "B" batteries. The curve indicated that at 370 volts applied voltage, the ions were sw ept out of the chamber to within one percent. This correction has been applied to the equation relating electrometer deflection to I_o.

The quanta absorbed photoelectrically in the ionization chamber produce photoelectrons and positive bromine ions. When a photoelectron is produced by ejecting an electron from an inner level of the bromine atom, this particular electronic level is left incomplete. Another electron may fall into this level with a consequent radiation of a bromine K quantum, if the K shell happens to be the particular level in question. Not all of the relatively short fluorescent bromine K rays will be absorbed before they reach the walls or ends of the chamber. Practically all of the L and M quanta so produced will be absorbed however. Furthermore, by the same process, there will be produced, in the front and back guard plate regions, quanta which are not part of the ~~chamber~~ original intensity, and there produce ions. This correction was made by J. C. Clark (19) for the particular ionization chamber used. The solution of this problem involves a mathematical formulation of the above considerations, having regard of the geometry of the ionization chamber, and necessitates graphical integration.

In order to evaluate the exponentials in equation 9, the thicknesses and absorption coefficients of the aluminum, balanced filters, mica window, and methyl bromide must be known. These are given below.

$t_1 = 0.193$ mm	$\mu_1 = 4.6\ cm^{-1}$
$t_2Cd = 0.0037$ cm	$\mu_2Cd = 81.0\ cm^{-1}$
$t_2Ag = 0.0033$ cm	$\mu_2Ag = 623\ cm^{-1}$
$t_3 = 0.0043$ cm	$\mu_3 = 4.658\ cm^{-1}$
$t_4 = 7.8$ cm	$\mu_4 = 0.0782$ at 68.3 cm Hg
$t_5 = 10.0$ cm	

The linear absorption values are from Compton and Allison (20) and are for a wave length of 0.474 Å. Substituting the above values in equation 9, we get

$$S = 0.146 \qquad (10)$$

This equation represents the ratio of the radiation of mean frequency $\nu = \frac{\nu Cd - \nu Ag}{2}$ measured to that produced at the target.

B. <u>Absolute Intensity Formula</u>.

As was pointed out, the electrometer was calibrated so that a given deflection corresponds to a known voltage; and this in turn to an amount of charge given by $Q = Cv$. Dividing by the charge on the electron gives the number of ions collected. Or, number of ions collected per second $= C\Delta V/et$, where

c = capacitance of electrometer system.

ΔV = voltage difference corresponding to $\delta Cd - \delta Ag$.
$\Delta V = 1/a(\delta Cd - \delta Ag) = 1/1097(\delta Cd - \delta Ag)$; and is then the produced voltage on the electrometer due to the quanta defined by the frequency limits of the cadmium and silver filters.

e = Charge on the electron.

t = Time the radiation is allowed to enter chamber.

Various measurements have been made to determine the energy required to produce a pair of ions in gases by the absorption of x-rays. Stockmeyer (21) has summarized the results concerning this constant, and, together with determinations of his own, gives, for methyl bromide,

$$\epsilon = 25.4 \text{ electron volts per ion pair.}$$

This is $\epsilon \times 1.6 \times 10^{-12}$ ergs per ion pair.

Thus, the energy measured per second passing through an aperture of unit area at a distance R from the target =

$$\frac{c \Delta v \epsilon 1.6 \times 10^{-12}}{a\ e\ t} . \qquad 11$$

Where "a" is the area of the defining aperture used in the experiment.

In this equation, the Δv term is directly proportional to the difference between the electrometer deflections with Cd and Ag filters in the path; that is,

$$\delta_{Cd} - \delta_{Ag} \sim \Delta v.$$

The energy thus produced at the target per second of mean frequency is equal to that measured (given by formula 11) divided by the ratio of that measured to that produced (given by formulas 9 and 10). We may express this as:

Energy produced per second at the target per unit area distant "R" from the target $= \dfrac{C \Delta v \epsilon 1.6 \times 10^{-12}}{S\ a\ e\ t}$

The number of electrons produced in the time interval "t" is given by

$$\frac{it}{e} \quad \text{, where is is the space current in the x-ray tube.}$$

The number of atoms per unit area of the target is

$$\frac{N \rho x_0}{A} \quad \text{, where}$$

N = Avogadro's number, ρ = density, x_0 = target thickness and A = Atomic weight of target material.

