A POWER SERIES APPROXIMATION OF THE DYNAMIC TRANSFER CURVE OF A VACUUM TUBE

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John O. Cheney

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This is to certify that the

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A POWER SERIES APPROXIMATION OF THE DYNAMIC TRANSFER

CURVE OF A VACUUM TUBE

By

John O. Cheney

AN ABSTRACT

Submitted to the College of Engineering of Michigan State

University of Agriculture and Applied Science in

partial fulfillment of the requirements

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ABSTRACT

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One of the major problems in vacuum tube circuits is making accurate calculations of the tube's output when the tube is operating in the non-linear region of the tube. By use of finite power series the dynamic transfer curve is approximated and this power series establishes a relationship between the grid voltage and the plate current. The coefficients of the corresponding terms for transfer curves of different plate voltages are compared and a means of plate modulation calculations is devised. In addition several examples are worked out including a single-ended amplifier operating class AB_1, a push-pull amplifier operating class AB_1, and a plate modulated rf. amplifier operating class C. Some other approximations are discussed.

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INTRODUCTION

Recently there have been papers written by serveral authors pertaining to circuits with non-linear circuit elements.[1,2] The method of solution has been to approximate the non-linearity by a power series of few terms and then proceed to solve the non-linear differential equation. This paper will show that vacuum tubes may be handled in a similar manner by approximating the dynamic transfer curve such that $i_p = f(E_{bb}, e_c)$.

In April of 1919 H. J. van der Bijl published a paper which provided a means to determine the plate current of a vacuum tube given the input voltages and the characteristics of the tube.[3] The essentials of his paper follow.

$I_o = \alpha(\gamma E_b + E_c + \epsilon + e \sin pt)^B$ where e sin pt was the input signal, α is a constant whose value is a conductance and depends upon the structure of the tube, $\gamma = 1/\mu$, and ϵ is a voltage and depends on several things such as the contact potential difference between the cathode and grid, and the power developed in the

1. Pipes, L. A.; Forced Oscillations of Non Linear Circuits; Communications and Electronics; September 1954; pp. 352-358.

2. Ku, Y. H.; Circuit with Non Linear Inductance and Capacitance; Communications and Electronics; January 1955; pp. 619-626.

3. van der Bijl, H. J.; Theory and Operating Characteristics of the Thermionic Amplifier; Proceedings of the Institute of Radio Engineers; Volume VII, Number 2; April 1919; pp. 97-128.

filament (which was the cathode in many cases in those days).

But $E_b = E_{bb} - I_oR$ where R is the load resistance of the tube and E_{bb} is the applied plate voltage of the tube.

Then $I_o = \alpha[\gamma(E_{bb} - I_oR) + E_c + \epsilon + e \sin pt]^B$.

Also let $V = E_b + E_c + \epsilon$.

van der Bijl then claimed that experiment showed B to be equal to 2 and that ϵ was very small compared to the rest of the expression for V. He then solved the expression for I_o and obtained the expression below:*

$$I_o = \frac{1 + \alpha 2\gamma R(V + e\sin pt) - \sqrt{1 + \alpha 4\gamma R(V + e\sin pt)}}{2\alpha\gamma^2R^2}$$

This paper is still accepted today. However, there have been several modifications that have greatly reduced its value for analytical computation. The first limitation is that the tube must be conducting and that the grid should not be positive. The second and more discouraging point is that B is not equal to 2 but is approximately 1.6. This makes an analytical solution for I_o somewhat difficult and complicated, if not impossible.

With the advent of multigrid tubes van der Bijl's equation became less and less useful for the solution of circuit problems. Others[4] tried various means most of which relied on the parameters of the tube. In as much as these parameters varied to some extent with the applied voltages the mathematics became very quickly highly involved.

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* See Appendix A for the steps in the solution of the equation.

4. Caporale, Peter; A Note on the Mathematical Theory of the Multi-electrode Tube; Proceedings of the Institute of Radio Engineers; Volume XVIII, Number 2; September 1930; pp. 1593-1599.

A popular way to handle these problems is a graphical method of solution. This, however, has several serious drawbacks. With tubes going into cutoff or saturation it becomes difficult to predict the d.c. current drawn if the signal applied to the grid is a.c. A good way is to assume that the characteristic is linear until the tube cuts off. Using this assumption it is possible to make an approximation of the output current by Fourier series. A similar assumption is made at saturation.

These methods have the disadvantage that the slightest change in any one of the applied voltages necessitates starting all over again. There is also no way to figure plate modulation. Hence the graphical solution has several drawbacks which become quite serious when an analytical solution is desired.

CHAPTER I

THE APPROXIMATION

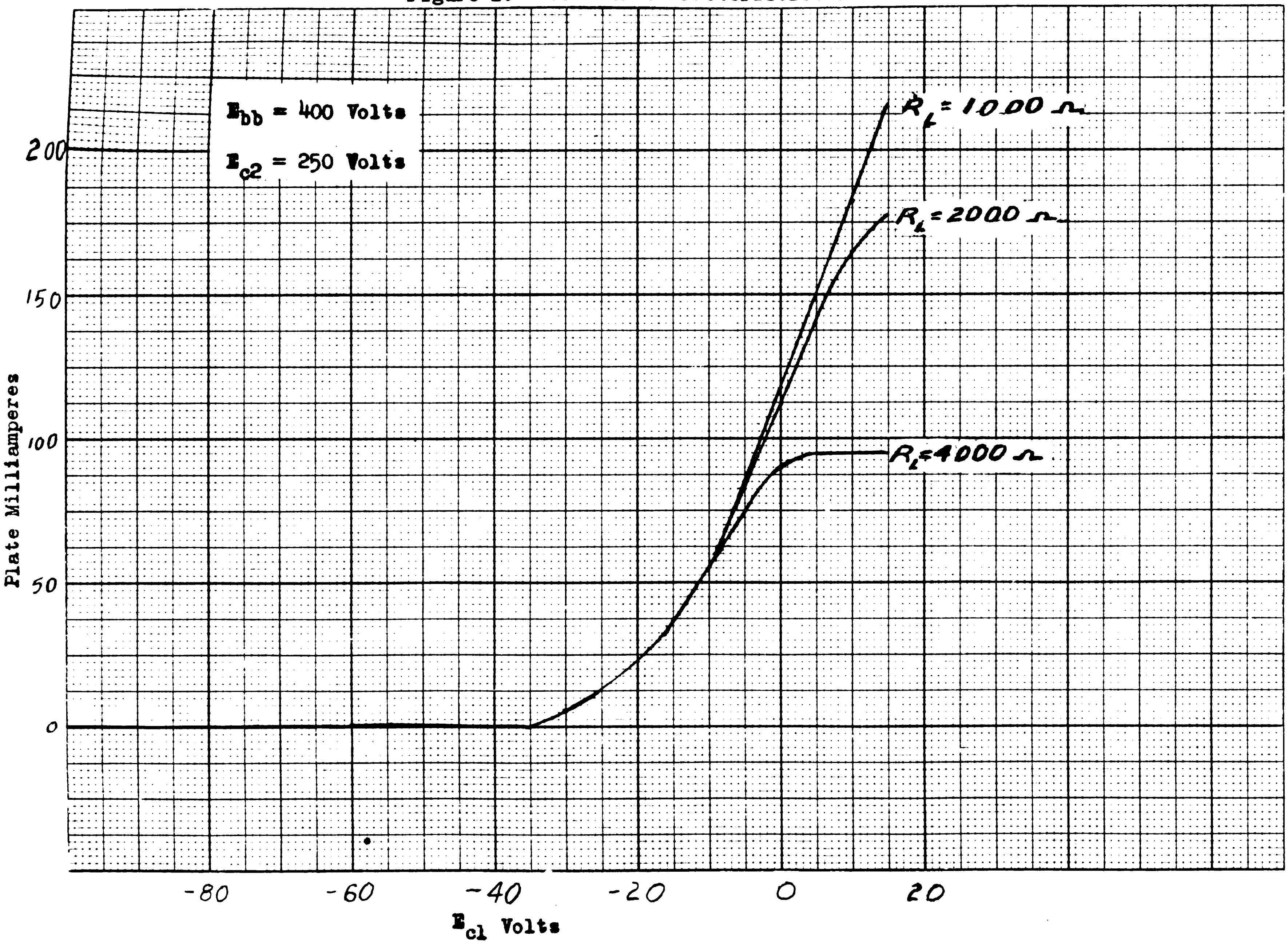
A solution is to approximate the tube curves with some mathematical expression. This solution poses two questions which must be answered before we can employ it. The first is, "What curve shall we approximate?" The second is, "Having selected the curve, how shall we approximate it?"

Since we are interested primarily in the tube as a circuit element we are most interested in its output for a given input. The transfer characteristic gives us the plate current for a given grid voltage if the plate voltage is held constant. Therefore the dynamic transfer characteristic will give us the curve we want.

Figure 1 shows us the curve we have to approximate. The ideal approximation would be an infinite convergent series. This series would then fit the curve on every point. Unfortunately such series are difficult to find unless the curve fits a known function. In our case this is not generally true. One may point out that the curve is very nearly that of a translated hyperbolic tangent. This is true but the variation from the hyperbolic tangent is on the very part that we are most interested in. If the tube goes both into saturation and cutoff, the hyperbolic tangent may well be the curve to use. This point will be discussed more fully later on.

