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DIFFRACTION OF LIGHT BY ULTRASONIC WAVES
OF VARIOUS STANDING WAVE RATIOS

Thesis for the Degree of M. S.
MICHIGAN STATE UNIVERSITY

Billy D. Cook

1959

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DIFFRACTION OF LIGHT BY ULTRASONIC WAVES OF
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Billy D. Cook

A Thesis

Submitted to the College of Science and Arts Michigan
State University of Agriculture and Applied
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DIFFRACTION OF LIGHT BY ULTRASONIC WAVES OF
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ABSTRACT

The theory for the diffraction of light by plane ultrasonic waves of various standing wave ratio is derived. The medium disturbed by the ultrasound is considered to act as an optical phase grating. By evaluating the diffraction integral for the light amplitude, expressions for the time dependent and time average light intensities are found for the diffraction spectrum. These expressions reduce to those known for progressive and stationary waves.

Measurement of time dependent and time average light intensities indicate that the theory is valid.

TABLE OF CONTENTS

INTRODUCTION	1
THEORY	6
General Procedure.	6
Diffraction of Light by Standing Waves	7
Doppler Shift of the Diffracted Light	11
Time Dependence of Light Intensity for Stationary Waves.	12
EXPERIMENTAL STUDY	14
Apparatus and Procedure	14
Time Dependent Intensity Measurements .	16
Average Light Intensity Measurements .	19
SUMMARY	23
BIBLIOGRAPHY	24

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FIGURES

Fig. 1.	Special Tank to Absorb Ultrasound . . .	15
Fig. 2.	Schematic Diagram of Experimental Apparatus	15
Fig. 3.	Time Dependent Light Intensity Observations	17
Fig. 4.	Theoretical Curves for Time Dependent Light Intensities	18
Fig. 5.	Experimental Results for Average Light Intensity of the Zero Order for the SWR equal one	20
Fig. 6.	Experimental Results for Average Light Intensity of the Zero Order for the SWR equal two.	21
Fig. 7.	Experimental Results for Average Light Intensity of the Zero Order for the SWR equal infinity	22

INTRODUCTION

Diffraction of light by ultrasonic waves has been studied for many years. These studies have considered two extreme cases: the progressive wave and the stationary wave. The object of this investigation is the study of diffraction of light by ultrasonic waves of various standing wave ratios.

Standing waves may be considered as a superposition of two periodic wave trains of the same frequency and travelling in opposite directions¹. If the wave trains are continuous sinusoidal progressive waves, a fixed distribution of nodes or partial nodes and antinodes occur in space. Stationary waves are standing waves in which the energy flux is zero at all points². Since in the field of ultrasonics the usual method of obtaining standing waves is by reflection, it is convenient to distinguish the different wave trains in terms of an incident wave and a reflected wave. Define the wave travelling in the direction of the net energy flow as the incident wave and the other as the reflected wave. The relative amplitudes of these waves are given by the standing wave ratio which is defined as the ratio of the incident wave amplitude to that of the

reflected wave. In the limiting cases, i.e., the stationary wave and the progressive wave, the SWR^* is equal to one and infinity, respectively.

The medium disturbed by an ultrasonic wave was considered by Raman and Nath^{3,4,5} to act like an optical phase grating. For both progressive and stationary plane waves, they predict for normal incidence of the light to the ultrasonic beam, that the light emerges at angles θ_n given by

$$\sin \theta_n = n \lambda / \lambda^* \quad n = 0, \pm 1, \pm 2, \dots \quad (1)$$

where λ is the wavelength of the light and λ^* the ultrasonic wavelength. In a later paper⁶, they predict a Doppler shift in the various orders of diffracted light. For progressive waves the incident light of frequency ν emerges in the n th order at the frequency of $\nu + n \nu^*$ where ν^* is the frequency of the ultrasonic waves. The intensity of the light in the n th order is given by

$$I_n = J_n^2(\nu) \quad (2)$$

* Hereinafter, the expression standing wave ratio will be abbreviated to SWR .

The parameter V is given by

$$V = \frac{2\pi L \mu}{\lambda} \quad (3)$$

where μ is the amplitude of the periodically changing index of refraction, L is the path length of the light in the ultrasonic beam, and J_n is the n th order Bessel function.

