

A STUDY OF THE UNIFORMITY OF
DISTRIBUTION FOR A MEDIUM PRESSURE
IRRIGATION SPRINKLER OF
OSCILLATING TYPE

Thesis for the Degree of M. S.
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Cheng-lung Chen

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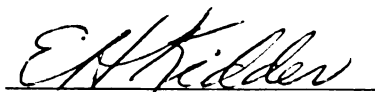
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A STUDY OF THE UNIFORMITY OF DISTRIBUTION FOR A MEDIUM
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By

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A THESIS

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INTRODUCTION

The purpose of a sprinkler is to distribute the water to the soil in the form of droplets so that it can be absorbed without running off. Many factors influence the efficiency and economy of sprinkler irrigation--one of the important factors, however, is uniformity of water distribution. Ideally, the water should be distributed uniformly over the area to be irrigated.

Since the development of the light-weight, portable pipe, and quick-coupling connectors, sprinkler systems have seen considerably more use. Nearly all of the sprinkler systems installed in the past ten years have utilized the rotating head or circular-spray sprinklers. However, since nearly all rotating sprinklers cover circular areas, an absolutely uniform application is not possible. Christian-sen [2], in his extensive and detailed experiments, concludes that the uniformity of distribution of water from this type of sprinkler varies greatly, depending upon pressure, wind, rotation speed of sprinkler, spacing, and many other factors. Careful study, however, indicates that approximately uniform application is possible when (1) the type of pattern produced is correct for the arrangement of sprinklers, (2) the sprinklers are correctly spaced, (3) the sprinklers rotate at a uniform rate, and (4) there is no appreciable

wind.

Since there is no control of a jet after it leaves the sprinkler, some manufacturers and experimenters have been striving for a sprinkler design that gives a uniform pattern over a large portion of the area covered, with a rather abrupt breaking off at the edges. Others have given more consideration to the effect of overlap and have tried to obtain a different pattern. We know that there are two desirable distribution patterns on this type of sprinklers: one is the triangular-shaped pattern and another, a trapezoidal pattern; nevertheless the best overlapping in simple geometric form (square or equilateral triangle arrangement) still can not attain uniformity of water distribution.

Staebner [6] states, "No matter how successfully they may distribute water over a circular area, they leave much to be desired, because if circles just touch one another a considerable area is left unwatered, and if they overlap a great amount of double coverage results." He further concludes, "More uniform distribution over a large area can be obtained with the overhead-pipe system (nozzle lines) than with any other type of spray irrigation equipment now available."

Nozzle lines consist essentially of parallel lines of pipe ($3/4$ to $1\frac{1}{2}$ inches in size) equipped with small brass nozzles, usually spaced 24 to 48 inches apart. To cover a strip of ground on both sides of the lines they are oscillated through an angle of about 90 degrees, generally by

means of a water-operated, oscillating motor. A complete line of equipment is available for nozzles, ranging from those that throw a round jet for maximum coverage, to others that have deflectors to break up the jet for narrower strips. Some nozzles, having orifices of triangular shape, are designed to distribute the water over a considerable area without oscillating the line. For best performance, however, nozzle lines should be equipped with oscillators that slowly rotate the line, resulting in a fairly uniform distribution of water when the lines are correctly spaced and under no wind conditions. Nozzle lines are usually operated at pressures of 25 to 40 pounds per square inch. The width of strip effectively covered increases with increase in pressure up to about 40 pounds per square inch. Above this there is little or no increase in width because of a greater dispersion of the jets. Where the pressure is adequate, the lines are spread about 50 feet apart.

Oscillating-back-and-forth sprinklers covering rectangular areas, which lend themselves well to overlapping in simple geometric arrangement without too much waste of water, should give a fairly good uniformity of water distribution. The desirable distribution pattern of an oscillating sprinkler would be a uniform amount of water over a large portion of the rectangular area covered, with a rather abrupt breaking off along the edges. More control probably can be achieved with a set of nozzles instead of a single nozzle.

To the author's knowledge, to date no detailed analysis has been made of this type of sprinkler to determine its characteristics. To determine the practicality of this type of sprinkler, and, if practical, to accumulate information for its further development is the objective of this study.

ANALYSIS OF PROBLEMS

General Consideration

A jet of water issuing from a sprinkler nozzle tends to be a broken-up mass of water drops instead of a solid jet. Therefore, there is a great deal of air between the drops, and the surface of each individual drop is acted upon by air resistance or drag. But this contact of the moving water particles with the stationary air, accelerates the air in the same direction in which the particles of water are moving, causes the velocity of the air in and around the spray to approach that of the droplets. At this time the resistance of the air to the moving water droplets will be a minimum and consequently the trajectory distance will be a maximum. However, if the jet of water is continually made to change its position in space, the drops will react like they were passing through relatively stagnant air. Under this condition, maximum drag will be encountered and the trajectory distance shortened.

Shear between the jet and the air largely determines the break-up and diffusion of the jet. The initial characteristics of the jet are, however, important and are determined by the nozzle characteristics. For this reason, a jet should be regulated before its issuing by a proper selection and operation of a nozzle. Before we decide to

choose one type of nozzles, we have to note some effects of characteristics of a nozzle on distribution patterns. The more important factors that affect the distribution of water should be water pressure and the sizes of nozzles.

Pressure is needed to provide the velocity which is required for two purposes: to secure distance to travel, and to break up the water into small drops that will be properly distributed over the desired area. However, both purposes can not be satisfied at the same time; the latter can be developed only at a sacrifice of the former. Let us consider what happens as the pressure is gradually increased in a single round sprinkler nozzle. With very low pressure the water issues in a solid stream, and all of it strikes the ground about the same distance from the nozzle. With an increase in pressure, the water becomes broken up into drops and covers a longer and wider area. At this time, the minimum fall-out of water occurs near the sprinkler and the fall-out of water increases until it reaches a maximum toward the outer portion of the trajectory distance. Beyond that, the rate of fall-out decreases very rapidly to the outer limit of the trajectory of the jet of water. Upon examining the drops that strike the ground, we find that the largest are carried to the outside edge of the area covered, while the smallest fall near the sprinkler. With further increase in pressure the area of coverage is out to the trajectory distance, and the entire area receives an almost equal amount of water. As the pressure continues to

increase, more of the water falls near the sprinkler and the average size of the drops becomes smaller.

It is noted that there is a definite relationship between the operating pressure and nozzle size of a sprinkler for these characteristic distribution patterns. Small nozzles at high pressures increase the stream break-up and tend to atomize the water. For the same discharge, large nozzles at low pressures decrease the stream break-up and result in large drops. The size of nozzle and the pressure combination required will thus be determined by the desired application rate and distribution pattern.

Many other factors such as the shape of nozzle, roughness and length of the cylinder, angle of taper affect the water distribution pattern, and have been studied by Bilanski [1]. Only one factor from one sprinkler with one nozzle was studied at a time; other factors being held constant insofar as possible. However, the sprinkler design can only determine the initial characteristics of the free jet, whereas the interaction between the jet and the ambient air determines the final pattern of fall-out. Sprinkler design exerts a somewhat indirect method of control.

The idea behind the oscillating sprinkler is that there is more possibility of direct control of the distribution pattern. If the distribution for all practicable angles of inclination be known, the rate of travel at the different angles can be adjusted arbitrarily--thus giving a measure of independent control of the distribution pattern. After the determination of the characteristic of

the characteristic of one particular nozzle and the angle of inclination, θ , with respect to the axis of oscillation, water distribution patterns from an oscillating sprinkler may implicitly be expressed as a function of limits of α -- a and b, non-uniformity of travel $\frac{d\alpha}{dt}$, and period t, even though a rigorous analysis of the general problem of water distribution is not yet possible.

$$\int_a^b f(d, p, x, y, \theta, \alpha, \frac{d\alpha}{dt}, t) d\alpha = C \quad (1)$$

As shown in Equation (1), provided the weather variables were neglected, the ideal ~~parabola~~ function, $f(d, p, x, y, \theta, \alpha, \frac{d\alpha}{dt}, t)$, of water distribution pattern must be uniform along both x axis and y axis. The relationships between the parameters involved, however, can only be defined empirically. Because of the difficulty of the experimental determination of these parameter, the problem should be restricted to the one containing the most pertinent variables. Therefore, the pressure and sprinkler geometry and the time factor were held constant throughout the test, and the study was restricted to relating the three parameters θ, α , and $\frac{d\alpha}{dt}$ to the distribution pattern.

A great deal of testing, relating nozzle pressure and size of nozzle to its effect on stream break-up and overall distribution of water, has been conducted during the past years. A general recommendation for best overall results is to use 35 psi, pressure with nozzle sizes up to and including 9/64 inch, and beginning with 5/32 inch to

add 5 psi pressure for every 1/32 inch increase in nozzle size. A decision was made to use a nozzle with diameter of 0.159 inch giving a 1/8-inch jet. Since medium pressure (from 20 to 50 psi) sprinklers cover large areas and the water jet is well broken up, for the primary test the pressure was held at 40 psi.

Now our function was simplified as $g(x, y, \theta, \alpha, \frac{d\alpha}{dt}, t)$ with only three parameters θ, α , and $\frac{d\alpha}{dt}$ to be analyzed.

Analysis of Idealized Jet

With a determination of the distribution pattern for a given size of nozzle, d , and water pressure, p at different angles α , the function $g(x, y, \theta, \alpha, \frac{d\alpha}{dt}, t)$ should be integrable between the limits of the α , a and b , such as shown in the Equation (2).

$$\int_a^b g(x, y, \theta, \alpha, \frac{d\alpha}{dt}, t) d\alpha = C \quad (2)$$

Where the limits of α , a and b , may be any values depending upon the area to be irrigated. Favorable values of a and b should be an angle of $\alpha = 45$ degrees. Besides $\frac{d\alpha}{dt}$, the theoretical relationships among x, y, θ , and α are known as analyzed in the Appendix I and Appendix III. In these analyses, for each θ the trajectory of an ideal water particle which strikes the ground is determined. Figure 41 summarizes the effect of θ . The values of θ beyond 8 degrees will not be considered in the following analysis because the areas irrigated would depart too much from a

rectangular shape.

Let $\theta = 0$ degrees and a nozzle of 0.159 inches in diameter be run at a certain fixed angle α for the primary test, the problem can thus be simplified to be one which only contains variables x , y , and t , while the other variable $\frac{d\alpha}{dt}$ is kept constant. Then:

$$\int_{\frac{1}{4}\pi}^{\frac{3}{4}\pi} h(x, y, \alpha, t) d\alpha = k(x, y, t) \quad (3)$$

The theoretical analysis of the distribution of an ideal water particle from a nozzle is discussed in the Appendix IV, in which the concentration of a water particle falling on the ground is calculated and y is not entered in the Equation (4) by assuming lateral uniformity. The period, t , is a linear factor, so is not included in the equation either.

$$C(x) = \frac{1}{\pi \sqrt{\left(\frac{v^2}{g}\right)^2 - x^2}} \quad (4)$$

Here we are interested in noting that the concentration is almost uniform along the greater portion of the distance from a nozzle and increases rapidly to infinity at the end of the trajectory. In the analysis the area covered by a droplet is related to $dx/d\alpha$. As the limit where $dx/d\alpha$ approaches zero, an area related to the drop size would be more realistic. Since even an ideal water particle from a nozzle does not give a uniform distribution at a constant oscillating speed, an actual jet will probably also require a non-uniform oscillation.

Because of a typically human failure to distribute measurements in a systematic manner without any theoretical guide, the preceding analysis was primarily studied for predicting the testing function of distribution $k(x, y, t)$. The theoretical concentration and the expected actual distribution are compared as shown in Figure 1.

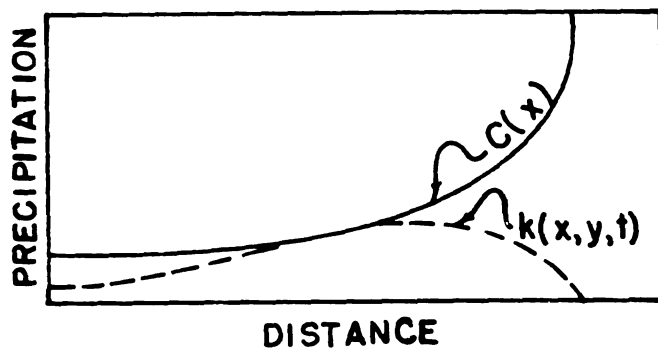


Figure 1. Theoretical $C(x)$ and the expected $k(x, y, t)$

EXPERIMENTATION

Equipment

The primary experiment was conducted in the Livestock Pavilion of the Animal Husbandry Department because of the ceiling limitation of the Land Development Laboratory in the Agricultural Engineering Department. Water was obtained from a steel storage tank, which was refilled from the University water system. A horizontal centrifugal pump with a rate of delivery of 150 gallons per minute was used to deliver the water from storage tank to the nozzle.

Rubber hose of $1\frac{1}{2}$ -inch diameter about 50 feet long made a flexible connection from the pump to the 4-inch pipe. Two globe valves were placed in the discharge line to control the pressure. A $\frac{1}{2}$ -inch nipple 2 inches in length was joined at right angles with the 4-inch pipe. A cap on the nipple with a hole 0.159 inch in diameter countersunk to be a sharp-edged orifice, resulted in a $1/8$ -inch diameter jet.

Fly screen was used as anti-splash device to minimize the recocheting of the water droplets as they struck the floor. The screen was mounted on a wooden frame 4 feet wide, 50 feet long, and 2 inches high.

All runs were made for 10 minutes. One quart oil cans with the tops cut out were used to catch the water falling out from the effluxing nozzle. The tops of the

oil cans were approximately level with the nozzle. The catch of water in each can was measured in a graduated volumetric cylinder.

Bourdon pressure gages were used to measure the pressure. A pressure gage with a 3-1 $\frac{1}{2}$ inches diameter dial and a scale ranging from zero to 100 pounds per square inch was placed in the delivery line 2 feet from the nozzle. The whole scheme of the equipment is shown in Figure 2.

Technique

To find the distribution at various angles of inclination, a 0.159-inch orifice nozzle was operated at fixed angles of 40, 50, 60, 70, 80, and 90 degrees from the horizontal. The water pressure was 40 psi in all tests. As shown in Figure 2, each inclined angle was fixed by a plumb hung from the center on the angle scale on one side of the 4-inch pipe. Water pressure was fixed by the control valve before the test and the run started and stopped with the shut-off valve. To eliminate the errors of any time-lag in starting and stopping, the running time was chosen as 10 minutes as measured by a stop watch.

The catch in each can was measured with a graduate calibrated in cubic centimeters and then converted into inches of water depth per hour.

Experimental Results

For each angle of inclination the catch from each can was plotted, and contours drawn. Figure 3 shows the precipitation contour of the vertical angle, $\alpha = 90$ degrees, and it is apparent that the distribution is a bell shape. Contours for angles of inclination of 80, 70, 60, 50, and 40 degrees are shown in Figures 4, 5, 6, 7, and 8, respectively. The patterns are elongated unsymmetrically with the greatest concentration further away from the nozzle for the less inclined angles.

