STUDY OF SYNTHESIS OF R C NETWORKS

Thesis for the Degree of M. S.

MICHIGAN STATE UNIVERSITY

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Abstract

Synthesis of RC network means designing a network with only RC elements in such a way that it responds in the given manner to the applied driving force.

But a complete theory on RC network does not exist today and many engineering questions are still unsolved, such as, complete characterization of all the three network functions of a two terminal pair network, synthesis technique using minimum number of elements, and methods of keeping element values within practical limit.

The transfer function of a four terminal network is of principal interest in the synthesis process. From the transfer function it is possible to find out the driving point admittance (or impedance) and transfer admittance (or impedance). The driving point admittance or impedance may be realized physically in such a way that the pair of terminals placed at the far end also realizes the transfer admittance (or impedance).

From the transfer function it is possible to design parallel ladder structure, symmetrical lattice, ladder or grounded two terminal pair networks.

The design procedure of a two terminal RC network is quite similar to that of the LC network as were suggested by R.M. Foster and W. Cauer. So the same methods may be followed to design an RC network from the prescribed driving point function.

STUDY

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SYNTHESIS OF RC NETWORKS.

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MAHBUBAR RAHMAN.

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Introduction:

In network analysis either the network and the applied impulse are given and we seek the response, or the network and the response are given and we seek the impulse.

But in network synthesis the response to a certain driving force is given and the problem consists in designing a network having such properties that it responds in the given manner to the applied driving force.

Network synthesis is steadily assuming increased scientific interest and technical importance especially in view of increasing needs in the electrical communication field. The first contribution to the systematic synthesis of networks wasmade by G. A. Campbell (11) in his paper on the "Physical theory of the Electric wave Filter" in the year 1922. After one year O. J. Zobel (73) published another paper on electric wave filter, which was put into the proper perspective by R. M. Foster (24) in his paper "A Reactance theorem" which, we can truly say touched off a modern network synthesis. Then Cauer, Darlington (14-15) Brune (9), Weinberg, Gewertz (25) etc, ~~have~~ described useful ways of synthesizing LC and RLC networks.

But even a few years back, RC networks were playing a very minor part in the design problem as compared to LC and RLC networks. At present RC networks are also becoming of increasing importance as the circuit techniques are extended to very low frequencies (such as in servo mechanism) where inductances of very high quality are difficult to manufacture.

As a complete theory of the RC networks under consideration does not exist today, we shall restrict ourselves to the network function which is usually of principal interest--the transfer function, defined as the ratio of the steady state output voltage to input voltage in the domain of complex frequency variable, P.

Exclusion of inductances from the network imposes some restrictions upon the circuit transfer function. Assuming that a suitably restricted transfer function has been designed to meet some given requirements, the problem remains of finding an RC network which has this transfer function when operated between a specified generator and load.

According to Fialkow (18), the transfer function of an RC network may be realized physically in parallel ladder, symmetrical lattice, ladder or grounded two terminal pair networks.

Different methods of such networks realization were suggested by Fialkow and Grest (18-23), Guillemin (28-35), Orchard (48-49), Weinberg (65-69), etc.

Here we shall discuss in detail the methods suggested by Guillemin to realize the transfer function in parallel ladder (32) and ladder (34) networks and the method of Orchard for the development of symmetrical lattice (48).

From the specified transfer function Guillemin (32) derived a network having the form of a set of separate ladder structures connected in parallel. In order to realize the network he was compelled to place two more restrictions on the problem. The first one was that the network is of minimum phase function (i.e. the zeros of the transfer function should be on the left half of the P = iw plane) and the second one was that the generator should be of zero internal impedance.

But in many filters and equalizers the most important quantity is the attenuation of the circuit and hence the first restriction is of no consequence. The second restriction, that the network should be driven by a generator of zero internal resistance may not be very easy to attain and so it needs some degree of approximation.

In the development of R.C. network Orchard overcame the above two restrictions imposed by Guillemin. Though the first restriction is not very important, the ability to depart from this restriction makes the design process more general. But to overcome these restrictions, he had to derive a lattice network which has the disadvantage that it is balanced with respect to earth. But the zeros of this network may be anywhere of the P plane and the generator and the load have equal resistive impedence.

In his second method, Guillemin (34) developed the RC network into a single ladder structure, from the given transfer function by simply shifting the zeros of driving point function which we shall discuss later in full.

In recent available literatures on RC networks, many of the authors (5, 13, 37, 38, 47, 57, 61, 64 and 70) discussed the design problem of parallel T resistance capacitance network, so we shall also discuss this problem at the end.

In every synthesis and design problem, the number of elements necessary in the network is very important because that decides the cost of the network. Fialkow has given a table (18) of the number of elements necessary for different type of networks to realize the transfer function:

$$A(P)=K\frac{P+.5}{(P+1)(P+3)(P+5)}$$

which is given below:

Table I

Network Structure	Max K	Actual K	No. of Elements
a. Parallel ladders:			
1. Guillemin	-	3.93	15
2. Fialkow - Crest	10	-	-
b. Symmetric lattice	0.86	0.86	17
c. Ladder	15	14	11
d. Grounded two terminal pair	28.4	22	16

Many engineering questions in R.C. network synthesis are still unsolved. The most important of them are the complete characterization of all the three network functions of a two terminal pair network, synthesis techniques using minimum number of elements, methods of keeping the element values within practical limits, etc.

In the following, we will first discuss the synthesis of two terminal networks and then extend it to four terminal networks. After deriving each method of network synthesis we shall solve one problem to illustrate the discussion.

Synthesis of two terminal networks:

If a finite number of electric circuit elements be connected in series, parallel or series-parallel forms so that two terminals are available for external connection, the impedance of the combined elements between the two terminals is known as the driving point impedance of the system.

According to R.M. Foster (24), it is possible to determine inductance-capacitance networks, if the driving point impedance of the network is given. But W. Cauer (30), showed that the same general method may also be applied for dissipative impedance functions when these correspond to networks containing only two kinds of elements (resistance-capacitance or resistance inductance).

So, we will use the same general method to find the two terminal resistance-capacitance network, as was used by Foster to realize L-C networks.

Let us consider the mesh and mutual impedences of an arbitrary dissipative network which may be written in the most general form by (29)

$$bik = Rik + Sik\,(jw)^{-1} \text{ ------------------(1)}$$

If we denote $j\omega = P$, it becomes more convenient to handle.

$$\therefore bik = Rik + Sik\,P^{-1} \text{ ---------------------(2)}$$

If the network is driven from the first mesh then the driving point impedance may be represented by

$Z11 = \frac{D}{[illegible]}$ ----- --------- -----

where D is the determinant of the network and D11 is the minor of its first row and first column.

It is clear that both D and D11 are polynomials in P but due to the fact that there is negative power in the element of eqn (1), these polynomials will also have negative powers. So to make it more convenient for manipulation, we eliminate the negative power by letting the impedance function,

$$b^{*}_{ik} = Pb_{ik} = R_{ik}P + S_{ik} \text{ -------}(4)$$

Then denoting the determinats of the elements bik* by D* and its monor by D11*, then we have,

$$D^{*} = P^{n}D \text{ ----- }(5)$$

$$\text{and } D_{11}^{*} = P^{n-1} D_{11} \text{ -----}(6)$$

assuming that there are n number of rows and columns (actually n independent meshes) in the determinant.

So, our driving point impedance function becomes,

$$Z_{11} = \frac{D}{D_{11}} = \frac{D^{*}}{P^{n}} \times \frac{P^{n-1}}{D_{11}^{*}}$$

$$\text{or, } Z_{11} = \frac{D^{*}}{PD_{11}^{*}} \text{ ----------}(7)$$

Here D* and D..* involving only positive powers of P and D* involves power ranging from n to zero inclusive and D11* involves power n-1 to zero inclusive.

Hence the most general driving point impedance function representing a network in which every mesh contains independent resistance and capacitance has the form,

$$Z11 = \frac{D^*}{PD11^*} = \frac{AnP^n + An\text{-}1P^{n-1} + \text{------------------} A1P + Ao}{P\ (Bn\text{-}1P^{n-1} + Bn\text{-}2P^{n-2} + \text{--------------} + B1P + Bo}$$
$$\text{----------(9)}$$

$$= \frac{AnP^n + An\text{-}1P^{n-1} + \text{----------} + A1P + Ao}{Bn\text{-}1P^n + Bn\text{-}2P^{n-1} + \text{----------} + B1P^2 + BoP}$$

These polynomials may be factored in terms of their roots. If P is considered to be the variable then the roots of these polynomials are real and negative (33).

Let us denote the roots of the numerator and denominator respectively by,

$$P_1,\ P_3 \text{---------} P_{2n-1}$$
and $$P_0,\ P_2, \text{-------}\ P_{2n-2} \qquad \text{---------(10)}$$

where Po denotes the zero root of the denominator.

The physical realizability of the network demands that the poles and zeros of a network should be alternate (30). The alternation of poles and zeros may be expressed by,

$$-\infty \leq P_{2n-1} < P_{2n-2} < \cdots \cdots < P_2 < P_1 < P_0 = 0 \qquad \cdots \quad \cdots \quad (11)$$

Which is known as the seperation property. This property indicates that the prescribed zeros and poles of the driving point function must separate each other, otherwise no corresponding network exists.

The factored form of the driving point impedance function may now be written as,

$$Z_{11}(P) = \frac{H\ (P-P_1)\ (P-P_3)\ \text{----------}(P-P2n-1)}{P\ (P-P_2)\ (P-P_4)\text{----------}(P-P2n-2)} \text{----(12)}$$

where $H = \frac{A_n}{B_{n-1}}$ ---------------(13)

Now, if we put jw in place of P then equ (12) becomes,

$$Z_{11}(jw) = H \frac{(W-W_1)(W-W_3)-------------(W-W_{2n-1})}{W(W-W_2)(W-W_4)------------(W-W_{2n-2})} -------(14)$$

The equs (9) and equ. (14) of the driving point impedance function may be identified with a physical network. But a partial fraction expansion of this function will represent the desired network by a series combination of simpler ones.

So, the partial fraction expansion of equ. (14) may be given by,

$$Z_{11}(jw) = H\left\{1 + \frac{A_0}{W} + \frac{A_2}{W-W_2} + -----------+ \frac{A_{2n-2}}{W-W_{2n-2}}\right\}$$
-----------------(15)

For the determination of the coefficients A_K's a frequency is taken very near to W_K (W_2,-W_4,...W_{2n-2}). For this frequency the particular term containing A_k becomes very high. So, we can say that when W approaches W_K, all other terms are negligible in comparison to the term containing the coefficient A_k. At the same time, the equ. (14) becomes extremely large on account of the factor $W-W_K$ in its denominator, which approaches zero.

So, the equ. (15) may be written when W approaches W_K by,

$$Z_{11}(jw)_{w \to w_k} = H \frac{A_k}{W-W_k} \text{------------(16)}$$

Which is also equal to the equ. (14) so we equate the equations (14) and equ. (16)

$$H \frac{A_K}{W-W_k} = H \frac{(W-W_1)(W-W_3) \text{----------} (W-W_{2n-1})}{W(W-W_2)(W-W_4) \text{----------} (W-W_{2n-2})} \text{-----(17)}$$

$$A_k = \frac{(W-W_1)(W-W_3) \text{--------------} (W-W_{2n-1})}{W(W-W_2) \text{---} (W-W_{k-1})(W-W_{k+1}) \text{--} (W-W_{2n-2})} \text{------(18)}$$

Now, to realize the network from the partial fraction expansion, let us consider the diagram of a parallel resistance capacitance circuit given in fig. (1)

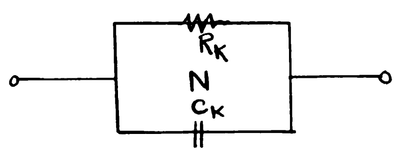


Figure 1

Its driving point impedance is given by,

$$Z_{11} = \frac{1}{\frac{1}{R_k} + jwc_k} \text{------------(19)}$$

$$= \frac{R_K}{1 + jwR_kC_k} \text{---------(20)}$$

But at the resonant frequency,

$$R_k = -\frac{1}{jW_kC_k} \text{-------(21)}$$

or, $P_k = jw_k = \frac{-1}{R_kC_k}$ ----(22)

So, equ. (20) may be written as,

$$Z_{11} = \frac{R_K}{1 + \frac{jw}{jw_k}} = \frac{R_k W_k}{W_k - W} \text{-------}(23)$$

Then again putting the value of W_k in the numerator of equ. (23)

$$Z_{11} = \frac{R_k \cdot \frac{1}{-R_k C_k}}{W_k - W}$$

$$= \frac{\frac{1}{C_k}}{W - W_k} \qquad \text{-------}(24)$$

Equ. (24) is comparable with individual terms of equ. (15) excepting the first two terms. So, terms of equ. (15) may be represented as a simple parallel resistance-capacitance network and when all of these two element networks are connected in series, we have the driving point impedance as given in equ. (16), excluding the first two terms.

We can find out the values of resistances and capacitances by the following way,

$$\frac{1}{C_k} = Z_{11} (W - W_k) \text{----}(25)$$

$$C_k = \frac{Y_{11}}{W - W_k} \text{-------}(26)$$

Where $Y_{11} = Z11^{-1}$ -----(27)

Corresponding resistance may be found out from equ. (22)

$$R_k = \frac{1}{-P_k C_k} \text{--------}(28)$$

Among the first two terms of the equ. (15) the first one is a constant and may be represented by a resistance. The second term $\frac{A_0}{\pi}$ may be represented by a capacitance

$$R_{2n} = H = \frac{A_n}{B_{n-1}} \quad \text{-------(29)}$$

$$\text{and } C_c = -A_0 \quad \text{-----------(30)}$$

Hence the physically realized network for the above driving point impedance function may be given by fig (2).

Now, we will solve a numerical problem by the above method.

Let the poles and zeros of the driving point function are specified at

$$P = 0, -4, 8 \quad \text{------(31)}$$
$$\text{and } P = -2, -6$$

So, we can write the driving point impedance as,

$$Z_{11} = (P) = \frac{H\,(P+2)\,(P+6)}{P\,(P+4)\,(P+8)} \quad \text{-----(32)}$$

$$= H\,\frac{P^2+8P+12}{P^3+12P^2+32P} \quad \ldots\ldots \quad (33)$$

Now if we put p=jw, the equation (35) becomes,

$$Zjw = H\,\frac{(-w^2+12)+8jw}{-12w2-j(w3-32w} \quad \text{-------(34)}$$

by rationalizing,

$$Z11\,(jw) = H\,\frac{4w4+112w2-j(w5-52w3-384s)}{w6+80w4+1024w2} \quad \text{-----(35)}$$

Let us assume the real part of z11 = 1, when

$w \rightarrow 0$

$$z11(jw) = H \frac{112}{1024} = 1 \qquad \text{-----}(36)$$

$$H = \frac{1024}{112} = 9.15 \qquad \text{-----}(37)$$

$$\text{so, } y11\ (jw) = .11 \ \frac{P(P+4)\ (P+8)}{(P+2)\ (P+6)} \qquad \text{----}(38)$$

From equ. (23)

$$Co = \frac{Y11}{P}\Big|_{P=0} = .11 \ \frac{4X8}{2X6} = .293$$

$$C4 = \frac{Y11}{P+4}\Big|_{P=-4} = .11 \frac{-4X4}{-2X2} = .44$$

$$C6 = \frac{Y11}{P+8}\Big|_{P=-8} = .11 \ \frac{-8X-4}{-6X-2} = .293$$

The resistances associated with these capacitances are calculated by equ. (28)

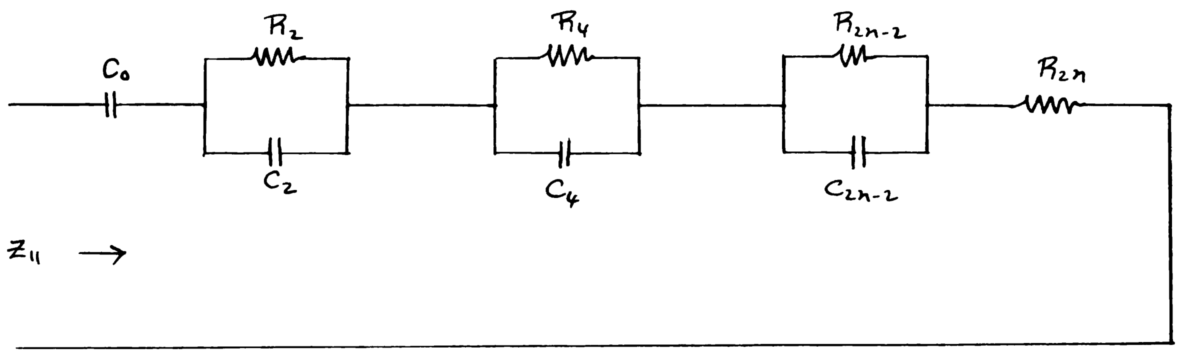
$$R2 = \frac{1}{.44X4} = \frac{1}{1.76} = .57$$

$$R4 = \frac{1}{.293X8} = \frac{1}{2.344} = .43$$

Here R2n=o because Z11 (P) vanishes when P= ∞

The network is shown in figure 3

The synthesis of the resistance-capacitance network is also possible by treating the admittance function in the



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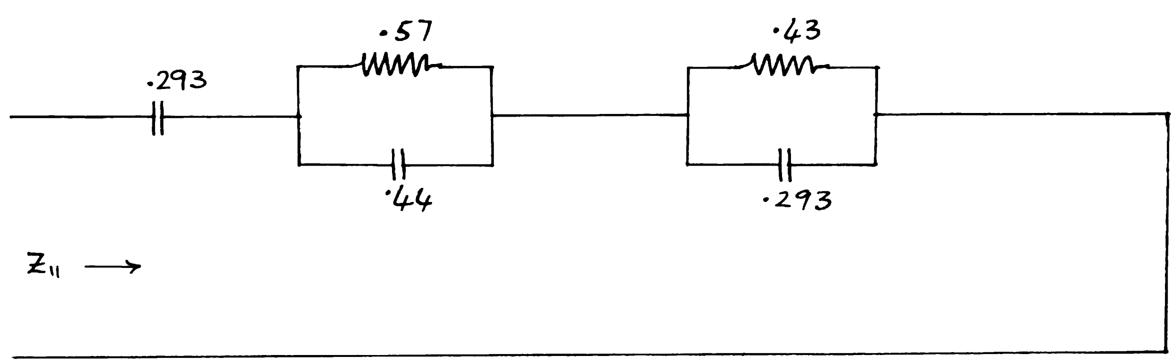

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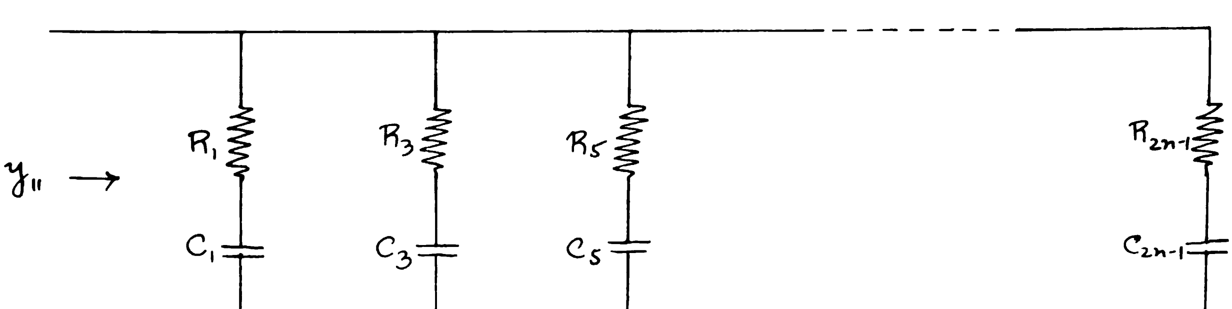


FiG - 4

same manner as we have already done by expanding the driving point impedance function into partial fraction and comparing the terms with simple component networks.

