THEORETICAL AND EXPERIMENTAL ANALYSIS OF AUTOMOBILE MUFFLER SYSTEMS

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ABSTRACT

THEORETICAL AND EXPERIMENTAL

ANALYSIS

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For many years, automobile mufflers have been designed, developed and ultimately approved or rejected on the basis of trial and error type approaches carried over from the era when accurate and reliable test instrumentation was not readily available.

The lengthy and sometimes wasteful development programs consisted of building a great number of handmade samples in the hope that ultimately the customer would be satisfied. With the advent of electronic instrumentation, and loudspeakers and microphones with favorable frequency response, research work in the field of acoustics became more promising. In 1954 a report was published by NACA, illustrating a possible theoretical approach to the muffler design problem. Building on the foundations laid in that report, this thesis relates the details of a research program on automobile mufflers. A reliable test set up was developed and a series of test mufflers was built. The test mufflers incorporated the basic circuit components frequently used in present day muffler designs. Analysis of these curcuits was done by comparing test results to analytically derived frequency response curves.

The scope of the investigation ranged from single chambers to the complex assemblies which constitute the modern mufflers.

The program revealed that by eliminating from the considerations some of the variables caused by the relatively unknown sound source; which is the automobile engine, good agreement can be found between theory and experiment on the true acoustical behavior of mufflers.

Reverse-flow features, cross-bleed chambers, and the use of louvers in mufflers were investigated. Systems of equations were developed for complete muffler systems. By the use of computers theoretical attenuation curves could be arrived at. Comparisons were made between analytical and experimental attenuation curves. From these comparisons it can be concluded that the test set up is adequate for the further exploration of muffler acoustics, and that it is possible to set up systems of equations which will predict muffler performance with substantial accuracy.

Recommendations have been made for the expansion of the test program to on-the-engine testing. It can be foreseen that muffler development will soon be able to benefit from the tools now readily available in laboratories and computer centers.

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ANALYSIS

OF AUTOMOBILE MUFFLER SYSTEMS

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PREFACE AND ACKNOWLEDGMENTS

The combination of strictly technical development problems and the need for passing an exclusively subjective final approval test makes muffler design work a rather complex art. The additional complications introduced by annual changes in system layout and/or characteristics of the sound source have resulted in lengthy and expensive programs of development. A concentrated effort in the direction of advancing the science of sound muffling is necessary. The program, which is dealt with in this thesis, illustrates one aspect of this effort.

In the capacity of chief development engineer of Hayes Industries, Inc., of Jackson, Michigan, I had the opportunity to be exposed to the complexities of the problem and to the unexplored territory ahead. The experience gained during the execution of the daily assignments was a very tangible help in the preparation of this thesis.

Further, the continued interest of top management, especially Mr. G. B. Vass, President, has been greatly appreciated. The very useful suggestions made by Dr. F. S. Tse, and Dr. G. Martin, and the ideal cooperation of the people at the Michigan State University Computer Laboratory, have greatly contributed to the final success of this effort.

To my wife, Sandra, however, I must attribute the credit for the finalization of this program. Her constant interest and concern have provided the stimulus required to complete this thesis and to chart the course for our future programs.

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INTRODUCTION

In a report published in 1954, engineers of the Langley Aeronautical Laboratory of NACA summarized theory and experimental approaches to engine-exhaust muffler performance and design.

Prior to that report very little information had been published on the specific subject of noise filtration in engine exhaust systems. In 1959, a series of papers on the subject of mufflers were presented at the annual passenger car convention of the SAE. These papers dealt with specific considerations of muffler design without generalizing theory for future reference.

The NACA report prompts us to recognize several important conclusions and/or limitations:

-Laboratory test methods were developed for muffler testing under "true acoustical" conditions, eliminating some important variables of the engine test conditions.

-Some correlation between "cold" and "hot" tests seemed possible.

-Substantial agreement was found between cold test results and attenuation values derived analytically.

-Mathematical analysis of common simple muffler chambers was achieved.

-Only relatively simple sound filters were considered.

-Only straight-through type silencers which permit through-flow of sound without turning around were considered.

-The physical dimensions of the mufflers used in the NACA study were such that they far exceeded dimensions normally encountered in automobile mufflers.

In an effort to determine to what extent the NACA experiments and theory had contributed to the understanding of the operation of automobile mufflers the author set up the following program:

-Develop an infinite tailpipe capable of retaining its basic characteristics after repeated exposure to sound levels of up to 160 DB in a frequency range from 50 to 1000 Cps. This infinite tailpipe should have a diameter of not more than 2 inches.

-Design an exhaust pipe (muffler inlet) of sufficient length to study standing waves in sound fields of 50 Cps. This exhaust pipe should further enable us to measure sound levels inside the pipe at random locations.

-Construct a sound generating system of high quality and intensity as the sound source for the test program.

-Design and build a series of mufflers incorporating parts and materials normally used in muffler design. This series of mufflers would become progressively more complicated as the units incorporate more and more features of the commercial mufflers.

-Study, experimentally and theoretically the behavior of the various mufflers, limiting the study to "cold testing."

-Evaluate test results against theoretical analysis.

-Consider variables in hot systems not accounted for in this program.

As a result of this program discuss the merits of cold testing in muffler research and suggest a program for combination of hot and cold testing.

It is this program which forms the basis for this thesis. The various formulae required to follow the mathematical analyses are derived in Chapter I. Chapter II is devoted to the test system and instrumentation. Armed with the mathematical background and equipped with satisfactory test apparatus, a basic test approach is developed, and the sequence of designs to be tested is discussed.

Actual calculations and test results are covered in a chapter where the contribution from computers is stressed. Finally, a correlation of results allows us to draw our conclusions and make recommendations for future programs. It should be recognized that no absolute and final answers could be obtained in this study. The objective was primarily that of gathering the theoretical background, the laboratory setup and the basic approach for a continuation of a research program. The author believes the results of this analytical and experimental investigation substantially increase the amount of technical data published on muffler research. All features of commercial mufflers were studied and their behavior analytically predicted with good agreement between computed and actual performance. It is believed that this analysis will contribute substantially to the goal of introducing a higher degree of predictability in muffler development.

Ultimate reductions in cost of development programs and of product designs are sure to follow.

CHAPTER I

ACOUSTICAL VARIABLES AND RELATIONSHIPS FOR USE IN SOUND TRANSMISSION LINES

The following definitions will be important in the interpretation and understanding of the acoustics theory as treated in this manuscript. Abbreviations and units of measurement are given in an effort to consolidate this information in one section.

1. Definitions and Symbols

Acoustics: The Science of Sound

Bel: A fundamental division of a logarithmic scale for expressing the ratio of two amounts of power. The number of bels denoting such a ratio is the logarithm to the base 10 of this ratio.

Decibel: (db) One tenth of a bel., with W_1 and W_2 designating two amounts of power, and n the number of decibels denoting their ratio $n(\text{in db}) = 10 \log_{10} \frac{W_1}{W_2}$

When impedances are such that ratios of pressures are the square roots of the corresponding power ratios, the number of decibels by which the corresponding powers differ is expressed by $n = 20 \log \frac{p_1}{p_2}$ db

Acoustic Impedance: See Chapter I Section 3

Particle Velocity: In a sound wave is the instantaneous velocity of a given infinitesimal part of the medium, relative to the medium as a whole, due to the passage of a sound wave. Units are cm/sec.

Loudness: That aspect of auditory sensation in terms of which sounds may be ordered on a scale running form "soft" to "loud". Loudness is chiefly a function of the intensity of a sound, but it is also dependent on the frequency and the composition. The unit is the sone.

Pitch: That aspect of auditory sensation in terms of which sounds may be ordered in a scale running from "low" to "high". Pitch is chiefly a function of the frequency of a sound, but it is also dependent on the intensity and composition. The unit is the mel.

Sound Pressure Level: Is applied to data taken by a sound pressure meter with a "flat" response. The reference pressure is 0.0002 dyne/cm^2.

Standing Waves: Constitute the wave system resulting from the interference of progressive waves of the same frequency and kind. They are characterized by the existence of nodes or partial nodes in the interference pattern. In order to obtain standing waves the interfering waves must have components traveling in opposite directions.

The following symbols are of importance in the development and discussion of the general wave equation.

x, y, z	coordinates of a particle of the medium.
ξ, η, ζ	component particle displacements along the x, y, and z axes, respectively.
u, v, w	component particle velocities, i.e., $u = \frac{\delta \xi}{\delta t}$ $v = \frac{\delta \eta}{\delta t}$ $w = \frac{\delta \zeta}{\delta t}$
ρ'	instantaneous density at any point.
ρ	constant mean density at any point.
s	condensation at any point, as defined by $s = \frac{\rho' - \rho}{\rho}$
p'	instantaneous pressure at any point.
p_o	constant mean pressure at any point
p	excess pressure or acoustic pressure at any point, as defined by $p = p' - p_o$
ϕ	velocity potential
c	velocity of propagation of the wave

2. Acoustic Plane Waves

Acoustic waves are disturbances propagated in a compressible fluid. The frequency of these waves is in the so-called audible range, roughly between twenty cycles per second and twenty thousand cycles per second. The propagation of acoustic waves in a fluid medium is generally three dimensional. The general wave equation, applicable to both liquids and gases is of the form

$$\frac{\delta^2 \phi}{\delta t^2} = c^2 \nabla^2 \phi$$

The solution of this equation represents a propagation of the velocity potential ϕ, the velocity of propagation being c.

The velocity potential $\phi = \phi(x,y,z,t)$ and[1]

$$u = \frac{\delta \phi}{\delta x} \quad v = \frac{\delta \phi}{\delta y} \quad w = \frac{\delta \phi}{\delta z}$$

$$p = -\rho \frac{\delta \phi}{\delta t} \qquad s = \frac{p}{\rho c^2}$$

Of particular importance in the study of transmission lines is the case in which the propagation of the wave is bounded in two dimensions. Such a wave is called a plane wave. When a plane acoustic wave travels in the x-direction, all particle motions are assumed to be in this direction.

Consequently

$$\frac{\delta \phi}{\delta y} = v = o \qquad \text{and} \qquad \frac{\delta \phi}{\delta z} = w = o$$

because the y and z components of velocity (v and w respectively) are zero.

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[1]Numbers refer to references in the Bibliography.

The velocity potential ϕ now becomes a function of x and t only.

The general wave equation then becomes

$$\frac{\delta^2 \phi}{\delta t^2} = c^2 \frac{\delta^2 \phi}{\delta x^2}$$

This equation has the general solution

$$\phi = f_1(ct - x) + f_2(ct + x)$$

where c is the velocity of propagation.

Expressing the motion as a function of harmonic waves with propagation in the positive and in the negative x direction,

$$\phi = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)} \qquad (1)$$

where A and B are complex amplitudes of waves traveling in the positive and in the negative x direction respectively, and where $\frac{\omega}{k}$ = propagation velocity.

Assume

$$\phi_+ = Ae^{j(\omega t - kx)}$$

and

$$\phi_- = Be^{j(\omega t + kx)}$$

then[1]

$$p = \frac{\rho \delta \phi}{\delta t} = -j\omega\rho(\phi_+ + \phi_-) \qquad (2)$$

$$s = \frac{p}{\rho c^2} = -j \frac{k}{c} (\phi_+ + \phi_-) \qquad (3)$$

$$u = \frac{\delta \phi}{\delta x} = -jk(\phi_+ - \phi_-) \qquad (4)$$

From these complex relationships we find that--acoustic pressure and condensation lag velocity potential by 90°. Also,

$$\xi = \int u dt = \frac{u}{j\omega} = -\frac{1}{c} (\phi_+ - \phi_-) \qquad (5)$$

$$\frac{\delta \xi}{\delta x} = -\frac{1}{c} (-jk\phi_+ - jk\phi_-) = \frac{jk}{c} (\phi_+ + \phi_-) \qquad (6)$$

and

$$s = \frac{-\delta \xi}{\delta x}$$

which indicates that a rare faction or a negative condensation is present in the medium whenever

$$\frac{\delta \xi}{\delta x}$$

is positive i.e., whenever particle displacement is increasing as x increases. Particle velocity leads particle displacement by 90°. For waves traveling in the positive x, the particle velocity lags the velocity potential by 90°. For waves traveling in the negative x direction the particle velocity leads the velocity potential by 90°.

The actual equations giving the various acoustic variables are the real parts of equations (1) through (6). Assuming the case where A and B are real constants A and B we find

velocity potential- $\phi = A \cos(\omega t - kx) + B \cos(\omega t + kx)$ (1a)

pressure (acoustic)- $p = \rho\omega A \sin(\omega t - kx) + \rho\omega B \sin(\omega t + kx)$ (2a)

condensation- $s = \frac{k}{c} A \sin(\omega t - kx) + \frac{k}{c} B \sin(\omega t + kx)$ (3a)

particle velocity- $u = k A \sin(\omega t - kx) - k B \sin(\omega t + kx)$ (4a)

particle displacement- $\xi = \frac{-A}{c} \cos(\omega t - kx) + \frac{B}{c} \cos(\omega t + kx)$ (5a)

strain- $\frac{\delta \xi}{\delta x} = \frac{-k}{c} A \sin(\omega t - kx) - \frac{k}{c} B \sin(\omega t + kx)$ (6a)

3. Acoustic Impedance

Important relationships between acoustical variables can be expressed using the concept of impedance.

A. Specific Acoustic Impedance.

The ratio of acoustic pressure in a medium to the associated particle velocity is defined as the specific acoustic impedance of the medium. For plane wave

$$z_+ = \frac{p+}{u_+} = \frac{-j\omega\rho\phi+}{-jk\phi_+} = \rho c$$

$$z_- = \frac{p-}{u_-} = \frac{-j\omega\rho\phi_-}{jk\phi_-} = -\rho c$$

This impedance is also called the characteristic impedance of the medium.

B. Acoustic Impedance.

The use of electrical analogues has resulted in the definition of acoustical variables facilitating the interpretation of the analogy.

The Acoustical analogue of a voltage across part of an electric circuit would be the pressure difference across an acoustic element.

Analogous of electric current at a point of the electric circuit would be the acoustic flux or volume velocity of the fluid.

If we assume particle displacement through a surface to be normal to that surface and the same at all points, we find the volume displacement

$$X = \xi S$$

where S is the cross section of the element. The velocity then

$$\frac{\delta X}{\delta t} = \frac{\delta \xi}{\delta t} S$$

The acoustic impedance Z of a fluid medium acting on or through a surface of given area is the complex quotient of the acoustic pressure at the surface divided by the volume velocity at the surface.

$$Z = \frac{p}{dX/dt}$$

In general, the acoustic impedance is equal to the mechanical impedance divided by the square of the area of the surface being considered. The acoustic ohm, unit of acoustic impedance, has dimensions of

$$\frac{\text{Pressure}}{\text{Volume Velocity}} = \frac{\text{Dynes}/\text{Cm}^2}{\text{Cm}^3/\text{Sec}} = \frac{\text{g (mass)}}{\text{Cm}^4\ \text{Sec}}$$

The acoustic resistance R is defined as the real component of the acoustic impedance. It is associated with the dissipation of energy. Acoustic Reactance X of a medium is the imaginary component of the acoustic impedance; it results from the effective mass and stiffness of the medium.

The acoustic impedance is related to the specific acoustic impedance at a surface by $Z = z/S$.

The acoustic compliance C of an element is defined as the volume displacement X that is produced by the application of unit pressure. The units of acoustic compliance are $\text{Cm}^4\ \text{Sec}^2\ /\ \text{g}$.

The acoustic inertance M of an element is defined as $M = m/S^2$ where m is the effective mass of the element. Acoustic inertance has dimensions of $\text{g}\ /\ \text{Cm}^4$.

4. Analysis of Acoustic Transmission Lines

Only the considerations which are of importance in the specific subjects handled in this thesis are summarized. In order to properly interpret the results of our test programs, it will be necessary that we understand such phenomena as reflection in pipes, resonance in pipes, the effect of changes in pipe cross section, and the basic theory of a side branch. These subjects are now dealt with in preparation of the considerations of the specific problems in our test program.