For stationary waves, the Raman and Nath theory predicts that for the n th order, the emerging radiation consists of frequencies

$$\nu + (2r+n)\nu^* \quad r = 0, \pm 1, \pm 2, \dots \quad (4)$$

The intensity I_{nr} of each sub-component of frequency $\nu + (2r+n)\nu^*$ is

$$I_{nr} = J_r^2(V) J_{r-n}^2(V) \quad (5)$$

for each of the progressive waves composing the stationary wave. The total intensity in the n th order is given by

$$I_n = \sum_{r=-\infty}^{\infty} J_r^2(V) J_{r-n}^2(V) \quad (6)$$

The theory of Raman and Nath considers the medium to act only as a phase grating and assumes that there is no amplitude modulation of the light by the ultrasonic waves. More general theoretical considerations by others⁷ have shown that the Raman and Nath theory is valid for

$$\pi L \lambda v / \mu_0 \lambda'' < 1 \quad (7)$$

where μ_0 is the index of refraction of the undisturbed medium.

Experimental confirmation in liquids of the Raman and Nath theory has been made by R. Bar⁸ and F. Sanders⁹. An asymmetry of the light intensity about the zero order in the diffraction pattern was detected for both progressive and stationary waves. Zankel and Hiedemann¹⁰ have explained this asymmetry for progressive waves to be caused by finite amplitude distortion.

Pande, Pancholy, and Parthasarathy¹¹ have shown that one may obtain optically a phase grating corresponding to that of a stationary wave without having an actual ultrasonic stationary wave. Using two piezoelectric transducers driven from the same high frequency oscillator, they produced identical ultrasonic progressive waves

travelling in opposite directions in two different vessels, the sides of which were close together and containing the same liquid. For normal incidence of the light to both ultrasonic beams the diffraction pattern was the same as one caused by a stationary wave.

A variation of the method described above may be used to obtain, in an optical sense, standing waves of various standing wave ratios. With separate control of the sound output of the transducers, one can produce a full range of SWR from one to infinity.

THEORY

General Procedure

In considering the ultrasonic diffraction of light caused by an optical phase grating, an integral method may be used to determine the diffraction phenomena. For collimated monochromatic light at normal incidence to a transmission phase grating, the amplitude distribution of the diffracted light is given by

$$A(\theta) = C e^{i\omega t} \int_{-D}^D e^{i \left[\frac{2\pi x}{\lambda} \sin \theta + v(x, t) \right]} dx \quad (8)$$

where $2D$ is the width of the light beam. λ is the wavelength of light, ω is the angular frequency of the light, and θ is the angle at which the light is diffracted. The term $v(x, t)$ is a dimensionless parameter describing the optical phase grating in space and time.

The value of $A(\theta)$ is in general a complex quantity. The time dependent light intensity I is given by

$$I = A A^* = |A|^2 \quad (9)$$

and the time average light intensity $\bar{I}$ is given by

$$\bar{I} = \frac{1}{T} \int_0^T |A|^2 dt. \quad (10)$$

The parameter $V(x, t)$ is related to the instantaneous sound pressure $p(x, t)$ through the periodic change of the index of refraction $\mu(x, t)$. For a given sound pressure $p(x, t)$,

$$V(x, t) = \frac{2\pi L}{\lambda} \mu(x, t) = \frac{2\pi L \chi}{\lambda} p(x, t) \quad (11)$$

where χ is the piezo-optic constant for the medium.

Diffraction of Light by Standing Waves

The instantaneous pressure of an ultrasonic sinusoidal standing wave may be written as

$$p = P \sin(\omega^* t - \kappa^* x) + \frac{P}{a} \sin(\omega^* t + \kappa^* x) \quad (12)$$

where ω^* and κ^* are the acoustic angular frequency and wave constant, respectively, and a is the standing wave ratio (SWR).

The optical grating corresponding to the ultrasonic

standing wave is described by

$$V = V \sin(\omega^* t - k^* x) + \frac{V}{2} \sin(\omega^* t + k^* x). \quad (13)$$

The diffraction integral, Equation (8), becomes

$$A(\theta) = C e^{i\omega t} \int_{-D}^D e^{iu l x} \chi e^{i[V \sin(k^* x - \omega^* t) + \frac{V}{2} \sin(k^* x + \omega^* t)]} dx. \quad (14)$$

where $u = 2\pi/\lambda$ and $l = \sin \theta$. Using the identity

$$e^{i b \sin f} = \sum_{r=-\infty}^{\infty} J_r(b) e^{i r f}. \quad (15)$$

Equation (14) reduces to an integrable form

$$A = C e^{i\omega t} \int_{-D}^D \sum_{s=-\infty}^{\infty} \sum_{r=-\infty}^{\infty} J_r(v) J_s\left(\frac{v}{2}\right) e^{i(r+s)\omega^* t + i(r-s)k^* x} dx. \quad (16)$$