Here $Y11 = \frac{1}{Z11}$

$$= \frac{B_{n-1}P^{n} + B_{n-2}P^{n-1} + \text{--------} + B_0 P}{A_n P^{n} + A_{n-1}P^{n-1} + \text{----------} + A_0} \quad \text{----}(39)$$

$$= \frac{P\,(B_{n-1}P^{n-1} + B_{n-2}P^{n-2} + \text{-------} + B_0)}{A_n P^{n} + A_{n-1}P^{n-1} + \text{----------------} A_0} \quad \text{----}(40)$$

This may be expanded into partial fraction expansion.

$$Y11 = P\left\{\frac{A1}{P-P1} + \frac{A3}{P-P3} + \text{-------} + \frac{A2n-1}{P-P2n-1}\right\} \quad \ldots (41)$$

When P approaches Pk (by the same argument given to derive equ. (16)), the admittance function Y11 may be written as,

$$Y11 = P\,\frac{AK}{P-Pk} \quad \text{----}(42)$$

$$AK = \frac{Y11\,(P-Pk)}{P} = \frac{P-Pk}{P\;Z11} \quad \text{-------}(43)$$

Now let us consider a series RC circuit as in Fig. 5

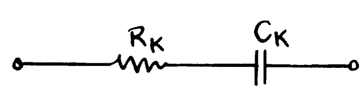


Figure 5

It's driving point admittance may be written as,

$$Y11 = Z11^{-1} = \frac{1}{Rk + \frac{1}{jwCk}} \text{-------}(44)$$

$$= \frac{jwCk}{1 + jwRkCk} \text{-------------}(45)$$

By equ. (22)

$$Y = \frac{jwck}{1 + jw/-jwk} = \frac{PCk \times Pk}{Pk-P} \text{-----}(46)$$

$$Y11 = \frac{PCk \times 1/-RkCk}{Pk-P} = \frac{P/Rk}{P-Pk} \text{----}(47)$$

Equ. (47) is comparable with individual terms of equ. (41). So a parallel combination of these networks will give equ. (41). The resistances and capacitances may be found by equ. (47), where,

$$Rk = \frac{P\ Z11}{P-Pk} \text{---------}(48)$$

Now by comparing with equ. (43), we have,

$$\frac{1}{RK} = Ak \text{------}(49)$$

and $Ck = -\dfrac{1}{PkRk}$

Hence, the network may be realized by the form shown in figure 4.

A numerical example is given below:

From the last example,

$$Y11\ (P) = \frac{.11P\ (P+4)\ (P+8)}{(P+2)\ (P+6)} \qquad \text{---------(50)}$$

$$= \frac{P(.11P^2 + 1.32P + 3.52)}{(P+)\ (P+6)} \qquad \text{-------(51)}$$

Expanding equ. 51) into partial fraction expansion,

$$Y11\ (P) = \frac{A2P}{P+2} + \frac{A6P}{P+6} + \underline{A10P} \qquad \text{-----(52)}$$

From equ. (47),

$$A2 = \frac{(P+2)}{P} \times \frac{.11P\ (P+4)\ (P+8)}{(P+2)\ \ (P+6)} \Bigg|\ P = -2$$

$$= \frac{.11 \times 2 \times 6}{4} = .33 \qquad \text{-------(53)}$$

$$A6 = \frac{(P+6)}{P} \times \frac{.11P\ (P+4)\ (P+8)}{(P+2)\ \ (P+6)} \Bigg| \qquad P = -6$$

$$= \frac{.11 \times -2 \times 2}{-4} = .11 \qquad \text{-----(54)}$$

By inspection it can be said that,

$$A10 = .11 \qquad \text{------(55)}$$

From equ. (49)

$$R2 = \frac{1}{A2} = \frac{1}{.44} = 2.28 \text{ ohms.} \qquad \text{-----(56)}$$

$$R6 = \frac{1}{A6} = \frac{1}{.11} = 9.1 \text{ ohms.} \qquad \text{------(57)}$$

and from equ. (56)

$$C2 = \frac{-1}{P2R2} = \frac{1}{2 \times 2.8} = .18 \text{ ------(58)}$$

$$C6 = -\frac{1}{P6R6} = \frac{1}{6 \times 9.1} = .0185 \text{ -----(59)}$$

and $C10 = A10 = .11$ --------(60)

The network is shown in figure (6)

Cauer's extension to Foster's reactance theorem (30) may also be applied to the synthesis of two terminal R.C. networks. So, we may get two cannonic forms of physically realizable network from the given driving point impedence function.

If we consider a ladder structure as shown in figure (7), it can be proved easily that the driving point impedence Z11 of this network may be expressed by alternate series and parallel combinations of the components of the network, starting with the right hand end and working back toward the terminals which is actually continued fraction.

So, if we manipulate the equ. (9) into continued fraction, we can realize a ladder type of network.

Starting with equ. (9),

$$Z11 = \frac{A_nP^n + A_{n-1}P^{n-1} + \text{----------} + A_1P + A_0}{B_{n-1}P^n + B_{n-2}P^{n-1} + \text{---------} + B_1P^2 + B_0P} = \frac{P}{Q}$$

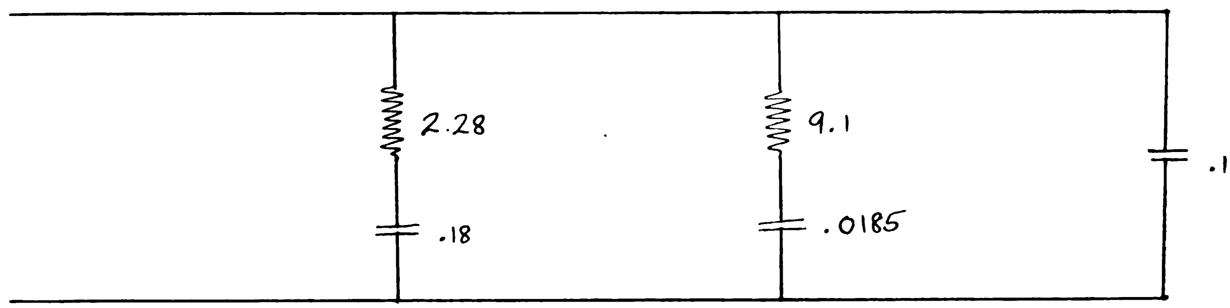


FIG-6

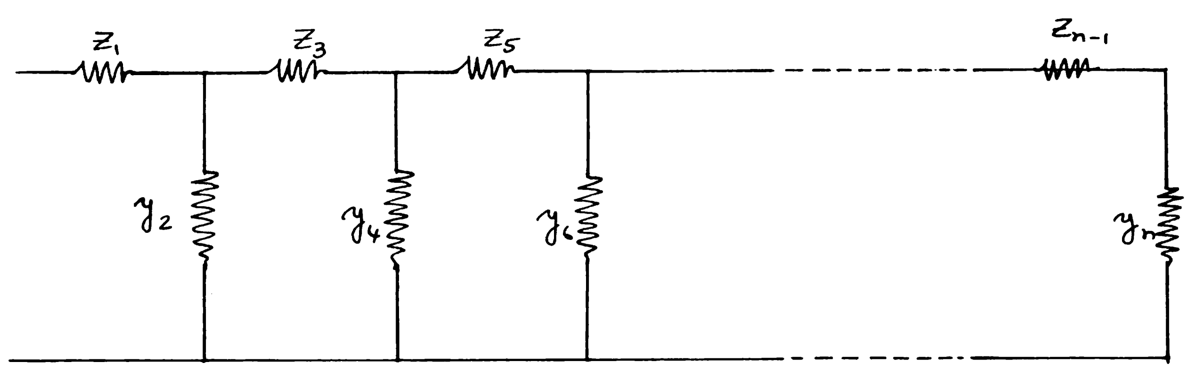


FIG-7

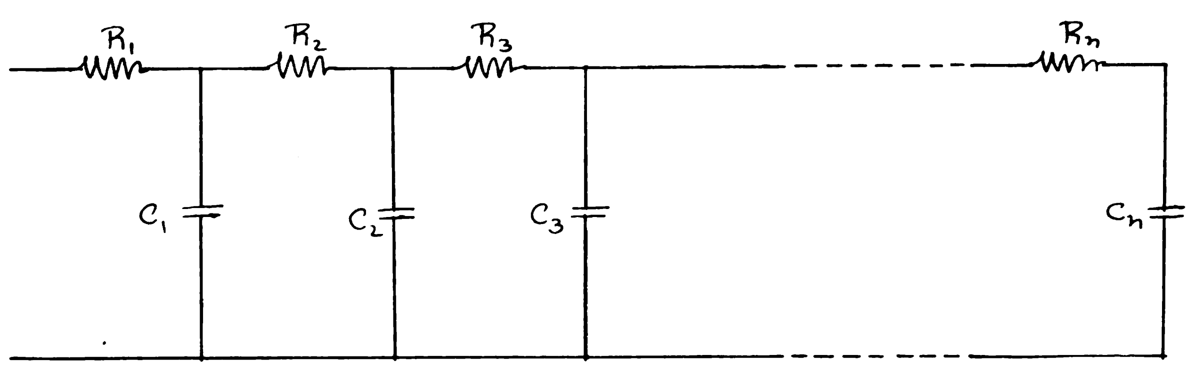


FIG-8

$$= R_1 + \frac{P_1}{Q} = R_1 + \cfrac{1}{\cfrac{Q}{P_1}} \quad \text{-------------(61a)}$$

where, $R_1 = \frac{A_n}{B_{n-1}}$ and $\frac{P_1}{Q}$ is the remainder P_1 is the Polynomial of $(n-1)$ degree. Now, dividing Q by P_1

$$Z_{11} = R_1 + \cfrac{1}{C_1 P + \cfrac{Q_1}{P_1}} = R_1 + \cfrac{1}{C_1 P + \cfrac{1}{\cfrac{P_1}{Q_1}}} \quad \text{---------(61b)}$$

where $C_1 = \frac{B_{n-1}}{A_{n-1}}$ and $\frac{Q_1}{P_1}$ is the remainder and Q_1 is the polynomial of $(n-1)$th degree.

So, it is clear that if we go on in this way, the process will terminate after 2n terms. The continued fraction therefore, reads,

$$Z_{11} = R_1 + \cfrac{1}{C_1 P + \cfrac{1}{R_2 + \cfrac{1}{C_2 P + \cdots + \cfrac{1}{R_n + \cfrac{1}{C_n P}}}}} \quad \text{--------------(62)}$$

Now, we can draw a ladder network the resistances of which are represented by R's and the capacitances of which are represented by C's in the equ. (62). The network may be realized in the form given in figure (3)

A numerical example is given below:

Let $$Z11 = \frac{(P+1)\ (P+3)\ (P+5)}{P\ (P+2)\ (P+4)} \text{ ---------(63)}$$

$$= \frac{P^3+9P^2+23P+15}{P^3+6P^2+8P} \text{ ------------- (64)}$$

The continued fraction is found out as,

$$Z11 = 1 + \cfrac{1}{\frac{1}{3}P + \cfrac{1}{3 + \cfrac{1}{\frac{1}{6}P + \cfrac{1}{12 + \cfrac{1}{\frac{1}{30}P}}}}} \text{ -------(65)}$$

So the network will be of the form shown in figure (9)

Another form of cannonic structure may be realized if we turn end for end both the numerator and denominator of equ. (9) and if the same process of division and invrsion is carried out, a continued fraction will result in which alternate terms involve P-1. Here, the impedence function is,

$$Z11 = \frac{Ao + A1P + \text{------------} + AnP^n}{BoP + B1P^2 + \text{-----------} + BnP^n} = \frac{x}{y} \text{ ----(66)}$$

$$= C1^{-1}\ P^{-1} + \frac{X1}{Y} = C1^{-1}\ P^{-1} + \cfrac{1}{\frac{Y}{X1}} \text{ ------(67)}$$

where $C1 = \frac{Ao}{Bo}$ and $\frac{X1}{Y}$ is the remainder.

The same process of division and inversion is carried out and we get,

$$Z11 = C1P1^{-1} + \cfrac{1}{R1^{-1} + \frac{Y1}{X1}}$$

where $R = \frac{B1}{A1}$

and $$\frac{Y1}{X1} = \frac{B2^{1}P^{2} + \text{------------} + Bn^{1}P^{n}}{A1^{1}P + \text{--------------} + AnP^{n}} \qquad \text{------}(63)$$

In this way working for 2n cycles, we have,

$$Z11 = C1^{-1}P^{-1} + \cfrac{1}{R1^{-1} + \cfrac{1}{C2^{-1}P^{-1} + \cfrac{1}{R2^{-1} + \cfrac{1}{Cn^{-1}P^{-1} + \cfrac{1}{Rn^{-1}}}}}}$$

The network for equ. (69) is given in figure (10)

A numerical example for the above procedure is shown below:

From the last example,

$$Z11 = \frac{15 + 23\,P + 9P2 + P3}{8P + 6P2 + P3} \text{-------------}(70)$$

$$= (.53)^{-1}\,P^{-1} + \frac{11.75P + 7.125P^{2} + P^{3}}{8P + 6P^{2} + P3}$$

Continuing the same process, we have at last

$$Z11 = (.53)^{-1} + \cfrac{1}{(.68)^{-1} + \cfrac{1}{(.98)^{-1}P^{-1} + \cfrac{1}{(3.5)^{-1} + \cfrac{1}{(.0051)^{-1}P^{-1} + \cfrac{1}{(50)}}}}}$$

The network for this function is shown in fig. 11.

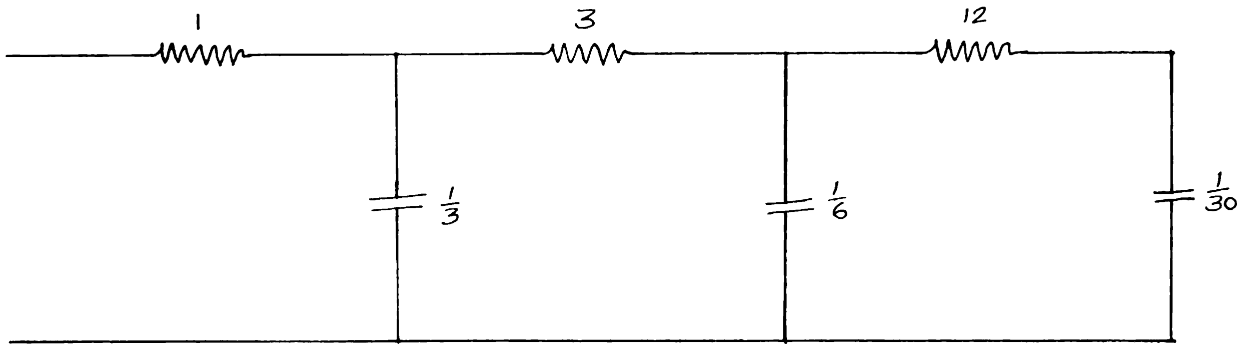


FiG-9

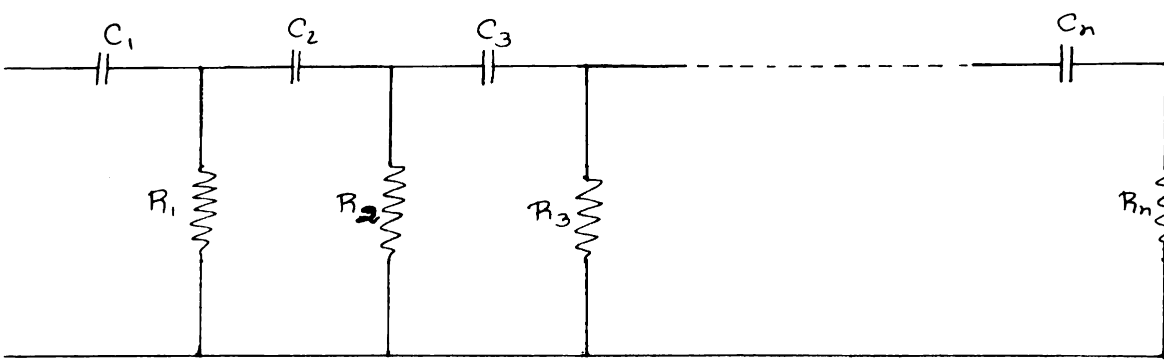


FiG-10

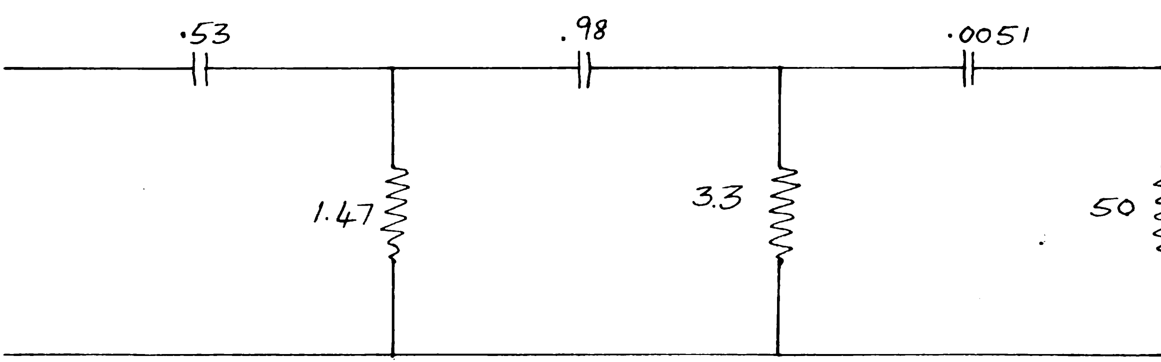


FiG-11

Synthesis of Four Terminal Networks:

A four terminal network may be described as a group of impedance elements (or element) having one pair of terminals for connection to a source of power or any other supply network (which is known as the input terminals) and a second pair of terminals for connection to a load or any other networks to which current is delivered. This pair of terminals is known as the output terminals.

These four terminal networks (which contain only resistances and capacitances in this particular case) may be physically realizable from the specified transfer function if this transfer function obeys certain restrictions (25)

There are a few different procedures to develop the four terminal R.C. networks from the given transfer function and three of them are given below:

1. Parallel Combination of Ladder Structures:

Let us consider the four terminal network shown in figure (12) and investigate the characters of its short circuit driving point and transfer admittance.

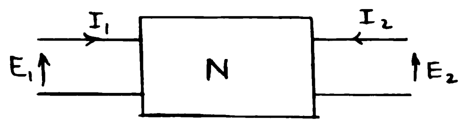


Figure 12

The relations between the terminal voltages and current of a four terminal network may be given by,

$$I_1 = Y_{11}E_1 + Y_{12}E_2$$
$$I_2 = Y_{12}E_1 + Y_{22}E_2 \qquad \text{------------}(71)$$

in which Y_{11} and Y_{22} are the short circuit driving point and $Y_{12} = Y_{21}$ (the network is assumed to be passive) is the short circuit transfer admittances.

If the network N contains only resistances and capacitances, then Y_{11} and Y_{22} are rational functions of complex frequency variable with simple poles and zeros lying alternately on the negative real axis of P plane and the smallest critical frequency is zero (30).