A. Reflection in Pipes

Assume that at some point x along a pipe the acoustic impedance changes from its characteristic value of

$$\frac{\rho c}{S} \text{ to } Z_x$$

where Z_x may be either real or complex. If an initial wave traveling in the positive x direction and represented by

$$p_i = Ae^{j(\omega t - kx)}$$

is incident at this point, a reflected wave

$$p_r = Be^{j(\omega t + kx)}$$

will in general be produced. The volume velocities of fluid flow corresponding to these two waves are given respectively by

$$\frac{dX_i}{dt} = \frac{p_i}{\rho c/S} \quad \text{and} \quad \frac{dX_r}{dt} = -\frac{p_r}{\rho c/S}$$

When both waves are present, the varying phase relationship between them causes the acoustic impedance to vary from point to point along the pipe, rather than remaining the same at all points as is true when only the incident wave is present. A general expression for acoustic impedance which includes the reflected wave is

$$Z = \frac{p_i + p_r}{\frac{dX_i}{dt} + \frac{dX_r}{dr}} = \frac{\rho c}{s} \frac{p_i + p_r}{p_i - p_r} \tag{7}$$

or

$$Z = \frac{\rho c}{s} \frac{Ae^{-jkx} + Be^{jkx}}{Ae^{-jkx} - Be^{jkx}} \tag{8}$$

At the cross section of the pipe where the impedance changes, the usual conditions of continuity of pressure and volume velocity may be replaced by a condition of continuity of their ratio, that is continuity of acoustic impedance, hence the phases and amplitudes of the incident and reflected waves must be so related as to cause Z in equation (8) to be equal to Z_x.

B. Resonance in Pipes

Assume that the fluid in a pipe of length l and area $s = \pi a^2$ is driven by a vibrating piston located at the left-hand end where x = o and that the pipe is terminated in an acoustic impedance Z_1 at the right-hand end where x = l. Application of equation (8) at x = l results in

$$Z_1 = \frac{\rho c}{s} \frac{Ae^{-jkl} + Be^{jkl}}{Ae^{-jkl} - Be^{jkl}} \tag{9}$$

This equation in effect determines the reflected pressure amplitude B in terms of the incident amplitude A. The input impedance Z_o at x = o is correspondingly given by

$$Z_o = \frac{\rho c}{s} \frac{A + B}{A - B} \tag{10}$$

Equations (9) and (10) may be combined to eliminate the complex pressure amplitudes A and B and then simplified to give

$$Z_o = \frac{\rho c}{s} \frac{Z_1 + j\frac{\rho c}{s}\tan kl}{\frac{\rho c}{s} + jZ_1 \tan kl} \tag{11}$$

It is apparent that the input impedance depends not only on the terminating impedance Z_1 but also on the length of the pipe l and the wave length constant k.

The resonant frequency of such a pipe may be defined as that at which the reactive component of the input impedance vanishes. At this frequency the input impedance is a minimum and the power radiated out of an open-ended tube is a maximum.

Assuming

$$Z_1 = \left(\frac{\rho c}{s}\right)(\alpha + j\beta)$$

then equation (11) may be rewritten as

$$Z_o = \frac{\rho c}{s} \; \frac{\alpha + j\,(\tan lk + \beta)}{(1 - \beta \tan kl) + j\alpha \tan kl} \qquad (12)$$

Application of the condition that $X_o = o$ gives

$$\beta \tan^2 kl + (\beta^2 + \alpha^2 - 1) \tan lk - \beta = o \qquad (13)$$

Two special cases are of particular interest. One is when the pipe is terminated at $x = l$ in an infinite flange. Both α and β are small as compared to unity. In that case equation (13) is approximated by $\tan kl = -\beta$ (at low frequencies).

For this condition β has been determined to be $\beta = \frac{8ka}{3\pi}$ *

where a is the pipe radius. We then have

$$\tan kl = -\, 8ka \,/\, 3\pi$$

satisfied by

$$\tan (n\pi - kl) = \frac{8ka}{3\pi}$$

where n is an integer. Hence

$$n\pi - kl = \frac{8ka}{3\pi}$$

and

$$f = \frac{nc}{2(1 + \frac{8a}{3\pi})}$$

or for $n = 1$ the fundamental resonant frequency is

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*Radiation reactance for flanged open pipe.

$$f_1 = \frac{c}{2(l + \frac{8a}{3\pi})}$$

This indicates that the "effective length" of the pipe is $l + \frac{8a}{3\pi}$ rather than l. Experiments have shown the end correction to be .82a. For unflanged pipes, this end correction is .6a.

The other special case is that where the pipe is closed by a rigid cap. In that condition $Z_1 = \infty$ and

$$Z_o = \frac{\rho c}{s} \frac{1}{j \tan kl} = -j \frac{\rho c}{s} \cot kl.$$

The reactance is zero when cot kl = o or for

$$kl = (2n - 1) \frac{\pi}{2}$$

n equals any integer and

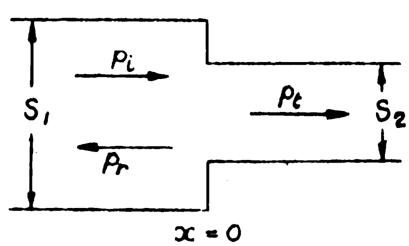
$$f = \frac{2n - 1}{4} \cdot \frac{c}{l}$$

For

$$n = 1 \quad f_r = \frac{c}{4l} .$$

C. Transmission from one pipe to another.

FIGURE 1



Let S_1 and S_2 be the cross sectional areas of two pipes, conductors of an acoustic wave. The incident pressure wave is p_i the reflected wave p_r and the transmitted wave p_t. If

$$p_i = A_1 e^{j(\omega t - kx)} \tag{14}$$

then we would have a reflected wave

$$p_r = B_1 e^{j(\omega t + kx)} \tag{15}$$

and a transmitted wave

$$p_t = A_2 e^{j(\omega t - kx)} \tag{16}$$

Since we are not considering any changes in the medium or its characteristic impedance, the values of the wave length constant k on both sides of the junction are identical. Let the junction be at $x = o$. We have a condition of continuity of pressure which gives

$$A_1 + B_1 = A_2 \tag{17}$$

In as much as there is also continuity of flow (volume flow) we can establish the following condition:

The total volume flow at any point in pipe 1 is

$$S_1 \, (u_i + u_r)$$

That in pipe 2 is

$$S_2 \, (u_t).$$

Replacing $u_i = \frac{p_i}{\rho c}$ and $u_r = - \frac{p_r}{\rho c}$ and $u_t = \frac{p_t}{\rho c}$ we have at $x = o$ to insure continuity of flow

$$S_1 \frac{(p_i - p_r)}{\rho c} = S_2 \frac{p_t}{\rho c}$$

or for $x = o$

$$S_1 \, (A_1 - B_1) = S_2 \, A_2$$

and

$$A_1 - B_1 = \frac{S_2}{S_1} \, A_2$$

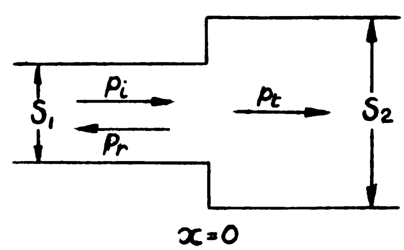
Similarly for a wave traveling from the smaller cross section to the larger one.

$$\frac{S_1\,(p_i - p_r)}{\rho c} = S_2\,\frac{p_t}{\rho c}$$

and

$$A_1 - B_1 = \frac{S_2}{S_1}\,A_2$$

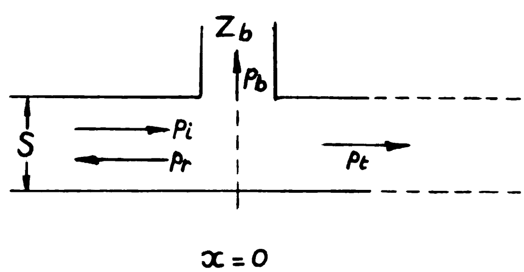
FIGURE 2



D. Theory of the Side Branch.

Consider a pipe of uniform cross section S to which is attached a side branch of input acoustic impedance. Z_b The pressure of such a branch causes the acoustic impedance at the junction to differ from the characteristic value $\frac{\rho c}{S}$. Consequently a reflected wave is produced. It is further possible that a portion of the incident acoustic energy is transmitted into and dissipated in the branch.

FIGURE 3



Let

$$p_i = A_1 e^{j(\omega t - kx)}$$

Then in general

$$p_r = B_1 e^{j(\omega t + kx)}$$

and

$$p_t = A_2 e^{(j\omega t - kx)}$$

Let x = o at the junction then

$$p_i = A_1 e^{j\omega t} \quad p_r = B_e^{j\omega t} \quad p_t = A_2 e^{j\omega t} .$$

The pressure at the branch entrance may be similarly shown

$$p_b = A_b e^{j\omega t}$$

Assuming pipe cross-sectional dimensions to be small in comparison with the wave length of the sound we have continuity of pressure and volume flow. Hence

$$p_i + p_r = p_t = p_b \tag{18}$$

Let the volume velocities be

$$\frac{dX_i}{dt} = \frac{p_i}{\rho c/s} \qquad \frac{dX_r}{dt} = - \frac{p_r}{\rho c/s}$$

$$\frac{dX_t}{dt} = \frac{p_+}{\rho c/s} \qquad \frac{dX_b}{dt} = \frac{p_b}{Z_b}$$

Then continuity of volume velocity means that

$$\frac{dX_i}{dt} + \frac{dX_r}{dt} = \frac{dX_t}{dt} + \frac{dX_b}{dt} \tag{19}$$

Dividing (19) by (18) we get

$$\frac{dX_i/dt + dX_r/dt}{p_i + p_r} = \frac{dX_t/dt}{p_i} + \frac{dX_b/dt}{p_b}$$

which by definition of acoustic impedance is the same as

$$\frac{1}{Z} = \frac{1}{Z_t} + \frac{1}{Z_b}$$

where

$$Z = \frac{\rho c}{s} \; \frac{A_1 + B_1}{A_1 - B_1}$$

(See equation 10). And

$$Z_t = \frac{\rho c}{s}$$

It should be noted that if $Z_b = \infty$ which corresponds to no branch, all incoming acoustic power is transmitted past $x = o$. If however $Z_{b\to o}$ which implies both $R_{b\to o}$ and $X_{b\to o}$ then we have the equivalent of $Z = o$ and no power is transmitted.* This does not mean that the branch Absorbs all energy. In fact for $R_b = o$ it absorbs no energy at all but reflects 100 percent of the incoming energy back to the source.

Also we recognize that if R_b has a finite value, some acoustic energy is dissipated in the branch; and some is transmitted.

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*We know that

$$Z = \frac{\rho c}{s} \; \frac{A_1 + B_1}{A_1 - B_1}$$

for

$$Z = 0 \quad A_1 = -B_1$$

and the magnitude of the reflected wave is equal to that of the incident wave.

CHAPTER II

TEST INSTRUMENTATION AND EQUIPMENT

The basic theory required to follow the reasoning of the following chapters was covered in Chapter I. Several details in connection with the test instrumentation are of very substantial importance. In fact, part of the original objective of this program was to develop a workable test setup, suitable for further research in this field. Consequently this chapter will be entirely devoted to details of instrumentation and construction of equipment.

1. Basic Test Circuit

The basic test circuit consists of the following major components:

a. Sound Source

b. Muffler Inlet (Suitable for Measuring Standing Waves)

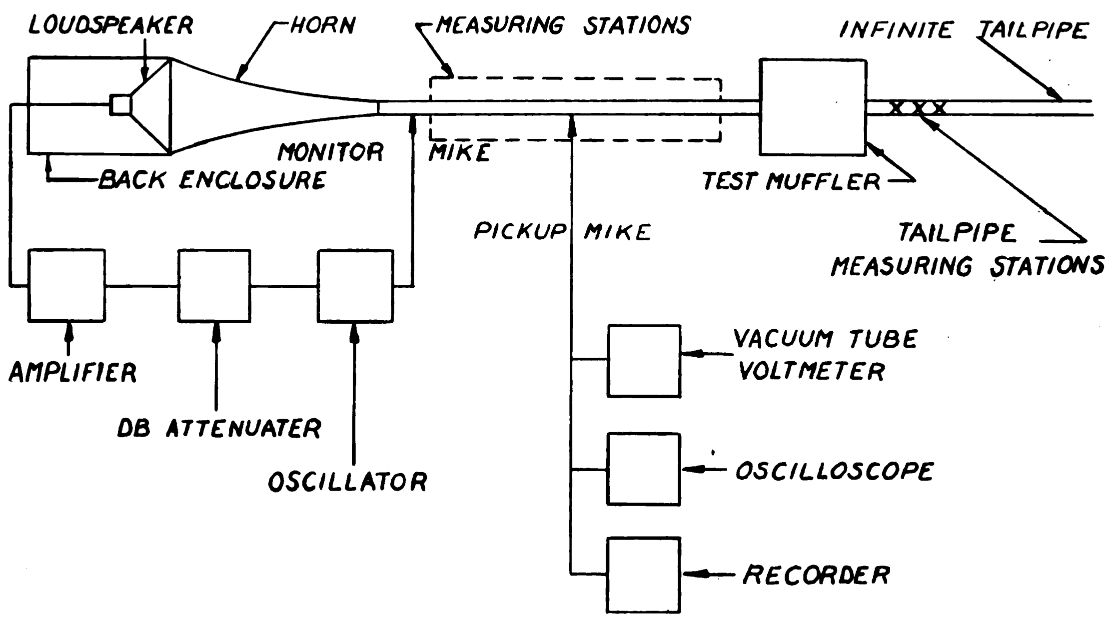
c. Test Muffler

d. Termination

e. Measuring Circuit

The schematic diagram is shown in Figure 4. The sound source consists of the power supply, the oscillator, db attenuator, the amplifier, and the loud speaker with exponential horn, back enclosure and monitoring circuit. The exponential horn was made to reduce the cross section of the wave front from 15" loud speaker diameter, to a muffler inlet of 1-3/4" I.D.

FIGURE 4



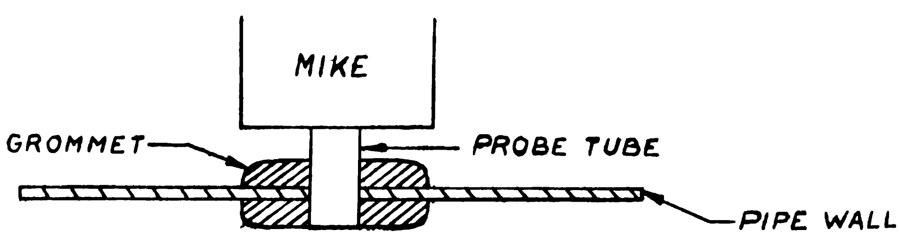
It was made of fiberglas with perfectly smooth inside surfaces. An average of four inches of concrete was poured around the fiberglas horn to reduce the possibility of setting up vibrations as a result of high intensity sound. A concrete back enclosure was also built to as much as possible avoid variation of the mechanical impedance of the source as a result of anything other than the conditions introduced by the load. (test muffler)

A monitor microphone was used to automatically reduce the oscillator output (through a compressor circuit) whenever sound levels in the system exceeded 160 db. This way adequate protection was afforded for the loudspeaker. The loudspeaker itself was a 15" diameter, high-intensity, low frequency driver built by Transducers, Inc. The microphones were high intensity condenser microphones built by Altec Lansing. The db attenuator was used to enable us to turn down the output power by predetermined steps and to subsequently switch back to the original output level with perfect accuracy.

A high fidelity McIntosh amplifier was used in the input circuit. The measuring circuit consisted of a vacuum tube volt meter (with DB scale), an oscilloscope to examine the degree of distortion and a recorder, automatically recording the sound levels as the microphone was moved from one station to another.