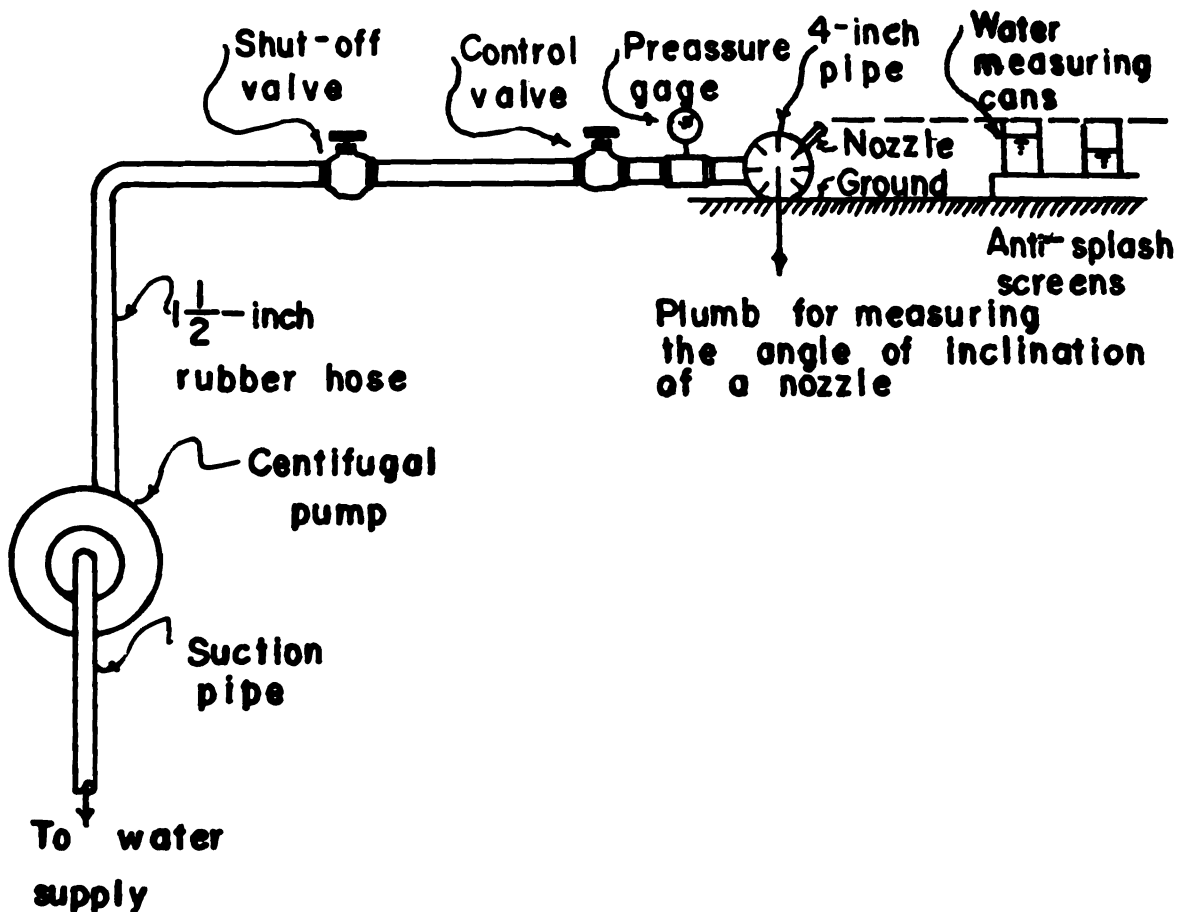


Figure 2. Schematic diagram of flow system

DISTRIBUTION ANALYSIS

Water Distribution Patterns from a Nozzle

After the function, $h(x, y, \alpha, t)$, of the water distribution patterns had been obtained for angles of inclination $\alpha = 40, 50, 60, 70, 80$, and 90 degrees, the data necessary for the integration of Equation (3) was available.

$$\int_{\pi/4}^{\pi/2} h(x, y, \alpha, t) d\alpha = k'(x, y, t) \quad (3d)$$

The problem was how to integrate $h(x, y, \alpha, t)$ from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ by using the experimental patterns. A simple addition of the distribution patterns for the different angles did not prove to be satisfactory. The distribution from a fixed angle was not a good approximation of the distribution from a jet moving through an angle of 10 degrees at a constant angular velocity. A graphical method was evolved which permitted a more accurate integration of the total distribution pattern. This method is described in detail in Appendix V. The function $k'(x, y, t)$ thus obtained is plotted in Figure 9. The precipitation contours and the lateral cross sections at certain distances from a nozzle were shown in Figures 10 and 11, respectively.

Water Distribution Patterns from a Set of Jets

A set of parallel jets. Consider a set of nozzles in a line, each normal to a comparatively long supply pipe which oscillates back and forth from 45 degrees to 135 degrees. An analysis was made to determine the desirable spacing of two adjacent nozzles to get a reasonably uniform pattern.

It was found that the maximum spacing from center to center of nozzles that could be used was 26 inches. For the nozzle size and pressure tested, the application rate would then be about 2 inches per 45 minutes (i.e., 2.67 inches per hour), which is a high rate of application. This high application rate would be undesirable on most irrigated soils. A lower application rate, however, could be obtained by choosing a smaller nozzle.

The distribution along the center line for the spacings of two adjacent nozzles analysed are shown in Figure 12. Longitudinal and lateral profiles for the 26-inch spacing are given in Figures 13 and 14.

A set of fan-shaped jets. Using the method of the Appendix V and the relationships of the Appendix I, the distribution from a jet with a lateral angle $\theta = 4$ or 8 degrees was obtained as shown in Figures 16 and 17. The distribution from a jet with $\theta = 0$ degree as shown in Figure 15 had already been obtained as a symmetric part of Figure 10; those of 2 and 6 degrees, were interpolated in the lateral direction at distances of 5, 10, 15, ..., 45,

and 50 feet from the nozzles. These relative distributions are plotted in Figure 18.

It is apparent from the figures that a set of equally sized orifice nozzles will not give the desired uniformity of distribution. Before determining the oscillating characteristics, the uniformity in lateral direction at each cross section should be adjusted by using large sizes of nozzles at the greater θ .

The determination of the sizes of the nine orifices could not be obtained mathematically because of the complicated distribution function as stated before. The empirical method, or trial and error method, as so far used was again resorted to. **Because** of the fact that the actual observation in fields showed a slight change in the irrigated areas for a small change in nozzle sizes, the following assumption was made:

$$\frac{c(\frac{1}{4}\pi d_1^2)Vt}{c(\frac{1}{4}\pi d_2^2)Vt} = \frac{e(B_1 L_1)D_1}{e(B_2 L_2)D_2} \quad (5)$$

where c and e are coefficients of discharge and distribution, respectively. All symbols are represented in Figure 19. Assuming $B_1 = B_2$ and $L_1 = L_2$, then we have:

$$\frac{d_1^2}{d_2^2} = \frac{D_1}{D_2} \quad (6)$$

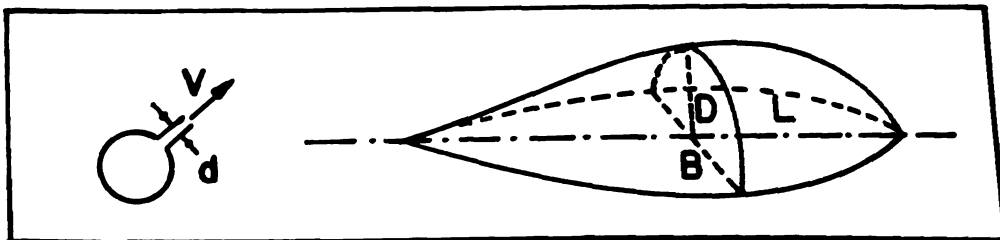


Figure 19. Relationship between d and D .

The water distribution patterns thus adjusted are shown in Figure 20. It is evident that complete uniformity cannot be achieved. However, the analysis made so far assumes non-interference between jets. The actual patterns should be somewhat different from the analysis, but no way to predict the interference effect is at hand except by experiment.

The sizes of the orifices were determined to be 0.199 inch in diameter at $\theta = 8$ degrees, 0.179 inch in diameter at $\theta = 6$ degrees, 0.159 inch in diameter at $\theta = 4$ degrees, 0.139 inch in diameter at $\theta = 2$ degrees, 0.119 inch in diameter at $\theta = 0$ degree.

Water Distribution Patterns from a Slot

From the analysis made for a set of orifices, it was considered that a slot might replace a set of orifices. A slot with wider edges and narrower middle part as shown in Figure 21 was expected to have a distribution as shown in Figure 22. A nearly uniform distribution at each lateral section was obtained. Water falling beyond 11 feet laterally from the slot was supposed to be overlapped by that of the adjacent slot. Now the only factor remaining was to adjust the oscillating speed. The longitudinal distribution of water with assumed lateral uniformity was shown in Figure 23. In the same way water falling out beyond 48 feet was considered to be overlapped.

From Figure 23, it was easily found that the

comparative concentrations between the most further part and the most near part from the slot would be 3.34 to 1. Unfortunately, there was no way to relate $\frac{d\alpha}{dt}$ to α so that conceivable changes in angular speeds at different inclined angles were empirically determined.

DISCUSSION

Development of the Proposed Distribution Patterns

Oscillating mechanism. Since fundamentally there seemed no expedient to relate $\frac{d\alpha}{dt}$ to α , the exact relative speed at any α was determined empirically by using the mechanics of a cam. From observation of the distribution of constant speed it was apparent that the relative oscillating speed should be 3.34 at $\alpha = 45$ and 135 degrees to 1 at $\alpha = 90$ degrees; other values of α between these two extremes being proportionally accelerated. The cam gives an oscillation of 45 degrees. This was doubled to 90 degrees by means of two gears having the relative numbers of teeth equal to $2:1$. This is shown in Figure 24.

For actual operation, the sprinkler should have a self-oscillating mechanism actuated by the flow of water. It was simpler for this test to oscillate the nozzle mechanically by an electric motor in order to get easy operation and good oscillation. A gear reducer integral with the motor resulted in a speed of 2.67 revolutions per minute. Since an even slower speed was desired to eliminate the effect of $\frac{d\alpha}{dt}$ on uniformity, two V-pulleys with ratio of one to two were introduced to give 1.335 revolutions per minute. The whole view of oscillating mechanism and flow system is shown in Figure 25.

Nozzles. Various types of nozzle system could be used with the oscillating sprinkler, but not all of them would give adequately desirable uniformity. Moreover, the actual water distribution patterns would not be expected to agree exactly with the analysis made. The following types of nozzles were made and tested to find out the most desirable one.

A set of holes to give a 16-degree fan-shaped spray:

- Type 1. 9 holes of the same size, each having a diameter 0.159 inch.
- Type 2. 9 holes of the different sizes, from the center to both ends, diameters being 0.119, 0.139, 0.159, 0.179, and 0.199 inch, respectively.
- Type 3. Holes of the same size 0.08 inch in diameter numbered according to the area of the hole in Type 2, from center to both ends, 2, 3, 4, 5, and 6, respectively.

A slot giving a 16-degree fan-shaped spray:

- Type 4. A slot with 0.119 inch at center and 0.199 inch at both ends as shown in Figure 21.
- Type 5. A slot with 0.119 inch opening width all through the whole length.
- Type 6. A very thin slot.

A jet from each nozzle was run and observed as shown in Figures 26, 27, 28, 29, 30, and 31.

From these figures, as from visual observation, the Type 1 seemed to give more uniformity than the other types.

The jets from the nine holes in this nozzle seemed to vibrate near the nozzle contributing to the uniformity of the fall-out on the ground.

With the slot opening, the jet tended to be divided into three parts as shown in Figure 32. This phenomenon was seen in all the slot nozzles (Types 4, 5, and 6). Although the slot nozzle could be improved by putting some guide walls in the expansion, a set of holes seems preferable as a sprinkler nozzle.

Applicability of the Proposed Distribution Patterns

As a guide to the further study, the Type 1 nozzle with the oscillating mechanism was run outdoors for 10 minutes under the lightest wind possible. The precipitation contours are plotted in Figure 33. The pattern is promising although not as good as was expected from visual observation. An approximately rectangular pattern was achieved but without sufficient uniformity laterally or longitudinally. A variation in size of holes, as first projected, and a correction in the cam would improve uniformity.

With the pattern obtained a triangular arrangement of oscillating sprinklers might be more effective than a rectangular pattern with reasonable uniformity of application and without too much waste of water to be overlapped. As shown in Figure 35, it is not hard to determine the spacings of sprinklers by using the result of the above mentioned test. The spacings, 15 by 52 feet, seemed

sufficient to cover the whole areas under this wind direction.

The distribution from the Type 2 nozzle was also determined, Figure 34, and was too concentrated at the edges as expected from visual observation. A set of holes intermediate between Types 1 and 2, would apparently be better.

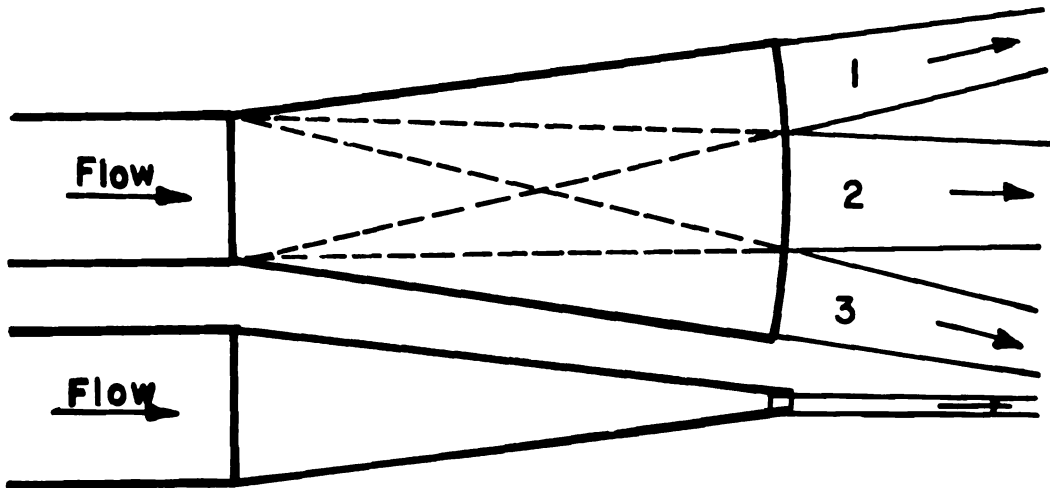


Figure 32. Flow diagram in a nozzle of a slot opening.

CONCLUSIONS

The final distribution patterns obtained in this study, Figures 33 and 34, demonstrate that a reasonably uniform, approximately rectangular pattern can be obtained from a set of nozzles oscillated in a controlled manner. The control that is possible in this type of sprinkler is indeed promising. However, certain difficulties in the further development of the oscillating sprinkler are also apparent from this study. Since the discharge from any one nozzle of a set is distributed to a smaller area than with a rotating sprinkler, either the individual nozzles must be smaller to reduce the discharge, or the time between moving the sprinkler must be reduced. Another possibility would be to use a single nozzle which shifts its setting in increments between ± 8 degrees as it oscillates.

These considerations as well as the mechanical design are developmental problems which are the next logical step following this investigation.

SUMMARY

It has been generally learned that one of the important factors which influence the efficiency and economy of sprinkling irrigation is the uniformity of the water distribution. Although the sprinkler irrigation has been greatly developed since 1920, there have still been many problems in getting the uniform distribution of the water.

Extensive and detailed experiments on the rotating-head sprinkling system have been conducted and studied, but many rotating-head sprinklers presently in use do not give the desired uniformity of water distribution. Since to date, to the author's knowledge, no detailed analysis has been made on the oscillating sprinkler to obtain a fairly uniform pattern with a smaller overlap of irrigated areas, the objective of this study was to make such an analysis.

The primary indoor tests were conducted with an orifice nozzle of 0.159-inch diameter (giving a 1/8-inch jet) at every 40, 50, 60, 70, 80, and 90 degrees in angle of inclination. The precipitation contours thereby obtained were plotted to make the distribution analysis. Only the medium pressure (40 psi) sprinklers were studied.

It was found that the probable maximum spacings of the oscillating nozzles attached to a pipe was 26 inches, and that 9 nozzles of varying sizes with the middle one at

the right angle and the others 2, 4, 6, and 8 degrees inclined with the right angle respectively to both sides would give a fairly uniform distribution of water. In analysis, the author was convinced that a slot with wider edges and narrower middle part seemed to give the more desirable pattern; on the contrary, the secondary test showed its inapplicability because of sensitivity.