To find out the character of Y_{12} we reduce the four terminal network into a two terminal network by using two transformers as shown in fig.(13)

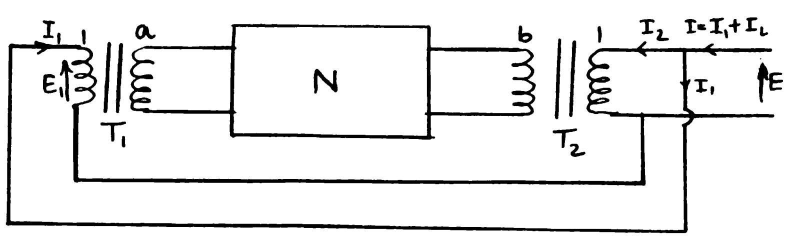


Fig.(13)

If the transformers are ideal then by equ. (72)

$I1 = a^2 Y11E1 + abY12E2$ (72a)

$I2 = abY12E1 + b^2 Y22E2$ (72b)

where a and b are turn ratios of transformers from fig (13)

$I = I1 + I2$ ---------(72c)

$E = E1 = E2$ --------(72d)

So by adding equ. (72a) and (72b), we have,

$I = a^2 Y11E + 2ab\ Y12\ E + b^2 Y22E$ -----(72e)

or we can write

$Y = \frac{I}{E} = a^2 y11 + 2ab\ y12 + b^2\ y22$ ----(72)

According to Brune (9) and Gewertz (20) y has the same property as Y11 or Y22 for all real values of a and b and therefore y12 can have only simple poles rectricted to the negative real axis but there is no restriction upon the zeros of y12 which may occur at any place.

Now, when P approaches Pk then (by the same argument used to derive equ. (16)) we have

$$\lim_{P \to Pk} Y = \frac{K}{P-Pk} \qquad \lim_{P \to Pk} Y11 = \frac{K11}{P-Pk}$$

$$\lim_{P \to Pk} Y12 = \frac{Ki1}{P-Pk} \qquad \lim_{P-Pk} Y22 = \frac{K22}{P-Pk}$$

where K, K11, Klw and K22 are residues at the pole P=Pk so eqn. (72) becomes,

$$K = K11\,a^2 + K12\,ab + K22\,b^2 \quad \text{-------}(73)$$

From Cauer's residue theorem, we know that,

$$K11 \geq 0 \qquad K22 \geq 0 \qquad \text{----}(74)$$

and $K11\,K22 - K12^2 \geq 0$ -------(75)

which is the desired condition for the physical realization of a four terminal R.C. network.

To satisfy the equ. (75), if at any point $K12 > 0$ then K11 and K22 must be greater than zero. So it is clear that if K11 or K22 has a pole at some value of P, K12 may or may not have the pole but if K12 has a pole at any value of P, K11 and K22 must have that pole. This may be expressed in another way that if the degree of numerator of y12 exceeds that of denominator (by one) then the numerator of both y11 and y22 are one degree higher than that of their denominators.

Now, let us consider the transfer function of a four terminal network with a load resistance of one ohm.

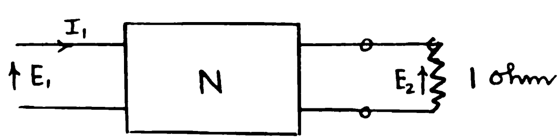


Figure (14)

The transfer function of this network may be written as,

$$Y12 = -\frac{E2}{E1} = \frac{I2}{E1} \quad \text{-------}(76)$$

$$= \frac{y12\ E1 + Y22\ E2}{E1}$$

$$= y12 + \frac{E2}{E1}\ Y22$$

$$= y12 - Y12\ y22$$

$$\text{or } Y12 = \frac{y12}{1+y22} \quad \text{-------}(77)$$

The quantity 1+ y22 in the denominator of Y12 is the driving point admittance of the network in figure (14). So the zeros of 1+ Y22 or poles of Y12 are simple and lie in the negative real axis of the P plane. It is clear from equ. (77) that the zeros of Y12 are either zeros of Y12 or poles of Y22. Also it is true that in the same network the poles of Y22 are identical with that of Y12, if the network begins with a series branch at the end of 2-2[1] excepting that branch is a pure resistance (32).

From the above discussion, we can conclude that the transfer function Y12 of any R.C. network terminated in a resistive load is the quotient of polynomials in which the degree of numerator does not exceed that of the denominator and the transfer function has simple poles lying on the negative real axis of P plane while the zeros may be anywhere in the plane.

So, the transfer admittance may be represented by,

$$Y12 = \frac{A(P)}{B(P)} = \frac{A_nP^n + A_{n-1}P^{n-1} + \text{-------} + A_o}{B_nP^n + B_{n-1}P^{n-1} + \text{----------} + B_o} \quad \text{-----}(78)$$

The denominator polynomial B (P) may be written as,

$$B(P) = B'(P) + B''(P) \quad \text{-----}(79)$$

So, the transfer function becomes,

$$Y12 = \frac{A(P)}{B'(P) + B''(P)} \quad \text{------------}(80)$$

$$= \frac{\frac{A(P)}{B'(P)}}{1 + \frac{B''(P)}{B'(P)}} \quad \text{---------------}(81)$$

comparing equ. (81) with equ. (77)

$$Y12 = \frac{A(P)}{B'(P)} \quad \text{----------}(82)$$

$$\text{and } Y22 = \frac{B''(P)}{B'(P)} \quad \text{-------}(83)$$

The polynomials B(P) should be parted in such a way that the ratio $B''(P)/B'(P)$ is physically realizable as driving point admittance of an R. C. network. The previous discussion showed that to realize the network, y22 should not start with shunt capacitance but here physical realizability also demands that y22 should not have any pole at P=D. This restriction can be easily met by separating B(P) in such a way that B'(P) and B" (P) both have the same degree of highest power.

The separation of B(P) may be shown in the following figure:

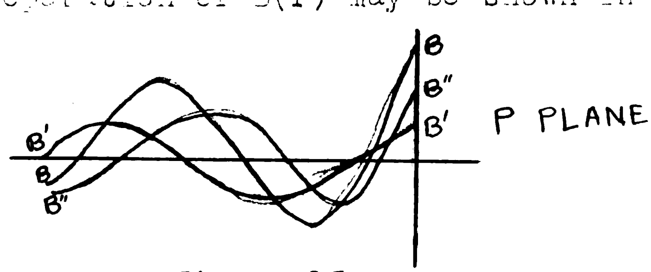


Figure 15

The curves of B, B' and B" are shown in the P plane. The polynomials B' (P) and B" (P) have simple zeros alternating on the negative axis. Now, since B = B' and B", if we draw the curves for B by adding the values of B' and B" at each point, we will find that the zeros of the polynomial B lie in between the zeros of the polynomials B' (P) and B" (P)

The equ. (82) may now be expanded to

$$Y12 = \frac{Ao}{B'(P)} + \frac{A1P}{B'(P)} + \text{--------} + \frac{AnPn}{B'(P)} \text{-----}(84)$$

It is a well known fact in the field of network synthesis that the driving point admittance function Y22 may be realized as a large variety of ladder structures with resistances and capacitances as either series or shunt branches and it may shown that different types of ladder networks which have the driving point admittance Y22 may be made such that their short circuit transfer admittances can be expressed by any one of the separate terms of equ. (84) and from this we may conclude that the synthesis work may be done through a set of such ladder networks which are connected in parallel and individually multiplied by a scale factor so as to obtain a resultant network with the admittance Y22.

Based on the above arguments we can start the synthesis process. Factorizing equ. (73) we have

$$Y12 = \frac{Ao + A_1P + \text{-------} + AnP^n}{Bn\ (P+d1)\ (P+d2)\text{--}(P+d2n-1)} \qquad (85)$$

where $0 < \alpha_1 < \alpha_3 \quad \cdots \quad \cdots \quad \cdots < \alpha_{2n-1} < \infty \quad \cdots \quad (86)$

The polynomial B'(P) will have the form

$$B'(P) = K'(P+\alpha_2)(P+\alpha_4) \cdots \cdots (P+\alpha_{2n}) \quad \cdots \quad \cdots (87)$$

The values of α_k's in the equ. (87) should be so chosen that,

$$0 < \alpha_1 < \alpha_2 < \alpha_3 \cdots \cdots < \alpha_{2n-1} < \alpha_{2n} < \infty \quad \cdots \quad \cdots (88)$$

and the value of K' is such that,

$$K'\alpha_2\alpha_4 \cdots \cdots \alpha_{2n} < B_o \leq B_n\alpha_1\alpha_3 \cdots \cdots \alpha_{2n-1} \quad \cdots (89)$$

We may now form the polynomial B"(P) by using equ. (79)

$$B''(P) = E(P) - B'(P) \text{ -----}(90)$$

$$= K''(P+\alpha_a)(P+\alpha_b) \text{ ---}(P+\alpha_z) \text{ -----}(91)$$

where, $0 < \alpha_a < \alpha_1 < \alpha_2 \cdots \cdots < \alpha_{2n-2} < \alpha_z < \alpha_{2n-1} < \alpha_{2n} \quad \cdots \cdots (92)$

Hence, from equ. (87) we have

$$Y22 = \frac{B''(P)}{B'(P)} - \frac{E_nP^n + E_{n-1}P^{n-1} + E_o}{D_nP^n + D_{n-1}P^{n-1} + D_o} \text{ -------}(93)$$

or we can write

$$Z22 = \frac{D_nP^n + D_{n-1}P^{n-1} + \text{ ------} + D_o}{E_nP^n + E_{n-1}P^{n-1} + \text{ ------} + E_o} = \frac{D(P)}{E(P)} \text{ ---}(94)$$

Dividing the denominator by numerator,

$$Z22 = R1 + \frac{D'(P)}{E(P)} \text{ -------}(95)$$

where $R1 = \frac{D_n}{E_n}$ and $\frac{D'(P)}{E(P)}$ is the remainder given by,

$$\frac{D'(P)}{E(P)} = \frac{D'_{n-1}P^{n-1} + D'_{n-2}P^{n-2} + \text{ --} + D'_o}{E_nP^n + E_{n-1}P^{n-1} + \text{ -------} + E_o} \text{ -----}(96)$$

By inverting the remainder,

$$Z22 = R1 + \cfrac{1}{\cfrac{E(P)}{D'(P)}} \qquad \text{-------------(97)}$$

Dividing E(P) by D'(P) we get,

$$Z22 = R1 + \cfrac{1}{C1P + \cfrac{e'(P)}{D'(P)}} \qquad \text{------(98)}$$

where the quotient $C1 = \frac{En}{D'n-1}$

and $\frac{E'}{D'} = \frac{E'n-1P^{n-1} + E'n-2P^{n-2} + \text{---} + Eo}{D'n-1P^{n-1} + D'n-2P^{n-2} + \text{----} + Do}$ -----(99)

The quotient of the polynomial $\frac{D'}{E'}$ is treated in the same way and after another cycle we have,

$$Z11 = R1 + \cfrac{1}{C1P + \cfrac{1}{R2 + \cfrac{1}{C2P + \cfrac{1}{\cfrac{D''}{E''}}}}}$$

In the same manner $\frac{D''}{E''}$ is dealt again and after n cycles of continued fraction the remainder is a constant and we obtain,

$$Z22 = R1 + \cfrac{1}{C.P + \cfrac{1}{R2 + \cfrac{1}{C2P + \cdots + \cfrac{1}{Rn + \cfrac{1}{CnP + \cfrac{1}{Rn+1}}}}}}$$

The form of the network realized is shown in figure (16) where Rk's and Ck's of equ. (101) are the resistances of the network.

This network has the short circuit driving point admittance Y22 and the short circuit transfer admittance $Y12 = \frac{Ao}{E'(P)}$ where Ao is a positive real constant which can be found out by writing the network admittance in the matrix form and using the formula of transfer admittance which is given by,

$$Y12 = \frac{D12}{D} \text{ -------(102)}$$

where D is the determinant of the network admittance and D12 is the minor of its first row and second column.

Next, let us consider the function Y22 again, but both numerator and denominator are turned end for end.

$$Y22 = \frac{Eo + E1P + \text{---------} + EnP^n}{Do + D1P + \text{-----------} + DnP^n} \text{ ------(103)}$$

$$= G1 + \frac{E'(P)}{D(P)} \text{ -------------(104)}$$

where $G1 = \frac{Eo}{Do}$ and the remainder

$$\frac{E'(P)}{D(P)} = \frac{E'_1P + E'_2P^2 + \text{------} + E'nP^n}{Do + D_1P + \text{-----------} + DnP^n} \text{ ------(105)}$$

Now, we divide D(P) by E'(P) So the equ. (104) becomes

$$Y22 = G1 + \frac{1}{\frac{1}{S}P^{-1} + \frac{D'(P)}{E'(P)}} \text{ ---------------(106)}$$

where the remainder is

$$\frac{D'(P)}{E'(P)} = \frac{D'_1 P + D'_2 P^2 + \text{------} + D'_n P^n}{E'_1(P) + E'_2 P^2 + \text{-------} + E'_n P^n} \quad \text{--------}(107)$$

Instead of repeating the same process, we go back to the previous method and manipulate the remainder in the same way as we treated $Z22 = \frac{D(P)}{E(P)}$ to find out the resistances Rk's and capacitances Ck's of figure (16). So, we can write the final form of the continued fraction directly to compare ing with the previous work.

$$Y22 = G_1 + \cfrac{1}{C_1 P^{-1} + \cfrac{1}{R_1 + \cfrac{1}{C_2 P + 1 - \cfrac{}{\ddots \cfrac{1}{R_n + \cfrac{1}{C_n P + \cfrac{1}{R_{n+1}}}}}}}} \quad \text{------------}(108)$$

The network realized by the above equation of Y22 is given in figure (17).

Here one point should be cleared, so that the values of Rk's and Ck's are quite different in different ladder structures.

The short circuit transfer admittance of this network also may be found by the same method as was given by equ. (102) and it can be shown that the transfer admittance of this network is

$$\underline{Y12} = \frac{A_1 P}{E'(P)} \quad \text{-------}(109)$$

where A_1 is a positive real constant and the short circuit driving point impedence is the same Y22 as given in equ. (97)

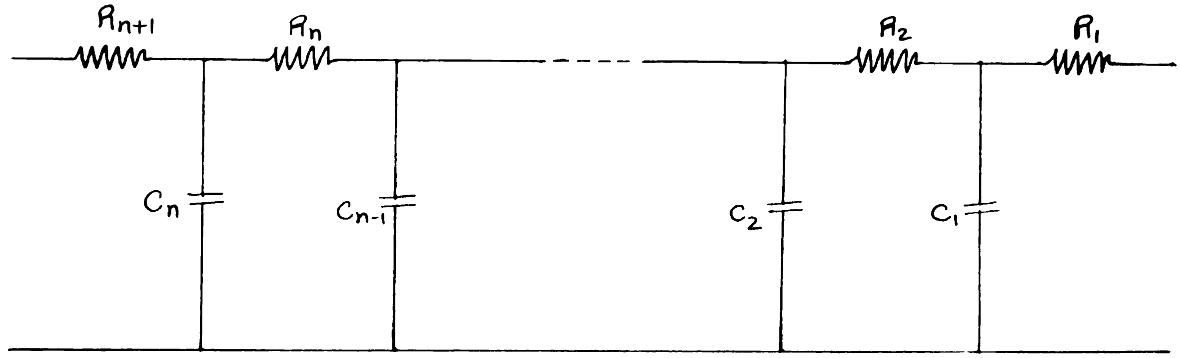


FIG-16

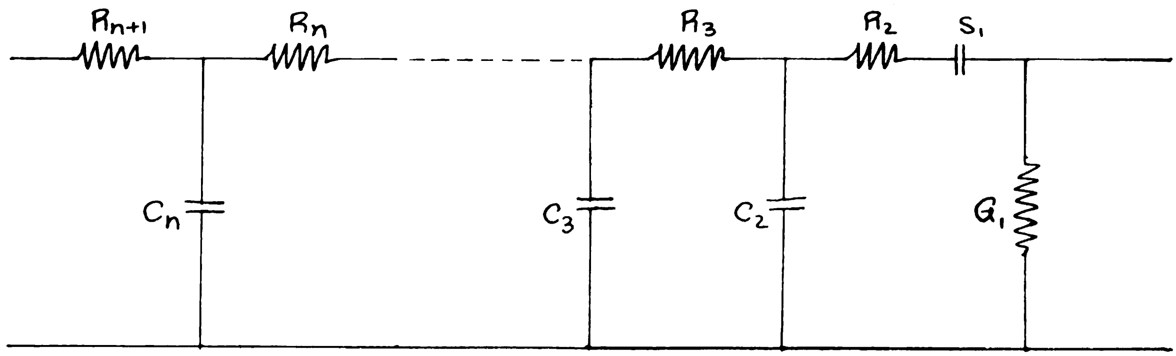


FIG-17

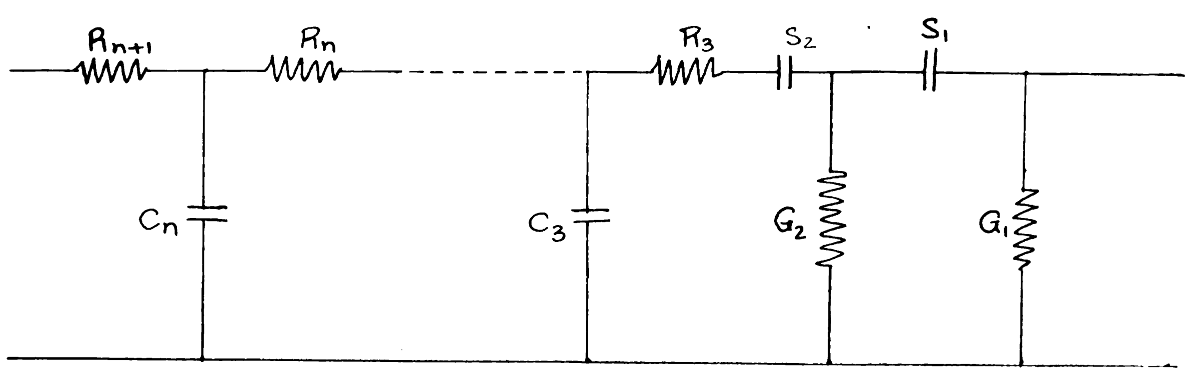


FIG-18

Now, again Y22 is considered and the first cycle, which was used to synthesize the last network, is continued for the first two cycles instead of one. ~~and~~ Then again repeating the original process of manipulation, we get,

$$Y22 = G_1 + \cfrac{1}{S_1P^{-1} + \cfrac{D'(P)}{E'(P)}} = G_1 + \cfrac{1}{S_1P^{-1} + \cfrac{1}{\cfrac{E'(P)}{D'(P)}}} \quad \text{----(110)}$$

$$= G_1 + \cfrac{1}{S_1P^{-1} + \cfrac{1}{G_2 + \cfrac{1}{S_2P^{-1} + \cfrac{D''(P)}{E''(P)}}}}$$

The remainder may be written as,

$$\frac{D''(P)}{E''(P)} = \frac{D''_2 + D''_3 P + \text{---} + D''_n P^{n-2}}{E''_2 + E''_3 P + \text{---} + E''_n P^{n-2}} \quad \text{-------(112)}$$

As already stated, it is now manipulated by the first method and we get

$$Y_{22} = G_1 + \cfrac{1}{S_1P^{-1} + \cfrac{1}{G_2 + \cfrac{1}{S_2P^{-1} + \cfrac{1}{R_n + \cfrac{1}{C_nP + \cfrac{1}{R_n + L}}}}}} \quad \text{---------(113)}$$

The above equation of Y22 is realized by the network in figure (18). S_1 and G_1 of this network have the same value as the last network and the transfer admittance calculated by the same way is given by

$$Y12 = \frac{A_2P^2}{E'(P)} \quad \text{------------(114)}$$

where A_2 is a positive real constant.