Because of the small pipe diameters, it became impossible to have the microphone move inside the pipe without having an influence on the propagated wave. Instead, a substantial number of holes were drilled at random locations in the exhaust pipe. Small rubber grommets were inserted in all of those holes and closely fitting plugs closed the holes off entirely, whenever they were not in use. Probe tube microphones inserted through the hole in the grommet were used to pick up sound levels inside the pipe. See Figure 5

FIGURE 5



It was assumed that a sufficient number of sound readings at the randomly located measuring stations would produce at least one reading properly indicating the maximum sound level in the pipe. Upon completion of a great number of experiments this assumption is considered to be fully justified. A detail of location of measuring stations is shown in Figure 6.

With no reflection present in the tailpipe (See Section 2 on infinite tailpipe) one reading in back of the muffler is adequate. Three measuring stations were provided to continuously verify the proper operation of the infinite tailpipe.

Heavy walled tubing was used for both tailpipe and exhaust pipe (muffler inlet). This would reduce the amount of transmission and dissipation through the walls. It is recognized however that under the circumstances transmission through the walls could not be entirely avoided.

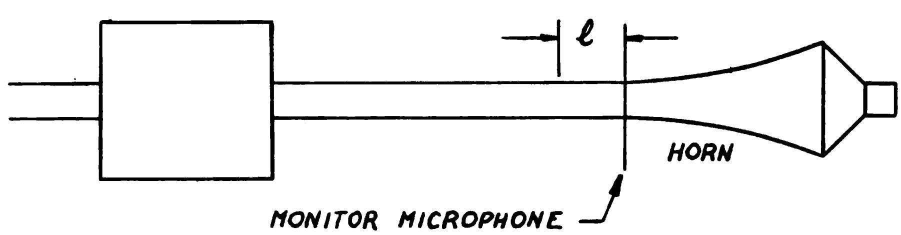
2. Infinite Tailpipe

As we discussed in Chapter I a sudden change in the cross section of a pipe transmitting a sound wave will cause a reflected wave to be set up. Any measurement of sound level in that pipe would be influenced by both the incident and the reflected waves. In order to determine the true incident sound level a detailed study of maximum and minimum sound levels measured in the standing wave as well as the distance from the cross sectional change would have to be made. An analysis of the effect of an open ended tailpipe on the acoustical transmission characteristics of a muffler system is made in reference No. 2. In studying the performance of an acoustical filter in detail, it becomes advantageous to eliminate the effect of an open ended tailpipe. In reference No. 2 an infinite tailpipe was constructed to this end. The objective is of course, to construct a termination to the system in such a way that no reflected wave is set up at any point in the tailpipe. In trying to

FIGURE 6

STATION NO.	DIMENSION ℓ	STATION NO	DIMENSION ℓ
1	$8\frac{3}{4}$"	11	$72\frac{1}{2}$"
2	$19\frac{3}{4}$"	12	75"
3	$30\frac{3}{4}$"	13	$78\frac{1}{4}$"
4	$41\frac{3}{4}$"	14	$81\frac{1}{2}$"
5	$47\frac{1}{4}$"	15	$84\frac{3}{4}$"
6	$52\frac{3}{4}$"	16	88"
7	60"	17	91"
8	$63\frac{3}{4}$"	18	$95\frac{1}{4}$"
9	$67\frac{1}{2}$"	19	99"
10	70"	20	$103\frac{1}{2}$"

ℓ = DISTANCE OF MEASURING STATION FROM END OF HORN

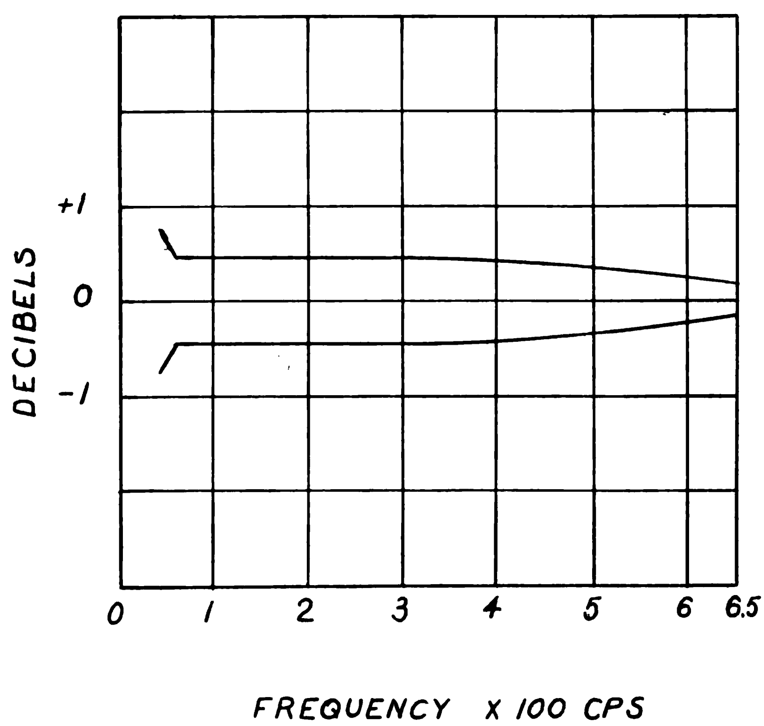


LOCATION OF MEASURING STATIONS IN EXHAUST PIPE construct such a termination cotton was first used in the pipe to absorb a maximum of sound energy without reflection. It was soon found that, after exposure to high intensity sound levels the cotton would pack to varying degrees of density, completely modifying the percentage of absorbed and the percentage of reflected power. The cotton was then separated into small sections or compartments. Even then, the performance of the infinite tailpipe was very erratic and to tally unsatisfactory. Finally after many approaches were tried, and many more investigated, a totally satisfactory solution was arrived at.

A mesh of flat, thin stainless steel wire was inserted into the pipe. The material was heavy and resilient to the extent that high intensity sound levels did not alter the density with which the material had originally been packed. It was soon found that, even though reasonable absorption was attained, a denser material should be used in a portion of the pipe, to effect total absorption. The final construction consisted of 12 feet of packed mesh followed by 15 feet of cotton prepacked and in fixed sections.

The drop of intensity across the mesh was sufficient to reduce pressure levels to a point where the cotton portion of the tailpipe performed in a consistent manner. The ultimate reflection set up by this termination was always within $\pm$ .75 DB. In the frequency range from 50 to 1000 cps. A typical calibration curve for the infinite tailpipe is shown in Figure 7.

FIGURE 7



CALIBRATION OF INFINITE TAILPIPE

3. Physical Layout

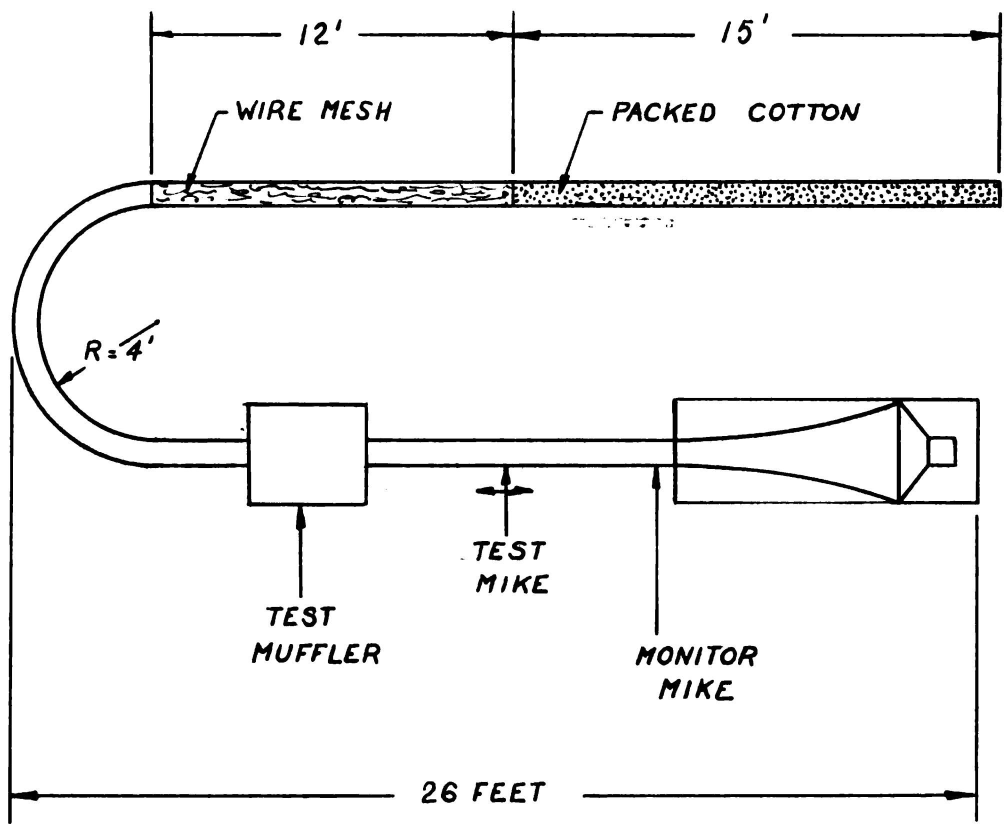
A substantial portion of the physical layout has now been described. It will suffice to mention that all sound tests were run in the acoustical laboratory of Hayes Industries, Inc. in Jackson, Michigan. A layout of the test set up in the anechoic room, with physical dimensions of the system is shown in Figure 8. It should be noted that all sound measurements were made inside the pipes, and the use of an anechoic room would not in itself have been required.

4. Calibration Details

Calibration of all electronic instrumentation was performed in accordance with the recommendations of the manufacturers. The calibration curves of the condenser microphones are shown in Figure 9 and the calibration of the infinite tailpipe in Figure 7. The sound source calibration was not deemed important. Indeed inasmuch as the muffler evaluation was based upon an analysis of the muffler performance at one frequency point at the time, the particular loudspeaker efficiency at that frequency was not important. With all measurements both in front and in back of the test muffler taken through the same microphone, the microphone calibration was also of no major importance. Readings were important as they related to other readings. The absolute value of these readings was only of interest to determine approximate sound pressure levels at which tests could proceed. These levels were between 130 and 160 DB. re .0002 dyne/cm^2

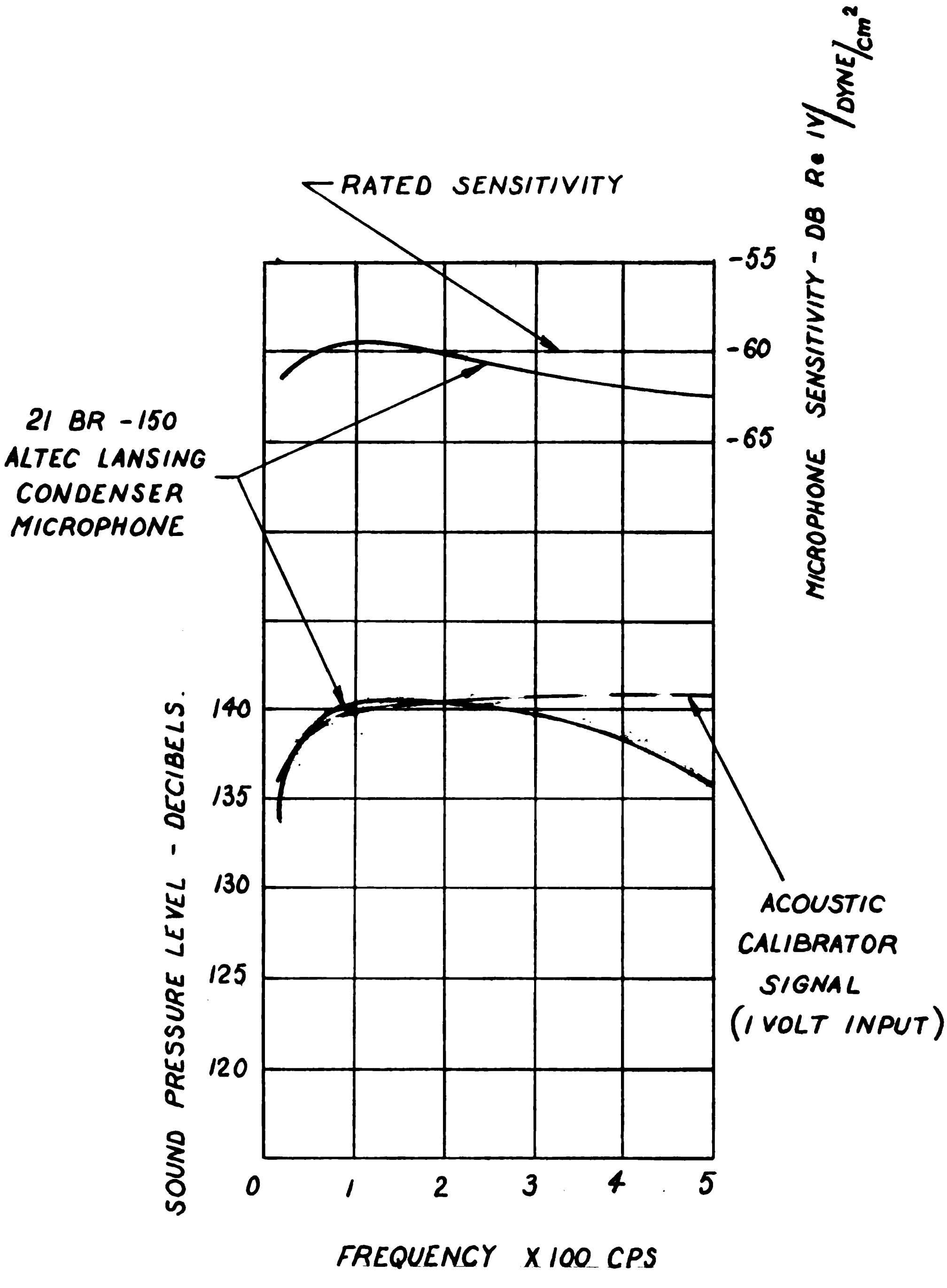
Calibration checks were made frequently however in order to make sure the microphones were functioning normally. The one very important

FIGURE 8



TEST SETUP IN THE ANECHOIC ROOM.

FIGURE 9



CALIBRATION CURVES OF CONDENSER MICROPHONES calibration, namely that of the infinite tailpipe was a source of extreme satisfaction. The consistency of the operation of the infinite tailpipe prompted us to disregard correcting tailpipe sound pressure levels. It was felt that the magnitude of the correction factor was well within acceptable limits for this program.

CHAPTER III

DETAILS OF MUFFLER TEST PROGRAM

More information must be given on how the tests were actually run, and how the measured values were interpreted. In addition it seems opportune to reiterate in detail the scope of this program and its true objective. This information then will set the stage for the analysis of the test results in Chapter IV and V. Correlation of experimental data with computer results will then enable us to arrive at our conclusions and recommendations covered in Chapter VI.

1. Method of Test

Upon calibration of the equipment and after setting the compressor circuit controls in such a way that sound levels at the monitor microphone never exceed 160 db over the entire frequency spectrum (from 50 to 650 cps) the recorder is started up and the tests can begin. The test microphone is moved from one measuring station to another. The sound level drops by more than 40 db as the microphone is pulled from one of the measuring points in the pipe. This gives us very clear distinction between the sound levels at each station. The recording of successive sound levels creates a good pattern of the standing wave system in the exhaust pipe. Continuous checks are made of the operating level of the power supply to the condenser microphones, calibration of the oscillator, and calibration of the recorder. At the same time the wave form at the measuring station gives an indication of the presence of harmonics of distortion of the wave. Even though in most cases distortion was negligible it seems as though a study of all distortions to the original sine wave would be of interest. Vibration of baffles inside the muffler as well as so-called "shell noise" vibration of the muffler shell seem to contribute to modifications of the sine wave signal. Upon measuring all sound levels in front of the muffler, the sound levels in the three stations in the tailpipe are also recorded.