Having ruled out infinite series we are reduced to a finite series of some type. One series to use would be a Taylor series. The form

Figure 1. Transfer Characteristics of a 6V6.



to be used would be:

$$i_p = a_1(e_{cl} + b) + a_3(e_{cl} + b)^3 + a_5(e_{cl} + b)^5 + \dots$$

The number of terms would depend on the number of points where the curve is to fit exactly and the number of points in turn would depend on the closeness of fit desired. Thus if n points were desired then the curve would have n terms (Note: At the point $b = -e_{cl}$, i_p would automatically equal zero and this point would not be included in the n points). Since the function contains only odd powers the curve would be symmetrical about the point $b = -e_{cl}$. This fact greatly expands the usefulness of the function. To determine the coefficients of the series we substitute the desired points into the equation, thus obtaining n equations with n unknowns, and then solve. Or by matrix notation:

$$\begin{bmatrix} i_{p1} \\ i_{p3} \\ \cdot \\ \cdot \\ \cdot \\ i_{p2n-1} \end{bmatrix} = \begin{bmatrix} c_1 & c_1^3 & c_1^5 & \cdots\cdots & c_1^{2n-1} \\ c_3 & c_3^3 & c_3^5 & \cdots\cdots & c_3^{2n-1} \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot \\ \cdots & & \cdot & \cdots\cdots & c_{2n-1}^{2n-1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_3 \\ \cdot \\ \cdot \\ \cdot \\ a_{2n-1} \end{bmatrix}$$

where $c_k = e_{cl} + b$ at the point k

and where i_{pk} = the current passing through the tube at that point.

Before actually solving a particular problem we shall first go about solving the general case. The method is that of organized substitution (or complete diagonalization of a matrix.).

The first step is write the c matrix and augment it by adding the i_p column to form an n + 1 by n rectangular matrix.

$$\begin{bmatrix} c_1 & c_1^3 & c_1^5 & \cdots\cdots & c_1^{2n-1} & i_{p1} \\ c_3 & c_3^3 & c_3^5 & \cdots\cdots & c_3^{2n-1} & i_{p3} \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ c_{2n-1} & c_{2n-1}^3 & c_3^5 & \cdots\cdots & c_{2n-1}^{2n-1} & i_{p2n-1} \end{bmatrix}$$

Divide the first row by c_1. Multiply this new first row by c_3 and subtact it from the second row. This will be the new second row. Repeat the last step except multiply by c_5 and subtract from the third row. Continue this process until the first column reads 1 0 0 0 ...

For simplicity let us call the number in the second column and second row of our new matrix d_3. Divide the second row of the new matrix (which will hereafter be referred to as the second matrix) by d_3. This will give us the second row of the third matrix. The first row of the third matrix is the same as the first row of the second matrix. Call the number in the second column and third row of the second matrix d_5. Divide and multiply the second row of the third matrix by d_5 and subtract it from the third row of the second matrix. This will be the third row of the third matrix. Repeat the operation until the second column reads ~~0~~1 0 0 0 ...

The process is repeated until principal diagonal of the last matrix (The diagonal that starts from the first term in the first row and contimues to the kth term in the kth row is the principal diagonal) consists only of ones and all terms below the principal diagonal are zero, or until the matrix looks as follows:

$$\begin{bmatrix} 1 & \gamma_{12} & \gamma_{13} & \gamma_{14} & \cdots\cdots & \gamma_{1n} & \alpha_1 \\ 0 & 1 & \gamma_{23} & \gamma_{24} & \cdots\cdots & \gamma_{2n} & \alpha_3 \\ 0 & 0 & 1 & \gamma_{34} & \cdots\cdots & \gamma_{3n} & \alpha_5 \\ \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots\cdots & 1 & a_{2n-1} \end{bmatrix}$$

The above matrix is called a half-diagonalized matrix. To completely diagonalize the matrix the same process as half-diagonalizing is used except that you start at the bottom and work up. A completely diagonalized matrix is shown below.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots\cdots & 0 & a_1 \\ 0 & 1 & 0 & 0 & 0 & \cdots\cdots & 0 & a_3 \\ 0 & 0 & 1 & 0 & 0 & \cdots\cdots & 0 & a_5 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdots\cdots & 1 & a_{2n-1} \end{bmatrix}$$

The reader will note that the n + 1 column is the term coefficients of the power series that we are looking for. (The justification for this manner of solution can be easily seen by placing a_{2k-1} after every term in the k column except the n + 1 column and a plus sign between columns except between the last two columns where there should be an equal sign. In this manner every row becomes and equation and the diagonalization process becomes merely an organized method of solution.)

Now we have decided what curve we are going to approximate and the general manner of way we are going to go about it. In the diagram below we have a single ended tretode amplifier. For the tube we shall use a 6V6 **. We will let E_{bb} equal 400 volts and E_{c2} be fixed at 250 volts. The load resistor will be 2,000 ohms.

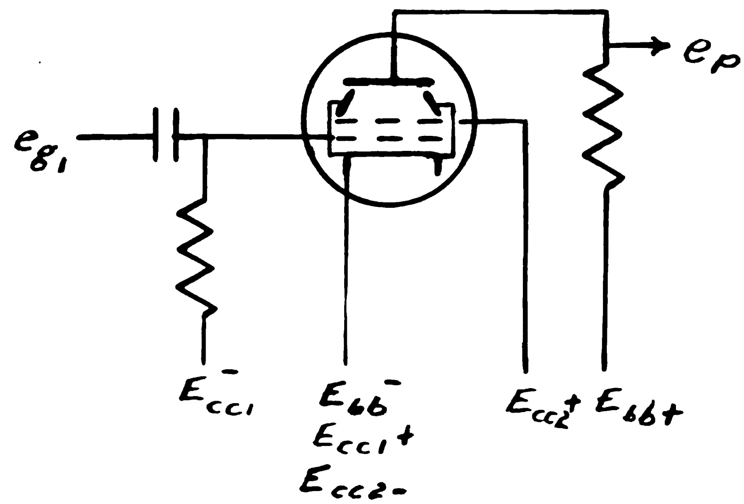


Figure 2: Basic tretode amplifier.

To obtain the dynamic transfer characteristic, a load line is drawn on the average plate characteristics chart (which is available from various tube manuals) and a curve of plate current vs. grid voltage is drawn along the load line.

For a first approximation we will choose the point $e_{c1} = -100$ volts as our origin. We will use five other points; one at cut-off, one just at saturation, one in the middle of the linear range, one well into

** The 6V6 is a beam power tretode. It behaves almost identically like a pentode.

saturation, and the last one approximately half way between cut-off and -100 volts. The points selected were e_{cl} = -60, -35, 0, +15, +50. For clarity the five equations will be written out in their entirity and solved by diagonalization. Each equation will be of the form:

$$i_p = a(e_{cl} + 100) + b(e_{cl} + 100)^3 + c(e_{cl} + 100)^5 + d(e_{cl} + 100)^7 + e(e_{cl} + 100)^9$$

(1) $0 = a\ 40 + b\ 64 \times 10^3 + c\ 102.4 \times 10^6 + d\ 0.16384 \times 10^{12} + e\ 0.000262 \times 10^{18}$

(2) $0 = a\ 65 + b\ 274.625 \times 10^3 + c\ 1160.3 \times 10^6 + d\ 4.902 \times 10^{12} + e\ 0.0207119 \times 10^{18}$

(3) $112.5 = a\ 100 + b\ 1000 \times 10^3 + c\ 100000 \times 10^6 + d\ 100 \times 10^{12} + e\ 1 \times 10^{18}$

(4) $177.5 = a\ 115 + b\ 1520.875 \times 10^3 + c\ 20113.6 \times 10^6 + d\ 266 \times 10^{12} + e\ 3.518 \times 10^{18}$

(5) $177.5 = a\ 150 + b\ 3375 \times 10^3 + c\ 75937.9 \times 10^6 + d\ 1708.6 \times 10^{12} + e\ 38.44 \times 10^{18}$.

In the matrix form:

$$(6)\quad \left[\begin{array}{cccccc} 40 & 64 \times 10^3 & 102.4 \times 10^6 & 0.16384 \times 10^{12} & 0.000262 \times 10^{18} & 0 \\ 65 & 274.625 \times 10^3 & 1160.3 \times 10^6 & 4.902 \times 10^{12} & 0.0207119 \times 10^{18} & 0 \\ 100 & 1000 \times 10^3 & 10000 \times 10^6 & 100 \times 10^{12} & 1 \times 10^{18} & 112.5 \\ 115 & 1520.875 \times 10^3 & 20113.6 \times 10^6 & 266 \times 10^{12} & 3.518 \times 10^{18} & 177.5 \\ 150 & 3375 \times 10^3 & 75937.9 \times 10^6 & 1708.6 \times 10^{12} & 38.44 \times 10^{18} & 177.5 \end{array}\right]$$

The first row is divided by 40 and then becomes the new first row for (7). The first row of (7) is multiplied by 65 and subtracted from the second row of (6) giving the second row of (7). The third row of (7) is obtained by multiplying the first row of (7) by 100 and subtracting it from the third row of (6). The fourth row of (7) is obtained by multiplying the first row of (7) by 115 and subtracting it from the fourth row of (6). In a similar manner the fifth row of (7) is found by multiplying the first row of (7) by 150 and subtracting it from the fifth row of (6).

$$(7)\begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 0.004096\times10^{12} & 0.00000655\times10^{18} & 0 \\ 0 & 170.625\times10^{3} & 993.9\times10^{6} & 4.6358\times10^{12} & 0.020286\times10^{18} & 0 \\ 0 & 840\times10^{3} & 9744\times10^{6} & 99.5904\times10^{12} & 0.999345\times10^{18} & 112.5 \\ 0 & 1336.875\times10^{3} & 19818.2\times10^{6} & 265.529\times10^{12} & 3.517247\times10^{18} & 177.5 \\ 0 & 3135\times10^{3} & 75553.9\times10^{6} & 1707.986\times10^{12} & 38.439\times10^{18} & 177.5 \end{bmatrix}$$

$$(8)\begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 \\ 0 & 1 & 5.825\times10^{3} & 0.02717\times10^{9} & 0.0001189\times10^{15} & 0 \\ 0 & 0 & 4.851 & 76.7676\times10^{3} & 0.899469\times10^{9} & 112.5\times10^{-9} \\ 0 & 0 & 12.032 & 229.206\times10^{3} & 3.358293\times10^{9} & 177.5\times10^{-9} \\ 0 & 0 & 57.292 & 1622.808\times10^{3} & 38.0662485\times10^{9} & 177.5\times10^{-9} \end{bmatrix}$$

(8) was obtained from (7) by rewriting the first row of (7) as the first row of (8); by dividing the second row of (7) by 170.625×10^{3} for the second row of (8); by multiplying the second row of (8) by 840×10^{3} and subtracting from the third row of (7) and then dividing by 10^{9} for the third row; by multiplying the second row of (8) by 1336.875×10^{3} and subtracting it from the fourth row of (7) and then dividing by 10^{9} for the fourth row; and by multiplying the second row of (8) by 3135×10^{3} and subtracting it from the fifth row of (7) and then dividing by 10^{9} for the fifth row of (8).