We thus see that the energy produced at the target per second, per electron, per atom, per unit area of the target, per unit area at the distance R from the target, is

$$\frac{\dfrac{C \epsilon \Delta v \, 1.6 \times 10^{-12}}{s \, a \, e \, t} \Big/ \dfrac{it}{e}}{N \rho x_0 / A}$$

This is more compactly written in equation 12. used for evaluating the experimentally determined data.

$$J_{\nu} \, d\nu = \frac{C \epsilon \Delta v \, A \, 1.6 \times 10^{-12}}{S \, a \, t^2 \, N \, x_0 \, i} \qquad 12$$

$$= \frac{1.60 \times 10^{-12} \, C \epsilon A}{S \, a \, N \rho x_0 \, i} \cdot \frac{\Delta V}{t^2}$$

The units associated with each of the terms used in expression 12 may well be stated here. These are given on the following page.

C = electrometer system capacity (Farads)

ϵ = energy per ion pair (electron volts)

Δv = potential difference corresponding to electrometer deflections $\delta_{Cd} - \delta_{Ag}$ (volts)

A = atomic weight of target (gms/gm atom)

S = ratio defined by equation 10

a = area of defining aperture (cm^2)

t = exposure time (sec)

N = Avogadro's number (atoms/gm atom)

X = Target thickness (cm)

i = x-ray tube current (coulombs/sec.)

Then, the units of $J_\nu d\nu$ are

$J_\nu d\nu$ = ergs per cm^2 at a specified distance from the target per sec per electron per atom per cm^2 of target.

VI.

Measurements

A. Target Thickness Measurements.

An interferometer plate, which has previously had a reflecting surface evaporated on it, is placed in the center of a square, the four targets at the corners. A portion of the plate through the center is blocked off by a 1/8 inch strip of mica. In this manner, a film of the thickness of the targets is evaporated on the two exposed portions each side of the center strip. When the plate is placed in a Michelson interferometer, the interference fringes of the center portion will be displaced with respect to those at the edges by an amount proportional to the optical path difference introduced by the deposited film. The distance between 2 adjacent fringes of the same set corresponds to an optical path difference of $\lambda/2$. The fraction of a fringe shift, then, corresponds to the actual thickness of the deposited film; and this thickness is the same fraction of $\lambda/2$. The photograph, figure 8, is of a fringe system produced as described above.

Fig 8

Calling the fringe displacement d' and the distance between fringes of the same set d, the equation for calculating the thickness of the deposited film is

$$x_0 = d'/d \; \lambda/2 \quad . \qquad 13$$

We see that the smaller λ is, the greater will be the relative displacement. All photographs were made with the interferometer adjusted for white light fringes, in order that any components adjacent to the principle line used for photographing should give an interference fringe in practically the same position as that produced by the principal component; and that there should be a maximum contrast between light and dark portions of the field. Lines used were those of mercury at $\lambda = 5461$ Å and $\lambda = 3654$ Å. Comparator readings were taken of the fringe shift for the calculation of thickness. The following Table II shows a typical set of measurements to determine target thickness.

Table II

Al 26E, Photographic Plate E, $\lambda = 3654$ Å

Distances between Fringes mm	Fringe Displacement mm
24	6
20	5
21	7
24	10
25	6
21	4
24	6
22	4
23	6
21	4
22	4
22	5
22	4
23	6
22	4
22	4
21	4
23	5
22	4
21	5
24	6
20	4
22	6

mean = 22.2 mm. mean = 5.1 mm

Mean fractional displacement = 5.1/22.2

Target thickness = $x_0 = 5.1/22.2 \times \frac{3654}{2}$ Å = 411 Å.

Table III summarizes the measurements made on target thickness for all the targets used.

Table III

Interferometer Plate Number	Lambda	Thickness	Weight Number of Fringes Used	Thickness x Weight of Observation.
Al 26E	5461 Å	344 Å	18	6192
Al 26E	3654 Å	380 Å	15	5700
Al 26E	3654 Å	411 Å	23	9453

Weighted mean thickness = 381 Å

B. <u>Absolute X-Ray Intensities</u>

Placing the appropriate values of the constants in equation 12 results in an equation for intensity of radiation in terms of v alone. This is herewith shown:

$$J_\nu \, d\nu = \frac{1.6 \times 10^{-12} \times 39.64 \times 10^{-12} \times 25.4 \times 26.96}{0.146 \times 0.912 \times 6.028 \times 10^{23} \times 2.70 \times 381 \times 10^{-8} \times 15 \times 10^{-8} \times 900} \cdot \Delta V ,$$

(all as denominator)

where for the conditions of the experiment

A = 26.96 gm/gram atom

a = 0.912 cm^2

N = 6.028 x 10^{23} atoms per gram atom

ρ = 2.70 gm/cm^3

x_o = 381 x 10^{-8} cm.

i = 15 x 10^{-8} amperes

t = 30 sec.

