HOW ARE THE MATHEMATICAL IDENTITIES OF LOW ACHIEVING SOUTH AFRICAN ELEVENTH GRADERS RELATED TO THEIR ABILITY TO SOLVE MATHEMATICAL TASKS?

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ABSTRACT

HOW ARE THE MATHEMATICAL IDENTITIES OF LOW ACHIEVING SOUTH AFRICAN ELEVENTH GRADERS RELATED TO THEIR ABILITY TO SOLVE MATHEMATICAL TASKS?

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The construct of mathematical identity has recently been widely used in mathematics education with the intention to understand how students relate to and engage (or disengage) with mathematics. Grootenboer and Zevenbergen (2008) define mathematical identity as the students' knowledge, abilities, skills, beliefs, dispositions, attitudes and emotions, that relates to mathematics and mathematics learning. A key part of this relationship that students have with mathematics is the students' evolving sense of self to understand how mathematics fits with this self (Black et al, 2010). Research shows that students' identity has many facets or multiple identities that are formed throughout their life history; engagement with their peers, family, and teachers; as well as engagement with mathematical tasks. Doing mathematics can be viewed as mathematical activity that involves integrating mathematical thinking by using mathematical facts and knowledge, and requires active student learning. Mathematical tasks used in the classroom form the basis for student's learning (Doyle, 1988) and different tasks are used to develop different types of skills and thinking (QCDA report). These tasks often appear in curricular or instructional materials in textbooks and should be within the current experiential space of students (Jurdak, 2008).

This study investigated how the mathematical identities of low achieving South African eleventh graders are related to their ability to solve mathematical tasks. I used a variety of

methods including mathematical stories, biographical interviews, focus-group interviews, mathematical tasks, and clinical interviews. These methods were used to investigate the salient features of mathematical identity, the performance in mathematical tasks, and the relationship between mathematical identity and mathematical performance.

This case study focused on five eleventh graders who were underperforming in mathematics at a low-socioeconomic school located in a working class community in the Western Cape Province in South Africa. The framework for analysis used concepts from Leont'ev's (1981) cultural historical activity theory, but build on the construct of a 'leading identity', which formed the basis for the study's theory of identity (Black et al., 2010). This notion of leading identity suggests a hierarchy of motives, with the most significant motive forming the leading identity.

My results show that the relationship between math identity and math performance are characterized by "*discontinuities and continuities"* (Stentoft & Valero, 2009) between these two components. This conceptualization helped in understanding and explaining the fragility of mathematical identity as produced in practice. I discuss the implications of this relationship between math identity and math performance for future research, teachers, and math education.

DEDICATION

I dedicate this dissertation to my darling wife Faith and my sons Dayle and Dominic, who were there for me throughout this journey, and always encouraged and supported me despite financial hardship and other challenges.
Furthermore I also dedicate this dissertation to all those students' who are struggling with mathematics, but who refuse to change to mathematical literacy.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1
INTRODUCTION

Rationale

Two of the greatest challenges that students of mathematics experience are self confidence and motivation to learn. In my own mathematical journey in high school I found that these attributes together with my attitude towards math had an affect on my actual performance in mathematical activity. In addition the classroom environment had a big impact on my attitude towards my school work, particularly in mathematics. I always had preconceived ideas about my mathematical ability to be successful in mathematics. I liked math and was fairly good at calculating and solving problems. I knew that I hated languages, so I understood the strength of these opposite emotions. My sister hated mathematics and dropped out of it as soon as she was able to. My mom always said that I was born with the math gene, while my sister was just not a math person. She believed in an innate ability to do mathematics, I did not understand this but felt special and different from my siblings and friends who did not have the math gene, because they could not do math or simply hated the subject.

As I moved through the primary grades 3 to 6, I vividly recalled my success in mathematics tests. Generally, I loved mathematics, because it was firstly fun and easy. The arithmetic and geometric curriculum was based on three components; speed of mental calculations, basic computations and calculations, and a word problem section. I remember scoring exceptionally well in the first two components and less well in the word problems. I believed my poor performance in the word problems was based on my dislike for studying language. In grade 3 to 6 we really had good mathematics teachers and I base this deduction on the following activities that resulted in good grades: we had fun activities; we engaged in mental problems regularly, to ensure quick accurate responses; we had lots of drill and practice, especially in computation and calculation; and we received rewards (incentives) for good quick accurate responses. Teachers encouraged and motivated us to help each other. They always reminded us that mathematics was the most important subject and we should do well and help our parents, who did not have schooling or very little of it. Collaborative and individual work was prioritized and balanced.

In grade 7 we had a teacher, dare I say it, from hell? She was moody and disliked by students. In fact, I remember this teacher as if she taught me yesterday. I disliked this teacher intensely. My grades in all subjects dropped below the expected level of development, even in mathematics, I was heading for failure in all three mathematical domains, and for failure in the grade. Unfortunately, in the eyes of our parents teachers could do no wrong. Avenues of complaints were closed and as students we suffered under this teacher who just did not care. My performance in math dropped to an all-time low and I was lucky to pass to grade 8, high school.

In grade 11, we were taught mathematics by a part-time wrestler. The students were afraid of him; they literally shook in their boots during the lesson. The stage was set for a real scary hectic year, but the experience of mathematics teaching and learning in grade 11 changed my future career choice to teaching. This wrestler turned out to be the most brilliant teacher of my school career. He never missed a period (even when he was sick), he never used a textbook, he used colored chalk in his explanations, he started his lesson as if we knew nothing (he made sure that all prior knowledge needed was understood before starting new content), he drew links and connected the mathematics to real life experiences, but most of all he made difficult concepts look so easy. In fact I will never forget his words when a student asked him why he did not give as much homework as other teachers. He replied (these are more or less his words), “If you understand what I did on the blackboard, why must I give you a hundred exercises, I’m not here to waste your time with unnecessary repetition, you may use your textbook and do as many exercises as you like and if you have problems come and see me." The class did extremely well in the final examinations.

It was during grade 11 that I decided to become a mathematics teacher because of the influence of the wrestler's mathematics teaching and his political views on how the apartheid government was oppressing "non-whites" through poor (mathematics) teachers. The apartheid government view was that non-whites could not do mathematics and in this way ensured that non-whites were unable to access core career areas, including engineering, medicine, etc. This was achieved by under-training mathematics teachers and by allowing un-qualified teachers to teach mathematics in 'non-white' schools. Unfortunately, during the first term in grade 12, Mr. Phillips accepted a mathematics Head of Department position at another school and we were left without one of the best mathematics teachers. In grade 12 we had three mathematics teachers in five months. The principal visited the class to observe each of the teacher's, and we did not see them again, because they were asked to resign. During the third term South African schools engaged in student boycotts and riots (the turmoil year of 1976[1]) and our school was the focus of unrest in the Western Cape. We received no further instruction and wrote the final matriculation examination under police protection, despite the fact that we never completed the syllabi.

I use my mathematical story to contextualize this study by introducing myself as a student of mathematics and to convey my interest in exploring the concept that researchers now call *mathematical identity* (Grootenboer & Zevenbergen, 2008; Anderson, 2007; Boaler &

[1] "Internal and external opposition to apartheid was fueled in 1976, when the Soweto uprising began with the protests of high-school students against the enforced use of Afrikaans-viewed by many Africans as the oppressor's language. The protests led to weeks of demonstrations, marches, and boycotts throughout South Africa. Violent clashes with police left more than 500 people dead, several thousand arrested, and thousands more seeking refuge outside South Africa" (One World Nations Online, The beginning of the end of apartheid, para. 1)

Greeno, 2000). Like Devlin (2000) and unlike my mom, I believe that "everyone has the math gene" (p.2). According to Anderson (2007), "learning mathematics is a complex endeavor that involves developing new ideas while transforming one's ways of doing, thinking, and being" (p. 7). The rationale for this study emerged out of exploring the construct *being*, based on Anderson's third view of learning mathematics that "involves becoming a 'certain type' of person with respect to the practices of a community" (2007, p. 7). It is within this focus on social relations and interactions we see how the complexities of race, gender, culture and community, socioeconomic status, and mathematical ability influence an individual's mathematics experiences.

Background

I became interested in research on mathematical identities and was motivated to conduct research within a low-socio-economic school community in South Africa. South Africa urgently needs a mathematically and scientifically literate population to ensure its survival and success in the global arena. Students throughout the country are finding themselves in situations that require high demands on them to master the mathematics content and to apply their knowledge and skills in everyday situations. The Department of Education (1995) envisaged in their new outcomes-based education system (introduced in 1998 and amended in 2001) that all learners have the ability to succeed in mathematics. The mathematics curriculum focuses on the acquisition of knowledge, subject skills, language skills, values, and attitudes, which is much broader than the traditional practices based solely on mathematical content mastery.

Furthermore, the country is still battling to overcome the consequences of the former apartheid education system that according to Kahn (2004) is still "catastrophic" (p. 149). Recently the Rhodes University vice-chancellor Saleem Badat called the state of education in the country a "tragedy" (Solomon, Times Live, 16 January, 2011). Researchers have noted that the new education system does not appear to be yielding satisfactory results and that the subject matter knowledge of the majority of students is alarming (van der Walt, 2009; Volmink, 2010; Khan, 2004; Mullis et al., 2008). In addition "only 36% of learners are achieving the reading and numeracy outcomes expected of a grade 3 learner ... the vast majority of learners in grade 3 (and grade 6) were performing two or three years below expectation" (WCED, 2006, p. 2). Other international studies have also shown that South African students perform poorly on numeracy tests (Trends in International Mathematics and Science Study, 2003; Taylor, Muller & Vinjevold, 2003). The bottom line is that our education system is failing our students (Cranfield, 2005; Cranfield, 2012). Some researchers have blamed these results on the troubled history of introducing the new curriculum with the practical problems of curriculum implementation at the classroom level and its influence on teachers and their classroom practice (Jita & Vandeyar, 2006; Jansen 2001; Taylor & Vinjevold, 1999; Howie, 2001; Howie, 2005; Howie, 2002).

As a consequence of poor student achievement the focus is on teachers' mathematical and pedagogical knowledge. Some researchers have focused their attention on teachers' identities in developing countries (Jansen, 2001; Jita, 1999, 2004; Jita & Vandeyar, 2006; Soudien, 2001). However, limited work has been done with subject-related identities, such as mathematical identities of teachers in developing countries (Jita & Vanderyar, 2006). According to Graven (2008) a key problem in school mathematics classrooms is that students do not want to be in these classes and their educational trajectory tends to be away from mathematics. It is within this context that I frame the research problem. My 19 years experience teaching mathematics seems to validate Graven's observation. Furthermore, I agree with Stentoft and Valero (2009) that the "mathematics learner is a natural focus when addressing identity" (p. 59) because it is within these social practices and interaction in the mathematics classroom that students' construct their mathematical knowledge and understandings.

Defining the Problem

"Sir, I hate geometry", "Sir, I hate working with indices", "Sir, these tasks are too difficult", "Sir, why do we have to do this", "Sir, I hate math", "Sir, I have never and will never pass math", and "Sir, my parents and even my primary school teachers even hate math". These were expressions so commonly expressed by my grade 10 students virtually throughout my 18 years of teaching senior mathematics. These exclamations, I believe, are also common to teachers in other schools. This hate for mathematics manifested itself in many different ways: in absolute disinterest in mathematics, in avoiding problem-solving activities and in poor results in mathematics tests and examinations. As a sub-examiner for 10 years and examiner for three years in the Senior Certificate Examination[2], I have witnessed the poor performance of students in the mathematics section of the matriculation examination. It is sad to see the errors that students made. Simply reading through the examiner's reports at the end of each year provided insight into this problem.

Was the above problem related to poor teaching? Was it related to the way students learn or understood mathematics? Was it a combination of the two? Why after a 'good' lesson did

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[2] "The Senior Certificate (SC) Examination, or Matric as it is popularly known, represents a high point of learning in South Africa. Most young people are encouraged to aspire to it. It holds great significance as a rite of passage, as it marks the culmination of twelve years of schooling. It is still, by far, the most popular determinant of access to higher education and, to a lesser extent, to the world of work. Perceived as a "high stakes" examination, the Senior Certificate year-end examination attracts a great deal of public interest. As a public measure of learners' performance, the quality of teaching and learning, and how well the system is doing, it is critical that the exam enjoys public confidence" (UMALUSI, 2004, p. 2)

some students not grasp or understand the task? Students are unique individuals and exposure to doing mathematics generates a range of emotions and feelings. As a teacher I needed to understand the basis for these responses, which undoubtedly contributed to their inability to learn mathematics. I had to realize that a great lesson did not always lead to great learning. This led to another string of questions. What happens when students are required to perform mathematical tasks and problems when they have not mastered the mathematical concepts, ideas, knowledge, facts, procedures and skills of the previous grade? More specifically: how do students' beliefs about mathematics, beliefs about their own mathematical ability, attitudes towards mathematics, motivation to engage mathematically, and a history of low achievement in mathematics affect the way they learn and do mathematics? What narrative emerges from the way these students learn and do mathematics? How does this narrative affect the individual students? What are the ways that students think about themselves in relation to the mathematics they are required to learn[3]? What is the extent to which these students develop a commitment to or see the value in, mathematics as they engage in the classroom mathematics?

I have been intrigued by this string of questions for many years. Connecting this string of questions to curriculum change in South Africa poses a challenging context for any research where all stakeholders operating within the education system are grappling with educational change. The purpose of this research study is to gain as much information as possible from the students' perspective, their mathematical stories, their views, and their opinions, especially where they constructed meaning as they related their mathematical story and as they engaged in doing mathematics. By operating as Creswell's (2003) constructivist researcher I made sense of

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[3] It is a mandatory for all students to enroll in one of two tracks called "mathematics" or "mathematical literacy" in grades 10, 11, and 12.

these meanings that students' had about their mathematical world. It is within this context that my research focused on exploring the mathematical identities of students and exploring how these mathematical identities related to their ability to solve mathematical tasks. I explored how students constructed their mathematical identity, how these identities were mediated by the students' culture and background, classroom interaction, and how these identities evolved as the students engaged in mathematical tasks.

Studies Relating to this Purpose

The construct of mathematical identity is being widely used in mathematics education with the intention understanding how students relate to and engage (or disengage) with mathematics. Grootenboer and Zevenbergen (2008) define mathematical identity as the students' "knowledge, abilities, skills, beliefs, dispositions, attitudes and emotions" (p. 244) that relate to mathematics and mathematics learning. As a key part of the relationship that students have with mathematics is the students' "evolving sense of self and their understanding of how mathematics fits with this" (Black et al, 2010, p. 56) and "the perceptions of individuals regarding their abilities to participate and perform effectively in mathematical contexts" (Nzuki, 2010, p. 79).

In addition Martin (2000) defines mathematical identity as being shaped by students' "beliefs about (a) their ability to perform in mathematical contexts, (b) the instrumental importance of mathematical knowledge, (c) the constraints and opportunities in mathematical contexts, and (d) the resulting motivations and strategies used to obtain mathematics knowledge" (p. 19). Taking this view of mathematical identity into consideration Nzuki (2010) argues that it is "imperative to draw from their past and perceived present experiences and to link these experiences to their anticipated future experiences". By "linking past, present, and future experiences, one can examine the students' trajectory of mathematical experiences given that it is within this trajectory that students' identities are developed and refined as a result of the cumulative effect of their mathematical experiences" (p. 95).

It is clear from the research that an identity has many definitions and that students' identities have many facets or multiple identities that are formed through their life history and engagement with their peers, family, and teachers (Sfard & Prusak, 2005; Sfard & Prusak, 2005a; Helms, 1998; Enyedy, Goldberg, & Welsh, 2006; Lave & Wenger, 1991; Lemke, 2000; Holland, Lachicotte, Skinner, & Cain, 1998; Gee, 2005; Grootenboer et al., 2006). A quick sample of the definitions represents the diversity of ideas about identity; identity is a sense of self (Helms, 1998); identity is a fluid, dynamic, recursive, discursive process (Enyedy, Goldberg, & Welsh, 2006); identity is a long-term, lived relations within a community of practice (Lave & Wenger, 1991; Lemke, 2000); identity is based on self understandings that have strong emotional resonance (Holland, Lachicotte, Skinner, & Cain, 1998); and identity is recognition by self or others as a certain 'kind of person' (Gee, 2005).

The activity of 'doing mathematics' is also widely used in math education research. Doing mathematics can be viewed as mathematical activity that involves integrating mathematical thinking by using mathematical facts and knowledge, and requires active student learning. Doyle (1988) argues that mathematical tasks used in the classroom form the basis for student's learning. Different tasks are used to develop different types of skills and thinking, but "yet all tasks should be about 'doing mathematics'" (QCDA report, p.2). Stein and Smith (1998) endorse this idea and define a mathematical task "as a segment of classroom activity that is devoted to the development of a particular mathematical idea" and it can "involve several related problems or extended work" (p. 269). These tasks often appear in curricular or instructional materials in textbooks. Also, these problem tasks should be “within the current experiential space of students” (Jurdak, 2008, p. 67).

The research question

I am particularly interested in researching the notion that “identity is a unifying and connective concept that brings together elements such as life histories, affective qualities and cognitive dimensions” (Grootenboer & Zevenbergen, 2008, p. 243). It is within this context that I frame my research question. The central focus of my research is aimed at answering the following question: ***How are the mathematical identities of low achieving South African eleventh graders related to their ability to solve mathematical tasks?***

The following list of sub-questions will be used to guide the study.

1. What are the salient features of the students’ mathematical identity?
2. How do students solve mathematical tasks?
3. How does their ability to solve tasks relate to their mathematical identity?

Importance of study

While the impetus for this study was provided by my experience as a student of mathematics, my own teacher education and my academic grounding, the motivation for this study stems from my concern for those students who have been marginalized because of their history of poor achievement and affective responses towards mathematics. I base this observation on my 18 years of teaching mathematics and eight years of school-based developmental work in primary and secondary schools. Recently, during the piloting of the research instruments, a teacher referred to some of these students as “no hopers,” and that I should avoid working with these types of students. Since these students seemingly show no interest in doing mathematics, some teachers simply allow them to occupy space in the class without any direct entrée into the classroom mathematical activity. In my practice I have encountered teachers whose key task was to point out their students' deficits in mathematics and defects in personality, rather than making attempts to understand the contingencies that create this problem of deficits and defects.

Weathers (2011) suggest that these demeaning stories result in children internalizing these attributes as part of their "dumb kid" (an umbrella term that includes bad, dumb, lazy, angry, ADHD) self-concept story, which becomes a self-defeating, self fulfilling prophecy. According to Weathers (2011) these self-concept stories are "not a logical, rational process" but, result from a "compilation of hundreds of stories slowly synthesized over time into a self-concept that is a collage of what they have heard about themselves from others" (p. 4). Lamotte (n.d) refers to the challenge of the "changing needs and demands that at-risk students place on the educational agenda" (p. 1), while the complexity of the problem lies in the lives of the at-risk students that demonstrate the multidimensional nature of low achievement in mathematics (Bracey, 2002).

Research has noted persistent issues relating to student participation and engagement in mathematics, but there seems to be little progress towards improving this situation where "students hold unhelpful and unhealthy views of mathematics" (Grootenboer & Zevenbergen, 2008, p. 243). It is precisely this phenomenon that led to researchers exploring a variety of *identity* conceptual frameworks, and recently focusing on mathematical identities, which includes "knowledge, abilities, skills, beliefs, dispositions, attitudes and emotions related to mathematics and mathematics learning" (Grootenboer & Zevenbergen, 2008). This notion of (mathematical) identity is becoming increasingly more common in research literature in mathematics education (Cobb et al., 2009). As with my narrative, everyone has experiences with mathematics from before attending primary school, through primary and high school, and through tertiary education. All these experiences combine in different ways to make up our mathematical identities.

Since the research sample will only include the mathematical identities of low-achieving students, I firmly believe that this study will contribute and add value to the limited research related to this subject in developing countries. We need to ascertain what factors are behind the choices that students' make to study or not study mathematics, but at the same time to be able to relate this with their learning of mathematics and ability to do mathematics, in this case the ability to complete mathematic tasks.

Overview of the dissertation

This dissertation is organized into seven chapters. In this first chapter I provide an introduction to the study by briefly presenting my mathematical journey by focusing on how I developed my mathematical identities. In addition I defined the problem by noting the complexities in attempting to define mathematical identity, as well as providing the research purpose and guiding questions.

In chapter 2, I present a review of the literature intended to locate my study within a particular body (or bodies) of scholarship in order to contextualize and give reasons and meaning for its conduct. This review and the conceptual framework provide the theoretical background for the study. I focus specifically on studies of the use of math stories that emphasize the development of mathematical identities, and how these identities are shaped by interactions, engagements, and contributions within mathematics classrooms as well as outside these classrooms. I present this review by relating the literature that informed each of my guiding sub-questions: constructing mathematical identity, performance in mathematical tasks, and the relationship between math identities and math performance. At the end of the chapter I draw on the body of research that led to the development of my conceptual framework.

In chapter 3, I describe my method. I begin by describing my research design, pilot studies, sampling procedures, data collection procedures, research instruments, coding the data, and the framework for data analysis. There are two male and three female eleventh grade students in my sample and the research site was in an impoverished region in Cape Town, South Africa. The data collection instruments included a structured mathematical story, interviews about their mathematical journey, mathematical tasks, clinical interviews about the tasks, and a focus-group interview. At the end of the chapter I describe my framework for analysis.

In chapter 4, I present my results on the salient features of each participant's mathematical identity, by implementing the framework that establishes the leading identity and cultural models (value-statements underpinning beliefs) for each participant. The purpose of this chapter is to present a mathematical identity profile for each participant. My focus is to provide the information in as full detail and in as a systematic way as possible. I make substantial use of each participant's mathematics story, individual interview, and the focus group interview.

In chapter 5, I present my results for the mathematical performance of each participant. The purpose of this analysis was to answer the second research question, "How do students solve mathematical tasks?" I analyzed each participant's response to the mathematical tasks. The structure of this chapter presents the results of the analysis of: the two quantitative methods called the Points-Percentage score and the Four- scale point score. I also present the results of the qualitative method, based on the clinical interview with each participant, that involves a detailed analysis of each student's solution strategies and mathematical thinking in some of the mathematical tasks based on the responses in the clinical interviews. The purpose of this chapter is to present a mathematical performance profile for each participant.

In chapter 6, I present my results for relating the mathematical identity profile with the mathematical performance profile. This analysis will answer the research question "How does their ability to solve tasks relate to their mathematical identity?" The key idea in this chapter is to relate these two components by focusing on the participant's beliefs and perceptions about math abilities that: 1) did not translate in the outcomes of learning, therefore 'discontinuity' (contradicting or not connecting) with the beliefs and perceptions; and 2) did translate in the outcomes of learning, therefore 'continuity' with the beliefs and perceptions. The purpose of this chapter is to report on connects/disconnects between math identity and math performance. Through this exploration it is possible to identify the strengths and weaknesses in the ways these mathematical identities were constructed.

Finally, in chapter 7, I present my discussion; conclusions; and implications for teachers, mathematics education, policy development, and limitations and directions for future research. The purpose of this chapter is to consolidate and synthesize the similarities in the participants' construction of math identity and math performance across the five cases. A cross-case analysis of the results was characterized in most situations by "discontinuity and disruptions". 'Discontinuity' was the focus in chapter 6 and in this chapter I interpret "disruptions" in a very narrow sense as the underlying reasons behind the discontinuity between math identity and math performance.

CHAPTER 2

THEORETICAL BACKGROUND

This chapter is divided into three parts: 1) research related to mathematical identities; 2) research related to learning mathematics and mathematics performance; and 3) research related to the relationship between mathematical identity and mathematical performance.

The study is located theoretically within a sociocultural epistemology which emphasizes the social basis of learning (Forman, 2003; Gee, 2001; Holland et al., 1998; Wenger, 1998); the interactive processes that promote social development (Renshaw, 1992); the interdependence of social and individual processes involved in the construction of knowledge (John-Steiner & Mahn, 1996); the recognition of the importance of both community and individual learning (Jaworski, 2007); and focusing on learning within communities of practice (Lave & Wenger, 1991). By locating this study within this sociocultural theory the purpose of the study is to explore and understand the link between mathematical identity that students develop and the mathematical knowledge that students acquire along their mathematical journey.

In recent years researchers have drawn from Erickson's early work on the development of identity (Erickson, 1959), and have built on this sociocultural perspective by making the notion of identity increasingly visible (Stentoft & Valero, 2009) and increasingly common (Cobb, Gresalfi, & Liao, 2009) in mathematics education research. The notion of mathematical identity is strongly connected to this sociocultural perspective with the intention to explore mathematical learning as a process of participating in communities of mathematics practice (Lave & Wenger 1991; Galbraith, Goos, Reinshaw, & Geiger, 1999). It is within these communities of practice that students interact with their peers and teacher, engage in mathematical tasks, and contribute to classroom norms and standards. According to Boaler and Greeno (2000) student identities are shaped by these interactions, engagements, and contributions. This means that students have been influenced in some way or the other by years of “in-school” and “out-of-school” experiences and consequently developed as a learner of mathematics. They have developed beliefs about teaching, teachers, schools, community-life, curriculum, pedagogy, peers, learning materials, and future careers all of which impact their learning of mathematics. Within this period of time the students confronted contradictions in the belief system, tensions, crisis, ups-and-downs, barriers to learning, and doubts about their ability to do math.

Some students may even see math as useful and relevant, or realize that mathematics is important to their future aspirations or integral to their daily lives. In the construction of mathematical performance we are able to see how students draw on the past and present experiences to define who they are; to define their relationship/dispositions towards mathematics; to define beliefs regarding their ability to do (engage in) mathematics; and to define their future aspirations. Mathematical identity, defined by Boaler (2002) as “the idea that students develop relationships with their knowledge” (p. 16), is a useful first-step in understanding this relationship between mathematical identity and mathematical performance. This idea suggests that as students engage in classroom and mathematical practices they not only develop knowledge and but they also develop a relationship with that knowledge, and now their mathematical identity includes the knowledge they possess, the ways in which they hold knowledge, the ways they use knowledge, the ways their mathematical beliefs interact with this knowledge, and the ways their work practices interact with this knowledge (Boaler, 2002).

Since this study is based on two components, a student's mathematical identity and his or her proficiency in mathematical tasks, it is expedient to focus on what mathematical knowledge was developed. It is within this relationship that the performance of students in math tasks provides an indication of successful or unsuccessful mathematics learning. Wenger (1998) concurs with this idea that mathematics learning is a process of identity formation.

Holland et al., (1998) argued that as students participated or engaged in their 'figured world', they operated in contested spaces that can be viewed as sites of tension or as sites of struggle. Despite operating in these spaces, identity according to Grootenboer et al., (2006), is still self-determined since the individual adapt or develop their identity to fit with situations within the classroom and their interaction with the teacher. By focusing on the individual's mathematics identities, we are able to observe that "the development of particular kinds of mathematics identities reflects how mathematics socialization experiences are interpreted and internalized to shape people's beliefs about mathematics and themselves as doers of mathematics (Martin, 2007, p. 207).

These are all facets that are important to the thesis of this study and in this chapter I now explore the bodies of literature that focus on my three research sub-questions:

1. What are the salient features of the students' mathematical identity?
2. How do students solve mathematical tasks?
3. How does their ability to solve tasks relate to their mathematical identity?

Research related to mathematical identities

Mathematical stories and narrative: The first research sub-question is aimed at exploring the salient features of a students' mathematical identity. One way of extracting these features is through (math) stories that children tell, but then the onus is on the researcher to make sense of the student's "trajectory of identity, their cultural history, their present experiences and their imagined futures" (Williams, Black, Hernandez-Martinez, Davis, Pampaka, and Wake, 2008, p. 6). In so doing a coherent narrative is constructed that "weave together" (Wlliams et al., 2008, p. 6) all the elements of the student's story. Recent research has emphasized the story of a student fulfilling a role in the mathematics classroom (Ingram, 2007), analyzed "maths and me" essays (Di Martino, 2009), focused on the emotional experiences of students engaging in mathematics (Hannula, 2003; Hannula, 2002), emphasized the activity of "storying one's self" and "accounting for one's aspirations" (Williams et al., 2008, p. 1), and viewed these stories as narratives of identity consisting of inter-connecting sub-stories (Black et al, 2007).

Children continuously hear stories about them from their families, peers, and teachers, and these stories may be positive or negative. These stories according to Weather's (2011) refer to the child's "problems, diagnoses, disabilities, conflicts, and failures" (p. 1) and the child will "mesh these stories together into one that describes themselves" (p. 2). Williams et al., (2008) argued that in these narratives "it is possible to understand how aspirations can evolve, how identities grow, and how key moments are said to deflect trajectories in significant ways for the students" (p. 6). It is through these types of stories that we can capture the salient features of the child's identity as it relates to mathematics. By drawing on Grootenboer et al's (2006) notion that identity is a connective construct that contains the elements of "beliefs, attitudes, emotions, cognitive capacities, and life histories" we can get a sense of how children "know and name themselves" (p. 612).

These math stories are factual accounts of students' past experiences and these experiences can be emotional and in some cases very evocative. By capturing the multiple mathematical identities of her sample Ingram (2007) argued that it provided a context to understand the affective responses in mathematics. Hannula (2003) found that within the same classroom students have different experiences and that students' development of their affects[4] may follow different paths. Hannula (2003) also noted that by "immersing into such vivid emotional experiences allow us to be more sensitive to emotions in the classroom, which otherwise might remain unnoticed" (p. 8). Individuals construct their identities through a narrative approach and use this to tell the stories that they develop, create, and in some cases modify these identities (McAdams, 1993). These stories often present individuals as certain "kinds of people" (Gee, 2000). Gee (2000) also suggested that these stories can also be used as a means of identification and labeling particular characteristics of identity. Esmonde, et al., (2009) sees these mathematics stories as identity resources, and they argue that these stories can be used to create "subject positions" that can be accepted, challenged, modified and altered.

A meticulous read through numerous research studies reveals that these math stories are told with the intention to accomplish something which equates to presenting a particular portrait or profile of a view of mathematics, and in some cases these stories may be told to convince the researcher of its authenticity or credibility. In this regard Hannula (2003) warns that in the study of emotions or any other subjective experience (all components of math identity) "each person's subjective experience is inaccessible to others" (p. 1). Furthermore, Hannula argued that there is a chasm between the inner worlds of the participant and researcher, and that this chasm must be bridged twice: between participant and researcher; and then by researcher and his audience. A possible gap in the research is to identify another mathematical activity that can serve as a means

[4] Includes attitudes towards mathematics and examples of emotions that arise from a situation (Hannula, 2003)

of verifying synergy between the stories and the math activity. In this study I use mathematical tasks as the math activity.

Components of mathematical identity: Researchers who study mathematical identity tends to agree that the notion of 'the self' or 'the identity' is multifaceted and often refers to multiple components as identities (Hogg et al., 1995). Cote and Levine's (2002) described identity in terms of a social, personal, and ego.

1. The social mathematical identity in which the focus is on learning about the communities that the individuals are part of in mathematics. This includes the mathematics classroom where doing mathematics is the primary activity. However, this does not exclude the home, the family, the friends, the race, and the nationality of individuals. All of these will influence the mathematical identity of children. For example, Nasir (2002) found that students' mathematical identities vary depending on cultural categories that include gender, class and ethnicity.
2. The personal mathematical identity answers the question "what makes one the person one is?" or "Who am I?" It is this identity that roughly makes you the unique individual, very different from other people. This is seen as the way you define yourself, your network of values, and beliefs that structure your life.
3. The mathematical ego-identity, which seems to be a more elusive construct compared to the social and personal identities, researchers (Boaler & Greeno, 2000; Sfard & Prusak, 2005, 2005a) tend to focus more on the behavior that individuals engage in because of their ego-identity. These behaviors include engaging, interacting, and affiliating. In addition I include the notion of 'belief' which is used interchangeably with "words such as attitude, perception, opinion, disposition, or conviction" (Hill,

2008, p. 17). I will use this notion of belief in a broad sense since it is difficult to define and differentiate between these words (Leder & Forgasz, 2002).

In this regard identity refers to how individuals define themselves as well as how those who they come into contact with will define them. Mathematical identities include an individual's perception of their experiences with others and their aspirations, and this relationship extends from the past to the future (Anderson, 2007). Once we have an idea of what the students' mathematical identity is, we can try to figure out how they developed this identity by exploring the ways in which students identify with mathematics and how this connects with their learning of mathematics.

Commitment or non-commitment to mathematics: This involves investigating the ways students have developed their commitment or non-commitment to doing mathematics. Black et al (2007) reflecting on previous research emphasized that 'identity' is a "useful tool for understanding student engagement (and disengagement) with mathematics" (p. 1). Black et al (2007) found that the students (math) stories show various ways in which they position themselves with or against mathematics and used Leontev's (1981) notion of 'leading identity' which seems to be significantly 'leading' the student's development and/or future-aspirations. They also referred to this as the 'imagined identity' of the student's future. These leading identities tend to emerge out of the students recollection of critical moments in their math history.

This 'imagined identity' is in synergy with Gee's (2001) version of 'imagination' as one of the four faces of identity of mathematics learning, the other faces includes 'engagement', 'alignment', and 'nature'. Imagination refers to the images we have of ourselves and of how mathematics fits into the greater experience of life, and "these images can contribute either positively or negatively to their identity as mathematics learners" (Anderson, 2007, p. 9). In contrast, engagement refers to the direct experiences that individuals have of their world and their active involvement with others (Wenger, 1998). Most of this 'engagement' takes place in engagement (with mathematics, teachers, and peers) in mathematics classrooms. It is this engagement that "aids student's development of an identity as capable mathematics learners" (Anderson, 2007, p. 9). Alignment takes place when "students align their energies within institutional boundaries and requirements" (Anderson, 2007, p. 10). These requirements are set by others, including, teachers, school districts, state education departments, and other institutions. By following these requirements, according to Gee (2001), the students see themselves as certain 'types of people'. These three faces of identity are "not mutually exclusive but interact to form and maintain a student's identity" (Anderson, 2007, p. 10). Lastly, the 'nature' face of identity refers to characteristics over which individuals have no control, including gender and the color of skin (Gee, 2001).

Identity and learning mathematics: In contrast to the definition of identity as 'who one is', Sfard and Prusak (2005) view identity-making as a form of communicational practice, and equate identities to reifying[5], endorsable[6], and significant[7] stories about an individual. This operational view of identity is used as an analytical tool in some research studies (Ingram, 2007; Juzwik, 2006). Since individuals have multiple identities, Sfard and Prusak (2005a) split these into two categories: *actual identity*, meaning stories about the actual state of affairs, and

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[5] Meaning the transformation of action into states through the use of the words *be, have, can, always, never, usually*

[6] Meaning the story has at least the tacit support of the identity-builder (the individual the story is about)

[7] Meaning if any change in it is likely to affect the storyteller's feelings about the identified person particularly with regard to membership of a community

designated identity, meaning a state of affairs *expected* to be the case now or in the future. By devising these two categories Sfard and Prusak argued that learning occurs when the gap between actual identity and designated identity is closed, and when there is a gap the individual may experience a sense of unhappiness. This brings the affective responses, not just unhappiness, but other emotions and feelings, into play which is fundamental to learning mathematics.

Researchers used this notion of identity to bring about greater understanding of learning mathematics (Grootenboer & Zevenbergen, 2008). In fact identity and learning math has been employed in a range of theoretical perspectives that includes the psychological or developmental (Marsh, Graven & Debus, 1991), the socio-cultural (Boaler, 2002; Op'T Eynde et al, 2006) and the post-structural (Walshaw, 2004). The psychological/developmental perspective focuses on the individual and attempts to compartmentalize and categorize aspects of identity with the intent to describe and understand it in relation to learning mathematics. In this regard identity formation is self-determined as the person adapts or develops to fit with events and situations of life (Grootenboer et al, 2006). The socio-cultural perspective focuses on interactions between individuals, culture and society and how these interact with each other to improve the learning of math. In this regard identity is located both within and external to the individual, 'steered' by society, navigated by individuals along predetermined passages, and is the sum of the relationship skills, emotions, and physical abilities (Cote & Levine, 2002; Grootenboer et al, 2006: Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991). The poststructural perspective, according to Faucault (1984) challenges this notion that identity is formed by either the individual or social phenomenon. This perspective views identity formation "as a continuing process of becoming", thus identity brings "with it connected and multiple discourses" (Grootenboer et al, 2006).

One ingredient in learning mathematics concerns the students' perception of the usefulness, relevance, and importance of mathematics. If students' perceive math in this way, they may be able to see mathematics as important or integral to their lives then they are "more likely to engage with the material and develop a mathematical identity grounded in utilitarianism" (Grootenboer & Zevenbergen, 2008, p. 247). Martin (2000) argued that mathematical identity as an individual's beliefs about their ability to perform in mathematical contexts; the instrumental importance of mathematical knowledge; the constraints and opportunities in mathematical contexts; the emerging motivations that flow out of this knowledge; and the strategies that were used to obtain this knowledge (p. 19). All these research ideas provide a context for to make some arguments for the nature of this study and constructing the salient features of an individual's math identity. It is this first part of Martin's (2000) argument regarding students' beliefs in their mathematical ability that raises student's self efficacy beliefs and their self esteem in regard to math. These issues of self efficacy and self esteem (Pajares, 2005; Watt, 2004) will be treated indirectly during this study. Martin (2000) looked at racialized experiences of students as a social construct to interpret the kinds of mathematical experiences and opportunities to learn and to determine those students who are mathematically literate and competent, and those who are not. Nzuki (2008) added that "the students' sense of identities are shaped and shape their mathematical experiences" (p. 47).

Leading Identity construct: The notion of identity used in this study is informed by Cultural-Historical Activity Theory (CHAT), because identity is conceived as being dialectically related to social practice. The framework for analysis use concepts from Leont'ev's (1981) cultural historical activity theory, but build on the construct of a 'leading identity', which formed a theory of identity. This construct, as described by Black et al (2010), is based on the idea of the existence of leading activities seen as activities “which are significant to the development of the individual’s psyche through the emergence of new motives for engagement” and “alongside new motives for engagement comes a new understanding of self – a leading identity” (Black et al., 2010, p. 55). This notion of leading identity suggests a hierarchy of motives. For Black et al., one of the students’ ‘imagined’ identity of the future became the ‘leading identity’ in the student’s narrative. Grounding this approach will help in understanding and explaining how identity is produced in practice (Williams et al, 2009).

By drawing on CHAT, the study adopts the construct of ‘leading activity’ with the aim to explore the key or significant activities that are “implicated in the development of students’ reflexive understanding of self and how this may offer differing relations with mathematics” (Black et al, 2010, p.55). In addition identity is viewed as “emerging from engagement in joint object-oriented and socio-cultural mediated ‘activity’” (Black et al, 2010, p. 56). In our daily lives we are a unique product of these activities and their associated motives, which forms part of our identity, and as we go through the process of constructing our mathematical identities, our engagement in the classroom.

Through biographic narratives and interviews the study will explore the students’ evolving sense of self and their relationship with mathematics, which together affects their mathematical identity as well as their level of engagement with mathematical activity (Black et al, 2010), which in this study involves the step by step solutions to mathematical tasks. It is through this exploration of how students’ participate in activities that we understand the nature of the identity. Wise (n. d) refers to engagement in activity “where we either accept or decline our role within community, developing our identity in the process”, where this “acceptance or denial is based on our own cognition, our perceptions of self, and our identity within the community”

(p. 3). Black et al (2010) explored how students perceive doing mathematics fits with their motives and how this connection between motives and mathematical identity changes over time as the students' progress in their mathematics career.

Some of these activities are more significant in our development than others and any activity that brings about a change in motive is called a 'leading activity', while the identity invoked by this leading activity and associated motives, is the 'leading identity'. By understanding an individual's leading identity we have a mechanism to understand the individual's self and trajectory, and provides a methodological tool for understanding periods of change (Black et al, 2010). A leading identity indicates an identity which is more significant than other identities because it results in changes in the hierarchy of motives and developmental change (Black et al, 2010). Motives are derived from aspirations (Black et al, 2010).

Some students may focus on studying with the intention to gain qualifications, while others study because of the future-aspirations. In these cases one can attribute these motives to study as based on the "use-value" of mathematics beyond high school. This leading identity gives students' the motivation to persist in their course of study. This becomes very important in the South African context, where unsuccessful or low-achieving students are forced to drop mathematics and take up the study of mathematical literacy, which may have disastrous consequences for their leading identity and future-aspirations. This questions the sustainability of the leading identity as students' transition from grade to grade during the Further Education and Training (FET) band (grades 10 to 12).

In addition to the leading identity, Black et al (2010) also draw on the construct of "identity in practice" (p. 56). These involve a distinction between: identities constructed in the doing of an activity; and the narrative self that is based on the stories that students construct about self. These two types of "engagements of identity are held together by *cultural models"* (Black et al., 2010) developed by participation in practice. These cultural models also emerge by reflecting on the narrative of self (Gee, 1999; Holland & Quinn, 1987). Cultural models, according to Black et al., (2010), "provide a resource – and a constraint – which we draw on in constructing stories about ourselves but which essentially are learnt in practice as a product of subjective experience" (p. 56). For example, familiar cultural models about mathematics are that math is 'hard' or math is 'challenging'.

Research related to learning mathematics and mathematics performance

There is now impressive research-based literature that seeks to explain learner performance through the acquisition of various forms of mathematics knowledge (Casey, Nuttall & Pezaris 2001), the impact of a mathematics curriculum (Long 1998, Riordan & Noyce 2001; McCaffrey et al., 2001), instructional practices of educators (Mack 2001; McCaffrey et al., 2001), mathematical learning environments (Noble et al., 2001) and the interest and motivation of learners (Köller & Baumert, 2001). Despite decades of mathematical reform including the introduction of 'fuzzy math' in the United States, which de-emphasizes computational skills, stresses emphasis on social issues and advocates 'cooperative learning' (Kilpatrick, 2001), researchers in mathematics are providing a wealth of recommendations, advice, suggestions and guidance, but numerous research reports on standardized testing (bearing in mind all the issues attached to this mode of assessment) reflect learner under-performance. The impact of international assessment studies on educational authorities in different countries provides a measure of insight in how these countries are grappling with the 'judgment' that their students are under-performing compared to other countries students. Here I refer specifically to the ranking tables that rank countries in order of performance from best performing country to low performing country. Of greater interest for me is the analysis of factors, including pupil level, classroom level and school level that influence performance and learning mathematics in different countries. Howie (2002) provides insight into an analysis of these levels as pertaining to achievement in the South African context. She also gives a list of recommendations that highlights the fact that more research is needed to answer questions relating to under-performance of South African students in these tests.