To get (9) we will rewrite the first and second rows of (8) as the first and second rows respectively of (9). Then divide the third row of (8) by 4.851 for the third row of (9). The fourth row is obtained by multiplying the third row of (9) by 57.292 and subtracting it from the fifth row of (8) and then dividing by 10^3.

$$(9)\quad \begin{bmatrix} 1 & 1.6\times10^3 & 2.56\times10^6 & 4.096\times10^9 & 6.55\times10^{12} & 0 \\ 0 & 1 & 5.825\times10^3 & 27.17\times10^6 & 118.9\times10^9 & 0 \\ 0 & 0 & 1 & 15.825\times10^3 & 0.1854\times10^9 & 23.191\times10^{-9} \\ 0 & 0 & 0 & 38.8 & 1.1276\times10^6 & -101.53\times10^{-12} \\ 0 & 0 & 0 & 716.162 & 27.444\times10^6 & -1151.6\times10^{-12} \end{bmatrix}$$

The next matrix, (10), is readily found in a similar manner. The first three rows of (10) are identical with those of (9). The fourth row of (10) is the fourth row of (9) divided by 38.8. The fifth row was obtained by multiplying the fourth row of (10) by 716.162 and subtracting it from the fifth row of (9) and dividing by 10^3.

$$(10)\quad \begin{bmatrix} 1 & 1.6\times10^3 & 2.56\times10^6 & 4.096\times10^9 & 6.55\times10^{12} & 0 \\ 0 & 1 & 5.825\times10^3 & 27.17\times10^6 & 118.9\times10^9 & 0 \\ 0 & 0 & 1 & 15.825\times10^3 & 185.4\times10^6 & 23.191\times10^{-9} \\ 0 & 0 & 0 & 1 & 0.02906\times10^6 & -2.617\times10^{-12} \\ 0 & 0 & 0 & 0 & 6632 & -723.04\times10^{-5} \end{bmatrix}$$

$$(11)\quad \begin{bmatrix} 1 & 1.6\times10^3 & 2.56\times10^6 & 4.096\times10^9 & 0 & -0.714\times10^{-3} \\ 0 & 1 & 5.825\times10^3 & 27.17\times10^6 & 0 & -12.96\times10^{-6} \\ 0 & 0 & 1 & 15.825\times10^3 & 0 & 2.982\times10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -5.785\times10^{-12} \\ 0 & 0 & 0 & 0 & 1 & 0.109\times10^{-15} \end{bmatrix}$$

Matrix (11) was obtained from (10) by dividing the fifth row of (10) by 6632 for the fifth row of (11); by multiplying the fifth row of (11) by 29.06×10^3 and subtracting it from the fourth row of (10) for the fourth row of (11); by multiplying the fifth row of (11) by 185.4×10^6 and subtracting it from the third row of (10) for the third row of (11); by multiplying the fifth row of (11) by 118.9×10^9 and subtracting it from the second row of (10) for the second row of (11); by multiplying the fifth row of (11) by 6.55×10^{12} and subtracting it from the first row of (10) for the first row of (11).

By this time the reader must be fairly familiar with the process. I will, therefore, merely outline the rest of the solution and give the final results. The next step would be to eliminate the terms other than one in the fourth column, and then the third, and lastly the second. The resultant matrix would be:

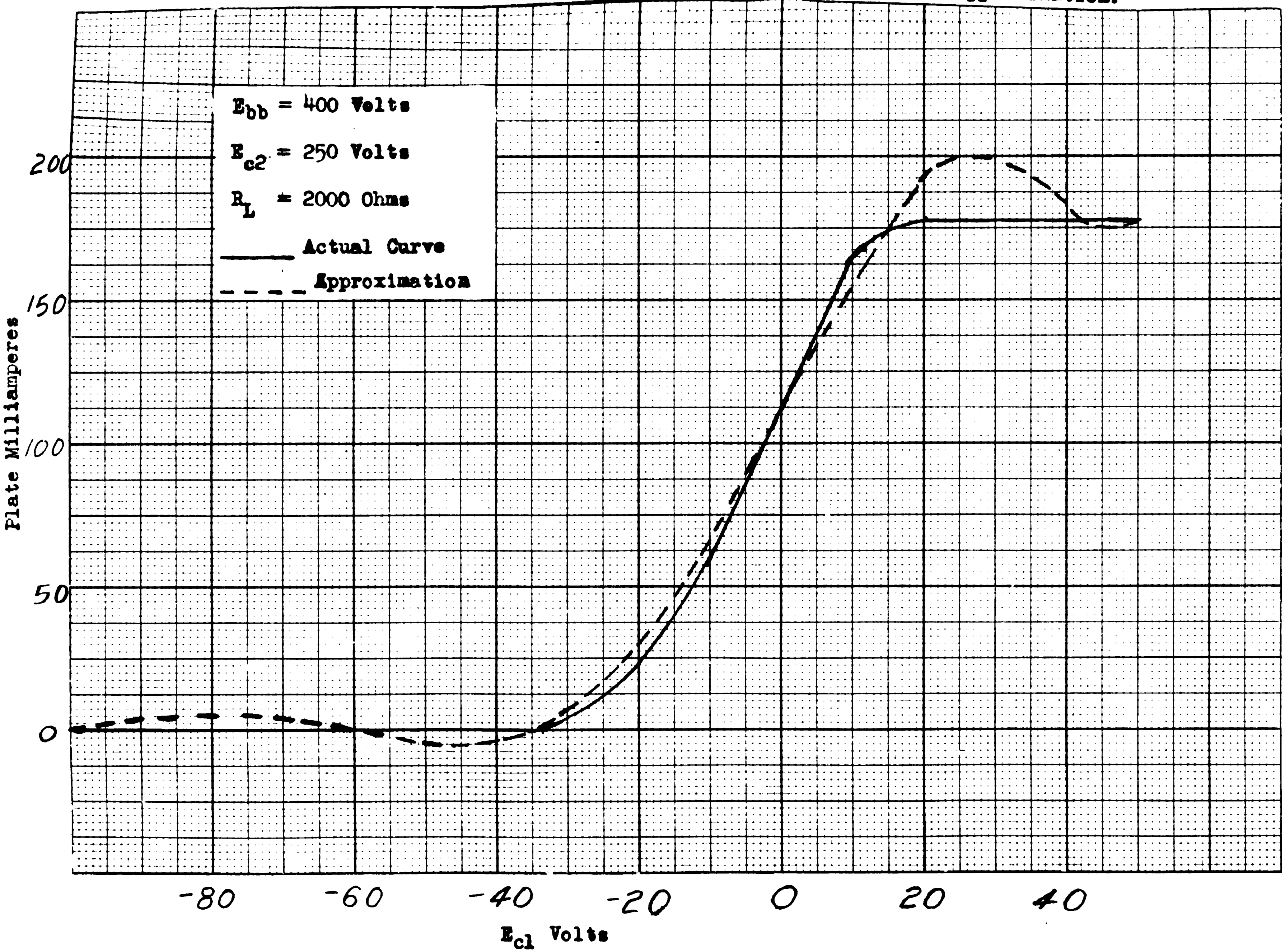
$$(12) \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 427.87 \times 10^{-3} \\ 0 & 1 & 0 & 0 & 0 & -406.42 \times 10^{-6} \\ 0 & 0 & 1 & 0 & 0 & 94.53 \times 10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -2.617 \times 10^{-12} \\ 0 & 0 & 0 & 0 & 1 & 0.109 \times 10^{-12} \end{bmatrix}$$

Our results can now be expressed in the equation that approximates our curve.

$$(13) \quad i_p = 0.428(e_{cl} + 100) - 0.496 \times 10^{-3}(e_{cl} + 100)^3 + 0.09453 \times 10^{-6}(e_{cl} + 100)^5 - 5.785 \times 10^{-12}(e_{cl} + 100)^7 + 109 \times 10^{-18}(e_{cl} + 100)^9$$

The next step is to plot the expression and compare it with the curve we are acutally trying to approximate. This is done on Figure 3.

Figure 3. A Transfer Characteristic of a 6V6 and its First Approximation.



In this particular case we note that the approximation is very close to that of the actual curve until saturation is reached. At this point there is considerable overshoot. Nevertheless the error is less than 15 percent (Based on the saturation current). Therefore we note that the curve for the most part is well fitted.

It is now possible to vary the grid voltage over a considerable range without making extensive new calculations. We still, however, are nailed down tight as far as plate voltage, screen voltage, and load resistor are concerned.

Let us next investigate the effects of changing the plate voltage. In order to be consistent we will use the same load resistor value and the same origin. For our first attempt we use a plate voltage of 300 volts and select the same points as we used in the 400 volt curve. Our augmented matrix appears as follows:

$$(14)\quad \begin{bmatrix} 40 & 64\times10^{3} & 102.4\times10^{6} & 0.16384\times10^{12} & 0.000262\times10^{18} & 0 \\ 65 & 274.625\times10^{3} & 1160.3\times10^{6} & 4.902\times10^{12} & 0.0207119\times10^{18} & 0 \\ 100 & 1000\times10^{3} & 10000\times10^{6} & 100\times10^{12} & 1\times10^{18} & 112.5 \\ 115 & 1520.875\times10^{3} & 20113.6\times10^{6} & 266\times10^{12} & 3.518\times10^{18} & 135 \\ 150 & 3375\times10^{3} & 75937.9\times10^{6} & 1708.6\times10^{12} & 38.44\times10^{18} & 135 \end{bmatrix}$$

This solves in the same manner as the previous matrix and the resulting equation is as follows:***

$$(15)\quad i_p = 0.60548(e_{c1} + 100) - 0.58153\times10^{-3}(e_{c1} + 100)^3 + 0.14259\times10^{-6}(e_{c1} + 100)^5 - 10.134\times10^{-12}(e_{c1} + 100)^7 + 221\times10^{-18}(e_{c1} + 100)^9.$$

***For the rest of the paper merely the first matrix and the solution will be given. See the Appendices for step by step solutions.