In this way Y22 may be expanded (n+1) times, each time we find G and S for one more cycle and then going back to the original process and the remainder is then expanded in terms of Rk's and Ck's. So, after (n+1) times, expanding y22 in terms of G and S for n cycles, the remainder will be a constant which may be represented by a resistance.

So, after (n+1) times, the driving point admittance becomes,

$$Y22 = G_1 + \cfrac{1}{S_1P^{-1} + \cfrac{1}{G_2 + \cfrac{1}{S_2P^{-1} + \cdots}}} \text{-----------------------(115)}$$

The network corresponding to equ. (115) is given by figure (20). The transfer admittance of this network is

$$Y12 = \frac{A_nP_n}{B'(P)} \text{----------(116)}$$

Now, if we connect all these ladder structures in parallel, the transfer admittance will not be the same as desired but if we multiply all Y12 (K) by different constants the desired transfer admittance may be found. So, if we multiply Y12's by $\frac{Ao}{A}$, $\frac{A_1}{a_1}$, -------$\frac{An}{A}$ then we have,

$$Y12 = \frac{Ao}{Ao}\, Y12 + \frac{A_1}{A_1}\, Y_{12} + \text{-------}\frac{An}{An}\, Y12 \text{-------(117)}$$

$$= \frac{Ao}{B} + \frac{A_1}{B'} + \text{------} + \frac{An}{B'} \text{----------------(118)}$$

But it is clear that since we have multiplied Y_{12}s by different constants, the desired Y_{22} at the same time was also multiplied by a constant

$$C = \frac{A_0}{K_0} + \frac{A_1}{K_1} + \text{-----} + \frac{A_n}{K_n} \text{-------------} (119)$$

But since this is not desirable, we multiply the transfer admittances by $\frac{A_0}{K_0} + \frac{A_1}{K_1} + \text{-----} + \frac{A_n}{K_n}$ etc. The parallel connection of these networks will produce the same short circuit driving point admittance (the given Y_{22}) but the transfer admittance will become $\frac{Y_{12}}{C}$

<u>Easier Method to find out the values of the constants A_k'</u>

When the value of n is large, A_k becomes very laborious to calculate. But if P is small, an easier method may be applied to find out these values. We know,

$$Y_{12} = \frac{A_k P^k}{D'} \text{---------} (120)$$

$$= \frac{A_k P^k}{D_n P^n + D_{n-1} P^{n-1} + \text{--} + D_o} \text{--------------------} (121)$$

But when $P \rightarrow 0$ -----------(122)

$$Y_{12} = \frac{A_k P^K}{D_o} \text{---------} (122)$$

Now, if we consider figure (19), then for $P \rightarrow 0$, the impedance of the shunt capacitances C_n---C_4 are much larger compared to the resistances R_{n+1}, R_n, ----R_4, and the current through the resistances are very small because there is a shunt elastance in series. So the voltage at the input of the network is equal to that just to the left of the elastance S_3. Therefore, the value of A_3 is only influenced by the network right to this point.

If E_1 is the input voltage and E_2 and E_3 are the voltages across the conductance G_2 and G_3, we can say that E_1 is the voltage to the left of S_3 and impedance looking to the right of G_3 is very large in comparison with $\frac{1}{G_3}$. So,

$$\frac{E_3}{E_1} = \frac{\frac{1}{G_3}}{\frac{S_3}{P} + \frac{1}{G_3}} = \frac{P}{G_3 S_3} \text{------------}(123)$$

In the same way,

$$\frac{E_2}{E_3} = \frac{\frac{1}{G_2}}{S_2/P + \frac{1}{G_2}} = \frac{P}{G_2 S_2} \text{---------}(124)$$

So, the current through the short circuited output terminal,

$$I_2 = \frac{E_2 P}{S_1} \text{-----------------}(125)$$

$$\text{or } E_2 = \frac{I_2 S_1}{P} \text{-------------}(126)$$

multiplying equ. (123) and (124), we have,

$$\frac{E_3}{E_1} \times \frac{E_2}{E_3} = \frac{P}{G_3 S_3} \cdot \frac{P}{G_2 S_2} = \frac{P^2}{G_2 G_3 S_2 S_3}$$

or $\frac{E_2}{E_1} \times \frac{E_2}{E_3} = \frac{P}{G_3S_3} \cdot \frac{P}{G_2S_2} = \frac{P^2}{G_2G_3S_2S_3}$

or $\frac{E_2}{E_1} = \frac{P^2}{G_2G_3S_2S_3}$ -------------(127)

putting the value of equ. (127) we get,

$I_2S_1 = \frac{P^2}{G_2G_3S_2S_3}$ ------------(129)

or, $\frac{I_2}{E_1} = \frac{P^3}{G_2G_3S_1S_2S_3}$

Putting the value of equ. (122) in equ. (130) we have

$\frac{A_3P^3}{Do} = \frac{P^3}{G_1G_2S_1S_2S_3}$ -----------(131)

or $A_3 = \frac{Do}{G_1G_2S_1S_2S_3}$ -------(132)

So, in a generalized form we may write this equation as,

$A_n = \frac{Do}{(G_1 --- G_{n-1})(S_1S_2 ----- Sn)}$ -------------(133)

Now, we will solve a numerical example by the above synthesis procedure:

Let the transfer function be given by

$Y12 = \frac{A(P)}{B(P)} = \frac{P+1}{(P+2)(P+4)(P+6)}$ ----(134)

$= \frac{P+1}{P^3+12P^2+44P+48}$ ------------(135)

we choose,

$$B'(P) = \frac{1}{3}(P+3)(P+5)(P+7) \text{ --------(136)}$$

$$= \frac{1}{3}P^3 + 5P^2 + 23.6P + 35 \text{ ----(137)}$$

Hence,

$$B''(P) = B'P) - B'(P)$$

$$= \frac{2}{3}P^3 + 7P^2 + 20.3P + 13 \text{ -----(138)}$$

$$Z22 = \frac{1}{Y_{22}} = \frac{1}{\frac{1}{3}P^3 + 5P^2 + 23.7P + 35} \text{ ------(139)}$$

$$= \frac{1}{2} + \cfrac{1}{.445P + \cfrac{1}{1.25 + \cfrac{1}{.4P + \cfrac{1}{.91 + \cfrac{1}{33P + \cfrac{1}{.008}}}}}}$$

The network is shown in figure (21). The transfer admittance is calculated.

$$Y_{12} = \frac{2}{\frac{1}{3}P^3 + 5P^2 + 23.7P + 35} \text{ -------(141)}$$

Considering Y_{22} again

$$y_{22} = \frac{\frac{2}{3}P^3 + 7P^2 + 20.3P + 13}{\frac{1}{3}P^3 + 5P^2 + 23.7P + 35} \text{ -------------(142)}$$

let us turn it end for end.

$$Y_{22} = \frac{13 + 20.3P + 7P^2 + \frac{2}{3}P^3}{35 + 23.7P + 5P^2 + \frac{1}{3}P^3} \text{ --------(143)}$$

$$= .371 + \cfrac{1}{2.92\,P^{-1} + .615 + \cfrac{1}{2.22P + \cfrac{1}{.076 + \cfrac{1}{63P + \cfrac{1}{.0042}}}}}$$

The network is given by figure (22) Y_{12} is calculated

$$\underline{Y}_{12} = \frac{6P}{\frac{1}{3}P^3 + 5P^2 + 23.7P + 35} \qquad \text{-------------(145)}$$

In this case

$$C = \frac{1}{2} + \frac{1}{6} = .667 \qquad \text{--------------(146)}$$

The admittance levels of the networks are multiplied respectively by

$$\frac{1}{2X.667} = .75 \text{ and } \frac{1}{6X.667} = .25$$

The final network of the parallel ladder structures terminated by a one ohm load is shown in figure (23)

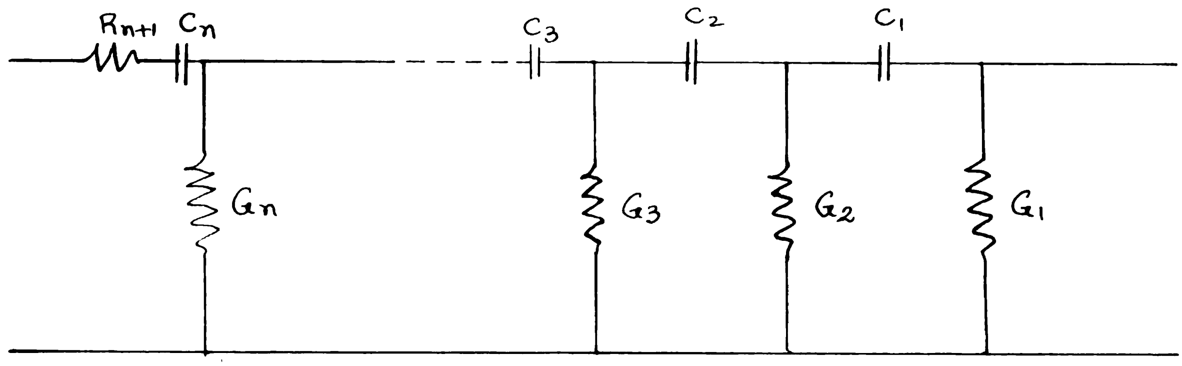


FiG-20

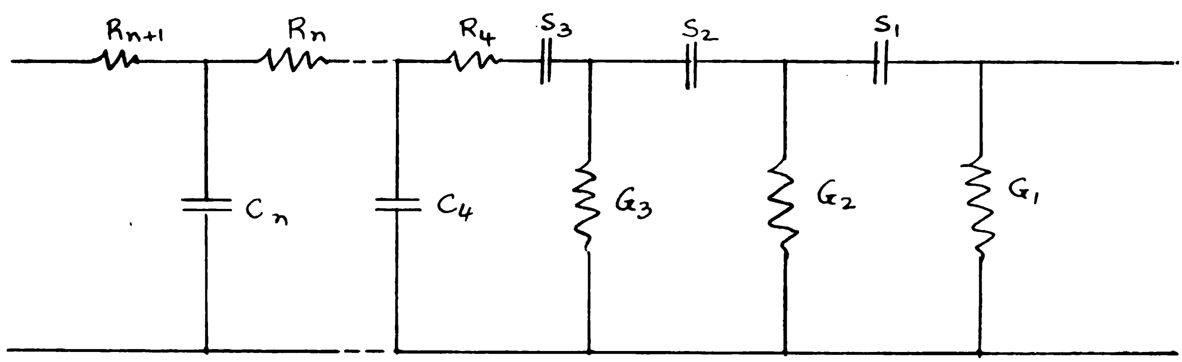


FiG-19

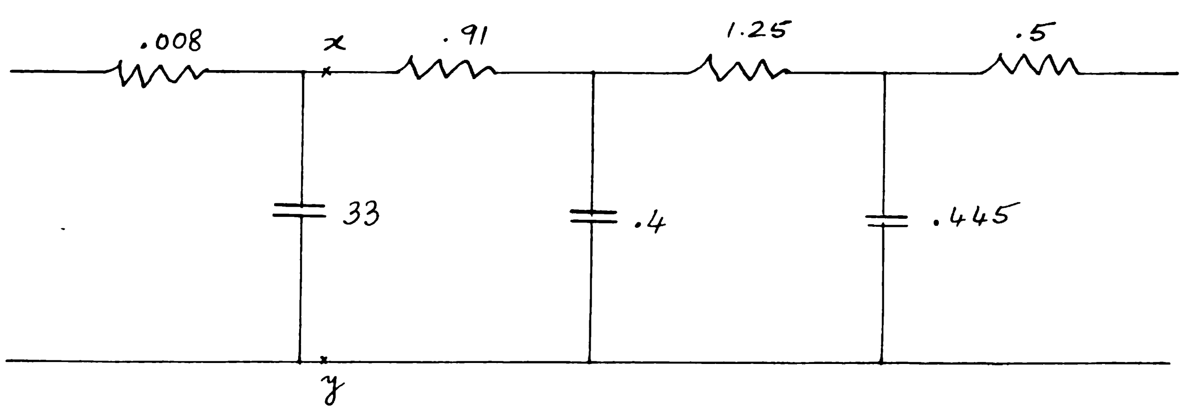


FiG-21

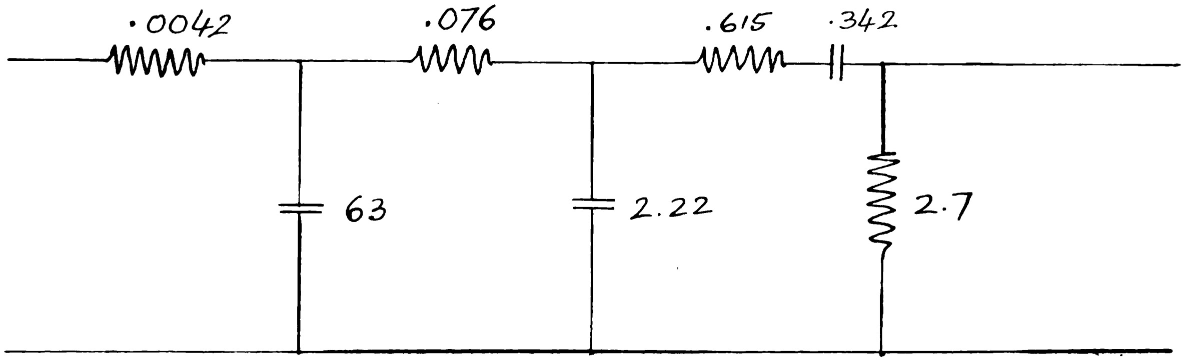

.0042
.076
.615
.342
63
2.22
2.7

FiG-22

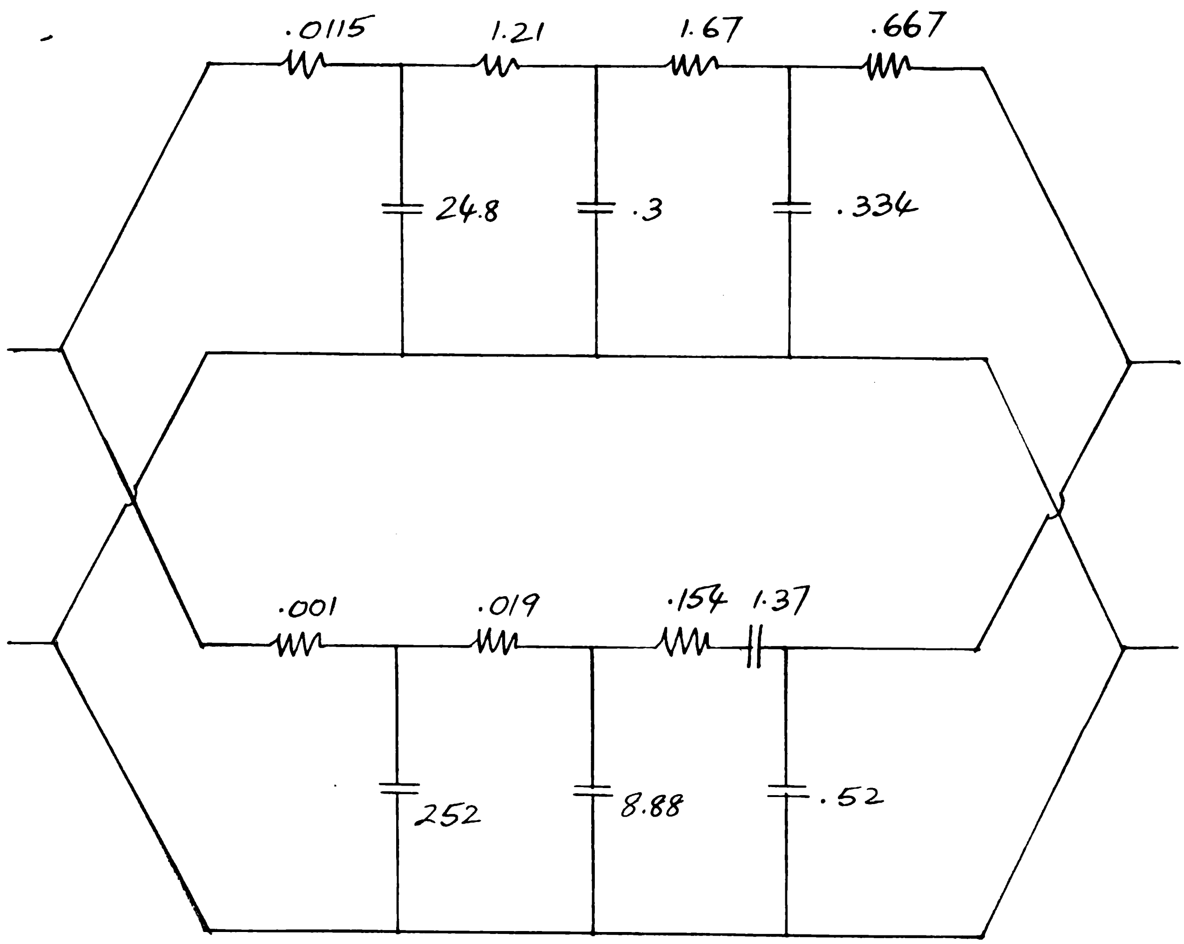

.0115
1.21
1.67
.667
24.8
.3
.334
.001
.019
.154
1.37
252
8.88
.52

FiG-23

Modified Method with Fewer Elements (32)

Let us consider any of the above ladder networks with transfer admittance,

$$Y_{12} = \frac{A_K P^K}{B'} \text{------------------} (147)$$

This ladder structure is drawn in figure (24) in such a way that the first two elements are shown and the others are shown by a Box B. But since $\frac{I_2}{E_1} = \frac{I_1}{E_2}$, the network may also be represented by figure (25)

Now E_1 is the voltage across the points A b (across parallel combinations of C_n and $\Gamma n+1$). So, if we interchange the elements C_n and $\gamma n+1$) as in figure (26), there is no effect on the voltage E but the short circuit current I, is changed.