Integrating, one obtains

$$A = 2 C e^{i\omega t} \left[\sum_{s=-\infty}^{\infty} \sum_{r=-\infty}^{\infty} J_r(v) J_s\left(\frac{v}{2}\right) \chi e^{i(r+s)\omega^* t} \frac{\sin[u l + (r-s)k^*]D}{[u l + (r-s)k^*]} \right]. \quad (17)$$

To normalize, it is assumed that the light amplitude at $\theta = 0$ equals unity for $v = 0$. Thus one finds

$$C = 1/2 D \quad (18)$$

Let $n = r - s$, and then Equation (17) becomes

$$A = \sum_{n=-\infty}^{\infty} \sum_{r=-\infty}^{\infty} J_r(v) J_{r-n} e^{i[(2r-n)\omega^* + \omega]t} W_n \quad (19)$$

where

$$W_n = \frac{\sin [(u l + n k^*) D]}{[(u l + n k^*) D]} . \quad (20)$$

For ideal diffraction, $D \rightarrow \infty$. W_n then has non-zero values only for

$$u l + n k^* = 0 \quad (21)$$

which is the same condition given in Equation (1).

Thus the light is diffracted into discrete orders of amplitude A_n given by

$$A_n = \sum_{r=-\infty}^{\infty} J_r(v) J\left(\frac{v}{2}\right) e^{i[(2r-n)\omega^* + \omega]t} \quad (22)$$

since W_n equals unity when the condition in Equation (21) is satisfied.

The time dependent intensity in the n th order is

$$I_n = \sum_{p=-\infty}^{\infty} \sum_{r=-\infty}^{\infty} J_r(v) J_p(v) J_{r-n}\left(\frac{v}{a}\right) J_{p-n}\left(\frac{v}{a}\right) \times e^{i2(r-p)\omega^*t} \quad (23)$$

and the average light intensity is

$$\bar{I}_n = \sum_{r=-\infty}^{\infty} J_r^2(v) J_{r-n}^2\left(\frac{v}{a}\right) \quad (24)$$

Equation (24) reduces to the Raman and Nath results for both progressive and stationary waves. For progressive waves $a \rightarrow \infty$, and Equation (24) reduces to

$$\bar{I}_n = J_n^2(v) \quad (25)$$

as

$$\begin{aligned} J_{r-n}(0) &= 0 & r \neq n \\ &= 1 & r = n \end{aligned} \quad (26)$$

For stationary waves $a = 1$, and Equation (24) becomes

$$\bar{I}_n = \sum_{r=-\infty}^{\infty} J_r^2(v) J_{r-n}^2(v) \quad (27)$$

Doppler Shift of the Diffracted Light

The motion of the optical phase grating produces a Doppler shift in the diffracted light. Equation (23) shows that for a standing wave the light in the n th order contains components of the angular frequency

$$\omega + (2r - n) \omega^*.$$
(28)

The average intensity of these non-coherent components is

$$J_r^2(\nu) J_{r-n}^2\left(\frac{\nu}{a}\right).$$
(29)

However, for a progressive wave the only component having a non-zero intensity is the one where $r=n$. Thus the angular frequency of light in the n th order for a progressive wave is

$$\omega + n \omega^*$$
(30)

with the average intensity given in Equation (25). The terms in the summation in Equation (24) are the intensities of the non-coherent components. However, the actual frequency shift is so small that it is not easily observable.

Time Dependence of Light Intensity for Stationary Waves

For stationary waves the optical grating may be described by

$$V(x, t) = V \cos \omega^* t \sin k^* x. \quad (31)$$

The diffraction integral then becomes

$$A = C e^{i\omega t} \int_{-D}^D e^{i u l x + i v \cos \omega^* t \sin k^* x} dx, \quad (32)$$

Using Equation (15), integrating, and normalizing as before, one obtains

$$A = \sum_{n=-\infty}^{\infty} J_n(v \cos \omega^* t) e^{i\omega t} W_n. \quad (33)$$

For $D \rightarrow \infty$, the light intensity in the n th order is

$$I_n = J_n^2(v \cos \omega^* t). \quad (34)$$

Raman and Nath obtained Equation (34) which is equivalent to Equation (23) when $a = 1$. Equation (34) is more suitable for physical interpretation and for time dependent calculations than Equation (23). Comparing

the light intensity for the progressive wave, Equation (25) and the light intensity for a stationary wave, Equation (34), one sees that they are similar. The argument of the Bessel function is modulated in time for the stationary wave. This modulation is expected since the stationary wave may be considered sinusoidal in space with a time varying amplitude.