Before checking this equation let us study the method of solution used in obtaining these last two equations and see if we can notice any similarity. First thing that we note is that the first five columns are correspondingly the same. Only the sixth columns are different. If we were to use the same points in approximating the 200 volt curve, again the first five columns would be identical and only the sixth column would be different. Instead of repeating all the work all over again for the first five columns the five by five matrix was augmented twice to make a five by seven matrix, and the 300 and 200 curves were solved for simultaneously.

The seventh column of the matrix was:

$$\begin{matrix} 0 \\ 0 \\ 82.5 \\ 87.5 \\ 87.5 \end{matrix} \tag{16}$$

The equation of the 200 volt curve was:

$$i_p = 0.4909(e_{cl} + 100) - 0.47357\text{x}10^{-3}(e_{cl} + 100)^3 + 0.11749\text{x}10^{-6}(e_{cl} + 100)^5 - 8.599\text{x}10^{-12}(e_{cl} + 100)^7 + 192\text{x}10^{-18}(e_{cl} + 100)^9. \tag{17}$$

Examination of the function plots as compared to the actual curves (Figures 4 - 5). Shows that the error in saturation is tremendous and renders the curve unfit for use in this area. Also the fit in the linear region could be improved. For these reasons we make a second approximation.

In hope that we can save some work we will again solve for the two curves simultaneously. This time we will select our points more in

Figure 4. A Transfer Characteristic of a 6V6 and its First Approximation.

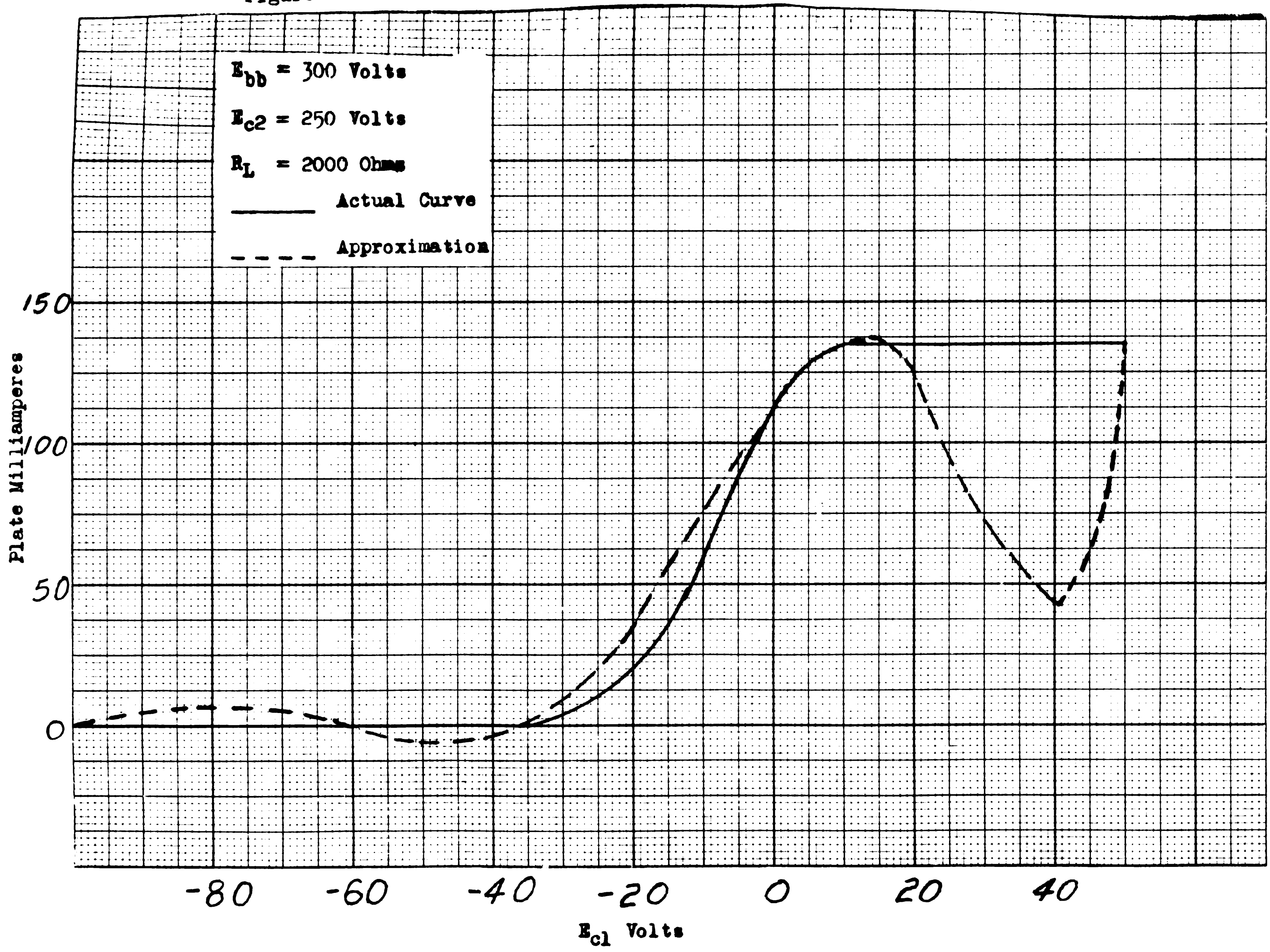
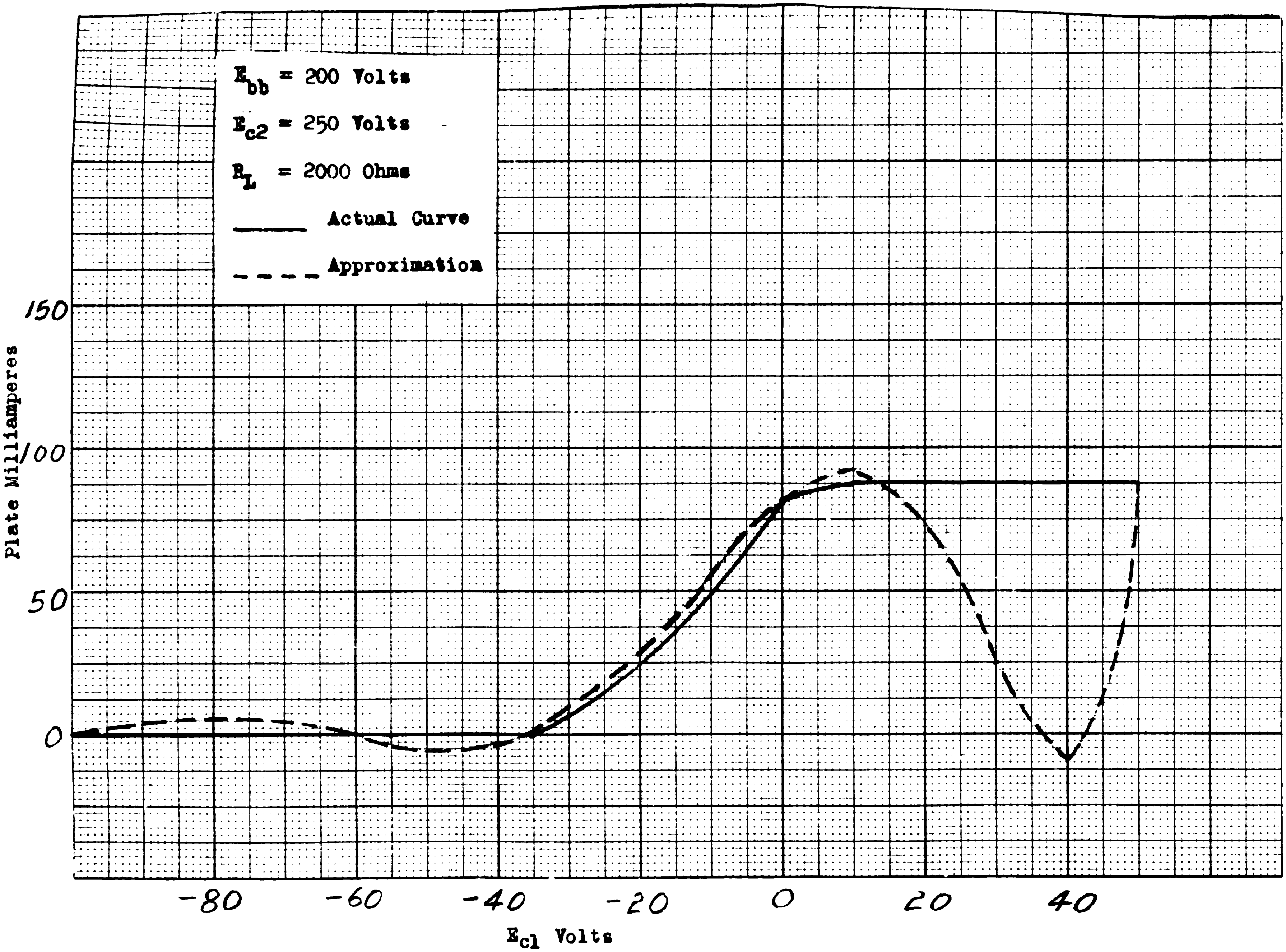


Figure 5. A Transfer Characteristic of a 6V6 and its First Approximation.



accordance with the principles we enumerated when approximating the 400 volt curve.