In figure (25) the current may be given by

$$I_1 = \frac{E}{Rn+1} \text{-----------} (148)$$

But in figure (26) the current is changed to

$$I_1' = ECnP \text{-------------} (149)$$

$$I_1' = Rn+1\ CnP \text{---------} (150)$$

$$I_1 = \frac{I_1'}{R_{n+1} CnP} \text{---------} (151)$$

But we know,

$$Y_{12}^{(K)} = \frac{I_1}{E2} \text{--------------} (152)$$

$$Y_{12}^{(K)} = \frac{I_1'}{E2R_{n+1}\ CnP} \text{---------} (153)$$

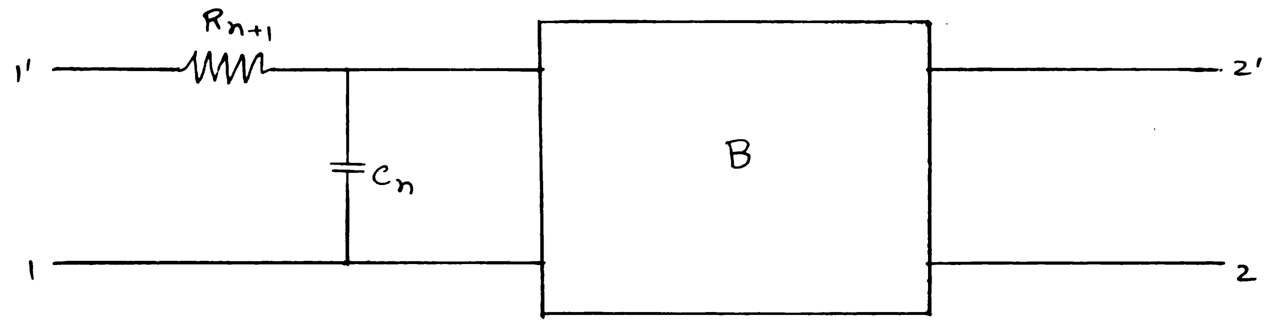


FIG -24

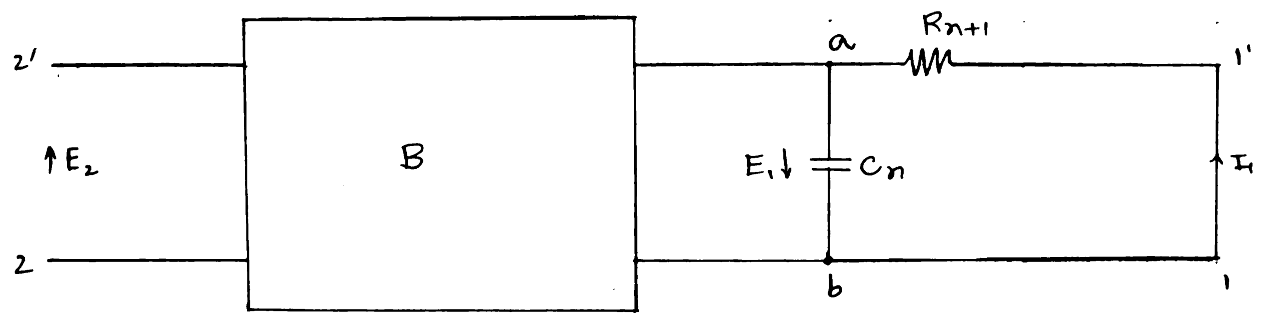


FIG -25

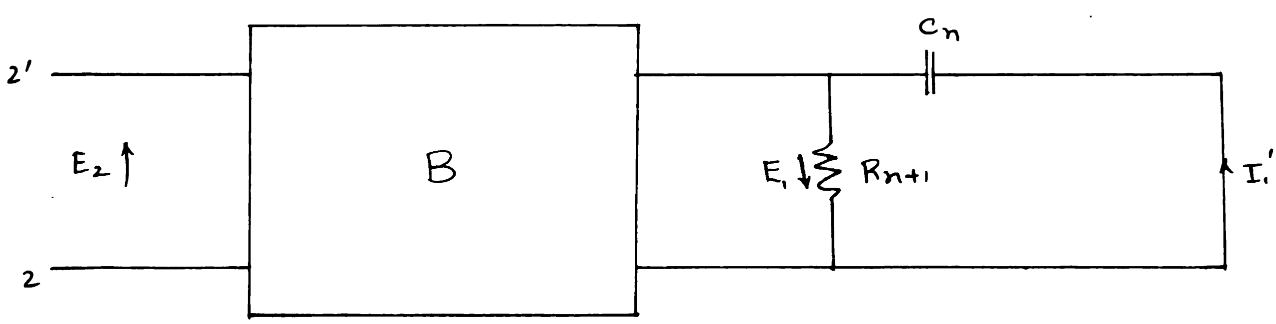


FIG-26

$$\text{or } \frac{I'_1}{E^2} = Y^{(K)}_{12} \times R_{n+1} C_n P \text{ ---------(154)}$$

which has the same form as $Y^{(K+1)}_{12}$ excepting the constant multiplier. From this result it is clear that a single ladder structure may yield a transfer admittance to the form

$$Y'^{(K)}_{12} \times \left(1 + \frac{A_{k+1} P}{A_k}\right) \text{ ----------------(155)}$$

This includes two terms of equ. (84) which means that the network needs only $\frac{n}{2}$ numbers of parallel ladder structures instead of n numbers used in the previous synthesis process.

So, if we consider the modified figure (27), we have

$$I''_1 = \left(\frac{A}{K_{n+1}} + bC_n P\right) E \text{ -----------(156)}$$

$$= a\, y^{(K)}_{12} \left(1 + \frac{bR_{n+1} C_n P}{a}\right) \text{ --------(157)}$$

where a and b have the value between 0 to 1 comparing equ. (157) with (155) we get

$$\frac{bR_{n+1} C_n}{a} = \frac{A_{k+1}}{A_k} \text{ --------------------(158)}$$

$$\text{or } \frac{a}{b} = \frac{A_k}{A_{k+1}} \times R_{n+1} C_n \text{ ----------------------(159)}$$

One more resistance or capacitance may be eliminated by assuming A = 1 if $\frac{a}{b} > 1$ or b = 1 if $\frac{a}{b} < 1$

From the figure it is clear that only two resistances and one capacitance or vice versa are needed in the terminating branch.

Thus by adding one more element to the ladder structure, we can eliminate another ladder structure. So, in the final network we can eliminate $\frac{n}{2}$ ladder structure by adding $\frac{n}{2}$ elements, one in one of the remaining ladders.

A numerical example will illustrate the procedure:

Let us attack the same problem we solved in the last example.

In the first ladder structure (figure 21) up to point xy the values of resistances and capacitances will be the same as calculated there. But the shape of the network to the left of xy will be changed as shown in figure (27)

Here,

$$\frac{A_1}{A_0} = \frac{1}{1} = 1 \text{ ---------(160)}$$

$$\frac{a}{b} = 1 \times R_{n+1} C_n \text{ ---------(161)}$$

$$= 1 \times .008 \times 33$$

$$= .264$$

Hence we choose $b = 1$ -----------(162)

$$a = .264 \text{ -------(163)}$$

So the shape of the network will be of form given in figure (28)

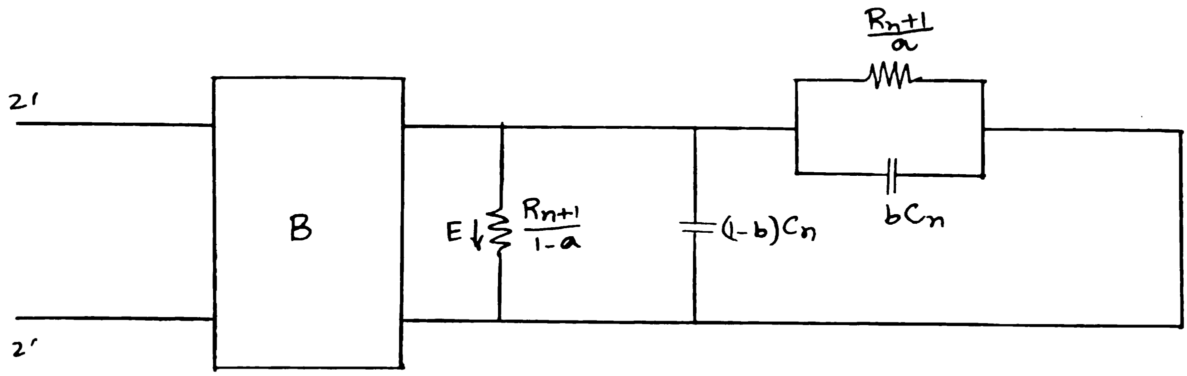


FIG-27

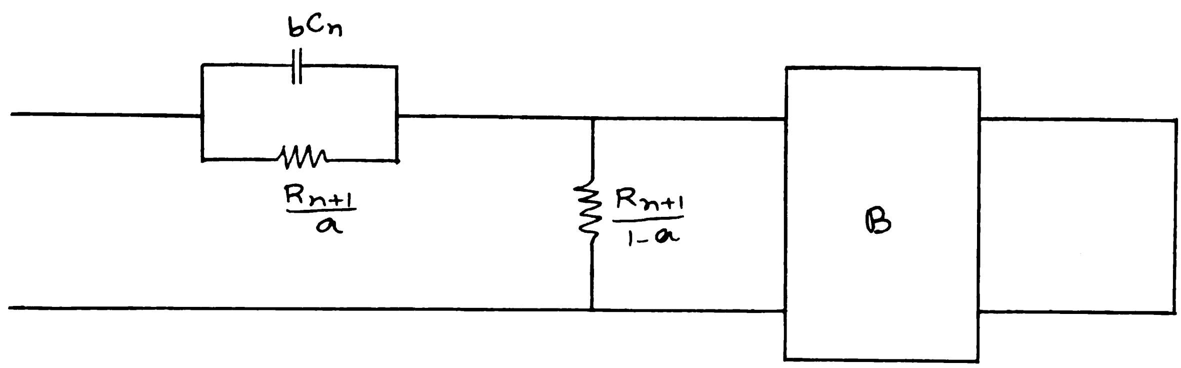


FIG-28

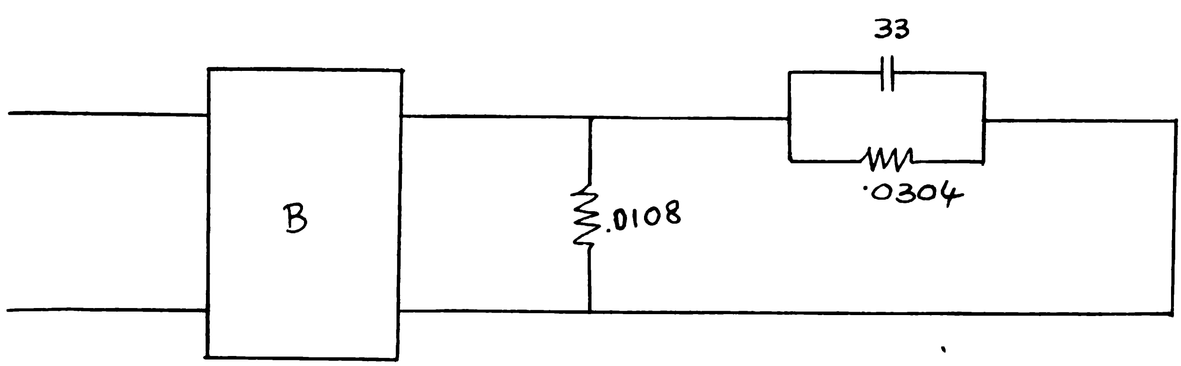


FIG-29

$$R_1 = \frac{.008}{1-.264} = \frac{.008}{.736} = .0108$$

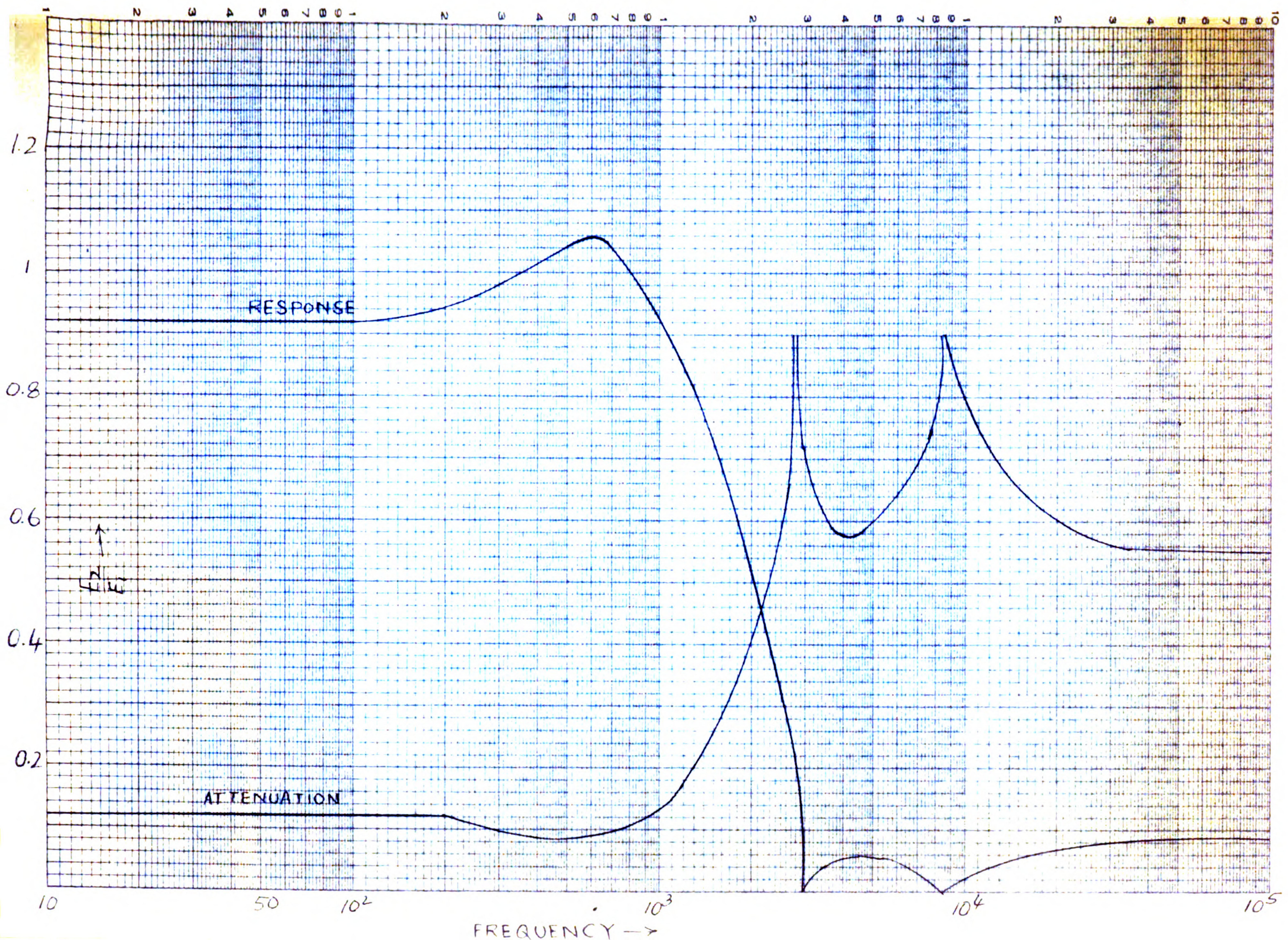
$$R_2 = \frac{.008}{.264} = .0304$$

$$C_2 = 33$$

The network is shown in figure (29)

Filter Design:

In order to illustrate what may be achieved with this type of network when used as a filter, experimental curves of a six pole polynomial B(P) are shown in the next page which are taken from the masters thesis of K. Patrick and M. Thomas entitled "A Design Procedure For Linear Passive RC Filter Networks." (74)


RESPONSE
ATTENUATION
$\frac{E_2}{E_1}$ ↑
1.2
1
0.8
0.6
0.4
0.2
10
50
10^2
10^3
10^4
10^5
FREQUENCY →

Development into a Lattice:

The driving point impedance or admittance of a four terminal R.C. network are functions of complex frequency variable. The poles and zeros of driving point functions are always simple/ and real and alternate along the negative real axis of the complex frequency plane (62).

Four terminal networks may be specified conveniently in terms of the Z or Y co-efficients occuring in the two pair of linear equations:

$$\begin{aligned} E_1 &= I_1 Z_{11} + I_2 Z_{12} \\ E_2 &= I_1 Z_{12} + I_2 Z_{22} \end{aligned} \qquad \text{------------------}(164)$$

$$\begin{aligned} I_1 &= E_1 Y_{11} + E_2 Y_{12} \\ I_2 &= E_1 Y_{12} + E_2 Y_{22} \end{aligned} \qquad \text{----------------}(165)$$

If the network is passive then $Z_{12} = Z_{21}$ and the driving point and transfer impedance (and admittance) functions may be expanded into partial fraction (43) and we have,

$$Z_{11} = A_{11}^{(D)} + \frac{A_{11}^{(O)}}{P} + \sum^{n} \frac{a_{11}^{(r)}}{P + Pr}$$

$$Z_{12} = a_{12}^{(D)} + \frac{A_{12}^{(o)}}{P} + \sum_{r=1}^{n} \frac{a_{12}^{(r)}}{P + Pr} \qquad (166)$$

$$Z_{22} = A_{22}^{(D)} + \frac{A_{22}^{(O)}}{P} + \sum_{r=1}^{n} \frac{A_{22}}{P + Pr}$$

$$Y_{11} = b_{11}^{(D)} P + b_{11}^{(0)} + P \sum_{r=1}^{n} \frac{b_{11}^{(r)}}{P + qr}$$

$$Y_{12} = b_{12}^{(D)} P + b_{12}^{(0)} + P \sum_{r=1}^{n} \frac{b_{12}'}{P + qr} \qquad \text{-----(167)}$$

$$Y_{22} = b_{22}^{(D)} P + b_{22}^{(0)} + P \sum_{r=1}^{n} \frac{b_{22}}{P + qR}$$

Then Cauer's residue theorem states that,

$$a_{11}^{(r)} > 0 \qquad a_{22}^{(r)} > 0 \qquad a_{11}a_{22} - a_{12}^{2} \geq 0 \qquad \text{------(168)}$$

and $b_{11}^{(r)} > 0 \qquad b_{22}^{(r)} > 0 \qquad b_{11}b_{22} - b_{12}^{2} \geq 0$ -------(169)

for all values of r

If such a four terminal network is placed between a pair of resistances, one acting as a generator and another as a load. Then the transfer impedance of the complete circuit may be written as,

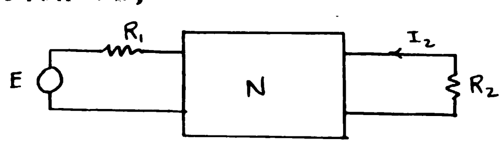


Figure 30

$$X_T = \frac{E}{I} \qquad \text{----------(170)}$$

where E and I are shown in fugure 30

The transfer impedance is a rational function of complex frequency P with real co-efficients (4). The zeros of this function are the same as/the driving point function and since the network contains only resistances and capacitances, these zeros will be simple and occur at the real negative axis of complex frequency plane. And, since the network

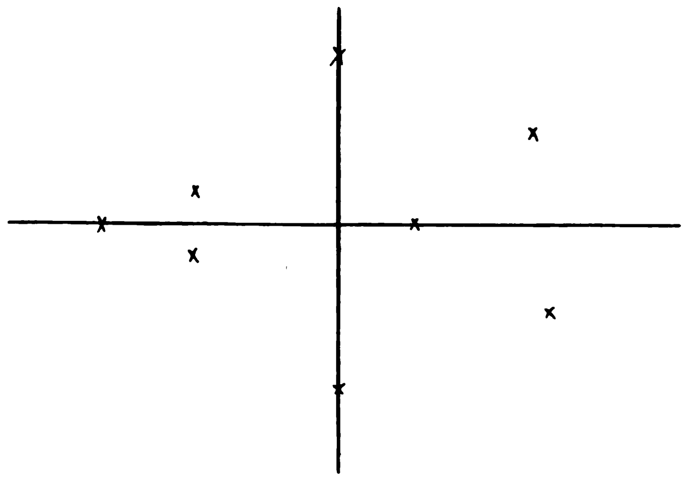


FiG - 31

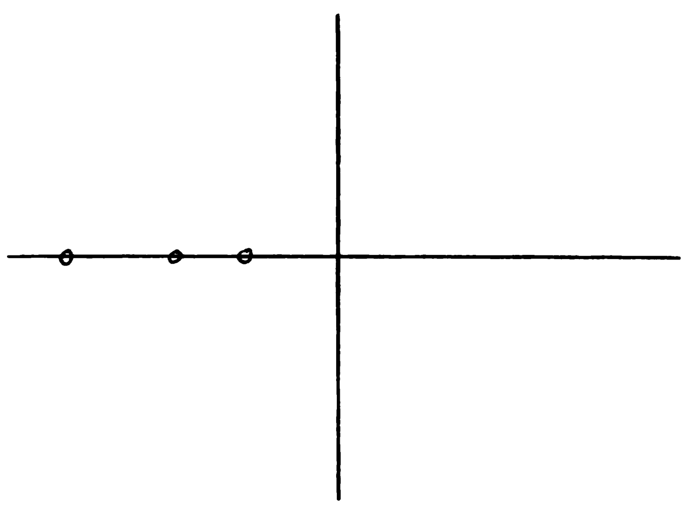


FiG - 32

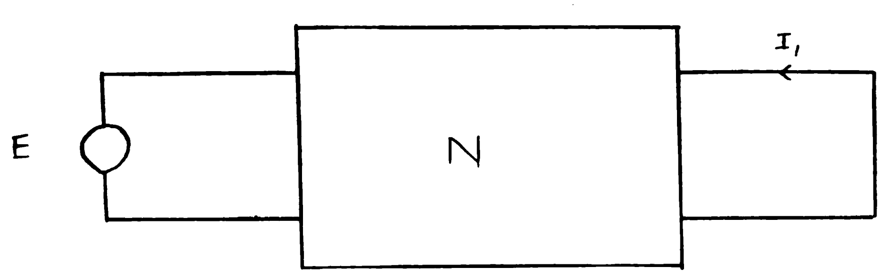


FiG - 33

contains resistances R_1 and R_2 in series with two meshes, the transfer admittance cannot have a zero at P = D.