Problem solving has become the focus for a substantial number of research studies of the last three decades (Schoenfeld 1985; Cai & Silver, 1995; Lesh & Zawojewski, 2007). Problem solving plays an important role in the learning of mathematics (NCTM, 1989), and is considered one of the five strands for mathematical proficiency[8] (Kilpatrick, Swafford, & Findell, 2001). Other researchers have focused on investigating students' mathematical problem-solving knowledge and skills (Hembree, 1992). However, according to Malloy and Jones (1998) there is a lack of empirical research available about African American students ability to solve math problems, and that "educational research has given marginal status to factors that contribute to mathematics achievement in diverse populations" (p. 161). This idea is supported by Martin (2000), and very recently by Larnell (2011). The situation is very similar in South Africa, where there is a lack of empirical evidence about learning mathematics within the diverse communities (Howie, 1997; Howie 2005; Cranfield, 2005). Since the sample for this study involves students from diverse communities in South Africa, with historically low-achievement, the following factors emerging out of the global research offers an opportunity not to accept lower expectations on my research sample.

[8] The five strands are: conceptual understanding; procedural fluency; strategic competence (problem-solving competence); adaptive reasoning; and productive disposition

Influence of different tasks on learning math: Every day student's spend their time in classrooms, working through prepared materials that include amongst others mathematics textbooks. The textbook is one of the main sources for covering content in mathematics classrooms (Valverde et al., 2002). Out of these textbooks different types of tasks include: tasks that make connections with what students already know, and tasks that emphasize relational understanding and instrumental understanding (Skemp, 1976), or more commonly called conceptual understanding, and procedural understanding (Hiebert & Carpenter, 1992). These tasks also provide the contexts in which the students learn to think about mathematics and some tasks places different cognitive demands on these students. Henningsen and Stein (1997) argued that students "develop their sense of what it means to "do mathematics" from their actual experiences with mathematics, and their primary opportunities to experience mathematics as a discipline are seated in the classroom activities in which they engage" (p.525).

Hiebert et al (1997) similarly argued that students "also form their perceptions of what a subject is all about from the kinds of tasks they do" (p.17), while their "perceptions of the subject are built from the kind of work they do, not from the exhortations of the teacher" (p. 18). Hiebert et al (1997) concluded that tasks are critical in the learning of mathematics. Pollard and Filer (1996) argued that effective students are 'produced' in these learning contexts when they are able to adapt their mathematical strategies and presentation of identities to these social environments. Osborn et al., (2003) refers to this social identity that students construct as a result of the class in which they are members or not members, and it is precisely these classes that produce the social identities that become available to individual students. Pepin and Haggerty (n.d) suggests that learner identity may be conditioned or influenced by mathematical tasks in textbook. According to Pepin and Haggerty:

> Mathematical tasks can and should be seen as a process that can potentially help to enhance mathematical understanding rather than simply a vehicle for content. Thus, the nature of the mathematical task becomes crucial since each task may induce a significantly different disposition towards the mathematics. It is argued that mathematical tasks play an important part in the mathematics education process, and that mathematical tasks may enhance or impede pupil understanding, both in the classroom, as well as in a wider sense in terms of pupils' perception of what mathematics is and in which ways it is practised (nd, p. 13).

Anderson (2007) also argued that the use of mathematics tasks allows students through 'engagement' to develop strategies that develop and support students' identities as mathematics learners. Researcher's have also looked at the meaning of "success" in math, and connected this idea to motivation, cognition, and behaviour (Wigfield & Eccles, 2002). Malloy and Jones (1998) found two problem-solving characteristics that related directly to learning preferences of African American students, namely that they frequently used "holistic reasoning" and students' were confident "in their mathematical abilities regardless of success" (p. 160). These differences can be ascribed to cognitive and affective realms (Malloy and Jones, 1998).

Influence of learning environments on doing and learning math: At the beginning of the year when teachers and students meet for the first time within a mathematics classroom setting every child begin the year with a mathematical identity that consists of a number of components as noted earlier. These environments determine largely the quality of learning mathematics that occurs, and the types of solution strategies that students will use when engaging in math activities, including task-solving. Cobb et al., (2009) proposed a "concrete approach for analyzing the identities that students are developing as they engage in mathematical activity in particular classrooms” (p. 41). It is within this context they claim mathematics education research literature has analyzed three distinct cases of classroom practice: students identify with classroom math activity; students merely cooperate with the teacher; and students resist engaging in classroom activities. These ideas are supported by the research of Boaler and Greeno (2000) and Martin (2000).

It is within this context that Grootenboer and Zevenbergen (2008) claim that “the classroom community is temporal, and it will be the mathematical identity that will remain” (p. 245), and that identity contains “cognitive capacities” (Grootenboer et al, 2006, p. 612) that largely depends on ability to do mathematics. The development of mathematical identities is dependent on the child’s experience in the classroom doing mathematics. Some children become excited doing mathematics, while others do not care and are really disinterested in doing mathematics. There is general concern that participation rates in mathematics are diminishing and according to Grootenboer and Zevenbergen (2008) “students are not rejecting mathematics per se, but rather the form of mathematics they experience at school” (p. 247), and this leads to “math anxiety” as a result of a students’ sense of powerlessness children experience when they think about ‘doing mathematics. For a minority of students doing mathematics conjures up different ideas that includes conjecturing, questioning, proving, and generalizing (Boaler & Greeno, 2000; Lampert, 1990), but for the majority of students doing mathematics means just following the rules given by the teacher (Lampert, 1990).

Hiebert and Wearne (2003) argued that the purpose of math tasks is to stimulate students’ thinking, hence their learning. How students’ learn math is a contested field, but there seems to be consensus that students from a traditional class believe that mathematics is learned by practicing procedures and that knowing mathematics, is based on recalling and applying these rules and procedures (Boaler & Greeno, 2000; Lampert, 1990; Resnick & Ford, 1981). Clements and Ellerton (1995) observed through one-to one interviews that students revealed strong conceptual knowledge of mathematical topics, but when afforded the opportunity to demonstrate this knowledge they were unable to do so. These students' responses showed two clear categories: 1) some students (approximately 25% of the sample) who gave correct responses generally did not have a sound understanding of mathematical knowledge, skills, concepts and principles which the tasks intended to cover; and 2) those who gave incorrect responses was given by students who possessed partial or good understanding. Perhaps, by exploring the relationship between mathematical identities and ability to solve tasks will provide more information to explore this phenomenon. This approach using clinical interviews for one-to-one task-based interaction between researcher and interviewee has become a professional tool for teachers of mathematics (Bobis, et al., 2005). As researcher's we should do our best to know how students get the answer as well as why they gave that answer, so analyzing their mathematical processes becomes an important part of these one-to-one clinical interviews.

The mathematics learning environment plays a major role in doing and learning mathematics. Boaler and Greeno (2000) found that successful student's at math, interacting and engaging in a discussion-based class, had more positive beliefs about doing mathematics than those students' from traditional classes. Furthermore they reported that the majority of the students from the traditional classes disliked mathematics, while the majority of students from the discussion-based classes liked mathematics. These two groups with their distinctive classroom type also produced a different set of beliefs about learning mathematics and what it means to be good at math. The classroom structure and organization is important for learning mathematics or how math is learned. Cobb, et al., (2009) argued for the use of an *interpretive* *scheme* that focused directly on the relationship between different classroom cultures and the student identities that develop within these classrooms. The interpretive scheme had two components: *normative identity*, meaning the "doer of mathematics that is established in the classroom"; and *personal identity*, meaning the extent to which individual students develop as they participate in classroom activities" (p. 43).

Influence of student-level factors (affective responses) towards learning math: Teachers are expected to accommodate the range of demands and needs of many students who are experiencing motivational, attitudinal, and emotional issues relating to mathematics. Exposure to mathematics over a period of time, experiencing different teachers' pedagogies, understandings, and beliefs about mathematics, seems to generate a range of different emotions, feelings and beliefs in students.

In many cases these responses are negative and thus influence both learning and achievement (Gomez-Chacon, 2000). Ingram (2007) argued that as students progress from grade to grade they seem to become less resilient to negative emotions and feelings about mathematics, but proposed that more research be conducted to understand the effect of these negative feelings on students' learning of mathematics. McLeod (1992) provides historical support for this kind of research, and fairly recently (Leder & Grootenboer, 2005) built on the affective domain theory by referring to the need to learn more about beliefs, attitudes and emotions. Recent research studies have focused on the relationships among student values, beliefs, attitudes, emotions and feelings instead of on one element of this affective domain (Schuck & Grootenboer, 2004; Hannula, Evans, Philippi, & Zan, 2004). Other researchers have reported on the role of affect and its relation to cognition (Dai & Sternberg, 2004); affect and motivation (McLeod, 1992); affect and its relation to math learning (Gomez-Chacon, 2000; Hannula, 2002; Zan, Brown, Evans &

Hannula, 2006); beliefs about mathematics (Schoenfeld, 1992; Pehkonen, 1995; Hill, 2008); self-concept and math problem solving (Pajares & Miller, 1994). These researchers concluded that the development of the affective domain is both individual and social, and that affect interacts with cognition.

Research related to the relationship between mathematical identity and mathematical performance.

Recent studies have extended the focus of mathematics education "to include the mathematical knowledge that co-develops with the classroom identity" (Nzuki, 2008, p. 49). Di Martino (2009) using a qualitative analysis of students' own description of their relationship with math found that there were "three strictly interconnected dimensions: the emotional disposition toward mathematics, the view of mathematics, the perceived competence in mathematics" (p. 55). Nasir (2002) explored the relationship between student's mathematical thinking and their shifting identities. Martin (2000) found that successful students displayed high levels of achievement by defying the negative influence of low expectations of achievement. Boaler and Greeno (2000) found that despite similar math experiences some students developed stronger math identities than their peers in their class. Malloy and Jones (1998) reported that African American students' "confidence and self esteem are high regardless of achievement" (p. 160) and that these findings supports the research of Steele (1995).

Emphasis on student identity, especially how students' view themselves as learners of mathematics are "a decisive parameter for their engagement and success in school" (Roesken, Hannula, & Pehkonen, 2011, p. 497). The relationship that students develop between their self-held construction of their mathematical identities and learning math depends largely on what transpires during classroom interactions. Cobb et al., (2009) found that in a classroom where the teacher is in authority, is responsible for methods that students use to solve tasks, is determining the legitimacy of student responses, the students mathematical activity involved giving answers that was teacher determined, reproducing procedures as enacted by the teacher. In the design experiment class where instruction and authority was distributed across the class, where teacher and students jointly determined contributions to the lesson, the students' mathematical activity was characterized by "conceptual prerogatives".

There are three identity types for mathematics learners that emerged out of different mathematics stories and these emphasized 'turning points', 'failing', and 'roller coaster' (Drake et al., 2001). These types can occur at any time within short periods of time or even yearly, showing that "identity formation is viewed as dynamic and somewhat unstable" (Grootenboer, Smith & Lowrie, 2006, p. 613). Each student represents a member of the mathematics community, they are compelled to interact and engage in this math classroom, but it is within this environment that different cultural identities meet. It is claimed that the students enter these classroom environments "with dimensions to their identity that are integral part of their mathematics learning" (Grootenboer & Zevenbergen, 2008, p. 244). Mathematical identities includes the development of mathematical knowledge and skills, as well as attitudes, beliefs, emotions and dispositions, and these are "complexly inter-related" and to focus on mathematical identity "means they can be considered simultaneously" (Grootenboer & Zevenbergen, 2008, p. 244).

Anderson (2007) argued that "the types of mathematical tasks, and teaching and learning structures used in the classroom contribute significantly to the development of students' mathematical identity" (p. 9). These mathematical identities can be clustered in very convenient categories that synergize closely with Martin's (2000) central themes that influence mathematical identities. Martin (2000) defines mathematical identity as an individual's beliefs about their ability to perform in mathematical contexts; the instrumental importance of mathematical knowledge; the constraints and opportunities in mathematical contexts; the emerging motivations that flow out of this knowledge; and the strategies that were used to obtain this knowledge (p. 19). These central themes are the basis for Martin's (2000) multi-level framework for conceptualizing and analyzing mathematics socialization processes, which are shaped by different contexts that includes socio-cultural, community, school, and intrapersonal. The intrapersonal characteristics fit succinctly with the objectives of this study. Under this intrapersonal context Martin (2000) relates four key factors that will influence the construction of mathematical identity: 1) personal identities and goals; 2) perceptions of school climate, peers, and teachers; 3) Beliefs about mathematics abilities and motivation to learn; and 4) beliefs about the instrumental importance of mathematics knowledge.

The key idea is to relate these two components: a theoretical subjective construction of mathematical identities (MI), and a practical illumination of outcomes of learning (mathematical performance). These two components can be characterized by relationships based on contradictions with some aspects synergy between mathematical identity and mathematical performance. For example, beliefs about math abilities did not translate in the outcomes of learning, contradicting the belief. The relationship between beliefs and ability; aspirations and ability; dispositions and ability were tenuous at best. Consequently, I framed my final analysis framework based on the post-structural perspective of identity as defined by Stentoft and Valero, (2009):

> This leads us to propose a conceptualization of the notion which encompasses the fragility and instability of identification processes as embedded in discourse. We contend that a notion of identity formulated from this perspective and emphasizing the dialectic relationship between identification[9] and discourse[10] offers interesting possibilities for interpretations of mathematical learning as a fragile process that is characterized more by discontinuities and disruptions than by continuity and stability: a process which cannot be taken for granted even when students and teachers are confined by the walls of the mathematics classroom (p. 58).

In particular I drew upon the notion that "A function of fragile identities may be, then, that they are part of the process of objectification which enables this refiguring, a site for reflection on how things are and why they are not as they should be" (Solomon, 2007, p. 9).

Stentoft and Valero (2009) reviewed research about identity, and elaborated on the "strengths and weaknesses" in how identity is constructed, and argued that the notion of "fragile identities in action allows us to bring attention to what is normally considered as 'noise' or 'impossibilities' in our understanding of mathematics education and classroom interaction" (p. 55). By making these noises and disruptions visible we are able to talk about issues that were

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[9] The action of engaging in constructing multiple identities or rather seen as "identities-in-action", to bring out the continuously shifting possibilities for new constructions as arising with changes in discourse (Stentoft & Valero, 2009, p. 65)

[10] Mathematical discourse, in this sense, "is the written and spoken language used by mathematicians and students of mathematics for communicating about mathematics. This is "communication" in a broad sense, including not only communication of definitions and proofs but also communication about approaches to problem solving, typical errors, and attitudes and behaviors connected with doing mathematics" (Wells, 2009, p. 1).

ignored or taken for granted (Stentoft and Valero, 2009). This perspective supports the idea that identity is not a “static and constant entity” (p. 62). Bauman (2004) supports this idea that identity is fragile and dynamic. He suggested that “one becomes aware that ‘belonging’ and ‘identity’ are not cut in rock that they are not secured by a lifelong guarantee ...” (p. 11). Within this conceptualization of identities-in-action Stentoft and Valero (2009) warns that when “Viewing identity as fragile emphasizes the vulnerability of identity to disturbances, as well as highlights the uncertainty inherent in its very construction” (p. 63).

CHAPTER 3

METHOD

In this chapter, I describe the methodology used in this study. I begin by describing my research design, including my grounded theory approach. I then describe how my pilot studies informed the finalization of my instruments. This is followed by describing my data collection procedures that included a description of the research site, the sample, the procedures and the research instruments. The final part of the chapter involves a description of my data preparation and analysis procedures by focusing on the transcriptions and coding of data. The coding of the data included coding the interviews; coding and scoring the mathematical tasks; and presenting the framework for analysis.

Overview of the Research Design

This section provides a description, and justification for, the research methods used that framed the collection of data, as well as the types of data that were collected for this study. It describes the design; the sampling method that was used, and how concerns of reliability and validity were addressed.

This study used qualitative data with an exploratory case-study design, since "case studies are the preferred strategy when 'how' and 'why' questions are being posed" (Yin, 2003, p. 1). The primary unit of analysis was the eleventh grade students. The five case studies relied on multiple sources of evidence that allowed me direct access to the subjective experiences of the five participants. Student behavior is fluid, dynamic and contextual, and since the goal of this study is to deepen our understanding, through exploration of the reality and complexity of students' relationship with mathematics and engagement in mathematical tasks, these qualitative case studies provided a 'wide-angle' lens required for this purpose. The intention of analyzing this data was not to find generalizable explanations but to provide rich, vivid and deep descriptions of the effects that mathematics learning has on the development of children, including their mathematical identity. Furthermore, despite using theoretical perspectives of affective responses and (mathematical) identity to inform this study, I also used a grounded theory approach in describing my strategy for research inquiry. The grounded theory approach for this study was beneficial because it allowed the research to be inductive. All decisions made during this data collection and analysis process were grounded in the information itself, the emerging categories, and emerging themes.

This chapter provides an overview of the research design including a brief synopsis of the two pilot studies; the data collection process including the research site, the sample, the research procedures, and the research instruments; the data preparation and analysis including information regarding transcriptions, and coding of the data; and a closing summary of the chapter. The 'coding the data' section focused on coding of the interviews, coding of the mathematical tasks, and the framework for analysis.

Pilot Studies

This process involved two iterations to improve the final tasks. The first pilot was implemented in South Africa and involved two students, and the second pilot was implemented in the USA. A brief description of each pilot is given to emphasize the development process of refining the two key instruments: the mathematics story and the mathematical tasks that focused on selecting the final tasks used in the study

First Pilot: Sample: Two female students were used in this pilot. They attended a school in a similar environment as proposed for this study. Both students were affiliated to the Muslim Faith and spoke English and Afrikaans fluently. They were drawn from the 60% to 75% mathematics achievement category at the school. The two students were conveniently selected because they volunteered to participate despite the fact that it was their midterm examination period. I was unable to find a volunteer in the 50% or below category. However, the two students proved to be suitable participants to test the research instruments. They worked independently on all the tasks, but all discussions took place in plenary. The plenary discussion worked very well because the students could respond to individual comments, as these resonated with how they felt about issues and the tasks.

Instruments: The two instruments that were piloted were the 'mathematics story' and the 'mathematical tasks'. The 'mathematics story' refers to the students relating their experiences in mathematics, the emotions and attitudes that they associate with mathematics, their ability to do mathematics, and their goals regarding mathematics. The instrument specifically ask the students to emphasize the critical events, that is, to describe their 'peak mathematical experience' as the high point in the story, as well as a 'nadir experience' as the low point in their story. They were also asked to describe any turning points in their mathematics experience, any other valuable scenes in their story, any challenges they faced in doing mathematics, and any alternative futures for their story (or future aspirations – describing a positive future and a negative future).

The mathematics tasks were selected based on the integrated South African National Curriculum that included topics involving number, algebra, geometry, measurement, data-handling, trigonometry, analytical geometry, and graphs. The source for the selection of the tasks was the Examination Aid Mathematics Grade 10 Papers (not dated), which consisted of all open-ended (constructed) response questions contained in previous examination papers from different examination boards across South Africa. In addition, the assessment standards for each selected item were also presented on the instrument. The instrument was divided into three ‘doing mathematics’ sections: section 1 included five tasks: number (1 task), algebra (2 tasks), measurement (1 task), and analytical geometry (1 task); section 2 included five tasks: number (1 task), algebra/graphs (1 task – could solve algebraically or graphically), geometry (2 tasks), and trigonometry (1 task); and section 3 included three tasks: algebra (1 task – with five parts), data-handling (1 task – with five parts), and graphs (1 task).

Implementation limitations: The timing of this pilot was problematic because all the schools in Cape Town were engaged in midterm examinations. However, the principal at the pilot school made a classroom available for me to work with the two students. Unfortunately, we were only given 2 hours to complete all the tasks. This resulted in limitations with regard to time for both instruments. The students were given 30 minutes to complete the first instrument and 90 minutes to complete the mathematical tasks. Both students admitted that they felt rushed during this process and would have preferred more time.

Two mathematics teachers were asked to assess the instrument and the feedback was very positive. Both teachers said that the three sets of assessment tasks covered the syllabus well and the selected tasks were in the academic range of all 10th graders. Some of the students may struggle because they lacked the procedural capability to solve the problems.

Recommendations for improvement of the instruments: The following recommendations emerged out of the first pilot:

1. Give students more time (at least 60 minutes) to respond to the ‘mathematics story’ instrument;

2. Emphasize to students to elaborate and to give as much detail as possible and examples to illustrate what they mean for the purpose of clarity;
3. Ask students to say more about the topics in mathematics that they find meaningful;
4. Administer the mathematics tasks close to the end of the academic year for 10th graders, because some teachers only complete certain topics much later in the year or to work with 11th graders' at the beginning of the next academic year.
5. Reduce the number of tasks and mathematical domains, for example, to leave out tasks involving trigonometry, and analytical geometry.

Second pilot: The mathematical tasks were revised and reduced to 11 items and piloted for further adjustments or improvements. Two 10th graders from the United States were invited to complete the 'mathematics tasks' instrument. Both students are home-schooled and, according to their parents, were apparently doing very well in mathematics. Both students preferred to complete all the tasks and then discuss matters arising out of the activity of completing the tasks.. The students were allowed to complete the tasks, consisting of 11 items and both completed it in 80 minutes. They worked well on all the tasks, but got bogged down with a geometry task. In fact they were getting very frustrated because they could not find the value of all the angles. Clearly, they wanted to solve this because the problem was familiar to them. I had to ask them to continue with the other problems and then return to that item after they had completed the other tasks. The reason for their frustration was based on the fact that the students did not notice that 'opposite sides are parallel' and hence could not find 'alternate angles', the key to solving the problem.

Recommendations for improvement of the instrument: The following recommendations emerged out of the second pilot:

1. Retain all six tasks, but juggle the order: Q3, Q2, Q1, Q5, Q6, Q4, for section 1. The items cover the three categories of potentially easy, average, and difficult, very well. Also the emotional shifts were in place with students feeling good to feeling frustrated. With these items their love for math and their ability to do math was under constant scrutiny.
2. The students based their analysis of the degree of difficulty of a task on their subjective experiences: potentially *easy* on the fact that they knew exactly what to do in solving the problem and felt comfortable with the question; potentially *average* on the fact that they had some idea about what to do, but were not convinced that their solutions were correct; and potentially *difficult* on the fact that they had no idea how to solve the problem or understand what they had to do. The easy and average tasks were familiar (they had experience doing that type of problem) to them, while the difficult tasks were unfamiliar to them (did not see that type of problem yet).
3. Retain all five tasks in section 2, and keep the same order. These items covered the three categories of easy, average, and difficult, as reported by the students, very well, but also had greater variety which the students really liked. Also the emotional shifts were in place with students feeling good to feeling slightly frustrated about the tasks. With these items their love for math and their ability to do math was under constant scrutiny as observed with section 1.

Based on these recommendations the final mathematics task instrument was finalized, ready for implementation in this study (see appendix D).

Data Collection

Research Site: The sample was drawn from a high school in Cape Town, South Africa. The school is located within a low-socio economic working class community in the Cape Flats. The school attracts students from diverse backgrounds, including language, ethnicity, religion, gender and race. There are approximately 140 eleventh-graders at this high school. At least 60% of these students are considered low-performing in mathematics that is, achieving less than 50% in the mathematics examination. In addition, there was only one mathematics class and three mathematical literacy classes. In the South African mathematics curriculum students are mandated in grades 10-12 to select either mathematics (seen as pure mathematics) or mathematical literacy, seen as applied mathematics. This resulted in limiting my choice of a sample, because I could only select from the (pure) mathematics class of 37 students.

I specifically chose (pure) mathematics students because I wanted to understand why students with a low achievement (under 50% in mathematics final examinations) were choosing this level of mathematics and how this choice impacted their mathematical identity. Mathematical Literacy was seen by many within the South African education community as an escape route for those who struggle with (pure) mathematics.

The principal made an office occupied by a Head of Department available to me for my four weeks of research. The principal also offered his support throughout the data gathering process. This venue proved to be problematic during intervals (recess) and end of periods, because noise was at a premium. Unfortunately there were no other venues available where this noise could be avoided. Consequently, I decided to work within this constraint.

Sample: I received the grade 10 class schedule (in the US, this would be called a transcript) for 2011, as well as the grade 9 class schedule for 2010. These schedules contained the performance of each student in every subject in grade 10 and in grade 9. My first criterion for my sample selection was based on the performance of the grade 10 students in 2011 who achieved less than 50 % in their final examination. I looked for all scores under 50%. This result yielded 29 students, 16 were female students and 13 were male students. The range of scores had a maximum score of 45 % and a minimum score of 23%.

The second criterion was to narrow this sample to 12 students, so I looked at the grade 9 class schedule to check on the achievement of the 29 students. I observed that 6 students did not have a grade 9 result in 2010. I excluded the 6 students from selection. Based on the above information I randomly selected 18 students from the remaining 23. The following day I went to the mathematics class to invite the students to participate, but 5 of the remaining 18 students selected were absent. I found out that some of the five were habitually absent. So they were excluded from my potential sample. Thus my potential sample size was n=13. Of these, six either chose not to participate or could not get parental permission. So my final sample was n=7. All seven students wrote their mathematical story. After reading the mathematical stories, I decided not to complete individual interviews with two students, because they wrote very little about their math journey and seemed disinterested in the process compared to the other five students who showed greater enthusiasm to participate. My projected case study sample was always 5 students, and it was clear from the math story that I would be able to get sufficient data from my sample of 5 students. I have provided pseudo names for this sample, namely, Yusuf, Avril, Moses, Katie, and Zena. There were 3 female students and 2 male students. Table 1 shows all the students who were initially selected by my random selection with the pseudo names of the students in the final sample shown written out, and the pseudo-initials of the other students given.

Table 3.1: Selection procedures: sample, gender, percent in grade 9 and 10, range in percent, and comments regarding selection

Name	Gender	Grade 10 percent correct	Grade 9 percent correct	Range in percent: Gr 9 percent correct less Gr 10 percent correct	Comments	
Katie	F	32	40	8	Selected	Consented
D W	M	35	41	6	Selected	Consented
Moses	M	45	51	6	Selected	Consented
N I	F	38	43	5	Selected	Withdrew
A M	M	36	40	4	Selected	Withdrew
T A	F	34	67	33	Absent	
N S	F	33	59	26	Selected	Withdrew
Yusuf	M	41	66	25	Selected	Consented
D L	M	39	62	23	Selected	Consented
Zena	F	41	62	21	Selected	Consented
D B	M	45	43	-2	Absent	
Avril	F	35	52	17	Selected	Consented
M J	F	36	53	17	Selected	Withdrew
A D	M	42	59	17	Selected	Withdrew
C M	F	33	46	13	Absent	
A H	F	30	42	12	Absent	
L M	M	35	34	-1	Absent	
G V	M	35	30	+5	Selected	Withdrew

I also observed that the majority of the remaining 23 students dropped significantly in their grade 10 results compared with their grade 9 results.

Procedure: I administered all the activities and handled all information. The first activity involved writing their mathematical story. They were invited to write their mathematical story, using the ‘mathematical story’ instrument (emphasizing their relationship with mathematics during their primary and early high school). I allocated 60 minutes, but all participants completed the activity in the range of 40 -55 minutes. I read each of the stories, because I wanted to use the stories as a way of getting to know the students before they engaged in the next activity. The following day the participants engaged in a focus-group activity. I used a semi-structured interview protocol with the intention to gain as much information from the students. This was video-taped as all students consented to it. Additionally, I also audio-taped the focus-group interview as a back-up in case there were problems with the video recorder. The key reason for using this focus-group strategy is to get students to respond by reflecting on what their peers are saying. According to Rabiee (2004) a feature of this focus-group interview is that the data generated through this social activity “are deeper and richer than those obtained from one-to-one interviews” and it illuminates “the differences in perspective between groups of individuals” (p. 656).

The next activity involved individual interviews, which took approximately 30 - 45 minutes per student, and these interviews were audio-taped. The following day the students were involved in writing step by step solutions to mathematical tasks. They did this over two sessions of 45 and 60 minutes respectively. The students were told that this was not a test and the results will not be used by the school. They were instructed to complete the tasks individually and to the best of their ability. They were not allowed to communicate with each other, but work alone. They were only allowed to use a scientific calculator in the financial math task. After the students completed the tasks I graded and coded the problems on a separate page so that the students could not see that the script was graded. Seeing a score or markings (suggesting correct or incorrect) may have been distracting or off-putting for the students, hence the decision to keep the grading separate. I wanted the students to discuss how they felt during the activity of completing the tasks and how they thought about solving each task. This was the essence of the next round of individual (clinical) interviews.

Instruments: The following data collection instruments were used during the research:

Mathematical story-narrative: This narrative provided me with the participants' accounts in their own words about their lives and mathematical activities. These narratives were personal, mathematical, emotional and evocative. I read their stories so that I got a better knowledge of each student and for the stories to partly guide the focus group interview. It was the first step towards a glimpse into the mathematical identity of the participants (see appendix A).

Interviews about the Mathematical Story: These were semi-structured individual interviews with the 5 case-study participants, and a focus group interview with the 7 participants. The focus group interview was conducted to allow participants to speak freely. A set of guiding questions were formulated to help focus the general direction of the discussion. These semi-structured questions also provided a plan for getting the kind of information that was sought to answer my research question. The individual interviews provided another layer of triangulating data, to check for internal verification. For example, were the students saying the same thing that they described in their story-narrative? These interviews focused on gaining data about each student's opinions, affective responses, beliefs, and experiences, the key elements of the mathematical identity of an individual. I was interested to see how students at this crucial juncture were experiencing mathematics at school and at home. (See appendices B & C).

Mathematical tasks: Students were asked to write step-by-step solutions to the mathematical tasks (see appendix D). These tasks were completed individually. The math tasks were divided into two sections that allowed coverage of a range of mathematical domains, because South Africa has an integrated curriculum.

Clinical Interviews about the mathematical tasks: In this regard, I used task-oriented, flexible interviews aimed at pursuing the students' math thinking by asking as many questions as needed to understand the reasons behind the students' solution strategies (Confrey, 1980). I hoped to learn more about the diversity of student understandings, underlying (mis)conceptions, understanding student errors, identifying knowledge gaps, clarifying mathematical processes needed to do mathematics, and examining specific areas of curricular difficulties. The key purpose was to find out how these students do mathematics and think about mathematics, as well as pursuing their statements of justification for their answers and thinking processes (Confrey, 1980). (See appendix E).

Data Preparation and Analysis

Transcriptions: My first core task was to transcribe all the interviews. I employed an assistant to help me with this process. I transcribed the focus-group interview by using both the audio and video recordings. In addition I also transcribed two of the individual interviews, as well as three clinical interviews. My assistant transcribed three individual interviews and two clinical interviews. We encountered the following difficulties transcribing these audio-tapes:

Noise: There was excessive noise in the background. The noise came from students who were not in their classroom during lessons, from students enjoying their interval, from knocks on the door of the interview room, from excessive use of the intercom system, and from the siren indicating the end of class periods or beginning and ending of interval. These were all picked up on the recording tapes, resulting in a few transcription problems. Unfortunately, there was no other venue available for this purpose.

Indistinct students: There were moments where the students' comments were indistinct during the interview sessions, and this made transcriptions very difficult. Sometimes students were unable to pronounce a word completely; sometimes they swallowed their words; sometimes they spoke very fast with a difficult English/Afrikaans accent; sometimes they mumbled when thinking aloud or were unclear about their thoughts; and sometimes they just lacked confidence in answering the question. They also spoke very softly at times.

Interview technicalities: Due to the nature of the interviews, students were also required to explain their thinking, their solution strategies, and their reasons for their solutions, and this resulted in a stumbling block for transcriptions. There were times when students were pointing or gesturing and pointing to sketches. Sometimes they had difficulty expressing themselves in words and used actions instead. Unfortunately some of these actions were lost during transcribing, since the students made a decision to only allow for the audio and not video recording of these interviews.

Personality of the students: All the students were eager and willing to participate in the interviews, but there were times where they displayed tiredness, especially during interviews that occurred later in the day. This resulted in just yes and no responses. Also when students were confident about their answers they spoke loudly and clearly, but when they weren't sure, their responses were often soft and inaudible. However, most of them were very positive throughout the interview, despite the time taken and the interview questions demanded further probing especially when students seemed unsure. The constant probing allowed forced the students to think deeply about their response. However, I had to make sure that the constant probing did not have a negative effect on the responses. This meant that I stopped probing when I felt that students were becoming agitated.

The next step in the process involved the attainment of inter-transcriber agreement. Each transcriber listened to the tapes that had been transcribed by the other person and checked if everything that was capturable was captured in each transcription. This ensured that the final transcription represented an accurate version of what was said during the interviews.

Coding the data

After I approved each transcription, my analysis started by conducting a general read through each transcription, in order to answer my first research questions: What are the salient features of the students' mathematical identity?

Coding Interviews: At this juncture I used Nvivo 9, an analysis tool to help organize and analyze the information. In particular, the software helps to work with unstructured information aimed at helping researchers make better decisions. In the coding of the data, I used pre-established codes (categories used to identify mathematical identities) by interacting with my literature review of mathematical identity and the theoretical frameworks that emerged out of the literature. In addition I also used open codes that emerged out of my own understanding of the data. I used Black et al's (2010) framework that refers to developing a 'leading identity', to inform my data analysis. The following procedure was followed:

1. The first activity was to make a specific list of key words, key phrases, and key sentences associated with initial categories or subcategories that included the participants' math-learning story, their self-recognized math identity and their performances. I searched for key words/phrases/sentences that are good indicators for a category or subcategory. I realized that I needed to broaden these categories.
2. I ended up with a second list of categories, subcategories to code. Once I established these categories/subcategories, I populated each with participants' transcript

responses. For example, I used a category A to represent "math identity" and chose the key word "difficult" as a code-indicator for that category A. I then combed the transcript for the word difficult and observed that it appeared in 2 or 3 other places.

3. Next I looked at the phrases/sentences in which this word occurred and then made further connections with my category A. I then put these coded sentences/phrases into a table of key words for each participant.

4. I also chose and interpreted whole segments of the transcript that seemed to fit into a category or another category based on my own interpretation of what the participant was saying. For example, in one transcript the following was said "My mother tells me every afternoon that I must sit with my books. I must do my homework, so I must see what I'm doing". My interpretive code was equated with pressure from family to make learning disciplined and regular. However, this can also feed into their math story as a description of the motivation to work and possibly doing math with a certain regularity or sustained discipline.

5. Through this process I managed to explore the different kinds of evidence and prioritized in terms of prime evidence to answer my research questions. This 'combing the data' procedure required my interpretation at different focal points.

The final selection of the general categories for mathematics identity included: Knowledge (the kinds of knowledge); Ability (what ability); skills; dispositions; attitude; emotions; beliefs about math; beliefs about learning; motivation; opportunities to learn. The subcategories for mathematics identity included: list of qualities; level of confidence (very confident/self-reliant, vulnerable, shaky, not confident at all, and doubting own abilities); constraints and limitations; resources (what are they or how did they use it); projections for the future; working style and work ethic; projections of success or failure at tasks/exams/tests; relationship with people through or because of math; relationship with themselves through or because of/in the context of math; classroom norms and impact on math learning; teacher characteristics and impact on math learning; and any other subcategory that emerged.

I put in as many categories and subcategories that helped me characterize math identity for each participant. This is not an exhaustive list of categories/subcategories. In each of the column/row cells, I placed data emerging out of the interviews. Not all cells were filled. Each cell was coded, for example, Y123, meaning Yusuf's interview transcript and the numbered line sentence. Some cells were coded the same, because of the potential of overlapping categories. For example, for the category "Knowledge of math", if you go down on the rows (subcategories), you can (based on my interpretation) apply all of them to "Knowledge" and I found data for most, but not for all of the cells. I did the same for each category, going down that column, row by row, cell by cell. This was just a design to help me organize the data and to guide my coding, which helped me in writing each case study narrative. In this way the break down of the data into more specific items, made it easier to refer to when I commented on their performance in the mathematical tasks.

Coding Mathematics Tasks: To answer the second research question, "How do students solve mathematical tasks?" I analyzed each participant's response to the mathematical tasks by applying three methods. The first two methods involved a holistic scoring scheme, which provided a quantitative analysis, while the third method involved a cognitive analysis scheme, which provided a qualitative analysis (Cai, 1995). This analysis emphasized the students' proficiency in doing math; in choosing appropriate strategies that included trial and error, graphical representation, arithmetic calculations, and algebraic equations; in knowledge and skills; and in making sense of their computational results (Cai, 1995). The three methods included:

Points-Percentage score: The grading procedure involved the grade 12 matriculation grading procedure which included an allocation of points given for a step by step calculation. I am familiar with this procedure because I was a sub-examiner for 12 years and examiner for 3 years in the Western Cape Province in South Africa. The value of doing this type of analysis is that it will provide a percentage that will determine whether the student did well in the test if the passing benchmark is 50%. A detailed memorandum to the math tasks with mark allocation for each step in every task was developed using the marking scheme provided by the Examination's Aid[11]. This memorandum and copies of the participants' written tasks were graded and coded by three mathematics education PhD candidates plus me, and was aimed at ensuring inter-rater reliability. For 20 of the 27 tasks the graders scored the items exactly the same. In the other 7 tasks, where there were differences of one or two points, the participant received the score given by the majority of graders. There was a tie between the four graders for only two of the 7 tasks. In these two cases I used my grade as the tie breaker, because I had more experience with these types of grading rubrics.

Table 3.2 shows the task numbers with task description, and the maximum points per task, and the names of each participant (the actual scores for Points-Percentage and Four-Scale rating codes will be given in this framework).

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[11] Examination Aid Mathematics Grade 10 Papers (n.d), the same book used to construct the mathematical tasks.

Table 3.2: Description of mathematical tasks and maximum points.

Task	Description	Max points.
1 a	Interpreting length in terms of x	1
b	Solve for x by comparing areas	10
2	Compound increase, give the yearly rate	11
3	Apply exponential laws	7
4	Using average concept	7
5 a	Patterns: draw 4^{th} fig.	1
b	Draw 7^{th} fig	1
c	Interpret results	2
d	Determine 40^{th} fig	6
6	Finding intersection: given two linear equations	8
7	Finding angles in a geometric figure:	12
8 a	Solve for x: linear fractional equation	4
b	Solve for x: quadratic equations	5
c	Solve for x: linear with letters (fraction)	7
d	Solve for x: exponential equations	5
e	Solve for x: linear inequality	8
9 a	Calculate the height in a volume problem	6
b	Find the volume if dimensions change	5
10 a	Interpreting graphs: parabola and line	13
b	Straight line graph	3
c	Solve for x, use graph	2
11 a	Draw bar graph	9
b	Determine sample	2
c	Arithmetic mean	6
d	Determine mode	2
e	Determine median	4
f	Interpret data	3
		150

Table 3.3 was used to show how the distribution of the points by mathematical domain. This gives an overview of the participants' weakest and strongest areas in mathematics. Table 3.3 was used for both the Percentage-Points score and the Four-Scale Rating codes.

Table 3.3: Mathematical Domains (MD): Tasks numbers per domain and maximum points for each domain.

Math Domains (MD)	Tasks per MD	Max pts for each domain
Algebra	3, 6, 8	44
Data Handling	4, 11	33
Financial Math	2,	11
Geometry	7,	12
Graphs	10	18
Measurement	1, 9	22
Patterning	5,	10
		150

Four-scale points score: This is a number representing performance on each task. The following codes were used: S0 – for no answer; S1 – for minimal response, including simply rewriting the problem with minor adjustments: showing limited understanding and conceptual errors; S2 – for a competent response, a good understanding, but incorrect answer due to arithmetic or conceptual errors; S3 – exemplary, fully understand the problem, correct reasoning and complete answers, but may have a minor arithmetical error in the solution strategy.

