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MULTINOMIAL LOGIT MODEL

BY

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THESIS

TABLE OF CONTENTS

ABSTRACT

MULTUINOMIAL LOGIT MODEL

By

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The principle of multinomial logit model is to model the choice behavior of individuals. Knowing the difficulty and great expense in obtaining survey data, the author attempts to use the widely available but more aggregated census tract data in U.S.A. for estimating the choice pattern of housing consumer in Chicago SMSA with special reference to its relation to transportation to the Central Business District (CBD) in Chicago.

The results review that the mode "drive along" has the largest effect on the people's choice of rental housing market location in the area as compared to the other four modes; carpool, bus, train and walk. The rent and travel cost show even more profound effect on the utility level of the choice of rental housing location.

INTRODUCTION

The widely used Multinomial Logit (MNL) Model in modeling housing market and transportation mode choice (with special purpose such as shopping, work, recreation, business) considers choice of housing as independent to the choice of mode with special travel purpose. These models assume sequential choice pattern, which is operationally more efficient than simultaneous choice pattern. In fact, not many research has been done on the difference between sequential choice model and simultaneous choice model. Manheim has shown that when certain conditions are satisfied, the two approaches are equivalent. He ponited out that

> "Ideally a single simultaneous-choice model incorporating all relevant choice dimension would be utilized. This is impractical because of the large number of dimensions in real situations. In practice, some sequential models must be utilized."(Manheim,M p.421)

Alex Anas in his Residential Location Market And Urban Transportation, attempts to consider both the choice of mode to working in CBD and the choice of housing location in Chicago SMSA in his model.

He employed 1970 census tracts data in the Chicago

SMSA to estimate the utility coefficients in his model. Zonal aggregated attributes as well as disaggregate attributes are used, and are compared in his work. He comes up with the probable conclusion that

> " errors in the estimated coefficients arising from the aggregation of certain attributes (such as rent) to the zonal level appear to be smaller or comparable in magnitude to errors arising purely from sampling"
> (Anas,A p.159)

Based on his observation, this paper attempts to use 1980 census data in the Chicago SMSA to estimate the residential location markets and urban transportation choice behavior in Chicago SMSA.

THE MODEL

The sequential choice pattern of the individual is assumed to take the following form

Step 1: Choice of the i<u>th</u> residential neighborhood from the existing residential market (in this case, the i<u>th</u> census tract)

Step 2: Choice of the k<u>th</u> dwelling unit given the choice of i<u>th</u> neighborhood

Step 3: Choice of the m<u>th</u> mode for work trip to CBD given the choice of ith neighborhood and the kth dwelling unit

Two utility functions appear in the choice model

$$U_i = \sum_u X_{iu} \cdot \alpha_u$$

is a measure of the utility level for each individual choosing the ith neighborhood. This utility depends on the socio-economic status of the individual which is incorporated into the vector attributes $\vec{X_i}$. Examples of these attributes include income, rent of housing, family size, housing quality etc. α's are the coefficients to be estimated.

$$U_{im} = \sum_v X_{imv} \cdot \beta_v$$

is a measure of the utility level for each individual choosing the ith neighborhood, mth mode of travel to the CBD. The vector $\vec{X}_{im}$ consists of zonal attributes that depend on mode of travel. Most commonly used attributes of this kind are travel cost, travel time and dummy variables for each mode.

The estimated probabilities that associate with the above three-step choice model are given by respectively, $\hat{P}_i$, $\hat{P}_{k|i}$, and $\hat{P}_{m|ik}$ (1). The magnitude of these probabilities depend on the utility which the individual can derive from his choice.

With the log-likelihood function defined as

$$\log L = \sum_i \sum_m \tilde{N}_{im} \log \hat{P}_{m|i} + \sum_i \tilde{N}_i \log \hat{P}_i + \text{constant}$$

one could solve for the coefficients β's and α's by the maximimun likelihood method. This is equivalent to finding the solutions for the following set of maximization equations(2)

$$\frac{\partial \log L_1}{\partial \beta_v} = 0 \qquad \text{for all } v$$

$$\frac{\partial \log L_2}{\partial \alpha_u} = 0 \qquad \text{for all } u$$

where L_2 is the marginal part of the likelihood function and is given by the second term of in the log-likelihood function, whereas L_1 is the conditional part of the function, and is given by the first term of the function.

By using two step method(3), the equations can be solved by using numerical method.

Newton-Raphson method is used for solving the above non-linear equations. The iteration equation for finding β's is as follows

$$\vec{\beta}_{n+1} = \vec{\beta}_{n} - (\nabla^2 \log L_1)^{-1} \cdot \nabla \log L_1$$

where n stands for the n<u>th</u> iteration. $\nabla \log L_1$ is the gradient of the log-likelihood function ,and $\nabla^2 \log L_1$ is the variance covariance matrix of the estimated coefficients.(4)

For testing the significance of the estimated coefficients, one can use the t-statistic to test the hypothesis

H_o : =0 for all v by using $t_1 = \beta_i / s_1$

where

s_1 is the square root of the i<u>th</u> diagonal elements in the variance-covariance matrix and t_1 is the standard t-statistic.

DATA

Table 1 summarizes the attributes that are being used in this paper.

Table 1

Attributes For Calibration

Attributes for zone specific utility u_i (X_{iu})	Attributes for zone-mode specific utility u_{im} (X_{imv})
Number of rooms in the census tract $(room)_i$	Dummy variable for each mode of travel (D_{im})
Per capita income for family $(income)_i$	Travel cost to CBD $(cost)_{im}$
	Travel time to CBD $(time)_{im}$
	Rent $(rent)_i$

Five modes are generally available to residents living in the Chicago SMSA: car(mode 1), car-pool(mode 2), bus(mode 3), train(mode 4), walk(mode 5).

For those residents who are not living in the CBD, it is assumed that mode 5(walk) is not in their choice set.