The remaining constants have already been given. This equation reduces to 13, the one used in calculating the data of this experiment.

$$J_\nu \, d\nu = 3.896 \times 10^{-34} \Delta V \qquad 13$$

where $J_\nu \, d\nu$ will be expressed in ergs per sec per cm^2, at distance R from target, per electron per atom per cm^2 of target, when Δv is expressed in volts.

The experimental data to be presented in this thesis was obtained from observations using four thin aluminum targets on cellophane backing films. These four targets were evaporated at the same time as the interferometer plate Al #26 E was made. The measurements covering the thickness of the targets, and made on this interferometer plate, are those presented above under section A of Part VI of this thesis. These four targets have been designated Al 26 A, B, C, D. A complete set of data for target Al 26 C is given in the following Table IV. The data shown in table V are the mean values of δCd - δAg for each x-ray tube voltage used on targets Al 26 A, B, and D. Curves showing the variation of $J_\nu \, d\nu$ as a function of bombarding electron energy are shown in figure 9. Also shown in figure 9 is a curve labled "mean" which is the mean values of $J_\nu d\nu$ of all data collected throughout the voltage range 31.7 to 44.0 kev.

Table IV

Al target No. 26 C. $\theta = 60^{o}$, exposure time 30 sec.
Tube current 0.15 microamperes.

Tube Voltage KV	δ_{Cd}(mm)	δ_{Ag}(mm)	$\delta_{Cd} - \delta_{Ag}$	Δv(volts) x 10^3	$J_\nu d\nu$(ergs, etc.) x 10^{36}
44.00	89.5	82.2	7.3		
44.00	88.2	81.0	7.2		
44.00	88.3	80.0	8.3		
44.00	88.9	83.6	5.3		
		Mean	7.02	6.75	2.63
41.40	74.0	69.0	5.0		
41.40	74.0	68.2	5.8		
41.40	74.9	68.5	6.4		
41.40	76.0	71.0	5.0		
		Mean	5.55	5.34	2.08
38.43	59.5	55.0	4.5		
38.43	60.8	55.7	5.1		
38.43	61.0	55.3	5.7		
38.43	60.0	55.6	4.4		
		Mean	4.92	4.73	1.84
34.77	44.3	40.8	3.5		
34.77	43.1	40.5	2.6		
34.77	43.2	40.6	2.6		
34.77	44.0	40.8	3.2		
		Mean	2.98	2.86	1.12
31.75	31.4	29.3	2.1		
31.75	31.2	29.0	2.2		
31.75	31.4	29.2	2.2		
31.75	30.8	28.9	1.9		
		Mean	2.10	2.02	0.79

Table V

$\theta = 60^{o}$, exposure time 30 sec., tube current 0.15 microamperes.

Al #26 Targets	Tube Voltage KV	Mean $\delta_{Cd} - \delta_{Ag}$ mm	Δv volts x 10^{3}	$J_{\nu}\, d\nu$ (ergs, et c.) x 10^{36}
A	31.75	2.60	2.50	0.97
B	31.75	1.46	1.40	0.55
D	31.75	2.00	1.92	.75
B	34.77	2.68	2.58	1.00
A	35.75	4.46	4.29	1.67
D	35.75	2.98	2.87	1.12
A	38.43	5.54	5.33	2.08
B	38.43	4.25	4.09	1.59
D	38.43	4.30	4.13	1.61
B	41.40	5.12	4.92	1.92
A	42.50	7.87	7.57	2.95
D	42.50	6.33	6.09	2.37
B	44.00	6.55	6.30	2.45