The points selected were $e_{cl} = -60, -35, -10, +10, +30$. This gave the following matrix.

$$(18)\quad \begin{bmatrix} 40 & 64\times 10^{3} & 102.4\times 10^{6} & 0.16384\times 10^{12} & 0.000262\times 10^{18} & 0 & 0 \\ 65 & 274.625\times 10^{3} & 1160.3\times 10^{6} & 4.902\times 10^{12} & 0.0207119\times 10^{18} & 0 & 0 \\ 90 & 729\times 10^{3} & 5904.9\times 10^{6} & 47.8297\times 10^{12} & 0.38742\times 10^{18} & 57.5 & 50.0 \\ 110 & 1331\times 10^{3} & 16105.1\times 10^{6} & 194.8717\times 10^{12} & 2.3579\times 10^{18} & 135.0 & 87.5 \\ 130 & 2197\times 10^{3} & 37129.3\times 10^{6} & 627.4852\times 10^{12} & 10.6045\times 10^{18} & 135.0 & 87.5 \end{bmatrix}$$

This leaves us with the following results:

For the 300 volt curve:

$$(19)\quad i_p = 0.364899(e_{cl}+100) - 0.342348\times 10^{-3}(e_{cl}+100)^3 + 0.07854\times 10^{-6}(e_{cl}+100)^5 - 4.563\times 10^{-12}(e_{cl}+100)^7 + 74.247\times 10^{-18}(e_{cl}+100)^9$$

For the 200 volt curve:

$$(20)\quad i_p = 0.457889(e_{cl}+100) - 0.444022\times 10^{-3}(e_{cl}+100)^3 + 0.111769\times 10^{-6}(e_{cl}+100)^5 - 8.532\times 10^{-12}(e_{cl}+100)^7 + 208.426\times 10^{-18}(e_{cl}+100)^9.$$

The comparison of these functions to the actual curves is shown in Figures 6 and 7. This set gives us a much more satisfactory approximation. There will be no attempt to compare coefficients of like terms at this time. Instead the next step will be to note the effects of varying the load resistance.

After comparing the transfer curves of various applied plate voltages with the range of load resistance desired it was decided that the 200 volt set would give us the most desireable information. (This set had all leads going into saturation which was not true with the 300 and 400 volt sets.)

Figure 6. A Transfer Characteristic of the 6V6 and its Second Approximation.

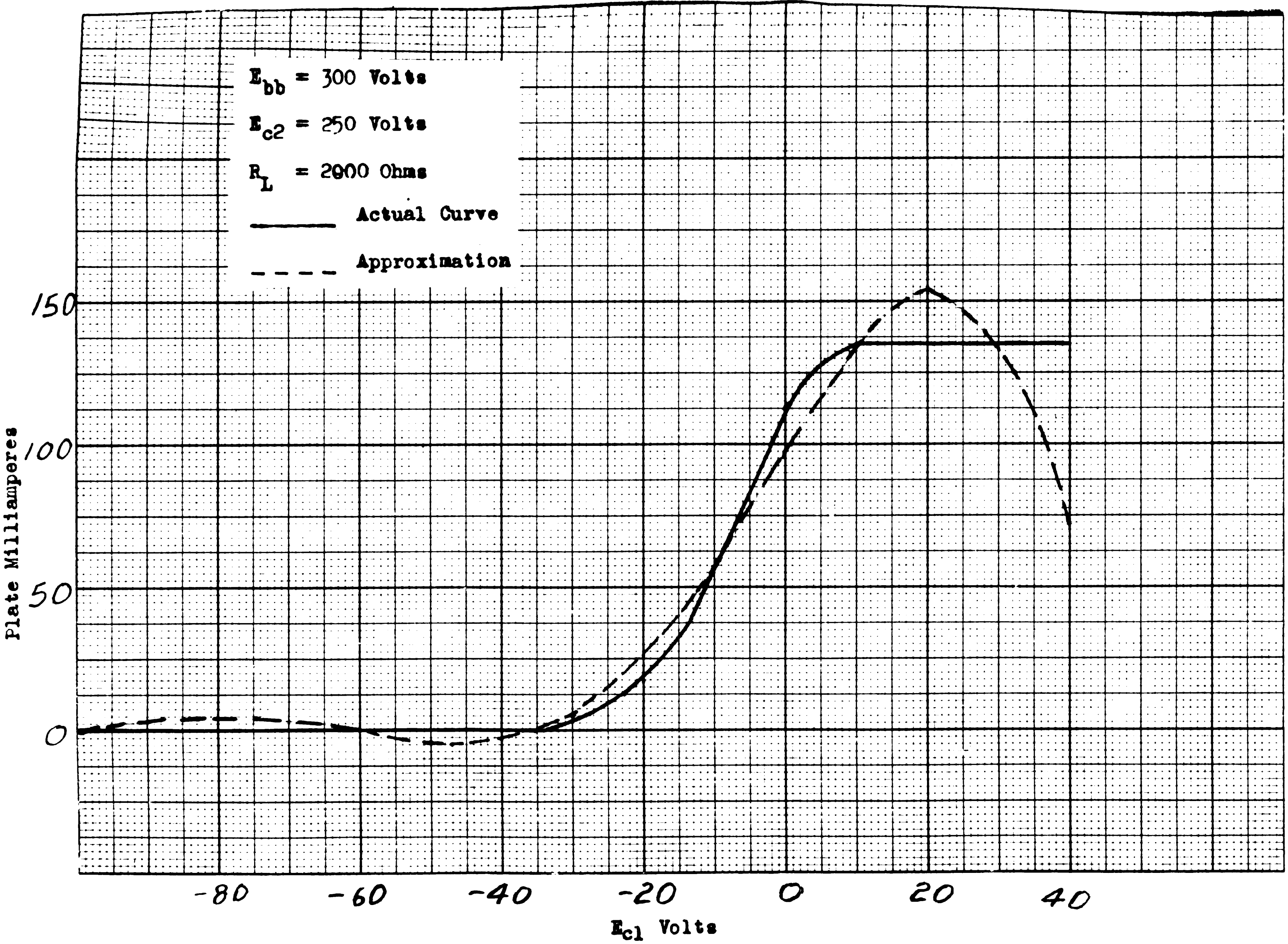
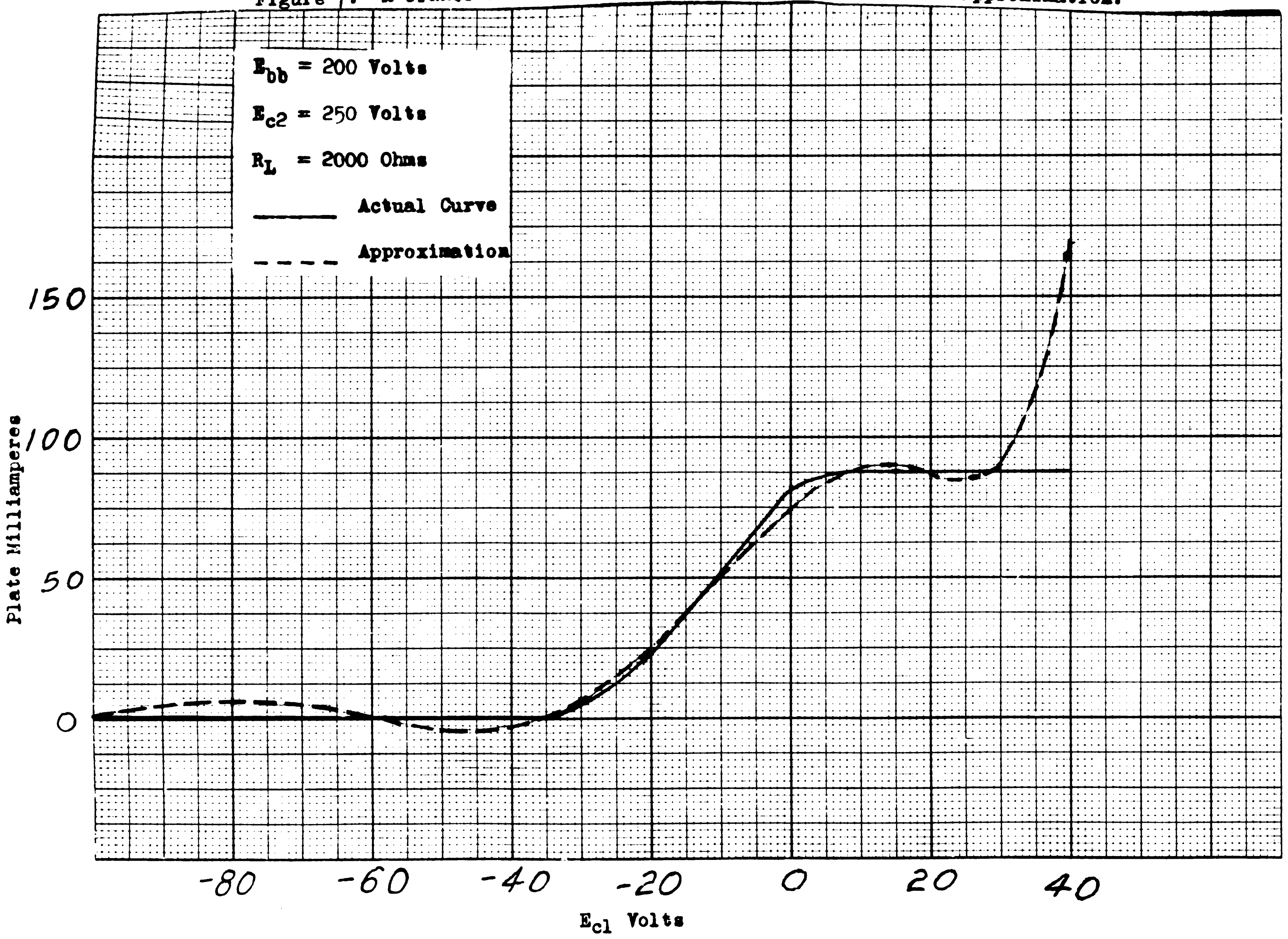


Figure 7. A Transfer Characteristic of a 6V6 and its Second Approximation.



Since the curves were so radically different near saturation it was felt that nothing could be gained by attempting to use the same set of points for both curves. The 4000 ohms curve was approximated using the points $e_{c1} = -60, -35, -15, -5,$ and $+20$. The 1000 ohms curve was approximated using the points $e_{c1} = -60, -35, 0, +10, +30$. Once again the initial matrices and the results will be given.