The rent for census tract i is calculated from

$$R_i = f_i r_i + (1-f_i)V_i /10$$

where f_i is the proportion of zone i's occupied dwellings that are renter occupied; r_i , the average annual rent of these renter occupied dewllings; and V_i ,the average annual market value of the owner occupied dwellings in zone i. This formula is a rule of thumb for estimating zonal rent.

The attributes, number of rooms, per capita income and rent are available in census. However, travel cost and time to CBD are not available. These data have to be collected from Chicago Transportation Authority Study(CATS). The CATS does not provide zonal travel cost and time. It provides a more aggregated seven-sector division of the Chicago SMSA, and from which the travel cost and time for each census are estimated. The seven sectors are, CBD, North Chicago, South Chicago, North-west Chicago, Western sector, South-west sector and South sector of Chicago SMSA.

It has been studied extensively that a sample size of about 300 can give accurate estimation for the result. A sample size of 293 census tracts is chosen randomly from the existing census tracts(about 4920) with the number of work trips to CBD from each tract as the weights of choosing them.

The utility function U_i and U_{im} read

$$U_i = \alpha_1 \cdot Income_i + \alpha_2 \cdot Room_i$$

$$U_{im} = \beta_1 \cdot \log(Rent_i + Cost_{im}) + \beta_2 \cdot (Time_{im}) + \beta_3 \cdot D_{i2} + \beta_4 \cdot D_{i3} + \beta_5 \cdot D_{i4} + \beta_6 \cdot D_{i5}$$

THE RESULT

Since the fortran program for this paper(see appendix) uses the Newton-Raphson approximation method to solve for β's, and the numerical method used does not gurantee feasibility of solution, the computation ends up with an approximation for β's(5) values as follows

(-256, 14.2, -6.05, -17.8, -16.4, -48.5)

Checking the sign and relative magnitude of the coefficients for the four dummy variables, -6.05(car pool), -17.8(bus), -16.4(rail), -48.5(walk), one can find that this is consistent with one's intuition in that all these modes give negative utility as compared to the reference mode, drive along(mode 1), which takes up zero for comparison purpose. The relative magnitude for each mode reveals the individuals' preferences(measured in terms of utility) towards different modes. The smaller the coefficient is, the less satisfaction one can derive from choosing the mode.

The coefficient for log(Rent+Travel Cost) is very small(-256), this means that people are very sensitive to the change in rent and travel cost. A small change in these values would decrease(or increase) their utility level by a great amount.

Since the iteration process in the numerical method does not converage to a unique solution, further inference on the values and the significance of these estimated coefficients are not possible.

CONCLUSIONS

Although the computation ends up with an estimation of the coefficients for the zone-mode specific utility U_{im} ,the result agrees with what most studies have identified by using similar model.

Because of the scope of this paper, the demand side of the model has been investigated while the supply side of the model was left aside. Thus care must be taken when one tries to compare the results of this paper to those obtained by A.Anas because he applied the equilibrium model on the 1970 data, which take care of both the demand side and the supply side of the model.

The aggregated data used in this paper, the census tract data, give reasonably well estimation. The use of aggregated data at the census tract level can be widely adopted if some of the necessary travel attributes such as zonal travel time for work trips to CBD and zonal travel cost to CBD for each modes, are also available in the census tract.

NOTES

1. The exact forms for these probabilities are

$$\hat{P}_i = S_i^{\gamma} \exp(U_i + (1-\sigma) I_i) / \sum_j S_j^{\gamma} \exp(U_j + (1-\sigma) I_j)$$

$$\hat{P}_{k|i} = 1/S_i$$

$$\hat{P}_{m|ik} = \exp(U_{im}) / \sum_m \exp(U_{im})$$

where $I_j = \log \sum_m \exp(U_{jm})$

with $\gamma = (1-\tau)$

$\sigma = 1-(1-\tau)(1-\theta)$

where σ is a measure of correlation among unobserved travel mode related attributes within a zone

τ is a measure of similarity of dwelling units in unobserved random attributes

θ is a measure of mode similarity in unobserved random utilities.

Notice that the $P_{k|i}$ is aggregated probabilities because within-zone variation of dwelling units cannot be observed with the census data

2. The exact forms of the equation read

$$\frac{\partial}{\partial\beta_v}\left(\sum_i \sum_m \tilde{N}_{im} \log \hat{P}_{m|i}\right)=\sum_i \tilde{P}_i \sum_m (\tilde{P}_{m|i}-\hat{P}_{m|i})X_{imv}$$

$$= 0 \quad \text{for all } v$$

$$\frac{\partial}{\partial\alpha_u}\left(\sum_i \tilde{N}_i \log \hat{P}_i\right) = \sum_i (\tilde{P}_i-\hat{P}_i)X_{iu}$$

$$=0 \quad \text{for all } u$$

$$\frac{\partial}{\partial\gamma}\left(\sum_i \tilde{N}_i \log \hat{P}_i\right) = \sum_i (\tilde{P}_i-\hat{P}_i)\log S_i$$

$$=0$$

$$\frac{\partial}{\partial\sigma}\left(\sum_i \tilde{N}_i \log \hat{P}_i\right) = \sum_i (\tilde{P}_i-\hat{P}_i)I_i$$

$$= 0$$

where $\hat{P}_{m|i}$ represents the estimation of the probability by using census data; $\tilde{P}_{m|i}$ and $\tilde{P}_i$ are given by

$$\tilde{P}_i = \sum_m \tilde{N}_{im}/\tilde{N}$$

$$\tilde{P}_{m|i} = \tilde{N}_{im}/\sum_m \tilde{N}_{im}$$

and

$\tilde{N}_i$ = total work trip to CBD from ith census tract

$\tilde{N}_{im}$ = total work trip to CBD from ith ecnsus tract through mode m

Forcing γ to equal 1 and forcing σ to equal 0, one needs only to solve the first two sets of the above equations. This is justified on the ground that γ and σ ususlly take up 1 and 0 respectively in most studies, and this could facilitate the calculation in this paper. The reader can find similar examples in A.Anas's book listed in the Bibliography.