The oscillating mechanism, a cam, was developed to oscillate back and forth with the different relative speeds, approximately 3.34 at $\alpha = 45$ and 135 degrees to 1 at $\alpha = 90$ degrees. A triangular arrangement of spacings in oscillating sprinklers was favorable.

For the further study, it was suggested that the spacings of this type of irrigation sprinklers should be determined to suit the variable wind velocities and directions.

APPENDIX

I. Relationships among α , θ , ϕ , and ψ

Let Y be an axis of oscillation for two nozzles, one in XZ plane and the other at an angle θ from the XZ plane. It is to be noted that the angle θ does not change during oscillation. At certain instant when the nozzle which is moving in the XZ plane makes an angle α with the XY plane, or X axis, the other nozzle makes an angle ϕ with the XY plane. The angle between the XZ plane and the vertical plane containing the nozzle set at an angle is denoted by the angle ψ . The relationships among these angles are shown in Figure 36.

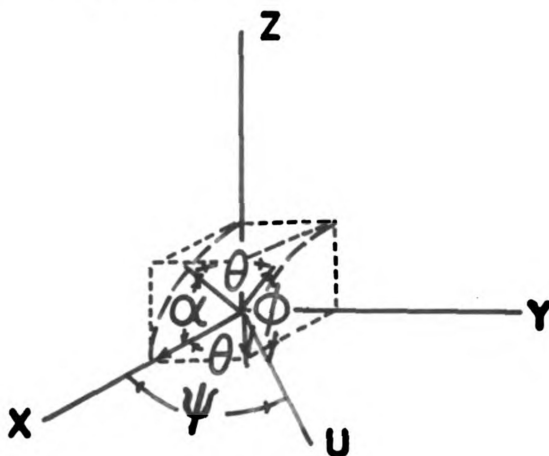


Figure 36. Relationships among α , θ , ϕ , and ψ

The geometric or trigonometric relationships are readily stated.

$$\sin \phi = \sin \alpha \cos \theta \quad (5)$$

$$\phi = \arcsin (\sin \alpha \cos \theta) \quad (6)$$

In particular, if $\theta = 0$ degree, then

$$\phi = \arcsin (\sin \alpha) = \alpha$$

Also

$$\tan \psi = \sec \alpha \tan \theta \quad (7)$$

$$\psi = \arcsin (\sec \alpha \tan \theta) \quad (8)$$

The relationships are shown graphically in Figures 37, 38, and 39.

II. Hydraulic Characteristics of a Jet from a Standard Nozzle

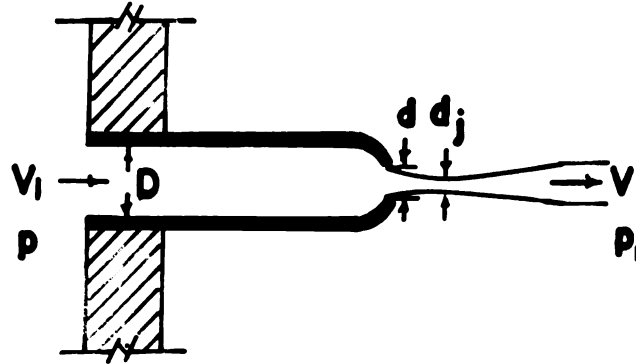


Figure 40. Diagram of a jet from a standard nozzle

$$p_1 - p = \Delta p = \frac{\rho}{2} (V^2 - V_1^2)$$

$$V_1 \frac{\pi D^2}{4} = V C_c \frac{\pi d_j^2}{4} = V \frac{\pi d_j^2}{4}$$

$$V = \frac{1}{\sqrt{1 - C_c^2 (d_j/D)^4}} \sqrt{2 \frac{\Delta p}{\rho}} \quad (9)$$

$$= \frac{1}{\sqrt{1 - (d_j/D)^4}} \sqrt{2 \frac{\Delta p}{\rho}} \quad (10)$$

Where

$$C_v = \frac{1}{\sqrt{1 - C_c^2 \left(\frac{d}{b}\right)^4}} = \frac{1}{\sqrt{1 - \left(\frac{d_i}{b}\right)^4}} \quad (11)$$

$$\begin{aligned} Q &= C_d \frac{\pi d^2}{4} \sqrt{\frac{2 \Delta p}{\rho}} \\ &= C_c C_v \frac{\pi d^2}{4} \sqrt{\frac{2 \Delta p}{\rho}} \\ &= \frac{C_c}{\sqrt{1 - C_c^2 \left(\frac{d}{b}\right)^4}} \frac{\pi d^2}{4} \sqrt{\frac{2 \Delta p}{\rho}} \end{aligned} \quad (12)$$

Where

$$C_d = C_c C_v = \frac{C_c}{\sqrt{1 - C_c^2 \left(\frac{d}{b}\right)^4}} = \frac{C_c}{\sqrt{1 - \left(\frac{d_i}{b}\right)^4}} \quad (13)$$

Let Δp be pressure difference expressed in pounds per square foot. Then

$$V = C_v \sqrt{\frac{2 \Delta p}{\rho}} \quad (14)$$

If Δp in pounds per square inch, then

$$V = 16.970 C_v \sqrt{\frac{\Delta p}{\rho}} \quad (15)$$

Generally, water has $g = 32.2 \text{ ft/sec}^2$ and $w = 62.4 \text{ lb/ft}^3$ at temperature equal to 32°F. , hence $\rho = 1.94 \text{ slug/ft}^3$.

Substituting the value of ρ in the Equation (15), then

$$V = 12.2 C_v \sqrt{\Delta p} \quad (16)$$

III. Ideal Relationships among x , y , α , and θ

Theoretically, a water particle which falls out on the ground from a nozzle of certain α and θ can be analyzed by assuming that it follows the trajectory of a projectile in a vacuum.

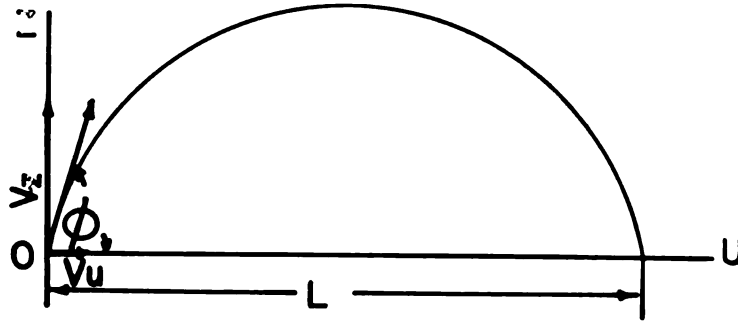


Figure 41. Interrelationship of velocity head and jet elevation

$$V_u = V \cos \phi$$

$$V_z = V \sin \phi$$

$$U = V_u t$$

$$t = U/V_u$$

$$Z = V_z t - \frac{1}{2} g t^2$$

$$= V_z \left(\frac{U}{V_u} \right) - \frac{1}{2} g \left(\frac{U}{V_u} \right)^2$$

$$= U \tan \phi - \frac{g U^2}{2 V^2 \cos^2 \phi} \quad (17)$$

In order to estimate the distance at which the jet strikes the ground, L_u , Z is set equal to zero in Equation (17). Then

$$L_u = \frac{2 V^2 \sin \phi \cos \phi}{g} = \frac{V^2 \sin 2 \phi}{g} \quad (18)$$

The exact location of the point which a water particle falls out on the XY plane can be expressed as:

$$x = L_u \cos \psi = \frac{V^2 \sin 2 \phi \cos \psi}{g} \quad (19)$$

$$y = L_u \sin \psi = \frac{V^2 \sin 2 \phi \sin \psi}{g} \quad (20)$$

Where

$$V = 16.970 C_v \sqrt{\frac{\Delta p}{\rho}}$$

$$\phi = \arcsin (\sin \alpha \cos \theta)$$

$$\psi = \arctan (\sec \alpha \tan \theta)$$

$$\begin{aligned} \text{From Appendix II, } C_v &= \frac{1}{\sqrt{1 - C_c^2 \left(\frac{d}{D}\right)^4}} \\ V &= \frac{16.970}{\sqrt{1 - C_c^2 \left(\frac{d}{D}\right)^4}} \sqrt{\frac{\Delta p}{\rho}} \end{aligned} \quad (21)$$

Substituting the values of V , ϕ , and ψ in Equation (19) and Equation (20), then we have in general

$$x = \frac{288 \Delta p}{\omega \left[1 - C_c^2 \left(\frac{d}{D}\right)^4 \right]} \sin 2\phi \cos \psi \quad (22)$$

$$y = \frac{288 \Delta p}{\omega \left[1 - C_c^2 \left(\frac{d}{D}\right)^4 \right]} \sin 2\phi \sin \psi \quad (23)$$

Where w = water density, pounds per cubic foot.

Therefore, if we know the diameter of the nozzle d , the pipe diameter D where water pressure in the pipe was measured, the contraction coefficient C_c which can be determined by the ratio of the nozzle diameter to the pipe diameter, i.e., d/D , the pressure difference Δp between the nozzle and the pipe, and the angles θ and α , then we can evaluate the exact theoretical position of a water particle hitting the ground surface.

Obviously $C_c^2 (d/D)^4 = (d_j/D)^4$ approaches zero if D is large enough compared to d , hence we can ignore the velocity coefficient C_v because it tends to be unity. Then Equations (19) and (20) become:

$$x = \frac{288 \Delta p}{\omega} \sin 2\phi \cos \psi \quad (24)$$

$$y = \frac{288 \Delta p}{\omega} \sin 2\phi \sin \psi \quad (25)$$

Where

$$V = 12.2 \sqrt{\Delta p}$$

Particularly if $\theta = 0$ degree, then simply

$$x = \frac{288 \Delta p \sin 2\alpha}{\omega}$$

$$y = 0$$

If we take $\omega = 62.4$ pounds per cubic foot at the water temperature 32°F. , then the foregoing equations will be simplified as follows:

$$x = 4.62 \Delta p \sin 2\phi \cos \psi \quad (26)$$

$$y = 4.62 \Delta p \sin 2\phi \sin \psi \quad (27)$$

In the same manner for $\theta = 0$ degree,

$$x = 4.62 \Delta p \sin 2\alpha$$

$$y = 0$$

Here we would like to show the interrelation of the x , y , θ , and α ; setting $C = \frac{288 \Delta p}{\omega}$

$$\frac{x}{C} = \sin 2\phi \cos \psi \quad (28)$$

$$\frac{y}{C} = \sin 2\phi \sin \psi \quad (29)$$

The relationships among x/c , y/c , θ , and α are plotted in Figure 42.

IV. Theoretical Analysis of the Distribution of a Water Particle from a Nozzle

Assuming that water particles are discharged at a constant rate from the nozzle, that the nozzle oscillates at a constant angular velocity between the limits 45 and 135 degrees, we can theoretically evaluate the concentration of water particles which hit the ground surface. The concentration will be $\frac{Qdt}{dx}$.

From Equation (18),

$$x = \frac{V^2 \sin 2\alpha}{g}$$

Hence

$$\sin 2\alpha = \frac{g}{V^2} x$$

$$\cos 2\alpha = \sqrt{1 - (\sin 2\alpha)^2} = \sqrt{1 - \left(\frac{g}{V^2}\right)^2 x^2}$$

$$\begin{aligned} \text{But } dx &= \frac{2V^2}{g} \cos 2\alpha d\alpha \\ &= 2\sqrt{\left(\frac{V^2}{g}\right)^2 - x^2} d\alpha \end{aligned} \quad (30)$$

Let λ be unknown, the density function $C(x)$ and the accumulative function $F(x)$ can be written as:

$$C(x) = \frac{\lambda}{\sqrt{(V^2/g)^2 - x^2}} \quad (31)$$

$$F(x) = \int_{-\frac{V^2}{g}}^x \frac{\lambda}{\sqrt{(V^2/g)^2 - x^2}} dx \quad (32)$$

Integrating $F(x)$, we have

$$F(x) = \lambda \left[\arcsin \frac{gx}{V^2} + \frac{\pi}{2} \right] \quad (33)$$

Since

$$F\left(\frac{V^2}{g}\right) = 1$$

$$1 = \lambda \left[\arcsin(1) + \frac{\pi}{2} \right]$$

Therefore,

$$\lambda = \frac{1}{\pi}$$

Consequently,

$$C(x) = \frac{1}{\pi \sqrt{\left(\frac{V^2}{g}\right)^2 - x^2}} \quad \text{for} \quad -\frac{V^2}{g} \leq x \leq \frac{V^2}{g} \quad (4)$$

$$= 0 \quad \text{otherwise}$$

$C(x)$ is plotted for $p = 40$ psi, $d_j = 1/8$ inch ($d = 0.159$ inch), and $V = 12.2 \sqrt{p} = 77.3$ ft/sec as shown in Figure 43.

Now

$$C(x) = \frac{1}{\pi \sqrt{34400 - x^2}} \quad \text{for} \quad -185.4 \text{ ft} \leq x \leq 185.4 \text{ ft.} \quad (34)$$

$$= 0 \quad \text{otherwise}$$

V. Integration of the Function $h(x, y, \alpha, t)$ with Respect to α

As an example, the longitudinal distributions of water from a standard nozzle on the center line, namely, $y = 0$, for $\alpha = 40, 50, 60, 70, 80$, and 90 degrees are plotted in Figure 44; other values of y will follow in the same way. Taking α as abscissa and precipitation rate as ordinate, we plot the α -precipitation curves for a certain value of x , for instance, $x = 0, 5, 10, 15, 20, 30, 40$, and 50 feet, respectively, from the nozzle as shown in

Figure 45. Then measuring the area involved by the α axis, $\alpha = 45$ degrees line, and α -precipitation curves in corresponding scale by planimeter, we can obtain the accumulated precipitation rate, for a nozzle oscillating from $\alpha = 45$ to 135 degrees in constant speed, of $y = 0$ and the foregoing values of x respectively; that of any other values of x will be found by interpolation.

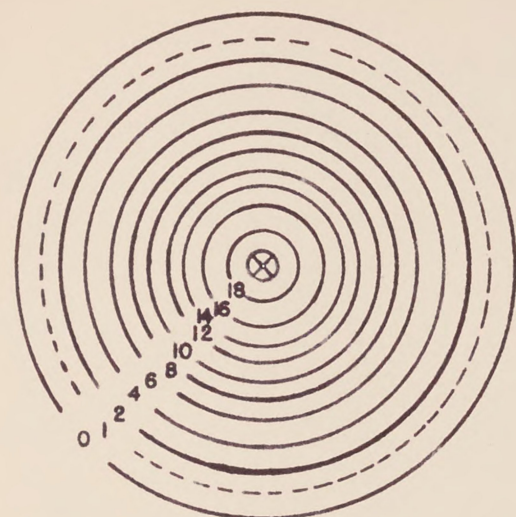
VI. Nomenclature

d	Size of the nozzle, inch.
p	Water pressure in the nozzle, pounds per square inch.
x	Longitudinal distance from the nozzle, feet.
y	Lateral distance from the nozzle, feet.
θ	Angle of the nozzle from the one which is fixed at a right angle with respect to the oscillating axis, degrees.
α	Inclined angle of the nozzle for $\theta = 0$ degree, degrees.
$\frac{d\alpha}{dt}$	Oscillating velocity, radians per second.
t	Period, seconds.
a	Lower limit of α .
b	Upper limit of α .
D	Precipitation rate, inches per hour.
V	Velocity of the jet, feet per second.
ϕ	Angle corresponding to the angle α of the nozzle which makes angle θ with respect to the one which is perpendicular to oscillating axis, degrees.

ψ	Angle between two vertical planes, each of which contains a nozzle and the vertical Z axis, degrees.
p_1	Water pressure in atmosphere, pounds per square inch.
Δp	Pressure difference between atmosphere and the nozzle, pounds per square inch.
V_1	Water velocity in the nozzle, feet per second.
C_c	Contraction coefficient of the nozzle.
d_j	Diameter of a jet, inch.
ρ	Density of water, slug per cubic foot.
C_v	Velocity coefficient.
Q	Water discharge, cubic feet per second.
C_d	Discharge coefficient.