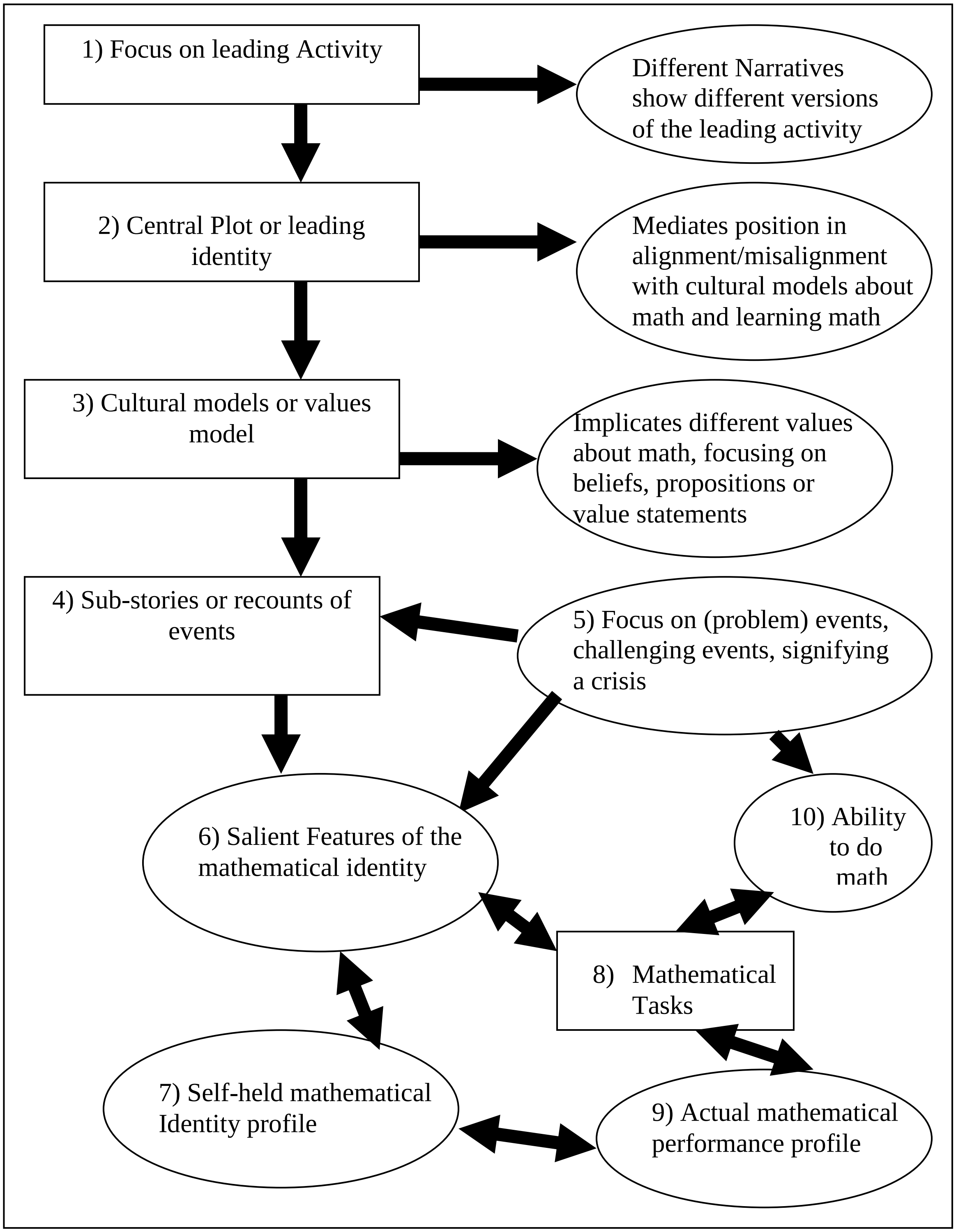
The aim of this type of analysis of the mathematical tasks was to extract a visible record of their solution processes, the mathematical knowledge base, strategies, and their solution justifications within multiple mathematical content areas that included: geometry, measurement, data-handling, algebra, patterning, and graphs. By using this approach I was able to examine each student's response in every task in terms of the cognitive aspects of their solution strategy or partial solution strategy. Both the quantitative and qualitative analysis were not exclusive processes but were interrelated and complementary in terms of providing a mathematical performance profile for each participant.

Framework for Data Analysis

Figure 3.1 represents the framework for analysis that was used specifically to write up the 5 case studies, by analyzing the math story, the three interviews (focus, individual, and clinical), and the performance in the tasks. Through this analysis I aim to answer my three research sub-questions: 1) what are the salient features of the students' mathematical identity? 2) How do students solve mathematical tasks? And 3) How does their ability to solve tasks relate to their mathematical identity? There are three distinct parts to this analysis: to construct a mathematical identity profile; to construct a mathematical performance profile; and relating mathematical identities to ability to solve math tasks.

To construct the mathematical identity profile, I use Black et al's., (2010) framework that focus on the following features, as presented in the theoretical background in chapter 2: leading activity; leading identity; cultural models, sub-stories (that include the math journey from primary to high school and emotional attachments along this journey). These parts 1-5 in figure 3.1 provide the bases for presenting the salient features of the mathematical identity (part 6 in figure 3.1) as constructed by each participant in their math story and interviews

Figure 3.1: Framework for analysis



This analysis is shown in chapter 4 where the main focus is to establish each participant's self-held mathematical identity profile (part 7 in figure 3.1)

To construct the actual mathematical performance profile, I used the participants' actual performance in the mathematical tasks together with their clinical interview. This analysis is shown in chapter 5 that presents the "*actual cognitive"* element, the way participants' completed the math tasks and his or her thinking about solving these tasks

To construct the relationship between the mathematical identity and participants' performance in mathematical tasks, I analyzed how math identity and math performance 'interacted' with each other. Figure 3.1 shows the relationship that different features of math identity impacted performance in math and vice-versa. To explain this relationship between math identity and math performance, I use Stentoft and Valero's (2009) conceptualization of identities-in-action (see chapter 2) by focusing on the idea that there's "continuity or discontinuity" between the self-held mathematical identity and mathematical performance of each participant. The focus of chapter 6 is to present this analysis.

CHAPTER 4

CASE STUDIES OF STUDENTS' MATHEMATICAL IDENTITIES

In this chapter 4, I directly address the first research sub-question that guided this study, namely, what are the salient features of the students' mathematical identity? This analysis also explore the ways in which students identify with mathematics and how this connect with their doing mathematics? In particular, it describes how the use of the *leading identity* and *cultural models* not only fitted into the students' mathematical journey, but seemed to guide this journey.

This chapter consists of five main sections. In each I briefly introduce one of the study's five participants, Yusuf, Avril, Moses, Katie, and Zena. For each student I describe the leading identity, the development of his or her math identity, mathematical journey from primary to high school, the emotional attachment in doing mathematics at high school, and cultural models. Cultural models, according to Black et al (2007) are culturally derived rules aimed at facilitating stories that individual's narrate about self and their activities. Cultural models are learnt during daily practice and daily activities with special emphasis about the classroom. Cultural models are also, narrowly, viewed as beliefs or value-statements (Black et al., 2007), a view that this study will employ in terms of cultural models.

The first part of each case study describes the student's perceptions of his or her own relative ability to do mathematics, and the socio-cultural context in which this sense of self was formed. The student's interaction with family and friends, his or her engagement in the mathematics classroom with teacher and peers, each participant's peak and nadir-experiences doing mathematics, and the reasons why he or she is doing mathematics as a course of study and not mathematical literacy[12] are also described.

My focus is to provide the information in as full detail and in as a systematic way as possible. In this chapter I make substantial use of each participant's mathematics story, individual interview, and the focus group interview. For the purpose of referencing I will use Math Story when a quote is taken from that piece of data. I will use, for example (Int. Y 123) to show that the extract is taken from the individual interview with Y representing the first letter of name of the participant and 123 representing the line number in the transcript. Similarly, I will use (FG. 23) to show that the extract is taken from the focus-group interview (FG) and the 23 representing the line number in the transcript. The interview quotes are given as evidence to support my interpretations.

Yusuf

Introducing Yusuf

Yusuf is from a very poor 'colored' community, who were dispossessed of their land by the apartheid government. Yusuf is the youngest of three children. He has two brothers, one who is married with two children and another brother who is 18 years old and currently completing his matriculation year at the same school as Yusuf. His parents are married and in their late forties. His father works as a tiler, while his mother is unemployed. He laughed about his height by commenting that he always helped his mother "take the stuff off the top of the cupboards".

Yusuf was family oriented; he noted that he loved being around his family and 'nice' friends. Furthermore, he noted that this group of family and friends had a great influence on his

[12] Each student in South Africa must choose one of the mathematics or mathematical literacy.

decision-making. His father did not complete his matriculation year, but his mother completed her matriculation year, but with mathematical literacy. His mother is his chief source of encouragement, “my mother tells me every afternoon that I must sit with my books. I must do my homework” (Int. Y 29). She wants him to do well at school. It is out of this conversation with Yusuf that I detected his strong sense regarding the value that he attached to school mathematics. Yusuf believes that the school principal is responsible for effective teaching and learning; “I think the school actually should let the teacher know that we are not liking the way that he is teaching us, and that he should have more activities with us and more activities after school” (Int. Y 389). Yusuf identified this as a problem that affected the learning of math in his class.

He said that there were two definite groupings of students within their math class namely, the good achievers in math called the ‘fast lane’ group and the ‘slow’ group consisting of low achievers. During the focus-group interview one of the students remarked that “the fast lane students are like students who always get high marks and know what is going on. So because we don’t know and we at the board and they will comment no it’s not that and that” (FG 288). In support of this another student said “the fast lane students would actually comment also still and try to like embarrass a person” (FG 277). This behavior stems from the way the teacher responds in the class, “it is not only that he [the teacher] gives you negative feedback but he will like also embarrass you” (FG 273). These two groups behaved in very different ways because the ‘fast lane’ group was viewed by the ‘slow’ group as the teacher’s ‘pets’, meaning that they received favorable accolades from the teacher. This fast-lane group was consistently engaged in the mathematical activities and interactions within the class, while the low-achieving group felt marginalized and received negative comments from the teacher and ‘fast lane’ peers. This classroom dynamic sets an important mathematical context as these students construct their mathematical identity, and according to Martin (2007) "a mathematics identity encompasses a person's self-understandings of himself or herself" (p. 41), thus the social context (math classroom) is important.

Yusuf identified a possible solution to this problem, "they [the school] could actually split the class. Maybe get another teacher for half of the class that's how it was last year, where we worked in smaller groups" (Int. Y 397). He wanted his class of 42 students to be split into two classes of 21 students. He thought these changes would help him be a better student of mathematics.

Through this introduction of Yusuf we are able to see the influence of habitus, which according to Johnson (2009) is "the internalised principles resulting from one's upbringing (structured structures) that result in an agent's action and view of the world, comprising dispositions that reflect the ongoing construction of an agent's social position (structuring structures)" (p. 60). We see the influences from the home and family directly from Yusuf's words, and the direct environment which reflects the low socio-economic status of the community including the friends, as well as a reflection on the influence of the school and their role in promoting effective teaching and learning. In the next sections we will see the influences from the classroom and the effect on beliefs, attitudes towards math and learning, and feelings toward math.

Leading identity

Throughout the interview Yusuf's mathematical identity emerged in many different ways. For example his relationship with his teacher(s) formed the basis for his beliefs about learning math, and his engagement with some math content formed the basis for his like or dislike of math. However, as he constructed his mathematical identity I tried to tease out from his narrative one[13] thing in particular that seems to drive him as a grade 11 student. I thought that the most significant part of his narrative, hence his leading identity, was based on the reason why he was doing mathematics. In this part I present evidence for this selection of his leading identity.

Yusuf chose pure mathematics in grade 10, because he wanted to "become an engineer one day" and because at primary school he "used to be one of the best students" (Interview, introductions). He wanted to become an engineer because he believed he was a good student in mathematics. He said very passionately that his "current career choice is mechanical engineering" and to do that he "must have pure math" (In. Y 120). He realized that doing pure mathematics was vital to his future ambition. This was further emphasized in his math story, when he wrote "I chose math because of the field I'm wanting to go into, engineering" (Math Story), as well as in the focus-group interview where he simply said "I would like to become a mechanical engineer" (FG. 80). It is this passion that characterizes his mathematical identity, which I believe is his *leading identity*.

His mother is his greatest source of encouragement, but he decided not to go into the career she planned for him. He said, "She helps me a lot. She wants me to go into the medical field, but I said no, rather into engineering to follow my grandfather's footsteps" (Int. Y 34). He wants to make his grandfather happy, but at the same time we note that his mother also had high expectations for him by wanting him to go into the medical field.

When asked the question if he will ever use the math that he is learning, he immediately reflected on his career choice of engineering, by saying, "not really, but in engineering, I hope to

[13] One thing – to see what defines his motive for studying math and/or shapes his relationship with math, and according to Black et al., (2010) the notion of a leading identity indicates an identity which is more significant than the other identities.

at least use some of the formulas" (Int. Y 114). Furthermore, he also emphasized his belief that math is just about knowing formulas by stating that anyone can do mathematics because "it's simple if you know the formulas, you can do it [mathematics]" (FG. 13).

To achieve this career goal he was adamant that his aim is "to pass matric with an A [grade – meaning 80% and above] for math, but won't get my hopes up for nothing" (Math story). He realizes that he must pass matric very well to pursue his career choice, and only some grade very close (for example B +) to an A-grade will suffice to fulfill his ambition. He shows a positive disposition towards engaging in math activities and this energy emerges out of the 'use value' principle. This positivity and strong aspiration about math was observed in how he defined himself as a student of mathematics, namely, "I think I'm quite good at it now" (Int. Y 228). In fact, on a scale of 1-10 he rated his motivation to do mathematics as an 8. Furthermore, he sees himself as "a bright person in mathematics, hardworking, lots of self esteem, confident, basically that" (Int. Y 482).

He admitted that even if he did not need mathematics in his career, he would still do it because "it's interesting" (Int. Y 237) and "it will help me in the future" (Int. Y 239). The way Yusuf sees himself as a student of mathematics contributed positively to his identity as a mathematics student. This fits with Andersen's (2007) imagination as a face of identity, where Yusuf's image of himself in relation to math, as well as his use of math in his future career influenced his imagination establishing his career as the chief reason for doing mathematics. Yusuf's response to this imagination face of identity is to align his choice of pure math to meet the basic requirements for mechanical engineering. According to Andersen (2007) Yusuf's identity is "maintained through both imagination and alignment" (p. 10).

Yusuf's response to the degree of difficulty in doing mathematics illustrates the 'fragility' (Stentoft & Valero, 2009) of his identity that reflects on his commitment. Yusuf admitted that if the "work gets too tough, then I am going to have to drop it" (Int. Y 131), referring to mathematics. So if the math gets more difficult he would rather take the easy way out by dropping pure math and taking on math literacy. Although Yusuf values mathematics as a subject and its importance for his future, he has a very different attitude towards what constitutes the purpose of mathematics. He articulated that "I don't actually see the purpose of doing mathematics with all the algebra in it because it is possible that in the near future we might not be able to use that also" (Int. Y 320). This negativity towards algebra started in high school "I never knew what was going on, starting with the algebraic letters. I was not taught that in primary school" (Int. Y 160) and it was "a bit weird to do math with letters" (Math story). This shows a limited understanding of the importance of algebra in his future math classes, and strongly based on how his teachers introduced algebra to him.

Yusuf also emphasized that he had a weakness for certain types of mathematics, meaning that he was not good in some areas of mathematics. He mentioned algebra again, fractions, linear equations, and some trigonometry that he did not understand. He claimed that his strong sections were working with Pythagorean problems and geometry. He went as far as saying "geometry, I'm good in that" (Int. Y 278). Yusuf had a limited understanding of mandatory mathematics content required at university to complete a degree in mechanical engineering.

Yusuf maintains a stable career mathematical identity throughout his narrative that drives his motive for continuing to study mathematics. Black (2009) refers to this as motives defined by the 'use value' of mathematics in the pursuit of this vocational objective.

Development of his math identity (Math journey)

Yusuf started his math story with the following reflection, “ever since primary school I had liked maths, not for the last 4 years. When I came to high school I started to unlike [meaning dislike] maths because of the teachers”. He was one of the top students in primary school, where he received 5 or six certificates for good achievement in mathematics. It was this achievement that motivated and encouraged him to do math, because of the high expectations placed on him by his parents and teachers. Furthermore, he articulated that the certificates he received “boost your self-confidence, but it’s just a piece of paper so, it actually has no meaning to what intelligence you have” (Int. Y 64), as well as “boost your self-esteem” (Int. Y 67). He is very measured in his response by acknowledging the powerful use of receiving certificates, but at the same time acknowledging that it is only a piece of paper and does not say anything about intelligence. This is a precursor, I believe, to the fact that he did not receive certificates at high school, but that fact does not mean he is not intelligent. In primary school there is a deliberate attempt to boost the confidence of as many students as possible through awards and certificates, but this is not a high school practice. This, for Yusuf, is very unfortunate. This particular aspect of the interview also highlights the fact that he did not receive recognition for his math work as he did in primary school. Yusuf’s perception of his good ability in mathematics was manifested in the fact that he received certificates as proof of his success. Consequently, he admitted “it made me feel like I should do something that’s going to be done with mathematics in the future, but I never really thought far about it at that time” (Int. Y 60).

From these early years he developed a positive disposition towards doing math. As soon as he started high school he started to dislike math, but still saw great value in pursuing math because of its importance in his future career. He is very clear that the reason for him not liking math is due to his math teachers. However, the fact that he loved math at primary school was not due to his primary school teachers. Although he loved math at primary school he did not attribute this love to these teachers, by noting that “the teacher never used to teach that good, but I used to sit a lot with my textbook and go through it with my mother”. He does not necessarily attribute his primary school success to the quality of teaching or qualities of the teacher, but emphasize his own ability to learn from the textbook with his mother at his side. This shows that his mother played a crucial role in his math learning. In addition, Yusuf also maintained that his achievement was also based on his own energy and ability, because he worked through the textbook. This showed that he was able to work independently in primary school, but acknowledged the role of his mother and sibling, by articulating that not only his mother played a role, but his older brother who “also used to help me now and then” (Int. Y 72). He emphasizes “now and then”, illustrating limited help from his brother. He was doing well in mathematics because of his own ability and not because of his teachers.

He admitted that the transition from primary school mathematics to high school mathematics was not that easy, because when he started at high school he “never knew what was going on, starting with the algebraic letters. I was not taught that in primary school” (Int. Y 160). He complained that the work in high school was “completely different to what I knew before” (Int. Y 170). This lack of a good or positive transitioning to high school showed the turning point in Yusuf’s mathematical experience from ‘liking’ math to ‘disliking’ math. He started his high school with limited math understanding due to gaps in learning, but also developing a hate for the section in mathematics, namely algebra, critical to his continued growth and conceptual understanding in mathematics. However, he admitted that this feeling toward algebra has changed a little now that he is in grade 11, because he was now getting used to ‘algebra’ in particular, but he still did not realize the importance of algebra in his future. He still believed that algebra will lose its usefulness. The transition from primary to high school was characterized by a lack of understanding in the new high school content.

These fluctuations in the development of a math identity imply that these identities are continuously under construction by the student and can change very easily. The role of the teacher within this construction is crucial to the development of a stable leading identity, which in Yusuf's case is based on the value that math will play in his future, future career or the choices that he must make.

Emotional attachment in doing mathematics at high school

Yusuf's emotional attachment to doing math was seen in grade 8, when he noted that "it was a bit weird to use letters instead of numbers", highlighting the fact that he really did not understand these 'pre-algebra ideas'. In grade 9 "the sums became even more complicated than before", because now he had to apply arithmetical operations to terms that included numbers and letters, hence felt confused. Despite these feelings of confusion in understanding basic algebra in grade 10 he still "chose maths because of the field I'm wanting to go into, engineering" (Math Story). He also said "I used to like triangles and shapes, used to like working it out and working out the area and the perimeter" (Int. Y 80). In particular he liked numbers and graphs, but "dislike the equations and all the long formulas" (Int. introductions).

The following emphasizes the importance of good grades and expectations related to examinations or tests. He said "a while ago we wrote maths and I thought I was going to fail the test, but I actually passed the test with 45% ... ever since I've been studying my math for 30 minutes, three times a week" (Math story). The fact that he was happy with 45% is very revealing in terms of his low expectations regarding a test result. As a result of this 45% he has changed his working habits by spending more time doing math. It seems as if 'not preparing' for a test or exam is a routine with Yusuf and his view on being a 'success in math' student is based on this story that he shared. He said:

> I was challenged last year with my final grade 10 exam. I never studied due to a bet I made with my friend. He told me if I could pass without studying he would give me R10. I passed with 57% but my friend never gave me a R10, he doubled it. The challenge strengthened my mind set towards maths. If you try hard enough you can succeed (Math Story).

One could consider his actions as reckless behavior, by gambling with his final examinations. More than that he said if you "try hard enough you can succeed", but he did not try because he claimed he did not study. These two attributes are in tension with each other since he did better when he did not study. However, he provided a motivation for this tension, by saying "sitting in class helps" (Int. Y 217), meaning that "the teacher must be effective at that time" (Int. Y 219), because "I remembered what was going on in class" (Int. Y 222). This articulation suggests that his success in the examination can be attributed, partly, to his teacher because he remembered what his teacher had done in class. It also emphasized how Yusuf learnt math, by doing what his teacher did. However, it can also be attributed to him, to his intelligence, because he knew what to take and how to manipulate the formulas from what the teacher said. It was difficult to ascertain exactly why he related this story, because it seems to contradict some earlier observations regarding his attitude towards his math teacher.

Cultural Models

The first cultural model emerged from the belief that math was easy or difficult. According to Yusuf he "can't say it's easy and can't say it's difficult, but if you know the formulas then it will be easy, if you don't know it, it will be difficult" (Int. Y 76). He added that

“I like learning new things in mathematics, like when they start a new formula, then I will like, take note of what the teacher is setting out” (Int. Y 352). Fundamentally Yusuf believed that math is all about knowing formulae, and this lack of formulae knowledge determines whether math is easy or difficult. This emphasized a very limiting view, because he never mentioned understanding concepts, but emphasized rote-learning of the formulae. The teacher gives the formula and the student learns it. If you do poorly in math then you did not study your formulae.

The second cultural model relates to his attitude and feelings towards math, math teaching and his work ethic, including his behavior in math class. It was difficult to separate attitude from a behavioral response on the one hand. Yusuf gives a positive statement like “I try my best”, but on the other hand qualifies it with a negative response like “at times” (Int. Y 143). His related behavior does not necessarily support his generally positive attitude. Yusuf admitted that in grade 8 he had a very strict teacher for math and disliked math as a result of the teacher. Consequently, he was “lacking in math”, meaning that he had gaps in his learning of math or did not know all his formulae, because “I was falling behind in my work”, meaning that he did not have all the necessary math content required for that period of time. His behavioral response to the strict teacher and his dislike for math was to stop “taking note in class” (Int. Y 175), meaning that he did not listen to his teacher.

A third cultural model was his belief that success in math is determined by your grade in math. He noted that “success basically means, you do well in your subject ... in mathematics now, if you reach an average of 60%, 70% plus ...you would be successful” (Int. Y283 - 286). This contradicted his earlier view that 45 % was a good result, but at the same time, this low percentage also showed that he was unaware that for his chosen career, he would only be accepted in a university if he achieved 75% and above in the matriculation examination.

However, he admitted that he was focused on achieving this success, but emphasized that “I try to aim for it but at times I don’t get up to 70%. I stay in the 60’s” (Int. Y 288). He firmly believed that if he gets above 75% in a math test or examination it will “motivate me and boost my confidence” (Int. Y 293), and in fact he “would feel quite excited about being able to do it, I will be more with my books and try to get higher, up into the 80’s” (Int. Y 290). His motivation to be successful in math is based on the belief that if he can just get a 75% it will motivate him to do better in subsequent tests/examinations. The problem with this belief is that he needs to work harder to get to 75% so that he can be motivated and develop the right attitude to achieve success.

Another cultural model regards Yusuf’s belief in homework and his responsibility to do math. Yusuf believes in the value of homework to improve his math learning, “I think the teacher should give us more homework, more often”. He believes that it is his responsibility “to take pride in my work, and study my work hard and at least try to pass, try my best” (Int. Y 362). He attempts to spend at least one or two hours every second or third day on mathematics. The impression is that he does not do math everyday.

Yusuf believes that the school and teacher should also play a pivotal role in his success. He stated that the school had a responsibility to tell the teachers that they must teach properly, they must give more math activities, they must give extra math classes after school, and they must be willing to work with small groups of about 5 students. This, he believed, would make him a better student. He believes that his peers “should actually just sit and be quiet in class and take note of what the teacher says” (Int. Y 405), and they “should all do their homework, and try to concentrate more on their mathematics” (Int. Y 408).

Another cultural model and a key element of this leading math identity was that Yusuf believed that he had the necessary qualities of an ‘excellent’ student as defined by Yusuf. In constructing his mathematical identities he viewed the relationship with the teacher as a basis for his beliefs about opportunities to learn. This relationship with the teacher is an important component of mathematical identity, especially for low-achieving students like Yusuf. Within this part of his narrative he emphasized that every student needed this quality to “being very cooperative with the teacher” (Int Y 245). Initially, I did not understand what Yusuf was trying to say, but he added that every student also “needs to have all the tools he [the student] needs, like the calculator, ruler, protractor and such things” (Int. Y 246), and that every student “needs to have confidence” (Int. Y 250). He said “I have most of the qualities [as noted above] except my having the proper calculator that the sir wants…” (Int. Y 260). He owned a Sharp calculator, but the teacher wanted all the students to have a Casio calculator. During this conversation I could feel Yusuf’s agitation and frustration with this issue, because he spoke abruptly “I am able to use it [his calculator] but most of the stuff he’s [the teacher] giving us I can’t do without a calculator [the teacher’s calculator]….” (Int. Y 265). His frustration was based on the fact that the teacher would not help him with the key-strokes for his calculator resulting in incorrect answers, because the teacher only showed the key strokes for the Casio calculator and not the Sharp calculator. This affected his ability to complete tasks, and affected his opportunities to learn in class, resulting in a math identity characterized by feelings of frustration based on the teacher’s request.

Yusuf also believed that ‘pacing’ in teaching influenced his opportunities to learn effectively. When asked about things that will help you succeed in math, Yusuf responded by saying that “most of the students in the class give help so now and again, because the teachers are too fast for the children" (Int. Y 89). He makes the clear distinction that the teacher's work too fast and the students help each other. However in his math story he noted that "in the beginning of grade 11, I change teachers for maths ... he is going a bit fast but I'm starting to get that feeling for maths again" (Math Story). Although this specific teacher is going too fast, he seems to like what this teacher is doing. So despite the pacing issue, he is getting that "feeling for maths again". The inference is that he is talking about regaining those primary school feelings that he had for math. However, he complained about the pace of teaching, and felt that his current math teacher was moving too fast, skipping certain sections, and skipping certain ways of working out a problem. He wanted to see more ways of solving that problem. He said "I feel it is not right and that they [the teachers] should go through everything thoroughly with the students" (Int. Y 191). He was very vocal during the focus-group interview when he said "I think that the math teacher we're getting now isn't suitable for us. We should get a better one, someone who is going to work with everyone in the class, because his attention is mainly on the fast lane students" (FG. 524). Yusuf blames his teacher for the lack of opportunities to learn, and the result is that he is now developing a negative disposition towards mathematics.

Yusuf also believed that mathematical knowledge is not only vested in the teacher, but sits outside of the teacher. After claiming that his teacher is the cause of his problems he found value in working through the textbook. He used his text book because "my text book has all the answers in the back of the text book, so I go through the question, then I try to work it out and once I have an answer, then I check if my answer is right, at the back of the text book" (Int. Y 195). He valued mathematical knowledge via the text book, because it gave him the 'right answers'. These 'right' answers confirmed that he was successful in his homework, hence successful in math. This strategy seems to work well for him because he doesn't "need the teacher to give the answers every time" (Int. Y 199). This particular issue, together with the teacher's insulting words, resulted in Yusuf displaying high levels of frustration and helplessness which impacted his own interest in doing mathematics in the classroom. He said that the teacher "checks the homework by letting the students write the answers down on the board, and if you write down the wrong answer, then he is a bit harsh on you" (Int. Y 376). Yusuf feared verbal punishment from the teacher and was too embarrassed to work on the blackboard. Yusuf reiterated that the use of the textbook provided him with a measure of confidence to complete the homework and at the same time it made him less dependent on the teacher.

Yusuf believed that his math ability was adversely affected by the size of the math classes, it should, according to him, be between 20 – 30 students. Currently he is in a class of 37 students, but they were originally 42 students. The number dropped to 37 because 5 students, after only two months into the academic year, were forced from the pure mathematics class to mathematical literacy. This is Yusuf's big fear that he will also be forced to drop pure mathematics and go to mathematical literacy classes where there are more than 40 students.

Summary of Yusuf's mathematical identities.

Yusuf's family has a great influence on his mathematical achievement. He developed a strong sense regarding the importance and use-value of mathematics. He is driven by his future-aspiration motive to become an engineer, hence his decision to do pure mathematics. He has high expectations for a place at a university. He believes he has the qualities to be successful in math. He aligns himself with a culture of performativity, and that he is confident and motivated to perform and be successful in math. He firmly believes that his teacher is the cause of his current mathematical failures because of negative feedback from the teacher and that his teacher loves to embarrass the students. In addition he believes that the school must play its part by making sure they provide effective teachers and they reduce the number of students in the math classes, by suggesting a change from 42 to 21.

Yusuf constructed his math identity as someone with a positive disposition towards engaging in mathematics activities. He believed he was good at mathematics: he was a ‘bright’ student, hardworking, high self-esteem and self confidence. He hated algebra, fractional work, linear equations and trigonometry, but loved geometry. He could not decide whether math was easy or difficult, but believed math was all about knowing your formulae. Yusuf feared failure and verbal punishment from the teacher, and preferred working with his textbook than working with his teacher.

Avril

Introducing Avril

Avril revealed a very positive image of herself when she humorously said “the one thing I like about myself is my friendly personality”. This humor was consistently displayed throughout the interview, with regular outbursts of laughter. These were not distracting or artificial, but sometimes it seemed like a nervous laughter. However, it was a key aspect of her fun-loving personality. She never resisted answering any question and was willing to provide a context for everything she said.

She was born in the same school community that was characterized by high levels of poverty and unemployment, while gangsterism, drugs and crime were on the increase. At two months old, her parents divorced and she stayed with her mother until her mother died a few years ago. Consequently, she moved in with her father and step-mother. She was the youngest of three children. Both her sister and brother were married. Her mother only made it to grade 11, but failed. Her father made it to grade 12, but dropped out of school because “the work got too much for him” (Int. A 27). Her mother worked in a manufacturing factory, but was forced to quit her work because of heart problems, bronchitis, and asthma. Her father works at a shipping company and moves around the country.

The pressure from the family was tremendous on Avril to complete her matriculation examinations, because neither her sister nor brother completed matric. She would be the first sibling in the family to achieve this seeming difficult objective. Her brother did not complete his matric because “I don’t know what went wrong there ... he kept changing his subjects ... took subjects he could not do” (Int. A 46). Apparently her brother made wrong choices and she wanted to avoid his mistakes.

Leading Identity

In relating her story I noticed a key component of her mathematical identities that might be considered her leading identity, because it seemed to be the key reason for her wanting to study mathematics. Avril chose mathematics and not mathematical literacy because of her field of interest, dermatology, and that she was always interested and good in mathematics in primary school. Avril reported that “My mommy is very driven, she wanted me to finish school, she has a big picture for me” (Int. A 41). She noted emphatically that “my mother really drives me to finish school because she said nowadays that without a matric you will not get a job” (Int. A 53). Her mother displayed great knowledge of the curriculum and knew that a matric is vital in terms of opening up job opportunities, as well as for entrance to a university or college. Her mother also noted that a matric without mathematics will not open many doors to the future. Avril, following on her mom’s advice concurred that “I know for a fact that I need math for my future. It is my dream to become a dermatologist and math is required to accomplish that dream” (Math

Story). This specialization in dermatology required a “good pass”, meaning a high percentage, in matric mathematics.

Avril constructed her mathematical identities along the lines of the importance of mathematics, and not out of an intrinsic love for math. She stated that “I’m not a fan of math, but if I do not succeed in mathematics, I won’t drop it”. She realizes the importance of math as a discipline, and even failure in math will not make her drop mathematics.

This idea of failing mathematics defines her emotional component of her mathematical identities and drives her narrative. In support of this idea she claimed that she had “a fear of failing it [math]”. She expanded on this by further articulating that “although I try not to think about it [failing], the thought that it could happen terrifies me” (Math story). She uses this strong word ‘terrifies’ because she is afraid that she will disappoint her parents and herself, as well as not being able to fulfill her dream of becoming a dermatologist. However, she declared that her big hope was to succeed in mathematics by “giving her all” and if she fails then “at least I know, deep down that I tried my best” (Math story).

In her narrative the passion to become a dermatologist, drives her to succeed in mathematics. Despite this desire to become a dermatologist she is not convinced that the current math that she is doing is relevant for this career, she noted that “if I do become a dermatologist, I would like to write books on how to use, maybe home remedies. You don’t have to waste money on expensive creams and stuff, so then you would have measurements and stuff, so I do think you will be using math. Not really that much” (Int. A 151). She emphasized that there’s “not anything that we’re doing now” in mathematics that she will use in dermatology, but she admitted that mathematics is “good to learn” (Int. A 160). Substantiation for constructing her leading identity about the use-value of mathematics is given conclusively in her statement that “I chose math because if I don’t qualify for that field, I can go into something else with maths” (Int. A 58).

Development of her math identity (Math journey)

Avril constructed her narrative by articulating a very positive relationship with mathematics that supports and is relevant to her leading identity. This positive relationship with mathematics defines her mathematical identities from primary school into high school. In primary school she always found math to be “fun and challenging” (Math story). In the early grades she was “basically good at everything, especially math” (Math story) and that her reasoning ability for problem solving “was a bit ahead of the rest of the class” and this resulted in her achieving the number one position in her class. She claimed that in grade 7 “math had become a breeze for me” (Math story). She showed high levels of self-confidence in her math ability by adding that “my logical reasoning was excellent”, “my teachers would regularly remind me of my thinking abilities”, “once again, I topped the class as the Mathematics Student of the Year [her emphasis]”, and she capped it all with “those were the good old days of mathematics” (Math story). Her teachers played a dominant role by ‘designating’ her, the best student, because she thrived on this knowledge. She said “That’s where I discovered my passion for Math. I found out that was my strong point, and I never got a low mark for it. Always get high marks” (Int. A 101). She was so excited when she articulated that “I never struggled with math. That is one subject I would never fail, ever” (Int. A 104) and “I was always 80%, 90%, so now it’s harder to get” (Int. A 106). She built on this “designated identity” (Sfard & Prusak, 2005) and she constructed her primary school mathematical identities as a supreme student of mathematics. She felt great when her teacher “gave us such a challenging problem-solving, that I was over the moon” (Math story). I believe that her intention was to convince me that she really loved problem-solving and mathematics. This period of schooling resulted in mathematical identities characterized by “a passion for numbers, graphs, patterns, etc” (Math story).

Grade 10 can be viewed as the year of dramatic change in her mathematical identities, because she felt “as if math had transformed overnight! It was as <u>if I didn’t understand anything</u> [her emphasis]” (Math story). She claimed that “the basics were easy, but then I found out <u>trigonometry was my breaking point</u> [her emphasis]” (Math story). The following response captured how her confidence levels towards mathematics since she started high school were decreasing.

> Sometimes if I don’t really understand, like I will just look at the work, and I’m just like, why am I doing this to myself? ... sometimes I don’t really get the stuff, but I do put effort in, because at home I’m always every day in my room, so I will just sit there and see if I need help and then I’ll look through the stuff that I don’t understand, and then I’ll leave that and go on to the next question, so then I can get a better, a clear idea of what should be done next time ... but I do try (Int. A 162)

All these are important in defining the key components of her constructed mathematical identities. She believes in understanding mathematics, it is not just about getting the answers correct as she noted:

> What’s the use you have the answer and you don’t have a way of getting to the answer. If we like have to go through the work as we did previously, like yesterday’s work, he [the teacher] will tell us to check the answer at the back of the book. And honestly I don’t think that’s how you teach math. You should actually show the way, if there’s maybe a longer way or

> a quicker way. Show us that because that can be useful. In the exams, because I still don't understand the use of the answer in the textbook, or show how you should get that answer, that will help you, but how you going to get there if you don't understand what you doing. (FG. 421)

She contextualized her struggles with math by declaring that in grades 8 and 9 the teachers "spoon-fed you" (Math story), but in grade 10 and 11, "this was the real deal that could either make or break you" (Math story). Perhaps in the early grade teachers were doing the work and giving the answers, and in so doing, math was made easy because the teachers helped students all the time. This depicts how, according to Avril, pedagogy or instructional practices play a huge rule in constructing mathematical identities. In grades 10 and 11, it was up to the student to do all the work and because of this pressure and burden on Avril to succeed in math, she "felt like dropping math" (Math story) for the first time, but realized that she "needed it [math] for the field I was going into". Once again substantiating the construction of her leading identity based on the use-value of mathematics.

Emotional attachment in doing mathematics at high school

The change in the teacher's pedagogy and instructional practices in high school resulted in mathematics becoming more difficult, challenging, and slowly destroying her positive spirit. Avril admitted that some of the students in the math class felt like "bunking [playing truant-skipping class]" because, it is "how the teacher teaches, you dislike maths immediately. From this year started I honestly started to dislike maths. I used to love it and now it is just not the same" (FG. 258)

Despite this development of a fragile mathematical identity (or a mathematical identity undergoing dramatic change from positive to negative feelings), she added extra motivation for not dropping mathematics. She claimed that “I also thought about overcoming my weaknesses” (Math story), and she was not going to “let one chapter put me behind. I was going to put everything in” (Math Story). She suggested that it was only one area in math that was causing the most problems and was confident that she could overcome the difficulty in the other domains of mathematics. The confidence she displayed was evident, and it was these features that defined her mathematical identities.

Avril admitted that since the beginning of grade 11 “I could feel my luck changing” (Math story). She claimed that she never understood math in grade 10, but never substantiated what she meant by not understanding. Although she linked this to her confidence and her ability not “to slack off” and stated that “now that I understand math more clearly, my thinking ability has also become clear and that was my turning point – when I rediscovered my passion for mathematics” (Math story). She places demands on herself to overcome any deficiencies or gaps in her mathematical knowledge and at the same time emphasize how she learns mathematics by describing her ability as a student of mathematics:

> I should work more at home in my free time. Do stuff I really don’t understand so I can get a better idea, because they say ‘practice makes perfect’. So I will practice how to become a better math student. I’m not bad at math, I’m an average level, like most of the children, but I think I can achieve better if I want to. (Int. A 174)

Despite this desire to work harder she still had difficulties in certain mathematical domains, namely “trigonometry – the word still haunts me” (Math Story). She admitted that when she was introduced to trigonometry, she was too “embarrassed to enquire more about trigonometry, it was not easy to overcome”. She received some help from her teacher and started to understand, but despite this slightly better understanding she did not change her feelings concerning mathematics. She clarified this by claiming that “I don’t hate math, but really enjoy it and I choose to do math, not because I have to, but because I want to” (Math Story). She used this as an opportunity to compare her primary and high school experience by reporting that:

> In primary school when the bell rang for Math, I got so excited because I knew that was something I could do. Most children couldn’t. And now in high school what I like about my Math classes, (laughs) how the sir embarrasses the children (laughs) ... (Int. A 117)

Her laugh was sarcastic as she related this, because she hated this approach by the teacher and used it to stress how the teacher embarrassed the students. She observed that “you can see they feel worthless ... and sometimes when someone tells him that, he gets very defensive and he tells you, you should get another teacher ... that’s not encouragement ... he should actually encourage a person, so he’s not really a good math teacher ... not to me” (Int. A 131).

Cultural models

The first cultural model is based on the belief that the relationship with her teacher formed an important component of her mathematical identities. She believed that a teacher must encourage and support students, and not break them down. She revealed the pedagogy of the teacher and his attitude towards those students who do not understand:

> He would like ask if you don’t understand something, he would do two or three examples on the board and if somebody still says they don’t understand, because now something is out of their understanding of what the examples mean from the board and then he would like go like. What! Are you stupid? (Int. A 124)

This had an impact on the construction of her mathematical identities because she had lost faith in the teacher's ability to help her, and to avoid embarrassment she rather kept away from the teacher. She believed that her peers in the class would be more helpful than the teacher, and that for her to be successful she basically only needed her calculator and peers:

> Well basically it's my calculator, it helps me, and sometimes if I don't really understand something I would ask my friends rather because I know if I had to ask my sir, then he's going to, like embarrass me in front of the whole class, and we're a big class, and you don't want to feel like that. I know how it feels ... I would like, ask my friends rather, and then mostly I have my calculator to help me (Int. A 136).

Another cultural model is based on her belief that success in mathematics should be rewarded by awarding certificates of merit to high school students to motivate them, but not only to the best students. When asked how she would feel if she received an award, she said:

> I'll feel on top of the world ... 'You don't really feel in charge of the students that are well ahead. It's almost like they not giving you a chance and you know while they're around that you won't really achieve that. So you don't also try. (Int. A 204)

She firmly believes that it is difficult to motivate yourself, but the award would motivate you to work harder, "It motivates you. So, I'm not really trying to get one, but if I do, I'll be happy because I want to throw everything in that I can, working overtime …" (Int. A 209).

Her fundamental cultural model is based on the belief that mathematics is not difficult. She expressed the following sentiment:

> Math is not difficult. Like, we spoke about that now the other day that anybody can do Math. It is just about your focus, logic thinking and stuff, and your reasoning also. So I don't think Math is difficult, but I don't think it is easy also. You have to work to get what you want. You can't just expect to get good marks if you don't work, so I think Math is in-between. (Int. A 109).

In expressing this belief she emphasizes her beliefs on the nature of mathematics. She believes that she possesses good thinking ability, reasoning for problem solving, and logical reasoning and these abilities separate her from the other students:

> [These attributes] means a lot because most children are not really on that level They can't do certain things, because sometimes they don't function or, like how can I say, they don't really try hard enough ... I wouldn't want to tell others I was one step ahead of them, because I can do all of this but they know they can't. This doesn't mean I have to look down on them, I can help them. (Int. A 235).

In addition she believes that logical thinking means that you have good understanding, but this must go together with good listening skills because it is important to "listen to people's stories, their domestic problems, anything, they just come to me, I would listen" (Int. A 242). She is positive about her ability, and she believes she can help others. Her self-confidence is high because she believes in what she is able to do, while other students do not know it. She positions herself as better than others in her intellectual category.

She related a class structural problem that involved the teacher giving greater attention to students in the "fast lane". This cultural model speaks directly to the belief about the ability to perform in a math class characterized by class discrimination. Avril believed that this arrangement was detrimental to a great classroom atmosphere, and resulted in negative attitudes towards mathematics and to engagement in mathematics. She reported that:

> ... at the moment we're work individually and like we said, now the other day, the fast lane students, they're in their group. They are on one side, and all the other children that really don't understand, are on the other ... they have a better understanding, we don't… (Int. A 316)

Avril claimed that the teacher showed interest in this "fast lane" group by allowing them more time at the blackboard, and more time to ask questions, because they understood most of the work. It also seems as if the "fast lane" students provide the teacher with credibility that some students follow and understand his teaching.