EXPERIMENTAL STUDY

Apparatus and Procedure

An optical phase grating corresponding to that of an ultrasonic standing wave is produced with two adjacent progressive waves travelling in opposite directions. The optical axis intercepts both sound beams. The two progressive waves are obtained by eliminating reflections in a specially designed tank. This tank is a modification of one described by Hargrove, Zankel, and Hiedemann¹². The sound travelling in each direction is absorbed in a castor oil termination. Figure 1 shows the tank with the castor oil terminations separated from the water by a thin membrane. As the specific impedance of castor oil and water are very nearly the same, there are essentially no reflections at the interfaces.

A schematic diagram of the experimental apparatus is shown in Figure 2. The mercury light source illuminates the source slit SL . The collimated beam produced by lens L , is normal to both sound beams. The light intensity of the diffracted orders is measured by a photomultiplier.

Two air backed quartz transducers are driven by a 500 watt crystal controlled radio frequency oscillator.

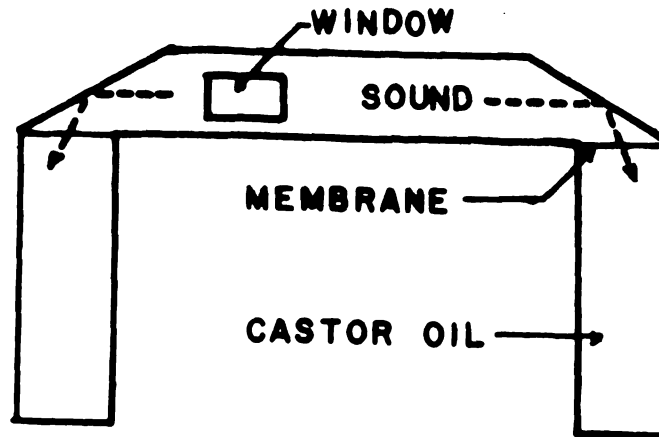


Fig. 1. Special Tank to Absorb Ultrasound.

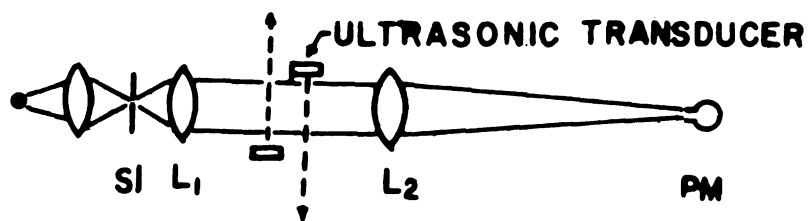


Fig. 2. Schematic Diagram of Experimental Apparatus.

The transducers are one inch square. The ultrasonic output of each transducer is controlled with a variable inductance used as a transformer to provide an impedance match between the transmitter and the transducer.

For time dependent measurements the output signal of the photomultiplier is amplified by two wide band amplifiers in cascade and displayed on an oscilloscope. The average light intensity is measured with a micro-photometer photomultiplier.

The measurements are made at 1.0 mc in water at room temperature.

Time Dependent Intensity Measurements

From Equation (34) one sees that for stationary waves, the light intensity of the diffracted orders varies periodically at the frequency of the sound. Figure 3 shows the oscilloscope displays of the time dependent intensity of the zero order and the first order. The approximate value of v as given in Equation (34) is 2.5 for the displays. Theoretical curves calculated for Equation (34) are shown in Figure 4. Comparison of Figures 3 and 4 shows the fair agreement of theory and experiment.

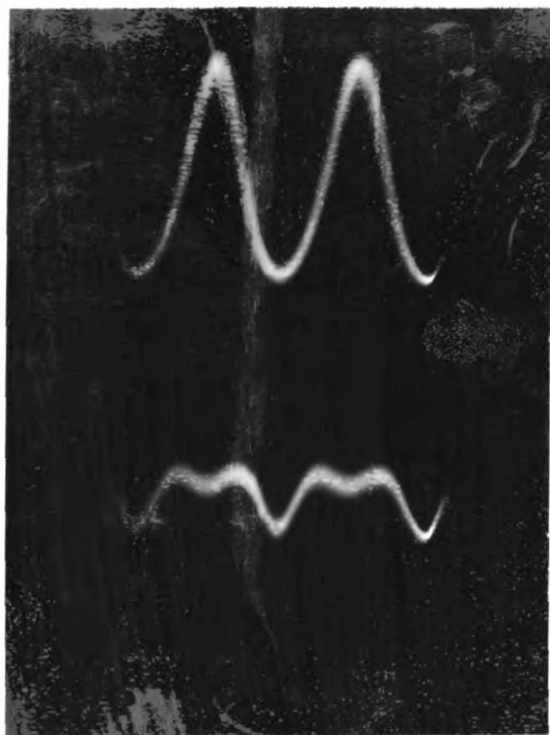


Figure 3 - Time dependent light intensity of zero and first orders of diffraction for stationary waves.