For the first approximation of the 4000 ohms curve:

$$(21)\quad \begin{bmatrix} 40 & 60 \times10^{3} & 102.4\times10^{6} & 0.16384\times10^{12} & 0.000262\times10^{18} & 0 \\ 65 & 274.625\times10^{3} & 1160.29\times10^{6} & 4.902\times10^{12} & 0.0207119\times10^{18} & 0 \\ 85 & 614.125\times10^{3} & 4437.05\times10^{6} & 32.058\times10^{12} & 0.23162\times10^{18} & 30 \\ 95 & 857.375\times10^{3} & 7737.81\times10^{6} & 69.983\times10^{12} & 0.63025\times10^{18} & 45 \\ 120 & 1728\times10^{3} & 24883.2\times10^{6} & 358.3181\times10^{12} & 5.515978\times10^{18} & 45 \end{bmatrix}$$

$$(22)\quad i_p = 0.41654(e_{c1}+100) - 0.40569\times10^{-3}(e_{c1}+100)^3 + 0.103113\times10^{-6}(e_{c1}+100)^5 - 7.952\times10^{-12}(e_{c1}+100)^7 + 177.6\times10^{-18}(e_{c1}+100)^9.$$

For the approximation of the 1000 ohms curve:

$$(23)\quad \begin{bmatrix} 40 & 64\times10^{3} & 102.4\times10^{6} & 0.16384\times10^{12} & 0.000262\times10^{18} & 0 \\ 65 & 274.625\times10^{3} & 1160.3\times10^{6} & 4.902\times10^{12} & 0.0207119\times10^{18} & 0 \\ 100 & 1000\times10^{3} & 10000\times10^{6} & 100\times10^{12} & 1\times10^{18} & 107 \\ 110 & 1331\times10^{3} & 16105.1\times10^{6} & 194.421\times10^{12} & 2.3579\times10^{18} & 160 \\ 130 & 2197\times10^{3} & 37129.3\times10^{6} & 627.4852\times10^{12} & 10.6045\times10^{18} & 160 \end{bmatrix}$$

$$(24)\quad i_p = 0.26735(e_{c1}+100) - 0.24008\times10^{-3}(e_{c1}+100)^3 + 0.04771\times10^{-6}(e_{c1}+100)^5 - 1.256\times10^{-12}(e_{c1}+100)^7 - 31.2\times10^{-18}(e_{c1}+100)^9.$$

These results are plotted along with the actual curves in Figures 8 and 9.

Before trying to interpret the results we have obtained let us examine a triode. The method will be the same as we used in investigating

Figure 8. A Transfer Characteristic of the 6V6 and its First Approximation.

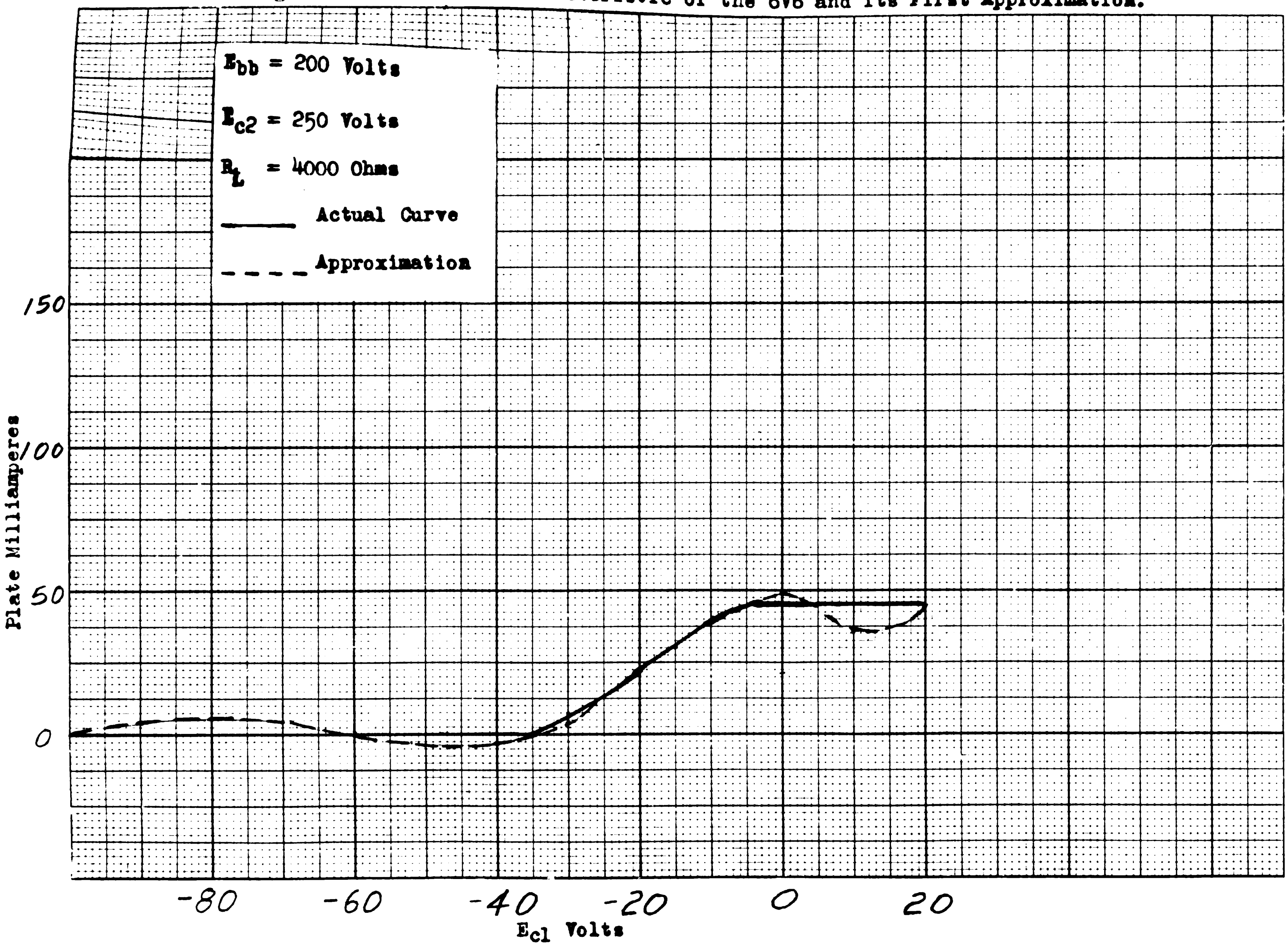
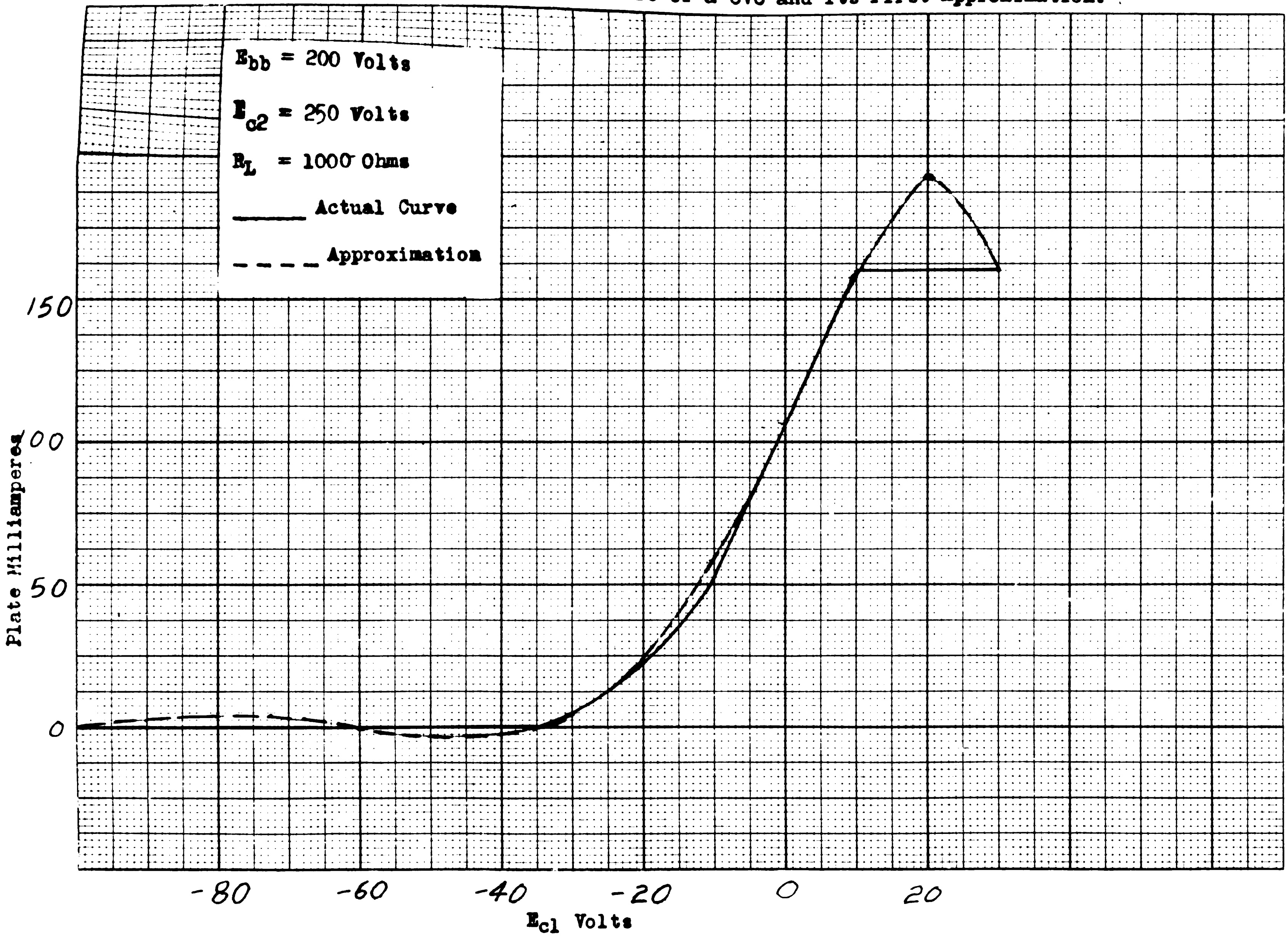


Figure 9. A Transfer Characteristic of a 6V6 and its First Approximation.



the 6V6. Once again we will use a common tube, the 6J5 or one-half of a 6SN7. The latter is a tube that is common to many television sets. We will first use a 20,000 ohm load resistor with plate voltages of 200, 300, and 400 volts. Then we will use a plate voltage of 200 volts with load resistors of 10,000 and 40,000 ohms. As usual we will give the initial matrix and the final result.