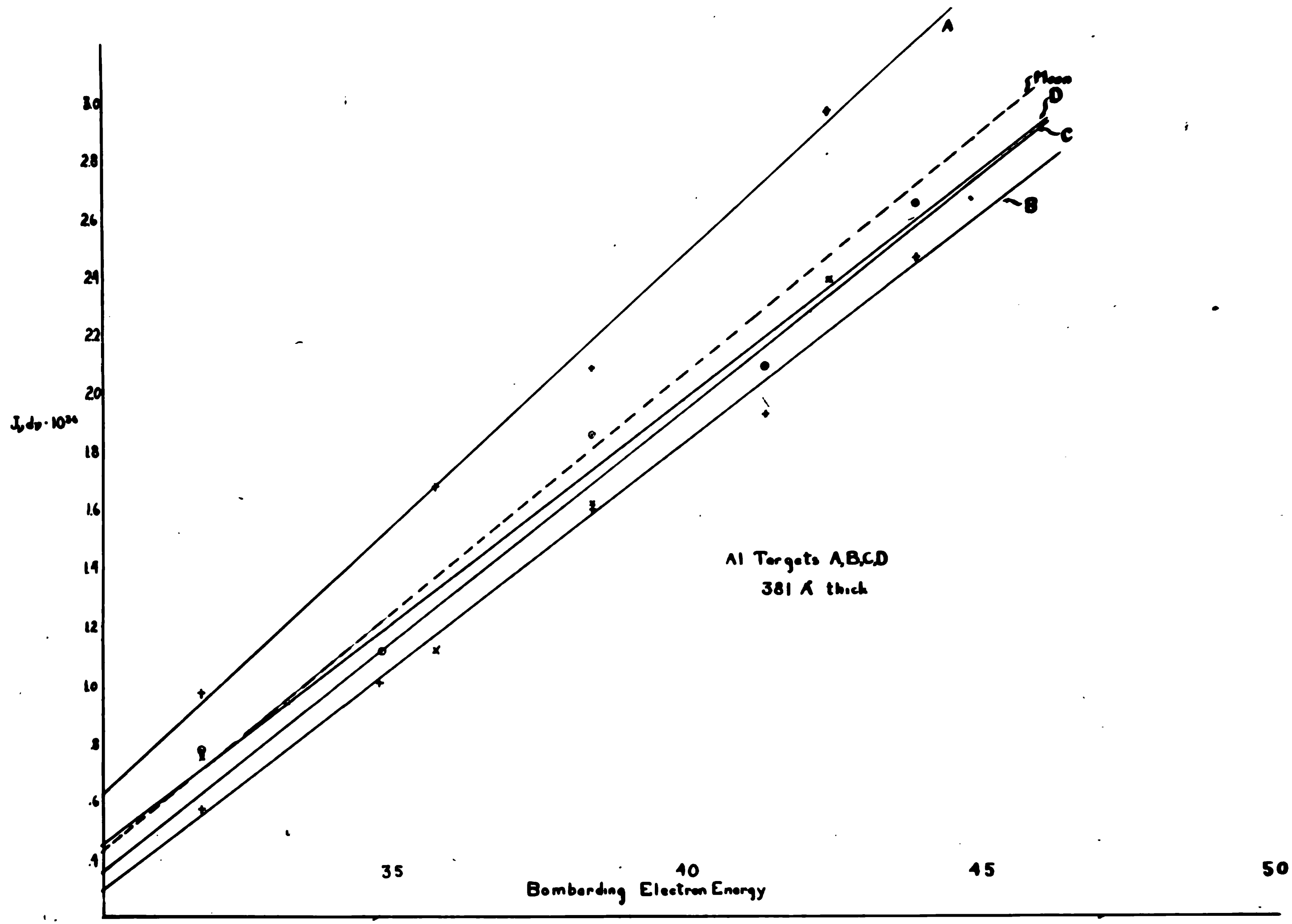

A
Mean
D
C
B
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
35
40
45
50
Bomberding Electron Energy
Al Targets A,B,C,D
381 Å thick

Fig 9

VII.

Comparison of Experiments with Theory

A. Intensity versus electron energy curves.

In figure 10 the results of the computations of $J_\nu d\nu$(corrected) and the mean values of $J_\nu d\nu$ determined experimentally are shown plotted as a function of the energy of the bombarding electron. As is seen, the computation for the theoretical values have been carried out for a voltage range from 30 to 80 KV, while the experimental results extend over a range from 31.75 to 44.0 KV. In these curves, the absolute values of x-ray intensity are expressed in the units used throughout this work; namely ergs per sec per cm^2 at a distance of 34 cm from the target per electron per atom per cm^2 of the target.