3. One step method involves solving the β's and α's at the same time whereas the two step method solve for β's and substitute these values into the other set of equations for solving the rest of the coefficients. The one step approach always give superior result to the two step approach, yet the computation is more complicated. This paper uses the two step approach for simplicity. Notes 3 indicates the two step approach that is used in this paper.

4. The exact form of the matrix is

$$=\sum_i \tilde{P}_i\left(\left(\sum_m X_{imv}\hat{P}_{m|i}\right)\left(\sum_m X_{imu}\hat{P}_{m|i}\right)-\sum_m X_{imv}X_{imu}\hat{P}_{m|i}\right)$$

for all u,v for β's

5. The corresponding values for $\frac{\partial \log L_1}{\partial \beta_v}$ are

(-0.0391, 0.0399, -0.0722, -0.1442, -0.1329, -0.0406)

which is reasonably close to zero vector.

APPENDIX A

EXPLAINATION OF SYMBOLS USED IN THE PROGRAM

I= i<u>th</u> census tract ,i= 1 to 293
M= m<u>th</u> mode ,m= 1 to 5

DA(I,1)= per capita income
DA(I,2)= number of rooms in census tract
DA(I,3)= number of dwelling unit in census tract(S_i)
DA(I,4)= $\tilde{N}_{i1}$
DA(I,5)= $\tilde{N}_{i2}$
DA(I,6)= $\tilde{N}_{i3}$
DA(I,7)= $\tilde{N}_{i4}$
DA(I,8)= $\tilde{N}_{i5}$

DATA(I,1)= annual rent
DATA(I,2)= travel mode

DLIKFU(U)=first derivative of log-likelihood with respect to β_u
D2(U,V)= variance-covariance matrix for solving α's
D2IN(U,V)= inverse of D2(U,V)

TIME(I,M)= travel time to CBD from zone i through mode m
COST(I,M)= travel cost(annual) to CBD from zone i through mode m

PI(I)= $\tilde{P}_i$
PIM(I,M)= $\tilde{P}_{mi}$
ESTI(I)= $\hat{P}_i$
ESTPIM(I,M)= $\hat{P}_{mi}$

EI(I)= I_i

ALPHA(U)= α's
BETA(U)= β's

FUNCTIONS:

UTILI(I)= U_i
UTILIM(I,M)= U_{im}
DI2= D_{i2}, DI3= D_{i3}, DI4= D_{i4}, DI5= D_{i5}

Others are variables necessary for the computation process, and they carry no specific meaning.