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LEGEND

⊗ SPRINKLER NOZZLE
 ~ CONTOUR IN in./hr.

SCALE

1 : 40

— 4 FEET —

FIG. 3 PRECIPITATION CONTOUR OF $\alpha = 90$ DEGREES FROM A NOZZLE OF 0.159 in. IN DIAMETER AT 40 psi. WATER PRESSURE

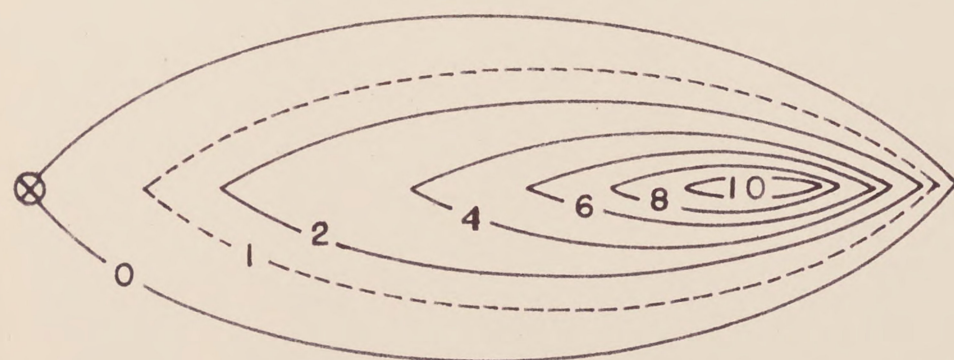


FIG. 4 PRECIPITATION CONTOUR OF $\alpha = 80$ DEGREES FROM A NOZZLE OF 0.159 in. IN DIAMETER AT 40 psi. WATER PRESSURE

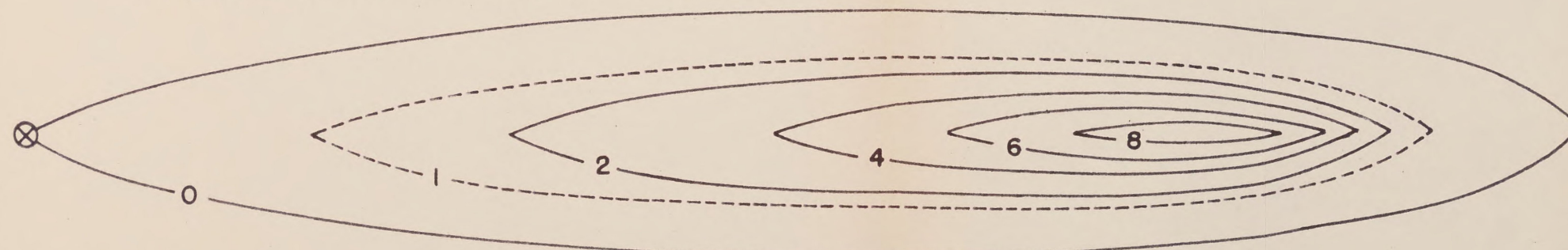


FIG. 5 PRECIPITATION CONTOUR OF $\alpha = 70$ DEGREES FROM A NOZZLE OF 0.159 in. IN DIAMETER AT 40 psi. WATER PRSSURE

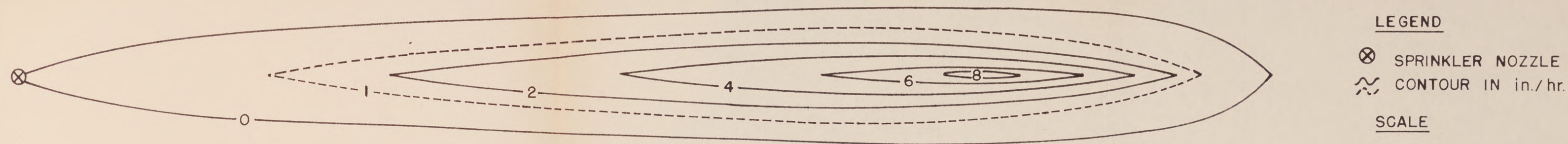


FIG. 6 PRECIPITATION CONTOUR OF $\alpha = 60$ DEGREES FROM A NOZZLE OF 0.159 in. IN DIAMETER AT 40 psi. WATER PRESSURE

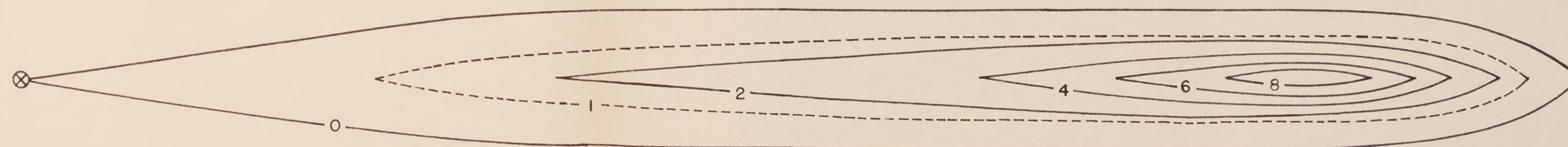


FIG. 7 PRECIPITATION CONTOUR OF $\alpha = 50$ DEGREES FROM A NOZZLE OF 0.159 in. IN DIAMETER AT 40 psi. WATER PRESSURE

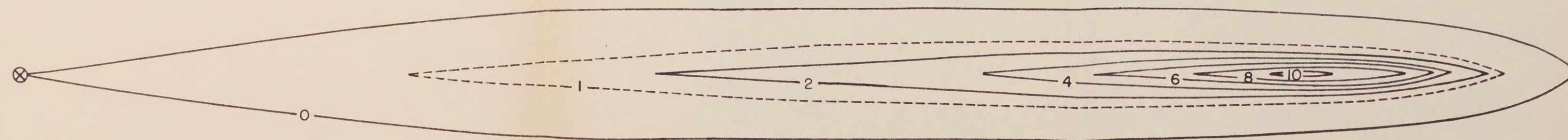


FIG. 8 PRECIPITATION CONTOUR OF $\alpha = 40$ DEGREES FROM A NOZZLE OF 0.159 in. IN DIAMETER AT 40 psi. WATER PRESSURE