For the 20,000 ohm set of curves, the points used were $e_c = -50$, -20, -2, $+10$, and $+30$. Once again $e_c = -100$ was chosen as the origin.

For the 400 volt curve:

$$(25)\quad \begin{bmatrix} 50 & 125 \times10^3 & 312.5\times10^6 & 0.78125\times10^{12} & 0.001953\times10^{18} & 0 \\ 80 & 512 \times10^3 & 3276.8\times10^6 & 20.97152\times10^{12} & 0.13421\times10^{18} & 1.2 \\ 98 & 941.192\times10^3 & 9039.2\times10^6 & 86.81255\times10^{12} & 0.83375\times10^{18} & 12.5 \\ 110 & 1331 \times10^3 & 16105.1\times10^6 & 194.87171\times10^{12} & 2.35795\times10^{18} & 19.3 \\ 130 & 2197 \times10^3 & 37129.3\times10^6 & 627.48517\times10^{12} & 10.6045\times10^{18} & 19.6 \end{bmatrix}$$

$$(26)\quad i_p = 0.263(e_c+100) - 0.17612\times10^{-3}(e_c+100)^3 + 0.03356\times10^{-6}(e_c+100)^5 - 2.1932\times10^{-12}(e_c+100)^7 + 47.4\times10^{-18}(e_c+100)^9.$$

Since the same points were used for the 300 and 200 volt curves the actual solution was preformed by using a five by eight matrix. For this reason for the 300 volt and 200 volt initial matrices only the last column is given. The 300 volt column is the first column and the 200 volt column is the second column.

$$(27)\quad \begin{matrix} 0 & 0 \\ 0 & 0 \\ 9.2 & 5.6 \\ 14.5 & 9.7 \\ 14.7 & 9.7 \end{matrix}$$

The equation for the 300 volt curve is:

$$(28)\quad i_p = 0.25861(e_c + 100) - 0.17111\text{x}10^{-3}(e_c + 100)^3 + 0.03207\text{x}10^{-6} (e_c + 100)^5 - 2.1197\text{x}10^{-12}(e_c + 100)^7 + 46.8\text{x}10^{-18}(e_c + 100)^9.$$

The equation for the 200 volt curve is:

$$(29)\quad i_p = 0.00392(e_{c1} + 100) - 0.07322\text{x}10^{-3}(e_{c1} + 100)^3 + 0.01282\text{x}10^{-6} (e_{c1} + 100)^5 - 0.7345\text{x}10^{-12}(e_{c1} + 100)^7 + 13.3\text{x}10^{-18}(e_{c1} + 100)^9.$$

These last three functions are plotted and compared with the actual curves of the tube in Figures 10 - 12.

For the different load resistors the first approximation used the points $e_c = -50, -12, 0, +10, +30$. Once again the origin was assumed at $e_c = -100$. Both curves were solved for simultaneously. The sixth column is the 10,000 ohm column and the seventh column is the 40,000 ohm column.

$$(30)\quad \begin{bmatrix} 50 & 125\ \text{x}10^3 & 312.5\text{x}10^6 & 0.78125\text{x}10^{12} & 0.001953\text{x}10^{18} & 0 & 0 \\ 88 & 681.472\text{x}10^3 & 5277.3\text{x}10^6 & 40.8678\ \text{x}10^{12} & 0.31648\ \text{x}10^{18} & 0 & 0 \\ 100 & 1000\ \text{x}10^3 & 10000\ \text{x}10^6 & 100\ \text{x}10^{12} & 1.0\ \text{x}10^{18} & 10.8 & 3.8 \\ 110 & 1331\ \text{x}10^3 & 16105.1\text{x}10^6 & 194.8717\ \text{x}10^{12} & 2.35795\ \text{x}10^{18} & 18.6 & 4.95 \\ 130 & 2197\ \text{x}10^3 & 37129.3\text{x}10^6 & 627.48517\text{x}10^{12} & 10.60450\ \text{x}10^{18} & 18.6 & 4.95 \end{bmatrix}$$

The solution for the 40,000 ohm curve comes out as follows:

$$(31)\quad i_p = 0.28609(e_c + 100) - 0.18677\text{x}10^{-3}(e_c + 100)^3 + 0.03462\text{x}10^{-6}(e_c + 100)^5 - 2.417\text{x}10^{-12}(e_c + 100)^7 + 57.5\text{x}10^{-18}(e_c + 100)^9$$

In solving for the 10,000 ohm curve the author must have made a slight error because the equation did not check at the points selected where the approximation should agree exactly with the curve. Resolving yielded the correct equation which very closely resembled the incorrect one. This serves as a warning that the coefficients must be calculated

Figure 10. A Transfer Characteristic of a 6J5 and its First Approximation.

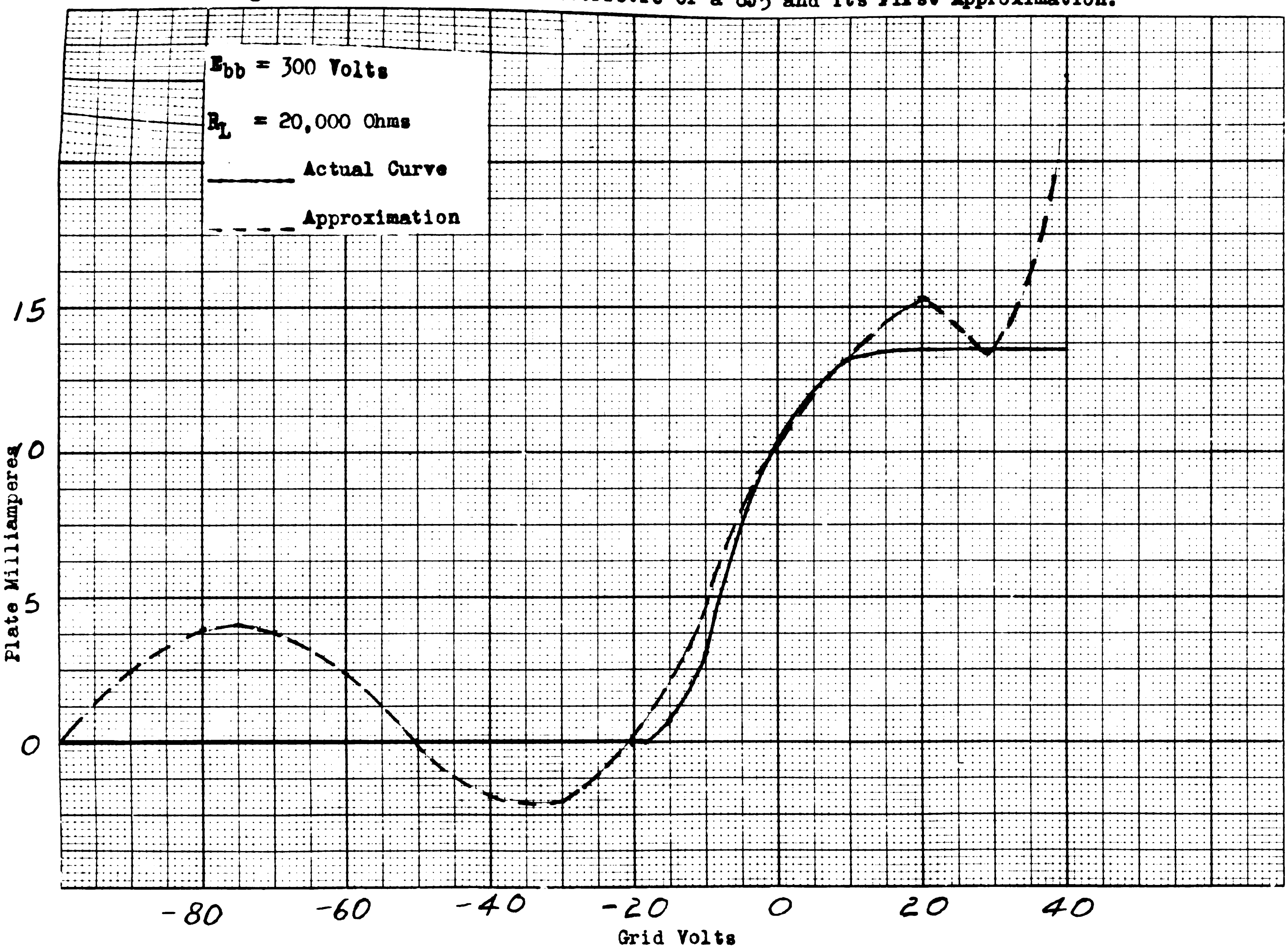


Figure 11. A Transfer Characteristic of a 6J5 and its First Approximation.