MAIN PROGRAM

```
       CHAN,PN1315683,rg2,jc499.
       attach,mnlda1,mnlA2.
       attach,MNLdat,mnlB2.
       ftn5.
       lgo.
       listty,i=tape6.
       PROGRAM MNL3

       real da(293,8),data(293,2),dlikfu(6),beta(6),residue(6)
       real delta(2),d1mafun(2),time(293,5),cost(293,5),PIM(293,5)
       real estpim(293,5),sum(293,6),d2(6,12),d2in(6,11),PI(293)
       real WTMASUM(2),W2MASUM(2,2),D2MAFUN(2,2),ALPHA(2)
       REAL EI(293),ESTPI(293)
       common/com1/beta,data,time,cost
       common/COM2/ALPHA,DA
       open(7,file='mnlda1')
       open(10,file='mnldat')
       OPEN(6,FILE='OUTPUT')
       do 101 i=1,293
101    read(7,102) DA(I,1),DA(I,2),DA(I,4),DA(I,5),DA(I,6),DA(I,7),
     C DA(I,8),DA(I,3)
102    format(F6.0,F3.1,6F5.0)
       do 103 I=1,293
103    read(10,104) DATA(I,1),DATA(I,2)
104    format(F6.0,9X,F2.0)
       DO 899 I=1,293
       DA(I,1)=DA(I,1)
899    DATA(I,1)=DATA(I,1)
       alpha(1)=0
       alpha(2)=-0.05
       do 111 i=1,6
111    residue(i)=0.0
       DO 898 I=1,293
       DA(I,1)=DA(I,1)/365
898    DATA(I,1)=DATA(I,1)/365
       BETA(1)=-0.05
       BETA(2)=-0.05
       BETA(3)=0.0
       BETA(4)=0.0
       BETA(5)=0.0
       BETA(6)=0.0
       do 105 I=1,293
       if(I.lt.10)then
       TIME(I,1)=0.82
       TIME(I,2)=0.82
       TIME(I,3)=0.77
```

```
      TIME(I,4)=0.63
      COST(I,1)=2.62
      COST(I,2)=2.92
      COST(I,3)=1.59
      COST(I,4)=1.35
      else if(I.lt.134)then
      TIME(I,1)=0.47
      TIME(I,2)=0.47
      TIME(I,3)=0.77
      TIME(I,4)=0.37
      TIME(I,5)=0.25
      COST(I,1)=2.12
      COST(I,2)=1.79
      COST(I,3)=1.59
      COST(I,4)=1.35
      COST(I,5)=0
      else if(I.lt.235)then
      TIME(I,1)=0.98
      TIME(I,2)=0.98
      TIME(I,3)=1.05
      TIME(I,4)=0.9
      COST(I,1)=2.65
      COST(I,2)=2.41
      COST(I,3)=1.59
      COST(I,4)=1.5
      else if(I.lt.272)then
      TIME(I,1)=1.67
      TIME(I,2)=1.67
      TIME(I,3)=1.25
      TIME(I,4)=0.9
      COST(I,1)=3.3
      COST(I,2)=3.00
      COST(I,3)=1.59
      COST(I,4)=1.5
      else if(I.lt.276)then
      TIME(I,1)=2.5
      TIME(I,2)=2.5
      TIME(I,3)=1.7
      TIME(I,4)=1.37
      COST(I,1)=4.13
      COST(I,2)=4.16
      COST(I,3)=1.59
      COST(I,4)=1.89
      else
      TIME(I,1)=1.57
      TIME(I,2)=1.57
      TIME(I,3)=1.2
      TIME(I,4)=1.28
      COST(I,1)=3.42
      COST(I,2)=3.16
      COST(I,3)=1.59
      COST(I,4)=1.81
      end if
105   continue
      TOTAL=0.0
      do 210 I=1,293
      PI(I)=0.0
      do 211 J=1,5
211   PI(I)=PI(I)+DA(I,J+3)
```

```
210     TOTAL=TOTAL+PI(I)
        do 311 I=1,293
        do 310 J=1,5
310     PIM(I,J)=DA(I,J+3)/PI(I)
311     PI(I)=PI(I)/TOTAL
999     do 212 I=1,293
        if(I.LT.10)THEN
        K=4
        else IF(I.GT.133)THEN
        K=4
        ELSE
        K=5
        end if
        A=0
        do 213 M=1,K
213     A=A+EXP(UTILIM(I,M))
        do 214 M=1,K
214     ESTPIM(I,M)=EXP(UTILIM(I,M))/A
212     continue
        do 777 I=1,293
        do 778 L=1,6
778     SUM(I,L)=0.0
777     continue
        do 216 I=1,293
        if(I.LT.10)THEN
        K=4
        else IF(I.GT.133)THEN
        K=4
        ELSE
        K=5
        end if
        do 215 M=1,K
        SUM(I,1)=(PIM(I,M)-ESTPIM(I,M))*LOG(DATA(I,1)+COST(I,M))+SUM(I,1)
        SUM(I,2)=SUM(I,2)+(PIM(I,M)-ESTPIM(I,M))*TIME(I,M)
        SUM(I,3)=SUM(I,3)+(PIM(I,M)-ESTPIM(I,M))*DI2(M)
        SUM(I,4)=SUM(I,4)+(PIM(I,M)-ESTPIM(I,M))*DI3(M)
        SUM(I,5)=SUM(I,5)+(PIM(I,M)-ESTPIM(I,M))*DI4(M)
215     SUM(I,6)=SUM(I,6)+(PIM(I,M)-ESTPIM(I,M))*DI5(M)
216     continue
        do 218 L=1,6
        DLIKFU(L)=0.0
        do 217 I=1,293
217     DLIKFU(L)=SUM(I,L)*PI(I)+DLIKFU(L)
218     continue
        WRITE(6,555)DLIKFU(1),DLIKFU(2),DLIKFU(3),DLIKFU(4),DLIKFU(5),
      C DLIKFU(6)
555     FORMAT(6F8.4)
        do 388 I=1,293
        do 389 L=1,6
389     SUM(I,L)=0
388     continue
        do 401 I=1,293
        if(I.LT.10)THEN
        K=4
        else IF(I.GT.133)THEN
        K=4
        ELSE
        K=5
        end if
```

```
      do 402 M=1,K
      SUM(I,1)=SUM(I,1)+ESTPIM(I,M)*LOG(DATA(I,1)+COST(I,M))
      SUM(I,2)=SUM(I,2)+ESTPIM(I,M)*TIME(I,M)
      SUM(I,3)=SUM(I,3)+ESTPIM(I,M)*DI2(M)
      SUM(I,4)=SUM(I,4)+ESTPIM(I,M)*DI3(M)
      SUM(I,5)=SUM(I,5)+ESTPIM(I,M)*DI4(M)
402   SUM(I,6)=SUM(I,6)+ESTPIM(I,M)*DI5(M)
401   continue
      do 701 L=1,6
      do 702 N=1,12
702   D2(L,N)=0
701   continue
      do 610 I=1,6
610   D2(I,I+6)=1.0
      do 603 L=1,6
      do 602 N=1,6
      do 601 I=1,293
601   D2(L,N)=D2(L,N)+SUM(I,L)*SUM(I,N)*PI(I)
602   continue
603   continue
      do 403 I=1,293
      if(I.LT.10)THEN
      K=4
      else IF(I.GT.133)THEN
      K=4
      ELSE
      K=5
      end if
      do 404 M=1,K
      D2(1,1)=D2(1,1)-LOG(DATA(I,1)+COST(I,M))*LOG(DATA(I,1)+