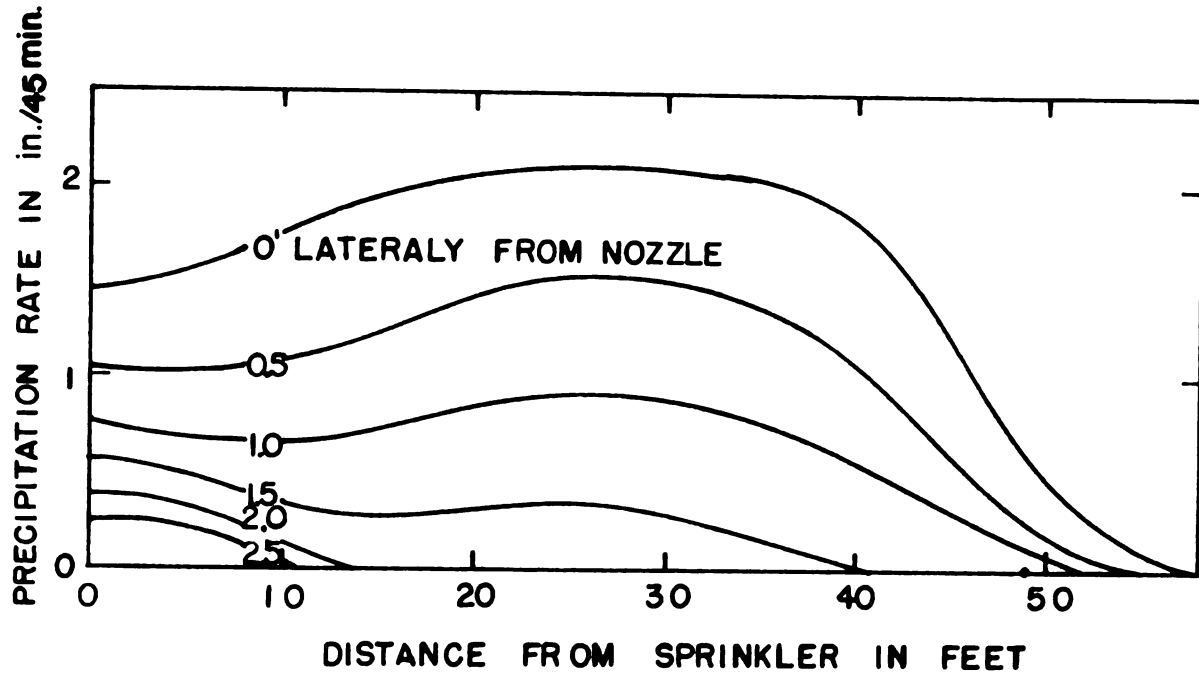


FIG. 9 $k'(x, y, t)$ IN LONGITUDINAL DIRECTION

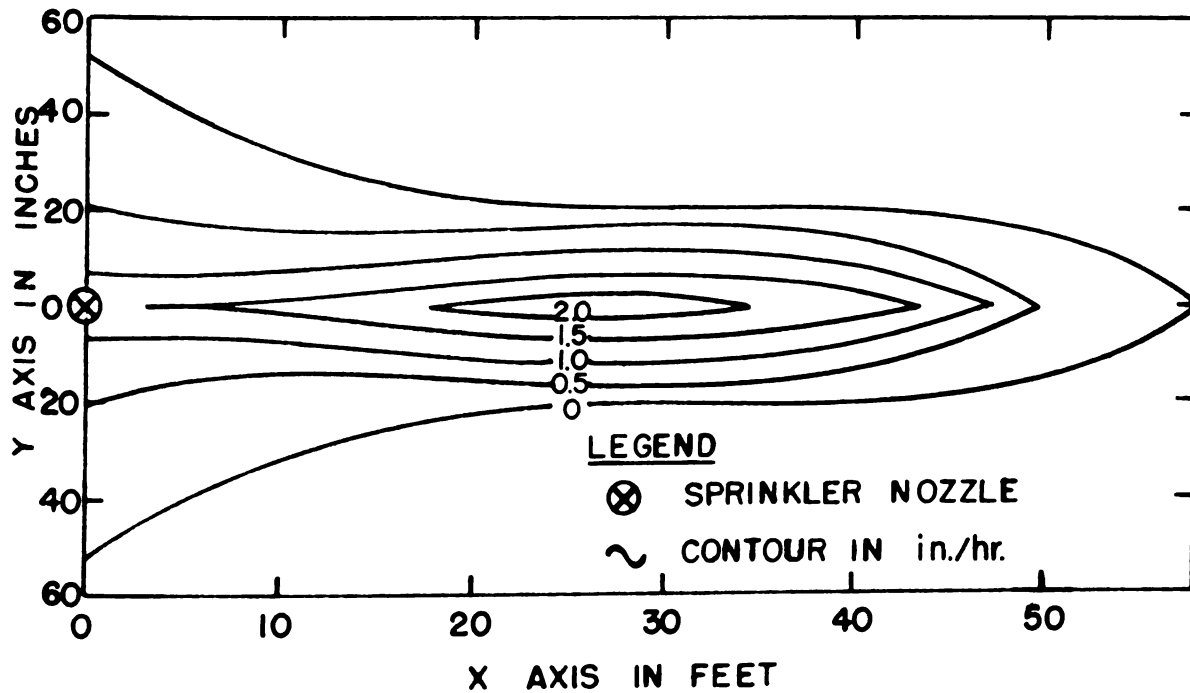


FIG. 10 PRECIPITATION CONTOUR OF $k'(x, y, t)$

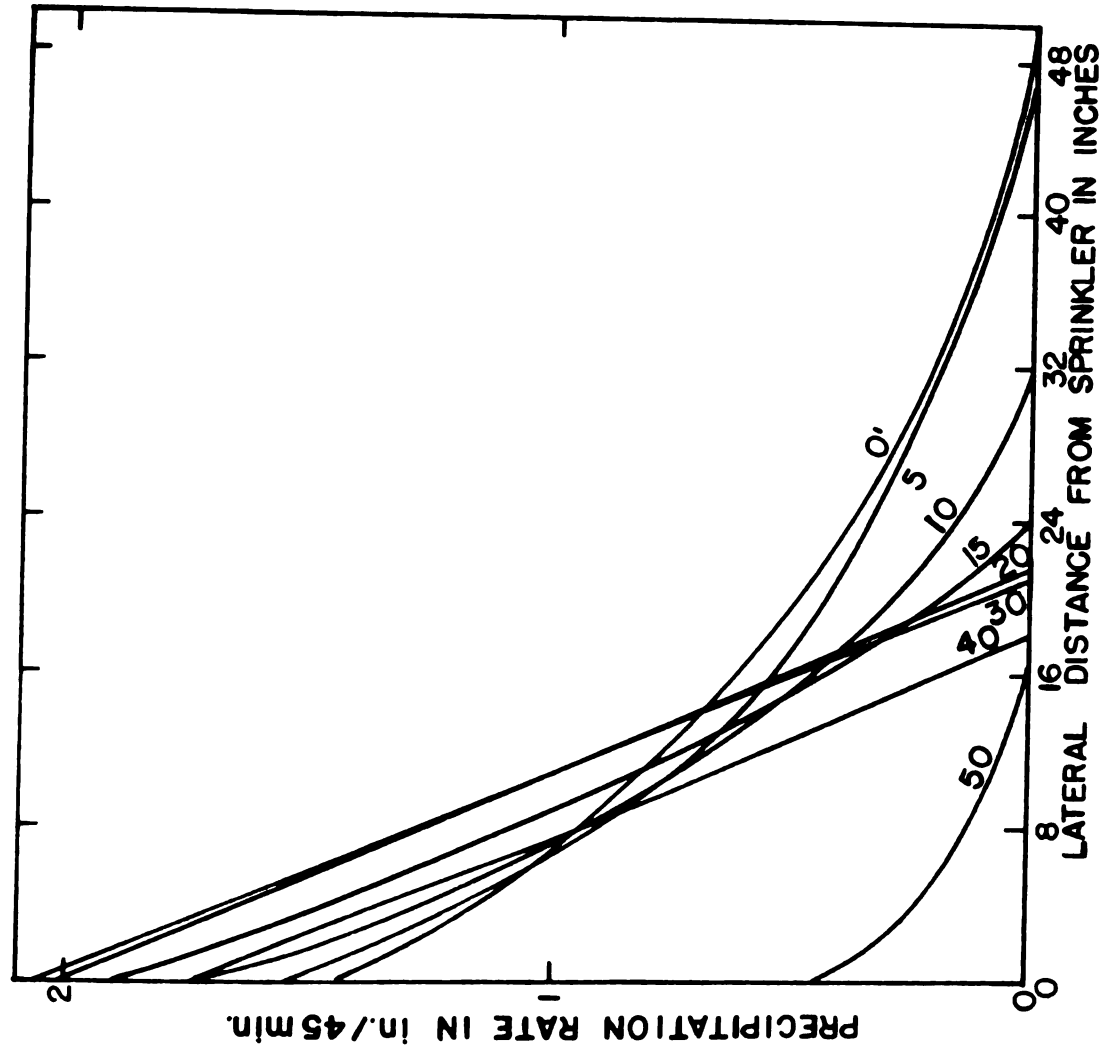


FIG. 11 $k'(x, y, t)$ IN LATERAL DIRECTION

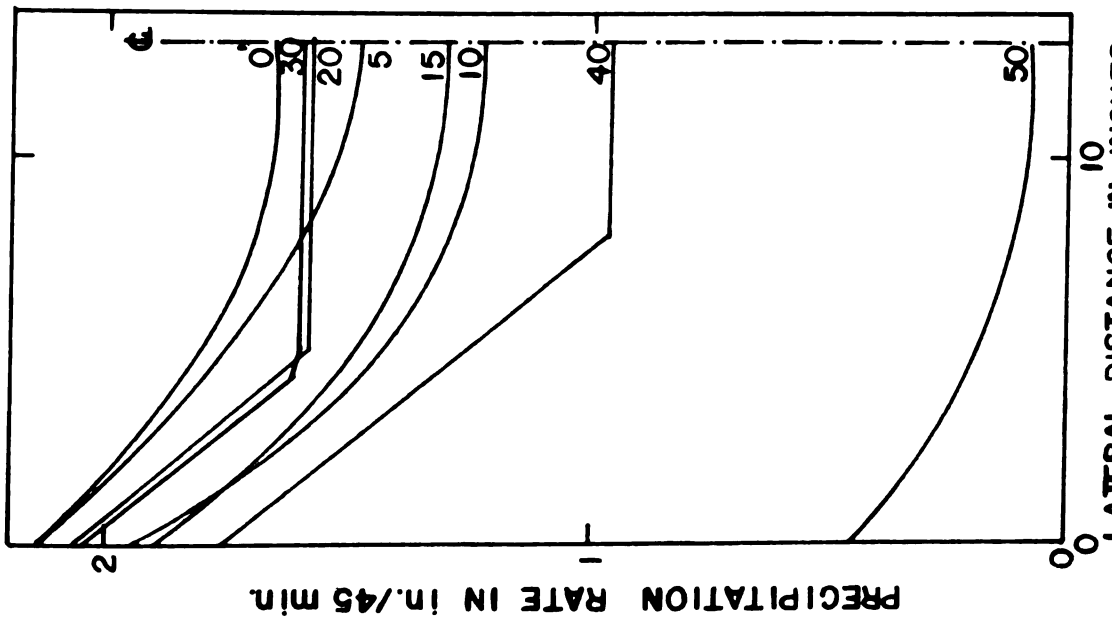


FIG. 14 ACCUMULATED WATER DIST. OF 26-in. SPACINGS IN LATERAL DIRECTION

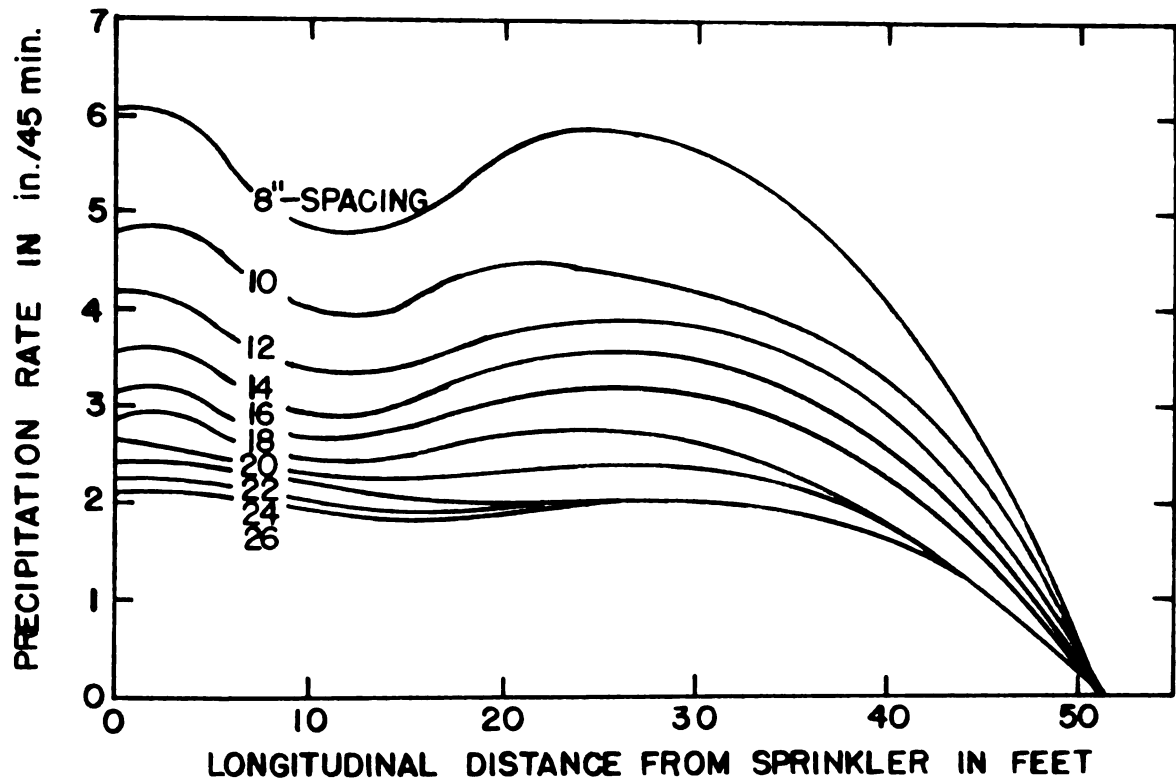


FIG. 12 ACCUMULATED WATER DISTRIBUTION OF SOME POSSIBLE SPACINGS OF TWO ADJACENT NOZZLES IN LONGITUDINAL DIRECTION

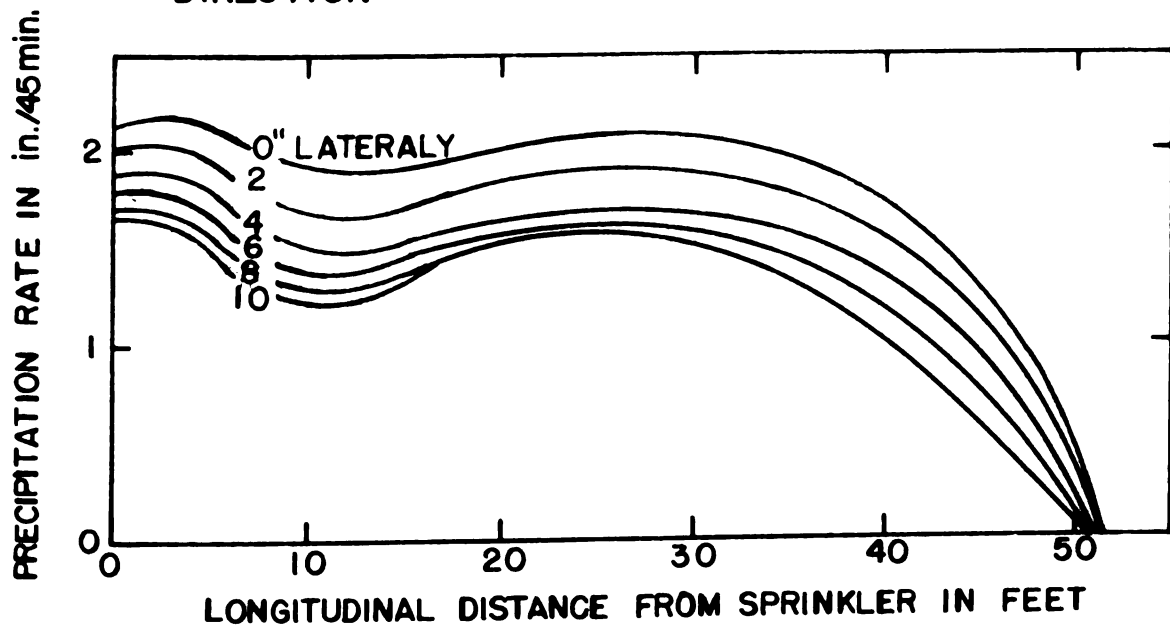


FIG. 13 ACCUMULATED WATER DISTRIBUTION OF 26-INCH SPACINGS IN LATERAL DIRECTION

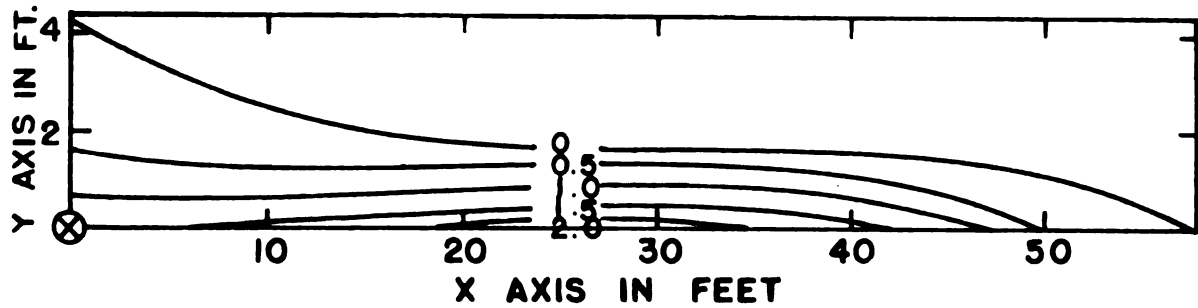


FIG. 15 ACCUMULATED PRECIPITATION CONTOUR FOR $\theta = 0^\circ$

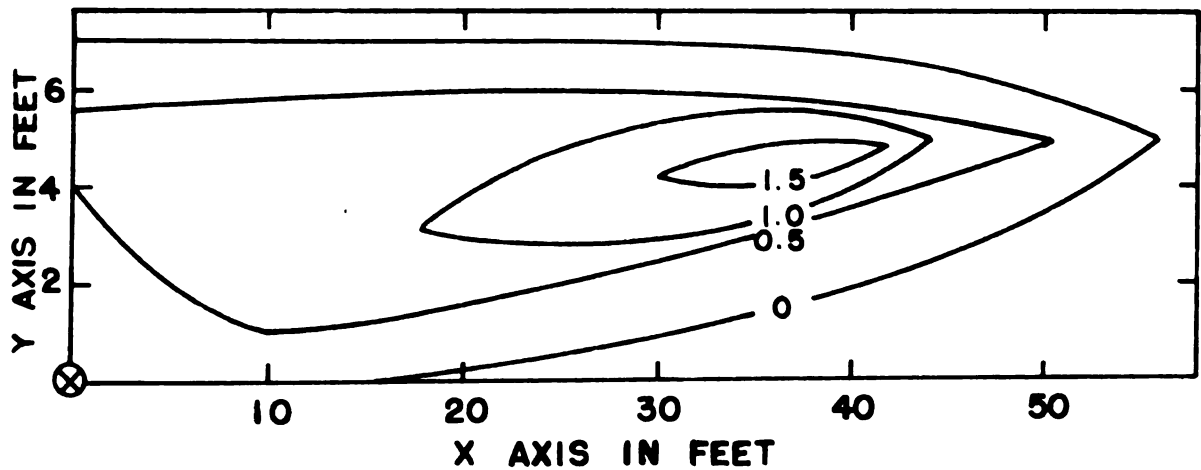


FIG. 16 ACCUMULATED PRECIPITATION CONTOUR FOR $\theta = 4^\circ$

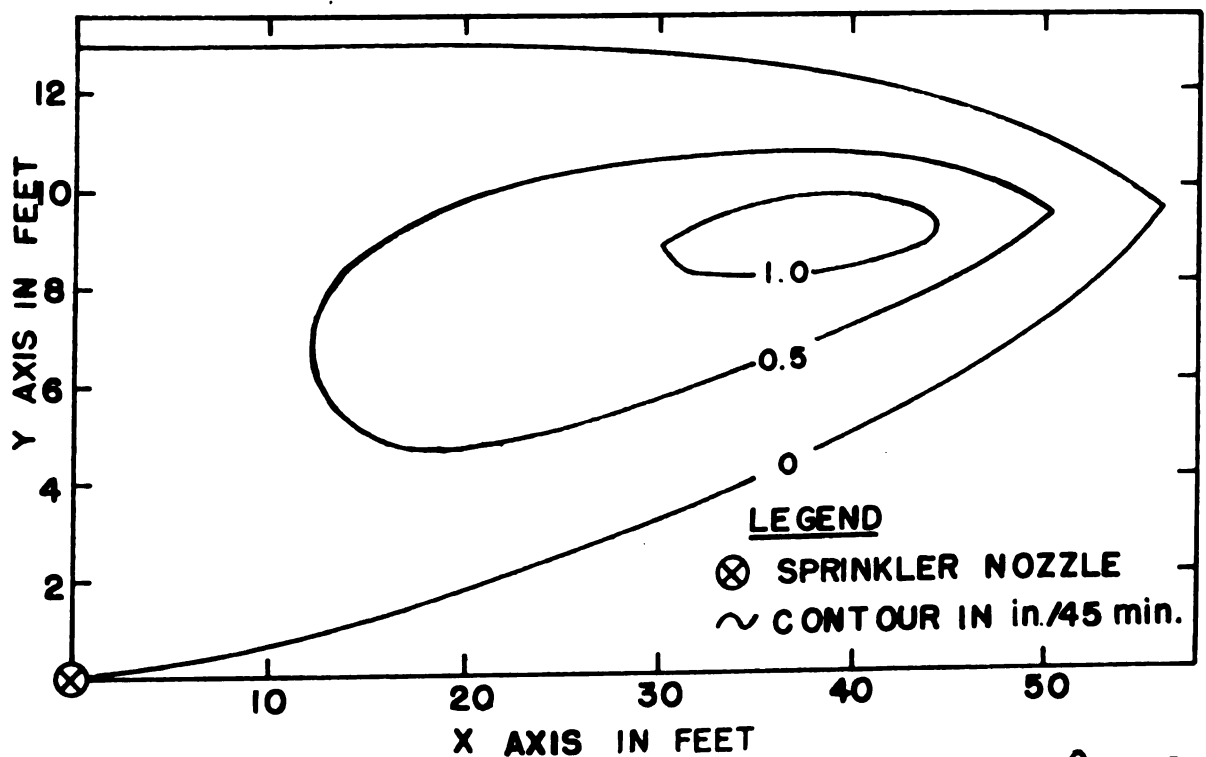


FIG. 17 ACCUMULATED PRECIPITATION CONTOUR FOR $\theta = 8^\circ$