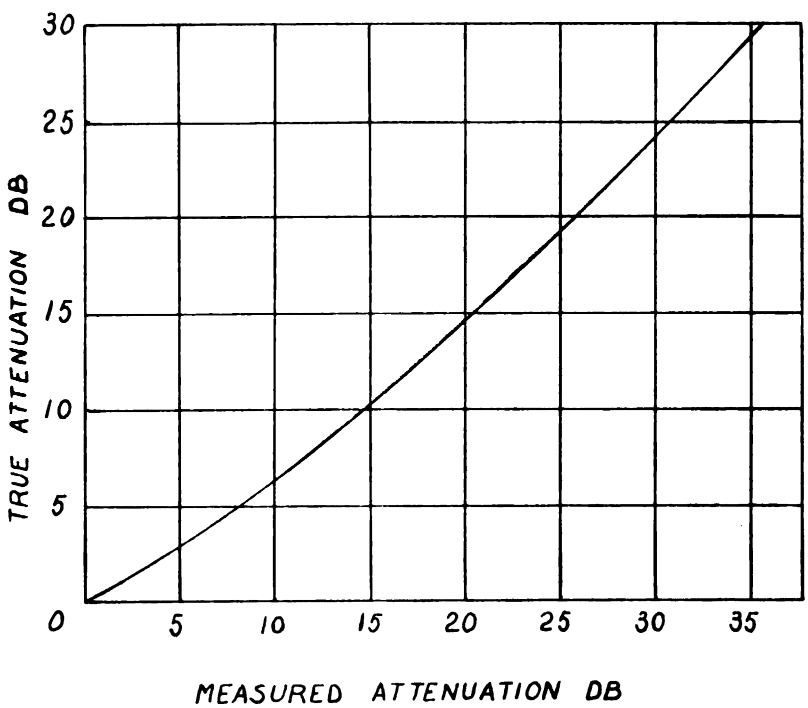
2. Analysis of Measurements

The maximum measured attenuation is the difference between the maximum sound level measured in the standing wave, and the sound level in the tailpipe. It is obvious that the maximum sound level in the standing wave is not representative of the magnitude of the incident wave. A rather complicated procedure of analysis is required to determine the true value of the incident wave. We would have to go through this procedure for every muffler tested, as well as for every frequency. To eliminate this tedious repetitive procedure, Reference No. 2 illustrates a "short cut" which is of sufficient accuracy in all cases and which allows for immediate correction of measured maximum attenuation to a reliable value of actual attenuation.

The reasoning on this short procedure is as follows:

Assume that all sound reflection takes place from a single point, and that the incident sound pressure is unity. Assume 20 percent of the incident wave is reflected; then 80 percent of the incident wave is

FIGURE 10



CORRECTION FACTORS FOR MEASURED ATTENUATION transmitted. The maximum sound level measured in the standing wave is then 120 percent of the incident wave or 1.2 . The true attenuation would be

$$10 \log_{10} \left(\frac{1}{.8}\right)^2 = 20 \log_{10} \frac{1}{.8} = 1.9382 \text{ db}$$

The maximum measured attenuation:

$$20 \log \frac{1.20}{.80} = 20 \log 1.5 = 20 \times .1761 = 3.522 \text{ db}$$

The correction required therefore would be:

$$3.522 - 1.938 = 1.58 \text{ db} \qquad \text{or } 20 \log \frac{1.2}{1} = 1.58 \text{ db}$$

This implies that the correction factor is in fact a function of the measured attenuation. This being the case, a curve, shown in Figure 10, was plotted and for all determinations of attenuation, the procedure of applying a correction factor to the maximum measured attenuation was used.

3. Reiteration of Objectives

To better illustrate the reasons for the choice of test units we repeat here in detail the objectives of the program.

a. Develop a test system capable of reliably "cold testing" muffler circuits of importance to automobile muffler design.

b. Restricting the test program to cold testing thereby eliminating the variables of gas flow and temperatures in the exhaust system determine the acoustical value of so called louvers as compared to holes of various sizes, in quantities however to produce the same total outlet area over equal length.

c. Extend the acoustical theory to the cross-bleed chamber and determine its' true nature as an acoustical filter.

d. Extend the theory and investigation to turn around chambers which are present in virtually every original equipment muffler.

e. Through correlation of experimental and theoretical analysis establish an approach for predicting the attenuation of a muffler circuit.

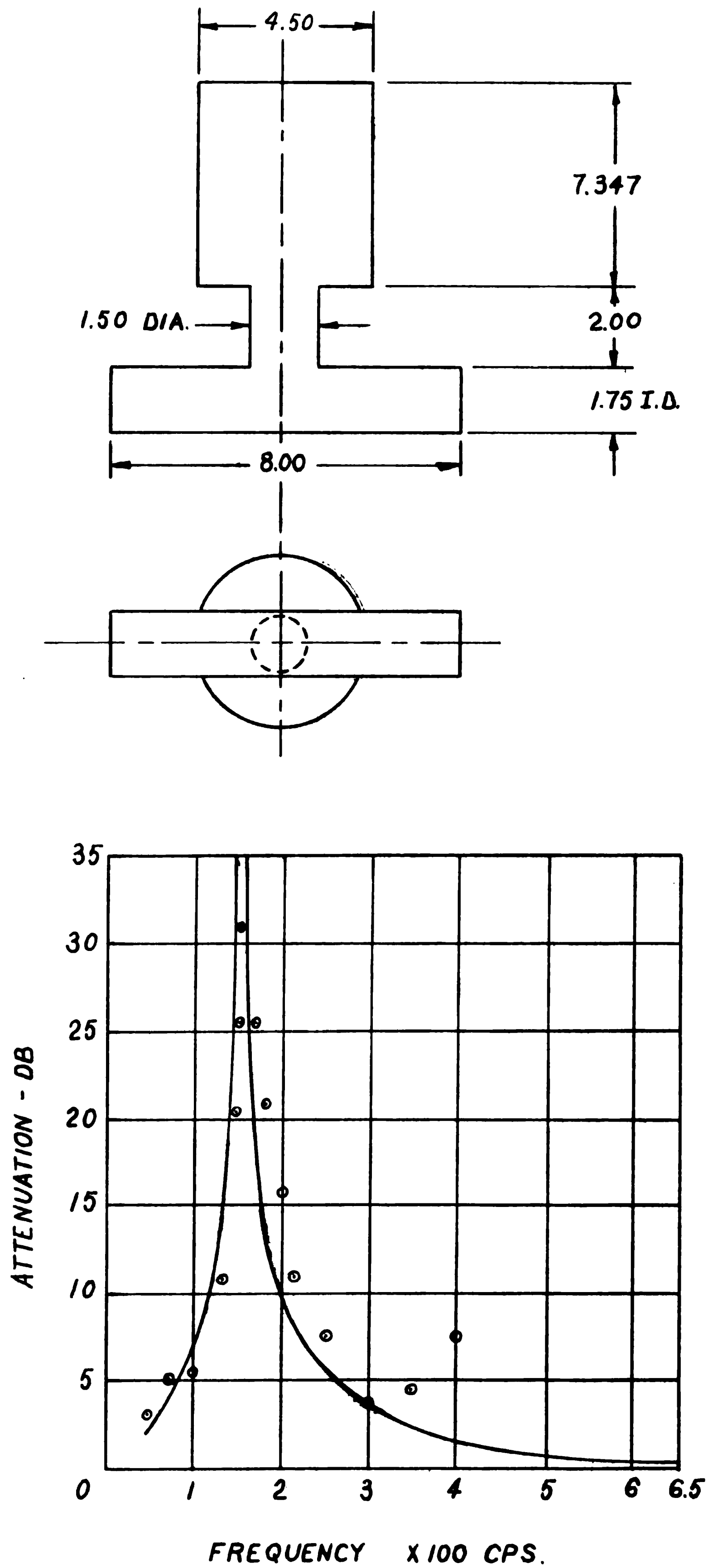
f. In gaining sufficient understanding of the important variables involved, discuss the anticipated influence of gas flow and elevated temperatures on the acoustical behavior of mufflers. Briefly discuss the subjective nature of all final judgment on sound. Make recommendations for future projects.

4. Series of Test Units

The first portion of the test program was planned in an effort to determine the reliability of the test procedure and the test apparatus. To this end three simple Helmholtz resonators were constructed. Their attenuation characteristics are easy to calculate, and they show a very sharp peak which would establish the degree of accuracy as a function of frequency. In an effort to determine the reliability of the computer programs, the performance curve of one of the NACA mufflers was computed and compared to the original NACA results.

The next step was to study the effect of louvers as compared to holes of various sizes. Figures 12 through 14 show mufflers which were built and tested. In addition to the study of the louver effect it was of interest to determine whether or not such a chamber would behave as a true expansion chamber. The true equivalent expansion chamber is shown in Figure 15. Test results and computer data are shown in Figure 12 through 15. It should be noted that the blind tubes a and b in Figure 15

FIGURE 11



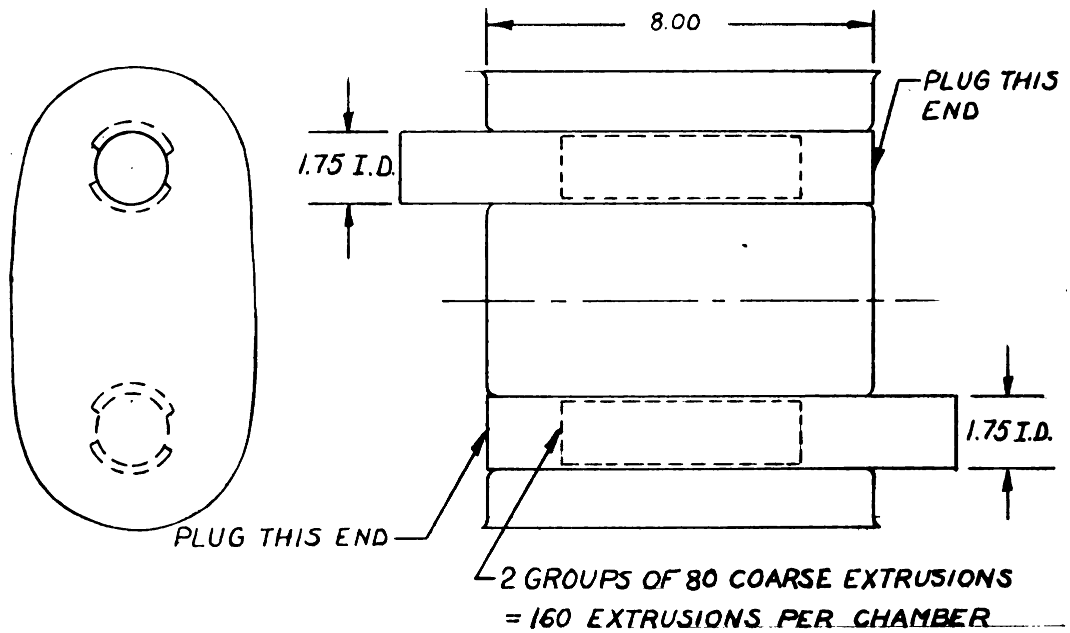
SIMPLE HELMHOLTZ RESONATOR

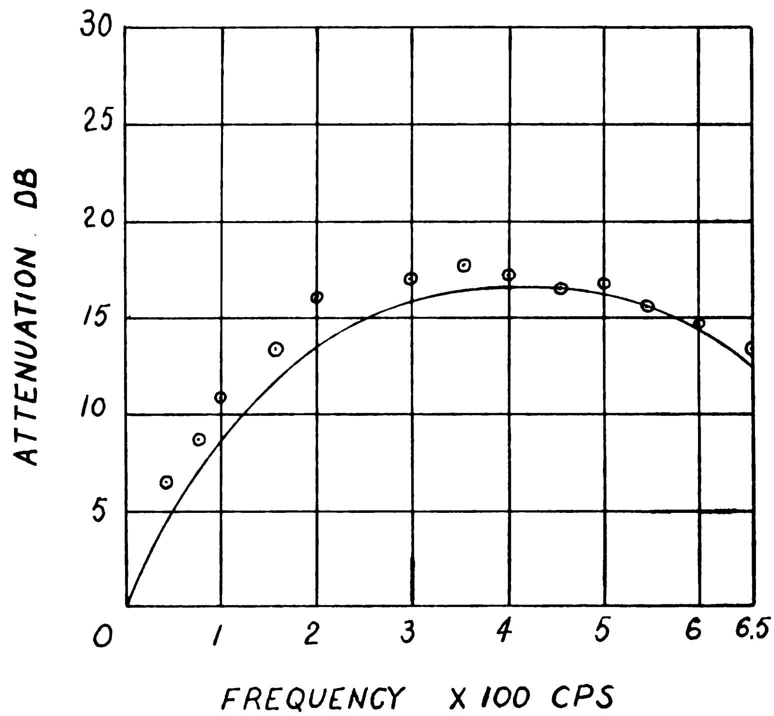
were installed only to maintain perfect similarity of construction with the other units and to keep the cross sections equal to those of the other test mufflers.

In the next series of mufflers we proceed from the single expansion chamber and the Helmholtz chamber to combinations of chambers. The entire series was built with the purpose of ultimately combining cross bleed and turn around chambers as shown in Figures 16 through 19 and as we normally find in commercial automobile mufflers. The double expansion chamber shown in Figure 16 is fed through one single pipe. The next muffler shown in Figure 17 feeds the second chamber through two pipes. In the following circuit, true reversal of flow is accomplished. In Figure 19 we progress to a triple chamber design and further to an additional reversal of flow in Figure 20, to our final muffler of the program.

In conclusion, a typical commercial muffler is sketched (Figure 25). This muffler was not actually constructed and tested. The purpose of showing it in this report is to illustrate which portions of a conventional muffler circuit were dealt with in detail. For the purposes of extending theory to this commerical muffler version, however, we have set up the applicable system of equations.