Examining the equation of transfer function it can be found that its poles are produced either byzeros Z_{12} or the poles of Z_{11} or Z_{22} which are not already contained by the zeros of Z_{12}. So there are no restrictions upon the zeros of transfer function because there is no restriction upon the poles of the transfer impedance Z_{12} excepting that they should occur in conjugate when complex.

The typical position of poles and zeros are shown in figures (31) and (32) respectively.

Synthesis Process:

Let us consider that the output terminals of a four terminal network is short circuited as shown in figure (33)

Putting $E_1 = E; I_2 = I$ & $E_2 = 0$ ----------------(171)

$$ZT = \frac{E1}{I_2} = \frac{E}{I} = \frac{1}{y_{12}} \quad \text{-------------------(172)}$$

This equation shows that only transfer impedance is not sufficient to specify the network because it contains only one of the three functions which specify a four-terminal network. The remaining functions Y_{11} and Y_{22} may be given by any values, so long as the three functions obey the residue theorem.

Let us consider here that the network is symmetrical and the equality sign of residue theorem holds here at each pole.

so, $Y_{11} = Y_{22}$ -----------(173)

and, $Y_{11}\ Y_{22} - Y^2_{12} = 0$ ----(174)

Equ. (173) tells us that the zeros of ZT are real, simple and negative. Around this pole Z_r^{-1} may be expanded into partial fractions of the form of equ. (162) and the co-efficients may be of any sign but real. Now, if both Y11 and Y_{22} are expanded similarly into partial fraction, according to our assumptions, we have,

$$b_{11}^{(r)} = b_{22}^{(r)} = (b_{12}^{(r)}) \text{ ---------------(175)}$$

For all values of r from this equation it is clear that from the expansion of $Y_{12} = Z^{-1}\ Y_{11}$ and Y_{22} may be derived only by making all residues positive. The admittance matrix of this network is now completely specified and this network is always realizable by means of a lattice structure.

The impedance matrix of this network may be found by solving equ. (165) and comparing it with equ. (164). From equ. (165)

$$Y_{22}\ I_1 = Y_{11}\ Y_{22}\ E_1 + Y_{22}\ Y_{12}\ E_2$$

$$Y_{12}\ I_2 = Y_{12}^2\ E_1 + Y_{22}\ Y_{12}\ E_2$$

Subtracting these equations, we have,

$$E_1\ (Y_{11}Y_{22} - Y_{12}^2) = Y_{22}I_1 - Y_{12}\ I_2$$

or, $E_1 = \frac{Y_{22}}{Y_{11}Y_{22}-Y_{12}^2} I + \frac{-Y_{12}}{Y_{11}Y_{22}-Y_{12}^2} I_2$

comparing with equ. (164)

$$Z_{11} = \frac{Y_{22}}{Y_{11}Y_{22}Y_{12}^2} \text{-------}(176) \qquad Z_{12} = \frac{-Y_{12}}{Y_{11}Y_{22}-Y_{12}^2} \text{---}(177)$$

similarly,

$$Z_{22} = \frac{Y_{11}}{Y_{11}Y_{22}-Y_{12}^2} \text{-------}(178)$$

The constant quantities are $a_{11}^{(D)}$, $a_{12}^{(D)}$ and $a_{22}^{(\infty)}$ of equ. (166) may now be written as,

$$\underset{P\to\infty}{\mathrm{dim}} Z_{11} = a_{11}(\infty) \text{-------}(179)$$

$$\underset{P\to\infty}{\mathrm{dim}} Z_{12} = a_{12}^{(\infty)} \text{-------}(180)$$

$$\underset{P\to\infty}{\mathrm{dim}} Z_{22} = a_{22}^{(\infty)} \text{-------}(181)$$

Now physical realizability demands that it should obey the residue theorem

$$a_{11}^{(\infty)} a_{22}^{(\infty)} - a_{12}^{(\infty)2} \geqq 0$$

Whether the equality or inequality sign holds here may be found readily by the limits applied above to the function $Z_{22}-Z_{12}^2$ and Z's replaced by Y's as given by equs. (176-178)

Hence,

$$a_{11}^{(\infty)}\, a_{22}^{(\infty)} - a_{12}^{(\infty)2} = \lim_{P\to\infty} (Z_{11}\, Z_{22} - Z_{12}^2) \quad \text{----(182)}$$

$$= \lim_{P\to\infty} \frac{Y_{11}Y_{22}-Y_{12}^2}{(Y_{11}Y_{22}-Y^2_{22})^2} \quad \text{----(183)}$$

$$= \lim_{P\to\infty} \frac{1}{(Y_{11}Y_{22}-Y_{12}^2)^2} \quad \text{----(184)}$$

This limit cannot be zero because then $Y_{11}Y_{22}-Y_{12}^2=D$ which is impossible because none of the Y's has pole at P=D. This can be proved by the fact that Z = 0 when P = D and so Y_{12} can not have a pole at P = ∞ and so $b_{12}^{(\infty)}=0$.

Also, it was specified that $B_{11}^{(\infty)} = b_{22}^{(\infty)} - b_{12}^{(\infty)2} = 0$. So, from these we can conclude that

$$\lim_{P\to\infty} Y_{11}\, Y_{22} - Y_{12}^2 = 0 \quad \text{-----(185)}$$

$$\text{and } a_{11}^{(\infty)}\, a_{22}^{(\infty)} - a_{12}^{(\infty)} > 0 \quad \text{------(186)}$$

The terms $a^{(\infty)}$ are the constant parts and by virtue of the inequality, we can extract from each end of the network a series resistance of magnitude

$$a_{11}^{(\infty)} - /a_{12}^{\infty}/ = a_{22}^{(\infty)} - /a_{12}^{\infty}/ \quad \text{-----(187)}$$

This is shown in fig. (34)

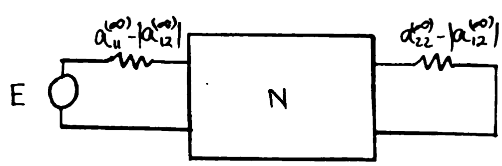


Fig. 34

The network which is left is still physically realizable because the only effect of extracting these resistances will be to make the equality rather than the inequality sign hold for the terms $a^{(D)}$ in Z function. This remaining network, still symmetirical, may now be realized physically as a lattice, the arms of which may be given by,

$$ZA = Z^*_{11} + Z_{12} \text{ ------ } (188)$$

$$ZB = Z^*_{11} - Z_{12} \text{ ----- } (189)$$

$$\text{where, } Z_{11} = Z_{11}^{*} - (a_{11}^{(\infty)} - /a_{12}^{(\infty)}/\ ) \text{ ---- } (190)$$

This synthesis may be done a little quicker (48) by remembering that the resistances are to be extracted from each end approximately of the magnitude of the smaller series resistance of $Z_{11} + Z_{12}$ and $Z_{11} - Z_{12}$, which are the arm impedances of the lattice before the extraction and equal to $(Y_{11} + Y_{12})^{-1}$ and $(Y_{11} - Y_{12})^{-1}$ respectively.

To illustrate the above procedure a numerical problem is solved below:

given

$$Z_T = \frac{(P+2)\ (P+3)\ (P+4)}{(P-2)\ (P-3)\ (P-4)} \text{ --------- } (191)$$

$$= \frac{P^3 + 9P^2 + 26P + 24}{P^3 - 9P^2 + 26P - 24} \text{ ---------- } (192)$$

$$Y_{12} = (Z_T)^{-1} = \frac{P^3 - 9P^2 + 26P - 24}{P^3 + 9P^2 + 26P + 24} \text{ ------- } (193)$$

$$= -1 + \frac{2P^3 + 52P}{(P+2)\ (P+3)\ (P+4)} \text{ -------- } (194)$$

$$= -1 + \frac{30P}{P+2} - \frac{78P}{P+3}\ \frac{42P}{P+4} \text{ ------------ } (195)$$

$$Y_{11} = Y_{22} = 1 + \frac{30P}{P+2} + \frac{70P}{P+3} + \frac{42P}{P+4} \text{ ---------(196)}$$

$$Y_A = Y_{11} + Y_{12} = 2\left(\frac{30P}{P+2} + \frac{42P}{P+4}\right) = \frac{144P^2 + 408P}{P^2 + 6P + 8} \text{ -------(197)}$$

$$Y_B = Y_{11} - Y_n = 2\left(1 + \frac{70P}{P+3}\right) = \frac{142P + 6}{P + 3} \text{ ----------(198)}$$

$$ZA = \frac{P^2 + 6P + 8}{144P^2 + 408P} \text{ ----------------------------(199)}$$

$$= \frac{1}{144} + \frac{3}{408P} + \frac{5}{4(144P+408)} \text{ --------------(200)}$$

$$ZB = \frac{P + 3}{142P + 6} \text{ -----------------------------(201)}$$

$$= \frac{1}{142} + \frac{105}{71(71P + 3)} \text{ -----------(202)}$$

Here smaller part is $\frac{1}{144}$

$$Z_A^* = \frac{3}{408P} + \frac{5}{4(144P + 408)} \text{ ----------(203)}$$

$$Z_B^* = \frac{1}{10224} + \frac{105}{71(71P+3)} \text{ ---------(204)}$$

To get proper values multiply equs. (203) and (204) by 144

$$Z_A^* = \frac{144}{51P} + \frac{180}{144P + 408} \text{ ------(205)}$$

$$Z_B^* = \frac{1}{71} + \frac{105 \times 144}{71(71P + 3)} \text{ ---------(206)}$$

$$R_1 = \frac{180}{408} \qquad C_1 = \frac{144}{180}$$

$$R_2 = \frac{105 \times 144}{71 \times 3} \qquad C_2 = \frac{71 \times 71}{105 \times 144}$$

$$R_3 = \frac{1}{71} \qquad C_3 = \frac{51}{144}$$

The network is shown in figure 35.

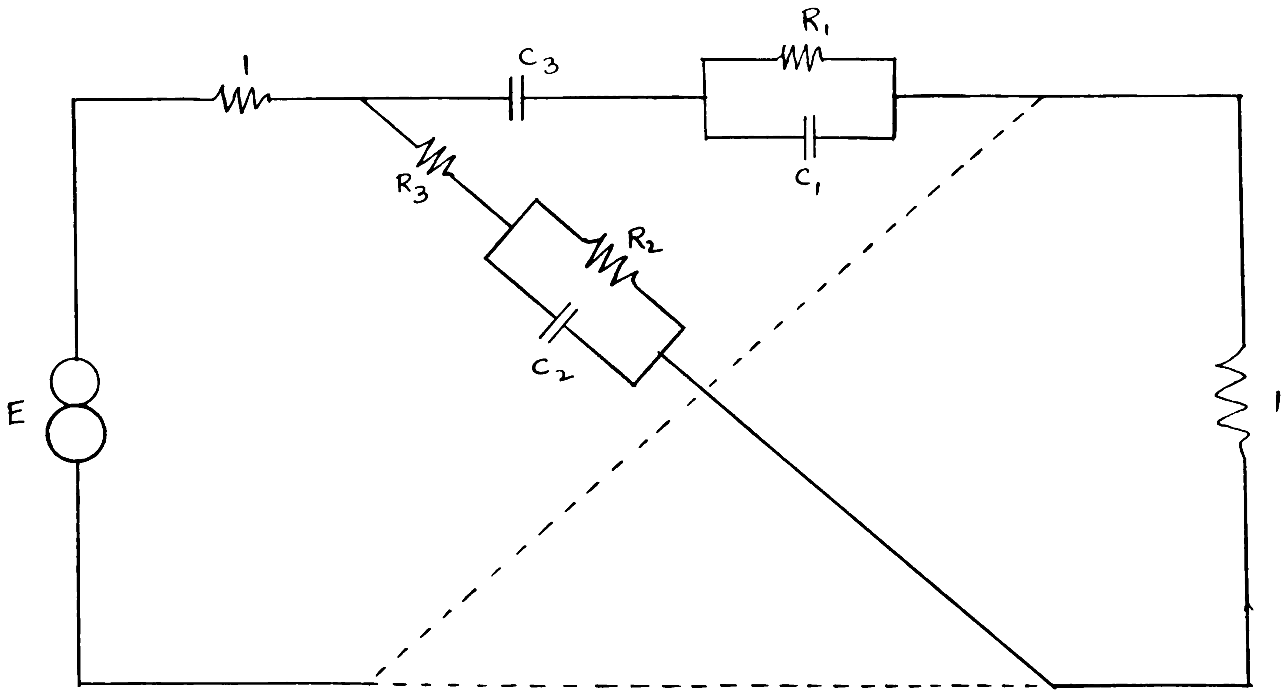


FIG - 35

Development into a single ladder structure:

We know that the transfer function of a network may be given by

$$Y_{12} = \frac{y_{12}}{1 + y_{22}} \text{ ----------------- } (77)$$

If the transfer function of a network is specified, it is possible to find out short circuit drilling point and transfer admittances.

From the driving point function it is always possible develop a ladder structure. But in four terminal cases this should be developed in such a way that the pair of terminals placed at the far end also realizes the transfer admittance Y_{12} (or Z_{12})

The poles of both Y_{22} and Y_{12} are complex natural frequencies of the network and the ladder development automatically gives a network in which the transfer function has proper poles.

The only thing left now is that of how one can produce the proper zeros of the transfer admittance. In a ladder network, it is possible to produce zero of the transfer function by making either the series branch impedance infinity or shunt branch impedance zero at the appropriate frequencies. This condition is a necessary one (but not sufficient) to produce zero of transfer function. This is not a sufficient condition because the transfer function of the network portion to the left of the branch we are considering may have pole at the same frequency, if the driving point function, looking at the left of this point, contain this pole.

This will be our method of producing zero at the desired frequency throughout the synthesis process.

A numerical example will show the whole procedure clearly:

Let us assume the functions:

$$Y_{22} = \frac{(P+1)\ (P+3)\ (P+6)}{(P+2)\ (P+4)\ (P+7)} \quad \text{------------(207)}$$

$$Y_{12} = \frac{(P+2.5)^2\ (P+7.5)}{(P+2)\ (P+4)\ (P+7)} \quad \text{-------------(208)}$$

From the equations it is clear that none of the zeros of the driving point admittance has fallen where transfer admittance has a zero. So, we follow the procedure below as remedy for it.

Let us calculate the value of

$$Y_{22}\ (-2.5) = \frac{7}{9} \quad \text{-----------------------(209)}$$

Now we calculate the value of Y_{22} at $P = 0$

$$Y_{22}\ (0) = \frac{9}{28}$$

If the zero frequency value of Y_{22} were larger than that of the value of Y_{22} at the point $P = -2.5$, we could shift a zero of Y22 to the point $P = 2.5$ simply by subtracting a positive real constant which is a conductance in the actual network. But this process isnot good here because

$Y_{22}\ (0) < Y_{22}\ (-2.5)$

So, we have to solve it in another way. This process is to shift the zero at $P = -3$ of Y_{22} to $P = -2.5$ and that is through the subtraction of an appropriate part of the pole at $P = -2$.

This pole may be given by the component admittance.

$$Y_1 = \frac{K_1 P}{P+2} \qquad \text{----------------(210)}$$

We can find out K_1 by equating $Y22\ (-2.5)$ with Y_1 at the point $P = -2.5$, and by subtracting this value of Y_1 from $Y22$, we can produce a zero at $P = 2.5$

$$Y_{22}\ (-2.5) = \left.\frac{K_1 P}{P+2}\right|_{p = 2.5} \qquad \text{---------(211)}$$

$$\text{or } K_1 = \frac{7}{45}$$

So the admittance to be subtracted from $Y22$ is

$$Y_1 = \frac{\frac{7}{45} P}{P+2} \qquad \left[\begin{array}{l}\text{where,}\\ R = 6.43 \quad C = .079\end{array}\right] \qquad \text{-----(212)}$$

Thus the procedure shows that we have to build a shunt branch admittance of the value given by Y_1, so that the network at the left of this point produce a zero at $P = -2.5$.

Hence the remaining admittance function is,

$$Y_{22}^{(a)} = Y_{22} - Y_1 \qquad \text{-----(213)}$$

$$= \frac{P^3 + 10P^2 + 27P + 18}{(P+2)(P+4)(P+7)} - \frac{\frac{7}{45} P}{P+2} \qquad \text{-----(214)}$$

$$= \frac{.845P^3 + 8.3P^2 + 22.5P + 18}{(P+2)(P+4)(P+7)} \qquad \text{-------(215)}$$

$$= \frac{(.345P^2 + 6.2P + 7)(P + 2.5)}{(P+2)(P+4)(P+7)} \qquad \text{---------(216)}$$

Now if we carefully observe the shunt branch admittance, we find that this does not produce a zero because the Y_{22}' to the left of this branch still contains a pole at $P = -2$ and the same is true for Y_{12}. Actually in removing the shunt branch Y_1, we removed only a part of the pole Y_{22} at $P = -2$. But if the whole pole was removed, the branch would have produced a zero at $P = -2$.

So, in the next steps, we follow the same zero shifting technique; but by removing a part of the admittance function, we cannot shift the poles of the function. This may be remedied by the followin way. Through the application of the shifting procedure first to the given function and the remainder is then inverted and zero of the inverted function is shifted, which is actually the pole of the original function. So, both the zeros and poles of a function may be shifted to any desired position regardless of where the critical frequencies originally were.