In conclusion, the following response revealed Avril's belief about the purpose of mathematics. This also defines and substantiates the choice of her leading identity. She said:

> I don't really know why we do math, but I know we do it and I know it's compulsory. Most jobs requirements include math, and you not really going to get a job if you don't have math. So if you have this big dream about becoming an engineer you going to need math. So I don't think you should have a choice about choosing math. If you could choose math or not do math, I don't think anybody should have a choice. Everybody should do math (Int. A 348) ... I am really motivated because I want to be the first person in my family to pass Math. That's the only subject I'm worried about. So when I define myself I would say I'm motivated, very confident …good self esteem … (Int. A 440)

Summary of Avril's mathematical identities

Avril will be the first of her family to complete matriculation and perhaps go to university if she fulfils her dream of becoming a dermatologist. She has accepted this responsibility to do what no other family member was able to do. There is no question in her mind that she will go to university, because she has a positive disposition towards math, as well as a positive perception regarding her own ability to do math. She constructed herself as having this positive relationship with mathematics as she drew on her positivity about her primary school achievement. Her imagined future is the motive for driving her to do well in mathematics. Avril constructed her mathematical identities along the lines of the importance of mathematics, and not out of an intrinsic love for math.

She is terrified about failing and disappointing herself and her family. Consequently she sees herself as hardworking and giving her all in the quest to pass mathematics with a good grade. She reported critical moments in her math experience and these included: entering high school and coming to grips with new teachers, new methods of teaching, and new norms and standards. Trigonometry was her breaking point, because she cannot master it. She considers her teacher as the key reason for her struggles in math, and for math becoming more challenging and difficult. She had lost faith in the teacher's ability to help her, and to avoid embarrassment she rather kept away from the teacher. She firmly believed that high schools should follow the primary school tradition of handing out certificates or rewards to all students for working hard, because it is a source of motivation.

Moses

Introducing Moses

Moses is from a very poor community characterized by a gang-ridden lower to middle working class 'colored' township about 20 minutes from the school. The community is known for its daily struggles for survival from intense poverty, and consequently the self-worth of the people is very low. Moses was very thoughtful and transparent throughout the interviews, not afraid to speak his mind. He has a big family of 6 children from his mother's side with two brothers and three sisters. Moses was the third oldest, with the twins older than him. The youngest sibling was 2 years old. He was quite emotional when he reported that "Most of my life I was staying with my mother, because my father was in prison. Due to my father's action and stuff, we as a family never got on well, but now have a tighter bond with each other. My mother was always there with us" (Int. M 6).

When asked about the education of his parents, he noted that "No, they didn't have any tertiary studies and stuff, but my father got into chemical engineering. He went for a course at SASOL [South Africa Synthetic Oil Liquid], but then he got into drugs and stuff …" (Int. M 18). At this point his voice went soft and inaudible. Unfortunately, Moses was not prepared to say much more about his father and prison. It made him very emotional just thinking about his father still in prison. Apparently, his father did not complete the SASOL course because of the development of bad habits that led to prison.

His mother was undoubtedly his source of inspiration and encouragement. Both parents succeeded in their matriculation examinations. His mother remarried 3 years ago. His mother is a house-wife and his step-father "is a very good man, he is nice to look up to, and he's an electrical engineer" (Int. M 27). These words, I believe, should not be taken lightly because Moses was looking for a father figure and I believe that his stepfather had a great influence on his identity and particularly his mathematical identity. Moses chose his words very well in describing his stepfather when he said “very good man”, not just good man, “nice to look up to” not just look up to, and “he’s an electrical engineer” not in prison. His mother was more than just a house-wife, but shaped his identity by setting clear standards. Moses reported that:

> She’s very strict you know, like for example, she bought me a Play Station, but then you know with that comes certain rules. She only allows me to play on the weekends. During the week it’s strictly school work. There are certain times I need to be inside, and everyday when I get home she asks if I have homework, if I did well, how my day was at school, you know, any little issue we will talk about (Int. M 33).

His mother cared about him, and his progress at school. She set uncompromising rules with a stable work ethic, and was very supportive and encouraged his ambitions for the future. He admitted that when he was younger he received “lots of attention”, and that his siblings helped him with math. He believed that this assistance helped him. He has learnt discipline and a good structure that supports an effective work ethic. These characteristics played an important role as he constructed this coherent narrative of himself as a student of mathematics.

Leading Identity

In his math story he noted that “my dream is to become an Air Traffic Controller” (Math story), then qualified the reason for this dream in the interview by saying “because I love geography and physics. I just love the whole sense of numbers … It always fascinated me, how they get that big object into the air, but I’m a bit afraid of height, that’s why I do not want to be a Pilot” (Int. M 119). He wanted me to know why he wanted to be a controller and it was because of the use-value of math and physics. He was “fascinated” with what keeps the plane in the air.

He emphasized that knowledge of mathematics was needed, "obviously you need to calculate where the plane is on the runway and all those issues you know, but ja, it's a lot of stuff" (Int. M 125). Moses realized that you needed to work hard in mathematics to get into a good university, and to fulfill your dream it is important to know "your work and stuff, and to ultimately get you a good job. Doing something you love doing. It shouldn't just be what you are forced to do it should be something you love" (Int. M 137). He wanted to reinforce this love for math by noting its relevance for everyone claiming that the "math we encounter everyday, outside and stuff in everyday life. We need it to actually understand why certain things are happening" (Int. M 207).

The mathematical identities of Moses were driven by his dream to become an air traffic controller because of the importance of mathematics based on the use-value of mathematics. He admitted that "if you want to get certain qualifications and get a good job, you need math, pure math, and get to do math" (FG. 41). He identified these areas that require certain qualifications "like engineering, you want to become a doctor, teachers, and stuff" (FG. 44). He emphasized both the importance of doing mathematics and why it is important to do it.

Moses was adamant that mathematics was useful and said that even if he did not need math for his career he emphasized that "I think I would still be doing it. It's a nice challenge, you know; keep your mind working all the time. Ja, I will still do it" (Int. M 153). He was very honest regarding mathematics becoming too difficult and that he was struggling to cope, but suggested some ideas for dealing with this. He would "first of all seek help, and if that didn't help me I would consider dropping" (Int. M 158) mathematics. He articulated that "if I can't cope with mathematics, then I will have to resort to mathematical literacy, which is something that I don't want to do" (Math story). He knew that this change from mathematics to mathematical literacy will not get him "anywhere in life with math literacy".

Encouragement and support from those around you is beneficial for success in mathematics, according to Moses. He consistently reiterated similar statements such as "support from your family and friends helps you a lot, as well as from everybody else. Just giving you support at the time you need to concentrate" (Int. M 143). He admitted that his sister and step-father "inspire" him, because his sister does electrical engineering and his step-father did chemical engineering. Together, they advised him to "stick to math" (Math story) and that engineering fields provided "good jobs". It is because of this prompting from his family "they always told me that is like good jobs, that you get and that's always something I want to do. That's why I want to do air-traffic control" (FG. 93).

Development of his mathematical identity (Math Journey)

He developed a positive start on this math journey as he recalled his feelings of elation in primary school, saying "I did like really well ... every year I would get honors for math, and the certificate ceremonies would be like a normal thing every year ... it was really fun, math" (Int. M 59). He was inspired by the fact that he was not only doing well in math, but he was enjoying himself. He also achieved good grades which "makes me feel happy and makes my parents, my mother feel happy also ... I feel very proud and stuff" (Int. M 69), but importantly good grades is "like, a reflection of you as a student like, um, it shows your capabilities and what you are showing to the teacher and everyone else" (Int. M 66). He used good positive language to support the idea that grades were important in the construction of his mathematical identities.

His primary school journey was characterized by a positive disposition towards mathematics and learning mathematics, which defined his mathematical identity during those early years. Moses thought that his mathematical peak "occurred in grade 7 where for the first 3 terms I averaged 90% in math" (math story). However, he admitted that his low point in math also occurred in grade 7, but during the last term[14]. This is a very significant moment in the construction of Moses's mathematical identities. During this term "the teacher just rushed the work, in you know, to prepare us for high school, so then you know you get a bit lost when they move too fast. I got a bit lost, and that was the only time when I actually struggled" (Int. M 62). He only struggled with math because of the change in the instructional practice of the teacher who focused on getting all the grade 7 math content covered before the students enrolled for high school in grade 8. The focal point in this story was the fact that it was the same teacher, for three of the four terms Moses had a positive disposition towards math, but in the last term everything changed and he developed a negative disposition because of the teacher. He was very bitter because of this negative experience at the end of grade 7. Instead of taking a positive disposition towards mathematics to grade 8, the first year of high school, he had feelings of discontent, unhappiness, and bitterness. The following transcript captures these feelings:

> At that time the organization for the school, they said that they were offering bursaries for high school, then you had to prepare for an examination ... then the teacher just rushed the last bit of the work, went a bit too fast, and when we came to the first examination … I did not do very well, because of that reason, I got lost. (Int. M 73)

His bitterness was a result of him missing out on the bursary, which he expected to get because he was a good student. He said "I got lost" meaning he did not have any idea about the math content for that last term. He blamed the school and especially the teacher for changing his positive view regarding math and this was the reason that led to him "getting low grades" (Math

[14] The South African school year consist of 4 terms: term 1 from January to March; term 2 from April to June; term 3 from July to September; and term 4 from October to December.

Story) and for him grades are important. As a result of these low-grades he was concerned how teachers viewed him, how his parents viewed him and how everyone else viewed him.

Emotional attachment in doing math at high school

In this construction of his mathematical identities he became emotional as he reflected on the transition from primary school to high school. He admitted that his "mind jumps all over the place when I don't know how to deal with something", and emphasized that "I panic a bit" (Math story). Moses gave the distinct impression that he was a very determined student, who believed that it is important to understand the math that you are doing. During the group interview Moses was particular emotional as he tried to explain his feelings when he did not understand what was going on in mathematics:

> It puts you into a slump like you feel in a hole you can't get out of and you falling further and further down. Then at one point you think, ok, you catching up now, you had a problem with this certain section ... then you think it's behind you now, but then it comes again then you have a problem. Just when you thought it's over, it persists. (FG 396)

Moses used the word 'panic' to describe his emotions during a difficult problem solving activity. When he encountered a difficult problem he said "I tend to panic a bit. I think, ok, this looks a bit foreign, then, I sit and think. This looks like another sum hidden, and you try to break it up and to do each piece on its own because sometimes it works and sometimes it doesn't". Despite this feeling of panic, Moses used this feeling to devise a solution strategy to help him out of this difficult situation.

He used to like working at the board, but he felt sad that his teacher made negative comments like "Are you stupid?", whenever he did anything incorrect on the board. He developed feelings of embarrassment and felt humiliated because “everybody starts laughing and stuff”. He has since lost his confidence to work at the board. Despite this lack of confidence in working at the board, Moses rated his motivation as 8 out of ten with ten being highly motivated. This made him feel good about doing mathematics and he reiterated that “when you like and understand something, then you’re relieved and you feel good about yourself. “Ok, I know this”, and I can relax now, so it’s fine” (Int. M 294)

Cultural Models

The first cultural model emerged from the belief that math was not difficult. He had a firm belief that mathematics is “not difficult ... if you understand it, it’s easy. I don’t actually find it very difficult, just in-between. I get confused, but I don’t find it very difficult” (Int. M 80). He associated difficulty in math with a lack of understanding, and if you understood your math then it cannot be difficult. He does not qualify the nature of this understanding. Although he has moments of confusion meaning a lack of knowing what to do in math, he does not link this confusion to math being difficult. Furthermore, he emphasized that “I know that if you actually work at it [math] you will actually pass” (Int. M 98) and “it’s paying attention in class, and when the teacher writes on the board I always take note first of what he is saying and then I write it down afterward. I first try to understand what the teacher is saying” (Int. M 101). His mathematical identity is defined by his belief about how you learn mathematics, which is by understanding. Consequently he believes math is easy as long as you understand the math.

The second cultural model is based on his belief about the qualities of a successful math student. He identified himself as a successful student in math because he was quite emphatic that “I am definitely hard-working and I do my homework and I work in class” (Int. M 166), that “when you’re successful you understand the work, you know what’s going on and can show someone else how to do it without making any mistakes" (Int. M 172), and that success is measured "by the grades you get ... for me the 80's and 90's [meaning percent]" (Int. M 175).

The third cultural model emerged based on his belief about motivation and instructional strategies used to gain mathematical knowledge. Moses believe in being rewarded in math, he feels that it motivates him to work harder by going home and drilling himself so that he can do better in the pedagogy of mental math. He stated that "the teacher will say 'ok, take out the page for the mental math', then she would give random numbers and stuff, then we had to write down the answer in a certain amount of time. That is what we enjoyed" (Int. M 83). The reward could simply be affirmation of good work by the teacher or be able to get the answer quicker than everyone else. He linked the strategy of mental math with enjoyment then qualified this enjoyment into 'happy' feelings that impacted his work ethic, by emphasizing that he "felt happy about it because it would make you go home and study your time-table and that, so you will know it for the next day" (Int. 87). In primary school he drew on cultural models which position mathematics and mathematics pedagogy as enjoyable and releasing feelings of happiness. So mathematics is fun as long as it is linked to rewards for answering correctly, even if it means drilling to know your times tables. In this regard he noted that "practice makes perfect" (FG. 146).

Another cultural model is based on the belief about his ability to perform within the classroom contexts. The following transcription highlights his belief in effective teaching approaches that worked for him. He is responding to the question: what kind of teaching approaches do you feel is effective and works well?

> Well, if the teacher works with the whole class and does not just focus on one area (group of students) then that would help everybody ...One

> area of students, yes ... Also having you do the work on your own before he gives you the answers and stuff. That actually helps a lot as well (Int. M 187).

The fact that his teacher only focused on one group, identified as 'fast lane' students, who the teacher gave 'special attention' to and focused more on their interactions than other members in the class, resulted in discontent amongst the students in the 'slow' group consisting of the majority of students. These students made derogatory remarks as other students engaged in board work. Moses endorsed the comment that "some children are very shy to be in front of the class and everybody is watching you and the pressure is on you ... they tend to get nervous and lose it" (FG 288). In fact he saw himself as one of those types of children who were shy, were nervous, and were unable to keep it together. This kind of 'favoritism' by the teacher made Moses very emotional and it impacted his opportunity to learn in the class.

Moses believed that it was very helpful to attempt the math work (tasks, exercises, and problems) individually before the teacher gave the answers. Apparently this was not happening and it caused great concern. The following observation draws on the cultural model that math can be very satisfying if it is taught effectively as defined by Moses: (A reflection from his best year, grade 9, in high school)

> The Sir, Mr. Awesome was very strict but I felt that that was a good thing. Even the lazy students were all working. There was no noise in the classroom and everybody was focused. When you came in everyday, he checked your homework. If you didn't do your homework you would have to stay in and stuff like that. He taught effectively, as there was

> concentration and no noise from outside because the classroom was on that side[15]" (Int. M 199).

Moses used the word "strict", because this resonated with the structure set up by his mother, who was also strict and disciplined. Furthermore, Moses mentioned "taught effectively" and this in turn seems to resonate with instructional approach used by that teacher. He identified the didactical approach that related to drill and practice because "practice makes perfect" (FG. 146), but included "practice and also your attitude towards work" (FG. 146).

Summary of Moses mathematical identity

There were two key features that characterized Moses's construction of his math identities: he wanted to be a better person than his father, who was still in jail; and he wanted to become an air-traffic controller, because of the use-value of math and physics. His respect for his mother and her rigid discipline formed the basis for how he lived his life. He learnt from his mother a discipline and a good structure that now supports his "effective" work ethic. These characteristics played an important role as he constructed this coherent narrative of himself as a student of mathematics. In primary school he was inspired by the fact that he was not only doing well in math, but he was enjoying himself. He wanted these attributes to carry on in high school, but blamed the teacher and his ineffective teaching style as the core problem. He complained about pacing of the lesson and the way the teacher spoke to students was embarrassing and very harsh. Moses gave the distinct impression that he was a very determined student. He believed in understanding the work, because he felt emotional about not understanding. He graphically explained these emotions as moments of 'panic' and how it impacted his learning.

[15] meaning an out-building classroom away from the noisy parts of the school

He rated his ability highly and perceived himself as high motivated, and he felt good about doing math activities. He firmly believed that math was not difficult and associated difficulty in math with a lack of understanding, and believed that if you understood your math then it cannot be difficult. He identified himself as a successful student in math, hardworking and that he valued doing homework. Moses believed in being rewarded in math, he felt that it motivated him to work harder by drilling himself so that he can do better in the pedagogy of mental math.

Katie

Introducing Katie

Katie lives within 10 minutes of the school in a community with high unemployment, high levels of poverty, and high levels of gangsterism and drugs. This area was also seen as a dumping ground for people dispossessed of their land during the Group Areas Act implemented during apartheid. Nowadays, the community strives to encourage this generation of young people to rise above their circumstances, to take the educational opportunities that their parents were denied, and become successful individuals. Katie is from a family of four with her younger sister in grade 6. Her father is a builder and her mother "works a lot so we don't see her much" (Int. K 5) she admitted. Her mother is a manager of a kitchen (of a restaurant) and the family only sees her on weekends.

Katie described herself as "outgoing ... like working hard and being with friends" (Int. K 8). Her father completed his matriculation examination, but did not study any further, while her mother dropped out of school in grade 10. Her parents were forced to work from a very early age because of financial problems.

Leading Identity

The following statement from Katie characterized so much about her disposition towards mathematics, as well as how her father wanted to influence her decision to do mathematics. She also gives an opening glimpse in her math story that she was not a positive learner of math:

> For me personally, they really want me to work hard and like I said with the pure Math, my father wants me to do pure Math but I don't feel like it's for me. He keeps on saying anyone can do it if you just work hard, and I believe that, but at times, they not there with us in the classroom so they don't know what we are going through. (Int. K 19)

This part of her narrative made it difficult to talk about one leading identity that was more significant than others. There were several components of her mathematical identities that could equally be classified as a leading identity. One component of her math identity is that she aims to satisfy her parents' wishes at the expense of her own. She said "they really want me to work hard", "my father wants me to do pure math", and "he keeps saying anyone can do it". She is not that convincing about her belief that her father is correct, because her experience with mathematics is very different and she said almost pleadingly that "they don't know what we are going through", suggesting that she is not the only one to feel that way about mathematics, but others in her cohort feel the same. While it was difficult to initially see why she constructed this negative disposition towards mathematics, it became clearer as she narrated her math story.

In grade 10 Katie chose mathematics instead of mathematical literacy, because "they all told me it would help me get into university" (Math story). She chose mathematics because others told her it is important for her future, especially if she wanted to go to university. Her imagined future had two complementary components, namely, that she wanted to become a chef and that she wanted to have her own business. In this regard she understood that doing mathematics, together with her accounting subject, will help her open her own business. Taking this imagined future into consideration, she wanted math to be her best subject, willing herself to believe this because then math will not be “a threat or a stop sign to my dream or goal” (Math story). Consequently, she is aware that mathematics is important for her dream.

This tension between the importance of mathematics, reinforced by her father’s wishes for her to do math, and the fact that she is not confident in being successful in mathematics permeates across her story. She is now currently in this math class because her father “really wants me to be part of math class”, and she claimed that “he doesn’t understand that I could fail because of it” (Math story). The fact that she is doing math and not math literacy is out of her hands because of her father’s wishes for her. She stated that she tried to explain to him that she would be better off doing mathematical literacy, but “he refuses to take my concern serious, he believes I can do it” (math story). Katie seems to have no confidence in her own ability, while her father seems to have all the confidence in her ability. Her main concern is a legitimate one, that she does not want to fail grade 11 because of math, and therefore she continues to try working very hard at passing math, if only to make her father happy.

Katie would like to become a chef in the future.

> The reason why I want to be a chef is that my mother worked as a chef at K at an early age and today she is the manager of her own kitchen. My father actually wants me to become an engineer, but that is not for me, so he does not agree with what I want to do, but the reason why I want to be a chef is because I find it interesting in creating new ideas and finding out how other cultures and countries think. (FG. 110)

Katie, not only wants to follow in her mother's footsteps, but also by aiming for this career, she may be able to avoid doing mathematics a mandatory subject for engineering as a career path. Her mother supports her dream, but Katie also claimed that it is what she wanted to become because "it's more what I see myself doing and what I enjoy doing, whereas my father's view is not for me" (Int. K 25). In fact, I could feel her anguish as she said "he wants me to be successful, but why can't he see me be successful in the dream that I want to do" (Int. K 28).

Despite her negative disposition towards mathematics and mixed feelings about continuing to do mathematics, Katie said that she is "really trying hard this term and if I pass Mathematics this term, I am going to stick with it" (Int. K 140). However, another possible component of her leading identity could also be defined by acknowledging the use-value of mathematics. Rather than completely dismissing mathematics as irrelevant to her future, she felt that she should struggle with it and simply work harder. She articulated that:

> At times, I don't think it's necessary for what I want to do, but because I don't have a choice, I just have to do it. Like, for the field that I want to go into, I don't really need this math that they are teaching me, but I think that maybe one day if I change, I think that if I go into another path, maybe it will benefit me (Int. K 145).

She may, in the future regret her decision to drop mathematics, so if she passed her exams this first term she will "stick with it". Within this conversation of the use-value of mathematics, she defined the purpose for doing math as "you need to know how to calculate things ... you need math everyday, even though you don't know it, you use it" (Int. K 251). Furthermore, she emphasized that she had the ability to do math, "like solving problems ... I like finding out how to get the answer, how many ways you can get to the same answer" (Int. K 256). It is precisely these beliefs regarding the purpose of math and her limited "like" for aspects of math, that shifted the focus in constructing one leading identity as a more significant mathematical identity.

She admitted that her exact position with math is that she is "on the edge, that's what I think, I'm in-between, so I don't know, I could fail or I could pass ... so I think math is difficult" (Int. K 332). However she stated that if she knew she would pass the math examination comfortably, then she "would be more motivated perhaps ... I would work even harder than I am already, I would involve myself more in the class, go to the board like they say, raise my hand when a question is read". If the situation at school was very different she would have a more positive disposition towards mathematics.

Development of her math identity (Math journey)

In primary school mathematics "was easier" and the teachers "took their time explaining the work" (Math story). Katie claimed that her teachers had a certain way to explain things to the class by using informal kinds of manipulatives that included "things like sweets or money, even people to explain fractions" (Math story). Katie believes that she is a visual learner because these manipulatives helped her to see things since "it made math easier for me", but it also helped her "become close to being accepted in the Star International Math Competition" (Math story). She claimed that this was the best time in her math journey. This represented the emergence of a positive disposition towards mathematics.

She claimed that in grade 8 the "math wasn't hard, because it was like revision of primary school", but in grade 9 things went bad because "the fractions started getting worse" (Math Story). Grade 10 and 11 was clumped together because "math is no walk in the park" (Math Story). Katie claimed that she struggled in math in these two grades because her teacher moved too fast with new work. However, the class-size, according to Katie is the ultimate problem, because “some children in the class learn faster and that means we move on faster” (Math Story).

Emotional attachment in doing mathematics at high school

Katie has a mix of emotions and strong feelings toward math stemming from a lack of confidence. She claimed that “I feel bad, because I’m worried that I might not make it in matric and sometimes feel that I should change to mathematical literacy” (Math story). She firmly believed that her turning point in mathematics was based on two reasons “the different teaching methods and load of work that got put on me now than in primary school” (Math story). These two reasons combined to make her “hate math”, and it is not for the lack of trying because she claimed that she, does try “very hard”. This effort with little reward, resigned her to say “still math is my enemy” (Math story). She encapsulates this transition from primary to high school beautifully, by stating that “in primary school I was excited to go to math class, now it’s just a living nightmare” (Math story).

She expressed the need to be encouraged and the need to be the centre of attention even if it is only for small periods of time. She said that her parents did not check on her homework because “they don’t necessarily check because of my sister ... they mostly focus on my sister, because they think I’m old enough, so I’m responsible” (Int. K 33) and “at times I would like them to ask me like, “do you have homework” and “are you having trouble with it or anything”” (Int. K 36). Her mathematical identity is defined by a low self esteem, and in this regard it seems as if most of the attention is on her sister. Katie seems to be envious of the attention given to her sister. In addition she claimed that she helped her sister “a lot” (Int. K 39) with mathematics and this in turn made her “really feel good” (Int. K 41). However, even when she helps her sister her low self esteem is prevalent, she feels she is always “supposed to be the example for her, and if I can’t do something, then I’m like, yo, what does she think of me” (Int. K 41). These feelings of inadequacy seem prevalent in this construction of her identity, she did not want to disappoint her parents and it was clear that she did not want to disappoint her sister. It is imperative for Katie to set the right example for her sister to emulate. The following transcript encapsulates these feelings:

> Yes, there’s lots of pressure on me. Because I feel like, if I don’t do the right thing, then she’s not going to do the right thing. So that’s why I do my homework, because if I’m not going to do it, she’s not going to do it.

She seeks affirmation from her parents that she’s doing a good job because she does not get it at school, and this may help her self-confidence. In fact when asked about how, on a scale of 1 to 10, she would rate her motivation to do math and her self-confidence with 10 the highest rating. She responded by giving herself a 5 for motivation and confessed that “I really don’t have much confidence in math, so I’ll say a 4”.

Cultural models

The first cultural model emerged from her belief that she is a particular type of learner of mathematics in primary school. She was clear about what she valued in primary school and reported that “I enjoyed the activities, the group work, the way we helped each other, we solved problems because we worked as a team ... you achieve more ... so I enjoyed that” (Int. K 83). The use of the cultural model that some students share is that mathematics is “enjoyable” and in Katie’s case this enjoyment’s source was vested in group work and working together to solve problems, because you learn from each other because you achieve more by listening to different points of view. Katie was emphatic about learning in this way when she said “Yes, I love working in groups” (Int. K 89), and she substantiated this ‘love’ by noting that:

> If there's a smaller group you work more together, but some people will learn faster than others. So if for example he [points to a student] does not understand, then I can help him. We all can help each other. (FG. 168)

The second cultural model emerged from her belief about constraints, troubles and opportunities to learn in mathematics in her classroom. The cultural model that builds on the notion that mathematics is "difficult" was framed very early in Katie's narrative when she articulated:

> In high school what distracts me the most is … how the teachers are, or how the teacher is, the way he speaks to us, because for me if you want respect, you've got to earn respect. (Int. K 95)

Katie related a story about another girl student, but I believe that she was talking about her own experience with her math teacher. She said a girl went to the black-board, and was struggling with the work. The teacher started "screaming and stuff" and the "class started laughing and stuff" (Int. K 101), this despite the fact that the girl did not want to go to the board in the first place, but the teacher insisted that she go to the board. Katie related how it affected her and she developed a negative disposition towards mathematics. She articulated that the girl was "embarrassed, and for me that is a distraction because it doesn't give us space to try. Now before she went up to the board, he's like, "don't worry, just try!" and he won't say anything, so she trusted him but then…" (Int. K 109) he broke that trust. This experience and many more experiences relating to this teacher resulted in Katie admitting that "I really don't like it in high school" (Int. 114).

The teacher's role in affecting Katie's mathematical identity cannot be underestimated as it is a barrier to her learning math since she has lost all respect for the teacher. In fact this relationship with the teacher has deteriorated so badly that Katie displayed sincere honesty in telling me that "I decided that I don't want to do math, because when we wrote the exams then he was like, yes, whoever failed should go to math lit [literacy], and he's not asking us to do it, he's telling us to do it" (Int. K 126) and she added:

> ... because he is showing me that I won't make it to the end of the year, and I feel that I am going to fail because of mathematics. So I thought maybe I should just switch to math lit, so what I'm doing is, I want to see my marks this term before I decide. (Int. K 130)

This switch from mathematics to mathematical literacy will not necessarily dampen her desire to become a chef because her future career does not require mathematics. It was clear that in constructing this negative disposition towards mathematics, Katie drew on the cultural model that "mathematics was difficult" and the teacher was to blame. Consequently, Katie did not see the point of continuing the study of mathematics.

She continued to build on this cultural model by highlighting the pedagogy of the teacher as another distraction to learning mathematics that resulted in her poor grades. In elucidating this experience she also emphasized her belief in how she learns mathematics. She constructed her mathematical identity around the importance of "understanding" in mathematics as the basis for her enjoyment and learning of math. She complained that her teacher:

> ... moves very fast, so if I don't know the answer or I don't know how to do that question ... I don't understand how he got that answer, because I don't know any of the steps, so it just doesn't work for me ... It makes me upset, especially on a Monday when we get double Math after each other ... So I really don't enjoy going to math class. (Int. K 162)

This lack of understanding, she claimed, led to feelings of frustration for her and her classmates on two levels; 1) they were frustrated because they did not understand the work; and 2) they were frustrated because "it's coming close to the exams and none of us know what is going on, and that's the time when we start panicking" (Int. K 174). There were limited opportunities to learn, because she could not even do her homework:

> ... but how can we do our homework if we don't understand. So he works faster because that group [fast lane group] understands the work, then he says we must go and do math literacy, but he is supposed to help us during intervals (FG. 202)

The overarching theme within this cultural model is that the students are threatened with being forced to do mathematical literacy if they do not understand since the teacher does not even allow for questions nor does he create other learning opportunities for those who are struggling. He treats everyone as if they are in the 'fast lane'.

Another source of frustration for her is in the area of assessment, testing, and examinations. She claimed that during assessment times she thinks that she knows the math, but gets a bad result. She said "when we wrote our last exam I had faith that I knew the work, but because of the trouble I have with math, and mostly fractions I didn't do as good as I thought I would" (Math story).

Summary of Katie's mathematics identity

Katie's existence seemed to be based on making other people happy. She was doing math because he father wanted her to do it. This tension between the importance of mathematics, reinforced by her father's wishes for her to do math, and the fact that she is not confident in being successful in mathematics permeated her story. She was adamant that she wanted to become a chef, and she knew that she did not require mathematics to enter into a career as a chef. She was not a positive learner of math, which made it difficult to decide on the most significant motive for her doing mathematics. She chose mathematics because others told her it is important for her future, especially if she wanted to go to university. However she also wanted to own her business, and for this reason she needed math. In this regard she understood that doing mathematics, together with her accounting subject, will help her open her own business.

Katie had no confidence in her own ability, while her father had all the confidence in her ability. She does not want to fail grade 11 because of math, and therefore she continues to try working very hard at passing math, if only to make her father happy. Despite her negative disposition towards mathematics and mixed feelings about continuing to do mathematics, she is trying hard to pass math, then she will continue to do math. Her mathematical identity is defined by a low self esteem and she displayed feelings of inadequacy in this construction of her identity. She believed math is difficult, although she found primary school math easier because the primary teachers accommodated her as a visual learner. Katie claimed that she struggled in math because her teacher moved too fast with new work, but she blamed class-size as the ultimate problem. She loved working in groups and felt that group work was not encouraged by math teachers at the school.

Zena

Introducing Zena

Zena lives with her parents and brother in a township on the Cape Flats created by the previous apartheid government for low-income colored families. It takes her about 15 minutes to get to school. The area is characterized by overcrowding, high levels of poverty, high incidence of crime and gangsterism. In this township it is easy to be a prisoner in the community with no role models to emulate. One's problems easily become the community's problems, so it made sense when Zena said "we like to talk about other peoples problems. So people always like come to me and my mommy for help and support. Like my friends will come to me. Her friends will go to her" (Int. Z 9).

Her mathematical story must be viewed within this social context. She reported that her mother "never finished school", while her dad "finished, then he went to university" (Int. Z 17). Unfortunately she did not know which university her father attended or what he studied. However, she said that "during the apartheid era, he was a teacher at M [a high school in the area], and then things got different then he couldn't do teaching anymore because he needed certain papers and things and he didn't have that, so he didn't continue as a teacher (Int. Z 20). He now works at a fish factory, while her mother "doesn't actually have a stable job. She's working at a shop in K" (Int. Z 27). She happily reported that her brother "finished matric, when I was in Grade 8, three years ago and he also did pure Mathematics, so if I need help I can go to him and he will help" (Int. Z 40). She gleefully noted that he is currently studying engineering. Consequently, she received good support and encouragement at home, and she reported that her mother, "... will always ask me if I have homework. Sometimes I tell her there is a lot of things I would like to tell her, then she still tell me, go to your room and finish your homework and then you come and talk to me" (Int. Z 34).

Leading Identity

Zena's leading identity seems to be less definite than some of the other participants, because of her view of mathematics and her lack of understanding that mathematics is a requirement for her field of study. Zena wants to be an accountant. She believes that mathematics is not mandatory for her career, she said “but you can also do math literacy, but I chose math” (Int. Z 117). However, for her to do accounting at university she must have mathematics. Mathematical literacy is not enough.

She said “Math is almost like accounting because it is a lot of numbers ... that both of them correspond with each other, and you have to do it” (FG. 54). This gives us a glimpse into how she sees the nature of mathematics. She substantiated this idea of still becoming an accountant, because “the things that we are doing like… I want to do accounting ... most of this stuff [referring to algebra and geometry] won’t actually help you because accounting is just like money and stuff that you must just fill out ... you’re not going to use 1000x^2” (Int. Z 107). Her mathematical identity is therefore defined by how she thinks of the nature of mathematics, and that some of the mathematics is irrelevant and will not help her in her career. Her career choice was straight and simple “I want to become an accountant because I like working with numbers”. However, she has another motive for wanting to become an accountant, namely “that’s where the money is” (FG 106). This is very true within the South African society and the belief that you can earn a big salary, which will help Zena and her family out of their poverty stricken community.

She claimed that in grade 9 she “started to be good in my mathematics again”, showing that her math journey had fluctuations of good and not so good achievement moments. It is due to this moment of change that in grade 10 she decided to “choose pure mathematics instead of mathematical literacy” (Math story). In grade 8 “math started difficult, and I got low marks, but afterward my marks went up and I could do pure math”. In addition she had self confidence in her ability as seen in a very short and sweet statement “I think I work hard enough to succeed” (Int. Z 120). Her goal is to “get an A [symbol] for mathematics in matric or in university” (Math story). She definitely does “not want to fail mathematics, as this is a very important subject” (Math story).

Mathematics is important for opening the doors to future careers, just in case the career of your choice does not materialize. She stated that “if you don’t do well in the career you have chosen, you can’t go into something else, but if you do mathematics you can go into anything that you want to do” (Int. Z 305). Since mathematics would open doors in the future, Zena chose mathematics for this reason. Thus, her leading identity is governed by her belief in the usefulness of mathematics.

Development of her mathematical identity (Math journey)

Zena claimed that in primary school she was “a very good learner, but not that good in mathematics”. She qualified this statement by adding that “I always knew my work” (Math story). She was a bit unsure about her achievement in primary school because she said “I think I did well in primary school mathematics ... I got two certificates” (Int. Z 58). She seemed to be evaluating her good achievement or success in mathematics based on how many certificates she received. Since she only received two, there is doubt about her actual ability to do mathematics. However, this doubt was washed away when she claimed that she did not struggle with mathematics because she “could do anything then” (Int. Z 60). Her mathematical identity was characterized by a sense of self-confidence.

She laughed and smiled broadly as she thought about her feelings in primary school towards mathematics by noting that “we had a lot of fun with group work and things ... mathematics was fun” (Int. Z 77). She gave the following reasons why group work was successful and fun, by putting “different children in a group, we could talk to each other while doing your work, and they wouldn’t mind because you’re actually socializing” (Int. Z 80).

Through group work, according to Zena, mathematics learning can be enhanced through socialization and working together. This was the same spirit that was predominant in her community.

As she thought about her math journey in high school she was unable to decide whether math was easy or difficult. She reported that math was “in the middle because some chapters are easy while some are probably difficult ...so we don’t actually know under what category it falls” (Int. Z 68). She made a clear distinction that mathematics is declared difficult based on the number of topics that you like.

Emotional attachment in doing mathematics at high school

Zena wanted to give the impression that she was hard working despite her weakness in mathematics. She was “so shocked” when she received an award in grade 6 for mathematics, because she had no idea how she achieved that award. She stated that “I wasn’t that good in it [math]”, but despite this she was “also very excited about this [achievement]” (Math story). She attempted to underplay her achievement, showing a lack of confidence in her own ability by saying that “I think it was my teacher that helped me get that award” (math story). Thus, giving the impression that she did not deserve this award, but her teacher perhaps wanted to motivate her, because she immediately added that due to this award “I knew that other good things will also come even though it doesn’t seem like it”.

Furthermore she felt “very excited and happy” after receiving these awards, showing that receiving an award of this nature does result in a more positive disposition towards mathematics. In this way “the children can know that they are good in something and so that the others can work harder to achieve something” (Int. Z 162). Zena suggested that as a math teacher she would give an award to “everyone who achieves over 70%” (Int. Z 166), because then “they can also boost up their self esteem to know I can work harder and harder” (Int. Z 170).

She stated that “when I came into high school I thought it would be a challenge, especially with mathematics, but I thought how good I was in primary school”. The awards boosted her confidence and she entered high school with great expectations and this was borne out during the first two terms in grade 8 where everything “went good, but then suddenly I slacked in my work and I failed the third term”. She had no idea how this happened, but she admitted that she “was so sad” and “this was a low point” (Math story) for her. She blamed herself for this failure by stating “I can blame no-one for this as it was my duty to do my work” (Math story).

She defined herself in a very positive way, thus defining her mathematical identity with an honest opinion of herself. She stated “for me, I’m a good learner, I am average in mathematics, I don’t do too well and I don’t do too badly. Sometimes I do good and I think I do my bit in class” (Int. Z 300). This sums up how she sees herself as a student of mathematics in a very positive way. When asked how she rated her motivation and self-confidence on a scale of 1 to 10 with 10 being the highest. She gave herself a 7 for her motivation to do math and 8 for self-confidence to do math.

Cultural Models

The first cultural model is based on the belief that math is difficult. She claimed that at the start of grade 10 everything went well because “the work was almost the same as the previous year”, but “as the year went on the work became very difficult and I struggled” (Math story). While the level of mathematics and the problems were familiar to her, mathematics was manageable and perhaps even easy. As she progressed in this grade her belief that math was difficult became entrenched in her construction of her math identities. Math became unmanageable. Her reasoning about dealing with this difficulty in a positive way was that "all people struggle sometimes in their life and they can make it, then why can't I" (Math Story). She used this reasoning to motivate herself by stating that "I then started to work harder and realized that the work wasn't so difficult after all and I just had to take note [listen attentively]" (Math story).

This cultural model emerged out of the belief that mathematics is hard work. She claimed that she works hard enough "I always take note of what the sir is saying and if he explains one and I don't get it, I tell him to explain it another way so that I can understand it to ... for myself" (Int. Z 122). She believed that hard work must be taken into consideration when teachers' pass or fail students. During the focus group interview she said that:

> ... if you need like 5%, then they [the teachers] can give it to you, but not more than that because you have to work. If a teacher take it that you not working and you need 5%, he's not going to give it to you. (FG 502)

She also believed that mathematics was challenging, by connecting this challenge to writing of mathematical examinations and tests. She claimed that her "single mathematical challenge" is "writing exams", because she "get stuck somewhere and don't know how to do a sum" (Math story). This challenge impacted on her experiences with mathematics "as it is always what I struggle with" (Math story).

She also believed that the learning of mathematics is foundational, meaning that you learn math in a step by step fashion in every grade. She stated that you cannot just come to grade 12 and expect to be able to do mathematics, "because there are lots of other things that come before, that you have to know in order to go further in mathematics" (FG 32).

This cultural model is based on the belief only correct answers are valued in the math class. She is only prepared to go to the black-board if she knows that her answer is correct, not to learn from the experience of working at the board and getting help when displaying difficulties in solving a problem. She stated that “if I know that my sum is right I will go to the board, but other times, not”. Her confidence and self-esteem is based on having the correct answers. However, this black-board experience may be different if she had a different math teacher because she explained that her current math teacher embarrasses the students in her class, makes her very nervous, and is unhelpful. She claimed that “when you do something wrong, he starts shouting at you and that makes you nervous and then you don’t want to do it any more because like, everybody knows that you’re doing wrong and they have the right answer” (Int. Z 138). She also claimed that:

> ...you are afraid to ask a question because of the sir. He is always harsh and you never know what his reaction will be, even though he seems in a good mood and you give a wrong answer, his mood just turns around on you. (FG. 266)

The cultural model that relates to the belief that students only hate/love aspects/sections of mathematics seems to be well constructed by Zena. She admitted that “fractions were and still are a thing in mathematics that I don’t like, but nowadays I seem to understand it and know what it’s really about” (Math story). She claimed that she loved to do problems that involved linear and quadratic equations, thus clearly favoring algebra over geometry. She also claimed that data handling is an easy section of mathematics, and that’s the reason why she ‘likes’ doing data handling problems.

Summary of Zena's mathematical identity

Zena wants to be an accountant and believed that mathematics is not mandatory for her career, but this idea is faulty because she needs pure mathematics to pursue her dream of becoming an accountant. Mathematical literacy is not enough to get her into a university accounting stream. She believed math is just working with numbers, and in this way it is like accounting, and she loves working with numbers. By implication she does not see the relevance of mathematics, especially algebra. In grade 8, math became difficult and she received a low percent in math, but now she has improved, raising her self confidence in her ability to do math. She believed she worked hard enough to succeed. Her ultimate goal is to receive an A (80-100%) symbol for mathematics in matric or in university.

She believed in group work and that mathematics learning can be enhanced through socialization and working together. She defined herself in a very positive way, and perceived herself as a good learner, average in mathematics, and achieves moderately average in math. She rated highly her motivation and self confidence to do math. While the level of mathematics and the problems were familiar to her, mathematics was manageable and even easy, otherwise math is difficult. She believed that hard work must be taken into consideration when teachers' pass or fail students, not just examination marks. She also believed that mathematics was challenging, because of the tests and examinations. It is this challenge that impacted on her experiences with mathematics.

CHAPTER 5

CASE STUDIES OF STUDENTS' MATHEMATICAL PERFORMANCE

In chapter 4 the case study analysis focused on the construction of the mathematical identities of each of the participants. This chapter focuses on the student's ability to solve mathematical tasks at one particular moment in time.