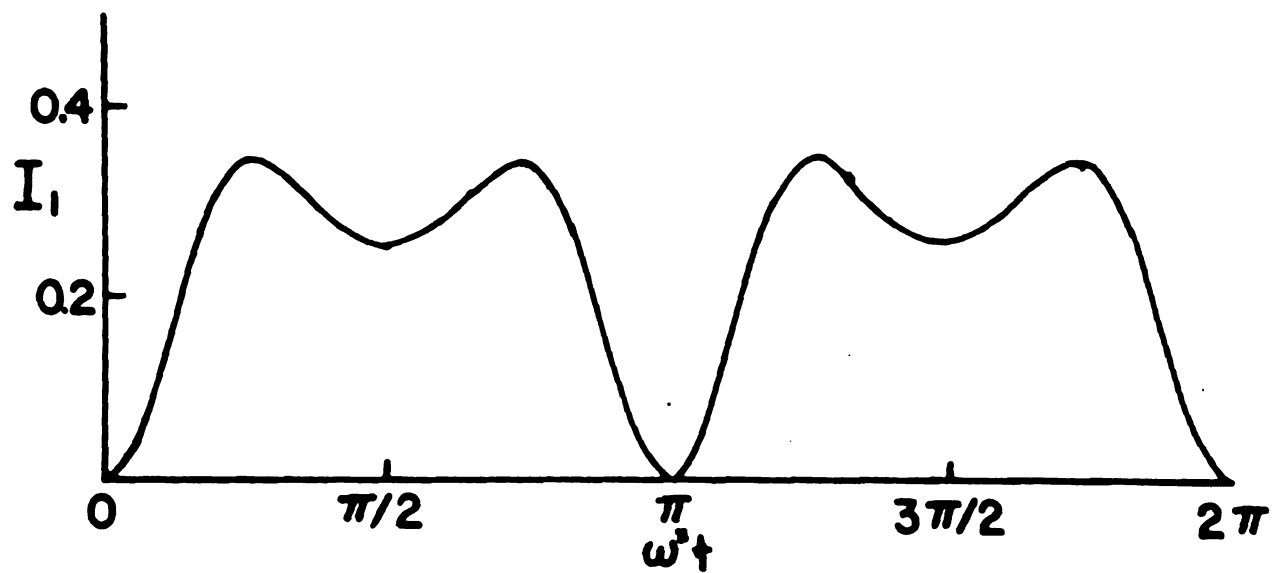
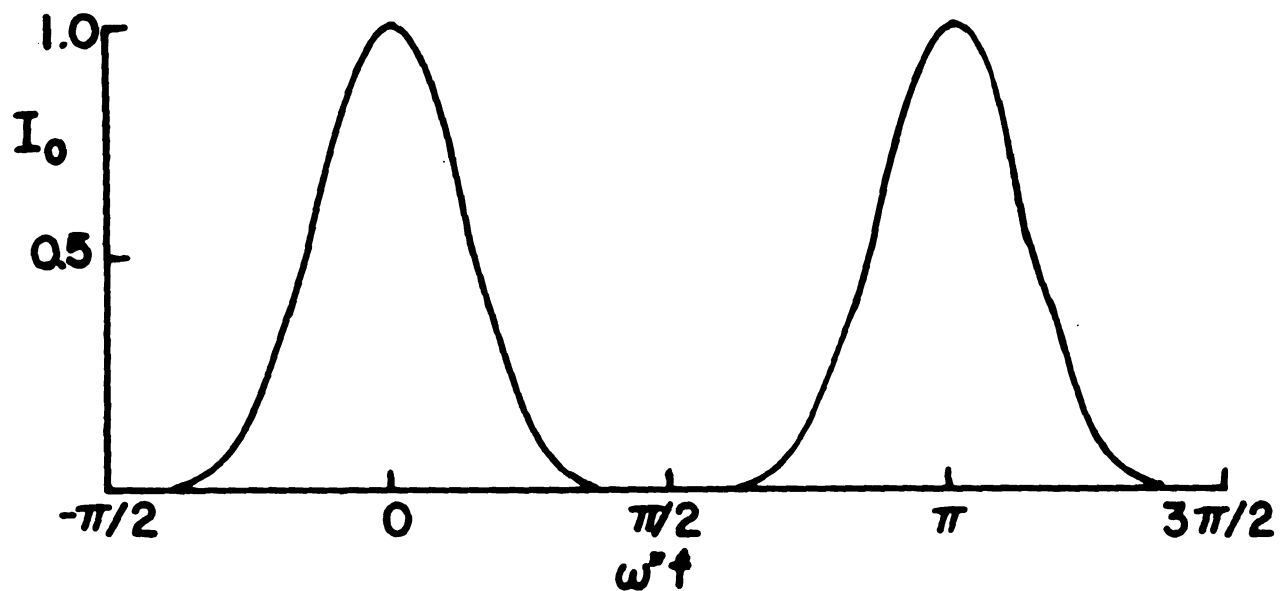


Fig. 4. Theoretical Curves for Time Dependent Light Intensities of Zero and First Orders. Stationary Waves of Amplitude $v=2.5$.