Now, we totally remove the pole at $P = -2.5$ from the inverted function because we have to produce a zero of Y_{12} at this point. So we calculate the residue of Z_{22}' at this point.

$$Z_{22}^{(a)} = \frac{1}{Y_{22}'}$$

$$= \frac{P^3 + 13P^2 + 50P + 56}{(.345P^2 + 6.25P + 2.5)} \qquad \text{-------(217)}$$

$$= \frac{A}{(.345P^2+6.25P+7)} \frac{B}{(P+2.5)} \qquad (218)$$

or, $B = 1$

So, a series branch impedance of value

$$Z_2 = \frac{1}{P+2.5} \text{ ----------- } (219)$$

is extracted following the shunt branch admittance

The remainder may be given by,

$$Z_{22}^{(6)} = Z_{22}^{(a)} - Z_2 =$$

$$Z_{22}^{(b)} = Z_{22}^{(a)} - Z_2 = \frac{(P+2.8)\ (P+6.8)}{(.345P^2+6.25P+7)\ (P+2.5)} \text{ ------- } (220)$$

In the above discussion Z_2 is a series branch, the values of resistance and capacitance of which are given below and this branch produces a zero at $P = -2.5$

$R = 0.4 \quad C = 1$

The first cycle of the network development is complete. We have partially developed the given Y_{22} and also we have been able to produce a zero of Y_{12}. So, we follow the same procedure to develop the network completely and to produce all the zeros of Y_{12}.

So, we start the second cycle by calculating the value of $Y_{22}^{(b)}$ at $P = -2.5$ again.

$$Y_{22}^{(b)}\ (-2.5) = -2.54 \text{ -------- } (221)$$

Since $Y_{22}^{(b)}$ (-2.5) is negative, we cannot subtract a positive constant (removal of conductance) from this value.

But the zero shifting procedure may be continued by partial removal of the pole at P = -2.8. This is done by subtracting a shunt branch admittance Y_3. By the method given above, the value of shunt branch admittance was found out.

$$Y_3 = \frac{.325}{P+2.3} \text{ ------------------(222)}$$

The values of the elements may be given by,

R= 3.1 and C= 0.01

The remainder is

$$Y_{22}(c) = Y_{22}(b) - Y_3 = \frac{.325\ (P+2.5)\ (P+5.4)}{(P+6.8)\ (P+2.8)} \text{ -----(223)}$$

Following the above cycle,

$$Z_{22}(c) = \frac{(P+6.8)\ (P+2.8)}{.325\ (P+2.5)\ (P+5.4)} \text{ ---------(224)}$$

From which a series branch impedance Z_4 may be subtracted to produce a zero of transmission at P = -2.5 and Z_4 may be calculated as

$$Z_4 = \frac{.85}{P+2.5} \text{ -----(225)}$$

where

R = .34 C = 1.18

The second cycle is now completed and the remainder is,

$$Z_{22}(d)\ Z_{22}(e) - Z_4 = \frac{1.9(P+6.6)}{(P+5.4)} \text{ ----------(226)}$$

$$Y_{22}(d) = \frac{(P+5.4)}{1.9(P+6.6)} \text{ -------(227)}$$

The only point left now at which Y_{12} is to be zero is $P = -7.5$. $Z_{22}^{(d)}$ can be made to have a zero at $P = -7.5$ by subtracting a positive real constant (removing a resistance) i.e.

$$Z_{22}(-7.5) = .8 \qquad \text{---------(228)}$$

$$\text{at } P \to D \quad Z_{22}(D) = 1.9 \qquad \text{---------(229)}$$

So, a resistance of .8ohm may be extracted from $Z_{22}^{(d)}$

$$R_5 = .8 \qquad \text{----------------(230)}$$

The remainder of $Z_{22}^{(d)}$ is

$$Z_{22}^{(e)} = Z_{22}^{(d)} - R_5 = \frac{1.9\,(P + 6.6) - .8}{P + 5.4} \qquad \text{-----(231)}$$

$$= \frac{1.1P + 7.9}{P + 5.4} \qquad \text{-------(232)}$$

$$\text{or } Y_{22}^{(e)} = \frac{.[illegible]13 + 4.[illegible]}{P + 6.8} \qquad \text{-------------------(233)}$$

$$= \frac{.28}{P + 6.8} + \qquad \text{---------(234)}$$

Equ. (244) shows that the next shunt branch may be represented by,

$$Y_5 = \frac{.28}{P + 6.8} \qquad \text{-------(235)}$$

R = 5 and C = .02[illegible]4

only portion left now is a constant and may be represented by

$$Z_7 = R_7 = \frac{1}{.72} = 1.39 \text{ ohms.}$$

The network is shown in figure (36). In the network it is clear that the zero producing branches are Z_2, Z_4, and Y_5. The remaining branches are zero shifting. But choosing of these zero producing and zero shifting branches are quite arbitrary

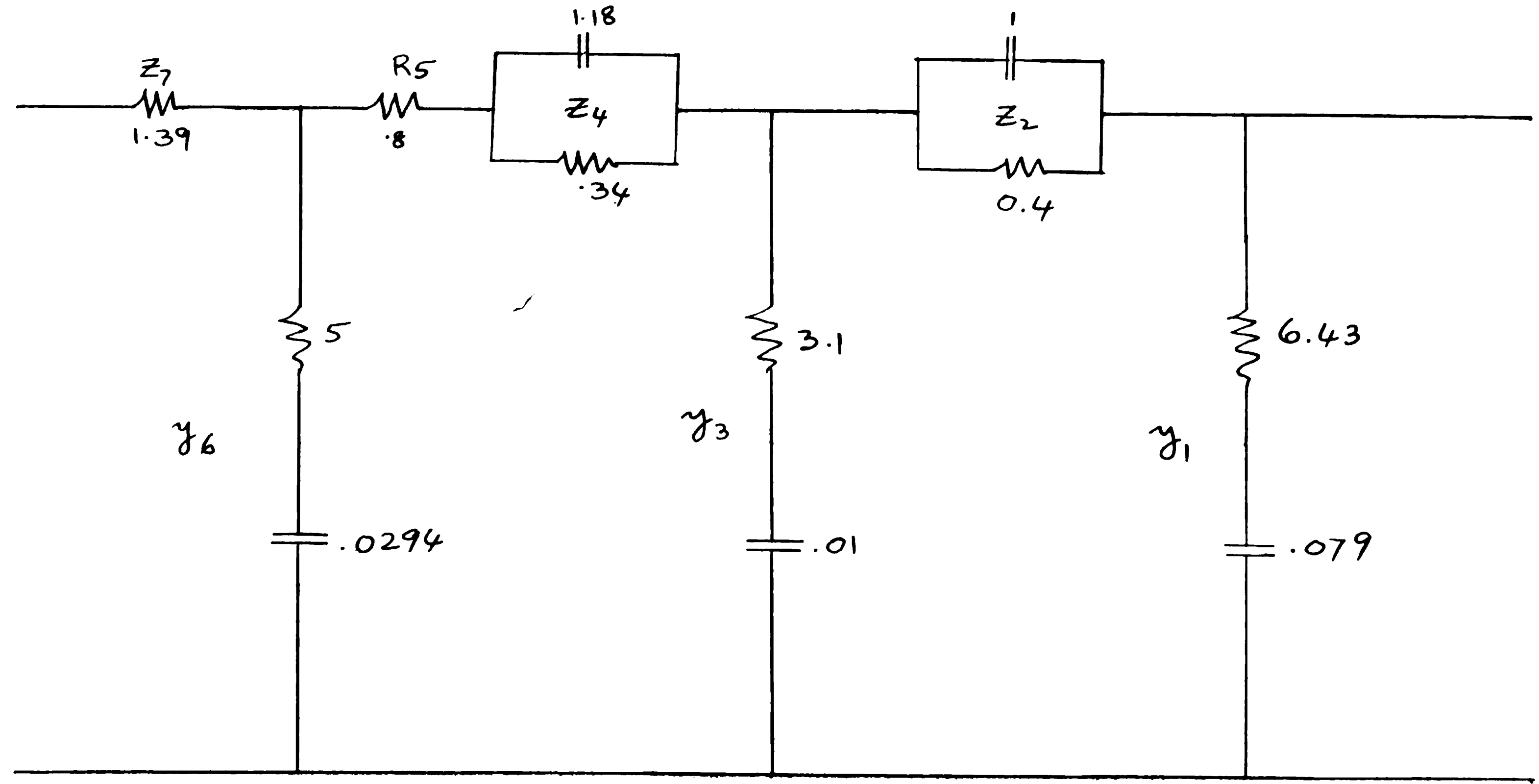


FIG - 36

Parallel T RC Network:

Recently, the electrical engineers have shown considerable interest in parallel T RC filters because this filter has improved rejection characteristic and can suppress a single frequency perfectly when used as band elimination filter. Parallel T RC network can also be used for measurements at radio frequency. Another advantage is that it can be made compact for low frequency applications. Highly selective parallel T RC network can be made for frequency as low as 10 C.P.S.

Most of the available literatures on parallel T RC network assumed that the generator in the input side should be of very low impedance and the load impedance should be infinite. But very recently Y oono (47) overcame this restriction. In the following, first we shall discuss the restricted parallel T and then the method suggested by Y. oono.

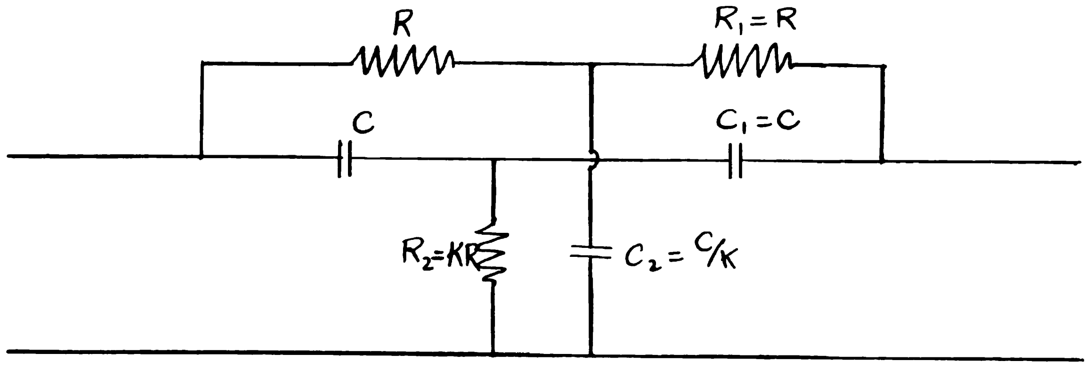


FiG-37

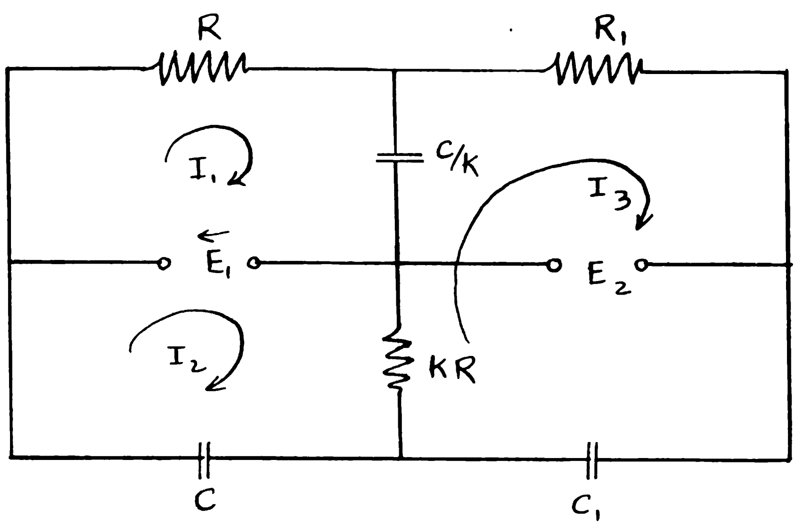


FiG-38

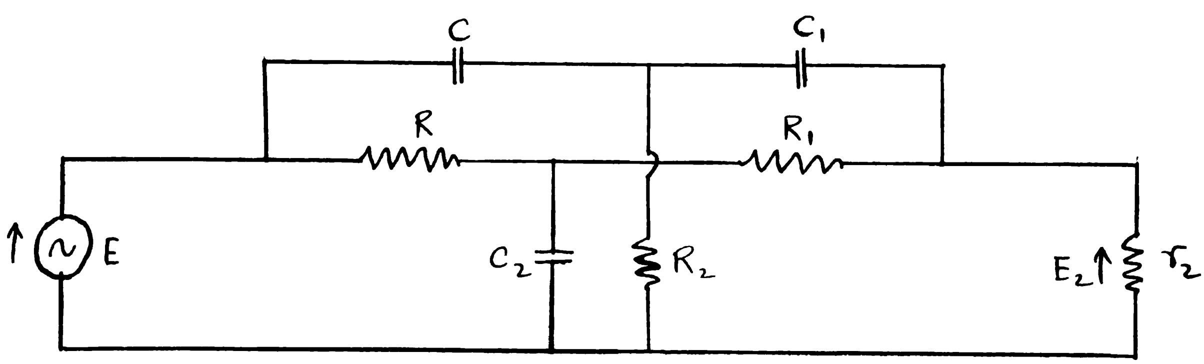


FiG-39

Band Pass Filter:

Let us consider the figure (27) and the figure (33), the equivalent network for parallel T. According to the procedure suggested by J. Tower (6) we assume that $R = R_1$, $C = C_1$, $R_2 = KR$ and $C_2 = \frac{C}{K}$. Now, if we substitute $E_2 = BE_1$ and write the loop equations, we have

$$E_1 - I_1\,(R - JK/WC) + I_3\,(-JK/WC) = 0 \text{ ------(236)}$$

$$E_1 + I_2\,(KR - J/WC) - I_3\,KR = 0 \text{ ---------(237)}$$

$$BE_1 + I_3\,(R - JK/WC) - I_1\,(-JK/WC) = 0 \text{ ------(238)}$$

$$BE_1 + I_2\,KR - I_3\,(KR - J/WC) = 0 \text{ -----------(239)}$$

From these equations, we can solve for B and we have,

$$B = \left\{KR^3 - (2K^2R/W^2C^2) + \left[J\,(K/W^3C^3) - (2K^2R^2/WC)\right]\right\}$$

$$\left\{KR^3 - (2KR/W^2C^2) = (2K^2R/W^2C^2) - (R/W^2C^2) + J\left[(K/W^3C^3) - (2K^2R^2/WC) - (2KR/WC) - (R^2/WC)\right]\right\} \text{ -------(240)}$$

If R is set equal to $1/w_0c$, then,

$$B = \left\{(K - 2K^2) + J(K - 2K^2)\right\} / \left\{(K - 2K - 2K2 - 1) + J(K - 2K - 2K^2 - 1)\right\} \text{ -----(241)}$$

which reduces to

$$B = \frac{2K^2 - K}{2K^2 + K + 1} \text{ -------(242)}$$

From this equation it is clear that at a frequency where $R = \frac{1}{W_0C}$, B is real and has a magnitude dependent upon only K when K = 0.5, then B = 0, which is the null point. At this point, the network also produces an abrupt phase reversal. If K is less than 0.5, there is only a partial null and network phase shift attains a value of 180° degrees as is shown by the real, negative value of B. Phase reversal occurs more gradually as the slope of the curve decreases with K. The phase shift and attentuation curves are shown in figures (40) and (41) which are taken from the paper of J. Bower (6). The curves were obtained experimentally. The response curve of such network is shown in fig. (42)

An example is shown below:

Let $F_c = 50$ C/S and B.W = 20

From the response curve of fig. (25) K = .25

$$\text{Now, } WC = \frac{1}{R}$$

$$\text{or, } C = \frac{1}{RW} = \frac{1}{2\pi.50.R}$$

$$\text{or, } R = \frac{1}{100\pi.C}$$

if we assume C = 0.1 GFd then,

$$R = \frac{1}{100.\pi.10-7} = \frac{10^5}{\pi} = 3.2X10^4$$

the elements are:-

$R = R_1 = 3.2X10^4$ $\qquad$ $C = C_1 = C10$ Gfd

$R_2 = .25 \text{ X } 3.2 \text{ X } 10^4$ $\qquad$ $C_2 = \frac{.1}{.25} = .4$ Gfd

$= 8 \text{ X } 10^3$

FIG-40

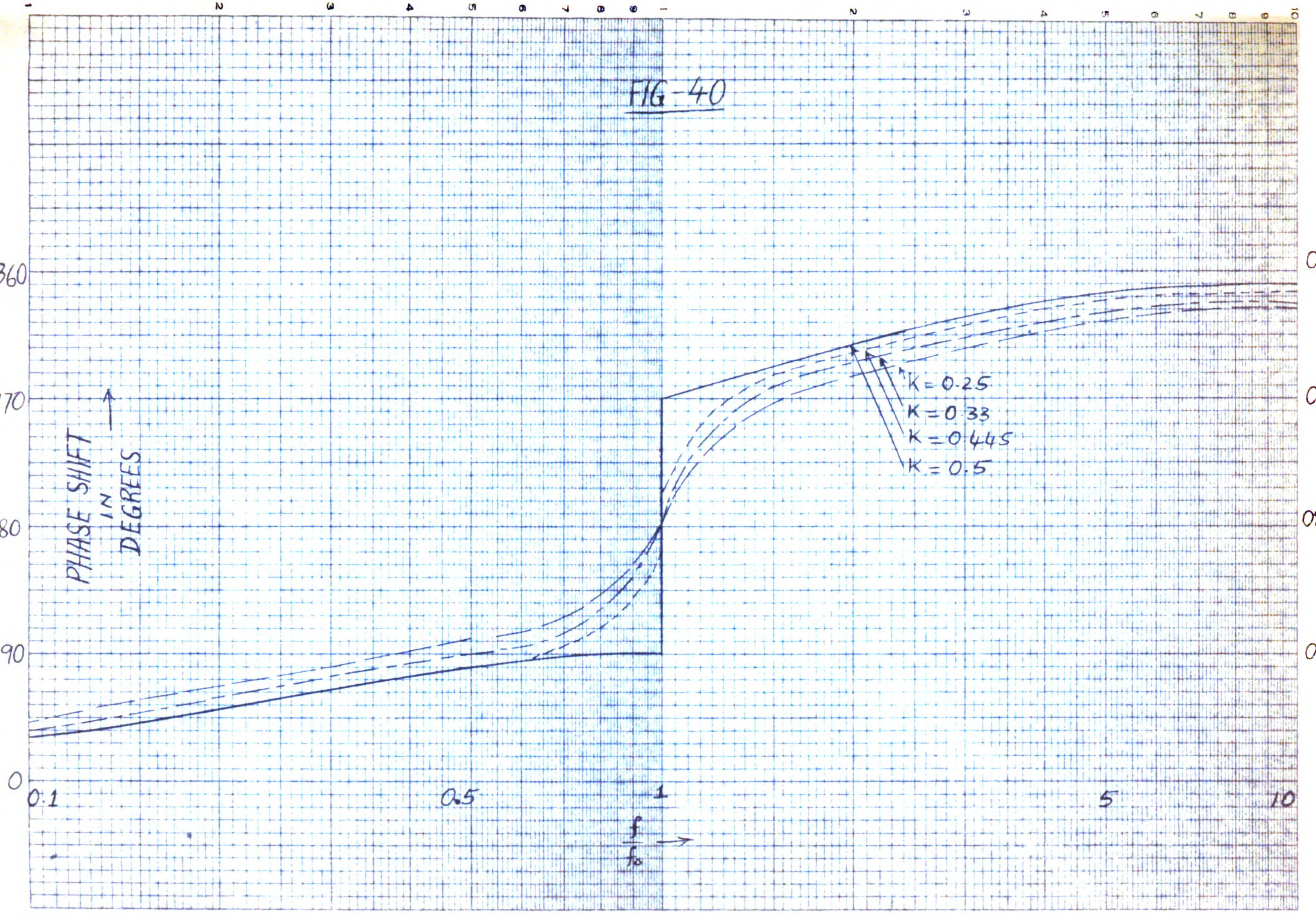

PHASE SHIFT IN DEGREES
360
270
180
90
0
0
90
180
270
K = 0.25
K = 0.33
K = 0.445
K = 0.5
0.1
0.5
1
5
10
$\frac{f}{f_0}$