The purpose of this analysis was to answer the second research question, "How do students solve mathematical tasks?" I analyzed each participant's response to the mathematical tasks by applying three methods. The first two methods provided a quantitative analysis, while the third method provided a qualitative analysis. Together, the three methods describe each student's ability to choose appropriate strategies, to produce graphical representations, to compute arithmetic calculations, and algebraic equations, to apply mathematical knowledge and skills; and to make sense of their results. Consequently, the analyses provide an understanding of students' gaps or deficits in their mathematical knowledge and skills, as well as (mis)conceptions in their mathematical thinking and understanding.

The structure of this chapter provides the results of the analysis of: the two quantitative methods called the Points-Percentage score and the Four- scale rating score; and the qualitative method, based on the clinical interview with each participant, that involves a detailed analysis of each student's solution strategies and mathematical thinking in the mathematical tasks.

Quantitative Results

The two methods are briefly summarized:

1. The *Points-Percentage (PP)* procedure involved a similar structure to that employed in the final grade 12 South African matriculation grading procedure, which included an allocation of points given for a step by step calculation. The percentage helps to determine whether the student did well relative to the passing benchmark of 50%.
2. The *Four-scale point (S)* procedure involved the allocation of a code number representing the student's performance on each task. The following codes were used:

    S0 – No answer;

    S1 – Minimal response, including simply rewriting the problem with minor adjustments: showing limited understanding and conceptual errors;

    S2 – A competent response, shows some understanding, but incorrect answer due to arithmetic or conceptual errors;

    S3 – exemplary, fully understands the problem, correct reasoning and complete answers, but may have one minor arithmetical error in the solution strategy

Points-Percentage procedure: The mathematical tasks used in the study are given in appendix D. These tasks were graded and coded by three mathematics education PhD candidates plus me, and was aimed at ensuring inter-rater reliability. For 20 of the 27 tasks the graders scored the items exactly the same. In the other 7 tasks, where there were differences of one or two points, the participant received the score given by the majority of graders. There was a tie between the four graders for two of the 7 tasks. In these two cases I used my grade as the tie breaker, because I had more experience with these types of grading rubrics. These scores are given in Table 5.1, which also shows the task number, a brief description of each task, the maximum number of points possible for each task, the score per task for each of participant, and the overall percentage for correct responses.

Table 5.1: Task description and Points-Percentage scores.

Task	Description	Max pts.	Yusuf	Avril	Moses	Katie	Zena
1 a	Interpreting length in terms of x	1	1	0	0	0	0
b	Solve for x by comparing areas	10	2	0	0	0	0
2	Compound increase, give the yearly rate	11	6	7	10	2	2
3	Apply exponential laws	7	4	2	6	0	6
4	Using average concept	7	6	0	5	0	0
5 a	Patterns: draw 4th fig.	1	1	1	1	0	1
b	Draw 7th fig	1	1	1	1	1	1
c	Interpret results	2	2	1	2	1	1
d	Determine 40th fig	6	5	0	6	0	0
6	Finding intersection: given two linear eq.	8	0	2	0	0	0
7	Finding angles in a geometric figure:	12	4	0	0	0	0
8 a	Solve for x: linear fractional equation	4	4	2	4	1	4
b	Solve for x: quad. eq.	5	5	5	5	1	5
c	Solve for x: linear with letters (fraction)	7	0	0	7	0	0
d	Solve for x: exponential equations	5	5	0	4	0	4
e	Solve for x: linear inequality	8	2	1	5	0	2
9 a	Calculate the height in a volume problem	6	0	0	6	0	0
b	Find the volume if dimensions change	5	0	0	0	0	0
10 a	Interpreting graphs: parabola and line	13	0	0	0	0	0
b	Straight line	3	0	0	0	0	0
c	Solve for x, use graph	2	0	0	0	0	0
11 a	Draw bar graph	9	4	0	0	8	8
b	Determine sample	2	2	0	2	1	2
c	Arithmetic mean	6	6	0	0	0	0
d	Determine mode	2	2	0	0	0	0
e	Determine median	4	0	0	0	0	0
f	Interpret data	3	2	1	0	0	1
		150	64	23	64	15	37
		percent	43	15	43	10	25

In task 1a, Yusuf was the only participant to score the one point which involved representing the length of a rectangle in terms of x, given that the breadth was x. However, he was unable to use this information to help him solve task 1 completely. He only managed 3 out of 11 points. The other four participants received 0 out of 11 for this task. They were unable to interpret both parts of the task. It seems as if the combination of algebra and measurement in the same task resulted in confusion and an incapability to even start solving this task correctly.

In task 2, Moses showed that he was able to solve this compound interest task, almost completely by scoring 10 points. He made a minor arithmetic error, but showed good understanding. Yusuf and Avril received more than 50% of the points, by managing to give some of the correct steps. They knew how to solve this task but made some errors in their computation. Katie and Zena displayed minimal understanding for this task, a task they should've been able to solve because compound interest has been in the curriculum since grade 9. Based on their low score of 2 out of a maximum 11, it shows that they did not know the formula needed to solve the problem and merely received points for correct substitution/interpretation of the principal and accumulated amounts used in the formula.

In task 3, Katie was the only one who failed to score a point. Moses and Zena were able to solve this competently with just a minor arithmetic error, hence losing 1 point in the step-by-step calculation. Yusuf and Avril showed moderate to minimal understanding of this type of exponent tasks. The fact that Yusuf received 4 points showed that he applied the exponent rules correctly, but made some errors in his computation, while Avril had difficulty applying these exponent rules and in her computation.

In task 4, it is difficult to understand why three of the participants failed to score a single point for this 8th grade task based on calculating the average. This type of task or a similar task was repeated in grades 9 and 10. Yusuf and Moses, failed to score maximum points meaning that they knew what they were doing, but made minor arithmetic errors. The fact that neither Yusuf nor Moses achieved full points for the task was due to the fact that they eliminated the top and bottom scores in the task, but divided the new total scores by the full number of students 10 instead of the 8.

Task 5, was one of the tasks in which all participants scored some points. In the first two parts (5a and 5b) only Katie failed to get the maximum two points. The others achieved maximum 2 points for being able to draw the 4^{th} and 7^{th} figure in this pattern. However, only Yusuf and Moses were able to interpret and explain their patterning results, as well as being able to find the number of balls in the 40^{th} figure. Avril, Katie, and Zena were unable to find the number of balls in the 40^{th} figure and scored 0 points.

Task 6 involved finding the point of intersection where two linear graphs meet. Only Avril managed to score a couple of points, while the other four participants failed to score a single point. This is a cause for concern because in grade 10 the students were exposed to algebraic and graphical methods to solve this type of task using linear equations. In grade 11 the focus is on finding the point(s) of intersection of the parabola and straight line.

In task 6 based on finding with reasons angles in a geometric figure that included a parallelogram with two isosceles triangles inside the parallelogram. Only Yusuf managed to get some points because he started correctly, but made some incorrect deductions that affected his performance. The other participants failed to score a single point, showing that this type of task is challenging for them. This is another poor result because they should be familiar with this kind of task.

In task 8a, Yusuf, Moses, and Zena solved the linear equation competently and received maximum points. Avril had some idea about solving for x, but only managed 50% of the required steps, while Katie struggled with this 8th grade task. Based on Katie's performance in this algebraic task, it seems as if she had difficulty with algebra.

In task 8b, only Katie failed to solve this quadratic equation competently. This was surprising because the equation was given in standard form with the right-hand-side equal to zero. All she had to do was find the quadratic factors and solve for x. The other four participants solved this task with maximum points.

In task 8c, only Moses scored maximum points, while none of the others were able to score a single point. Once again this was surprising because this was another 8th grade task and they were required to find the lowest-common-denominator and multiply each term by it to get rid of the fraction or they could have used the idea of equivalent fractions. It seems as if this kind of task involving solving for x in an algebraic fraction was a difficult exercise for the students.

Task 8d, seems to confirm that Yusuf, Moses, and Zena were fairly competent in using exponential rules even when solving for x. They were able to solve this task competently that required them to reduce both sides to a common base 2 and then equate exponents to solve for x. Moses and Zena made a minor arithmetic error and lost one point. Katie and Avril failed to score a single point.

Task 8e proved to be beyond their capabilities, because all of them struggled, except Moses who managed to get 5 out of the 8 points. This quadratic inequality required that the students firstly remove the brackets by multiplying using the FOIL-method, then taking all the terms to one side of the equation and add like terms, but this algorithmic approach was not applied and where it was applied, it was incorrect.

Task 9a required that the students calculate the height of a rectangular tank given the volume and the dimensions of the base, while 9b required that the students interpret that information and work with doubling of the volume. Only Moses managed to solve the first part competently, but did not know how use that information to solve the second part. All the others failed to score a single point showing that their knowledge of these types of mensuration tasks was non-existent, because none of the other participants were able to score a single point.

The task 10 a required that students find the x-intercepts, y-intercept and turning point of the parabola, but none of the participants were able to do this. In addition, they could not determine the equation of a straight line in task 10b. This inability to complete both these tasks made it difficult to solve task 10c. The fact that none of these students scored a single point for graphs is a cause of great concern for their success in math.

Task 11a required students to draw a compound horizontal bar chart. Katie and Zena lost one point out of the maximum 9, because they drew a vertical bar chart instead of the required horizontal one. Yusuf was able to score 4 points for being able to set up the graph. Avril and Moses were unable to score a single point.

In task 11(b, c, d, e, and f) only Yusuf was able to answer the questions that required knowledge of the mean and mode, but could not find the median. None of the others were able to calculate mean, mode, and median. Although Katie and Zena were able to draw the bar graph, they were unable to build on this knowledge. Moses and Avril found this task 11 very difficult. Yusuf, showed a better understanding of these data-handling concepts, than the other four participants.

The fact that all five participants achieved below 50% showed that there were deficits in their mathematical knowledge and raises questions regarding their learning of math. In addition, Avril, Katie, and Zena achieved less than 30%, which does not auger well for their continuation in mathematics. The score of 0 on so many items for all participants is alarming. For example Yusuf scored 0 eight times, Avril scored 0 sixteen times, Moses scored 0 thirteen times, Katie scored 0 twenty times, and Zena scored 0 fifteen times. These results were particularly alarming for Katie who only managed to score points in 7 out of the 27 tasks. Furthermore, she scored 10 points for data handling, and only 5 points for the rest of the tasks.

In the next section the distribution of the points by mathematical domain is provided. This gives an overview of the participants' weakest and strongest areas in mathematics. This analysis is shown in table 5.2.

Table 5.2: Mathematical Domains (MD) and percentage correct for each participant.

Math Domains (MD)	Tasks per MD	Max pts for each domain	Yusuf %	Avril %	Moses %	Katie %	Zena %
Algebra	3, 6, 8	44	45	27	70	5	48
Data Handling	4, 11	33	67	3	21	27	33
Financial Math	2,	11	55	64	91	18	18
Geometry	7,	12	33	0	0	0	0
Graphs	10	18	0	0	0	0	0
Measurement	1, 9	22	14	0	27	0	0
Patterning	5,	10	90	30	100	20	30
		150	43	15	43	10	25

Yusuf showed that his strongest section in mathematics is patterning, followed by data handling and financial math. However, the fact that he received 0% for graphs is a major concern, because graphs and algebra are the important math domains in the grade 11 and grade 12 curricula. His scores in algebra is not a passing score in South Africa. The low percentage in geometry and measurement is also a concern because of its importance in the final examination in both grade 11, and grade 12, since grade 12 examinations is based on content from grades 10-

12. Additionally the low score in geometry is revealing, showing some inadequacies in his learning of geometry.

Avril showed that her strongest section in mathematics is financial math, followed by patterning. In addition she managed a very low percent for data handling, and an unacceptably low score for algebra. She did not receive a single point for the other three mathematical domains: measurement, geometry and graphs. The fact that she could not score any points in these areas is a cause of concern, and similar to Yusuf's dilemma in these domains. The 0% for measurement and geometry is surprising because the bulk of the paper 2 work in the grade 11 and grade 12 final examinations involves these two domains.

Moses showed that his strongest section in mathematics is patterning, followed by financial math. In addition he is more competent in algebra than the other four participants. He attempted some of the data-handling items, but only managed an unacceptably low percentage. He did not receive a single point for the other two mathematical domains, geometry and graphs. He managed a very low score for measurement, and seemed to have difficulties when algebra was connected with the measurement solution strategy as in task 1b. The fact that he could not score any points in these areas is a cause of great concern.

Katie showed that her strongest domain in mathematics is data-handling, followed by patterning, but both of these were still less than 30%. This low achievement in the mathematical domains is very alarming because they were tasks typically familiar to grade 9 and 10. She received a poor score for algebra and for financial math which strongly indicates that she is struggling in these core areas of mathematics. She is not competent in any of the mathematical domains.

Zena showed that her strongest section in mathematics is algebra. However the percentage is very low and below passing. In addition she only managed a low percent patterning and for financial math. She did not receive a single point for measurement, geometry and graphs. As with the other participants this is a cause of great concern.

The Four-scale points (S) score: In this section the focus is on an analysis using the four-scale rating points for each of the mathematical tasks. Table 5.3 illustrates the distribution of the four-scale point codes for each of the tasks. The purpose of this rating scale was to provide additional information regarding the participants' performance. Table 5.3 represents the task number, the description of each task, and the rating given to each participant's response.

Table 5.3: Four-point rating scale applied to the tasks for each participant

Task	Description	Yusuf	Avril	Moses	Katie	Zena
(1) a	Interpreting length in terms of x	S3	S0	S0	S0	S0
b	Solve for x by comparing areas	S1	S0	S1	S0	S0
(2)	Compound increase, give the yearly rate	S1	S2	S3	S1	S1
(3)	Apply exponential laws	S2	S1	S3	S1	S3
(4)	Using average concept	S2	S1	S2	S1	S0
(5) a	Patterns: draw 4th fig.	S3	S3	S3	S1	S3
b	Draw 7th fig	S3	S3	S3	S3	S3
c	Interpret results	S3	S1	S1	S1	S2
d	Determine 40th fig	S2	S1	S3	S1	S1
(6)	Finding intersection: given two linear eq.	S0	S1	S1	S1	S0
(7)	Finding angles in a geometric figure:	S1	S0	S0	S0	S0
(8) a	Solve for x: linear fractional equation	S3	S1	S3	S1	S3
b	Solve for x: quadratic eq.	S3	S3	S3	S1	S3
c	Solve for x: linear with letters (fraction)	S0	S1	S3	S0	S0
d	Solve for x: exponential equations	S3	S1	S3	S1	S3
e	Solve linear inequality	S1	S1	S2	S1	S1

Table 5.3 (cont'd)

(9) a	Calculate the height in a volume problem	S0	S0	S3	S1	S1
b	Find the volume if dimensions change	S1	S0	S0	S1	S0
(10)a	Interpreting graphs: parabola and line	S0	S0	S0	S0	S0
b	Straight line	S0	S0	S0	S0	S0
c	Solve for x, use graph	S0	S0	S0	S0	S0
(11)a	Draw bar graph	S1	S1	S0	S3	S3
b	Determine sample	S3	S1	S3	S2	S3
c	Arithmetic mean	S3	S1	S1	S0	S1
d	Determine mode	S3	S1	S0	S1	S1
e	Determine median	S1	S1	S0	S1	S1
f	Interpret data	S2	S1	S0	S1	S1

In the graph tasks (10a, 10b, and 10c) all the participants received a rating of S0 for each component of the task showing that they did not even attempt these tasks. All of them did not know how to determine the x-intercepts or y-intercept of the parabola. Yusuf did not attempt six of the tasks. The S3 rating for some of the algebra tasks, especially solving both linear and quadratic equations (tasks 8a and 8b) showed that Yusuf competently solved these types of tasks. The S2 and S3 ratings for the two exponential tasks (tasks 3 and 8d) also showed that Yusuf competently solved these. Although he managed to find three angles out of 7, he received a rating of S1 because he did not provide any reasons for his solution. He scored a rating of S2 for his ability to partially solve the reasoning part of the patterning task.

Avril did not attempt 8 of the tasks scoring a rating of S0 for each of those items. She only received an S3 rating for being able to draw the two figures (the 4^{th} and 7^{th}) task 5a and 5b (patterning), and for being able to solve the quadratic equation. Furthermore she only achieved a rating of S2 for showing a fair understanding in terms of finding the interest rate, but did not solve the task correctly. Based on these few S3 and S2 ratings it seems as if Avril was unable to solve the majority of the math tasks competently.

Moses did not attempt 10 of the tasks scoring a rating of S0 for each of those items. He received an S3 rating for being able to apply the exponential rules correctly and solving the task competently; for being able to calculate the interest rate given the values for A, P and n in the compound increase task competently, for being able to draw the 4^{th} and 7^{th} figure in the patterning task competently; for being able to solve the reasoning part to the patterning task; and for solving 5 of the 7 of the algebra tasks competently. He received an S2 rating for applying the 'average' concept in task 4 and for partially solving the linear inequality.

Katie did not attempt 8 of the tasks scoring a rating of S0 for each of those items. She only received an S3 rating for being able to draw the bar graph (task 11a) and for being able draw the 4^{th} figure (task 5b) in the given pattern. Furthermore she only got an S2 rating for partially finding the sample for the data handling task. For the rest of the tasks she received an S1, and displaying very little understanding of how to solve the majority of the tasks competently. .

Zena did not attempt 10 of the tasks scoring a rating of S0 for each of those items. She received an S3 rating for being able to apply her exponential rules correctly; for being able to draw the 4^{th} and 7^{th} figure in the given pattern; for solving both linear and quadratic equations; for solving the exponential equation; and for being able to draw the bar graph and give the sample for the data handling task. She received an S2 rating for being able to interpret the pattern structure (task 5a and 5b). For the rest of the problems she received a rating of S1.

Table 5.4 provides an analysis of the number of times and in which mathematical domain the participants received the highest and lowest ratings. Table 5.4 represents the 7 mathematical domains that were assessed; the four-scale rating for the participants' responses within each domain; and a total score for each rating for each participant.

Table 5.4: Frequency distribution of performance by mathematical domains for each participant

M Domain	Yusuf				Avril				Moses				Katie				Zena			
	S0	S1	S2	S3	S0	S1	S2	S3	S0	S1	S2	S3	S0	S1	S2	S3	S0	S1	S2	S3
Algebra (7)	2	1	1	3		6		1		1	1	5	1	6			2	1		4
Data (7)		2	2	3		7			4	1	1	1	1	4	1	1	1	4		2
Financial (1)		1					1					1		1				1		
geometry (1)		1			1				1				1				1			
graphs (3)	3				3				3				3				3			
measure-ment (4)	1	2		1	4				2	1		1	2	2			3	1		
patterns (4)			1	3		2		2		1		3		3		1		1	1	2
Total	6	7	4	10	8	15	1	3	10	4	2	11	8	16	1	2	10	8	1	8

From table 5.4, Yusuf's competency in algebra, data, and patterns are immediately obvious. Out of the 27 items Yusuf was able to answer 10 items with a good understanding of the task by providing correct answers or partially correct answers, but in some cases with a minor arithmetical error. On the other end of the spectrum he was unable to answer 6 out of the 27 items, showing that he lacked the math knowledge to even attempt these problems across algebra, graphs, and measurement. Furthermore, together with the inadequate responses he showed that he had difficulties in every math domain, except in patterning. He was unable to solve any tasks in financial math, geometry, and graphs.

Out of the 27 task items Avril was able to answer 3 items with good understanding (S3-code) of the tasks by providing correct solutions or partially correct solutions, but with a minor arithmetical error. On the other end of the spectrum she was unable to solve 8 out of the 27 items (S0-code), showing that she did not have any idea of how to even start the problem. She managed to partially solve the financial math task, and able to solve only 1 of the 7 algebraic tasks. She showed a lack of math knowledge in all the domains. This lack of math competence is problematic for her future progress.

In 10 of the 27 items Moses received a rating of S0 showing that he did not even attempt 11 problems. He did not attempt the 1 geometry task; the 3 graph tasks; 4 out of the 7 data handling tasks; and 2 of the 4 measurement tasks. Out of the 27 items Moses was able to answer 11 items with good understanding of the task by providing correct answers or partially complete answers, but with a minor arithmetical error. Furthermore, he also provided 4 out of 27 inadequate responses across each of algebra, data handling, measurement and patterning. Moses showed limited competence in five math domains by successfully answering some of the tasks in those domains.

Out of the 27 task items Katie was able to answer 2 tasks with good understanding (S3-code) of the tasks by providing correct solutions or partially complete solutions, but with a minor arithmetical error. These 2 tasks came from data handing and patterning domains. She displayed very little understanding, both procedurally and conceptually, across all the math domains.

Zena, in 10 of the 27 tasks, received a rating of S0, showing that she did not even attempt the 10 tasks that included 2 out of 7 from algebra; 1 out of 7 from data handling; the 1 geometry task; the 3 graph tasks; and 3 out of 4 measurement tasks. Out of the 27 task items Zena was able to answer 8 items with good understanding (S3-code), four of these were in the algebra domain; 2 out of 7 from data handling; and 2 out of 4 from patterning.

Table 5.4 showed that all the participants struggled with the majority of tasks across all the domains, showing a lack of math competence in solving tasks. One of the key points of interest was to ascertain why so many of the tasks were not even attempted. This result was both surprising, because these tasks should have been familiar to all participants, and unexpected, because all of the participants were fairly positive about their ability to do mathematics. In the next section I analyze the thinking and solution strategies of each participant in an attempt to understand the poor performance in the math tasks.

Qualitative Analysis

This section reports students' reflections during clinical interviews of their ability to solve the mathematical tasks, and descriptions of their thinking behind their solution strategies. It serves as an aid to make inferences about students' cognitive processes.

Yusuf

For the first task 1a and 1b, Yusuf said "I thought it was easy, but we never actually went through such work before in the class" (Int. Y 495). He also wrote that the problem was easy, yet he was unable to solve it. He defended himself by saying that they did not do this type of problem recently in class, but that they did this in grade 9. He was able to interpret the information and managed to correctly write the length and breadth in algebraic terms, but even though he knew the formula to calculate the area of a rectangle, he just could not explain this, despite prompting from the researcher. So the problem looked easy, but "not knowing how to work out the area" (Int. Y 524) made it very difficult for him to proceed with the task. He admitted that he "was a little frustrated at the end when I had to find the area" (Int. Y 535). He said that the area of a rectangle is length times breadth, and despite my prompting to get him to say that you multiply x+4 (the length) with x-1 (the breadth), he said two times that "I'm not sure". It seemed as if he was frustrated that the length and breadth were given in terms of algebraic expressions, and this resulted in a barrier to his learning.

In the second problem Yusuf was able to apply his belief that all you need in math is to know the formula. He was really happy with himself that he knew the formula to calculate the compound interest, although in this case he was asked to find the yearly rate of interest. He was able to write down the formula, substitute for the different variables, and was able to explain how to make something the subject of the formula, in this case finding 'r'. It must be noted that in his formula he used what looked like the 'X' multiplication sign, when it should have been the '+', but during the solution he treated it as a '+' sign. He admitted that his only difficulty was that "I was actually struggling with the calculator with that one [meaning this task]" (Int. Y 548). His level of enthusiasm and excitement to explain how he solved this problem increased compared to his 'stuttering' for words to explain the first problem where he raised his voice to say "I'm not sure". I could feel his confidence in this explanation.

When Yusuf was asked to explain task 3, regarding simplification of the exponential rules, he immediately responded by saying:

> I thought it was an average problem, but usually I, like we done that
> last year in grade 10, beginning of grade 10. That time I was not much
> in the class due to athletics that was happening in the school.
> So I don't really know how to simplify it, but I tried. (Int. Y 563).

This opening salvo suggests a lack of confidence in his ability. However, Yusuf was able to explain the solution strategy correctly. His thinking about the problem revealed a good level of understanding the procedure to solve the task. He only made a minor arithmetic error when he multiplied 2x1 to obtain 4, and he subtracted the powers incorrectly. Procedurally he had everything correct, but he did not believe that he had the task correct. He claimed that "I think it was actually quite difficult ... when I came to the second step I did not know what to do ... so I thought it was getting a bit difficult at that stage” (Int. Y 288). He also graded his own problem as 2 out of 5, but it was graded as 4 out of 5 with only one point deducted for the minor error. He also added that he preferred the first task to the exponent one, even though he was unable to solve the first task and only received 3 out of 11. He claimed that he liked the first problem because “I’m a bit more familiar with that type of algebra” (Int. Y 605). He noted that he was absent doing athletics when exponents were taught by the teacher. Thus, giving the impression that he did not get the opportunity to become familiar with the type of exponent problem.

Task 4 produced the following interaction between Yusuf and interviewer:

Yusuf:	I thought it was easy because I like to do this type of, how do we call this now, hmmm ... data [thinking about it]
Corvell:	Data-handling
Yusuf:	Yes. I multiplied 87 by 10, and got 870. Then I subtracted 55 and 95 respectively from 870 then I got 770. Then I divided 720 by 10 and got 72.
Corvell:	Right, so why did you divide by 10?
Yusuf:	Because there are 10 test scores.
Corvell:	Because there are 10 test scores? But you mentioned that you actually took two away. So will there still be ten scores?
Yusuf:	[silence –mumbling inaudible]
Corvell:	So explain what you just thought of now?
Yusuf:	I’m not supposed to divide by 10, but I’m supposed to divide by 8.
Corvell:	Divide by 8
Yusuf:	Yes

Once again Yusuf’s confidence is increased because he knows that he is able to do this problem very easily and explained his thinking with aplomb. However, when he realized that he made an error, he was shocked. This showed that he had pride in his work and was disturbed by making silly errors. He felt good after solving this item correctly.

In task 5 he continued to display confidence in his ability to solve the task. The following extract shows a good understanding of how to solve the task, and it shows that this type of task was familiar to him, but he used match sticks in similar tasks in class, instead of balls.

Yusuf: By the first one, they say draw the 4^{th} figure. I saw that by the first three the first one has three, the second one has four and the third one has five [referring to the bottom row in each pattern of balls]. So definitely the 4^{th} one must have six in the base. So that's how I actually got to that one [pointing to his 4^{th} figure] and when they said to draw the 7^{th} figure. Then I checked by the 4^{th} one there was six, then I added 3 to the base and got 7. That's how I got the 7^{th} one.
Corvell: You feel that the way you did it, was correct?
Yusuf: Yes
Corvell: Did you feel good about that problem? Have you done some of these in class?
Yusuf: Actually we did some of these in the final exam last year, but it was with the match-sticks.

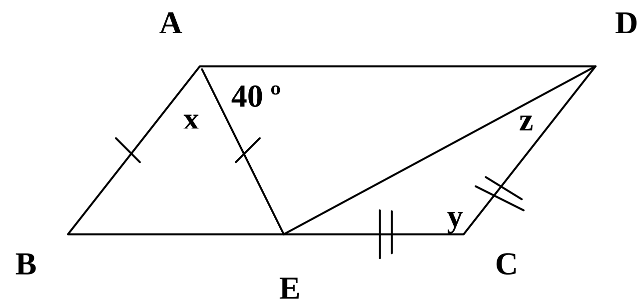
In task 6, Yusuf's response was very revealing in terms of why the task was so difficult and why he was unable to attempt it:

Corvell: Oh, with match-sticks. Let's look at number 6. Find the point of intersection. You wrote that one as difficult. Can you explain why you think it is difficult?
Yusuf: Because I don't understand that part of the work.
Corvell: So did you do a section called simultaneous equations? And do you know how to solve simultaneous equations?
Yusuf: Yes.
Corvell: So why do you think it is so difficult?
Yusuf: Because I'm not familiar with these types of questions.
Corvell: So you have not done it this year?
Yusuf: We've done a few, I think we've done about 30 or 35 in class, but that time I was also busy with athletics, so I did not actually get a number [meaning he missed out on a lot of the exercises]
Corvell: So it's because you were out of class? Do you think it will make sense to you once you've got it?
Yusuf: Yes.

He observed that they did many problems, say 30 or 35, but he was absent. It is unclear how much he missed out on during the teaching of this concept. A question to ponder at this stage was how did Yusuf deal with work after he was absent from class.

In task 7, Yusuf conveyed his confusion regarding geometric terms and definitions, as well as a lack of understanding in how to work with this type of task. I've included the question so it's easier to follow Yusuf's logic and problem solving ability.

7) ABCD is a parallelogram. AB = AE and EC = DC. Calculate x, y and z.



Yusuf:	Well, actually I never knew what was going on here. So I just tried anyway, but tried to get 180 degrees in each triangle.
Corvell:	So you tried to get 180 degrees in this triangle [pointing to ABE]. So which angle did you calculate first?
Yusuf:	I actually went with x first.
Corvell:	You went with x, which is angle BAE, and how did you get to the 100 degrees.
Yusuf:	I saw that AE is parallel to AB, so I knew that is 40 and the other side is 40 [referring to the base angles of triangle ABE] so that is 100 [pointing to BAE]
Corvell:	So you said AE is parallel to AB. How do you define parallel lines?
Yusuf:	You can see it on the triangle.
Corvell:	So you looking at the indication marks, and that indicates parallel lines to you?
Yusuf:	Yes, that’s how I actually learn’t it.
Corvell:	So EC and CD will also be parallel lines?
Yusuf:	Yes.
Corvell:	You’ve given it [referring to the task] a grading or rating of difficult. Is it because you unfamiliar with the question?
Yusuf:	Yes, I actually used to skip that one single question with the parallelogram in the test. I always used to skip that one.
Corvell:	So you don’t like working with parallelograms?
Yusuf:	No, don’t like it.
Corvell:	And what would your reason be?
Yusuf:	I never used to like it since primary school. I think I should actually start taking more note of the parallelograms.
Corvell:	So if I should ask you what is a parallelogram? Will you be able to tell me?
Yusuf:	Yes, it is a slanted square or rectangle

Summary: The development of his algebraic ideas was slow and he was not at the expected level mandatory for this grade. He was more comfortable with solutions to tasks that involved patterning and data handling, covered during earlier grades. On the tasks that required specific algebraic skills he was able to translate some task information into symbolic expressions and solve equations. It would be safe to conclude that based on his performance Yusuf was mathematically competent in only less than half of the mathematical tasks and only in certain mathematical domains. After completing half the number of tasks he wrote the following on his script:

> I thought it was all average questions. I liked it at first but when I saw the last question [referring to the simultaneous equation]. I was shocked I never knew what to do. I never remembered how to do a few of the problems. A few was easy, some was difficult. It was a nice test.

He predicted that he would score between 35% - 40 %, but actually achieved 43 percent.

Avril

Avril displayed very little understanding of some key concepts, especially in measurement, geometry and graph work. Although she was procedurally fluent in a few tasks, she made many arithmetic errors as well as conceptual errors. These errors resulted in incorrect solution strategies. For the very first task Avril said “That one I forgot sir. I just blanked out when I saw that one” (Int. A 471). When asked the reason for blanking out, she admitted that it was “the question sir, because we never did that in a long time” (Int. A 474). She said that they did this kind of task in grade 8 and/or grade 7; she was unsure but noted that she did not do this type of task again after these grades. When probed about her understanding of the question as it related to the rectangle she said “I was trying, as sir can see here [pointing to the erased part-on her script], it didn’t make sense, so I rubbed it out. I did not know the formula or anything” (Int. A 480). There were signs that she erased something. When asked whether she knew the formula to calculate the area of a rectangle, she replied “area is length times breadth” (Int. A 483). Since she knew the formula, she was asked if she could use this knowledge to try and solve it. I expected her to say at least the breadth is x because that was given, then the length would be 2 meters more, therefore $x + 2$, but this seemed out of her reach because she replied “I don’t think so sir” (Int. A 487) She admitted that this was completely foreign to her. When asked if she struggles with geometry she responded “a bit” and added that she becomes:

> Frustrated, because I know if I attempt it, it is going to be wrong, but I still try. Sometimes I forget also what is happening a bit. So I may do something like this then I’ll forget a step as I go about it, I’ll remember the first few things, but then I’ll forget and then I’ll feel frustrated because I don’t know what to do now, and I sit with an incomplete answer (Int. A 494).

She definitely lacks the confidence to attempt this type of tasks because she preempts that she will get it wrong. Strangely she says “but I still try”, and clearly in this task she did not try. Furthermore she used “forget” three times in different ways; she forgets what is happening referring what to do with the task; she forgets a step during these multi-step solutions; and forgets how to complete the rest of the problem. This indicates that she probably study solutions by rote learning. If she forgets something she’s unable to calculate through understanding. These ‘forgets’ lead to feelings of frustration because she’s unable to complete the solution. Avril rated this type of task as average, yet she did not attempt to solve it. She felt that she should have been able to solve it, she said “because I thought I could do it” (Int. A 500) and that caused frustration as well, but as she attempted it “it was difficult” (Int. A 502). However, there was no evidence of the attempt except the small part that was erased.

Avril seemed more confident on the second task, as indicated by the following transcript.

Corvell: So let's look at the second one, number 2, the one dealing with interest. Explain what you did? How you went about solving it?
Avril: Sir, as you can see here, we normally do it like this, when you compound, the power of n must be outside the brackets, and that's what I did. And then seeing the cumulative amount 4900, I wasn't sure if it was, but I put that as the cumulative amount and that's how I think it works.
Corvell: So you pointing at the 4900 as the cumulative, and the 3200 as ...
Avril: The principal amount.
Corvell: The principal amount.
Avril: And then obviously the n has to be 6 years and that's how I went about doing this because we did not have time
Corvell: So why is n, six years? How did you get n to be 6 years?
Avril: Doesn't n determine the year's sir, the yearly rate of interest?
Corvell: So determine the yearly rate of interest. So we got that for six years and when you look at compounded half-yearly, what does that mean for you?
Avril: Isn't it .... ummm ... [thinking about it] I think it's 3, supposed to be 3. If it's compounded yearly it means six months.
Corvell: You mean if it's compounded half yearly, then its six months?
Avril: Yes, six months.
Corvell: So then it's every six months. So will you still agree with your value of n?
Avril: [laughs] naaaa [this is an unsure gesture, showing confusion] because I didn't actually like read through it with a focus. I had most of the stuff and thought that was right.

At the beginning of her explanation she used the word "normally" referring to the use of the formula for calculating compound increase/decrease, but it was clear that she was not very sure about the role of 'n' in the formula, and she spoke as if she needed reassurance that this was correct as she looked at me saying "that's what I did". She also showed a lack of confidence in her ability by displaying feelings of confusion about the correct positioning in the formula of the "cumulative amount" and the "principal amount". The probing relating to the value of 'n' that she believed was 6 years, but conceptually this was not correct because of compounding half yearly. Her final response was to laugh as she realized that she made an error, blaming it on the idea that she did not "read through it with a focus". She displayed sadness because she really "thought that was right". She scored 7 out of 11 points for this task, because she made the conceptual error for the value of ‘n’. She was asked to explain how she made ‘i’ the subject of the formula. She said “Mr QQ normally showed us that what you do on the right hand side you do to the left hand side, meaning that this thing to get the six away, but then you have to do it this side ...” (Int. A 535). Her explanations were good and spot on in terms of her calculation, but she said “Mr. QQ normally showed us”, again displaying a lack of confidence in her own ability, where if anything was wrong in her explanation then it was the teacher’s fault and not her fault. This idea was reinforced as she continued to explain her solution strategy “you have to bring the 1 over and then minus it ... that’s what Mr. QQ showed us and then you have to do the whole thing in the calculator” (Int. A 535); “... he [Mr. QQ] showed us that if you put this in a calculator, sir, then you get this answer ...” (Int. A 544); and “then he [Mr. QQ] told us then what you do is round off” (Int. A 547). Throughout this explanation she did not take ownership of the solution, because she wanted to reproduce what Mr QQ had done in the class, not reflecting on her own learning. She did admit that this type of task “is something that I got used to doing. First I was a little confused, but then I decided how to do it. Now I’m ok with it” (Int. A 555).

When asked about her solution strategy for the third task she blurted out “I hate these stuff, sir, but I tried it because we also got something similar to this in the test, and I honestly did not know what I was doing, but I got it right [laughs]” (Int. A 579). She stressed her familiarity with this type of task, but was utterly convinced that she hated these exponent tasks. Once again she used Mr. QQ in her explanation of her solution strategy:

> Yes, I got it right [sounding very excited about her achievement - in the class test] and then I just remembered that you had to do something like this [pointing to her solution] Mr. QQ normally tells us if the bases can be the same then you should make it like so. Here I went like 2 times 2

> times 2 is 8, then I made it 2 to the power 3 brackets then n +1. And he told us to look at the bottom also and I don't know this I got wrong or something, but it's the way I did it.

She started off correctly by explaining her method to resolve the composite numbers (8 and 4) into a base 2. She misapplied the distributive property by writing 3(n + 1) = 3n + 1 and 2(2n + 1) = 4n + 1. So she ended up with all numbers written with a base 2. However, she made a major conceptual error by multiplying the three base 2's together and adding the powers. When asked about this part of her solution strategy she said:

> I then rewrote the powers in one line, so I times the whole thing with that and with that [trying to explain the steps in multiplying to get the 8] seeing that was the constant base so I did not count that [pointing to the 2 to the power 1 – which the thought was 0 power because the power 1 was not written]. I times all of that with the powers because Mr. QQ told us that when you multiply the powers then you just add on or something like that.

Despite some prompting she did not realize her error. She intimated that she doesn't like indices/ exponent tasks because "maths has a lot of rules and formulae to remember" (Int. A 622). She admitted that "I will try it, but I will know it's not going to be right so I'll leave it like that" (Int. A 624).

When asked about task 4 (the one dealing with average), she admitted that the last time she did this type of task was in grade 9 and noted that:

> I could not remember what was the average and the mode and stuff like that. So I forgot. I know something it says here about the highest and the lowest and something like that with mode and range and stuff and then the

other one is the median, in the middle or something and I just forgot the processes (Int. A 635).

She was not sure how to start this problem. She wrote on her script was 95 – 55 = 40 (she subtracted bottom score from top score), which is finding the range, but this was not the question. In addition to admitting that she forgot, she complained about the fact that "I would have to go back two years about these stuff", and she struggled "because the memorizing of what to do, uncertain for certain parts was a bit difficult" (Int. A 646). She noted that she must memorize "like which formula you got to use like the average, median, range and stuff like that". (Int. A 649). This emphasized how she learns mathematics, you need to memorize formulae and stuff, not necessarily understand so you can remember what to do.

The next task about patterning seemed to raise Avril's self-confidence. She looked very contented when she said "I like these patterns sir, something I really like about maths. So when I saw I had to do the pattern stuff, normally we have to do about matching and stuff and continue the drawing. I was a bit hating it, but then I thought I could do this" (Int. A 664). Her confidence in doing this task was based on the idea that she did not have to remember anything, and could just look and logically attempt to answer the questions. She confirmed this idea by stating that "I was looking at this [referring to the pattern of balls] and how it's done, you add one more to the base [referring to one more ball] which mean you had to add one more each time. So you do the same thing here, and I knew I had to add another one from here onwards [referring to one more ball in each row]" (Int. A 673). Despite her confidence in doing this type of task she only achieved 4 out of 10 points, she was able to predict the first few terms of the sequence, but could not logically make connections to find the 40th term. She did attempt it but admitted, while laughing nervously, that "I don't know sir, I was a bit blank there, but I tried" (Int. A 702).

She just looked at the next task (task 6), which was a simultaneous equation, which could be solved algebraically in a number of different ways, and shouted “I hate graphs”, “I hate it” (Int. A 712). The following transcript was an attempt to find out the source of this ‘hate’ for graphs:

Corvell:	At which point did you get to hate graphs, because you are quite strong in your hate for graphs?
Avril:	It’s just [burst out laughing] the numbers really put you off sir. Sometimes you could work on your answer, but then you flop at it.
Corvell:	Why?
Avril:	Because I was not sure what to do here, just draw lines
Corvell:	But you knew you had to draw straight lines? How did you know it was straight lines that you had to draw?
Avril:	I didn’t sir, I just drew it. That’s why I just honestly hate graphs.
Corvell:	So if you look at the question when you read it the first time what went through your mind?
Avril:	There’s a graph, just draw it straight enough.
Corvell:	That’s good. So you saw that it was a graph?
Avril:	Yes sir.
Corvell:	Was there anything about the question that you felt comfortable with? That you’ve done this type of question in class?
Avril:	We did this, but I honestly forgot. But we only started now again with graphs and it is not the straight line. So we started with the hyperbola and stuff like that again. And this was stuff we did last year, mid year or something, but I forgot all of it.

The concerning part of this interaction is that she just drew straight lines without knowing why you had to draw these lines. She did not make the connection between linear equations, reducing to standard form, and determining the gradient and y-intercept. Furthermore she tried to explain the source of her dislike for graphs.

Corvell:	... At which point did you really hate graphs? Was it in grade 8 and why, or in grade 9, or even in primary school?
Avril:	It started last year.
Corvell:	last year?
Avril:	Yes, because I was a bit playful last year and I did not really get much out of the stuff, but I was always asking sir [not Mr. QQ], and he would tell me. So only then I would understand, but now I guess I forgot everything. Honestly I don’t feel comfortable asking mr.QQ because he is highly critical of us. I’m not up for him.

Corvell: Alright, so your hate for graphs started last year
Avril: Yes sir.
Corvell: You said you were a little bit playful?
Avril: yes
Corvell: And is it because you just did not follow what was happening or something?
Avril: Yes sir. The reason I really hate graphs is because I did not understand really what you must do and what is expected of me, because I was playful and did not really take note. When I came across the sums I like didn't know what to do.

There are three key elements involved in her dislike for graphs: 1) she was playful, and missed out on learning the concepts; 2) she did not understand and because of this lack of understanding became playful and did not make the effort to learn; and she's fearful of her teacher and afraid to ask him to explain or re-teach these concepts related to graphs.

Task 7 relates to geometry and the following transcript reveals much about Avril's knowledge of geometric shapes and related definitions, axioms, and properties of quadrilaterals.