FIG. 18 ACCUMULATED WATER DISTRIBUTION IN LATERAL DIRECTION FOR $\theta = 0, 2, 4, 6, \text{ AND } 8 \text{ DEGREES}$

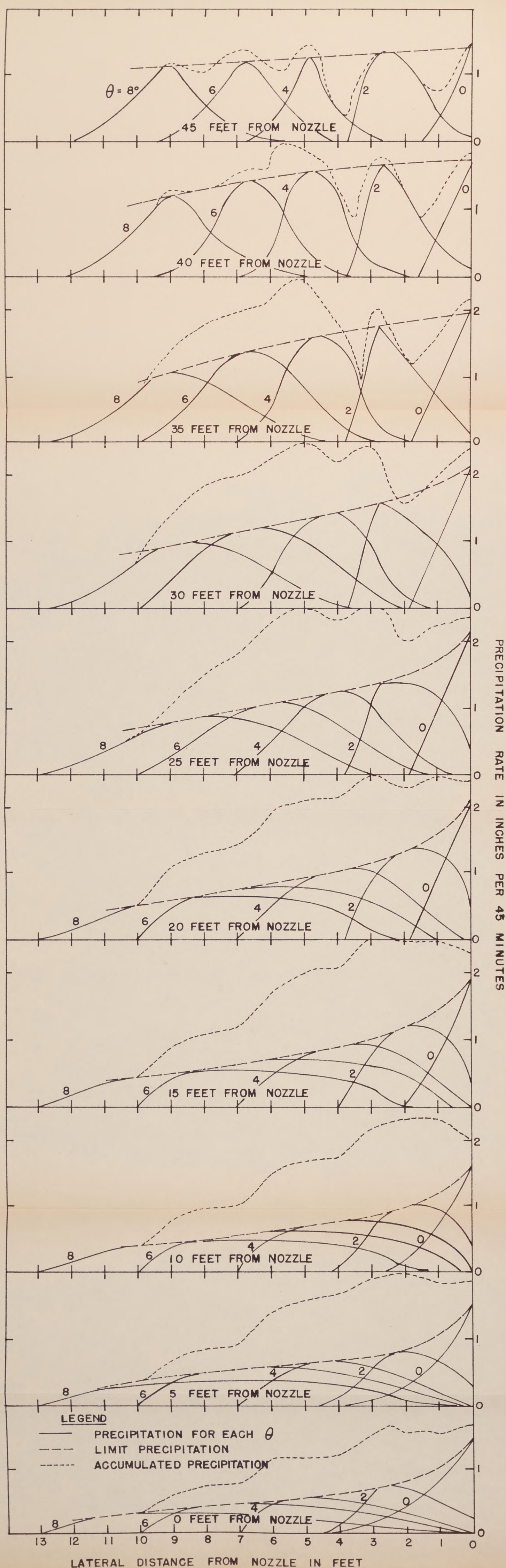


FIG. 20 ADJUSTED WATER DISTRIBUTION IN LATERAL DIRECTION FOR THE PROPOSED SETS OF JETS

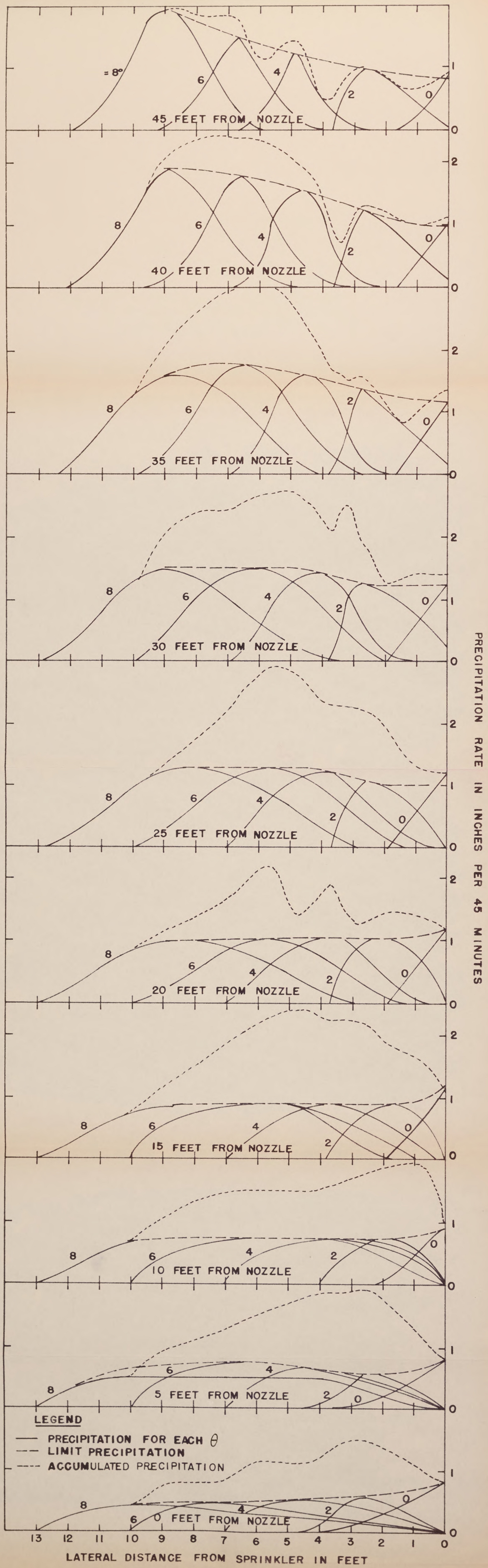
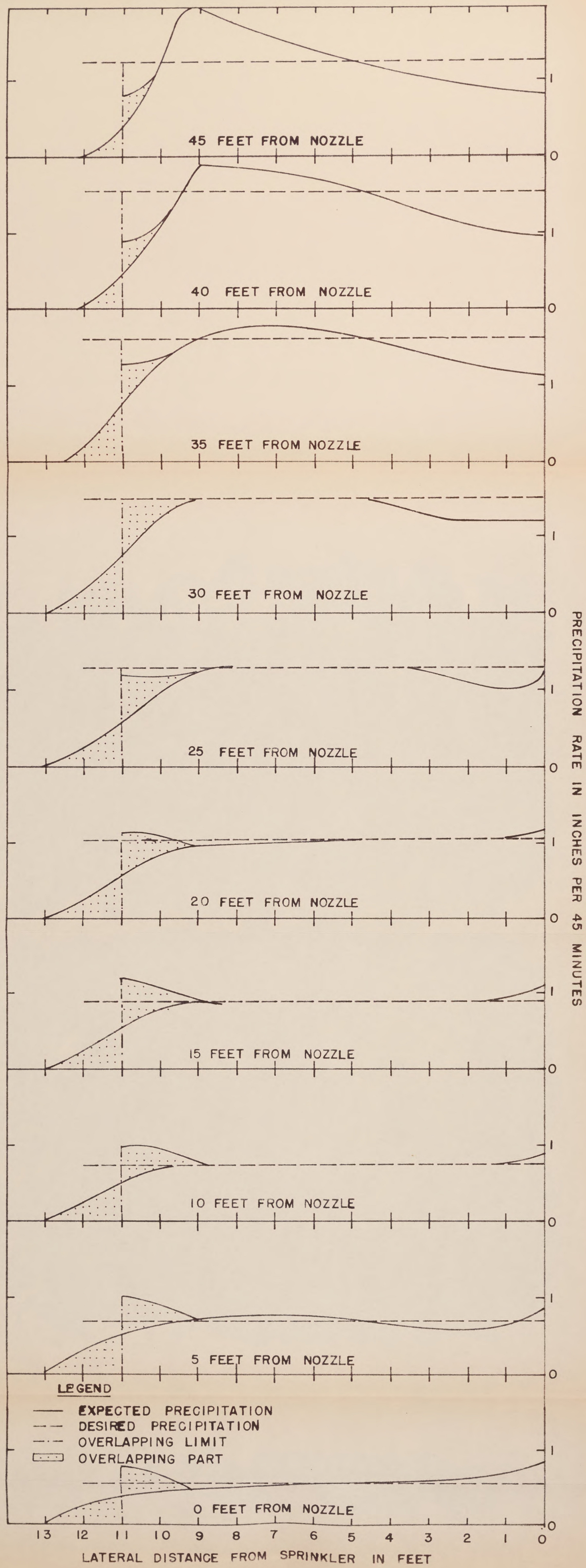


FIG. 22 ACCUMULATED WATER DISTRIBUTION IN LATERAL DIRECTION FOR A PROPOSED SLOT



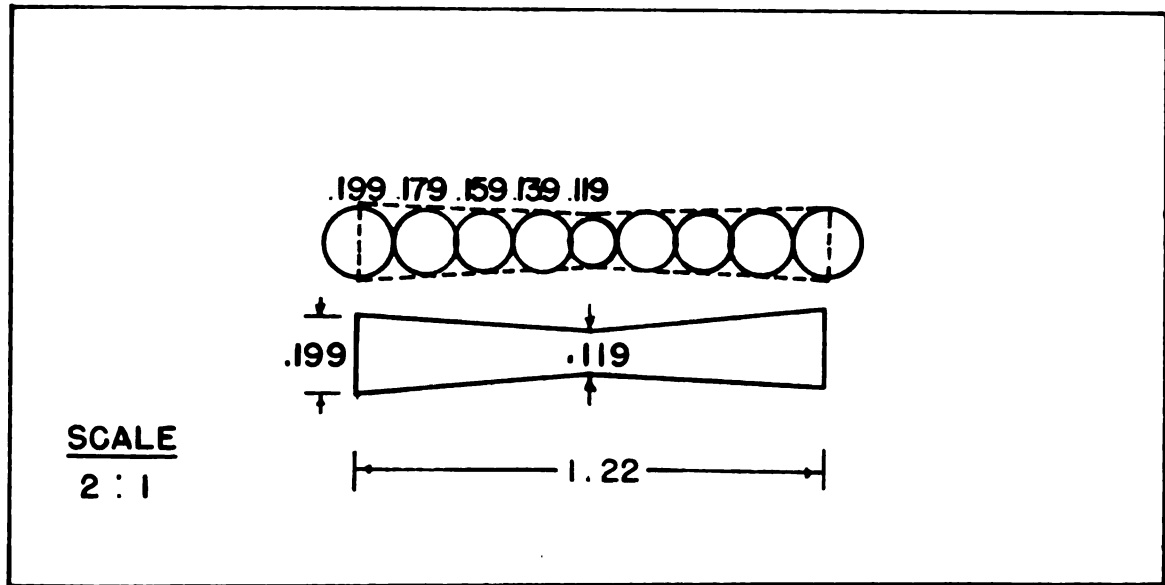


FIG. 21 PROPOSED SETS OF JETS AND A CORRESPONDING SLOT

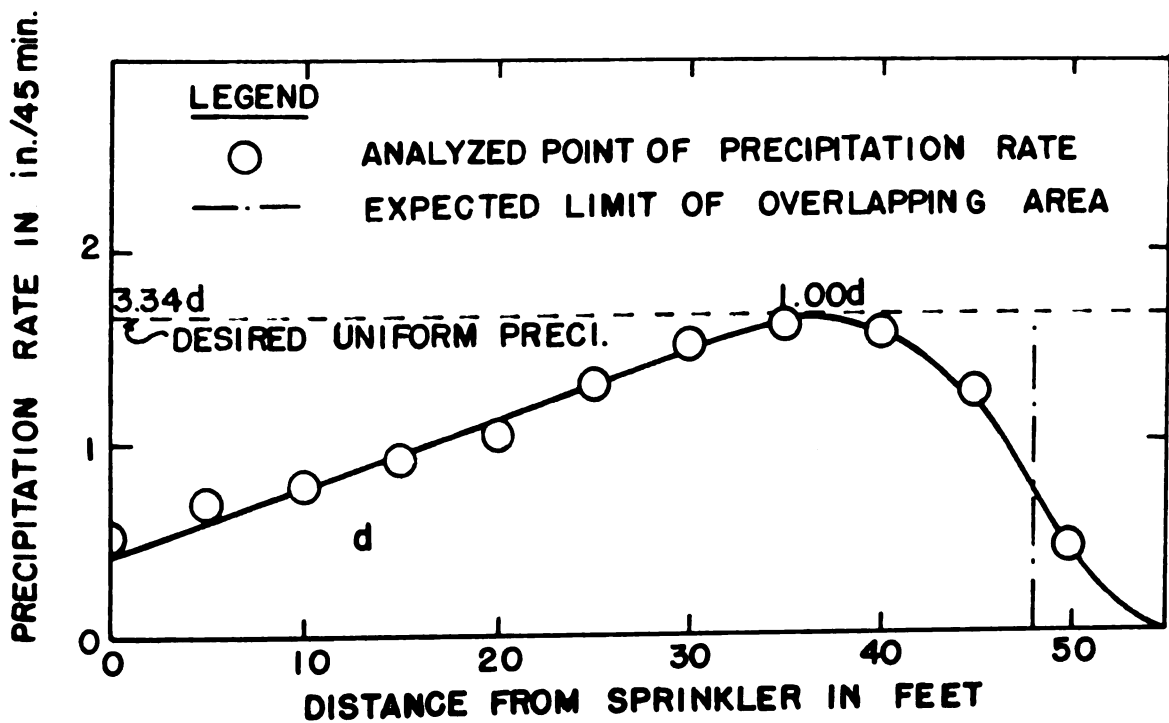
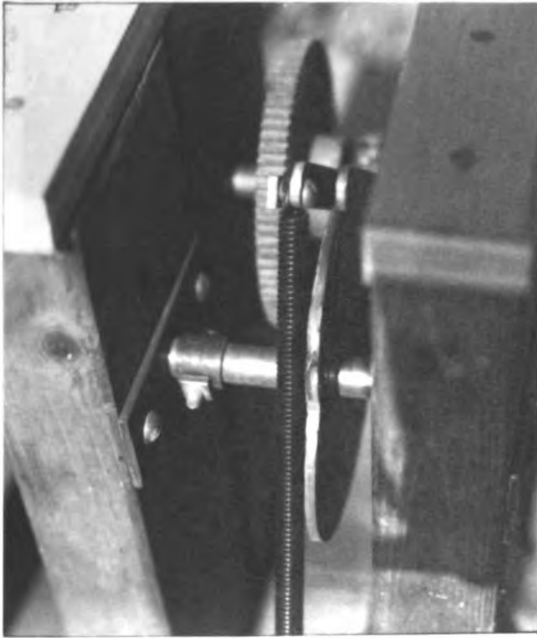
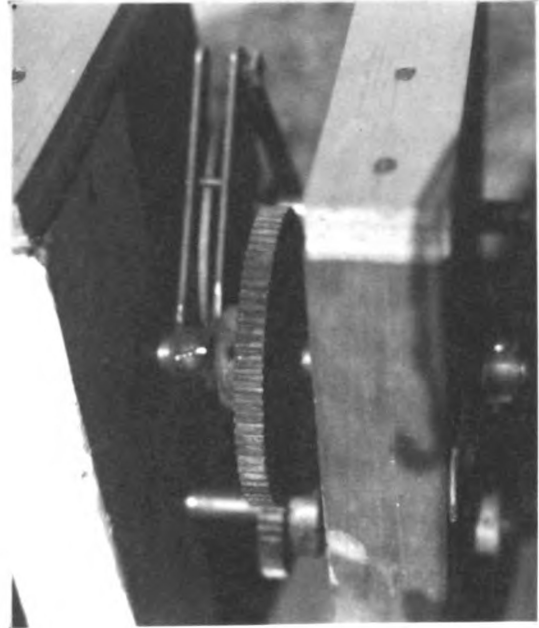


FIG. 23 ACCUMULATED WATER DISTRIBUTION IN LONGITUDINAL DIRECTION FOR A PROPOSED SLOT



Front view



Back view

Figure 24. View of oscillating mechanism.

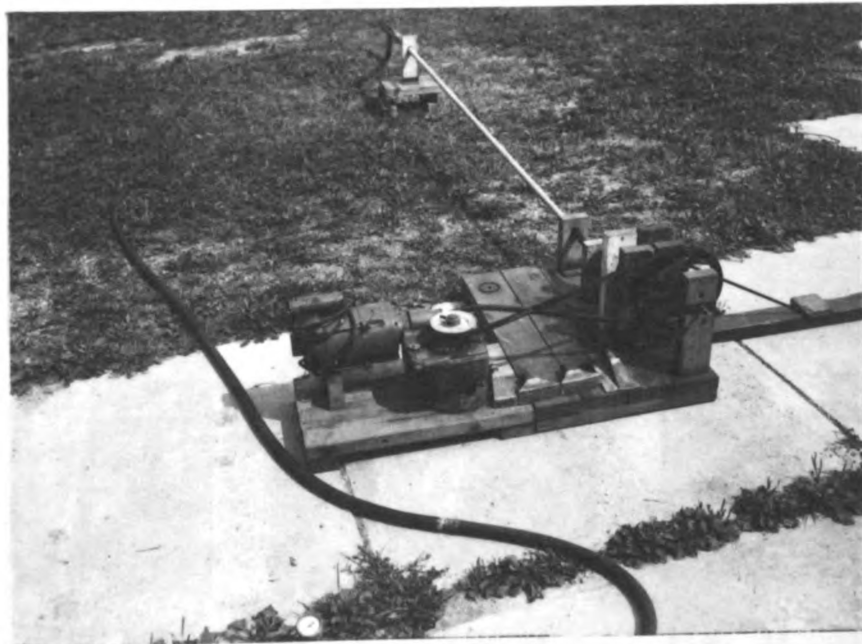


Figure 25. General view of oscillating mechanism and flow system.



Fig. 26 View of a jet from a nozzle of Type 1.



Fig. 27 View of a jet from a nozzle of Type 2.



Fig. 28 View of a jet from a nozzle of Type 3.



Fig. 29 View of a jet from a nozzle of Type 4.