FIG-41

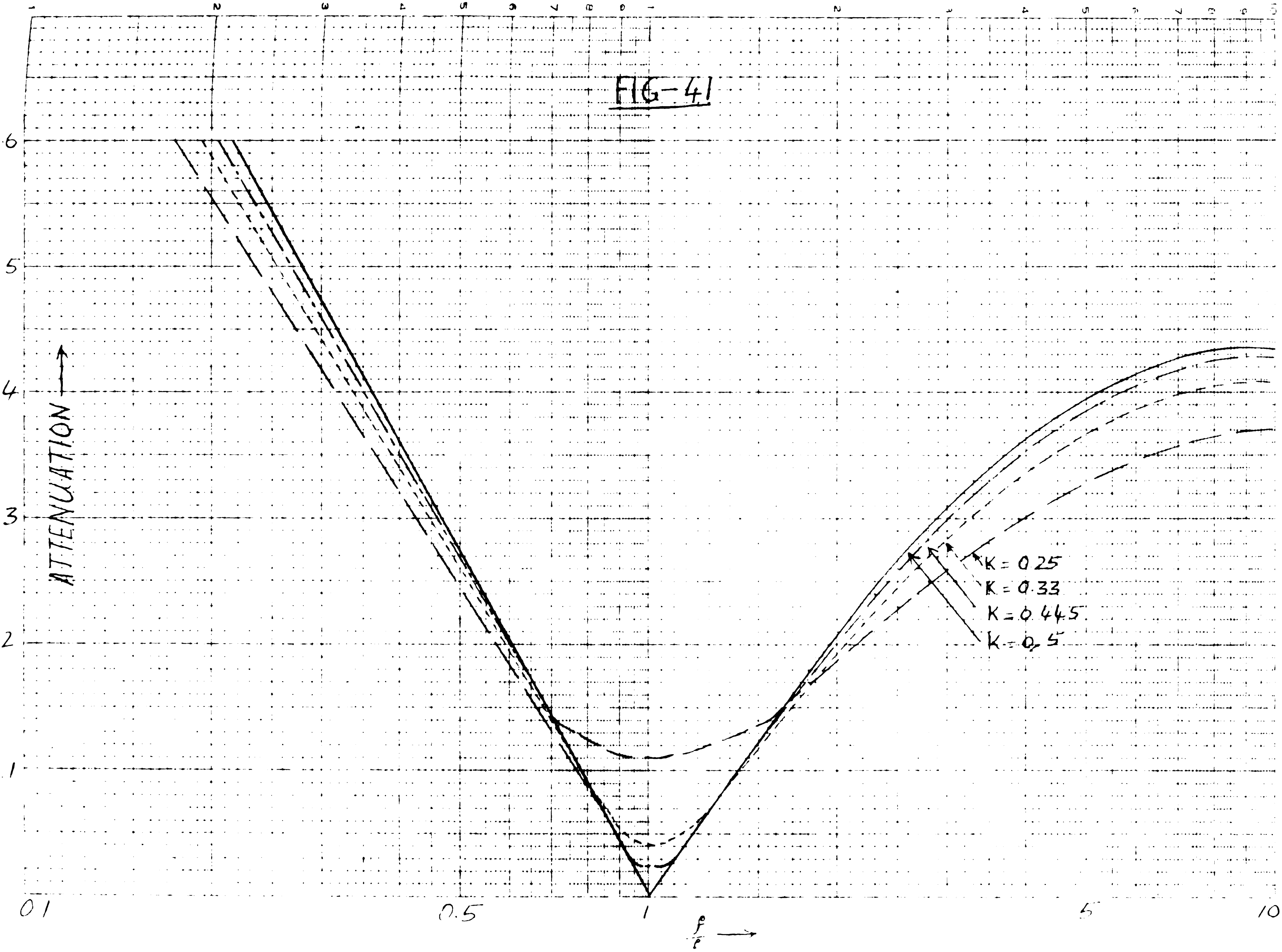
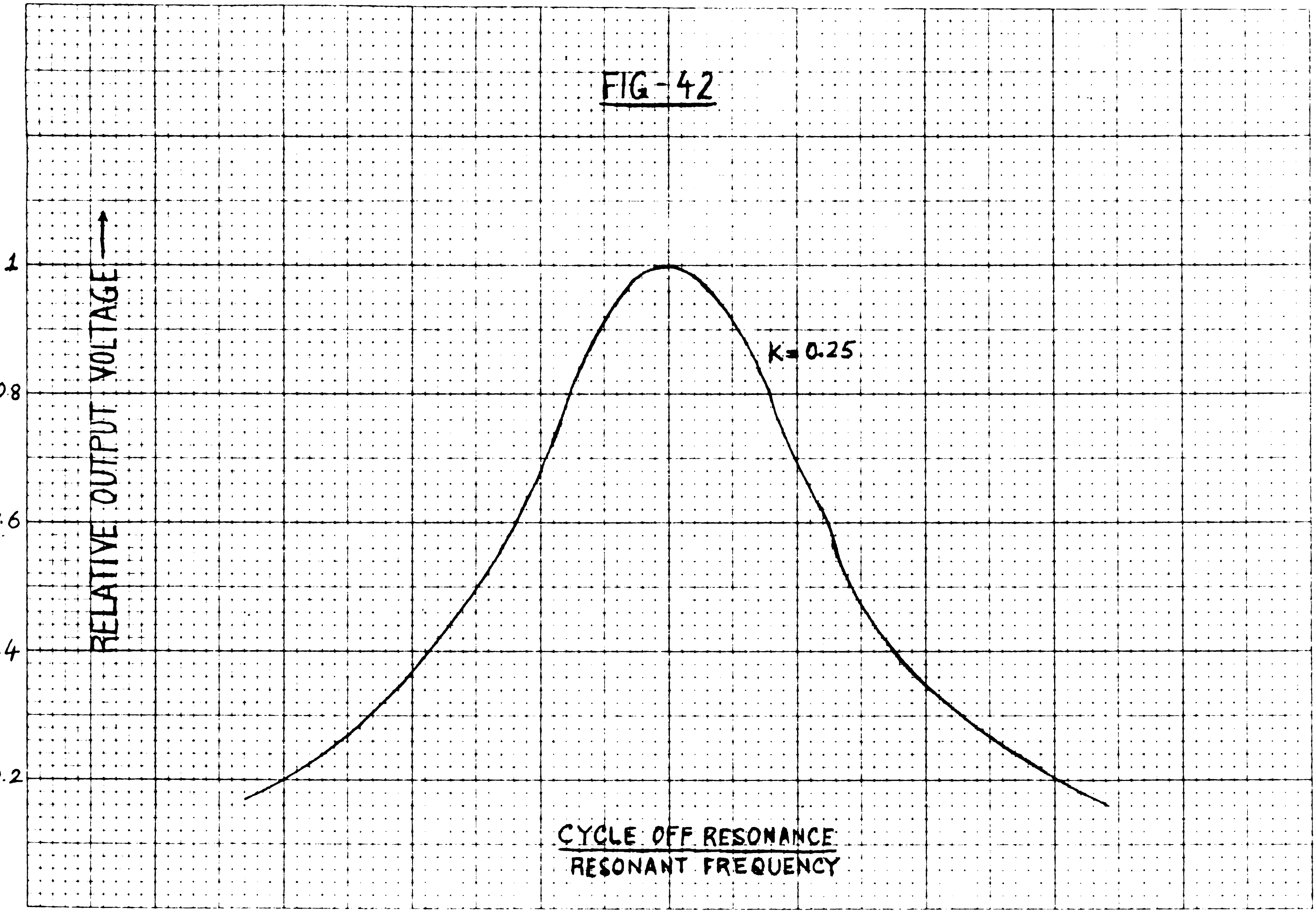


FIG - 42



RELATIVE OUTPUT VOLTAGE
1
0.8
.6
.4
.2
K = 0.25
CYCLE OFF RESONANCE
RESONANT FREQUENCY

Band Elimination Filter:

This more general method was suggested by Y. Oono(47) very recently. Let us consider the fig. (39) where $\gamma_1 > 0$ and $\gamma_2 < 0$. It is assumed that the transfer characteristic of the network should be symmetrical with respect to the resonant (infinite attenuation) frequency and

$$\frac{R_1}{R} = \frac{C}{C_1} = K \text{ -------(243)}$$

or, $R_1 = KR$ and $C_1 = \frac{C}{K}$ ---------(244)

From Stanton's (61) paper let,

$$\left.\begin{array}{ll} a = \frac{R}{R + R_1} & P = J\frac{f}{f_o} \\ \alpha = \frac{R_1}{R + R_1} & b = \frac{X}{X + 1} \\ q = \frac{X + X_1}{R + R_1} & \beta = \frac{X_1}{X + X_1} \end{array}\right\} \qquad (245)$$

$$R_2 = b\beta q^2 (R + R_1) = RK/\mu^2(1+K) \qquad (246)$$

$$C_2 = \frac{a\alpha}{Kq}(R+R_1) = C(1+K)/\mu^2 K \qquad (247)$$

where, $\mu = 2\pi f_o RC$ ... ... (248)

f_o = Resonant frequency

The transfer function of the network is calculated as

$$\frac{E_2}{\quad} = \frac{P^2 + 1}{FP^2 + GP + H} \qquad (249)$$

where $F = (1+\mu)^2 (1+K) \frac{r_1}{KR} + \left(1+\frac{r_1}{r_2}\right)$ ... ... (250)

$$G = \frac{1}{\mu}(1+\mu^2)(1+K)\left(\frac{1}{K}+\frac{r_1}{r_2}+\frac{r_1}{KR}\right) + \frac{1}{\mu}(1+K)\frac{R}{r_2} \quad \ldots \quad (251)$$

$$H = (1+K)\frac{R}{r_2} + \left(1+\frac{r_1}{r_2}\right) \quad (252)$$

Since the response is symmetrical with respect to resonant frequency, we have,

$$\frac{E_2}{E_1}\Big|_P = \frac{E_2}{E_1}\Big|_{1/P} \quad (253)$$

so, $F = H$ --------(254)

or, $(1+\mu^2)\, r_1/KR = \frac{R}{r_2}$ ... ... (255)

Let $r_1 = n r_2$ & $R = x r_1$ ... ... (256)

$$\mu^2 = nKx^2 - 1 \quad \ldots \quad \ldots \quad (257)$$

our transfer function becomes

$$\frac{E_2}{E_1} = \frac{1}{M}\,\frac{P^2+1}{P^2+\Delta P+1} = \frac{1}{M}\,\frac{1}{1+\Delta\frac{P}{P^2+1}} \quad \ldots \quad \ldots (258)$$

where, $M = 1+n+(1+K)\,nx$ (259)

$$\Delta = \frac{1}{M\mu}(1+K)(nKx+x+2)\,nx \quad \ldots \quad (260)$$

now, $\frac{E_2}{E_1}\Big|_{P=0} = \frac{E_2}{E_1}\Big|_{P=\infty} = \frac{1}{M}$ (261)

which M represents the loss at extreme frequencies. Let f_1 and f_2 ($> f_1$) be the frequencies for which

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{2M}} \quad \text{------}(262)$$

then it can be shown that,

$$\Delta = \frac{f_2 - f_1}{f_0} \quad (263)$$

which is the band width.

The roots of the denominator of equ. (258) must be real and negative and so $\Delta > 2$.

So, if M and Δ are prescribed the network elements can be found out. But if it is desirable to find minimum M for prescribed Δ or vice versa then from equ. (259)

$$K = (M-1-n-nx)/nx \quad \cdots \quad \cdots \quad (264)$$

and $$\mu^2 = -nx^2 + (M-1-n)x - 1 \quad \cdots \quad (265)$$

or $$\Delta = \frac{M-1-n}{M} \, \frac{(1-n)x + M + 1 - n}{-nx^2 + (M-1-n)x - 1} \quad \cdots \quad (266)$$

It can be shown that Δ has a positive minimum value and μ^2 is positive for some positive value of x only if,

$$M > (1+\sqrt{n})^2 \quad \cdots \quad \cdots \quad (267)$$

The value of x for which Δ is minimum is found,

$$x = \frac{M^2 - 2Mn + (1-n)^2}{M(1+n) - (1-n)^2} \quad \cdots \quad (268)$$

and $$\Delta = \frac{M-(1+n)}{\sqrt{M^2 - 2(1+n)M + (1-n)^2}} \quad \cdots \quad \cdots \quad (269)$$

minimum value of M is found

$$M = 1 + n + 2\sqrt{n}\,\frac{\Delta}{\sqrt{\Delta^2-4}} \quad \cdots \quad \cdots \quad (270)$$

The relations (269) and (270) are shown in fig. (43). the values of X, K and μ are determined as, $x = \frac{\alpha}{\beta}$, $K = \frac{\gamma}{\alpha}$ & $\mu = \sqrt{nKx^2-1}$

where $$\alpha = M^2 + 2nM + (1-n^2) = 2n\frac{\Delta^2+4}{\Delta^2-4} + 4\sqrt{n}\frac{\Delta}{\sqrt{\Delta^2-4}} + 2$$

$$\beta = M(1+n) - (1-n)^2 = 4n + 2\sqrt{n}\,(1+n)\frac{\Delta}{\sqrt{\Delta^2-4}}$$

$$\gamma = \frac{1}{n}\left[M^2 - 2M + (1-n)^2\right] = 2\frac{\Delta^2+4}{\Delta^2-4} + 4\sqrt{n}\frac{\Delta}{\sqrt{\Delta^2-4}} + 2n$$

An example is shown below:

Let $f_0 = 50$ $\quad \Delta = 3$

From the graph we chose

$\frac{1}{n} = 2.2$ and

$n = .455$ and $M = 3.2$

$$\alpha = M^2 - 2nM + (1-n)^2 = 10 - 2.9 + 0.3 = 7.4$$

$$\beta = M(1+n) - (1-n)^2 = 3.2(1.455) - 0.3 = 4.4$$

$$\gamma = \frac{1}{n}\left[M^2 - 2M + (1-n)^2\right] = 2.2\left[10 - 6.4 + 0.3\right] = 8.6$$

$$x = \frac{\alpha}{\beta} = \frac{7.4}{4.4} = 1.68$$

$$K = \frac{\gamma}{\alpha} = \frac{8.6}{7.4} = 1.16$$

$$u = \sqrt{nKx^2 - 1} = \sqrt{1.5 - 1} = .707$$

Let $C = 0.01\ \mu$fd

then $C_1 = \frac{C}{K} = .0086\ \mu$fd $\qquad C_2 = C\frac{1+K}{u^2K} = .04\ \mu$fd.

$R = \frac{u}{\omega_0 C} = 2.2 \times 10^5$ ohms $\quad R_1 = KR = 2.56 \times 10^5$ ohms.

$R_2 = \frac{RK}{(1+K)u^2} = 2.4 \times 10^5$ ohms.

$r_1 = \frac{R}{x} = 1.31 \times 10^5 \qquad r_2 = n^{-1} r_1 = 2.9 \times 10^5$ ohms.

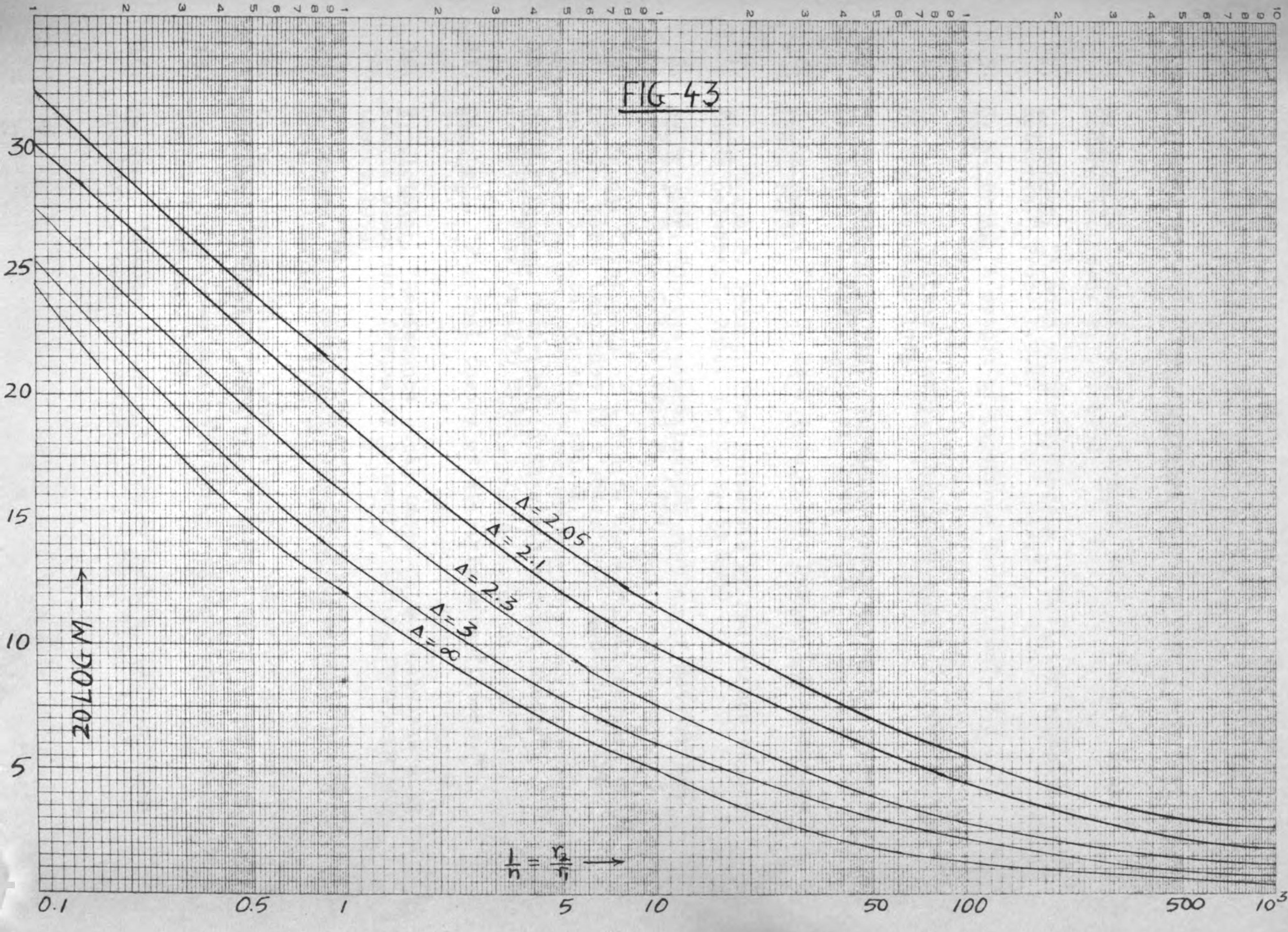


FIG-43
Δ = 2.05
Δ = 2.1
Δ = 2.3
Δ = 3
Δ = ∞
20 LOG M →
$\frac{1}{n} = \frac{r_2}{r_1}$ →
30
25
20
15
10
5
0.1
0.5
1
5
10
50
100
500
10^3

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