Average Light Intensity Measurements

The average light intensity of the zero diffraction orders are shown for standing wave ratios of one, two, and infinity in Figures 5, 6, and 7. For progressive waves, $SWR = \infty$, the theoretical curves predict zero light intensity in the orders for certain values of V . These zero values are not obtained experimentally. This discrepancy may be attributed to the Fresnel field intensity variation of the sound beam and to finite amplitude effects. Since the standing wave is a composition of two progressive waves, the deviations from the theory may be attributed to the same effects. Other sources of error are the non-linear response of the microphotometer and the mechanical vibration of the optical system. Small vibrations of the components of the optical system move the images of the diffraction orders about the entrance slit of the photomultiplier. Within experimental accuracy the intensity of the orders is symmetric about the zero order.

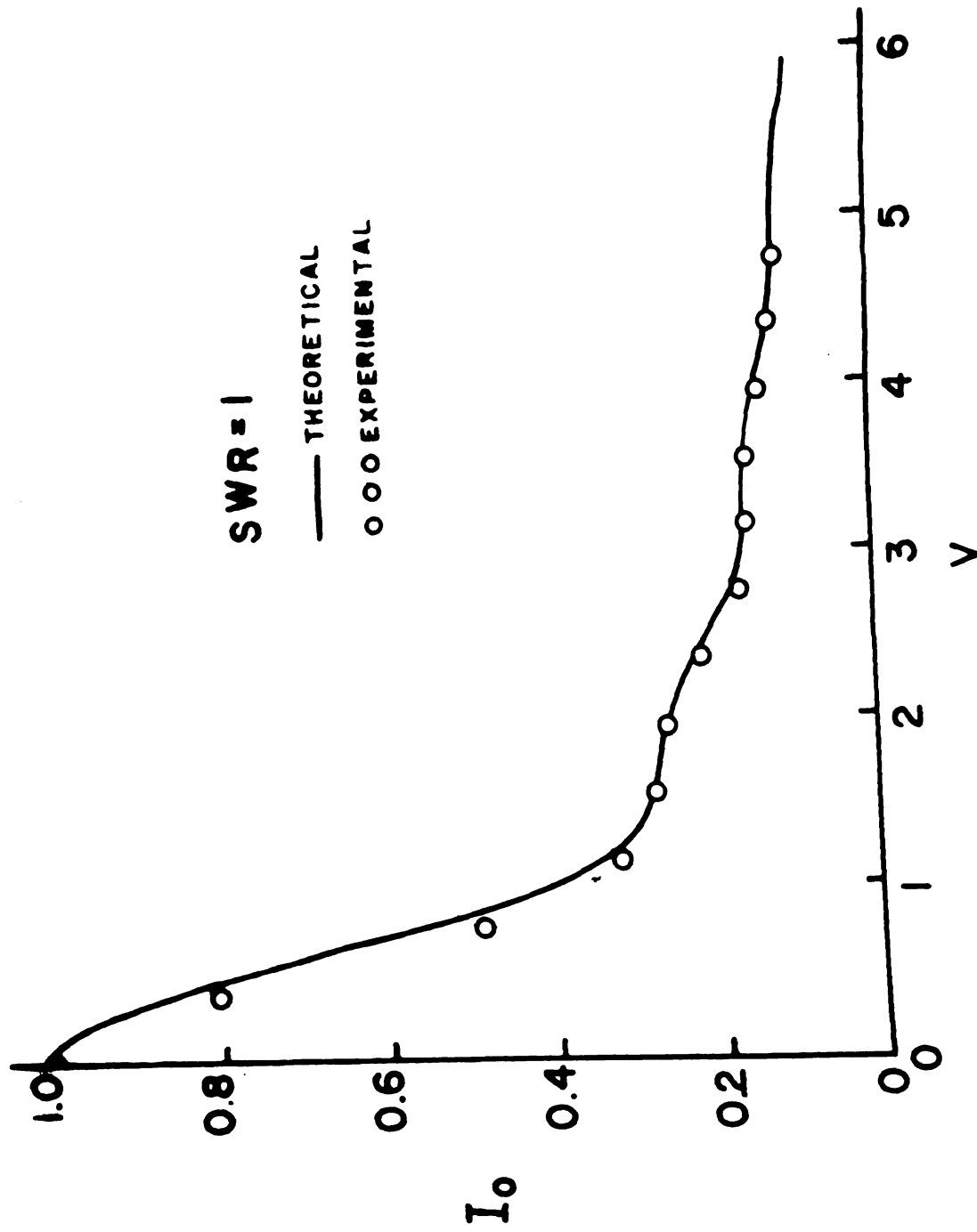


Fig. 5. Average Light Intensity of Zero Order for SWR equal one.