Fig. 30 View of a jet from a nozzle of Type 5.



Fig. 31 View of a jet from a nozzle of Type 6.

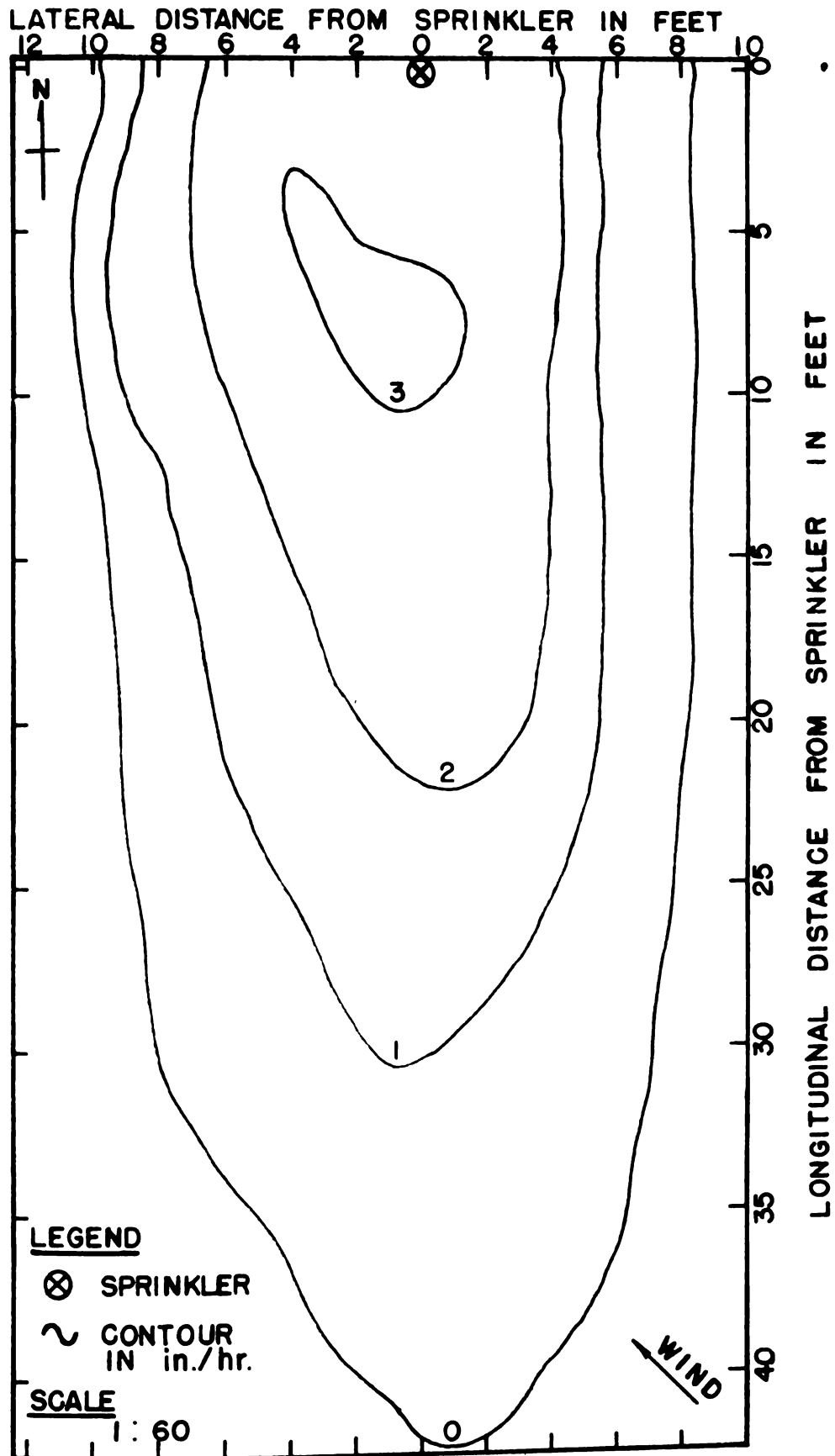


FIG. 33 PRECIPITATION CONTOURS OF A NOZZLE OF TYPE 1
OSCILLATING FOR 10 MINUTES UNDER 40 PSI. PRESSURE

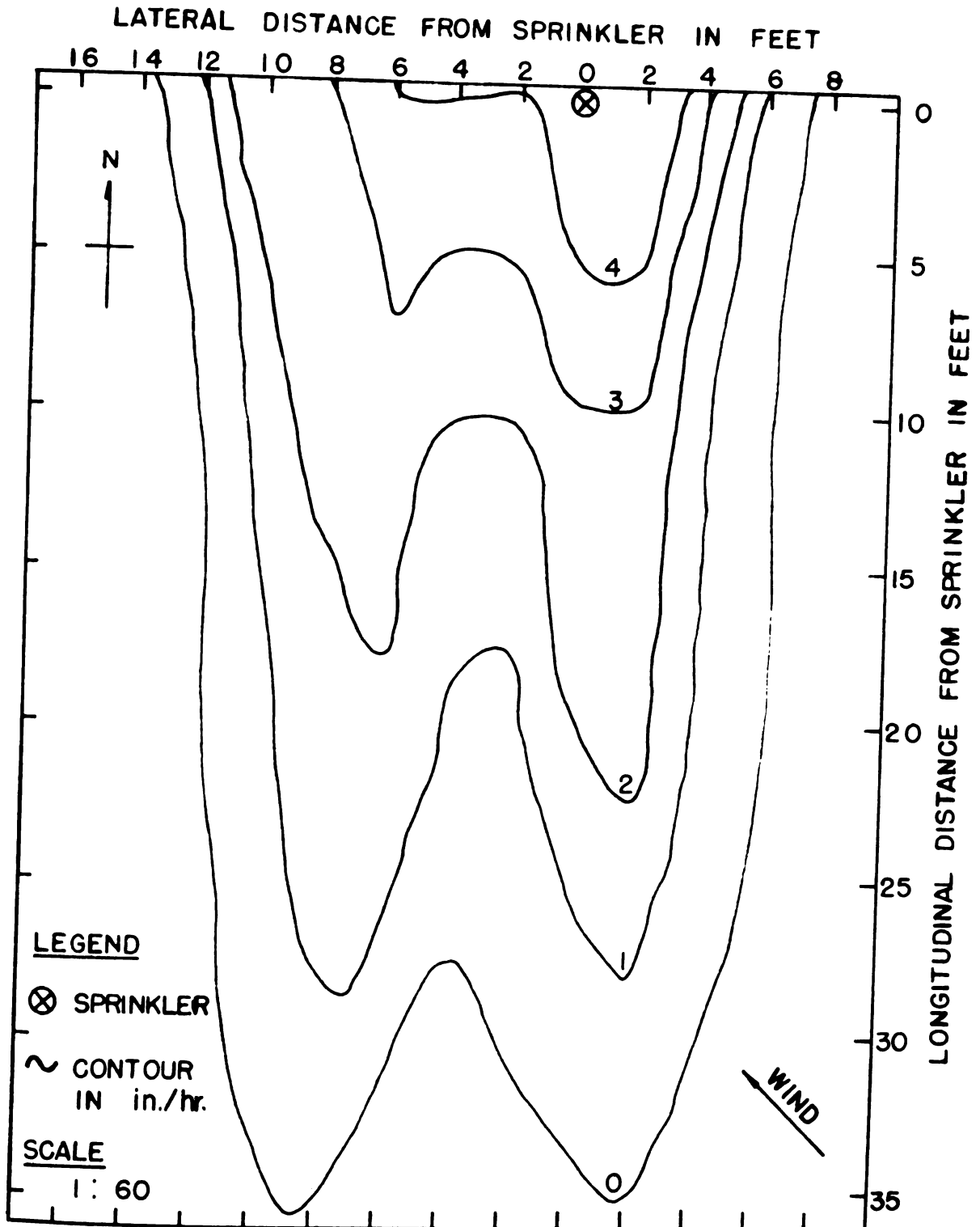


FIG. 34 PRECIPITATION CONTOURS OF A NOZZLE OF TYPE 2 OSCILLATING FOR 10 MIN. UNDER 40 psi. PRESSURE

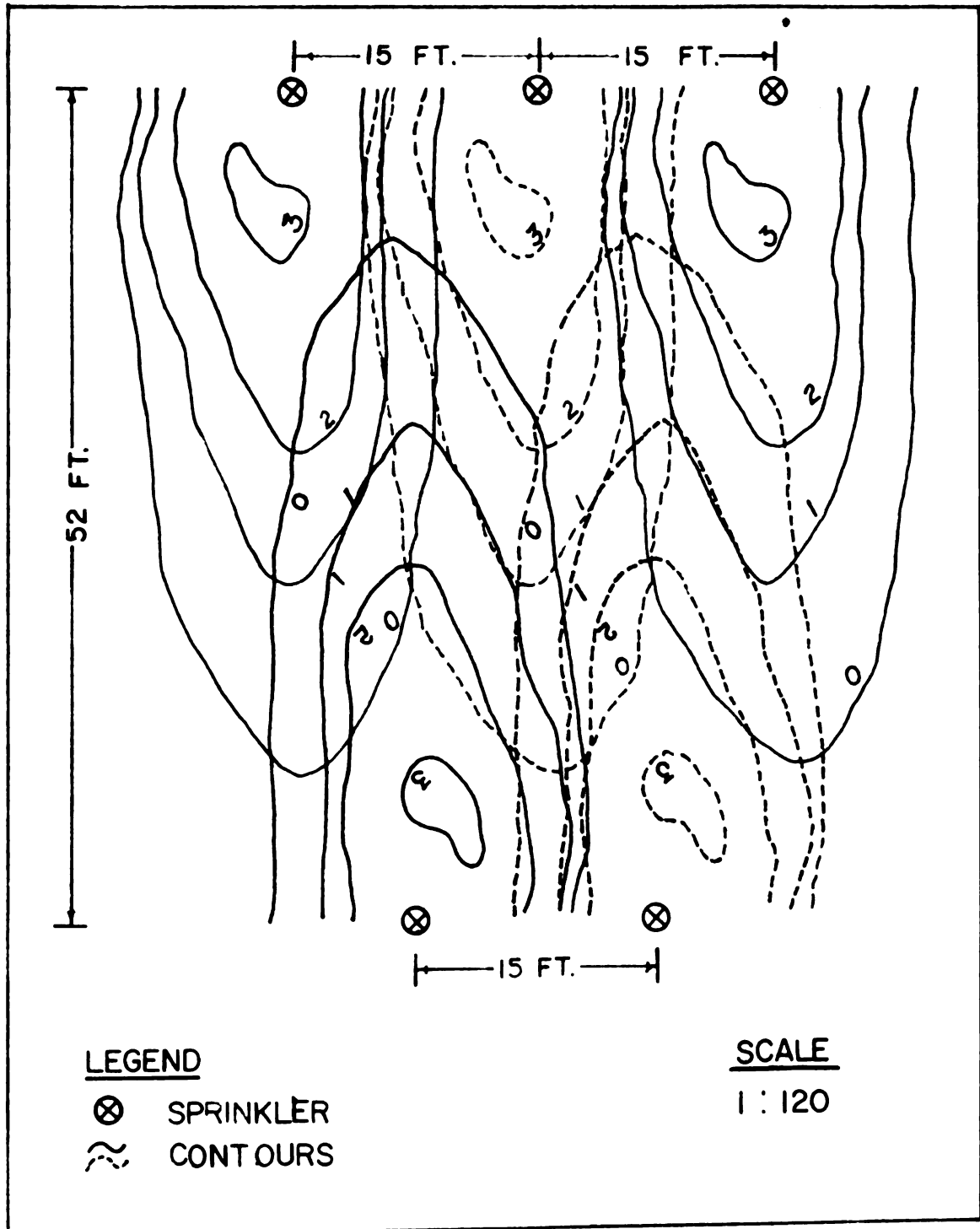


FIG. 35 SPACINGS OF SPRINKLER IN TRIANGULAR ARRANGEMENT

FIG. 37 RELATIONSHIP BETWEEN α & ϕ

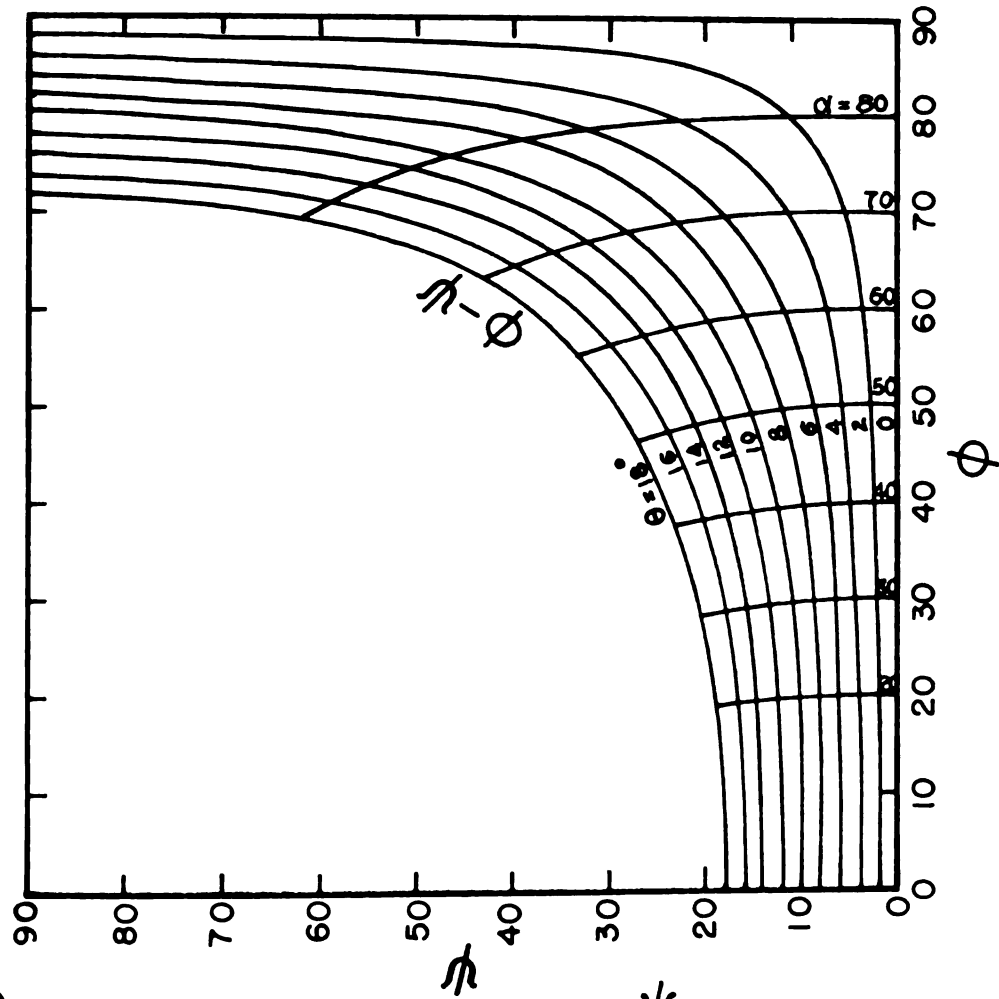
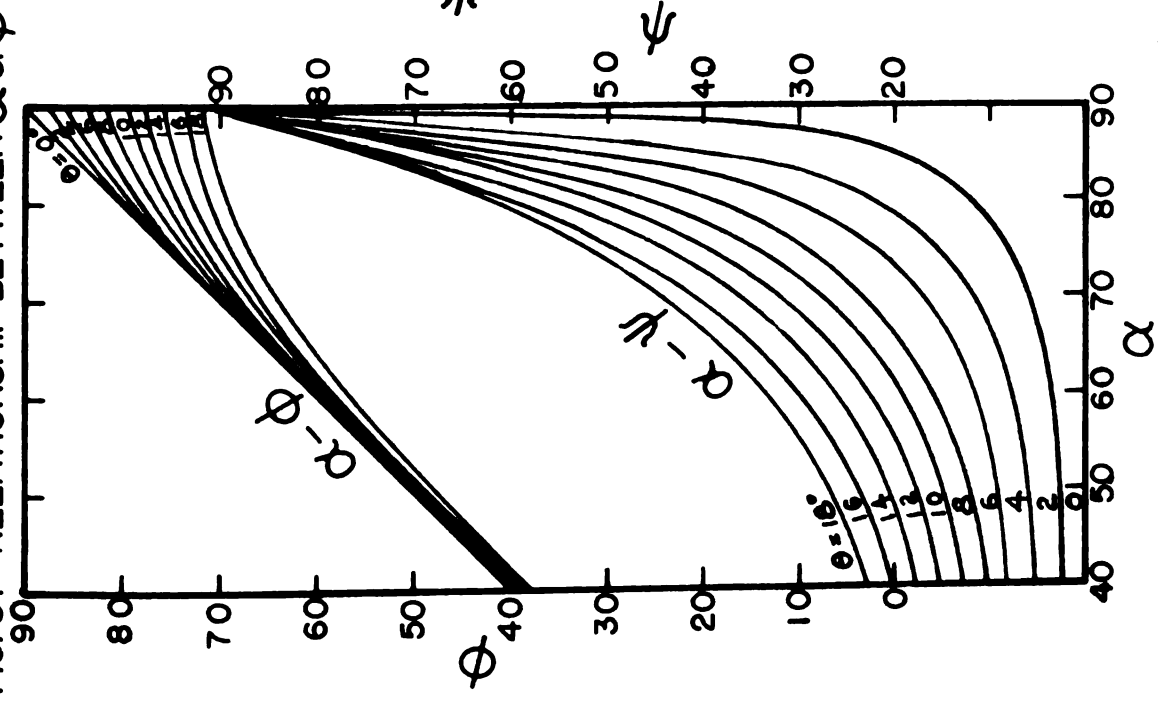