     C COST(I,M))*PI(I)*ESTPIM(I,M)
      D2(1,2)=D2(1,2)-PI(I)*LOG(DATA(I,1)+COST(I,M))*TIME(I,M)*
     C ESTPIM(I,M)
      D2(1,3)=D2(1,3)-PI(I)*LOG(DATA(I,1)+COST(I,M))*DI2(M)*
     C ESTPIM(I,M)
      D2(1,4)=D2(1,4)-PI(I)*LOG(DATA(I,1)+COST(I,M))*DI3(M)*
     C ESTPIM(I,M)
      D2(1,5)=D2(1,5)-PI(I)*LOG(DATA(I,1)+COST(I,M))*DI4(M)*
     C ESTPIM(I,M)
      D2(1,6)=D2(1,6)-PI(I)*LOG(DATA(I,1)+COST(I,M))*DI5(M)*
     C ESTPIM(I,M)
      D2(2,2)=D2(2,2)-TIME(I,M)*TIME(I,M)*PI(I)*ESTPIM(I,M)
      D2(2,3)=D2(2,3)-TIME(I,M)*DI2(M)*PI(I)*ESTPIM(I,M)
      D2(2,4)=D2(2,4)-TIME(I,M)*DI3(M)*PI(I)*ESTPIM(I,M)
      D2(2,5)=D2(2,5)-TIME(I,M)*DI4(M)*PI(I)*ESTPIM(I,M)
      D2(2,6)=D2(2,6)-TIME(I,M)*DI5(M)*PI(I)*ESTPIM(I,M)
      D2(3,3)=D2(3,3)-DI2(M)*PI(I)*ESTPIM(I,M)
      D2(4,4)=D2(4,4)-DI3(M)*PI(I)*ESTPIM(I,M)
      D2(5,5)=D2(5,5)-DI4(M)*PI(I)*ESTPIM(I,M)
404   D2(6,6)=D2(6,6)-DI5(M)*PI(I)*ESTPIM(I,M)
403   continue
      do 405 I=1,2
      K=I+1
      do 406 J=K,6
406   D2(J,I)=D2(I,J)
405   continue
      DO 443 I=1,6
443   WRITE(6,444)D2(I,1),D2(I,2),D2(I,3),D2(I,4),D2(I,5),D2(I,6)
444   FORMAT(6F10.4)
```

```
      do 709 M=1,6
      J=12-M
      do 704 L=1,J
704   D2IN(6,L)=D2(1,L+1)/D2(1,1)
      do 705 I=1,5
      do 706 L=1,J
706   D2IN(I,L)=D2(I+1,L+1)-D2(1,L+1)*D2(I+1,1)/D2(1,1)
705   continue
      do 707 K=1,6
      do 708 N=1,J
708   D2(K,N)=D2IN(K,N)
707   continue
709   continue
      EPSILON=0
      do 714 J=1,6
      RESIDUE(J)=0.0
      do 713 I=1,6
713   RESIDUE(J)=D2(J,I)*DLIKFU(I)+RESIDUE(J)
      BETA(J)=BETA(J)-RESIDUE(J)
714   EPSILON=EPSILON+RESIDUE(J)*RESIDUE(J)/(BETA(J)*BETA(J))
      EPSILON=SQRT(EPSILON/6.0)
      WRITE(6,888)BETA(1),BETA(2),BETA(3),BETA(4),BETA(5),BETA(6),
     C EPSILON
888   FORMAT(7F8.4)
      IF(EPSILON.GT.0.001)GO TO 999
      do 802 I=1,293
802   EI(I)=0.0
      do 804 I=1,293
      if(I.LT.10)THEN
      K=4
      else IF(I.GT.133)THEN
      K=4
      ELSE
      K=5
      end if
      do 803 M=1,K
803   EI(I)=EI(I)+EXP(UTILIM(I,M))
      EI(I)=LOG(EI(I))
804   continue
998   ASUM=0.0
      do 710 I=1,2
      do 711 J=1,2
711   W2MASUM(I,J)=0.0
710   WTMASUM(I)=0.0
      do 801 I=1,293
801   ASUM=ASUM+DA(I,3)*EXP(UTILI(I)+Ei(I))
      DO 824 K=1,2
      do 822 I=1,293
822   WTMASUM(K)=WTMASUM(K)+DA(I,K)*DA(I,3)*EXP(UTILI(I)+EI(I))
824   continue
      DO 834 L=1,2
      DO 832 K=1,2
      do 833 I=1,293
833   W2MASUM(K,L)=W2MASUM(K,L)+DA(I,K)*DA(I,L)*DA(I,3)*
     C EXP(UTILI(I)+EI(I))
832   continue
834   continue
      do 806 I=1,2
      do 805 J=1,2
```

```
 805      D2MAFUN(I,J)=(WTMASUM(I)*WTMASUM(J)-W2MASUM(I,J))/ASUM
 806      continue
          DET=D2MAFUN(1,1)*D2MAFUN(2,2)-D2MAFUN(1,2)*D2MAFUN(2,1)
          D2IN(1,1)=D2MAFUN(2,2)/DET
          D2IN(2,2)=D2MAFUN(1,1)/DET
          D2IN(1,2)=(0.0-D2MAFUN(1,2))/DET
          D2IN(2,1)=(0.0-D2MAFUN(2,1))/DET
          do 809 I=1,293
 809      ESTPI(I)=DA(I,3)*EXP(UTILI(I)+EI(I))/ASUM
          do 811 J=1,2
          D1MAFUN(J)=0.0
          do 810 I=1,293
 810      D1MAFUN(J)=D1MAFUN(J)+(PI(I)-ESTPI(I))*DA(I,J)
 811      continue
          DIFF=0.0
          do 813 I=1,2
          DELTA(I)=0.0
          do 812 J=1,2
 812      DELTA(I)=D2IN(I,J)*D1MAFUN(J)+DELTA(I)
          ALPHA(I)=ALPHA(I)-DELTA(I)
 813      DIFF=DIFF+DELTA(I)*DELTA(I)/(ALPHA(I)*ALPHA(I))
          DIFF=SQRT(DIFF/2.0)
          if(DIFF.gt.0.001)GO TO 998
          XLIK=0.0
          do 814 I=1,293
          do 815 M=1,5
 815      XLIK=XLIK+DA(I,I+3)*LOG(DA(I,M+3)/TOTAL)
 814      continue
          ESTLIK=0.0
          do 825 I=1,293
          if(I.LT.10)THEN
          K=4
          ELSE IF(I.GT.133)THEN
          K=4
          else
          K=5
          end if
          do 816 M=1,K
 816      ESTLIK=ESTLIK+DA(I,M+3)*LOG(PI(I)*ESTPIM(I,M))
 825      continue
          CHISQ=2.0*(ESTLIK-XLIK)
          write(6,818)ALPHA(1),ALPHA(2),XLIK,ESTLIK,CHISQ
 818      format(2F8.4,3F10.4)
          write(6,819)BETA(1),BETA(2),BETA(3),BETA(4),BETA(5),BETA(6)
 819      format(8F10.4)
          stop
          end
          function UTILIM(I,M)
          real DATA(293,2),BETA(6),COST(293,5),TIME(293,5)
          COMMON/COM1/BETA,DATA,TIME,COST
          UTILIM=BETA(1)*LOG(DATA(I,1)+COST(I,M))+BETA(2)*TIME(I,M)+
        C BETA(3)*DI2(M)+BETA(4)*DI3(M)+BETA(5)*DI4(M)+
        C BETA(6)*DI5(M)
          return
          end
          function DI2(M)
          DI2=(M-1)*(M-3)*(M-4)*(M-5)/(-6.0)
          return
          ENTRY DI3(M)
```

```
DI3=(M-1)*(M-2)*(M-4)*(M-5)/(4.0)
return
ENTRY DI4(M)
DI4=(M-1)*(M-2)*(M-3)*(M-5)/(-6.0)
return
ENTRY DI5(M)
DI5=(M-1)*(M-2)*(M-3)*(M-4)/(24.0)
return
end
function UTILI(I)
real ALPHA(2),DA(293,8)
common/COM2/ALPHA,DA
UTILI=ALPHA(1)*DA(I,1)+ALPHA(2)*DA(I,2)
return
end
```