Corvell: ... Now it is obviously a geometry question. How did you feel when you read it for the first time?
Avril: I was like, no [says that emphatically] I don't like such stuff sir.
Corvell: You don't like the geometry?
Avril: Not something like this, if it's just one triangle then its fine, but not all this.
Corvell: Did the figure look familiar to you? Did you guys do it at some point?
Avril: Yes sir. We did do it early last year. So that was last year and I forgot it.
Corvell: You forgot it? So when you read ABCD is a parallelogram, what did you think of?
Avril: [laughing] I can't remember sir, I just knew it was something difficult, do we really have to prove this now.
Corvell: So if we just cover everything up, and show you triangle ABE, what are you able to tell me?
Avril: These two sides are equal.
Corvell: Those two sides are equal. So what kind of triangle will that be?
Avril: Ummmm ... isosceles
Corvell: Isosceles. And what is the main property of the isosceles triangle?
Avril: I honestly forgot.
Corvell: You forgot it completely?
Avril: Yes, but I know something like that [pointing nowhere in particular]
Corvell: Alright, and the parallelogram. If I should ask you to define a parallelogram, will you be able to do that?
Avril: No
Corvell: Not at all. Any property that you know about the parallelogram?

Avril: No sir, I'm going to learn to pick it up again. I obviously forget what is happening and most of us don't know how to do that. We like confused with this stuff.

In task 8a, Avril multiplied $4(x – 5)$ correctly and took the 50 over to the right hand side to give her $4x$ -20 -50 = 0, showing that she had a fair idea about the process for solving the task. However, in the next step she factorized to get (x – 5)(x + 5) = 0, which was incorrect. This is her explanation:

> I first thought that we had to make the sum like a quadratic equation or something, because that is the way Mr. QQ taught us. Take the number and multiply the one that's outside, into the brackets, and that is obviously your answer, then it's like that and then you take the 50 over and make it equal to 0. So I did something like that, but I wasn't sure if that was correct.

She was not attempting to solve the task, but trying to recall what her teacher did, thus confusing her understanding of how to deal with a linear and a quadratic equation. She mixed the two methods. But in task 8b, she solved for x in the quadratic equation successfully, and gave the following response:

Corvell: You also said that it was an average problem. Why?
A: Because it was already in the form that I had to do it, something like that Sir, so I knew it wasn't a lot of steps that I had to do. I just had to go straight to this one [pointing to the two brackets]
Corvell: Ok, and were you more confident about that one than a)?
A: I am more, a bit, ja (giggles)
Corvell: Ok, why do you feel more confident? (Siren sounds)
A: Because I know for a fact that, that times that is going to be x squared and I didn't know about the x part though, I was like -10 + 5 is -5, so I just got the -5, but I didn't know about the x, and I think that times that is 50.
Corvell: So you got a greater sense that what you are doing there is correct. So you felt a little better about that one?
A: Yes Sir.
Corvell: Ok, and is that because you had done a lot of that in class? Why do you feel good about that problem?
A: I see most of that stuff in class, as I was a bit confident when I did it because I knew what I should do, and it didn't look foreign to me.

She acknowledges that the form of the equation is crucial to her being able to solve it. She emphasizes that they did a quite a number of these types of tasks, and it makes her confident as long as the equation is given in standard form.

Summary: Through this method of grading Avril achieved her 15%. The development of her algebraic ideas was non-existent, and she was not at the expected level of performance that is mandatory for this grade. She was not even comfortable with solutions to tasks covered during earlier grades, tasks that she was expected to know and solve. On the tasks that required specific algebraic and graph skills she was unable to translate task information into symbolic expression and solve equations. She lacked the ability to integrate specific skills for more complex tasks, and lacked the necessary knowledge or algorithms to even start solving the simple tasks. It would be safe to conclude that based on her performance in these tasks Avril was not mathematically proficient.

After completing half the number of the tasks she wrote the following on her script:

> Not again! I so hate graphs!! But I had fun ... at least I wasn't helpless.
>
> And it was unprepared so I had no idea what was coming in. But I think I did average although I feel I can do better. But I didn't try hard enough.

After completing the rest of the tasks she wrote:

> I felt a bit nervous when I saw graphs. I hate graphs! So I wasn't too happy with the questions, but I'm still trying.

Moses

Moses displayed limited understanding of some key concepts, especially in geometry and graph work. Although he was procedurally fluent in some of the tasks, he was guilty of far too many minor arithmetic errors, resulting in incorrect answers. For the first task Moses said “I was a bit puzzled because it sounded a bit confusing at first” (Int. M 306). He was asked about what puzzled him the most about the question. He read the question and gave the reason “the length of the rectangle is 2 meters more than its breadth, you know do the length in terms of it. I didn’t understand the instructions” (Int. M 308). It seemed as if the structure and wording of the question were problematic. By the structure I imply the integration of algebra and geometry/measurement, while the wording deals with the phrasing “2meters more than its breadth”. When asked what he would do to solve the question, he was talking to himself trying to understand what to do “If the breadth is x [hmmm-thinking/mumbling to himself] that would be [hmmm] the length would be 2 meters more than the breadth. It would be like [hmm] I’m not sure” (Int. M 313). Even after a direct prompt “So if they say, the breadth is x. then you know that the breadth is x, right? So what would the length be? Will it be 2x or x + 2?” he responded after a long pause “I’m not sure” (Int. M 317). I posed this question to him because on his script he wrote the length as $2x$, and I wanted greater clarification but he could not provide a reason. It must be noted that the home language of Moses was not English, but Afrikaans, although his school was an English medium school with all instruction in English. However, these instructions should not have caused so much confusion because he would have experienced tasks of this nature. He classified this task as difficult.

Moses very competently described his solution strategy for task 2, based on finding the interest rate in a compound increase problem. He remembered the formula, was able to substitute correctly, make ‘i’ the subject of the formula, and use his calculator correctly. He only lost 1 point for rounding incorrectly. He was able to interpret this question correctly and knew that to compound half-yearly you must multiply the 6 by 2. He admitted that he liked “financial math.

The problem was very easy to understand, and I took out the key elements of what they were looking for like the principal amount and the accumulated amount" (Int. M 354).

Moses competently solved the exponent problem and only lost 1 point for a minor arithmetic error. He lost the point for adding -1 + 3 + 1 – 2 incorrectly. He equated this to 0. A key moment in this transcript was when Moses was asked the reason for changing the plus or minus signs when dividing and multiplying like bases, he simply said "that's just the rule, I think" (Int. M 382). He was not very confident about his answer because he said he'll give himself 3 out of 5, but he could not identify where he went wrong in his computation, showing a lack of confidence in his ability. This idea was supported when he admitted that "for the first couple of steps I knew what to do, but when it came to the end I didn't really know", but he did everything correct except for the minor error. In addition when asked whether he knew the rules for exponents he responded "vaguely", yet he applied the rules competently.

Moses competently solved task 4 based on calculating the average. The following transcript shows Moses competently explaining his solution strategy and then discovering that he made an error after some prompting.

M: There are key elements in this also, like these 10 test scores that you given is an average of 87. Then 87 ×10 will give you 870 and that's how I got that. Then I added these 2 up, 55 and 95, then I got that, I got 150. Then the question was, if you remove those 2 then what will the average be? Then 87 – 150 gives me 720 ÷ 10 will give you an average of 72.

Corvell: So if you check the question, it said there, what is the average of the remaining set of scores? So, you have 720, are you still working with 10 scores?

M: It should have been 8.

Corvell: It should have been 8.Why?

M: Because they ask for the remaining……., I don't think I read it properly.

As he discovered his error he realized that he did not read this task correctly and missed the key part that involved subtracting two scores. He also said that this kind of problem made him feel confident about mathematics because he initially thought he had the problem fully correct. He also admitted that the task could be classified as easy.

Moses showed that he was competent in solving tasks that were related to patterning where he achieved full points for his effort. Once again he competently explained his solution strategy, showing that he does not struggle with these types of tasks. He admitted that he was confident about his solution and that these types of tasks were familiar to him.

Moses immediate response to task 6 was "Well, I don't like graphs" (Int. M 460). The word graph in the question seemed to distract Moses because he did not realize that he could solve this task algebraically. Moses also knew that he needed to write the linear equations into the standard form y = mx + c, but did not know what to do with these equations. The following transcript highlights this confusion between solving the simultaneous equations graphically or algebraically

Corvell: Because you're using the formula, y = mx + c. So you knew it was …
M: And the same on that side, but then I didn't really know how to go further from there.
Corvell: Ok, and once you thought you had the two equations, what were you thinking of doing?
M: I didn't really know.
Corvell: Have you done any simultaneous equations as well, this year or last year or the year before?
M: In last year, the beginning of last year.
Corvell: And have you done quite a number of them? With the graphs and with algebraic or….
M: Just with the algebraic not……
Corvell: Just algebraic not graphs. Ok, um, do you feel confident working with graphs?
M: Not really.
Corvell: Do you feel confident doing the algebraic ones, way of solving?
M: I'm not sure.

Despite efforts to get more information out of Moses and his ability to solve this task, he responded with a lack of confidence based a lack of understanding or knowledge how to move forward and solve the task.

A similar conversation occurred with task 7, based on geometrical shapes. Once again Moses did not attempt the task and the following extract shows his lack of geometrical knowledge.

Corvell:	... So we have another geometry problem. I see you did not attempt it. So when you read the problem, what were you thinking?
M:	... I did not know how to start.
Corvell:	Did not know how to start? So if I should say, M, what is a parallelogram could you tell me what it is? You're not familiar with the word?
M:	I am familiar, I just don't know, I forgot what it means …
Corvell:	And none of the properties? So is it because you did not know that, [referring to the parallelogram] that you did not do it?
M;	yeah don't know it

Moses was not going to say anything regarding his lack of geometric knowledge, because he really had no idea how and where to start.

Moses displayed good self confidence in his ability to solve the algebra tasks (8a, b, and c). He was also able to explain his thinking and strategy for solving. He was procedurally fluent in these tasks and showed good understanding. He expressed this self confidence by emphasizing the easy nature of the tasks by declaring that he felt "very confident" (Int. M 504) about solving task 8a, and task 8b "was just a trinomial and that was fairly easy, because we did quite a number of those, so I'm quite familiar with that" (Int. M 506). For task 8c he competently explained how to work with an algebraic equation, and declared how easy it was.

M:	And then I got this. I put 7x down, then 3, I left that, ja, 5x + x + 5, then I brought that over in the same step to be -5x and I left the numbers on the other side, 3 + 5, 2x = 8 and you just divide by that.
Corvell:	Ok, and you felt good working with the fractions? Is it because you know exactly what to do?
M:	Ja, I think that is easy. That wasn't a tricky one.

Corvell: Because I see you wrote easy. You find that kind of problem easy ...
M: Not all of them but that one was fairly easy.

For task 8d, Moses showed how he worked through a solution strategy:

Corvell: ... What did you do in d)? You wrote that it was an average problem. What were you thinking when you looked at it the first time?
M: What is going on here? I'm first thinking, how do I solve this? Then I thought ok no, this is the same like the exponential, and then you just get the bases the same, then you cancel the bases, and it would be like that. You bring down the 2x [meaning 2 to the power x] then 2 is a base and that will give you 16. The exponential will give you 4, then you just add 4x + 1, and then that is what you get if you cancel the bases …….

At this juncture Moses made a minor arithmetic error by multiplying $4(x+1) = 4x + 1$.

Corvell; So why did the bases just disappear? I see you cancelled it [referring to the line drawn through the 2 on both sides of the equation.
M: They were the same. I think that ….
Corvell: So, if you for example, have 2 to the power x $\times$ 2 to the power y, could you do that? [showing the cancelling of the 2's] And this is the explanation on the sheet where I wrote 2 power x $\times$ 2 power y. Ok, so you could do that and the answer would be xy. Is that what you are saying?
M: Ja.
Corvell: Ok, and that's for you, the reason why the bases cancel, and then obviously you just solve for x. You seem quite confident with your strategy, but you wrote that your answer is 'not sure'. Do you recognize maybe what could have gone wrong? Where you could have made an error?
M: Well, here, I wasn't sure here if I should take that as 4x or just as 4. Ok, but then when you multiply I think that should have been a 4 as well. Ja, I think that is where I went wrong [he saw his error]

Moses response during the task 10 showed that he was willing to make excuses for his inability to solve any of the graph tasks.

Corvell: ... So it's number 10, the graph. What went through you mind as you read that?
M: I don't know what's going on. (Laughs)
Corvell: Is it because the task is not familiar to you? You haven't done that?
M: It‘s actually not familiar to me, because I missed out on this section of work last year, when I damaged my ankle ligaments.
Corvell: And so you were away for a while? And you did not try to work through it, you just left it.
M: By the time I came back, it was time for examinations already and I had to come to school on crutches.

Corvell: Was that towards the end of the year?
M: It was June.
Corvell: And generally, how do you feel about graph work?
M: I don't like it. It's not one of my strong points.

Summary: The development of his algebraic ideas was much better than the other participants, but still not at the expected level mandatory for this grade, since these problems were selected from the 10th grade content. Although, generally speaking, he was more comfortable with solutions to tasks that were covered during earlier grades, like patterning for example. On the tasks that required specific algebraic and graph skills he was unable to translate task information into symbolic expression and solve equations, like the simultaneous equations. However, he lacked the ability to integrate specific skills for more complex tasks, and lacked the necessary knowledge or algorithms to even start solving a task. After completing the tasks he wrote the following on his script:

> A bit of a mixed feeling. Not sure. There were rough patches. I don't think I did very well.

He predicted that he will get 50%, but he only managed to score 43%.

Katie

Katie displayed very little understanding of the majority of key concepts, especially in algebra, measurement, geometry and graph work. She was not procedurally fluent in the majority of tasks. Her solution strategies were characterized by incorrect selection of formulae, far too many arithmetic errors, as well as conceptual errors. These errors resulted in incorrect solution strategies. For the first task Katie said "I just thought that I did not know how to do it, because I didn't do it in a very long time. We don't do that in Math" (Int. K 347). The type of task was last done in grade 8, but she admitted that it was familiar to her "it isn't exactly foreign it is just that I couldn't remember the formula, and so I couldn't do it" (Int. K 360). When asked if she could, in general calculate the area of a rectangle, she surprisingly said "not really, I don't think so" (Int. K 355). She forgot the formula. She also said that she considered this type of task as difficult, and in general she did not like geometry, meaning mixing geometry with algebra.

In task 2, the compound increase problem, she "just thought of what we did in class. I tried it but wasn't sure if it was correct" (Int. K 366). She tried to think of what the class or teacher did and not out of any accumulated knowledge of solving the problem. She had no confidence in her ability to do this and thought her response was incorrect. Her confusion was displayed in her solution that consisted of two strategies, each strategy starting with a different formula. When questioned about this she said "When I thought of 'compound' I couldn't remember which formula to use. Should I put the 'year' inside the brackets, or should I put the 'year' outside the brackets? So I was kind of confused, so I attempted both" (Int. K 369). She selected the second formula as the one closest to be correct. In fact this would be correct, based on her first step. Her formula was $A = (1 + i)^n$, but when she substituted she had $4900 = 3200\ (1 + i)^6$ showing a value for the principal amount. She was working from the premise of trying to do what she remembered, but not from a basis of understanding or following computing steps to solve. Her lack of understanding and level of confusion was high because she did a strange calculation, where she divided the accumulated amount (A) by the principal amount (P) and used her calculator to get a correct value of 1.53, but then did something very illogical by substituting this value for 'i', the interest rate. This illogical approach continued when she tried to explain why 'n' was equal to six. She said "because in a year there are 12 months, and they said half so that would be 6, and in a quarter, there would be 4" (Int. K 398). She insisted that her error was that she had to write n = 12 for 12 months. She did not understand that her calculation should have been 6 x 2 = 12 where you multiply the years by 2, because calculations are based on half-yearly computations. Thus, giving 12 periods of compounding interest so the value of 'n' should be 12. She only received 3 points for correct substitution. Despite this level of confusion she admitted that she preferred this type of task, because "it is easier. I think it is easier" and that they "do it a lot in math" (Int. K 413). She said they did financial mathematics for the last three years and doing this type of problem makes her feel good about mathematics.

She said the third task based on exponents was difficult. The following transcript shows an attempt to try and understand her solution strategy and to follow her reasoning.

K:	It looked like a fraction, so I found it easy … (laughs) ….. . Yes so what I did, I multiplied that on top……
Corvell:	So you multiplied the 2, the 8, and the 2, the bases, ok?
K:	Yes, and then I got to 32, and then I just wrote the powers like that….
Corvell:	Yes, but what I was trying to see was what the powers were. Maybe you could write it for me, because it looked like you had scratched it out there.
K:	And then it was here as well. So what I did was, I cancelled out the powers.
Corvell:	So you wrote 2n – 1. Is it that one? Ok.
K:	And then I cancelled, and I just divided. (Background noise as siren sounds)
Corvell:	Alright, so how did you cancel the powers? Did you just cancel the whole thing like, 2n – 1 cancelled? Oh, this is also 2n – 1. So you cancelled, and your answer is 32 ÷ 4, which is? 32 ÷ 4 will give you 8. Right? Again you were not sure about the answer. At which point do you think there was an error?
K:	When it comes to fractions I really don't know what the answer is. I am always unsure about my fractions.
Corvell:	Ok, is it more the fraction part that worries you or the fact that you are working with exponents?
K:	The fractions.
Corvell:	Because it was over 4 to the power of 2 and stuff like that?
K:	Yes.
Corvell:	Do you think that you know all your indices and exponent rules?
K:	No.

The fact that the exponent problem was presented in a fraction distracted Katie so much that she was not concerned about the exponent definition and rules. Her computation did not make any mathematical sense or follow any logical sequence of steps; the way she tried to cancel showed that she did not distinguish between an exponent and a number. She was just interested in cancelling. Her level of competence in this task raises many questions about her mathematical knowledge and ability.

For task 4 Katie alluded to the idea that this was a “relaxing question”, she noted that they did not do many of these type of tasks, but she thought the task was “actually the break in the paper because we don’t get sums like this often, so we need to get something that is more relaxing” (Int. K 449). She admitted that she enjoyed this question, but truthfully she had no idea how to solve it. The following transcript shows how she defines average.

Corvell:	So tell me what you were doing and thinking?
K:	The last time I remember getting a similar sum was in grade 9.
Corvell:	So you started by saying that 55 – 95. So you subtracted the two. Why did you subtract the two numbers?
K:	Because I took the lowest score and the highest.
Corvell:	So, how do we find average? How would you calculate the average?
K:	Um, take the highest amount and the lowest amount
Corvell:	Alright, so you take the highest and the lowest and you subtract, and the answer would be your average.
K:	Yes.
Corvell:	So you enjoy those kinds of problems, because I think you wrote easy. You felt good about doing it?
K:	Yes, I did.

Task 5 dealt with patterning and she once again admitted that she “enjoy that, I like it” (Int.K 467). The reason why she enjoyed the task was due to its visual impact “because I could see the figures and I could actually count how much there was and stuff. Ja, physically it was in front of me” (Int. K 496). Despite feeling good about the task, she was unable to solve it, not even the very first question that asked for identifying the pattern by counting the balls. It does seem as if there’s no connection between Katie liking a task and her ability to solve it.

The transcript for task 6, dealing with finding the intersection of two straight line graphs or solving simultaneously, reveal a bit more about Katie’s ability to solve the task.

Corvell: Ok, let's look at number 6. Find the point of intersection. So when you read the question, what went through your mind first, even before you wrote?
K: I knew I should put 0 on one side. (Laughs)
Corvell: You knew you should put 0 on one side? So you knew everything else had to go to the other side.
K: But then I just went blank again.
Corvell: Ok, so explain what you were doing there. This is basically what you then did, minus 5 ……..
K: I moved it over, and then I went blank and didn't know what to do anymore.
Corvell: Right, and the same with the other one.

At this point she was explaining the rule to transpose terms from one side of the equation to make one side equal to zero.

K: Yes.
Corvell: You went blank, because you are not familiar with the question? What do you think caused you to go blank?
K: I just didn't know what to do. I don't know I panicked.
Corvell: Is it the way the question was given, where we said find the point of intersection where the two graphs intersect?
K: No, it wasn't that. I don't know, maybe just the way I thought about it, and because I wasn't sure
Corvell: You know that these are kind of called 'simultaneous equations' where you do it algebraic too and graphically. Have you done both equations in class?
K: Yes we did. We work it out
Corvell: So you work it out algebraically and then show it graphically. So you have done some of them at least.
K: Yes.
Corvell: Do you like those kinds of questions?
K: No.

These types of tasks were familiar to her, but she forgot the rules, the definitions, and the algorithms, thus her ability to solve tasks were very limited.

In task 7 she really expressed her attitude towards geometry as well as her geometrical knowledge.

K: ... I was trying to remember what I was supposed to do there because I didn't know what to do because I did not do that in a while.
Corvell: So you were just totally confused? Have you done this kind of problem before?
K: Yes, I did because we did some of it … you're saying it is parallel to each other.
Corvell: So you are talking about the line EC and the line DC?

K: I knew they were parallel, but I did not know the amount.
Corvell: Ok, those two lines are parallel. Any thing else that stands out for you?
K: No (So much noise!)
Corvell: So the marks on the line indicate parallel lines
K: Yes it does
Corvell: Alright, so if I should say, K, what about the parallelogram? How would you define parallelogram?
K: That they are all equal, I guess?
Corvell: What is all equal?
K: The sides.
Corvell: The sides are all equal. Anything else? You're not sure? Ok, so generally that's just a difficult problem for you.
K: Yes.
Corvell: And do you feel good about looking at the problem?
K: No! (Laughs)
Corvell: How do you feel when you see a geometry problem?
K: I just don't like it.
Corvell: Do you think it is because you don't do a lot of geometry?
K: We don't do a lot of geometry, and I don't practice it so I don't…..

Her last response reflects a negative attitude towards geometry.

In task 8a Katie shows her confusion and lack of procedural understanding as she mixes the linear equation solution procedure with the quadratic equation procedure.

K: I multiplied in the brackets, because I know you should get that in the brackets, then I moved everything over. Then I tried to make a square bracket, but then I couldn't get the number that multiplies (inaudible – spoke too softly)
Corvell: So you tried to get the two sets of brackets factoring that…..
K: I know that amount and that amount should add.
Corvell: Ok, and you wrote it as an average problem?
K: Yes, because we did it a lot…
Corvell: You did it a lot and you find it difficult to work with?
K: Yes, I do. The way we are being taught about it, we're rushing.
Corvell: Ok, but you would prefer this kind of problem to the geometry,
K: Yes.

The key thrust of her solution strategy was to find factors from: $4x - 20 - 50 = 0$, she wrote in her next step $(2x \quad)(\quad)$, showing that she does not understand, hence unable to solve the task.

In task 8b she had a clear sense of the procedure, but lacked the necessary algebraic and arithmetic knowledge to solve the task competently. She knew that the quadratic equation required her to give the factors, which she did, but got the signs in the bracket incorrect. She admitted that to get the middle term correct was causing her difficulty.

In task 8c, Katie reflected on her frustration with doing tasks with fractions:

K:	Fractions! (Laughs)
Corvell:	And you love fractions?
K:	No!
Corvell:	So as soon as you see fractions you get uncomfortable.
K:	Yes, I feel frustrated.
Corvell:	Ok, so if you think, here's a fraction problem, how do you think you may have started that? [referring to the task]
K:	Usually I know … I just didn't know where to start.
Corvell:	Ok, so it just threw you, because I see you wrote that it is 'difficult'. Is that one familiar to you in class? You do a few of them in class?
K:	Yes, we do.

In task 8d, she made an error that displayed her lack of knowledge regarding exponents where she treated exponents and bases as ordinary numbers. Her method was illogical and showed that lacked a procedure to apply. She used some general ideas based on some math that she remembered:

K:	d) was ok, I guess. I tried to, I took over the two, and I took all the powers to the other side [she took the base 2 over to the other side and subtracted this from base 16, and took the exponent $x + 1$ from base 16 over to the other side, meaning that she ended up with exponents (left hand side) = bases (on the right hand side)]
Corvell:	So you took the 2 on the left hand side over…..
K:	To the right…..
Corvell:	And then you subtracted the 2 from the 16. So what did you do with the powers?
K:	I moved the powers.
Corvell:	So you moved the numbers to the one side and the powers to the other side. Again, was that question familiar to you? Have you done it in class?
K:	yes

In task 8e she did the following:

$$3(x - 2)(x - 3) \geq (3x + 1)(x - 7)$$

$$= 3x - 6 + 3x - 9 \quad = 3x - 6 + x^2 + 6$$

$= 6x – 3$ or $= x^2 + 3x$

This computation reveals a poor understanding of basic mathematical skills or concepts at many levels, despite the fact that her attempt at solving the task was incorrect. She knew that she needed to remove the brackets, but was unable to multiply correctly. She added: -6 -9 = -3 also incorrectly.

The following transcript is based on her interaction explaining her solution for task 9. It emphasizes once again her lack of mathematical knowledge and the consequent difficulties in trying to solve the task. She could not remember her formulae.

Corvell: ... let us look at number 9. It's a geometry one. Tell me what you were thinking.
K: I was trying to think back to when we did it, some of the formulas.
Corvell: When last did you do that?
K: Grades 8 and 9.
Corvell: So you did not do any of that in Grade 10?
K: No, but I tried.
Corvell: ... The height is equal to length times the breadth ...[pointing to what she wrote on her script]
K: The H is for height and the L is for length and the b is for depth. So I took the b, 20, and I multiplied and I got to 60 cm
Corvell: And that would give you the height?
K: Yes.
Corvell: And that is the formula for finding the height?
K: I'm not sure.
Corvell: Ok, what about a formula for volume? Do you remember?
K: The first thing that came to mind is length times the breadth plus the height.
Corvell: Length times the breadth plus the height, and that is what you did in the second part. How comfortable did you feel with what you did there? If you would grade it out of 5, what would you give yourself?
K: for a) 2
Corvell: a) 2? So you are not confident with the way you solved it or not confident about the formula you used?
K: About the formula.
Corvell: You're not sure that you got the formula right. Anything else you want to say about this problem?
K: No.

Katie's dislike for graph work was noted in her response to task 10.

Corvell: ... number 10, the graph. What did you think?
K: It didn't look nice. It didn't look enjoyable. (Laughs)
Corvell: So you did not attempt it. Have you seen it before?
K: It is familiar. We did it last year.
Corvell: You did it last year with the parabola and the line
K: With the graph. So I didn't actually remember it. We are starting to do it again, now.
Corvell: So you just forgot everything and that is why you did not do it?
K: Yes.
Corvell: Ok, and do you like graphs?
K: No, I like straight lines, not swirly lines. (Laughs)

In task 11 she competently drew the compound graph, but drew the bars vertically instead of horizontally as required and scored 8 out of the maximum 9 points. The transcript shows how she struggled to answer the questions relating to mean, mode and median.

Corvell: ... What about number 11?
K: It was alright. We do this and it's the last question in our exams [referring to the final exams in grades 8-12]
Corvell: So it's the last question in your exam and kind of familiar to you. Did you understand what was meant by 'compound horizontal'?
K: Not really.
Corvell: Is that word 'compound', familiar to you?
K: Only in financial mathematics.
Corvell: So only in that context. Do you have any questions about the rest of the problem?
K: I was trying to remember how to get the mode and the median.
Corvell: So how do you think you calculate the mode?
K: I wasn't sure...something about all the marks
Corvell: All the marks. Oh, so you added all of that up? For the 'mode', you wrote 14. How did you get 14?
K; I don't know how I got 14.
Corvell: What about the median? What do we mean by the median?
K: I didn't know how to do that …..
Corvell: So you forgot the arithmetic mean, …
K: Amount in the middle ...
Corvell: For the median? Ok, the median is the middle amount. And what about the arithmetic mean?
K: I never wrote that [pointing to her script that showed no answer for the mean]
Corvell: You never heard that before?
K: No.

Summary: Her mathematical knowledge was to a large extent not visible throughout her performance across all the math domains. Her lack algebraic knowledge, in particular, made it difficult to follow her solution strategies and her basic skills were below the expected level mandatory for this grade. She was uncomfortable with solutions to tasks covered during earlier grades, tasks that she was expected to know and solve. If you take away the 9 points scored for data-handling, she only scored 6 points for completing the other tasks. She lacked the ability to integrate specific skills for more complex tasks, and lacked the necessary knowledge or algorithms to even start solving the simple tasks. It would be safe to conclude that based on her performance in these tasks Katie was not mathematically proficient and her learning of mathematics is questionable. After completing half the number of tasks she wrote the following on her script:

> I felt frustrated and frighten when I saw the fraction. I just went blank for a moment so that made me upset with myself, makes me not confident to do the rest of the sums. I feel dumb because I went blank and didn't know all the answers to the questions.

After completing the rest of the tasks she wrote the following:

> I felt pressured because I didn't know what to expect. I also felt upset because I couldn't do all the questions, and felt disappointed in myself because I feel like I could've done much better. And a little frustrated because of the things I didn't understand.

Zena

Zena displayed limited understanding of some key concepts, especially in geometry and graph work. Although she was procedurally fluent in a few tasks she still made far too many minor arithmetic errors, resulting in incorrect answers. In addition she did not know some formulae. For the very first task she said “For me this is difficult, because things like this you don’t really know because it only gives you 2 meters. You don’t know what to do there” (Int. Z 335). She claimed that she worked with similar problems in grade 9. She drew the rectangle, but it was difficult trying to get more out of how she was thinking about this problem. Despite using prompts and leading her on with examples, she could not understand what to do. The following transcript shows this interaction.

Corvell:	So if we talk about a rectangle, and I see you drew a rectangle, how would you interpret the first question? It says, the length of a rectangle is 2 meters more than its breadth. What does that mean?
Z:	That the one side is 2 meters smaller than the other side.
Corvell:	So if the one side is say, 4…..
Z:	The other side will be 6.
Corvell:	Ok, so if the breadth is x, are you able to find the length?
Z:	I didn’t know what to do.
Corvell:	But now, if you analyze it like that, does it make sense to you? Do you know what the area of the rectangle is?
Z:	No.
Corvell:	You don’t know how to calculate the area? So it is difficult because the problem is unfamiliar to you? ...
Z:	I wasn’t familiar with it and didn’t know if it was correct.

It does seem as if the fact that algebraic language was included in the question, caused a distraction and a barrier to understanding for Zena, which she failed to overcome.

Zena forgot the formula required to solve the compound increase task. She gave the formula: $A = P (1 + i \cdot n)$, which was incorrect. She substituted correctly and achieved 2 points for this task. Her strategy was incorrect. She tried to explain her steps but these steps were illogical and incorrect, and very difficult to follow. The strangest part of this conversation was based on her admitting that she liked this financial math type of problem, because “It makes me feel good, because I actually know what I am doing” (Int. Z 387). This is a very strange illusion or false sense of security in her ability, because based on her explanations and solution strategy there is no way she was doing the right thing.

Her explanation for the exponent task 3 was much better in terms of accuracy and application of the rules, she made a minor arithmetic error and had 1 point deducted, otherwise her method was 100% correct. She lacks confidence in her ability to do these exponent problems, because she knows her rules well, but she still prefers the financial math task, because “that is much easier” (Int. Z 411), yet she got that task incorrect.

The next transcript was short in terms of conversation because she skipped task 4 based on finding the average and was not offering much by way of explanation

Corvell:	Number 4, here we’ve got the average of Ed’s ten test scores, which is 87. Tell me what happened there.
Z:	I didn’t know what to do there, so I just skipped it.
Corvell:	You didn’t have a clue what to do? Do you know how to calculate the average of something?
Z:	Ja, isn’t that when you add everything and then divide by the total
Corvell:	Ok, yes, so that’s the average. Was this problem familiar to you? Have you seen it before?
Z:	Yes.
Corvell:	Which grade?
Z:	In grade 10.
Corvell:	In grade 10? And you did not know what to do? If you look at it now, do you think you have some idea of what to do?
Z:	No.

In the patterning task 5 she showed some level of competency by solving only part of it, but struggling to do the more complex parts. Throughout the conversation she kept saying “I don’t know”, “I’m not sure”, and “that I found difficult” when asked to think a bit deeply about the solution. I tried to lead her through examples to try to get her to think more regarding her strategy, but that did not work.

This trend continued as she complained that the simultaneous equations task 6 was too difficult “because we do this but I wasn’t actually sure about how to handle it” (Int. Z 466). When pushed to give some solution strategy she said “I felt like I did not know what was going on” (Int. Z 473).

The following transcript picks up this theme of ‘not knowing’ anything (task 7):

Corvell: ... Now that is a geometry problem. You said that it is difficult, why?
Z: Because I am not sure how to work this out and stuff.
Corvell: So you didn’t know how to start, so you didn’t do any calculations. So it is difficult because you do not know where to start. Is the problem familiar to you, and have you done them?
Z: We did something at the beginning of last year, I think.
Corvell: Ok, so if I would say, ABCD is a parallelogram, what is a parallelogram? Do you know the definition?
Z: No.
Corvell: Do you know any of the properties of the parallelogram, maybe?
Z: (No response)
Corvell: Alright, so it’s just totally gone.

In the next two algebraic problems (tasks 8 a and 8 b) she displayed more enthusiasm “This is alright, because we did this in class, so I felt like I know what’s going on” (Int. Z 495). However, it was a strange sense of self-confidence because when asked about whether her answer was correct she said “I’m not sure” (Int. Z. 497). She was able to competently explain her solution strategy for both tasks and achieved full points for both. The following interaction shows her confidence in her explanations.

Z: Here you must FOIL it out, then you get that. So to work out it out , you must take 20 over, then the sign changes, then it’s 70 and then you bring the 4 over and divide by it.
Corvell: You bring the 4 over, but you’re dividing by 4 right? And what is your answer there?
Z: 17 1⁄2.
Corvell: Seventeen and a half. Ok, what about b)?

Z:	This is actually like, not completing the square, but factors, that is what you must do to get this and to make sure you check, I can say -10 $\times$ 50 will give me -50, -10 + 5 is 5.
Corvell:	So b) would be correct?
Z:	Yes.
Corvell:	You wrote that a) is an average problem and b) is an easy problem. Why would a) be average?
Z:	Because sometimes when you know what to do
Corvell:	Alright, and why would b) be easy?
Z:	Because you just have to find out the factors of the equation
Corvell:	And are you more familiar with a) or b).
Z:	Both actually.

She had no idea how to solve task 8c, even though she had an idea how to start but could not determine the common denominator.

Corvell:	... What about c)?
Z:	Ja, I had no idea what was going on there.
Corvell:	... How would you think about solving that? What do you think would be step1?
Z:	I think, finding the common factor or something.
Corvell:	What would the common factor be? Do you know how to find it?
Z:	Ja, we learned how to find it, but in here I couldn't find it.
Corvell:	So you wrote that it is an average problem, why not a difficult problem?
Z:	Because it's not much more difficult than the others.
Corvell:	So you feel that you should be able to do it though?
Z:	Yes.

In task 8 (d) she lost a point for a minor arithmetic error that was considered a slip-up, when she transposed numbers across the equal to sign, but forgot to change the positive sign to a negative sign. Another error occurred in task 8 (e) when she did the following: $3(x^2 - 3x - 2x + 6) = 3x^2 - 3x - 2x + 6$. Zena showed that she had difficulty in thinking through this type of inequality. She also did not say too much about this challenge.

Corvell:	What about e)? What did you think when you looked at the problem?
Z:	I thought that this was going to be difficult and it was difficult, because we didn't actually do this a lot so I did not know how to handle it.
Corvell:	What was the first thing that you did there? [pointing to the script]
Z:	I filled out the two brackets, then I multiplied the 3 into the brackets.

Corvell: So you got something on the left side and something on the right side, and at that point you did not know what to do. If you attempted a next step, what do you think it would be, based on what you have there?
Z: … and put the x's on the one side and all the numbers on the other side
Corvell: So you would get x's over and maybe keep all the numbers on the other side? Would you maybe try and get everything on one side? Would that work?
Z: I'm not sure.

In the following transcript Zena emphasizes the idea that task 9 type relating to calculation using the 'volume' concept, was done a year ago and they did not do it for a while, hence why she forgot how to solve it. However, her solution strategy showed that she did not know the formula for finding volume, and the transcript provides some thoughts on this task. She again showed that she was not convinced with her strategies.

Corvell: ... What about number 9? I see that you tried it. When you read the problem, what did you think and how did you feel?
Z: I think I was going to get stuck there and I did, so that's why I think that my answer is incorrect.
Corvell: Is that problem familiar to you? Have you seen it before?
Z: Yes, I've seen it before, but we didn't do much of this also.
Corvell: In which grade?
Z: Now, and last year we actually didn't do that much.
Corvell: So did you do that more in Grade 9, or when?
Z: Grade 9.
Corvell: So if I should say that these are 'volume' problems, would you know how to calculate the volume of the rectangular prism?
Z: (Long pause), no.
Corvell: You forgot it completely or you didn't do it?
Z: We did do it, but I just forgot, because it was a while back that we did that.
Corvell: Ok, and that's fine. So what did you do over here, because you did try to solve it? Is that $h = l \times b$? So you're saying that the height is equal to the length times the breadth, and then you substituted?
Z: Yes
Corvell: So you've got 36, or 30 and 20, is that right? 30 and 20, and you've got 900 and 400, 1300 and then, what did you do here?[on her script she had $30^2 / 20^2$ = 900 x 400 = 1300, then she found the square root of 1300 to get 36cm]
Z: Because I couldn't get a answer like that, so I had to square it to get the smaller answer that is there.

She used the incorrect formula and did not get any points for her effort.

Summary: She was uncomfortable with solutions to tasks covered during earlier grades, because she had forgotten the formulae and procedures. On the tasks that required specific algebraic skills, she was unable to translate task information into symbolic expressions and found it difficult to solve equations. She lacked the ability to integrate specific skills for more complex tasks, and lacked the necessary knowledge or algorithms to even start solving a task. After completing the tasks she wrote the following on her script:

> The little sums in here are actually challenging. I felt good while doing some because I knew what I was doing. This paper is actually tough. When I looked at the paper it looked easy but as I went on it didn't go to well. All the work in here we did do in class but I forgot. I was a little nervous doing this paper. I don't think I did well.

She also predicted that she would get 50% for this paper, but only managed 25%. She over rated her ability and potential to do well.

CHAPTER 6

RELATING MATHEMATICAL IDENTITIES WITH MATHEMATICAL PERFORMANCE

In mathematics stories and interviews participants constructed their mathematical identity by drawing on past and present experiences to define who they are; to define their relationship and disposition towards mathematics; to define beliefs regarding their ability to do (engage in) mathematics; and to define their future aspirations. In chapter 4 I described these past, present, and future experiences to give a sense of how the students developed their mathematical identities. The result of this analysis will be referred to as the participant's Mathematical Identity Profile (MIP).

The participants used their mathematical knowledge and skills to solve a set of mathematical tasks. In chapter 5 I described their performance on these tasks, their thinking behind their solution strategies, and their deficits in their mathematical knowledge and skills. The result of this analysis will be referred to as their Mathematical Performance Profile (MPP).

In this chapter I use "the idea that students develop relationships with their knowledge" (p. 16), a notion of mathematical identity that was used by Boaler (2002). This idea suggests that as students engage in classroom and mathematical practices they not only develop knowledge, but they also develop a relationship with that knowledge. Their mathematical identity includes the knowledge they possess, the ways in which they hold that knowledge, the ways they use that knowledge, the ways their mathematical beliefs interact with that knowledge, and the ways their work practices interact with that knowledge (Boaler, 2002).

Specifically, the analysis in this chapter will answer the last research question "How does students' ability to solve tasks relate to their mathematical identity?" This ability to solve tasks is dependent on and an indication of 'successful' mathematics learning and is strongly connected to Wenger's (1998) idea that mathematics learning is a process of identity formation.

The key idea in this chapter is to relate the two profiles: a theoretical subjective construction of mathematical identities (Mathematical Identity Profile), and a practical display of mathematical performance (Mathematical Performance Profile), representing the outcomes of learning. As I related these two profiles with each other, I observed that the students' beliefs and perceptions about their math abilities did not necessarily translate in the outcomes of learning, and in some cases contradicted their beliefs. In some cases there were disconnects between MIP and MPP. The relationship between beliefs and ability; aspirations and ability; dispositions and ability were tenuous at best. As a result I framed this chapter's analysis and discussion based on the post-structural perspective of identity as used by Stentoft and Valero (2009) that focused on "Exploring the fragility of discourse and identity in learning mathematics" (p. 55). Through this exploration it is possible to identify the strengths and weaknesses in the ways these mathematical identities were constructed (Stentoft & Valero, 2009). In particular I used Stentoft and Valero's contention that:

> ... a notion of identity formulated from this perspective and emphasizing the dialectic relationship between identification[16] and discourse[17] offers interesting possibilities for interpretations of mathematical learning as a

[16] The action of engaging in constructing multiple identities or rather seen as "identities-in-action", to bring out the continuously shifting possibilities for new constructions as arising with changes in discourse (Stentoft & Valero, 2009, p. 65)

[17] Mathematical discourse, in this sense, "is the written and spoken language used by mathematicians and students of mathematics for communicating about mathematics. This is "communication" in a broad sense, including not only communication of definitions and proofs but also communication about approaches to problem solving, typical errors, and attitudes and behaviors connected with doing mathematics" (Wells, 2009, p. 1).

> fragile process that is characterized more by discontinuities and disruptions than by continuity and stability: a process which cannot be taken for granted even when students and teachers are confined by the walls of the mathematics classroom (p. 58).

In this chapter I used *discontinuity* to describe disconnects between MIP and MPP, and *continuity* to describe connects between MIP and MPP. I analyzed data for this chapter using two approaches: Firstly I worked with the MIP of each student and followed some connections that were exhibited in MPP. For example, participant Moses, had a feature, say A of this MIP, then I combed the clinical interview about his math performance to look for aspects that reflected feature A; Second, I worked from MPP and followed connections in MIP. In this way I was able to match some features as "discontinuities", while others matched as "continuities" between MIP and MPP. From the analysis for each participant six features of the relationship between MIP and MPP emerged: leading identity and importance of math; self confidence; attitude towards reasoning through the tasks; role of previous instruction; resources for solving new tasks; and image of self as a learner of math. These features are discussed for each participant.