FIG. 38 RELATIONSHIP BETWEEN α & ψ

FIG. 39 RELATIONSHIP BETWEEN ϕ AND ψ

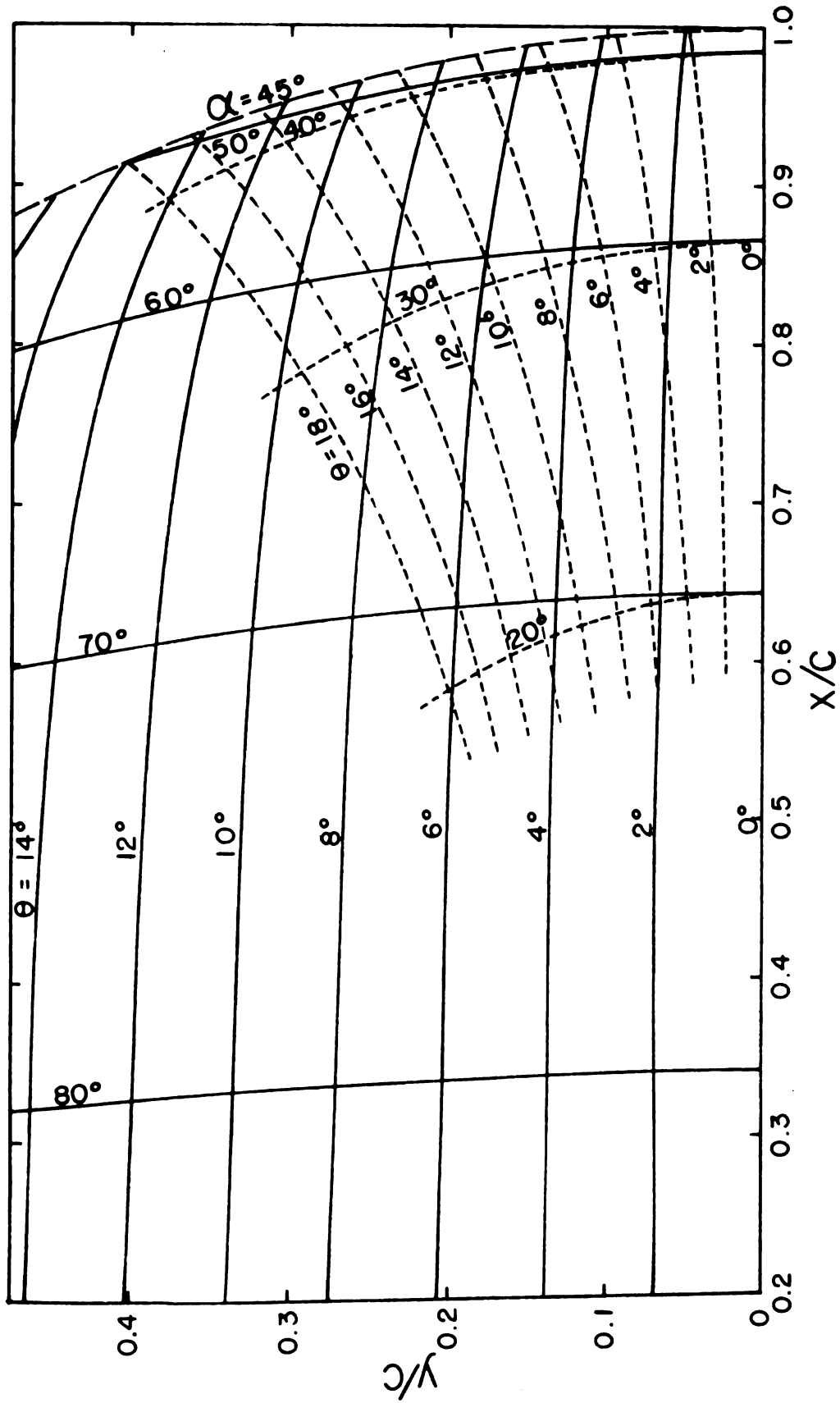


FIG. 42 RELATIONSHIP AMONG x/c , y/c , θ , & α

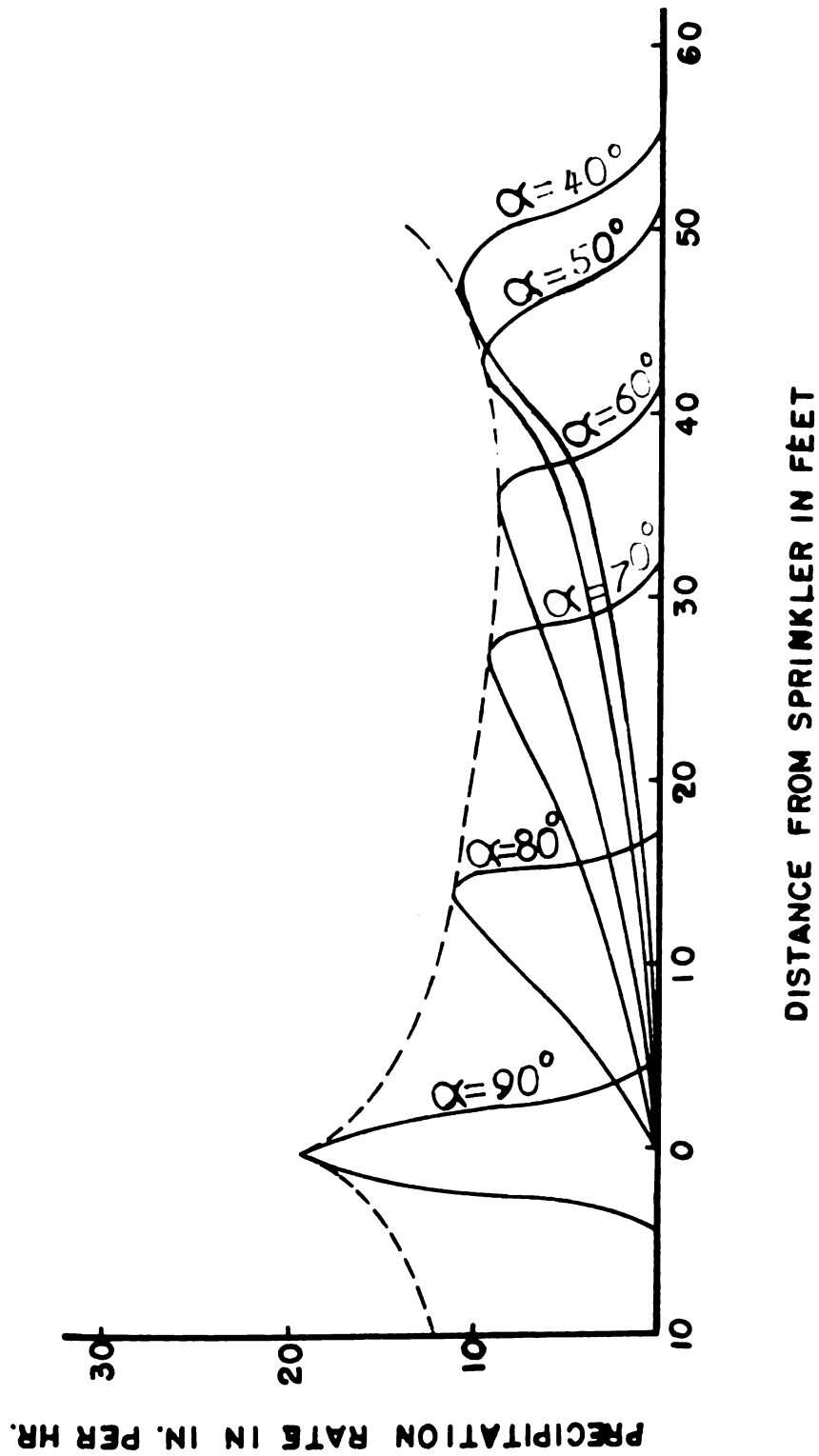
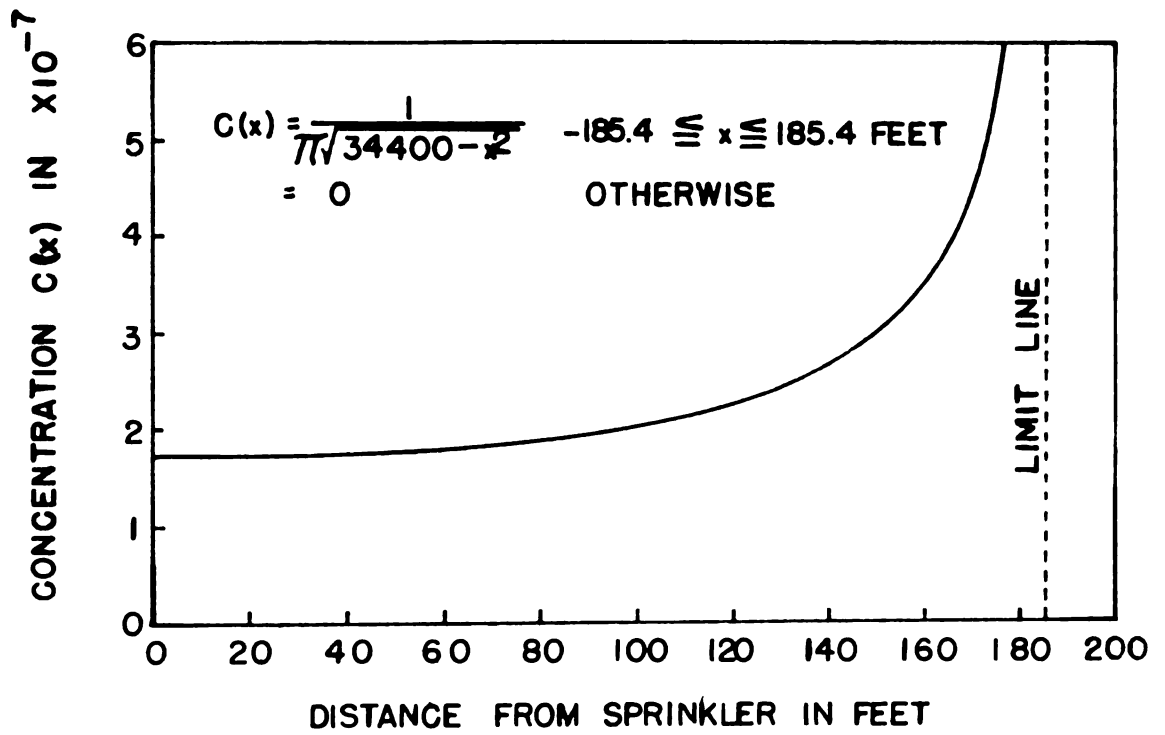
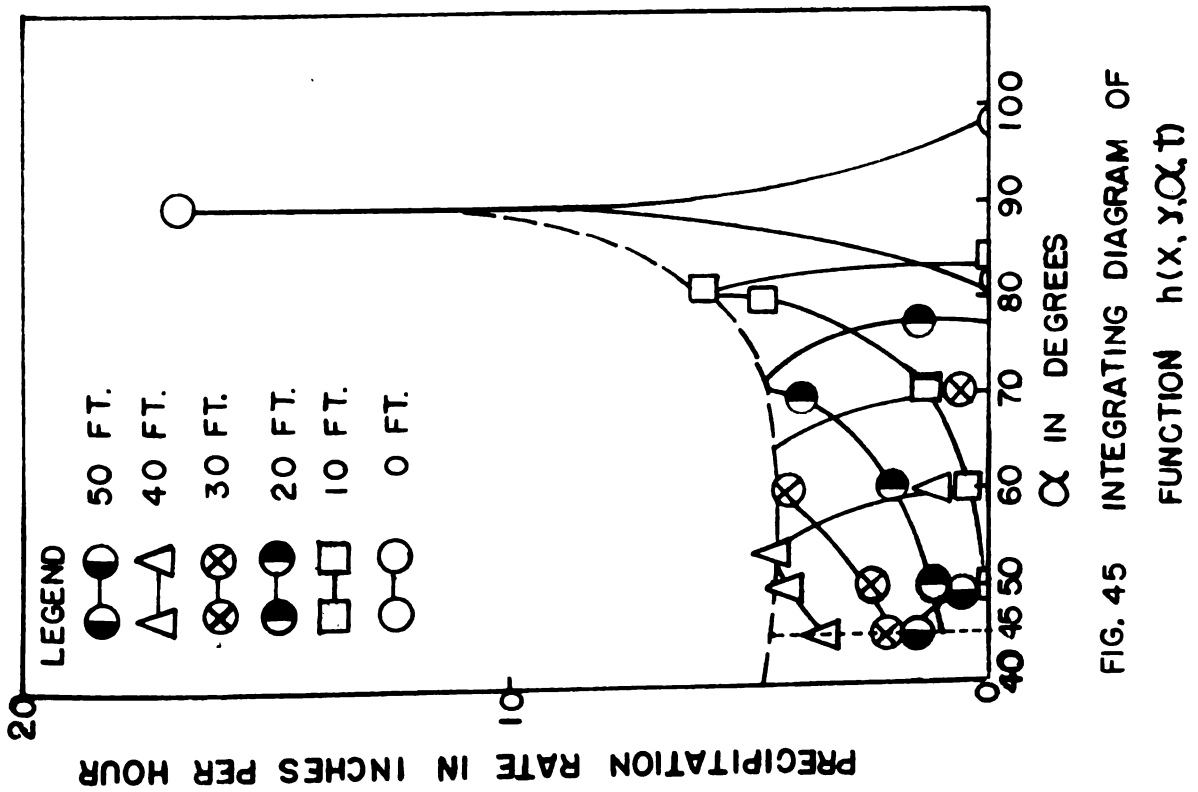


FIG. 44 LONGITUDINAL DISTRIBUTION OF WATER FROM A NOZZLE OF 0.159 INCH IN DIAMETER UNDER 40 PSI. PRESSURE

FIG. 43 CONCENTRATION CURVE OF $C(x)$ 

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