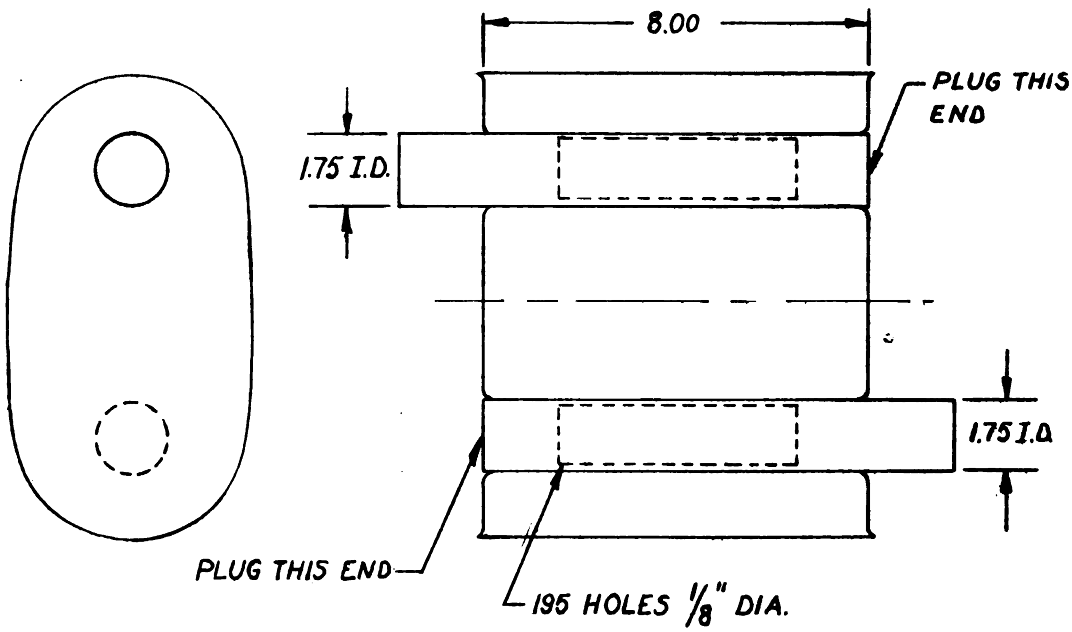
FIGURE 12

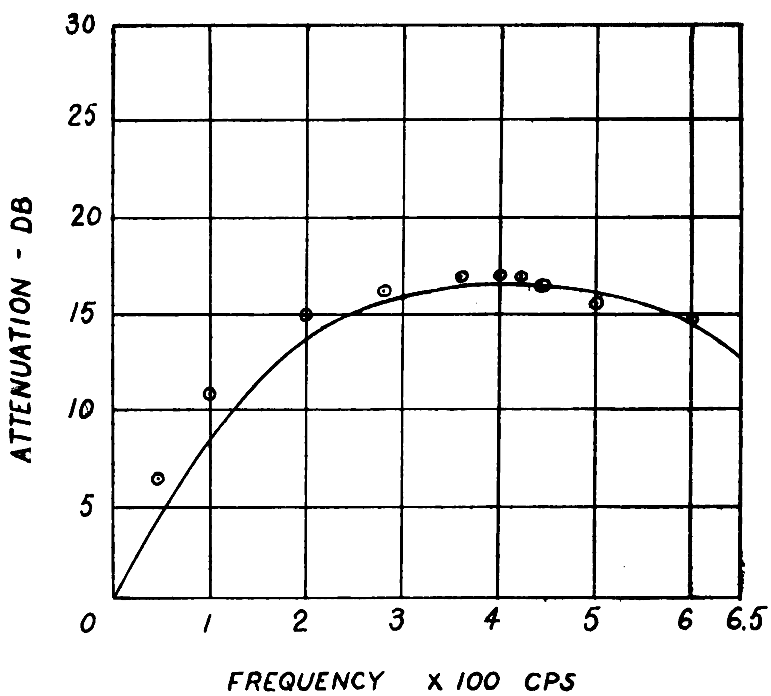




LOUVER FED EXPANSION CHAMBER

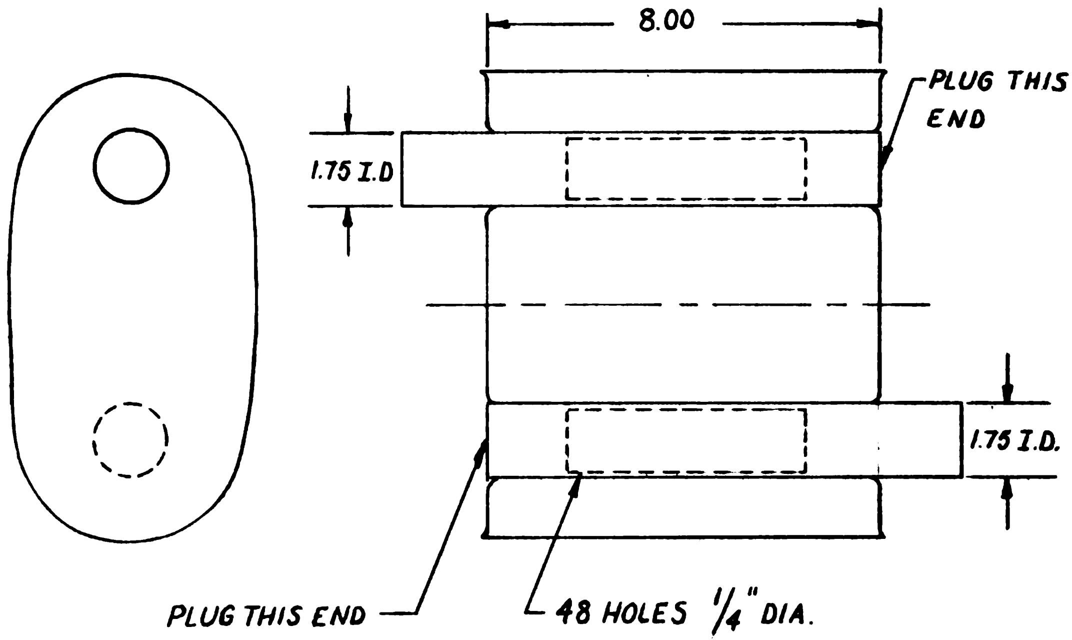
FIGURE 13

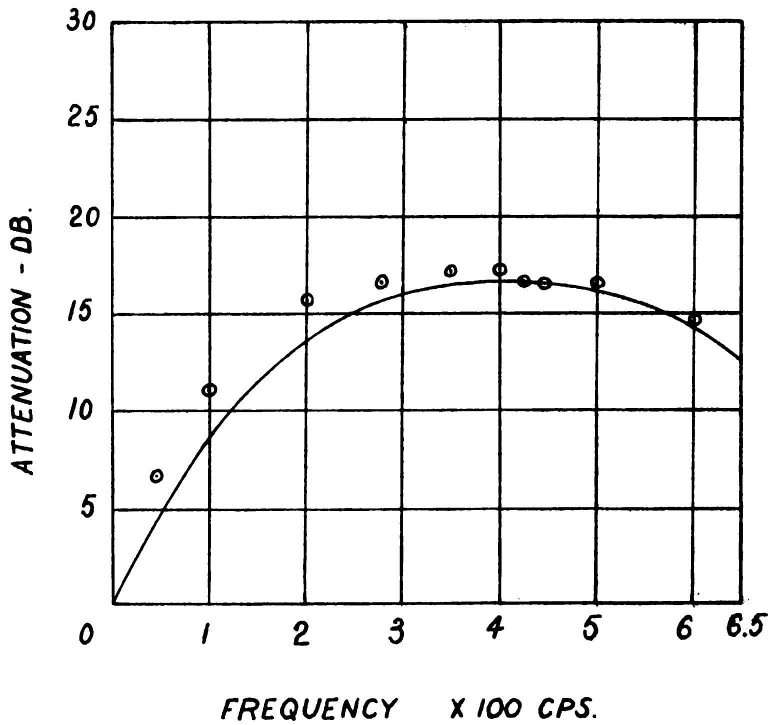




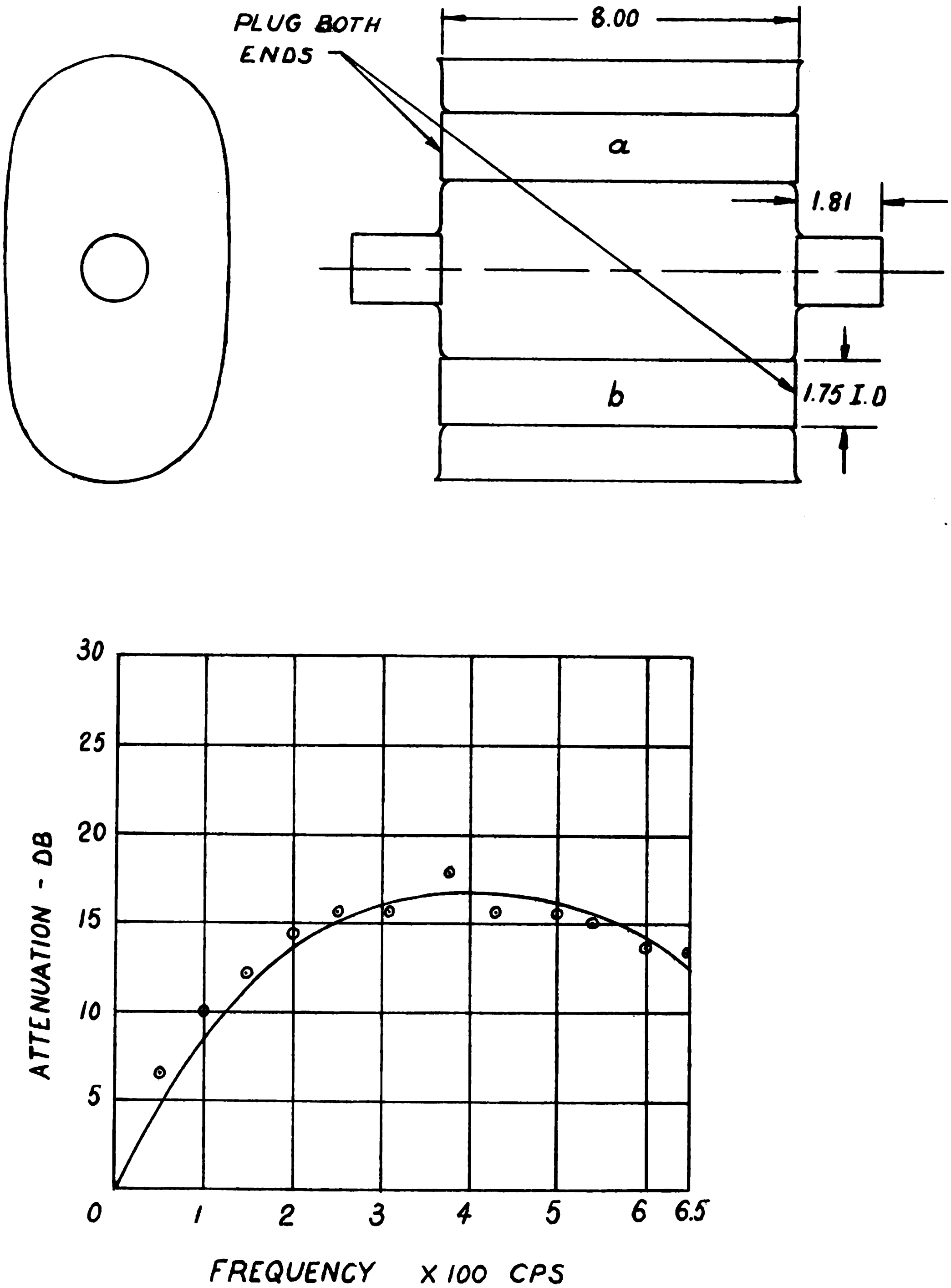
EQUIVALENT EXPANSION CHAMBER WITH 1/8 INCH HOLES

FIGURE 14



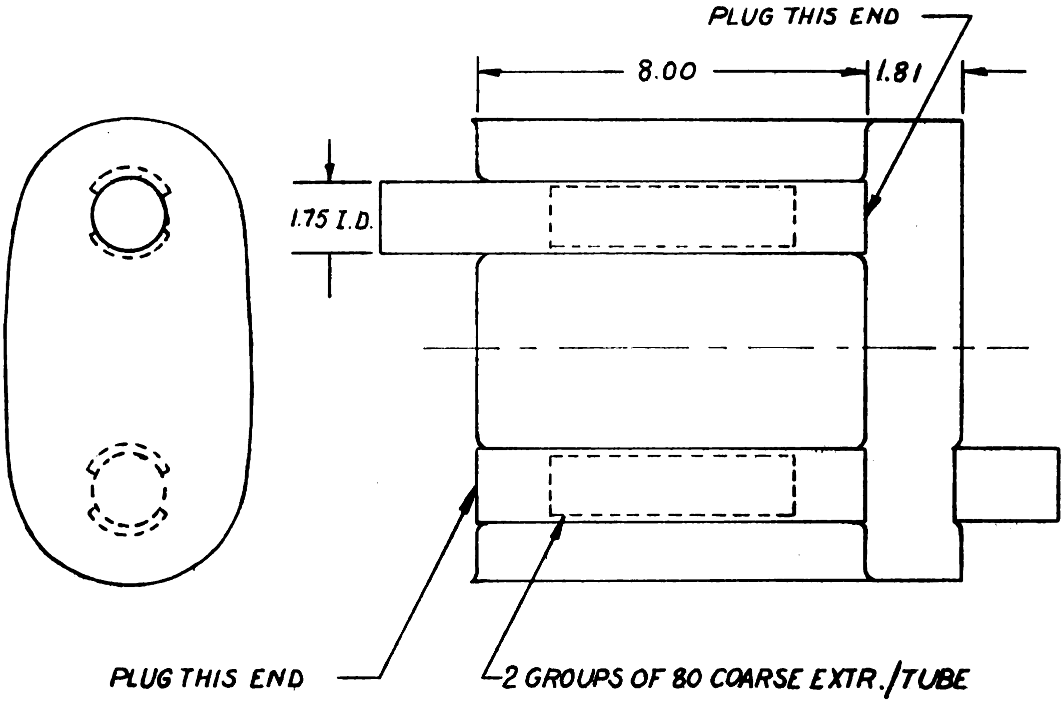


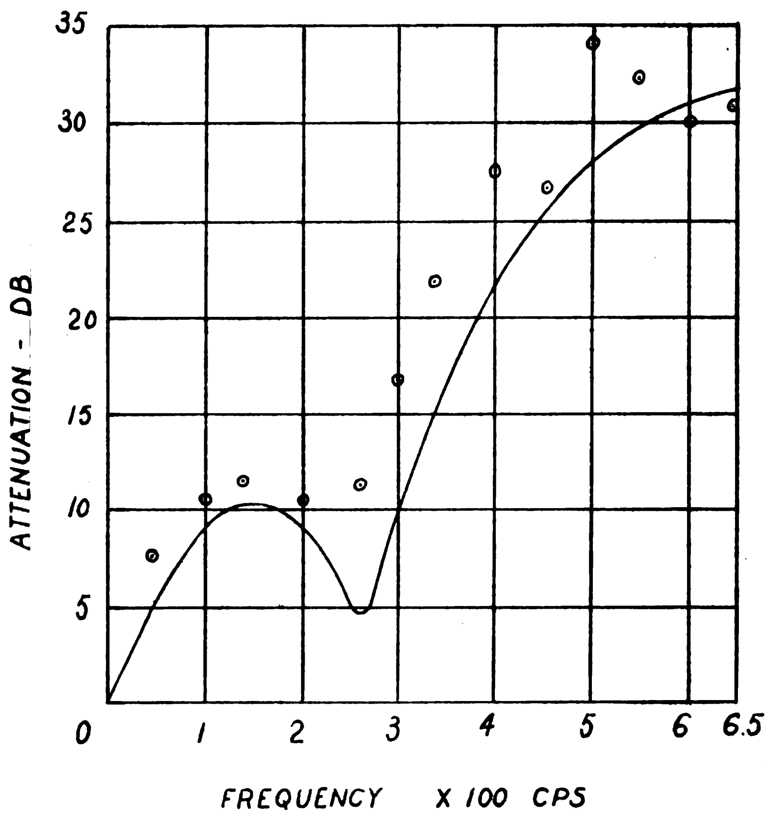
EQUIVALENT EXPANSION CHAMBER WITH 1/4 INCH HOLES