APPENDIX B

DATA FILE 1

COLUMN 7 = DA(I,3)
COLUMN 1 = ROOM
COLUMN 2 = DA(I,4)
COLUMN 3 = DA(I,5)
COLUMN 4 = DA(I,6)
COLUMN 5 = DA(I,7)
COLUMN 6 = DA(I,8)
COLUMN 8 = INCOME

4.9	413	82	9	43	24	451	9282
5.0	1334	273	13	296	81	1912	11042
6.7	1750	315	17	224	63	1630	11131
5.0	1229	399	134	313	110	2310	9870
5.2	791	195	79	251	115	1373	8932
4.3	1205	314	111	395	210	2187	7591
5.3	1867	394	127	289	147	2298	9181
5.6	3568	822	154	91	80	3141	7717
5.0	2155	598	170	66	122	2854	9170
4.2	1780	638	544	1038	400	3970	8636
3.6	691	263	522	754	378	2363	8382
4.0	1022	408	351	1295	260	3269	7805
4.9	293	85	108	151	83	486	8180
5.1	2116	779	353	467	124	3765	11592
5.1	2093	830	605	385	229	3647	9281
5.1	2492	994	860	464	330	4373	8772
3.1	1783	745	1630	1552	547	7566	11023
4.1	568	153	304	150	174	1523	7747
2.8	1368	528	1402	1064	196	6360	10268
5.0	912	490	446	504	281	2441	8098
3.0	1158	430	1022	441	191	4248	9351
2.8	841	560	1012	432	574	4775	5068
2.9	746	710	990	484	251	4589	5615
5.4	866	203	209	160	71	1129	10114
3.9	400	162	244	138	142	1202	6852
5.2	1160	402	454	350	275	2388	7838
5.0	682	270	225	180	50	1334	8044
3.4	902	494	1687	163	115	3644	14779
4.6	200	141	106	95	100	612	7132
3.0	1067	554	2702	212	173	5458	14962
4.9	244	51	118	134	91	599	7426
3.8	516	177	279	647	472	1955	10415
2.9	1146	500	3020	72	394	6272	17190
2.7	1270	559	3029	141	623	6329	17877
4.3	529	300	331	573	279	1775	10884
5.0	496	226	123	765	246	1565	12747
3.2	824	284	1287	52	238	3335	18014
5.1	406	369	278	55	37	927	17621
3.4	1235	434	1605	149	852	5249	30320
2.1	376	100	658	168	1009	2943	8398
3.4	974	320	1052	337	1341	5551	28636

1.4	91	12	367	50	823	2073	15192
5.3	369	103	50	99	44	558	9345
5.6	1153	296	113	152	59	1244	9684
5.1	1361	420	218	308	151	2107	9361
6.1	1226	326	152	253	105	1552	12640
5.4	1135	351	386	212	133	1868	9889
5.3	1414	368	242	251	270	2203	8973
4.6	1336	629	665	555	202	2906	6636
5.1	1639	510	506	535	194	2956	8922
5.0	1790	761	507	292	121	2774	8423
5.1	554	148	110	147	97	911	8907
4.5	1487	610	543	307	157	2848	7687
4.8	456	153	186	98	119	945	8660
5.2	1572	489	443	130	105	2449	8771
4.7	971	299	351	135	178	2002	8567
5.3	410	168	128	24	87	556	8325
5.1	317	63	140	43	74	610	6243
4.4	1229	403	809	347	172	2909	6412
4.3	1164	586	1016	941	359	3850	6585
4.9	176	205	430	336	128	1187	4902
4.9	1028	446	716	27	274	2677	5496
5.0	1309	403	806	82	74	3174	3564
4.1	418	393	494	182	128	1780	4423
4.8	560	212	504	100	282	1678	6901
4.7	315	195	363	89	171	1214	3688
5.7	888	480	572	125	61	1536	6725
4.6	871	337	582	252	102	2250	4374
5.3	2005	675	1158	137	75	3861	4578
5.7	510	301	232	43	42	1184	3880
4.0	117	32	177	36	21	438	5620
5.0	360	122	209	51	94	1103	3037
4.2	600	484	382	119	223	1624	3957
5.3	371	176	170	96	89	738	5661
4.5	353	248	225	37	88	986	4849
4.1	409	959	462	32	443	1801	5151
3.2	426	113	343	43	52	807	14223
3.9	422	137	670	76	105	2842	3081
3.7	74	83	154	15	25	967	3778
5.9	340	175	261	54	90	727	12403
4.4	162	106	148	130	10	1458	3458
4.3	286	171	227	101	382	917	6952
3.6	469	226	353	310	470	2068	15879
4.4	402	85	352	203	64	1690	4608
4.0	2470	761	1633	361	168	6545	7500
3.9	804	290	859	243	69	2736	5502
4.5	361	201	383	83	104	981	6396
4.4	1682	614	1617	431	123	5174	5366
4.0	1749	792	1534	552	87	4965	6570
5.2	1305	435	483	539	47	2337	9362
4.8	96	81	52	31	6	199	7129
4.6	709	420	385	415	209	2041	6763
5.3	836	344	555	167	58	1656	6839
3.3	292	100	113	44	32	1041	4609
5.8	2379	691	628	279	161	2622	6754
5.4	526	181	180	240	11	964	8612
5.6	1037	342	374	331	70	1567	6081
5.8	1291	524	392	218	30	1613	6215
4.6	427	120	50	131	151	1062	9494
4.6	487	265	256	86	212	1347	5039