Yusuf

Leading identity and importance of math

Yusuf's leading math identity was based on how he valued mathematics and the importance of it. The key reason for choosing mathematics was to fulfill his dream of one day becoming an engineer, because he "must have pure mathematics". He connected his perception of his own mathematics ability to his mathematics trajectory that he thought would work best for him. For example he declared himself as a "bright person in mathematics, hardworking, lots of self esteem, and confident" and he connected this with his aim to pass the grade 12 matriculation examination with an A-symbol, meaning above 80%. Yusuf only achieved 43% on the math tasks used in this study. He was mathematically competent on 14 tasks of 27 tasks.

This result questions Yusuf's perception of his own mathematics ability that he is a "bright" student in mathematics. These results are particularly strong indicators of contradictions about his perceptions of his own mathematics ability. This is Yusuf's conundrum. He has a history of low achievement in mathematics with a decreasing trend in his final examinations with scores of 66% in grade 9 and 41% in grade 10. Mathematics is a prerequisite for graduation, but more than that Yusuf must not only pass the mathematics examination with 50%, but he must pass it well to get into engineering at any of the South African universities. This type of performances will undoubtedly put pressure on Yusuf and heighten his anxiety and may force a re-construction of his mathematical identity. The fact that Yusuf is operating on the borders of continuing a mathematics career or switching to mathematical literacy will cause "discontinuities" in the (re)construction of his mathematical abilities. If he should switch to mathematical literacy his leading identity will change, and how will he then view the 'use-value' of mathematics is an important question. These results may help him transform his perceptions of his ability to do mathematics, or he may redefine his definition of hard work, by working much harder or more effectively to achieve better results. Because of this "discontinuity" in his mathematical identities Yusuf will engage in the activity of reconstructing his mathematical identities. In conclusion there is discontinuity between Yusuf's leading identity and his ability to do math.

Self confidence

Yusuf's beliefs about his mathematics abilities were based on his view regarding the nature of mathematics. He said "Can't say it's easy and can't say it's difficult, but if you know the formulas then it will be easy: if you don't know it, it will be difficult". He suggested that in engineering "I hope to at least use some of the formulas". This view is based on the instrumentalist perspective that occurs in traditional mathematics classroom and is simply knowledge of a collection of math terms, rules and formulae without knowing how it is connected to other aspects of mathematics. He positioned himself as a capable doer of mathematics and expressed confidence in his interactions with mathematics. He was enthusiastic talking about his math abilities, "I think I'm quite good at it now". He rated his motivation to do math as an 8 out of 10.

During the clinical interview, he was less excited and enthusiastic when he had to explain the thinking behind his mathematics strategies. During the tasks Yusuf struggled with geometry definitions, and did not know some of the formulae or rules needed to solve some of the tasks. He was unable to distinguish between parallel lines and equal sides of an isosceles triangle. When asked to define parallel lines, he offered the 'indication marks //' as a means to identify parallel marks, and stated that "that's how I actually learnt it". However, we use the 'indication marks >>' to show lines parallel. This is one example of his confusion with geometrical terms.

He never mentioned understanding concepts during the interview, but emphasized rote-learning of the formulae. In Yusuf's world the teacher gives the formula and the student simply learns it. If you do poorly in math then it means you did not study your formulae. It seems as if he used his mathematical identity to shield his true ability or under-achievement in mathematics from me. Despite his positive portrayal about his positive disposition towards his math ability as he constructed his mathematical identities, he struggled with doing the mathematics tasks. He expressed more negative views about mathematics during the clinical interview where he became frustrated with his lack of mathematical knowledge that impacted his self confidence. Analysis of individual tasks provided specific information about what Yusuf knew and was able to do. It also gave a good sense of his mathematical learning with respect to his prior learning and ability. This is evidenced by his ability to only solve 50% of the math tasks, meaning that his level of mathematical competence based on knowledge of formulae, rules, definitions, and procedures, was only at 50%. However, his prior knowledge regarding solution strategies to solve financial mathematics and patterning tasks fit strongly in his belief about his mathematics ability. These questions were all familiar to Yusuf and he was exposed to it at different grade levels.

From Yusuf's construction of his mathematical identities he developed these beliefs throughout primary school and into high school with some adaptations and shifts in his mathematics identities. Consequently, these beliefs affected his ability to do mathematics and his self confidence, because his clinical interview was punctuated with "I'm not sure". In conclusion there exists discontinuity between MIP and MPP as we relate it to his self confidence. He over-rated his math ability during his construction of his mathematical identity.

There were brief vignettes where Yusuf's self-confidence in MPP can be considered in continuity with his MIP. Yusuf was very confident about his ability in some tasks. In these cases success in the tasks can be related to his self confidence, but he was also self confident that he applied a correct procedure, but that procedure proved to be incorrect. He admitted that he liked doing some tasks (referring to data handling tasks) during the examinations, it boost his self confidence, but he is disappointed that they don't do more of these during class hours because "it is easy work". Yet, he was unable to draw the bar-graph and find the median. In these tasks his self confidence is manifested in his view of success, even if his procedures are incorrect.

Attitude towards reasoning through the problems

As a first example, I raise Yusuf's learning experience with algebra as a conduit for this discussion. Throughout his construction of his mathematics identity he was adamant that he had learning deficits regarding the learning of algebra, and these deficits produced a negative disposition towards his ability to learn algebra. He said "I never knew what was going on, starting with the algebraic letters. I was not taught that in primary school. He had negative beliefs about his own ability to do algebra. Yusuf only scored 45% correct in the algebra domain and he was able to only solve 4 out of the 7 algebra tasks suggesting that he was not mathematically competent in algebra. He also lacked confidence when explaining his solution strategies, even when he had it correct. For example, when he was asked whether he understood the exponent laws, he replied "not actually" even though he applied these laws correctly.

Yusuf's negative belief in his algebraic ability was in continuity with his mathematical identities. However, since algebra is one of the most important domains in his high school career, these negative perceptions regarding his ability can impact his future success in high school and in tertiary education. This low percentage in algebra showed his mathematical skill level was still below expected levels of performance for grade 11 students. In conclusion there was continuity between his ability to reason through algebra in MPP and the way he constructed himself as a student of algebra.

For a second example, Yusuf displayed self-confidence and a positive attitude in his ability to reason through tasks or simply just to do math. He believed that the challenge to do well strengthened his mindset towards mathematics and believed that if you try hard enough success is possible. He emphasized that he just need more self confidence to do better, because he perceived himself as hardworking. He even emphasized that he aims to get over 70% for tests, but admitted that he does not achieve more than 60%. This seems like an over estimation of his ability, but he believes he can do it. His levels of excitement increased as he fantasized about getting 75% in a test because it would motivate him and boost his confidence.

Yusuf admitted that the tasks were mostly average questions, familiar to him, and easy. However, he struggled with some of the tasks because he only remembered fragments of what his teacher did. He actually saw these math tasks as a test and admitted that he enjoyed it because the tasks contained some data-handling, a domain in math that he liked.

Yusuf's explanations during the tasks to explain his thinking and describing his solution strategies or lack thereof showed confusion, lack of self-confidence in his ability, inability to recall key terms and formulae, and in general his failure to know what to do was in contradiction to the way he presented his belief about his self-confidence. Unfortunately, these high levels of motivation to learn that he constructed in during his construction of MIP did not translate into learning mathematics effectively. During the clinical interview he displayed self-confidence as he explained some of his solution strategies to tasks that were familiar to him and that he remembered how to do them. For example, I could see how motivated he was when he described solution strategies for financial mathematics and patterning, because he liked these kinds of problems. Yusuf was motivated to do the mathematics that he liked, but it showed in the analysis of the math tasks that he was not fully sure what sections he liked. For example, he said he loved geometry, but only received 33% for it. In conclusion it does seem that Yusuf is strong in his belief that he is motivated to learn and self-confident, but there is discontinuity between MIP and MPP. He claims that he is motivated to learn, and consequently work hard, but he is not working effectively hard, meaning that he is perhaps working hard by simply giving more time to do math, but not learning mathematics. It is this aspect that makes the relationship between his belief and practical reality tenuous at best.

Role of Previous instruction

Yusuf always displayed a positive attitude towards mathematics but his related behavior did not support this positive attitude. He admitted that he developed deficits in his learning because of poor teaching as one of the reasons for poor performance. For example, he had a strict teacher and as a result he disliked mathematics resulting in him not taking notes or just not working, even though he emphasized that he always tried his best. He said “I think having the right attitude towards it [math problems] would actually help a lot, because if you don’t have the attitude you’re not going to remedy it” and “You should have a positive attitude toward mathematics. You should actually try your best to sit with your work after school, and not go play in the road”. However, key to Yusuf’s learning is the use of the textbook he says “my text book has all the answers in the back of the text book. So I go through the question, then I try to work it out and once I have an answer, then I check if my answer is right at the back of the text book ... I don’t need the teacher to give the answers every time”. For Yusuf getting the correct answer is important.

During the clinical interview, Yusuf said that he knew how to solve simultaneous equations, but he claimed that he was not familiar with the task on the mathematical tasks instrument. In the same breath he admitted that in class “We’ve done a few, I think we’ve done about 30 or 35 in class, but that time I was also busy with athletics, so I did not actually get a number [meaning he missed out on a lot of the exercises]” (Int. Y 660). So he could not recall the method, but Yusuf did not use his textbook in this regard.

Absenteeism from class seems to explain why he developed deficits in learning mathematics. During the athletics term, athletes get permission to train during school time, but the onus is on them to catch-up with outstanding work. The trend that I noticed during my 18 years of high school teaching was that the athletes did not do this, and some teachers did not go out of their way to help students' catch-up. There is discontinuity between Yusuf's positive beliefs in his ability, but this is not backed up by effective behavior practices resulting in gaps in learning.

These deficits in learning were in evidence during his inability to start a solution strategy for the graph work. He did not know how to find the x-intercepts or the y-intercept of the parabola. He was unable to start a typical grade 8 fraction task where finding the lowest common multiple should not have posed any difficulty. In conclusion there was discontinuity between MIP and MPP as it related to learning from previous instruction.

Furthermore, Yusuf also showed glimpses of permanently relying on previous instruction. We have seen that Yusuf was able to rely on previous instruction to solve some of the tasks, but he also remembered fragments of math knowledge. He achieved full or partial points for being able to recall from memory. Evidence of this is seen in this response: "That's how we done it in class. The sir never actually explained that to us. That, we actually done last year already, but at that time I was thrown out of class". Another example to show that Yusuf depended on fragments of memory was when he admitted that "Something I never knew, but then it actually hit me". In this way Yusuf displayed features of successful task-solving. In these types of vignettes we see MPP in continuity with MIP

Yusuf also displayed knowledge of previous success on a similar task that caused him to remember the solution strategy used at that time and needed to be reproduced in a similar task.

However, Yusuf confessed that he did not quite understand what he did. Yusuf said “like we done that last year in grade 10, beginning of grade 10. That time I was not much in the class due to athletics that was happening in the school. So I don’t really know how to simplify it, but I tried”. In this case he did not know how to simplify, but he also repeated previous calculation mistakes. This feature provides an opportunity to explain his failure in math. However, a phenomenon happens here that makes this feature one that actually, if it doesn’t support his success, at least it is remarkable that it does not deter him from considering the task a successful one: He makes some mistakes in his computations, but he seems to have some image of what they are, he is conscious he made the same kind of mistakes before. Evidence is seen in the comments “This one I got wrong here, the 12^{th} root of ... [pointing to 4^{th} step in his solution], but not sure” and “Then I multiplied it with all the powers by adding the powers together. And I don’t think that last step is wrong that I got, where I subtracted the top powers from the bottom powers. Ummm I made a mistake there [pointing to the answer]”.

Resources for solving new tasks

During his construction of MIP, Yusuf’s perceptions of how important it is to do mathematics undoubtedly influenced his expectations of the future, expectations of his success in mathematics, as well as his achievement in mathematics. Yusuf expressed the personal importance of doing well, he expressed his enjoyment only in doing certain domains in mathematics, This means that the most important resource for learning is Yusuf himself, because he expressed much energy on explaining the utility value of doing mathematics as opposed to doing mathematical literacy, and that mathematics forms a significant part of his future plans.

During the clinical interview it was apparent that the most important resource is the teacher and previous instruction of procedures, but the distinction from MIP is the addition of the role of memory. Yusuf depended completely on his ability to remember what his teacher did and said. In conclusion there was discontinuity between MIP and MPP as it relates to resources for solving new math tasks.

Image of self as a learner of math

From the early years of primary school Yusuf developed a positive disposition towards doing math. As soon as he started high school he started to dislike math, but still saw great value in pursuing math because of its importance in his future career. During the individual interview he gave the reason for not liking math, due to his high school math teachers. His mathematical identities reflected negative views about his math teachers, hence about mathematics. This led to some constraints on his participation during math class and a negative image of himself as a task solver. He claimed that he was unmotivated to study, unmotivated to participate, and unmotivated to talk to the teacher who clearly, according to Yusuf, did not have his interests in mind. This math classroom offered limited possibilities for learning mathematics with the teacher's pedagogy and attitude a not so important mediator between Yusuf's beliefs about learning mathematics and his love for mathematics. His learning was characterized by simply replicating the algorithms and procedures given by the teacher, and performing calculations on a calculator designated by the teacher. Yusuf did not receive help with his calculations because he had the wrong brand of calculator.

Based on Yusuf's performance in the math tasks, his MPP was filled with emotional attributes that included frustration as he highlighted that "I thought it was easy, but we never actually went through such work before in the class”. His perceptions of his teacher impacting on his learning, is continuous with his MIP, but the image of being a successful learner is not continuous with MIP. The question is whether he struggled in class or did he simply opt out by not participating. Either way his 43% in the math tasks was a clear indication that he was not learning at the appropriate level. He demonstrated very low skill levels for 50% of the math tasks. Yusuf’s opportunities to learn within this environment were constrained. In conclusion there is discontinuity between MIP and MPP as it relates to his image of self as a math task solver where the emphasis is on the role of emotions in math identity and performance.

Avril

Leading Identity and Importance of math

Like Yusuf, Avril’s leading math identity was based on how she valued mathematics. She knew that mathematics was important for her intended field of study, dermatology. To achieve this goal she must have pure mathematics with a high grade of at least 75% to access a university in South Africa. Her perception of her own ability was that she will do her best “give her all” to succeed in mathematics, because she had a fear of failing, it “terrified” her. She also believed that if she cannot qualify to do dermatology, with pure mathematics she will be able to do something else with mathematics. Avril has the same conundrum as Yusuf. She has a history of low achievement with a decreasing trend in her final examinations with scores of 52% in grade 9 and 35% in grade 10. Her percentage-point score in the mathematics tasks was only 15%.

This result is in contradiction to her statement that she tries hard in mathematics and it also questions her ability to be successful in mathematics. She was unable to carry out procedural operations and did not know her formulae. There does not seem to be any relation regarding

Avril’s positive disposition (from MIP) towards mathematics and her ability to do math (from MPP). Her performance in the tasks revealed a lack of mathematical knowledge in all the mathematical domains. There exists a chasm between her mathematical knowledge, knowledge that should have been cemented at this stage of her math journey and her constructed mathematical identities.

At this rate, failing mathematics seems to be a reality, something Avril wanted to avoid at all costs. Relating her ability to do math is “discontinuous” with her leading identity is and may require a re-construction of her mathematical identities, because this discontinuity may result in a change within her hierarchy of motives. If she should switch to mathematical literacy her passion based on her motive to become a dermatologist will fade, hence a re-construction of a new leading identity. The question arises regarding how she will view the ‘use-value’ of mathematics, after a switch to mathematical literacy. In conclusion there is discontinuity between leading identity and her ability to do math.

Self Confidence

Avril’s self confidence was based on her external motivation focused on her ideas that she must learn “this stuff” referring to math to pass the examinations and to do a particular career. Furthermore she displayed determination with “I don’t give up” and coupled this with her frequently stated trust in her math qualities. These qualities included the value she attached to understanding stating that “I always try to understand a problem”, and the value she attached to her ability to apply logical thinking and reasoning through a problem.

During the clinical interview Avril becomes very emotional using verbs like “frustrated”, “struggle”, and “blanked out”. All of these denote an emotional and subjective attitude towards these tasks that prevailed over her ability to reason. For Avril "thinking through the problem" means the ability to recreate the solution strategy from memory based on previously seen examples or teacher explanations. The most used verb during the clinical interview was "I forgot it". He attempts to recreate the solution steps from memory and then forgetting the exact steps resulted in a blockage. These attempts can also be viewed as her refusal to reason by herself. For example she remembers the formula for calculating the area of a rectangle, yet she refuses to replace the length and the width with the given quantities, that is, to write the equation of the new relationship. Furthermore, this assertion "because I know if I attempt it, it is going to be wrong", shows her lack of confidence in reasoning by herself as opposed to reproducing a solution from memory or from previous teacher instruction.

There is discontinuity relating to self confidence where her MPP invalidates the image of self confidence that was visible in her MIP. In MIP self confidence was based on external motivation, determination, belief in her own ability to reason, and her inclination to look for understanding. In MPP she relies on memory and ability to recall, tries to reproduce memorized solutions from previous instruction, and becomes emotional. This emotion blocks her abilities to reasoning through the problem by herself. In conclusion there is discontinuity between MIP where she emphasized a lot of confidence-based on these external factors and MPP where she showed no confidence to proceed on her own when she realizes that she does not remember previous instruction strategies or procedures.

Attitude towards reasoning through the problems

Her confidence towards doing math and in her logical abilities was reinforced by her primary school teachers and experience. Consequently, she believed that reasoning and understanding a problem is the base of successful performance in math. She looks to previous instruction and solution strategies for understanding math. During the construction of MIP she believes in transferring knowledge learned from one problem to another, not by reproducing the strategy exactly, but by understanding the essential ideas and applying them to a new problem.

Avril relies on her memory, especially memorized previous solution strategies. If she cannot remember, she refuses to reason through a new problem. She actually denies her own logical abilities "You must try. I will give you my book, but you must try first". Avril conveys the main features of "the ideal" problem solver or model math learner in MIP by emphasizing that students must be driven; must have logic, reasoning, comprehension; must not take on solutions without understanding them; must use previous instruction for deepening their understanding; not to copy a model strategy and reproduce it blindly; must be able to listen; must ask for help; must learn from others, must work well with peers (group work); must spend time working thorough many similar problems to assimilate a strategy; and must have a positive attitude towards the sort of math required in exams.

Avril seems to express a rather heavy reliance on analogies with previous problems done in class (similar problems-solution strategies). In this respect, she talks about the "previous models shown in class" as sources for understanding a solution strategy, not as models to be reproduced by heart in the next problem. Her emotional attitudes towards certain math (like her hate of graphs) become ways to legitimize why she won't even make the effort in certain directions. This is more than "I can't solve it because I do not remember the formula/the strategy". Consequently there is discontinuity between MIP and MPP as it relates to attitude towards reasoning through a problem.

Role of previous instruction

Emerging out of the previous discussion a synthesis of MIP is that previous instruction, based on similar or familiar problems served as a resource for understanding and acquiring knowledge that Avril could use and apply, by using her own reasoning to a new problem. A synthesis of the analysis of the value of previous instruction provided solution strategies that were reproduced by memory.

Through her construction of MIP Avril suggested that learners must have a critical attitude towards instruction in the sense that, if instruction does not fulfill its goal, the learner must look for additional support/instruction. They must ask questions and they must seek additional resources. These solution strategies shown in class are not to be simply memorized and reproduced. Instead, she sees these as an opportunity for enhancing comprehension. In this respect, she talks about the “previous models shown in class” as sources for understanding a solution strategy, not as knowledge to be reproduced by heart when she attempts to solve the next problem. In conclusion there is discontinuity between MIP and MPP as it relates to the role of previous instruction.

Resources for solving new math tasks

The most important resource for learning math is Avril herself, and this learning is based on reasoning, logical thinking, and understanding. The other key resources include teachers who provide good instruction, peer interaction, and textbooks. The most important resource is the teacher and previous instruction of procedures, but the distinction from MIP is the addition of the role of memory. Avril depends completely on her ability to remember what her teacher did and said.

Throughout the construction of her math story and the interview the common verbs used were “reasoning, logical thinking, understanding”. The most common words used during the clinical interview was “forgetting, being frustrated”. In conclusion there is discontinuity between MIP and MPP as it relates to resources for solving new math tasks.

Image of self as a learner of math

Avril projected the image about herself as a mostly successful task solver, based on her descriptions of her own logical abilities, on her ideas that she is highly motivated, on external support, and on her determination to succeed. During the clinical interview her attributes are very emotional because she uses verbs like “hate this stuff”, “love this stuff”, “being frustrated”, “blank out”, “struggle, and “feel bad”.

This emotion prevails over the external factors that should be motivating her to try to reason. For example, I invited her to try and reason from the formula given to her in task 1, and she refused on the basis of her emotional blockage. In conclusion there is discontinuity between MIP and MPP as related to her image of self as a math task solver where the emphasis is on the role of emotions in math identity and performance.

Moses

Leading Identity and Importance of math

Moses’ leading math identity was based on how he valued mathematics. He spoke about a fascination with numbers, and calculations in real life. He was also driven by his ‘imagined’ future as a traffic-controller. He knew that mathematics was important for this field of study. To achieve this goal he must at least pass pure mathematics with a high grade of 75%. His perception of his own ability was based on “working hard” and doing something that he loved, mathematics. He knew that he needed to succeed in mathematics.

However, like Yusuf and Avril, Moses has a conundrum. He has a history of low achievement with a decreasing trend in his final examinations with scores of 51% in grade 9 and 45% in grade 10. His percentage-point score on the mathematics tasks was only 43%. This result is in continuity with his belief that mathematics was “becoming too difficult and that he was struggling to cope”, but in discontinuity with his dream to become an air-traffic controller. He was realistic about coping with math or if not then having to change to math literacy. He was able to carry out some procedural operations, especially in algebra, but did not know all his formulae, geometric definitions and graph-work. There seems to be a tenuous relation regarding Moses fairly positive disposition towards mathematics and his ability to do math. His performance in the tasks revealed a lack of mathematical knowledge in some of the mathematical domains.

At this rate of achievement, failing to achieve that higher grade in mathematics seems to be a reality, something Moses wanted to avoid at all costs. Relating his ability to do math with his leading identity is “discontinuous”, and may require a re-construction of his mathematical identities. This discontinuity may result in a forced change within his hierarchy of motives. If he should switch to mathematical literacy his passion based on his motive to become a traffic-controller will fade, hence a re-construction of a new leading identity. This change will also affect his perceptions of his ability to do mathematics.

Self Confidence

For Moses his self confidence was based on the attainment of “good grades”, because these grades are a “reflection of you”, and it “shows your capabilities”. High grades imply high levels of self confidence, and it makes him feel happy and he is proud of his achievement. In addition his happiness and self confidence is high because he also made his parents feel happy and proud of his achievements. He also measured his success as a math student based on “the grades you get” and for Moses this was achieving at the 80 and 90 percent levels. He noted that his low point in mathematics was when he received low grades. He also feels self confident “when you like and understand something, then you’re relieved and you feel good about yourself. “Ok, I know this”, and I can relax now, so it’s fine”. He also rated his motivation as 8 out of 10, because he felt so good about doing math.

During the clinical interview I saw a different Moses because he lacked confidence in solving the tasks and used emotive words to reflect his true feelings by saying I’m “a bit puzzled”, “a bit confusing”, “not sure”, and “don’t know”. These feelings made it difficult for him to reason through the task. At times he struggled to read and understand the task.

Moses beliefs about his mathematics abilities are framed around his view that grades are important. He displayed feelings of disappointment at the thought of not doing well in these tasks, because he did not want to give anybody a bad impression of his ability. The tension in this belief construct is based on the fact that he is not getting the expected grade hence he showed low levels of motivation. There is discontinuity between MIP and MPP as it related to self confidence. In MIP he displayed a positive image of self confidence by declaring that he is determined and motivated, but during MPP these characteristics were replaced by “not sure” and “don’t know, and even with prompting and given a choice of two alternatives he was unable to make a decision. In conclusion there is discontinuity between MIP and MPP with regard to self confidence.

Attitude towards reasoning through the problems

His confidence towards doing math is seen in his declaration that math is “not difficult”, and “if you understand it, it’s easy. I don’t actually find it very difficult, just in-between. I get confused, but I don’t find it difficult. However, he also described his attitude towards doing “easy” and “difficult” tasks. For the easy tasks he claimed that he “get a good feeling, because this is like simple”, but for the difficult tasks he “tend to panic a bit, I think, ok, this looks a bit foreign, then, I sit and think. This looks like another sum hidden, and you try to break it up and to do each piece on its own because sometimes it works and sometimes it doesn’t”. He described how he would deal with these types of tasks, he may panic but he will still attempt to solve it.

This attitude is only partially visible during the clinical interview, where Moses was able to apply his theory to the easy tasks, but basically gave up on tasks that he considered average to difficult. He liked the financial math task, and considered it “easy to understand” and he was confident of about solving it because he was familiar with these types of tasks doing so many of them. The fluency of his explanations reflected his confidence and positive attitude, based on the fact that he knew exactly what to do.

There seems to be continuity between MIP and MPP as it related to his attitude towards reasoning through “easy” problems, but there is discontinuity between MIP and MPP as it related to difficult tasks, where instead of breaking-up the task into smaller understandable bits, he gave up. In the geometry task he could not decide whether his struggle to solve the task was based on the words used in the question or just with the content of geometry. During the exponent tasks, we see the interaction between a positive attitude and a lack of self confidence. Moses declares that these tasks are easy and as constructed in MIP shows this positive attitude to solving and in the clinical interview gives a fluent explanation of his thinking, but he displayed a lack of confidence when faced with a bit of doubt. As soon as I questioned what he was saying/doing he thought that he made an error and lost confidence by saying "not very sure about that", although he was perfectly correct. In conclusion: Two responses: There is continuity between MIP and MPP as it relates to attitude towards reasoning through "easy" tasks there is discontinuity between MIP and MPP as it related to attitude towards "difficult" tasks.

Role of previous instruction

During construction of MIP, Moses expressed his belief in the instructional approach that "practice makes perfect", and that "practice and also your attitude is important". In addition Moses believes that you must write down everything the teacher writes on the board, he showed frustration as he explained that "I told the sir to give us time to write it [the math examples] down, but he just said there's no time and ignored the question, and he was just going on". Because Moses believed in "practice", he placed high value in homework. Throughout the individual interview he emphasized that "I am definitely hard-working and I do my homework"; that an excellent math student is "determined, hardworking, pay attention in class, and do your homework"; and that an excellent effective teacher "checked your homework".

When questioned about a task he said "that's just the rule I think"; "for the first couple of steps I knew what to do, but when it came to the end I didn't really know", and "I am familiar, I just don't know, I forgot what it was". In addition to this display of lack of confidence in his ability, this is also about how he learns math, the rules. He lacks the confidence in application because he tries to respond via his memory of the rule.

Through the individual interview and math story Moses highlighted the ideas that you learn math through practice, and it is important to emphasize the value of homework, because this is the space for practicing what the teacher taught, hence the importance for writing down everything. The conundrum for Moses is that without good understanding of what you doing, you will be prone to forgetting. In conclusion there is discontinuity between MIP and MPP as it relates to the role of previous instruction.

Resources for solving new tasks

Moses claims that he is the most important resource for learning math. He needed “to understand what’s going on, that is the main thing and applying that to daily life”. What this entails is paying attention in class, but also “to always take note first of what he [the teacher] is saying and then I write it down afterward. I first try to understand what the teacher is saying”. His MIP is therefore not only about learning rules and procedures, but understanding.

However, during the clinical interview Moses qualified that this understanding is based on understanding procedures and applying rules and formulae and not about connections and conceptual understanding. In this regard Moses’ key resource was his memory, which showed lapses in application. Throughout the construction of MIP the common verbs used were “to understand”, while in MPP he used “didn’t really know”. In conclusion there is discontinuity between MIP and MPP as it relates to the role of resources for solving new tasks.

Image of self as a learner of math

Moses projected the image about himself as a fairly confident person that I can do the work and whatever was required. He saw himself as above average ability, but he just needed to put in a bit more time and work a bit harder so that he could cross the threshold “to get to that ‘excellence’, you know”. He projected this image that he was close to being declared an excellent student. Furthermore, Moses admitted that “I always try to give 100%”, but added that “when you get distracted it’s a bit hard and stuff. I always try my best”.

During the clinical interview he is much more emotional about his ability and effort to do better in the tasks. His effort to solve was punctuated with similar sounding responses as “At first it did make sense, but I think after the second step I didn’t know” and “I don’t know what’s going on. - It‘s actually not familiar to me, because I missed out on this section of work last year, when I damaged my ankle ligaments. In the second response Moses provides a reason for his inability to solve the task. In conclusion there is discontinuity between MIP and MPP as it related to his image of self as a math task solver.

Katie

Leading Identity and Importance of math

It was difficult to find one significant motive that can be classified as Katie’s leading identity, because she is torn between her father’s wishes for her and her own ambitions. The key reason for her doing pure math is aimed at making her father happy. For her own ambition or future career, becoming a chef, she does not require mathematics, and for this reason she constructed her MIP around a negative disposition towards mathematics. However, she admitted that math was important (not necessary) for her future, but she was forcing herself to do well in math because others were telling her that she must do it and she convinced herself that math will not be “a threat or a stop sign to my dream or goal.

Katie has a history of low achievement with a decreasing trend in her final examinations with scores of 40% in grade 9 and 32% in grade 10. She only achieved 10% on the mathematical tasks suggesting that she was not mathematically competent. She admitted that “she don’t enjoy math class”, and she has constant feelings of “frustration” especially “because it’s coming close to the exams and none of us [her classmates] know what is going on, and that’s the time when we start panicking”. Her dislike for math and feelings of frustration characterized her performance in virtually every task. Her performance in MPP proved that for Katie learning mathematics was a complex and a time-consuming activity characterized by these feelings of frustration resulted in her inability to understand rules, concepts, definitions, procedures, and solution strategies. In conclusion there is continuity between her negative disposition towards math and her ability to do math.

Despite her negative disposition towards mathematics and doubt about continuing with math, Katie also constructed herself as around her work ethic that she was “really trying hard this term and if I pass Mathematics this term, I am going to stick with it”. She displayed signs of determination to pass math, and did not dismiss math as totally irrelevant. “I think that maybe one day if I change [suggesting a change in her career – not a chef], I think that if I go into another path, maybe it will benefit me”. This signifies that she is not under-scoring the use-value of math, because she noted that “you need to know how to calculate things ... you need math everyday, even though you don’t know it, you use it”.

During the clinical interview she displayed very little math knowledge, even of essential foundation arithmetical knowledge and skills. She struggled in every math domain, even with prompting. Because she failed to develop basic math skills and knowledge she was unable to attempt tasks that were slightly more complex in nature, for example, the measurement tasks.

She claimed that she had the ability to do math that included “solving problems” and she liked “finding out how to get the answer, how many ways you can get to the same answer”. However, during the clinical interview these attributes were invisible, hence there was discontinuity in her belief that she was doing better in math by working harder. In conclusion there is discontinuity between her belief that she was working harder and her performance.

Self Confidence

Katie’s self confidence is based on her motivation to please her father, and her sister, and listening to what others (community and teachers) have to say. This is seen in her responses such as “They really want me to work hard”, “He keeps on saying anyone can do it [math]”, “They don’t know what we going through”, “I’m supposed to be her example”, and “what does she think of me”. All these external responses and internal feelings work together to create a person with low levels of confidence. She expressed the need to “help each other and encourage each other”. This spirit of community and her preference for group work in class is foundational for her self-confidence and self-esteem.

During the clinical interview there was no community or group to help. She was the individual learner with no help and it was easy to see and even feel her frustration as she used emotional words like “I don’t know I panicked”, “I went blank”, “just don’t like it”, “I feel frustrated”, and “it didn’t look nice”. Her most used phrase was “I forgot”. The following response captures very aptly her feelings and confidence during the clinical interview she said “I felt frustrated and frightened when I see the fractions. I just went blank and that made me upset with myself, makes me not confident to do the rest of the sums. I feel dumb because I went blank and did not know all the answers to the questions".

There is continuity between MIP and MPP as it relates to self confidence. During the math story and individual interview her self confidence was based on being in a collective with her peers, and not operating as an individual. She preferred working in groups. During the clinical interview her self confidence was based on individual performance and she had no self confidence to work on the tasks. She rated her motivation to learn as 5 out of 10, and for self confidence she gave herself 4 out of 10. In conclusion there is discontinuity between MIP where she emphasized a lot of confidence-based on a spirit of community (external factors) and MPP where she showed no confidence to proceed on her own understanding when she realized that she did not remember previous instruction strategies or procedures.

Attitude towards reasoning through a problem

Katie's confidence towards doing math was based only on early experiences during primary school when she felt "math is great", "this is like, my subject", "this is what I want", "makes me feel good", "I achieved something", and "I enjoyed the activities, the group work". She emphasized that it was in primary school that "we [class mates] helped each other", "we solved problems, because we worked as a team", and "so I enjoyed that". In addition she noted that she was a visual learner, because her teachers in primary school used a lot of "manipulatives". She also believes that her confidence and ability is based on understanding because "it does not make sense to me, you need to understand". However, the problem here is that she claimed that in her class "we [class mates] to afraid to say no, so we just say yes even though we don't understand, so we can't really learn on our own".

However, during the clinical interview she tried to rely almost completely on her memory, based on memorized solution strategies. If she cannot remember, she just gives up. She said that "It isn't exactly foreign it is just that I couldn't remember the formula, and so I couldn't do it" and "I just thought of what we did in class. I tried it but wasn't sure if it was correct". These responses show her lack of understanding. Katie expresses the main features of the good problem solver, but does not apply this during the clinical interview. She does her best to try and recall what her teacher did, and when she fails she develops an emotional attitude towards doing these tasks. In some she attempts to legitimize her lack of effort by highlighting her 'dislike' for domains of math and 'feelings of frustration" for forgetting her procedures. In conclusion there is discontinuity between MIP and MPP as it relates to attitude towards reasoning through a problem.

Role of previous instruction

Katie does not say much about the type of instruction that she received, but her attitude and behavior reflects on her learning as a student of math. She admitted that she does not try hard enough in class because she cannot "focus properly" since her teacher "moves too fast". She complained that because of this pacing "if I don't know the answer or I don't know how to do that question, I just have to move on, then I'm like, "ok, I have to get back to that", then by the time he is done, I don't understand how he got that answer, because I don't know any of the steps, so it just doesn't work for me. We try telling him to work slower, and then it's like, that day he works slower and the next day it's just back to normal". She illustrated how her teacher's mode of instruction impacted her learning of math.

She admitted that she "just thought of what we did in class. I tried it but wasn't sure if it was correct". She was drawing on her memory and not on understanding. Consequently she remembered fragments of procedures. She had difficulty in selecting a formula, despite the fact that they did so many similar problems. The following response emphasized her difficulty with instruction practices and ability to recall. She noted that “When I thought of ‘compound’ I couldn’t remember which formula to use. Should I put the ‘year’ inside the brackets, or should I put the ‘year’ outside the brackets? So I was kind of confused, so I attempted both”. This level of confusion was common throughout the construction of MPP. In conclusion there is continuity between MIP and MPP as it relates to the role of previous instruction.

Katie showed that she did not possess the required math knowledge needed to operate smoothly through the tasks, but occasionally her key strategy was to rely on fragments of previous instruction. This is seen in her response: “I just thought of what we did in class. I tried it but wasn’t sure if it was correct”. She also highlighted the fact that during class time she did understand or know how to do it and this affected her reliance on previous instruction: “I don’t know, maybe just the way I was taught about it, and because I wasn’t sure of that in class already, so I did not do it”. Even in this regard there is continuity between MIP and MPP.

Resources for solving new math tasks

The most important resource for Katie was her set of friends. She seemed to have already given up on her teacher because of his attitude towards slower-track learners. She claimed that “he’s not really worried. He doesn’t ask us like personally, do we understand? For me, if he sees we are not doing well in our examination, why doesn’t he come and talk to us and find out what the problem is”. She emphasized her belief that “this group encouragement will help me a lot to work harder. I will learn from their mistakes and they can correct me” and “that it will be great to work together, but then they not there with you in the exams ... so you got to work independently also".

Her only resource was Katie herself, working independently. By working independently she produced some illogical computations, expressed feelings of frustration with memory lapses, resulting in panic and feelings of "giving up". In conclusion there is continuity between MIP and MPP as it relates to resources for solving new math tasks.

Image of self as a learner of math

Katie projected a dual image about herself as: Firstly, a person that will "try and hope that I'll get it right". She also believed that the right attitude is important because "you feel good about yourself that you can do it and you tell your mind, I'm going to do this sum, and then you are going to be able to do it. If you are hard-working you will do it". Second, a person that is not excited "about doing math", or "doing math at home", because math is the "last thing I want to deal with". She admitted that "at times I think I work hard and, at times I think I could have done better".

During the clinical interview her attributes are based on emotional responses and that she struggled with tasks that she should be able to solve. She complained that she did not do a type of task "in a very long time", but at the same time added that "we don't do that in math". Perhaps these were times when she did not do her work or homework, which is plausible judging from the poor performance in the math tasks. In conclusion there is discontinuity between MIP and MPP as related to her image of self as a math task solver where the emphasis is on the role of emotions in math identity and performance.

Zena

Leading Identity and Importance of math

Zena views math that it is the same as accounting, because it is just a lot of numbers. This is a key feature in her leading identity. Her imagined future is to become an accountant because accounting is more meaningful and relevant than math, because most of math is just about algebra and geometry and will not help you in the future. Her leading identity is therefore defined by how she thinks of the nature of mathematics, and that some of mathematics is irrelevant and will not help her in her career. She does acknowledge that mathematics is important for opening the doors to future careers, just in case the career of her choice, accounting, does not materialize. She stated that “if you don’t do well in the career you have chosen, you can’t go into something else, but if you do mathematics you can go into anything that you want to do”. Since mathematics would open doors in the future, Zena chose mathematics for this reason. Thus, her leading identity is also governed by her belief in the usefulness of mathematics. Like the other four participants Zena also has a history of low achievement with a decreasing trend in her final examinations with scores of 62% in grade 9 and 41% in grade 10. She only achieved 25% on the mathematics tasks.

She intimated that success in math depends on her and how she helps herself. She also admitted that she does better in her other subjects than in math and that for her career she can also do math literacy, which is a false assertion because she must have pure math to enter the accounting field. This indifference was noted in her response to math being easy or difficult she said “it’s in the middle because some chapters are easy while some are probably difficult. So we don’t actually know under what category it falls”. She definitely approached the math tasks with a casual approach to task-solving, and so relating her ability to do math is “discontinuous” with her leading identity based on her future goal, and may require a re-construction of her mathematical identities, because this discontinuity may result in a change within her hierarchy of motives by switching to mathematical literacy. In conclusion there is discontinuity between leading identity and her ability to do math.

Self confidence

She claimed that in grade 9 she "started to be good in my mathematics again", showing that her mathematical identity had fluctuations of good and not so good achievement moments. In grade 8 "math started difficult, and I got low marks, but afterward my marks went up and I could do pure math". In addition she had self confidence in her ability as seen in a very short and sweet statement "I think I work hard enough to succeed". Her goal is to "get an A [symbol] for mathematics in matric or in university" and she definitely does "not want to fail mathematics, as this is a very important subject". Her self confidence is based on her motivation to pass the grade 12 examinations. She suggested that she had the right attitude by always taking "note of what the sir is saying and if he explains one and I don't get it, I tell him to explain it another way so that I can understand it for myself, so that I can get used to him and learn".

During the clinical interview Zena was not that responsive and made use of short sentences to express her frustration about her inability to solve the task. She became visibly emotional about this because it happened so habitually. She responded consistently in sentences that characterized her lack of confidence: "I felt like I did not know what was going on"; "No, that I found very difficult, because we do this but I wasn't actually sure about how to handle it"; "because I am not sure how to work this out and stuff"; and "I didn't know what to do there, so I just skipped it". Yet when I questioned her, by challenging her response, saying "you did not have a clue", about not knowing what to do, she responded with self confidence "Ja, isn't that when you add everything and then divide by the total".

It seems as if she tried to use "I forgot", and "don't know" to avoid explaining some of the tasks that she lacked confidence in, because she very ably explained those math procedures that she knew. There is discontinuity relating to self confidence where her MPP invalidates the image of self confidence that was visible in her MIP. In MIP self confidence was based on her desire to do well. In conclusion there is discontinuity between MIP and MPP as it related to self confidence.

Attitude towards reasoning through the problem

She admitted that she was more motivated in primary school, had a positive disposition towards math, and did not struggle with math. Her confidence was at a high as she related that "I could do anything then". Her current attitude towards math was based on following the advice that "the teacher like always says we must try to do it even if it is wrong, at least you tried, so that next time you know that what you did wrong you can always fix it". She constructed the belief that you do not quit, but work harder and try harder, because "the work load becomes more, and it becomes more difficult" as you progress through the grades.

Zena depended on her memory to complete the tasks. The reality is that her math knowledge and skills did not allow her to navigate through the tasks competently, and instead of trying harder, she gave up. In fact she did not even attempt 10 of the 27 items. Zena conveyed that it was important not to give up, but to try even if you made errors. In most of the tasks she admitted that they did similar tasks, but her memory let her down. Evidence is seen in this response "we did do it, but I just forgot, because it was a while back that we did that". This shows that her method of learning math was to memorize procedures and not to understand them, which is discontinuous with her belief that “you need to understand” as constructed in MIP. In conclusion there is discontinuity between MIP and MPP as it relates to attitude towards reasoning through a problem.