CONVENTIONAL EXPANSION CHAMBER

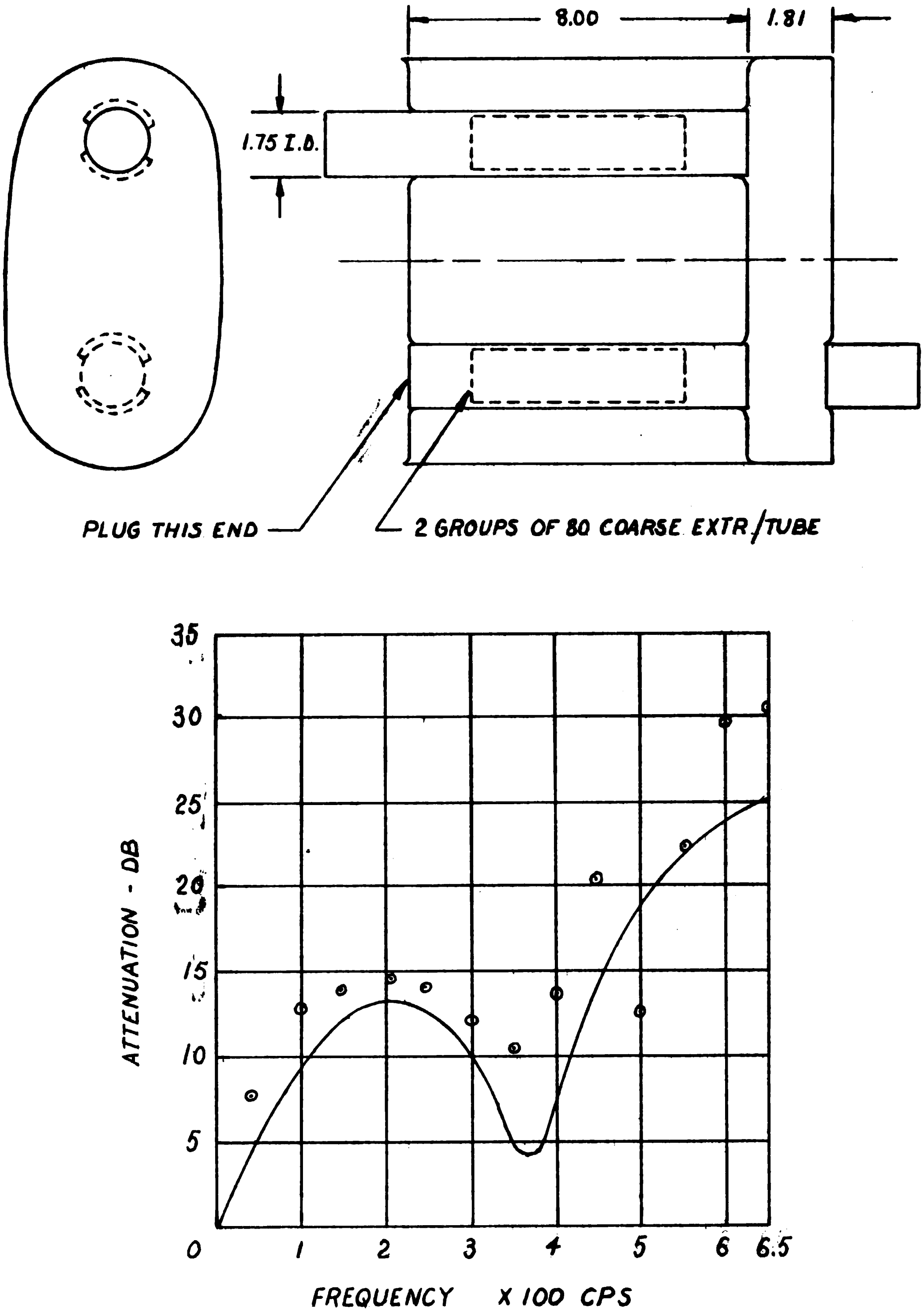
FIGURE 16





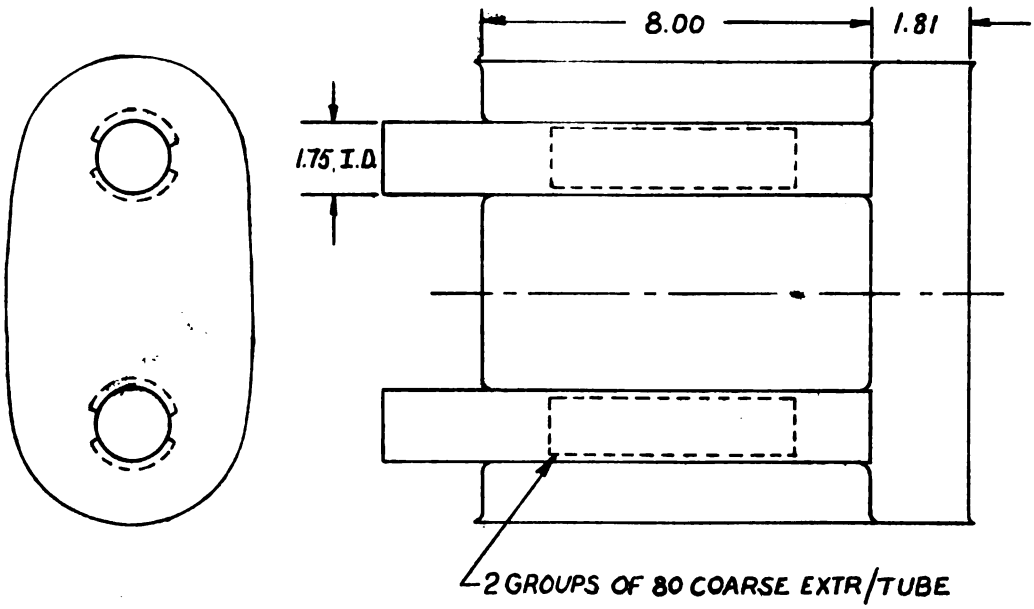
DOUBLE EXPANSION CHAMBER

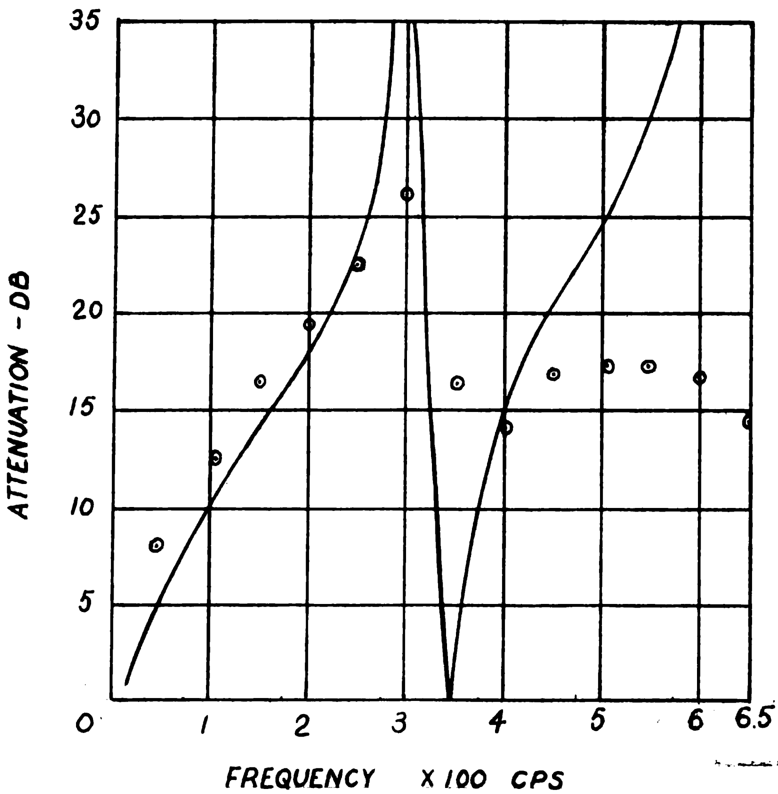
FIGURE 17



DOUBLE EXPANSION CHAMBER

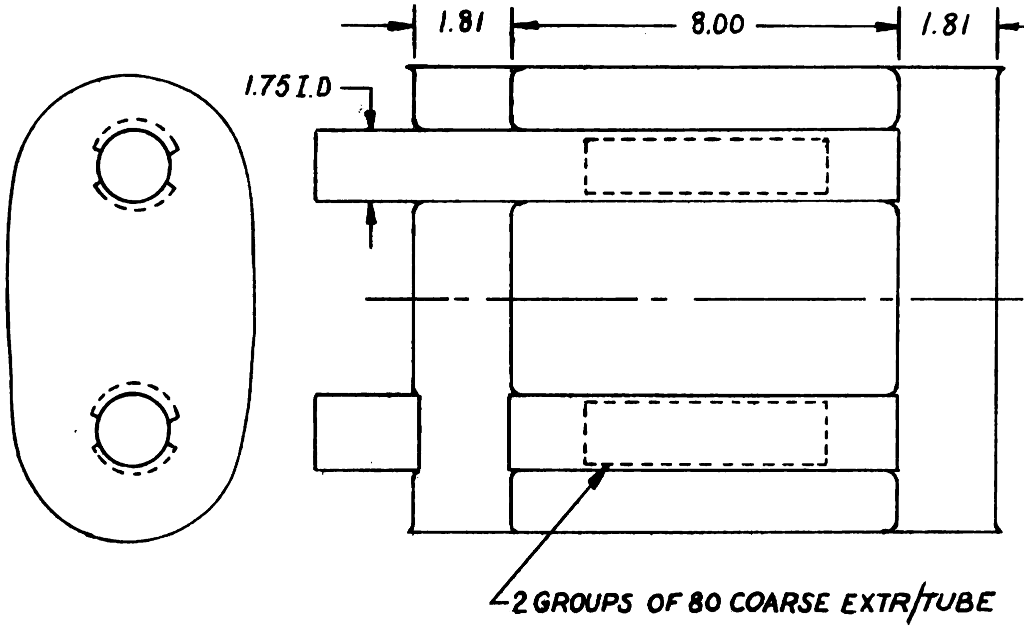
FIGURE 18

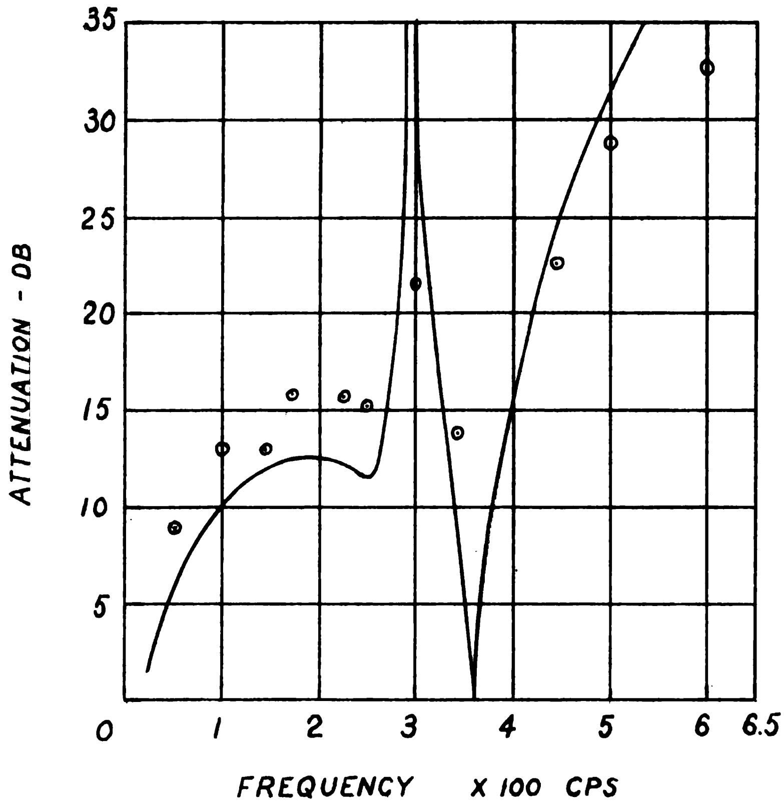




EXPANSION CHAMBER AND TURN-AROUND CHAMBER

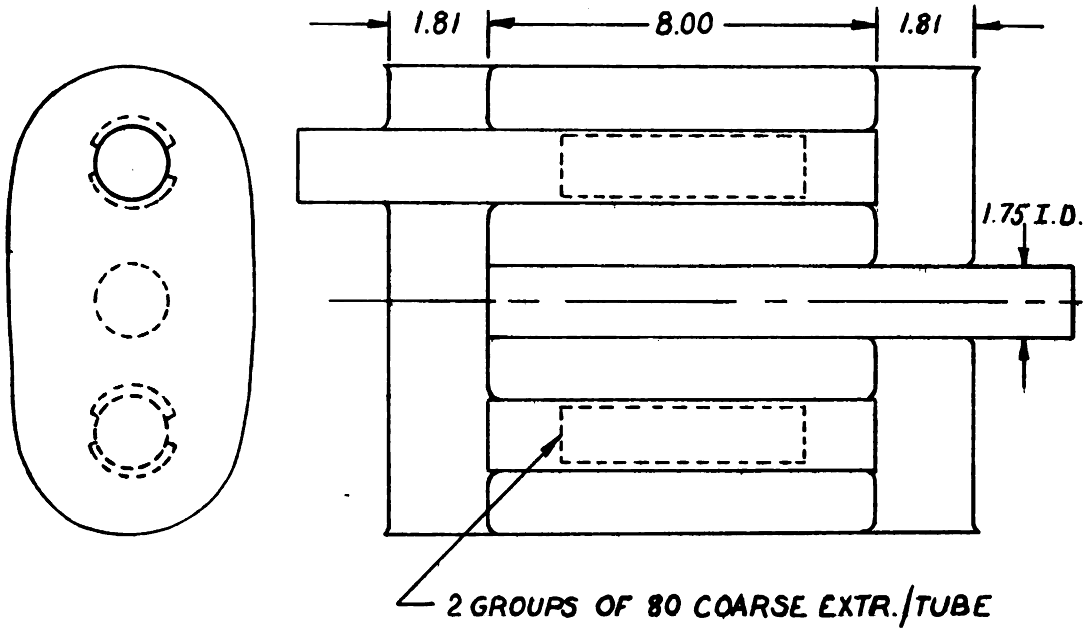
FIGURE 19

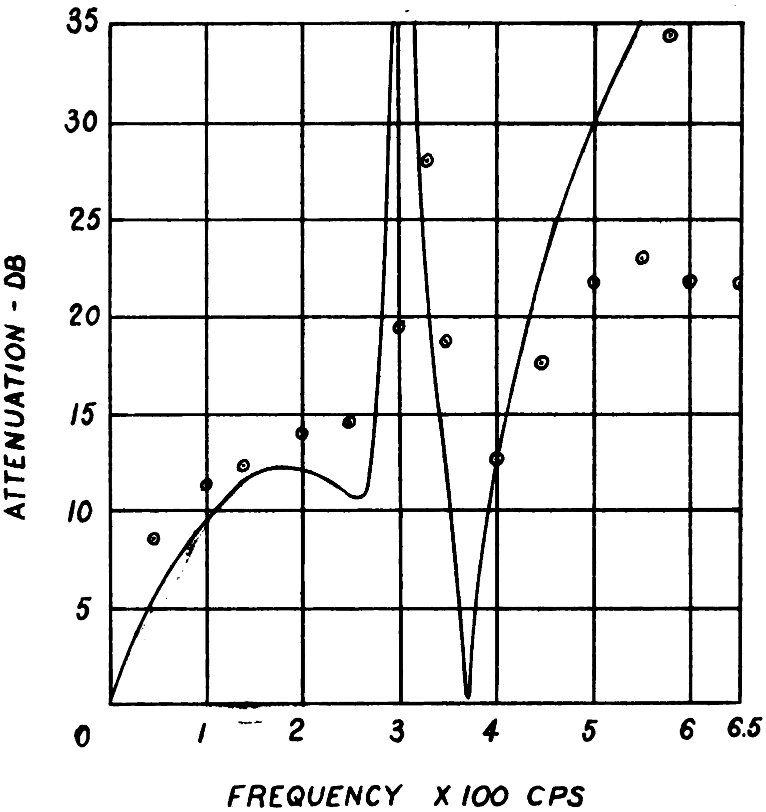




TRIPLE CHAMBER MUFFLER

FIGURE 20





TRIPLE CHAMBER MUFFLER WITH DOUBLE REVERSAL OF FLOW

CHAPTER IV

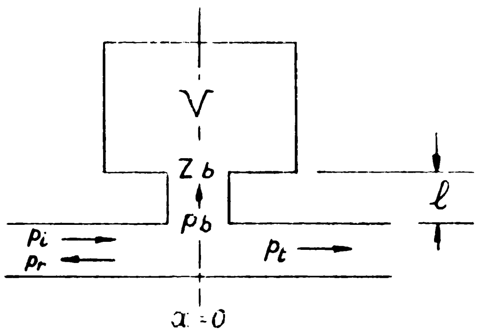
MATHEMATICAL ANALYSIS OF THE MUFFLER SYSTEMS

Utilizing the background gained in Chapter I we will now proceed to develop the mathematical solutions to the various muffler circuits considered. In most cases it will suffice to arrive at a system of equations for the units, leaving the solution of that system of equations to the computers. We will start with the simplest configurations, gradually moving into some of the more elaborate circuits.

1. Helmholtz Resonator

The theory of a side branch has left us the background to analyze the Helmholtz Resonator. This acoustical filter consists of a volume v and a neck or "tuning tube" C.