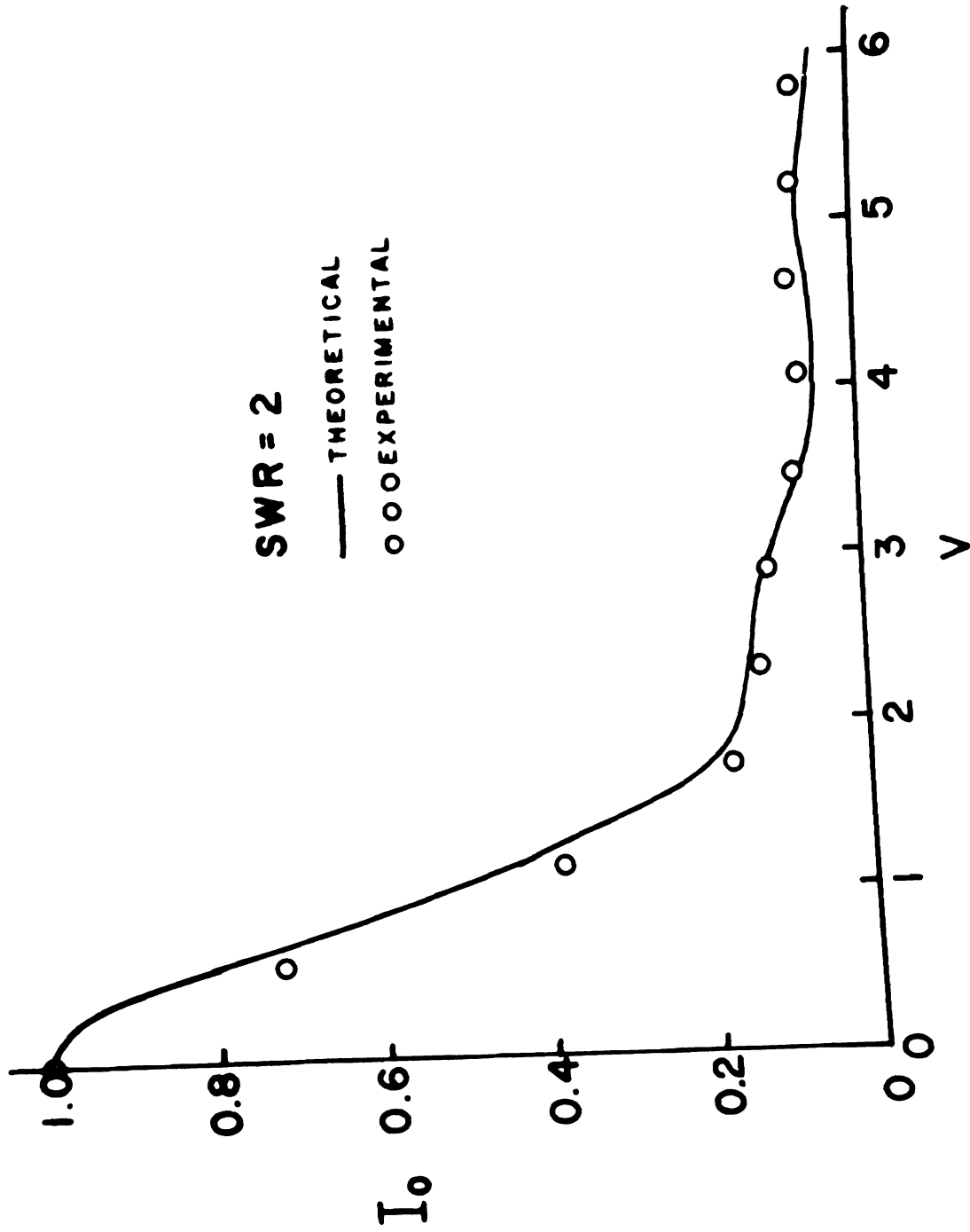


Fig. 6. Average Light Intensity of Zero Order for SWR equal two.