Role of previous instruction

Zena’s construction of MIP is based the principle that practice makes perfect, where the teacher completes a host of tasks and the students are then required to solve tasks similar to these. Consequently, the students respond to tasks that are familiar to them. These familiar tasks serve as a resource for Zena. Zena’s mathematical activity is to memorize and recall at the appropriate times.

During the clinical interview Zena responded by drawing from these experiences in class. She clarified her feelings in the following way: “when I looked at the paper it looked easy, but as I went on it didn’t go to well. All the work in here we did do in class, but I forgot. I was a little nervous doing this paper. I don’t think I did too well”. She predicted that she should score 50% for these tasks, but only achieved 25%. She ended the conversation by emphasizing the role of memory by saying “it’s just that I forgot all the stuff”. In conclusion there is discontinuity between MIP and MPP as it relates to the role of previous instruction.

Resources for solving new math tasks

The most important resource is Zena herself. She believed that it was important for her to complete all her homework, because it will help in understanding so that she can respond positively in class. She did complain that her teacher was the cause of her difficulties with math.

She wanted her to teacher "to explain properly, so that everybody can understand" but in class she was "afraid" to ask questions because her teacher was too "harsh" and displayed unpredictable behavior because "you never know what his reaction will be, even though he seems in a good mood and you give a wrong answer, his mood just turns around on you". Consequently, Zena represented the best resource to solve these tasks.

During the clinical interview the key resource was the teacher and how he explained or taught procedures. Zena dependent completely on what she could remember from the teacher instructions. During the math story and individual interview the key verb was "to understand", while in the clinical interview the key verb was "I forgot". In conclusion there was discontinuity between MIP and MPP as it relates to resources for solving new math tasks.

Images of self as a learner of math

Zena projected the image of herself as a hardworking individual, doing "good" in mathematics. Since the workload was much more she saw herself as someone who is able to "pick up fast", meaning she learnt quickly, and she worked with her other class mates. She also emphasized that she "took note in class" meaning she listened to what the teacher was saying, and taking "good notes". All these were features of someone determined to succeed in math.

During the clinical interview she displayed a different set of feelings about her emotions, and used phrases like "I don't actually like graphs". The following interaction highlights this 'hate', when she was asked "Do you think that you will work on graphs", and she responded "No, because I dislike it now". In addition she also did not know or failed to recall definitions, meaning that she did not work hard enough on the definitions in geometry. When asked if she knew the definition and the properties of the parallelogram, she said an emphatic "no" twice. In conclusion there is discontinuity between MIP and MPP as related to her image of self as a math task solver where the emphasis is on the role of emotions in math identity and performance.

Summary

The individual mathematical identity profile established in chapter 4 and mathematical performance profile established in chapter 5 are compelling stories of the participants' experiences with mathematics. This chapter focused on the *continuity* or *discontinuity* between mathematical identity and mathematical performance for each participant.

The result of this analysis build on the notion of "identities-in-action" (Stentoft and Valero, 2009) as we relate math performance to math identity. In the cases of Yusuf, Avril, and Moses we see disconnects in this relationship in all six features: leading identity and importance of math; self confidence; attitude towards reasoning through the tasks; role of previous instruction; resources for solving new tasks; and image of self as a learner of math. All three participants' had constructed a mathematical identity that was based on a positive disposition toward mathematics; on developing a leading identity in relation to their imagined futures; on beliefs that they possess the qualities to be successful in mathematics; and on a perception that they have the correct work ethic to be successful. However, these self-held attributes were not visible during the mathematical tasks. Their mathematical performance was characterized by a struggle to apply their knowledge and skills to familiar tasks. They lacked the mathematical competence to perform at their grade-level. Their learning style based on using their ability to recall what the teacher did, proved to be the major challenge because they failed to recall procedures and only remembered fragments of procedures.

In the case of Katie we see greater continuity between her math identity and her performance. Her math identity was characterized by negative feelings toward mathematics. In addition her imagined future was not based on the need for mathematics, meaning that she did not have to work as hard as the others. This does not mean that she did not see the importance of math, but did not view it as important for her. By constructing this negative disposition there is no pressure being successful in math. Her math identity and math performance was in continuity in most of the six features

In the case of Zena we observed discontinuity between her math identity and math performance on all six features, but the difference was based on her belief that math was not important for her future-aspirations. Like Yusuf, Avril, and Moses her self-held beliefs about math, her perceptions of her own ability, and her disposition about math was characterized by positivity at all levels.

These results suggest that mathematical identities are fragile and undergo change when identity and learning intersect (Stentoft and Valero, 2009). It seems as if the gap in learning between math identity and math performance, must decrease for greater continuity to exist between the two. This finding supports the view of Sfard and Prusak (2005) who argued that learning math is the act of closing this gap between imagined identity and future identity.

CHAPTER 7

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

Summary

In this chapter, I return to the focus of my study: *How are the mathematical identities of low achieving South African eleventh graders related to their ability to solve mathematical tasks?* The primary goal of this dissertation was to shed light on the complex and messy construct of mathematical identities, especially among low-achieving students living within diverse communities.

In chapter 1 I discussed my rationale for this study as well as to introduce the construct of mathematical identity. In chapter 2 I explored and described the theoretical background by focusing on the bodies of research literature relevant to this study, and I introduced the conceptualization for my analytical framework. I relied on theories in the research literature in mathematics education and other domains, including psychology that emphasized mathematics identity, mathematical tasks, and mathematical learning.

In chapter 3 I outlined my methods that included my research design and analytical framework. Each of my five research participants wrote their math story emphasizing their mathematical journey from primary to high school and focusing on critical moments, turning points, high points and nadir moments. They also solved mathematical tasks drawn from the South African mathematics curriculum of grades 8, 9, and 10. I also conducted a series of semi-structured interviews that included an individual, a focus-group, and a clinical interview based on the mathematical tasks with my five participants aimed at exploring more about their mathematics story.

In chapter 4 I addressed my first research sub-question: What are the salient features of the students' mathematical identity? I described the students' mathematical journey based on their past and present mathematics experiences, and future-aspirations. In particular I used the notion of leading identity and cultural models to extract the salient features of each participant's mathematical identity profile. Two key findings in this regard include, taking the South African context as an important factor.

Firstly three of the five students selected engineering, dermatology and traffic-controller as their future career choices that were dependent on mathematics as a course of study. For low-achieving students' within low-income South African communities, these selections may start to debunk ideas that these 'types' of students do not dream big. They saw themselves as future university students, and this is a shift for students in those communities, because they had deep-seated intentions to engage in careers that required considerable mathematics knowledge. Mathematics has always been seen as a gate-keeper, and prevented students' from selecting high profile careers. Yusuf, Avril, and Moses based their leading identity on the "use-value" of mathematics which was the driving force behind their future aspirations or "imagined future" (Anderson, 2007).

The second factor is related to students' transitioning from primary to high school. This transition resulted in major shifts in their mathematical identities. All five participants had developed a positive disposition towards math during primary school, and all of them changed to a less positive disposition in high school, showing that identity construction is a fragile process. All the participants reported that they developed positive attitudes towards math in primary school because they received merit certificates of performance by doing well in math, and they liked the way they were taught. This suggests that primary school teachers tended to relate their teaching to the needs of their students. For example, they use manipulatives for visual learners. This transition from primary school to high school was characterized by: 1) a lack of recognition to students who are hard-working; 2) a lack of synergy between primary and high school math content, which according to the students' was too complex and required accommodations in the curriculum; 3) a lack of synergy between instructional practices, and pacing of the curriculum, where students complained that everything was so different; 4) the focus on individual learning practices as opposed to group work, which is the preferred student approach; and 5) the lack of empathy by the high school teacher for struggling students, which resulted in frustration.

In chapter 5 I addressed my second research sub-question: How do students' solve mathematics tasks? I described each student's ability to choose appropriate strategies, to produce graphical representations, to compute arithmetic calculations, and algebraic equations, to apply mathematical knowledge and skills; and to make sense of their results. The analyses provided an understanding of students' gaps or deficits in their mathematical knowledge and skills, as well as (mis)conceptions in their mathematical thinking and understanding. Using this information I constructed a mathematical performance profile for each participant. Three key findings emerged from this analysis.

Firstly, by reading the math story and listening to the participants' during the interview, I expected to see highly competent responses to the math tasks from Yusuf, Avril, Moses, and Zena, because they required a high grade percentage in mathematics to be allowed into their chosen fields of study. Their performance on the math tasks potentially is a point of crisis. The students' need to rethink or change their motives that may in turn lead to a new leading identity, because the results in their math performance profile, presents a challenge to their future aspirations. This finding that emphasizes the discontinuity between future aspirations and poor performance in mathematics concurs with Black et al's (2010) finding that the "motive for studying mathematics is now subordinated in pursuit of the wider goal" (p. 69). A limitation of this finding is based on the fact that I can only consider this as a "potential" point of crisis, because the participants may be able to improve their performance within the few months before writing their final examinations and still fulfill their dreams. At the same time this limitation also serves as a recommendation for future research where I may be able to interview the students at the end of their academic year to measure and evaluate any changes to their math identity profile and math performance profile.

Second, involved how all the participants were learning math. A key factor that seems to count for the participants' partial success on tasks, as well as for their approaching a task even though they think they may eventually fail to solve the task correctly, is that he or she remembered fragments of direct instruction from the teacher. The key element of this finding is that the participants made no reference to remembering rules that he or she studied or learned by themselves. They indicated that some of their math learning came from textbooks focused on what the teacher did not explain well. However, in the actual solution strategy taken to solve the task, they tended to fall back on what they remembered from the teacher's direct instruction in class on a similar procedural strategy. Students may have acquired some knowledge of formal rules, but when it comes to applying them in a task, each participant looked for what he or she could remember from the direct instruction on procedures in similar cases. They recalled fragments of knowledge rather than reasoning through the problem. Reliance on previous knowledge –mostly trying to remember previous instruction and models- played both in favor of a successful math performance and as a deterrent. In favor: when he or she could do analogies or remember previous instruction. Not in favor: when he or she could not remember.

All the participants believed that mathematics is very important and that "everybody should do math", and that the curriculum should make mathematics a mandatory subject. However, this acknowledgement is not enough for academic success because it is not about doing math, but it is about learning math and doing well, that is, they should be competent students of mathematics. The finding in this regard was the (mis)understanding that students had about 'doing' math, which was dominated by the belief in simply repeating/recalling what the teacher did, compared to learning math, which is based on both procedural and conceptual understanding. For the sake of brevity and to avoid the debate suggesting a choice between basic skills versus conceptual understanding, I argue for Wu's (1999) assertion that "skills and understanding are completely intertwined ... the precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding" (p. 1).

Third, the emotional attitude towards certain math concepts and questions played a greater role in the process of resolving math tasks than the participants thought. For example, Avril admitted to some dislikes (towards trigonometry for example), but she did not see that her dislikes were factors that would deter her from performing to the best of her abilities. I also observed in the math tasks that every major failure (where she got 0 points) was preceded by "I hate this", and in most cases a refusal to reason through the task. It is reasonable to assume that one's mathematics identity would have some impact on the performance - at least it would reflect in some aspects of how one approaches math tasks and learning. I noted that performance itself, even a one-time, limited in purpose, non-consequential setting like the mathematics tasks proposed to subjects in this research, had a visible impact on the participant's self-held math identity. Once confronted with tasks that they could not solve, their math identity changed. I observed that emotional attitudes towards certain math (like the hate of graphs) became ways to legitimize why the participant's did not even make the effort. It turns out that math performance profile and math identity profile do not have clear boundaries but keep influencing each other.

In chapter 6, I addressed the third research sub-question: How does students' ability to solve tasks relate to their mathematical identity? I describe the relationship between math identity and math performance by focusing on the discontinuities or continuities that exist between the two components. Two findings that emerged from this analysis include:

Firstly, all the participants' claimed they were "hard-working" and spent time doing their homework. However, there was discontinuity between their identity defined s hard-working and their ability to do math. The nature of this hard-work was not clarified, but it is connected with 'effective' studying skills and ability to learn mathematics. It could also be an issue about time they spend doing mathematics homework. However, I argue that it is concerned with the nature of their 'understanding' mathematics. By taking performance in the tasks into account it showed that the participants' lacked both procedural and conceptual understanding. They believed in understanding mathematics, emphasizing procedural knowledge because they believed that the teacher "should actually show the way" meaning that there's an element of rote-learning of procedures. Despite prioritizing rote learning, they were not procedurally fluent in many of the tasks. It seems as if they used the positive disposition towards mathematics to shield their true ability in mathematics.

Second, all the participants' were unanimous in their criticism of their math teacher and they blamed him for their shifting disposition towards mathematics, from a positive to a negative one. They believed that the relationship with their teacher formed an important component of their mathematical identities. Based on their performance in the tasks, coupled with their fear of the teacher, there exists continuity between their mathematical identity and ability. There is a direct relationship between their ability and the lack of opportunities to learn, because they had lost faith in the teacher's ability to help them. As they constructed their mathematical identity profile, they were consistent that learning mathematics was a challenge in this math class, because participation in class was limited. The lack of faith in the teacher, as well as the self held perception of a lack of participation in the classroom impacted negatively on their ability to learn math. They believed that their ability to perform in a math class characterized by class discrimination based on academic achievement was unacceptable and resulted in perpetual frustration. They claimed throughout the interviews that the teacher only focused on the 'fast lane' students and this impacted their learning of mathematics because it instilled negative attitudes about mathematics. They claimed that the 'slow' group "really don't understand" the mathematics and their performance on the math tasks served as evidence that 'understanding' how to solve the tasks was the core problem relating ability to a positive disposition toward math. The lack of effort in some of the math domains is evidence of this 'grade 11' attitude, which emphasized a shift in mathematical identity from a positive disposition to a negative disposition towards mathematics; but the cause of this shift is directed at the teacher, and the learning deficits were based on the fact that the students perceived the teacher as unapproachable and only concentrated on the 'fast lane' students.

Conclusions

Each participant's input was undoubtedly unique, but there were similarities in their narratives that became apparent when looking across the five cases. The aim of this small-scale qualitative study is not to make any generalizable conclusions regarding the relationship between mathematical identities and performance in math tasks of low-achieving students in high school, but to extract themes of meaning to interpret these narratives based on past, present and future experiences. I report these results to not only assist in making connections to my own teaching context, but to provide meanings that will aid the readers of this study and those interested in the study of mathematical identities to decide on the relevance of these results to their context. Consideration should be given to the dynamic nature of researching mathematical identities because these student narratives that evolved during this study will "continue to evolve for each participant" (Martin, 2000, p. 31). The data analyzed and results of this study are mere "in-the-moment" (Goldin et al., 2011) episodes of math identity profile and math performance profile and its consequences for mathematical learning.

It is not only important what we find about one or the other of the subjects emphasized in this study (discontinuities; continuities; invalidations; disruptions; and elements of math identity) that makes the students' perform well or badly, but what we get by organizing these findings under these math identity and math performance profiles. By examining these profiles you get to see how the mathematical identity profile and mathematical performance profile relate to each other and must be seen together and used to adjust each other. You get to see that every feature of one's mathematical identity has a "theoretical discourse" to it, meaning that this is the discourse that tells what the student "believe" that feature to be. In addition, you get to see "how it plays in practice" discourse (data made visible in the students' performance during the clinical interview). These will be the two "discourses" on mathematical identities, both of which are in fact held by the participant equally, meaning that the two profiles together should make the picture of mathematical identities, dare I say, complete.

However, one should take into the consideration that this is not a "static picture", and the actual mathematical identities may even change on the basis of some mathematical tasks that the student confronts at one moment in time. Furthermore, this is not necessarily a “coherent picture”, because with the inclusion of the leading identity the math identity profile is what the participant’s “desires” to be in the future, but in math performance profile the student has the opportunity to confront what that mathematical identity really is in practice and how it works for them in practice. This means that we see cases of a series of discontinuities, continuities of the math identity statements and sometimes even new features added to the initial principles stated in math identities.

The issue is not about incoherence per se because in itself is nothing unexpected, but in this regard is based on how the principle of incoherence may be used to research an individual’s mathematical identities and then how to use that to perhaps improve their learning. It is within this last comment that recommendations to future research can emerge, because one key objective of mathematics education is to find ways and mechanisms to improve the mathematics learning of all students, but especially so with low achieving students.

It is hoped that this study is seen as a method for the future, using mathematical identity as a factor of pedagogy, which, based on my reading of mathematics research and anecdotal observations in most curricular conceptions, the mathematical identity of our students’ is too often completely ignored, when in fact, you can see it vividly with the participants’ in this study. Furthermore, it played a major factor in what the participants’ actually did during their solutions to the mathematical tasks.

Implications

Implications for Teachers

Although the students all have their unique systems of beliefs, emotions and dispositions towards mathematics, there are still elements of their math identities that are similar. In this study all the students were passionate about their future aspirations, based on very similar motives. Teachers should "recognise the varied motives students have for studying mathematics" (Black et al., 2010, p. 2). This study concurred with Black et al (2010) that teachers' through an understanding of student motives and identity can gain useful information and will be able to offer and provide "tailored support" (p. 2) to promote ways of learning math. This study goes beyond the findings of Black et al, in that through analyzing the relationship between mathematical identity profile and mathematical performance profile, teachers are able not only to provide support, but can intervene early in high school by providing the correct guidance to students aiming to enter high profile careers, but who are not achieving at appropriate or expected levels of attainment.

This study can help teachers by guiding their decisions when assisting low achieving students in math. The observations and comments from the participating students suggested that they would benefit more from teachers who show empathy towards their needs and struggles, who put forth efforts to be inclusive in their classroom practice by not discriminating on the basis of academic excellence; who are prepared to sacrifice time after school hours to tutor or assist periodically based on individual needs, and to teach effectively so that all students can understand. Understanding the taught material should not be based on memorization of procedures, because this mode of learning proved to be the most problematic because all the participants had a common recall problem and their mathematical performance profile was characterized by the words “I forgot”. However, students must take responsibility for their own learning. They depended on the teacher to take the initiative to teach them after a period of absence from the class. If a student is absent from class it is their responsibility to find out what content was missed and to make the effort to learn this.

Furthermore, teachers are encouraged to create an environment in which they can develop positive working relationships with low achieving students, recognize their strengths and weaknesses, and build on their mathematical identities to foster positive dispositions towards mathematics. Learning with understanding can be enhanced through classroom interactions as students make conjectures and learn to evaluate their own thinking and in this way develop mathematical reasoning skills. This fact underlies teachers’ new role in providing experiences that help students make sense of mathematics. Students learn what they are given opportunities to learn and it is much more than simply receiving information. Kilpatrick (2001) talks about proficient teaching, which demands conceptual understanding of math, fluency in applying basic instructional routines, and strategic competence in planning effective instruction. This is crucial for students to learn math, and I propose Kilpatrick et al., (2001) model of mathematical proficiency to guide our professional development program. This can result in practices that will empower students and encourage engagement with mathematics as a counter narrative to Grootenboer and Jorgensen’s (2009) argument that there are mathematical practices at schools that create ways of working in mathematics that are not empowering our learners, but causing them to disengage in the discipline in mathematics. The results of this study contributes to the theoretical discourse that “learning and a sense of identity are inseparable: They are the same phenomenon” (Lave & Wenger, 1991, p. 115).

By focusing on students' theoretical construction and practical application of mathematical identities, teachers are afforded the opportunity to gain a better understanding of the "contested spaces" (the math classroom) in which these students interact and engage. This will provide teachers with an opportunity to deal with factors that affect the participation of students in the discipline of mathematics. I found that just listening to the passion in which the students described their mathematical journey and stories, have fore-grounded the importance of what affected the students' perceptions of themselves, their struggles in applying the knowledge that they acquired and their future aspirations.

As an intervention strategy it is important, in the South African context with very large class sizes, to strive towards affording every student with maximum opportunities to learn, but giving special consideration at the more individual level to ascertain factors that affect their positive/negative mathematical identities. As educators we should use strategies that foster the development and the identification aimed at assisting students with negative dispositions towards mathematics. Some students, like Katie, may require numerous opportunities to learn and experience success in math. These opportunities, may involve the primary school strategies of recognizing and providing incentives for seemingly minor accomplishments, but these may serve as great motivation to low-achieving students as noted in this study.

Implications for Policy Development

One recommendation for policy-makers is to develop policy that will consistently aid schools to consider effective ways to bridge the divide between teaching for understanding and learning math. The study showed that these low achieving students were struggling with what has been termed as the "math curse" that is, having the students to depend largely on their memory and ability to recall procedures that were taught to them devoid of understanding the math principles foundational to these procedures. Teachers must be re-trained and encouraged to ensure that their teaching methodologies and strategies are geared towards using activities that emphasizes effective learning of mathematics. Lappan and Phillips (2009) have argued for "retooling mathematics teachers" (p.2). A recommendation in this regard to implement Kilpatrick et al's (2001) strands for mathematical proficiency, and this can be achieved by using Cranfield's (2012) framework.

Policy-makers and school administrators must make every effort to reduce the "fear of failing" of students, by limiting the power of teachers to force students into a mathematical literacy course without giving every student the opportunity to prove that they are unable to learn pure mathematics. Cranfield (2012) reiterated that "mathematics provides a foundation for students to access different fields of study, and as a country we need to move beyond using mathematics as a gatekeeper to success" and that "through ML [math literacy] as a new subject the country is still at risk of disempowering students by allowing three years of ML learning that does not lead to opening doors, but serve as a dead-end for a number of career choices" (p. 224). In this regard I propose that students who choose pure mathematics in grade 10 must not have the option to switch to mathematical literacy during the FET phase. This will place the responsibility on both the teacher and student to work at achieving the best result for the student. There will not be an escape valve for the student and they will be forced to work harder at achieving the required percentage. The teacher will have to put in greater effort because 'poor results' will affect their rating. At this point in time policy supports the teacher by giving them the power to force students into math literacy. This study showed that there are low-achieving students who have dreams and future aspirations that involve learning mathematics, and are not benefiting from positive learning environments to support these ambitions instead school-policy allows teachers to be empowered with forcing these types of students to drop mathematics. Volmink (1990), many years ago argued that "mathematics is therefore a significant means of empowerment. To deny some students access to the process of mathematics is also to predetermine who in society will move ahead and who will stay behind, but at the same time mathematics as it is taught in schools has been disempowering" (p. 98). The findings of this study have reinforced the notion that very little has changed and unless we have policy intervention the status quo will remain. I propose that the mathematical literacy curriculum be abolished and that South Africa reintroduce the higher and standard grade mathematics curricula as before 2005. Aspects of the math literacy curriculum can be included in the higher and standard grade curricula. By streamlining the policy in this way, will equip all students with a functional knowledge of mathematics that includes an integration of mathematics and mathematical literacy into one subject.

Implications for Mathematics Education Research

Mathematical Identity is a current topic in math education research and gives an opportunity to reflect away from looking at the notion of under-achievement in mathematics as a deficit-laden phenomenon. This study was aimed at contributing knowledge about how mathematical identities are related to the ability to solve mathematical tasks of low-achieving eleventh graders. This study on mathematical identity offers another perspective on examining mathematical learning and achievement of high school students. It provides an opportunity to observe how students' views of their mathematical identities interact with their actual engagement in mathematical tasks. Through this interaction and engagement we able to analyze the “discontinuity or continuity” (Stentoft & Valero, 2009) between mathematical identity profile and mathematical performance profile, and report on the reasons behind any discontinuity between the two. The results of doing this study will provide some necessary information that will help and support intervention strategies to improve learning of math and to develop better students of mathematics. This is in line with Martin’s (2000) view that these shifts in mathematical identity will instill a sense of agency in which students can act and change their mathematical experiences despite the difficulties they experience with the constraints and opportunities to learn. When students are confronted with the usefulness and value of mathematics and its impact on their future careers, they seem to construct a more positive disposition towards mathematics, and vice versa.

This analysis incorporates previous research ideas that identity is multi-faceted based on the ways that students built up their mathematical identities through writing and talking about their math journey and self. But, through the introduction of the leading identity, I argued that there exists an identity that is more significant than others because different motives are brought into play or “they reveal a hierarchical organization of motives” (Black et al., 2010, p. 69). This study is distinct from Black et al (2010) in that participant’s performance on the math tasks raised points of crisis and disturbances to this subjective self portrait of mathematical identities.

This study adds to our learning more about the educational problems that includes low engagement and low participation rates of students, poor motivation and attitudes, but especially low achievement in mathematics. Some researchers have argued for using sociocultural approaches to research issues relating to equity that focus on how students learn math, how they value math, and the context in which they learn math (Martin, 2000; Nasir & Cobb, 2002; Grootenboer & Zevenbergen, 2008). From this sociocultural approach some researchers have branched into emphasizing how students construct their mathematical identities and how they locate themselves within particular communities of practice, where low achieving students can operate in a learning classroom that is aimed at motivating and encouraging students. Schools, teachers, and policy-makers could work together to create these environments so that all students realize that their future aspirations and opportunities in life is largely dependent on their current level of performance in mathematics.

Limitations and Directions for Future Research

This study drew on the experiences of only five students from a low socio-economic school and from a community characterized by poverty and high levels of unemployment, so the results cannot be widely generalized. However, this context is common for the majority of South African learners who have been marginalized for many years because of apartheid policies of the previous government. It is hoped that this study will be acknowledged for providing entry points into these schools and communities, and to allow some student voices to be heard and to see that students within these environments have dreams and aspirations and can have positive dispositions towards mathematics.

A second limitation has to do with the month long duration of the study. Although I collected all the data that was required by the design, a second iteration at the end of the year would have resulted in a deeper understanding of shifts in mathematical identities after another 8 months had passed. This study was limited to a snap-shot experience, a one-moment-in-time opportunity. A recommendation for future research will involve a similar methodological design with the same students, especially to see if their future aspirations have changed, and to monitor improvement in their mathematical performance profile by comparing results. In addition, another study can include some classroom observational data that will help authenticate some mathematical identity profile observations about the teacher, peers and classroom environment, interactions and engagement. This will assist the analysis in terms of influence of the classroom on mathematical identity profile.

Further recommendations for future research are based on extending the methodology and theoretical framework of this study with other sets of historically marginalized groups based on race, gender, and/or language. Research of this nature will be valuable in terms of analyzing the effects of a post-apartheid community in terms of diversity as it relates to race, gender, and language. A longitudinal study of black students traveling from townships into the city to schools outside of their community will make for a valuable research study on a number of levels. One such level would be to analyze mathematical identities of these black students learning mathematics from a white or colored teacher, and being taught in their second or third language, but not taught in their home language. How do these students learn math? Consideration can also be given the construction of other components of their identity, including, gender, race, language, in addition to their mathematical identities, but focusing on the interrelationship among these components and their ability to solve math tasks. This type of study will be able to analyze how mathematics is used as a filter or gateway for students to gain entry into a range of careers that their parents were denied entry into because of the issue of ‘race’ during apartheid days.

APPENDICES

Appendix A: Data Collection Instruments to Capture the Mathematics Story

Mathematics Story Assignment Name: ……………

Please answer the following questions. I am interested in learning more about your formative experiences in mathematics, the emotions and attitudes that you associate with mathematics, your ability to do mathematics, and your goals for yourself regarding mathematics. Provide examples and illustrations to support your response. Elaborate as much as you can.

1. **Critical Events**

- **Event 1, Peak Experience**: A *peak experience* is a high point in your story about mathematics in your life. It is a moment or episode in which you experienced extremely positive emotions, such as joy, excitement, great happiness, or even inner peace, after a mathematics experience. Describe what happened, where it happened, who was involved, what you did, what you were thinking and feeling, and what impact this experience had on you.
- **Event 2, Nadir (negative) Experience**: A *nadir* is a low point. Describe a moment or episode that you consider a low point in your experience of mathematics. As before, describe what happened, where it happened, who was involved, what you did, what you were thinking and feeling, and what impact this experience had on you.

2. **Turning Point**

Sometimes when people look back over their lives, they can identify critical points at which an important change occurred. Was there a point in your life when you experienced a turning point, that is, your understanding of or feelings about mathematics changed in some way? Describe a specific time when you experienced a turning point. If you think you have not experienced a turning point, describe an event that comes closer than any other to qualifying as a turning point.

3. **Other Important Scenes**

Describe at least two more scenes involving mathematics in your life that stand out as especially important or significant. Think in terms of scenes from your childhood, adolescence, and adulthood.

4**. Single Greatest Challenge**

What is the single greatest challenge that you have faced in mathematics? How have you faced, handled, or dealt with this challenge? Have other people assisted you in dealing with this challenge? How has the challenge had an impact on your experiences with mathematics?

5. **Alternative Futures for Your Mathematics Story**

- **Positive Future**: Describe what you would like to have happen in the future regarding your interactions with mathematics, including what goals and dreams you might accomplish or realize.
- **Negative Future**: Describe a highly undesirable future for yourself regarding your interactions with mathematics, one that you fear could happen to you but that you hope does not happen.

Appendix B: Semi-structured focus-group interview protocols

Focus group interview (semi-structured)

Thank you for your willingness to participate in this interview, I really appreciate your time. I am interested in learning more about how students think about mathematics, how they do mathematics, and if they enjoy mathematics. Your identity will be anonymous, but your responses will be very valuable for my research. So please let me know what you think. I will give each of you a number so that I can identify who is speaking.

1) Why do you think mathematics is important or not important for your future careers or future plans? Will math be useful in these plans?
2) Why did you choose to do that in the future? On what did you base your decision/ or who/what influenced your decision?
3) Are basic skills in mathematics important? Discuss and explain? Why did you choose mathematics and not mathematical literacy?
4) What are the important qualities to succeed in math? Why are these important? Will you study math after you leave high school?
5) Do all of you think you possess these qualities? Explain?
6) Should students who fail math (say obtain less than 50%) remain in that grade? Discuss and explain? Are grades important to you?
7) Why is the teacher important/not important in your success?
8) Do you like working in groups? Explain?
9) Do your peers play an important role in your success/un-success in math? If yes, please describe the role(s) your peers play.
10) What kinds of math tasks/exercises do you particularly like? Why? Describe these?
11) Would you say math is hard or challenging? Did you always think this or does this opinion change?
12) Is there anything else that I should know about your experience in engaging in math?

Appendix C: Individual interview protocols

Individual Interview Protocol **Name:…………**

I want you to know that your identity will be anonymous and that the contents of this interview will be very useful in my research, as I attempt to learn more about students' of mathematics. Please answer these questions as accurately and as honestly as you can. Also feel free to say that you'd prefer not answer a particular question.

Interview 1

A. Personal and academic history

1. Tell me a bit about yourself? Where are you from? About your family? Brothers and sisters? Where do you fit in, are you close in age?
2. Did your parents go to university/college/technical colleges? In what fields?
3. What do they do for work?
4. Do they encourage you to do your homework and to work hard?
5. What are your favorite things? Hobbies? Sport?
6. Tell me about your primary school mathematics achievement? Did you do well or not well in mathematics? Did you struggle with math? Explain?
7. Is mathematics an easy or difficult subject? Why?
8. What did you enjoy most about your mathematics classes in primary school? In high school?
9. What things in class help you succeed? What things distract you from succeeding?
10. Do you feel like you will ever use the math you are learning? How? When?
11. What is your career choice and why? How might math be used in that career?
12. Do you think you try hard enough in class? What are the reasons why you try or not try?
13. What can you do to help yourself achieve at a higher level.
14. Do you think you acquired the basic math skills for high school? Explain why this is/is not important.

Thank You

Interview 2:

A. Perspectives on mathematics Learning

1. What are the necessary qualities of an excellent mathematics learner?
2. Which of these qualities do you feel is your strongest? Please explain.
3. Which of the qualities do you feel is your weakest? Please explain.
4. Explain what it means to be a successful mathematics learner.
5. Describe what your favorite mathematics classroom will look like? Why do you like this?
6. Describe teaching approaches that you feel are effective? Why?

B. Perspectives on Mathematics

1. Why do we learn mathematics? What is the purpose of doing mathematics?
2. What do you like most about doing mathematics? Explain?
3. What mathematics do you enjoy? Be specific and explain why you find these engaging.
4. What is it that you do not like about mathematics? Why?

C. Perspectives on responsibilities

1. What do you feel are your responsibilities in learning/doing mathematics?
2. What do you feel are the responsibilities of a mathematics teacher?
3. What do you think are the responsibilities of the school?
4. What do you think are the responsibilities of your peers in the mathematics classroom?

D. Perspectives on doing mathematics

1. How do you respond to easy math problems?
2. How do you feel when you encounter difficult problems?
3. Why is it important to have the correct skills and knowledge to do the math task and why is it important to have the correct attitude?

Thank you

Appendix D: Mathematical Tasks

Doing mathematics section1

Instructions: 1) Solve each of the problems.
2) Show all your calculations.
3) Do not use a calculator unless instructed to do so.

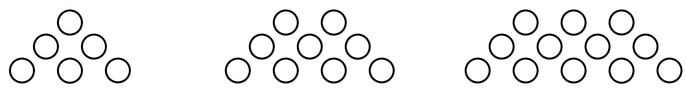
1) a) The length of a rectangle is 2 meters more than its breadth. If the breadth is x, give the length in terms of x in meters.
b) If the length is increased by 2 meters and the breadth is reduced by 1 meter the area remains unchanged. Determine the original length and breadth of the rectangle in meters.

2) R3200 is deposited and after 6 years there is R4900 in the account. Interest is compounded half yearly. Determine the yearly rate of interest, correct to two decimal digits. (you may use your calculator)

3) Simplify (Give answers with positive exponents): $\dfrac{2^{2n-1}.8^{n+1}.2}{4^{2n+1}}$

4) The average of Ed's ten test scores is 87. The teacher throws out the top and bottom scores, which are 55 and 95. What is the average of the remaining set of scores?

5) Look at the pattern below.



(1) (2) (3)

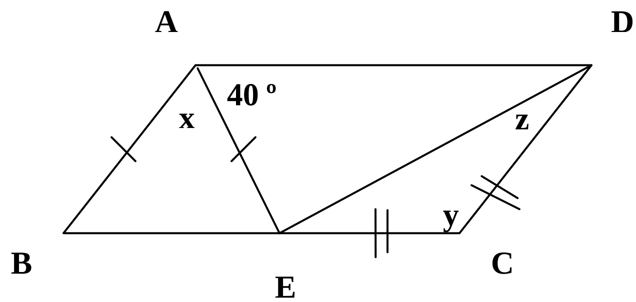
A. Draw the 4^{th} figure.
B. Draw the seventh figure.
C. Describe how you knew what the 7th figure would look like.
D. How many balls will there be in the 40th figure.

6) Find the point of intersection where the two graphs intersect:
$4x + y = - 5$ and $-3x + 4y = 18$.

SECTION 2

Instructions: 1) Solve each of the problems
2) Show all your calculations
3) Do not use a calculator unless instructed to do so.

7) ABCD is a parallelogram. AB = AE and EC = DC. Calculate x, y and z.



8) Solve for x in each of the following:

a) $4(x-5)=50$

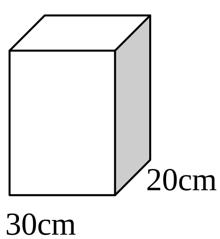
b) $x^2 - 5x - 50 = 0$

c) $\frac{x}{5} = \frac{3}{35} + \frac{x+1}{7}$

d) $2^{x} = 16^{x+1}$

e) $3(x-2)(x-3) \geq (3x+1)(x-7)$ Illustrate your answer on a number line.

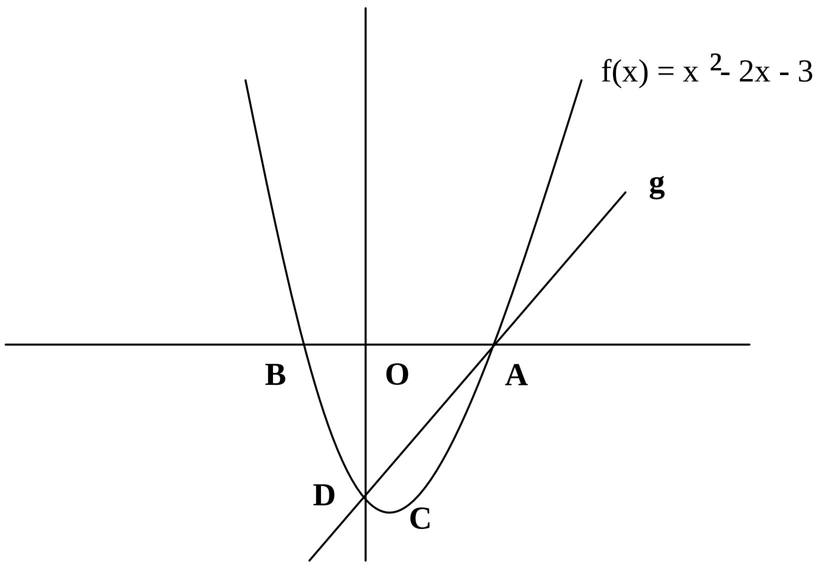
9) A mechanic uses 36000 cm ^{3} of oil to fill a tank which is in the shape of a rectangular prism. The base of the tank measures 30 cm by 20 cm.



a) Calculate the height of the tank

b) By how much will the volume of the tank increase if each of the length and breadth of the tank is doubled and the height remains the same

10) You are given that $f(x) = x^2 - 2x - 3$ and $g(x) = mx + c$.



a) Determine the coordinates of A, B, C, and D.
b) Determine the values of m and c.
c) For which values of x will $f(x) < g(x)$?

11) Test marks for a group of learners are as follows:

Mark out of 10	Number of boys	Number of girls
4	2	1
5	2	1
6	4	3
7	16	5
8	10	7
9	4	2
10	2	1

a) Use the information to draw a compound horizontal bar chart.
b) How many learners were in the group?
c) What is the arithmetic mean mark for the whole group?
d) What is the mode mark for boys?
e) What is the median mark for girls?
f) Only 1 girl had 10 out of 10; 2 boys had 10 out of 10. Does this mean that the boys are more clever than the girls? Motivate.

SOURCE: Examination Aid Mathematics Grade 10 Papers (not dated) with solutions (a collection of questions contained in previous examination papers from different Examination Boards). These Aids are important curricular support materials for teachers in preparing students for final examinations in grades 10 – 12.

Appendix E: Semi-structured clinical interview protocols

I will make a careful study of the learner's response to the tasks and select from the following semi-structured clinical interview questions to guide my questions. The student will receive a copy of his or her responses and use it as a frame of reference.

1. How do you feel about this problem?
2. How would you classify this problem? Easy or difficult? Explain?
3. Did you know what to do? What was going through your mind?
4. Explain your answer?
5. What mathematics do you need to solve this? Does this problem make you feel good about mathematics?
6. Why do you think this answer is correct? Are you familiar with this question/figure?
7. Do you spot an error that you made? What is it?
8. Explain your reasoning in this problem?
9. How did you go about answering this problem? What mathematics do you need here?
10. Why do you think this answer is correct? Do you know other ways to solve this?
11. Why did you choose this strategy?
12. What do you mean by that response?
13. Why can't you solve this problem?
14. Do you think you have the tools (mathematical knowledge) to solve these questions?
15. How do you really feel about this task?

All interviews will be conducted at the participating school.

Appendix F: Guardian Authorization Letter

1618 Spartan Village Apt G
Michigan State University
East Lansing
Michigan, 4882

February 14, 2012

Dear Parent/Guardian

Your child is invited to participate in a research study conducted by Corvell Cranfield, from Michigan State University's College of Education and College of Natural Sciences. I hope to learn more about the relationship between students' mathematical ability and the development of their mathematical identity in the context of solving mathematical tasks. Your child was selected as a possible participant in this study because he/she was randomly selected from a group that satisfied the criteria, in this case, the mathematical achievement category.

If you decide to allow your child to participate, the child will be required to write his or her mathematical story based on experiences in primary and high schools. These stories are important for understanding how your child feels about doing mathematics and how they developed their mathematical identity. Participants will also be asked to solve a number of mathematical tasks so that we can identify their mathematical strengths, their errors or misconceptions, their strategies, and their feelings when doing these types of tasks. They will also participate in a focus group interview, as well as in individual interviews. These activities will take place at the school, over a three to four week period, and will not interfere with their academic program at school. Some of these activities will be audio or videotaped only for the purpose to ensure that we capture all the essential data for this research.

There will be no risks and discomforts during the research. However, there will be some inconveniences in terms of time, since the activities will take place during breaks, before or after school, and free-time. Your child will be allowed the freedom of deciding the time suitable for these activities to take place. Every decision made during these activities will involve the principal of the school and permission will be asked so that the school is not impacted because of the activities. I am conducting this research so I can continue to improve my own teaching practices and provide insight to students, teachers, and parents about this complicated problem of understanding student mathematical identity and their ability to solve mathematical tasks.

However, I cannot guarantee that your child personally will receive any benefits from this research.

Any information that is obtained in connection with this study and that can be identified with your child will remain confidential and will be disclosed only with your permission or as required by law. Subject identities will be kept confidential in my written report and any work samples used will not include their names. Students will be referred to as a pseudo-name or a number in the report. Your child's participation is voluntary. Your decision whether or not to allow your child to participate will not affect you or your child's relationship with the school. If you decide to allow your child to participate, you and/or your child are free to withdraw your consent and discontinue participation at any time without penalty.

If you have any questions about the study, please feel free to contact Corvell Cranfield [email- cranfield@msu.edu: address- 9 Bromley Road, Gleemoor, Athlone, Cape Town: phone- to be announced]. If you have questions regarding your rights as a research subject, please contact the IRB (irb@msu.edu). You will be offered a copy of this form to keep. Your signature indicates that you have read and understand the information provided above, that you willingly agree to allow your child to participate, that you and/or your child may withdraw your consent at any time and discontinue participation without penalty, that you will receive a copy of this form, and that you are not waiving any legal claims.

Sincerely,

Corvell Cranfield

PhD student in the Division of Science and Mathematics education (Michigan State University)

Please complete the bottom portion of this letter and return it to me by (February 16, 2012).

Student's name ______________________________________ Date: ________________

Parent's signature ___________________________________ Date: ________________

My child can participate in this research project. YES ____ NO ____

My child is willing to participate. Yes_____ No______

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