FIGURE 21



Let the incident pressure be

$$p_i = A_1 e^{j\omega t} \qquad \text{at} \quad x = 0$$

Then

$$p_r = B_1 e^{j\omega t}$$

$$p_t = A_2 e^{j\omega t}$$

$$p_b = A_b e^{j\omega t}$$

If we can neglect energy losses due to viscosity no net dissipation of energy into the resonator takes place. This means we are assuming

$$R_b = 0 \ .$$

Let the cross sectional area of the neck be $S = \pi a^2$, the length of the neck l, and the chamber volume = V.

Referring to our definitions of acoustic compliance and acoustic inertance it is clear that the effective mass of the air in the neck acts as an inertance, and the volume in the chamber as a compliance. The mass in the neck can be considered to move as a piston energized by the sound intensity in the pipe and working against the springlike action of the volume in the chamber.

Let l' be the effective length of the mass in the neck. The mass is then $\rho S l'$ and the acoustic inertance

$$M = \frac{\rho S l'}{S^2} = \frac{\rho l'}{S}$$

the reactance

$$X = \omega M - \frac{1}{\omega C}$$

the acoustic compliance for the volume V: $\quad C = \dfrac{V}{\rho c^2}$.

Hence

$$X_b = \rho\left(\frac{\omega l'}{S} - \frac{c^2}{\omega V}\right) \tag{20}$$

the value of the effective length l' is arrived at as follows:

Assuming the air in the entire neck to be moving as a piston we must apply an end correction Δl to each end of that piston in order to allow for the mass in the pipe and that in the chamber V immediately subjected to and actually part of this vibration of the piston. This end correction Δl is identical to that used in Chapter I page 12 for radiation from an open pipe. In Chapter I page 12 we found $\beta = \frac{8ak}{3\pi}$ the correction factor being $\frac{8a}{3\pi} = \Delta l$. Hence

$$l' = l + 2\,\Delta l = l + \frac{16a}{3\pi} = \underline{\underline{l + 1.7a}}$$

now we know

$$p_i + p_r = p_b = p_t \tag{21}$$

$$\frac{dX_i}{dt} + \frac{dX_r}{dt} = \frac{dX_t}{dt} + \frac{dX_b}{dt}$$

or

$$\frac{1}{Z} = \frac{1}{Z_t} + \frac{1}{Z_b} \tag{22}$$

and

$$\frac{p_i}{\rho c/s} - \frac{p_r}{\rho c/s} = \frac{p_b}{Z_b} + \frac{p_t}{\rho c/s}$$

but because

$$p_b = p_t \qquad \frac{p_i - p_r}{\rho c/s} = p_t\ \left(\frac{1}{Z_b} + \frac{1}{\rho c/s}\right)$$

which is identical to

$$\frac{p_i - p_r}{Z} = p_t\ \left(\frac{1}{Z_b} + \frac{1}{Z}\right) \tag{23}$$

where Z is the characteristic impedance of the tube. Solving (21) and (23) simultaneously we find

$$\frac{p_i}{p_t} = 1 + \frac{Z}{2Z_b} = 1 + \frac{Z}{2(R_b + jX_b)}$$

Neglecting viscosity losses (R_b=o) we find

$$\frac{p_i}{p_t} = 1 + \frac{Z}{2jX_b} \tag{24}$$

And if we define attenuation across a filter as attenuation = 10 $\log_{10}$

$$\left|\frac{A_i}{A_t}\right|^2$$

or

$$10 \log \left|\frac{p_i}{p_r}\right|^2 = 10 \log_{10} \left(1 + \frac{Z^2}{4X_b^2}\right) \qquad (25)$$

to determine the resonant frequency we set

$$X_b = 0 \quad \text{or} \quad \rho = \left(\frac{\omega l'}{s} - \frac{c^2}{\omega V}\right) = 0$$

and

$$\frac{\omega l'}{s} = \frac{c}{\omega V}$$

and

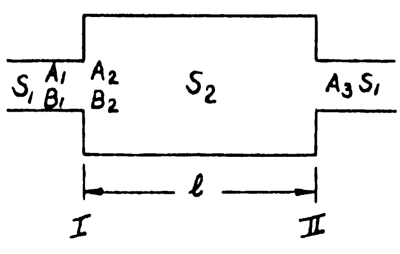
$$\omega = \sqrt{\frac{Sc^2}{l'V}} \text{ and } f_R = \frac{c}{2\pi}\sqrt{\frac{S}{l'V}}$$

At $f = f_R$ the theoretical attenuation is infinity. This in effect is the only point where the assumption $R_b = 0$ produces any serious degree of inaccuracy.

2. Expansion Chamber

Consider a chamber of the type shown in Figure 22. Assume a sound wave of $p_i = A_1 e^{j\omega t}$ at x = o resulting at that sudden change of cross section into $p_r = B_1 e^{j\omega t}$. The transmitted wave, however cannot be considered traveling in one direction only. Indeed because of the presence of junction II a reflected wave is present at junction I.

FIGURE 22



$$\frac{S_2}{S_1} = m$$

In general we can assume

A_1 and B_1 pressure magnitudes to the left of the junction I.

A_2 and B_2 magnitudes to the right of junction I.

Once the wave has entered the pipe of cross-section S_2 no changes occur until junction II is reached. Because the wave has travelled over a distance l, however, we have the incoming wave at junction II =

$$A_2 e^{j(\omega t - kl)}$$

and

$$B_2 e^{j(\omega t + kl)}$$

to the right of junction II we assume no reflections hence A_3 only. Conditions of continuity of pressure enable us to conclude

$$A_1 + B_1 = A_2 + B_2 \tag{26}$$

$$A_2 e^{-jkl} + B_2 e^{jkl} = A_3 \tag{27}$$

continuity of flow gives

$$S_1 (A_1 - B_1) = S_2 (A_2 - B_2) \text{ or for } \frac{S_2}{S_1} = m, (A_1 - B_1) = m (A_2 - B_2)$$

and

$$S_2 (A_2 e^{-jkl} - B_2 e^{jkl}) = S_1 A_3 \tag{28}$$

or

$$m (A_2 e^{-jkl} - B_2 e^{jkl}) = A_3 \tag{29}$$

The simultaneous solution of equations 26 - 27 - 28 - 29 for $\frac{A_1}{A_3}$ lead to the attenuation =

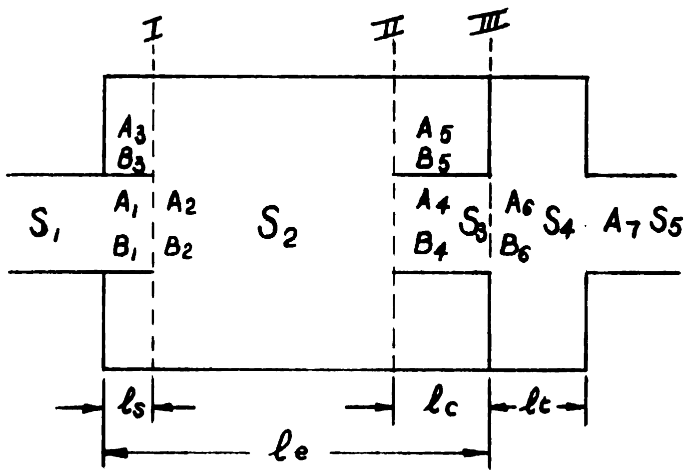
$$10 \log_{10} \left| \frac{A_1}{A_3} \right|^2 = 10 \log_{10} [1 + \tfrac{1}{4} (m - \tfrac{1}{m})^2 \sin^2 kl]$$

3. Combinations of Chambers

Various combinations of chambers make up the average automobile muffler. We are giving here the systems of equations for some of the mufflers in the test series.

The muffler as shown in Figure 16 consists essentially of two expansion chambers. An equivalent circuit (for low frequencies for which side inlet presents no noticeable difference from center inlet) is shown in Figure 23.

FIGURE 23



The conditions of continuity of pressure and volume flow give the following system of equations:

(30) $A_1 + B_1 = A_2 + B_2 = A_3 + B_3$

(31) $B_3 = A_3 e^{j2kls}$

(32) $S_1 (A_1 - B_1) = (A_2 - B_2) S_2 - (S_2 - S_1)(A_3 - B_3)$

(33) $A_2 e^{-jk(l_e - l_s - l_c)} + B_2 e^{jk(l_e - l_s - l_c)} = A_4 + B_4 = A_5 + B_5$

(34) $B_5 = A_5 e^{-j2kl_c}$

(35) $S_2[A_2e^{-jk(l_e - l_s - l_c)} - B_2e^{jk(l_e - l_s - l_c)}] =$

$$S_3\,(A_4 - B_4) + (A_5 - B_5)\,(S_2 - S_3)$$

(36) $A_4e^{-jkl_c} + B_4e^{jkl_c} = A_6 + B_6$

(37) $S_3[A_4e^{-jkl_c} - B_4e^{jkl_c}] = [A_6 - B_6]\,S_4$

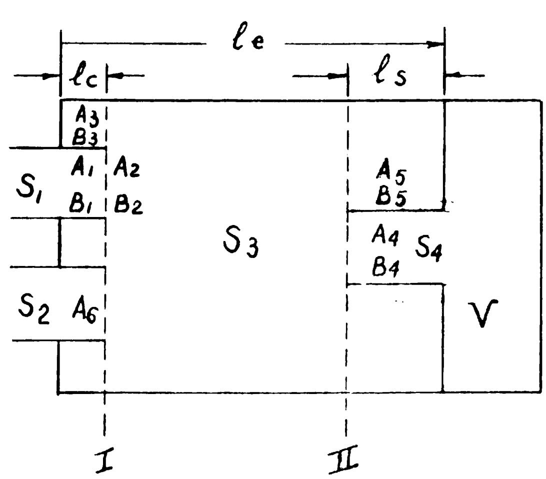
(38) $A_6e^{-jkl_t} + B_6e^{jkl_t} = A_7$

(39) $S_4\,(A_6e^{-jkl_t} - B_6e^{jkl_t}) = S_5\,A_7$

The muffler shown in Figure 18 was constructed to study the acoustics of a turn around chamber. With Chamber I receiving the acoustical signal through louvers in the inlet pipe and allowing "through passage" through louvers in the outlet tube, this chamber behaves as an expansion chamber.

Figure 24 shows a theoretically equivalent circuit (argumentation in Chapter V). The circuit presents a combination expansion chamber and Helmholtz resonator. The equations are listed following Figure 24.

FIGURE 24



(40) $A_1 + B_1 = A_2 + B_2 = A_3 + B_3 = A_6$ $\qquad m_1 = \frac{S_3}{S_1}$

(41) $B_3 = A_3 \cos 2kl_c + j A_3 \sin 2kl_c$

(42) $m_1 (A_2 - B_2) = (A_1 - B_1) + (m_1 - 2)(A_3 - B_3) - A_6$

(43) $(A_2 + B_2) \cos k (l_e - l_s - l_c) - j (A_2 - B_2) \sin k (l_e - l_s - l_c) =$

$A_4 + B_4 = A_5 + B_5$ $\qquad m_2 = \frac{S_3}{S_4}$

(44) $m_2 (A_2 - B_2) \cos k (l_e - l_s - l_c) - j\, m_2 (A_2 + B_2) \sin k (l_e - l_s - l_c) =$

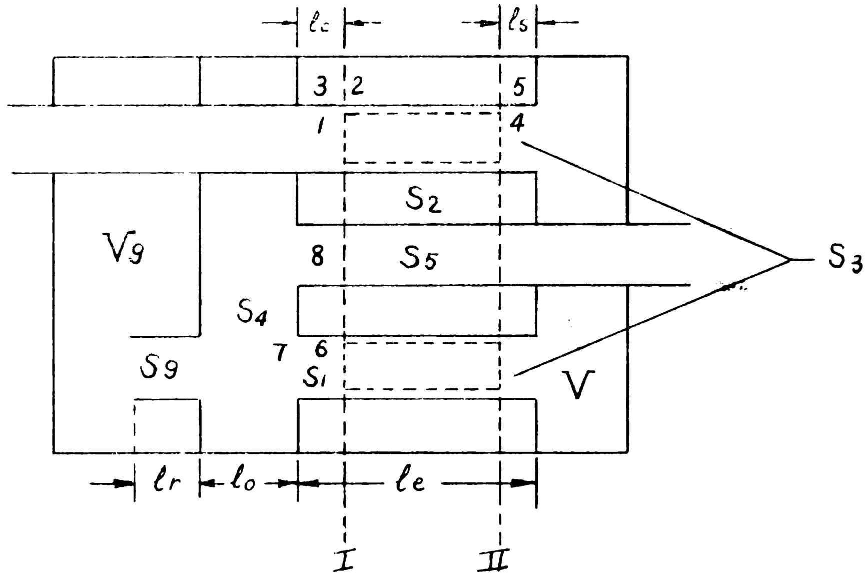
$(A_4 - B_4) + (m_2 - 1)(A_5 - B_5)$

(45) $B_5 = A_5 \cos 2kl_s - j A_5 \sin 2kl_s$

(46) $(A_4 + B_4) = (A_4 - B_4)\, j \left(\frac{2\pi f l s}{c} - \frac{cS}{2\pi f V}\right)$

In a similar manner, equations are set up for the mufflers in Figures 19 and 20. Various considerations, of importance in the analysis of the circuits are covered in Chapter V. The muffler shown in Figure 25 is an accurate duplication of a commercial muffler design. As mentioned previously this design was not constructed or tested but the system of equations which follows illustrates the application of the theory presented to complete muffler designs.

FIGURE 25



$$\frac{S_2}{S_1} = m_1 \qquad \frac{S_2}{S_3} = m_2 \qquad \frac{S_4}{S_1} = m_3 \qquad \frac{S_4}{S_5} = m_4$$

(47) $A_1 + B_1 = A_2 + B_2 = A_3 + B_3 = (A_6 + B_6)$

(48) $B_3 = A_3 \cos 2kl_c + j\, A_3 \sin 2kl_c$

(49) $m_1\,(A_2 - B_2) = (A_1 - B_1) + (m_1 - 2)\,(A_3 - B_3) - (A_6 - B_6)$

(50) $(A_2 + B_2) \cos k\,(l_e - l_s - l_c) - j\,(A_2 - B_2) \sin k\,(l_e - l_s - l_c) =$

$A_4 + B_4$

(51) $A_4 + B_4 = A_5 + B_5$

(52) $m_2(A_2 - B_2) \cos k\,(l_e - l_s - l_c) - j\, m_2\,(A_2 + B_2) \sin k\,(l_e - l_s - l_c) =$

$(A_4 - B_4) + (m_2 - 1)\,(A_5 - B_5)$

(53) $B_5 = A_5 \cos 2kl_s - j\, A_5 \sin 2kl_s$

(54) $(A_4 + B_4) = (A_4 - B_4)\, j\, \left(\frac{2\pi f ls}{c} - \frac{2cS_1}{2\pi f V}\right)$

(55) $(A_6 + B_6) \cos kl_c - j\,(A_6 - B_6) \sin kl_c = A_7 + B_7$

(56) $A_7 + B_7 = A_8$

(57) $(A_6 - B_6) \cos kl_c - j\,(A_6 + B_6) \sin kl_c = m_3\,(A_7 - B_7) - A_8$

$$(58) \quad (A_7 + B_7) \cos kl_o - j\,(A_7 - B_7) \sin kl_o = A_9 + B_9 =$$

$$(A_9 - B_9)\; j\,\left(\frac{2\pi f l r}{c} - \frac{cS_9}{2\pi f V9}\right)$$

$$(59) \quad m_4\,(A_7 - B_7) \cos kl_o - j\, m_4\,(A_7 - B_7) \sin kl_o = (A_9 - B_9)$$

CHAPTER V

DISCUSSION OF TEST AND COMPUTER RESULTS

The comparison between test results and theoretical attenuation can be made in major divisions, concentrating on the specific objective of each part of the program. Proceeding in this manner, this chapter will consist of the following sections:

1 - Reliability of the test set up

2 - Acoustical value of "louvers"

3 - Discussion on the cross-bleed chamber

4 - Turn-around chambers

5 - General approach to predicting muffler performance

6 - Major variables not accounted for

1. Reliability of the Test Setup

In Figure 7 the calibration curve for the infinite tailpipe was illustrated in the chapter on the test apparatus. We have elaborated on the anticipated or known inaccuracies in the performance of the various pieces of equipment. It was concluded then that as long as the performance of the infinite tailpipe remained within the limits of variation originally accepted, most other correction factors due to frequency response of microphones, etc., could be disregarded because of the fact that all measured data were used in comparison with other values obtained with the same setting on all electronic equipment. In this manner, if the "measured" maximum value of sound pressure level is for instance 3 db less than the actual maximum value because of lack of response of the microphone at that particular frequency, we can reasonably assume that the measured sound pressure level in the tailpipe, taken within seconds after the first reading was taken, would also be 3 db low. From this we conclude that as long as the microphone operates within reasonable limits from its' original calibration curve, corrections for calibration will not be necessary. The test results on a resonator are illustrated in Figure 11. They show very good agreement with the calculated values. In general, attenuation will be higher than that calculated, in frequency ranges showing very little attenuation. This is true primarily because of the transmission of sound through the walls of the pipes and the walls of the resonator. Obviously, at the point of resonance, the calculations show infinite attenuation which cannot be obtained in practice because of the presence of some resistance in the branch.