5.6	1197	461	628	166	47	2326	4448
4.8	1844	451	395	266	147	3240	8014
4.8	168	41	55	7	24	240	7882
5.1	1607	480	516	46	165	1964	8896
4.9	935	302	598	7	200	1707	8828
4.7	584	238	488	0	121	1384	8627
4.4	733	392	526	13	127	1731	7261
4.7	481	120	307	0	139	957	7436
5.0	373	146	430	6	35	893	6950
5.2	549	237	362	20	136	1322	6989
4.4	445	231	337	10	195	1203	6049
5.0	642	248	429	0	69	1046	8193
4.7	816	351	324	19	143	1647	7734
5.0	1742	552	488	38	127	2210	8582
4.9	1305	466	497	0	140	2104	8718
4.9	981	279	230	6	147	1287	9164
5.4	787	292	338	67	97	1348	7351
5.0	1186	522	394	77	141	2044	7526
5.5	213	56	88	75	21	632	3371
5.4	639	141	441	324	41	2192	4103
5.1	643	349	633	153	44	2130	4052
5.4	108	0	83	44	39	131	5461
5.1	841	284	509	169	71	1764	6503
5.3	3595	1208	635	180	257	3606	9021
5.5	2929	762	547	365	184	3512	8801
5.0	1199	342	654	171	34	1940	6584
5.3	854	285	472	174	55	1454	5875
5.6	752	158	328	156	11	912	6543
6.5	1023	307	38	305	111	1354	10158
5.3	2351	912	991	510	91	3680	6260
5.7	1014	304	336	225	34	1279	7211
5.8	1124	435	246	250	47	1681	5789
5.7	1357	496	441	411	102	2142	6899
5.5	1799	489	67	93	50	2173	8064
4.7	839	220	127	149	177	1488	7848
5.0	1828	499	103	168	99	2400	8422
5.1	1739	313	0	155	44	1805	8816
5.7	1717	441	26	146	136	1848	9996
5.6	1858	593	30	244	69	1865	8949
4.4	2508	684	332	318	257	3498	8893
6.1	872	241	105	465	96	1633	14229
3.0	469	99	74	324	573	1952	12018
5.6	575	144	66	483	126	1175	16437
4.7	1004	607	239	514	238	2174	8518
5.6	1550	378	160	149	67	1844	9272
3.6	1561	368	85	688	286	3006	11744
8.1	1362	249	0	145	11	1098	12657
5.4	368	69	0	62	6	458	14589
5.1	928	232	26	156	43	1186	6159
6.2	1556	370	6	345	27	1549	10587
6.9	4402	894	19	231	118	3358	9388
5.9	1189	314	32	457	67	1627	11105
6.6	1251	333	23	259	81	1389	11062
6.6	1045	245	15	316	142	1323	11983
6.6	2791	699	147	209	190	2747	16670
6.4	3034	690	20	651	85	2938	8843
5.4	1287	236	6	209	61	1381	8011
6.1	1662	329	97	130	61	1469	11835
5.3	4367	1026	29	409	156	4509	10763

6.6	890	316	20	73	41	870	9876
6.8	1574	244	15	324	49	1594	17483
6.9	2988	419	37	460	146	2770	13074
6.8	478	163	0	104	0	502	8141
5.4	2121	603	78	342	45	2120	1053
6.1	1329	389	211	405	64	1760	8928
7.2	884	244	16	485	149	1256	12181
4.2	809	241	43	510	111	1763	10633
6.1	994	344	104	530	91	1559	7735
6.8	5510	1326	52	480	97	5098	10124
5.3	2446	474	29	372	161	2528	9819
5.6	3893	755	130	124	78	3470	10314
5.2	1637	461	20	451	154	2333	8419
6.1	2626	455	65	362	144	2821	13414
6.2	1435	327	99	153	69	1437	11104
4.4	1318	263	22	725	80	2508	9412
6.1	1682	281	50	470	284	2267	12620
5.3	2031	524	8	187	19	541	9731
5.8	2697	722	123	258	91	4293	13334
6.2	2078	506	171	292	107	2024	12379
5.1	1607	401	141	161	103	2310	11501
6.9	2754	570	70	252	70	372	11231
6.2	3785	915	0	461	68	1898	8059
5.4	1614	333	67	194	53	1911	10518
6.4	935	182	0	306	27	1046	12959
7.0	1774	259	82	328	56	1591	14587
7.0	754	330	51	534	182	1592	13875
8.5	630	187	6	657	149	1377	25894
8.5	518	113	5	369	39	883	24968
8.5	460	65	13	409	36	846	26622
4.5	3394	606	0	337	44	3306	10882
6.8	2340	408	18	524	191	2921	15243
4.3	8213	1771	345	728	253	9172	9874
5.9	2692	626	60	840	143	3720	12059
5.0	2153	386	62	140	129	2236	8582
4.8	1791	496	142	205	222	2239	8416
5.7	2630	788	0	96	229	2952	7873
5.3	1748	420	36	97	41	2108	8477
7.2	1820	242	6	476	54	1837	15442
5.8	4107	698	6	200	137	3526	10237
7.4	1407	189	27	590	75	1696	21322
7.4	3966	726	7	528	19	3957	16948
7.0	1940	407	19	360	107	1961	16204
4.9	2063	480	15	91	49	2066	10267
6.0	4428	997	44	712	235	4244	11271
6.1	3987	798	21	437	59	3727	10799
6.7	1924	364	6	429	136	1982	11741
7.4	1131	219	0	116	11	1152	14179
5.8	3735	686	10	341	94	3606	13404
5.4	4776	1478	14	554	167	5196	7193
4.9	2404	554	12	113	45	2382	11125
6.3	5251	1594	48	439	189	4743	8092
5.2	4729	1105	35	416	186	4925	10693
4.7	5090	979	57	355	204	4421	10049
6.3	3360	795	110	497	286	3480	10570
6.1	1823	478	127	209	73	1891	10987
4.6	2037	444	186	529	136	3209	9137
5.3	1198	216	41	148	75	1410	9062
6.5	2309	411	11	343	123	2222	15894