B. Discussion of Results.

As is seen in figure 10, the results of these experiments are in agreement with the theory only in the order of magnitude of the absolute values of x-ray intensities. Many errors exist in the various measurements, as well as in such quantities not measured, but used from the literature, such as energy required to produce a pair of ions and the density of the very thin aluminum targets produced by the evaporation process. These errors are all involved in the final values of $J_\nu d\nu$, so a discrepancy between theory and experiment of the absolute value of $J_\nu d\nu$ would not be surprising, assuming the theory correct. The above mentioned errors remain practically the same for all bombarding electron

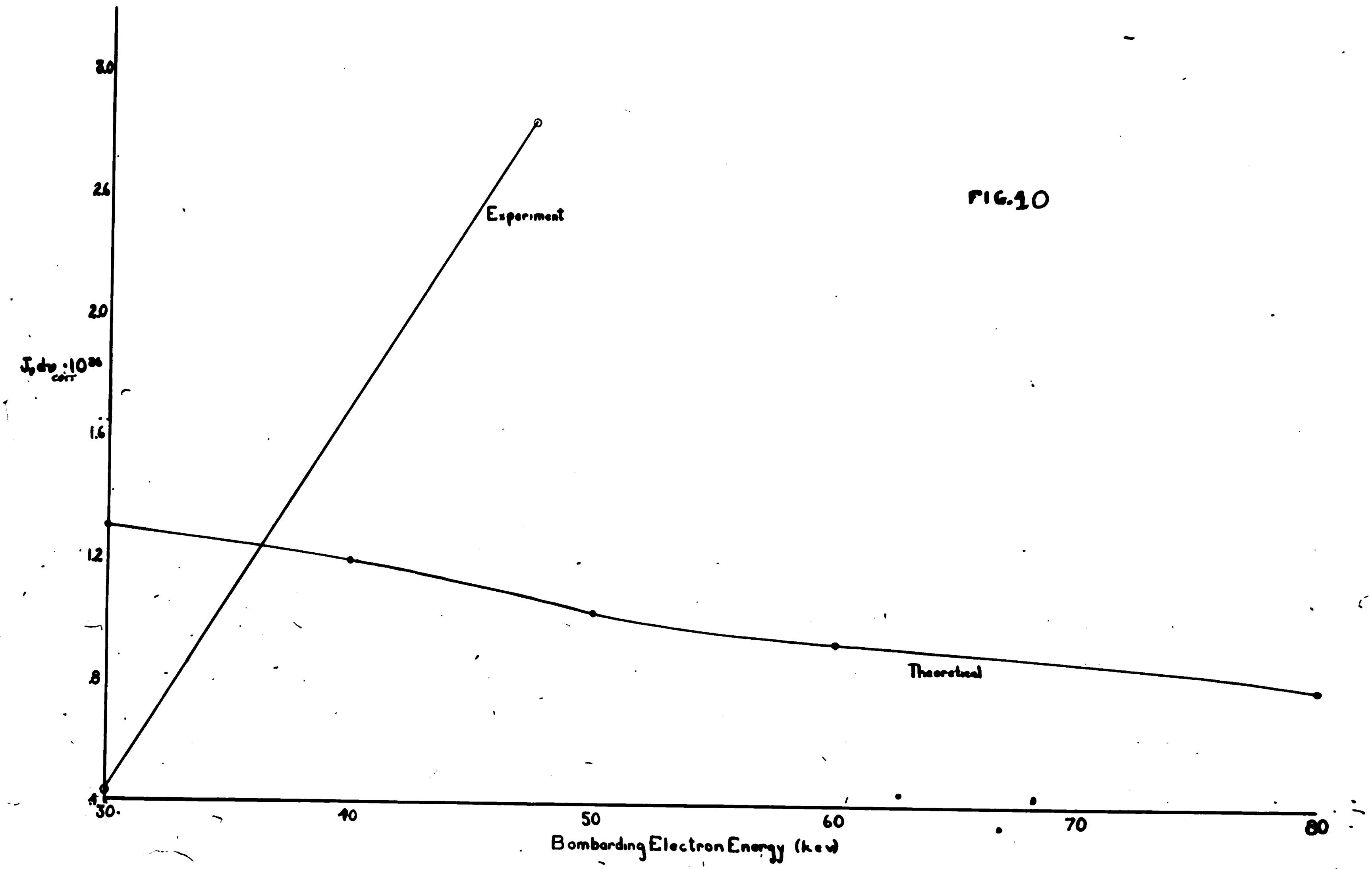

3.0
2.6
2.0
1.6
1.2
.8
.4
30.
40
50
60
70
80
Experiment
Theoretical
Bombarding Electron Energy (kev)

FIG. 10

energies, and the discrepancy between the experiments and theory in this respect is more disturbing. No other similar experiments have as yet been reported. Indeed only one other experiment (13) is reported which allows a comparison with this to be made is reported. These authors show an agreement between theory and experiment for absolute values of intensity for Nickel targets at electron energies of 15 Kev. More experimental results than those reported here are obviously required before definite conclusions can be drawn.

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