The computer program was checked against a NACA sample and found to be in perfect accord with NACA results. In general, experimental and analytical approaches were judged within the limits of reliability hoped for.

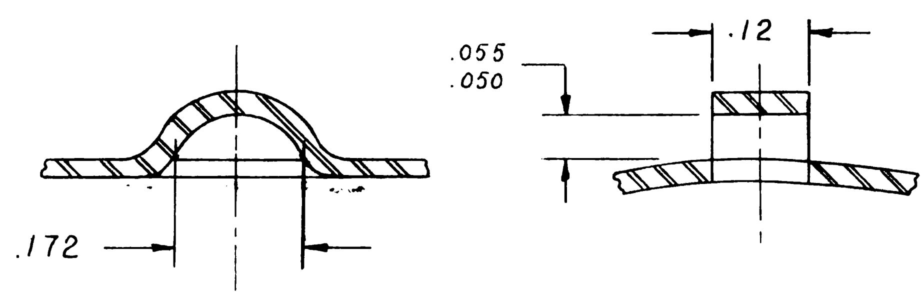
An important remark should be made in connection with sound velocity. With the fluctuations in temperature in the test room slight fluctuations in sound velocity resulted. Inasmuch as most of the testing took place in temperatures between 55° and 60° F a sound velocity of 1115 feet/sec was chosen for all calculations. It is recognized that minor inaccuracies may result from not adjusting sound velocities; these inaccuracies are however kept to a minimum with the narrow range of temperature fluctuation.

2. Acoustical Value of Louvers

The test results on the mufflers of Figures 12 and 13 have been compared to those of muffler of Figure 14. The results show identical performance over a frequency range from 50 to 650 cps. Because of the fact that by the time 650 cps is reached the physical dimensions of the mufflers are no longer negligible in proportion to wave lengths, plane wave theory is not probably acceptable in that range. Further more the objectionable and troublesome sound intensities in automobile mufflers lie in a frequency range below 650 cps. Hence no discussion or evaluation of performance beyond 650 cps was attempted.

From these test results, it can be concluded that, in the absence of gas flow, the commonly used louver designs, a cross-section of which is shown in Figure 26, show no acoustical value other than that shown by a series of holes distributed over the same length of pipe, and having the same outlet area, we can further conclude that the diameter of these holes is not of importance within the limits from 1/8" to 1/4" .

FIGURE 26



It should be recognized that the above statement does not detract from the value of louvers in actual engine-exhaust systems.

3. Cross-Bleed Chamber

Referring again to the muffler of Figure 12 and comparing its' attenuation curve to that of the unit in Figure 15 as well as to the calculated value of the curve of this expansion chamber it can be concluded that the louver fed muffler of Figure 12 behaves in a fashion identical to the characteristics of an expansion chamber. This muffler however constitutes the center section of the so-called cross-bleed chamber.

In order to further investigate the overall characteristic of the cross-bleed chamber, the mufflers in Figures 16 and 17 were constructed.

The analytical results closely duplicate test figures for both units. One important consideration is that end corrections must be applied to the length of the internal connectors between the two chambers. It should further be emphasized that the actual length of these connectors, which in fact determines the location of the louver groups has a definite influence on the muffler frequency response. The standard features of the cross-bleed chamber definitely result into characteristics of expansion chambers.

4. Turn-around Chambers

The next muffler in the series is shown in Figure 18. This muffler incorporates both the cross-bleed feature and the so-called turn-around chamber. Closer analysis of this design reveals that the turn-around chamber does not in itself allow through-passage of the sound waves. Instead the volume V of the turn-around section is excited by the sound wave through the "connector length" l of the internal pipes. To this excitation the volume V reacts as a spring and we have in fact a true

Helmholtz Resonator fed through two tuning tubes. Treating this chamber accordingly reasonable agreement is found between calculated and test values. Progressing in the direction of the complete muffler we now investigate the mufflers in Figures 19 and 20. Both of these mufflers consist of the cross-bleed and turn-around chambers in addition to a third chamber. This third chamber is a simple expansion chamber in Figure 19. But in Figure 20 this chamber becomes a second type of turn-around chamber. Here the sound waves are in effect bounced off the end wall and the reflected wave becomes the source of the muffler output signal or the transmitted wave. This chamber continues to act as an expansion chamber however.

5. General Appro ch to Predicting Muffler Performance

As mentioned in Chapter IV, a complete muffler design is shown in Figure 25. Identical reasoning as that which resulted into successful mathematical prediction of muffler frequency response was used to arrive at this muffler's system of equations. This final example was shown to illustrate that the various theoretical approaches which were developed in this program can be applied to commerical designs. To our knowledge this is the first time that a report on an attempt at capturing all theoretical aspects of a commercial muffler was published.

As a final note it should be pointed out that the muffler in Figure 20, the last one of our test series begins to show good broad-band attenuation characteristics required of a good silencer. It becomes understandable that with the introduction of a finite tailpipe, extra attenuation may be required to eliminate altogether the objectionable effects of tailpipe resonance.

Any actual muffler system should be analyzed in total. Both the muffler and the tailpipe can be coupled in one system of equations. Some day it may even be possible to properly introduce the characteristics of the source in the matrix. To this end more research will be necessary.

6. Major Variables Not Accounted For

It has been stressed that the investigation which forms the subject of this thesis has been deliberately restricted to "cold" evaluation of the muffler circuits. It may be argued that such an evaluation cannot be conclusive in its contribution to practical muffler development.

The basic objective was to determine how useful cold testing really can be, and how through a purely analytical approach, with the help of computers, the true acoustical value of complex muffler circuits could be predicted. Having been successful in this effort, we would now like to discuss the major variables which were ignored in this study.

In commercial engine exhaust systems, we are always confronted with at least four major considerations, not considered in this study.

1 - Elevated and widely fluctuating temperatures.

2 - Moving medium of a density different from air.

3 - Finite termination.

4 - A sound source generating a very irregular wave pattern at intensities higher than those used in the study.

Add to these considerations the fact that restrictions and cross section changes in the pipes are likely to have some importance, and that the ultimate sound source, at the junction of the cross over pipe, is even more irregular and unpredictable than the individual cylinder exhaust system, then it is easy to see that we now have a system which is far removed from the "ideal" exhaust system used for this study.

Looking at these complications of the problem one by one, the task of ultimately making useful predictions does not seem impossible.

Inasmuch as a muffler would ideally present a very substantial degree of attenuation between 50 and 500 cps (frequencies most likely to be objectionable in automobile exhaust systems) the job of designing a good muffler system can be partially solved by recognizing the important features of the specific installation. For instance, we know that the finite termination will result into a tailpipe resonance. This far away from the engine, it is possible to establish an expected temperature range for all driving conditions up to say 35 miles per hour.* On the basis of this temperature range (which, by means of thermocouples can easily be established under actual conditions), and a correction factor to accommodate the moving medium, the tailpipe resonance range is quite easily calculated. On page 1 of Appendix 1 a sample calculation is made showing the tailpipe resonance for a 4 feet long tailpipe under the assumed temperature range of from 200 - 400° and gas flow from 100 to 200 feet / second to be from 166 cycles / second to 199 cycles / second. The resonant frequency of a Helmholtz Resonator is a linear function of sound velocity, and the frequency of maximum attenuation of an expansion chamber is determined by an attenuation function

$$\sin^2 kl = \sin^2 \frac{2\pi fl}{c} = 1$$

This means that if a Helmholtz Resonator was properly tuned to 166 cp at a resonator temperature equivalent to a 200° F tailpipe temperature, then the resonant frequency at a resonator temperature equivalent to a 400° F tailpipe temperature would be 199 cps. and with the assumption of proportional temperatures (and gas velocities - hence sound velocities) our resonator is still tailored for the job. Similarly let $\sin^2 \frac{2\pi fl}{c} = 1$.

*The 35 mile per hour limit was chosen for two reasons, at higher speeds, noises other than exhaust are becoming more and more important, for

$$f = 166 \text{ cps and } c = 1360 \text{ ft/sec}$$

in the tailpipe, then

$$\sin^2 \frac{2\pi f l}{c} = 1 \text{ for } f = 199 \text{ cps}$$

and

$$c = 1635 \text{ ft/sec}$$

as assumed in the sample calculations.

The conclusion then is that a finite termination can be allowed for in the muffler design even accommodating fluctuations in gas velocities and in temperature.

This example of analysis illustrates that, to the extent that temperature changes, gas velocities, and gas densities affect sound velocity, they do not present a major departure from the theory applicable to cold testing.

Gas velocities through an extremely complicated path of flow, such as that present in an average passenger car muffler system will however also cause substantial variations in medium densities. Even though we did not elaborate on the implications of transmission of an acoustical wave through various media, it can be readily understood that, if the media show a fairly strong discontinuity, characteristic impedances in the system will change and new, unaccounted for reflections will be set up. It is our belief that most of the pressure changes in the system, even though severe will have as their major effect localized fluctuations in sound velocities; and that from an acoustical standpoint, predictability of performance would not be irrevocably lost. The most important problem in our efforts to "design mufflers by computer" is probably that of the always recurring subjective nature of sound, or of its' interpretation.

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*while exhaust frequencies are generally getting high enough to make the average muffler quite adequate.

It is unfortunate that, of these considerations which affect our subjective rating of a sound the most, the ones which are the least predictable with present methods are those which seem to be the most affected by the engine environment. Specifically loudness, frequency, pitch, and timbre may be considered the vital characteristics in human interpretation of a sound, or a sound level.

Of these, loudness is the one we attack squarely by introducing high attenuation. By locating the point of highest attenuation in the range of the most objectionable frequency, we are in effect hitting two out of the four characteristics quite successfully. Recognizing that pitch shows a fair relationship to frequency, we may even be successful in bringing the pitch of the exhausted sound within acceptable limits. Timbre, however, is a very complex characteristic. Although it is primarily a function of the wave form, itis also a function of intensity and frequency. Various adjectives such as mellow, brilliant, brassy, tinny, etc., describe best the effect of timbre on our hearing. We know that the engine as a sound source was never designed to make muffling easy. (A consideration which may in effect be very useful as a topic of study. Indeed, the efficiency of an engine does, at least to some extent, depend upon the efficiency of the exhaust system, and that efficiency will be greater if maximum advantage is taken of the analysis of exhaust acoustics and muffling).

Wave forms are a result of engine design (including manifold design), but they are not a consideration during the design process. Also, the side-effect of conducting gases at high velocity, is a generation of sound as the gases pass obstacles, "whistle" by open holes etc. It is for this consideration that louvers have been introduced into muffler design. They give us the equivalent of holes, necessary in acoustical chambers while eliminating the danger of the above generation of sound.

It must be recognized, that the flow of gas in an exhaust system is of a pulsating nature. These pulsations are caused by the sudden introduction of a new "slug" of gas at each opening of an exhaust port. These pulsations establish in the exhaust system, the frequency commonly called the firing frequency (firing frequency is equal to exhaust-port opening frequency. It is, however, for the sound in an exhaust system a not too accurate term). It is primarily at this frequency (and some of its' harmonics) that we are trying to reduce the magnitude of the sound level. To reduce the magnitude of the pulsations is to silence the exhaust system noise. With the high volume of gas discharged, we can consider this problem as a combination problem for the expert in acoustics and the expert in gas dynamics. A good intimate knowledge of the characteristics of the muffler chambers (a study presented in this thesis) will ultimately lay the ground work for good economical muffler design. The particular features which are required to contribute to the subjective approval of muffler design can be better identified and incorporated in the design considerations. The nature of the sound source must be studied in more detail.

A criterion of analysis must be established for this to enable the muffler designer to take into account the features of importance to him. With due consideration to the sound source, and the introduction of features to make the exhausted sound level "more pleasing" the basic sound attenuating circuit should ultimately be designed by mathematical analysis and solution.

Computer programs, now set up should be utilized to determine the effect of the many parameters involved. Upon completion of that program the optimum combination can be slowly approached first mathematically, then physically. The reward is undoubtedly a smaller muffler for equal efficiency, or a better muffler for equal size.

The economical implications of this result are substantial. The time that muffler design was done by trial and error only is rapidly coming to an end.

CONCLUSIONS AND RECOMMENDATIONS

The following basic conclusions can be reached as a result of this study.

- The theories, developed in ref. No. 2 can be applied to silencing systems of physical dimensions prevailing in passenger car exhaust systems.
- It was possible to build a test system with sufficient reliability and consistency to conduct test programs of long duration at sound levels up to 160 db. The infinite tailpipe construction finally arrived at behaved in a very satisfactory manner.
- The electronic and electrical apparatus used showed itself equal to the task. The protective circuit (monitor microphone system) is very strongly recommended to afford protection for the vulnerable loud speaker system.
- The test program shed more light on the contribution of louvers, cross-bleed chambers, turn-around chambers and resonators.
- In addition, with every one of these chambers analyzed individually, it now is possible to use the same approach to set up systems of equations of practically any combination of these chambers. In summary, we can conclude that cross-bleed chambers are essentially expansion chambers, turn-around sections are basically resonators (if they are used on either side of a cross-bleed chamber which in itself already allows turn-around). Louvers, of the dimensions used, do not have any acoustical value as long as gas flow is not introduced as a variable.
- Finally system analysis can now be extended to commerical units.

The approach to system synthesis will require additional design criteria; the basic building blocks are available however to predict success for such an effort.

As recommendations for future programs it would seem extremely interesting to utilize the knowledge gained in this study to develop muffler systems which would show good theoretical attenuation characteristics for a specific exhaust system. These units should consequently be "hot tested" to study what auxiliary chambers and features can be introduced to make the exhausted noise more pleasing. After it has been established what is needed to accomplish this, it is anticipated that muffler design work will become quicker and more successful while the cost of the design program as well as the cost of the ultimate configuration will be substantially reduced. At least the design would, in all its details, be scientifically justifiable, something which is certainly not true with the results of present endeavors.

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APPENDIX

SAMPLE CALCULATION OF TAILPIPE RESONANCE

Let the tailpipe length l = 4 Feet

Diameter = 4 Inches

or Radius A = 2 Inches

Let the tailpipe temperature vary from 200°F to 400°F and the gas velocity from 100 ft/sec to 200 ft/sec.

From Chapter I page 14 we know

$$f_R = \frac{c}{2(l + .6a)}$$

For an open ended unflanged pipe.

c = sound velocity = $\sqrt{49}$ T or

$$C_{min} = 1260 + 100 = 1360 \text{ ft/sec}$$

$$C_{max} = 1435 + 200 = 1635 \text{ ft/sec}$$

$$l + .6a = 4 + .6x\ .167 = 4.1$$

So $$f_{R\ min} = \frac{1360}{8.2} = 166 \text{ cycles/sec}$$

$$f_{R\ max} = \frac{1635}{8.2} = 199 \text{ cycles/sec}$$