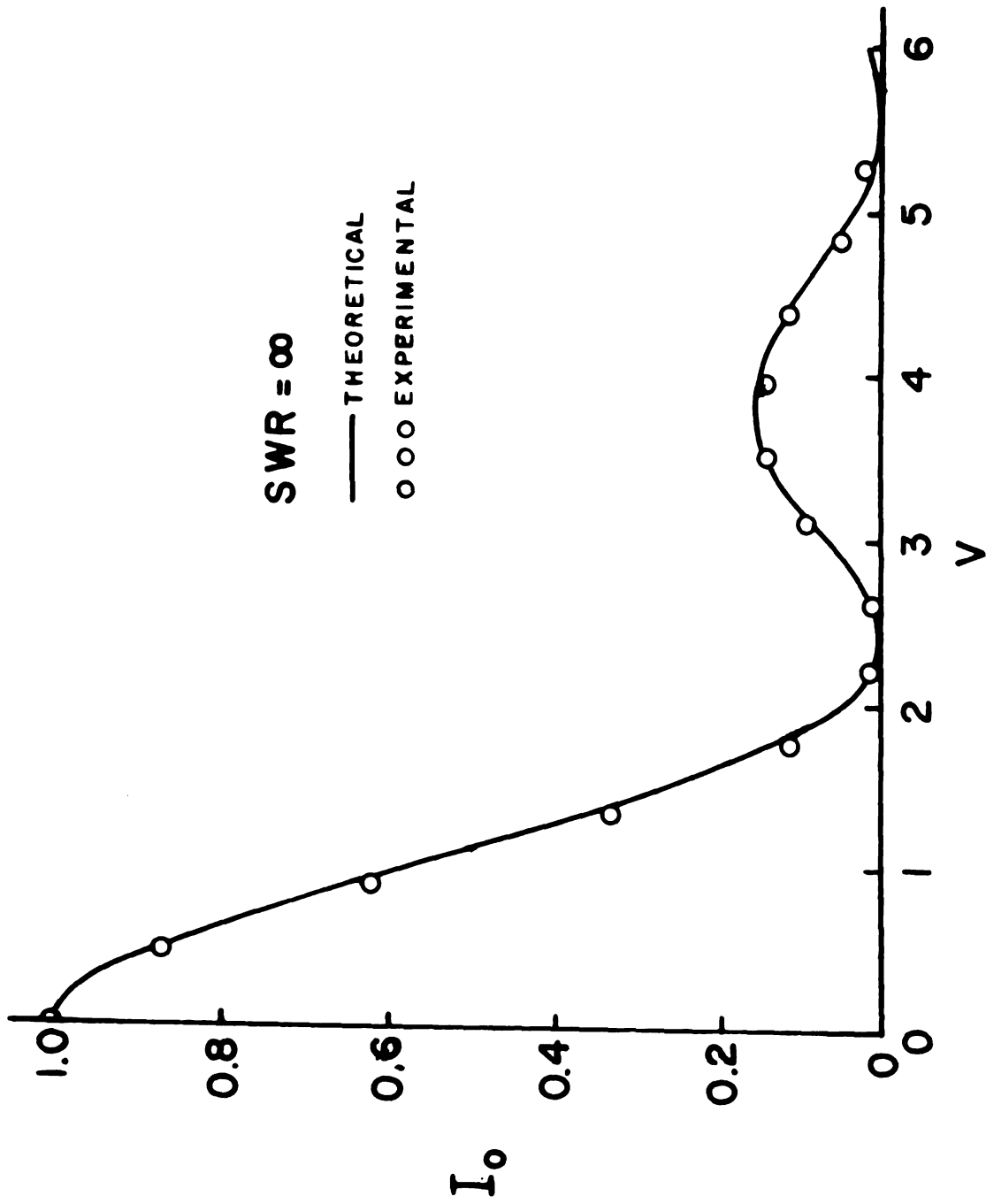


Fig. 7. Average Light Intensity of Zero Order for SWR Equal Infinity.

SUMMARY

Expressions for the diffraction of light by plane ultrasonic waves of various standing wave ratios are derived using the method of Raman and Nath. This method consists of evaluating a diffraction integral for an optical phase grating. The expressions of the light intensity of the diffraction spectrum reduce to those given by Raman and Nath for progressive and stationary waves.

The light intensity in the diffracted orders is found to be time dependent for standing waves with finite standing wave ratio (SWR). In application to diffraction type stroboscopes, the amount of light modulation in the diffracted orders can now be determined for various SWR.

It is theoretically shown that the magnitudes of non-coherent components resulting from Doppler shift depend on the SWR. Further, for finite SWR, the frequency shift is found to be independent of SWR.

Measurements of time dependent and average light intensities indicate that the theoretical results are valid.

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