5.2	3475	903	106	51	197	3532	7608
4.3	1606	439	111	477	227	2728	7715
4.8	3466	811	122	398	139	4112	8246
4.3	1775	300	36	29	83	2133	8420
6.8	6309	1505	52	510	112	5778	10038
6.0	5655	1270	8	716	120	5522	8749
6.2	616	176	0	82	19	588	7219
6.6	1726	388	11	413	30	1586	7761
5.6	1674	561	40	185	83	1865	6541
5.7	2460	653	88	420	133	2625	9481
6.9	2754	570	70	252	70	2311	11231
5.2	2082	514	27	582	135	2990	9467
7.2	1670	279	12	394	31	1611	10893
6.2	2978	612	26	522	95	2785	10900
7.0	1724	315	9	671	61	2272	17486
6.6	4854	932	26	1127	139	4775	11303
5.0	2749	655	0	83	161	2421	8309
7.1	2575	551	0	306	17	2085	10030
5.4	1973	535	38	651	182	2835	11072
5.7	1749	405	40	186	149	1882	9995
6.5	3037	631	88	233	159	8440	11281
6.9	1435	254	17	351	153	1476	13456
6.2	1897	611	0	230	17	2029	9498
5.2	2142	461	6	604	164	2350	9630
5.7	2756	487	27	186	61	2510	8801
7.7	2183	653	62	501	64	2229	10782
6.9	2624	572	24	495	222	2891	10350
5.6	1206	234	14	85	87	1162	8819
4.2	639	115	0	92	6	691	9531
5.5	3444	696	13	856	319	4101	10598
7.7	1174	259	7	179	33	910	9426
6.2	2365	349	0	251	144	2009	11071
8.3	925	149	0	178	20	836	14390
4.8	2450	386	14	219	122	2588	11730
4.5	1458	367	0	275	21	1851	11642
7.6	1069	257	0	167	32	909	10032
6.3	2969	499	7	310	149	2498	10713
4.5	3493	755	5	301	38	3622	9571
6.4	6192	1290	20	616	123	5822	9504
6.6	7961	1811	25	898	112	7815	9388
6.5	3243	685	0	494	138	2994	10910
5.6	3602	710	13	880	325	4289	10506
7.3	3705	739	14	487	98	3012	11062
4.8	3907	744	22	488	239	3817	11685
5.9	2076	326	9	126	89	2267	14669
5.5	1982	544	38	675	182	2870	11101
4.4	2694	714	29	605	64	3444	11049
5.4	3124	766	17	759	126	3638	10340
5.7	4435	1016	23	378	65	4424	11229
4.4	1736	385	0	327	59	1875	11478
7.7	2211	672	62	509	64	2254	10807
7.5	2630	490	7	466	94	2149	10024
6.5	786	139	0	92	6	945	11237
5.3	1643	790	32	51	23	1635	7253
6.3	2536	505	16	120	90	2415	9784
6.4	295	105	0	12	33	320	10365
5.1	1083	288	15	34	120	1266	6718
7.2	2062	332	31	414	94	1947	12945
8.0	985	161	23	501	27	1361	26688

6.8	1027	213	0	291	376	1346	16829
7.1	2251	610	7	206	64	2214	11201
5.5	1557	310	0	10	35	1460	9187
6.6	1303	274	4	293	112	1672	14845
5.3	2650	873	0	21	50	2627	8065
6.4	457	62	0	15	29	442	10621
7.3	2136	467	47	356	38	2052	17679
6.1	3363	1019	17	62	346	3797	14428
6.3	2532	651	12	291	182	2767	9420
6.1	6888	1916	33	878	343	7416	9416
6.5	5530	1803	52	465	113	5639	8045
6.9	4364	933	11	165	55	3955	8656
6.0	2736	496	17	411	48	2734	8394
6.3	5767	1917	48	574	109	6228	8149
5.1	787	497	28	11	6	1428	5262
5.3	908	278	0	253	24	1182	9857

APPENDIX C

DATA FILE 2

COLUMN 2 = ANNUAL RENT
COLUMN 1 = TRAVEL MODE

1	6524
2	6005
2	9526
2	4857
1	4977
4	3950
1	5426
1	5563
1	3704
4	3511
4	3884
1	3512
5	3715
3	6427
3	4977
1	4398
1	3763
5	3450
4	3522
4	4116
4	4786
3	2513
3	2809
1	6813
4	2830
4	3338
1	2877
3	5336
3	2673
3	4987
1	2654
4	4321
3	6526
5	7471
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BIBLIOGRAPHY

1. M.Ben-Akiva and Lerman, DISCRETE CHOICE MODEL, MIT Press, 1985

2. A.Anas, RESIDENTIAL LOCATION MARKET AND URBAN TRANSPORTATION, Academic Press, 1982

3. Chicago Area Transportation Study, PERSONAL ENERGY CONSUMPTION, 1980

4. G.Ingram(ed), RESIDENTIAL LOCATION AND URBAN HOUSING MARKETS, Cambridge,Mass: Ballinger, 1977

5. M.Manheim, FUNDAMENTAL OF TRANSPORTATION SYSTEMS ANALYSIS Vol 1:Basic Concept, MIT Press, 1979

6. D.Segal(ed), THE ECONOMICS OF NEIGHBORHOOD, Academic Press, 1979

7. U.S.Census Bureau, CHICAGO SMSA CENSUS TRACT, 1980