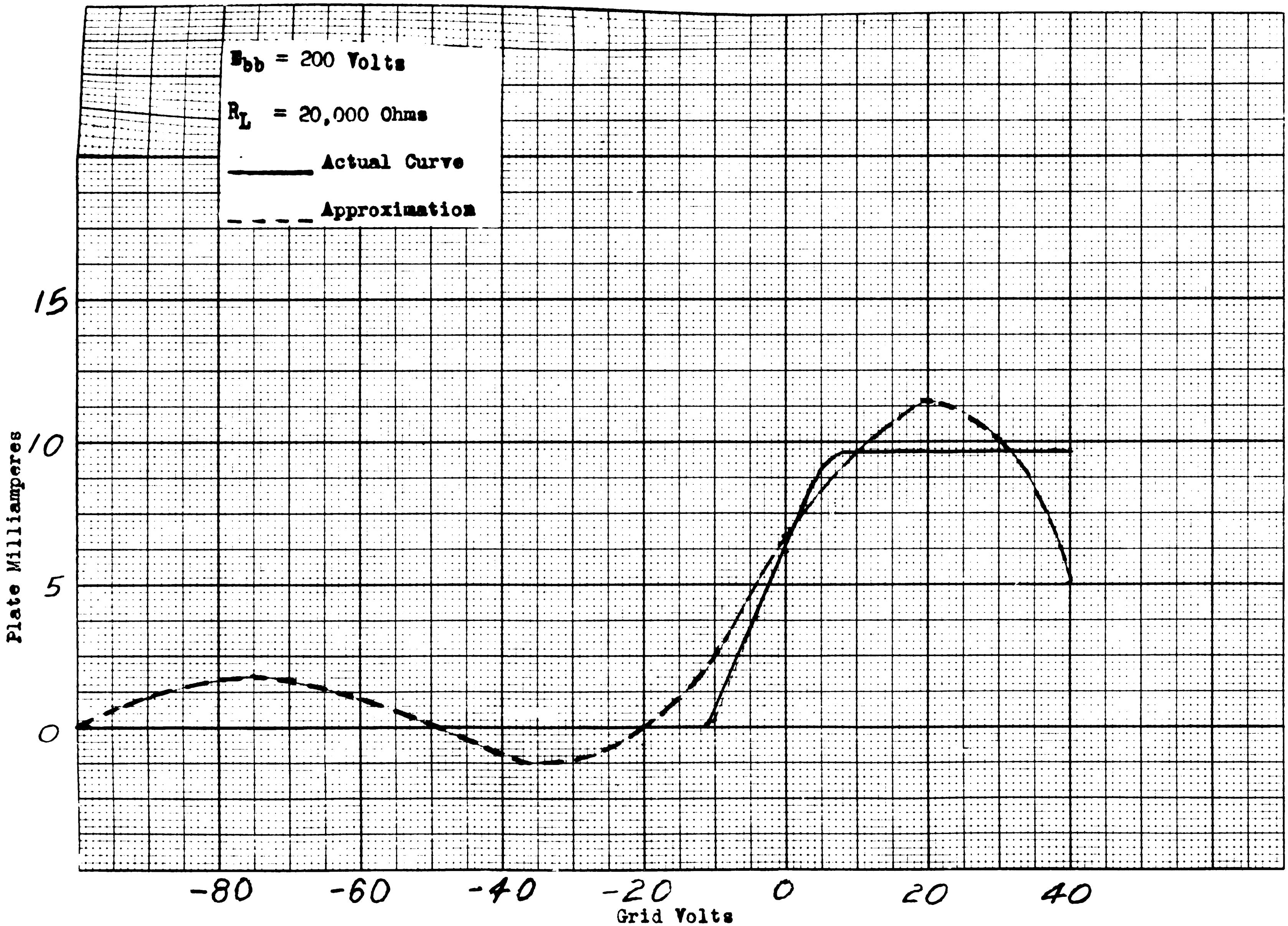
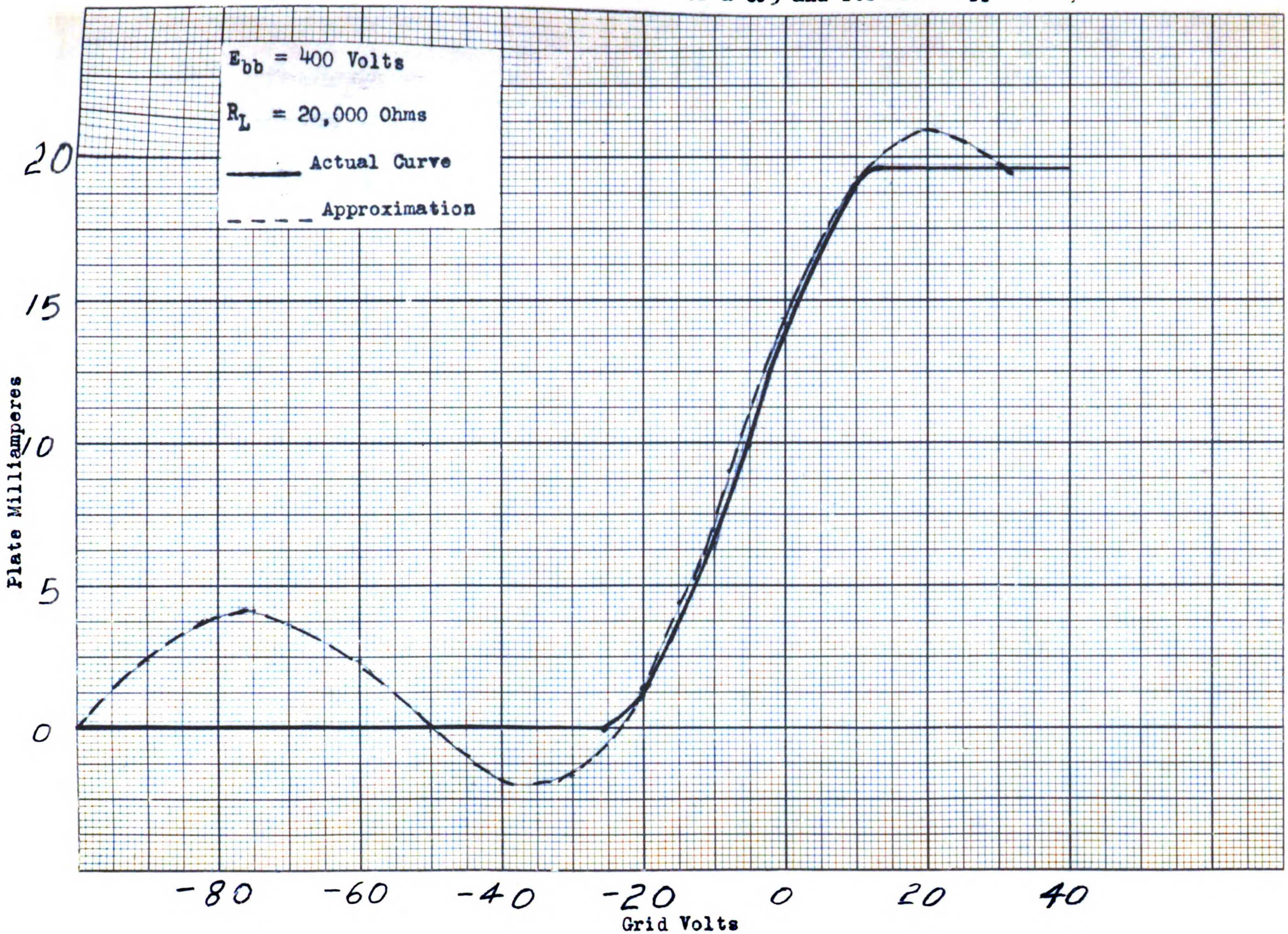


Figure 12. A Transfer Characteristic of a 6J5 and its First Approximation.



to more than three places. In order that the reader might compare the two equations both are given below. Equation (32) is incorrect while (33) is correct.

$$(32)\quad i_p = 0.51965(e_c+100) - 0.32788\times10^{-3}(e_c+100)^3 + 0.05625\times10^{-6}(e_c+100)^5 - 3.483\times10^{-12}(e_c+100)^7 + 73.2\times10^{-18}(e_c+100)^9 \quad \text{(INCORRECT!)}$$

$$(33)\quad i_p = 0.51196(e_c+100) - 0.32400\times10^{-3}(e_c+100)^3 + 0.05597\times10^{-6}(e_c+100)^5 - 3.498\times10^{-12}(e_c+100)^7 + 73.7\times10^{-18}(e_c+100)^9 \quad \text{(CORRECT!)}$$

The justification from this apparent idiosyncracy is that the errors in each term must be cumulative and hence the difficulty.

The correct approximations are plotted in Figures 13 and 14.

For a second approximation the same points that were used in the 20,000 ohm set were used in the 10,000 and 40,000 ohm set. The initial matrix is shown below with column 6 as the 10,000 ohm coefficients and column 7 as the 40,000 ohm coefficients.

$$(34)\quad \begin{bmatrix} 50 & 125\times10^3 & 312.5\times10^6 & 0.78125\times10^{12} & 0.001953\times10^{18} & 0 & 0 \\ 80 & 512\times10^3 & 3276.8\times10^6 & 20.97152\times10^{12} & 0.13421\times10^{18} & 0 & 0 \\ 98 & 941.192\times10^3 & 9039.2\times10^6 & 86.81255\times10^{12} & 0.83375\times10^{18} & 8.7 & 2.8 \\ 110 & 1331\times10^3 & 16105.1\times10^6 & 194.87171\times10^{12} & 2.35795\times10^{18} & 18.6 & 4.95 \\ 130 & 2197\times10^3 & 37129.3\times10^6 & 627.48517\times10^{12} & 10.6045\times10^{18} & 19.1 & 4.95 \end{bmatrix}$$

The results were as follows: (35) is the 10,000 ohm curve while (36) is the 40,000 ohm curve.

$$(35)\quad i_p = 0.08257(e_c+100) - 0.04557\times10^{-3}(e_c+100)^3 + 0.00471\times10^{-6}(e_c+100)^5 + 0.16469\times10^{-12}(e_c+100)^7 - 15.99\times10^{-18}(e_c+100)^9.$$

$$(36)\quad i_p = 0.06061(e_c+100) - 0.03908\times10^{-3}(e_c+100)^3 + 0.00689\times10^{-6}(e_c+100)^5 - 0.4020\times10^{-12}(e_c+100)^7 + 7.47\times10^{-18}(e_c+100)^9.$$

These approximations are plotted in Figures 15 and 16.

Figure 13. A Transfer Characteristic of a 6J5 and its First Approximation.

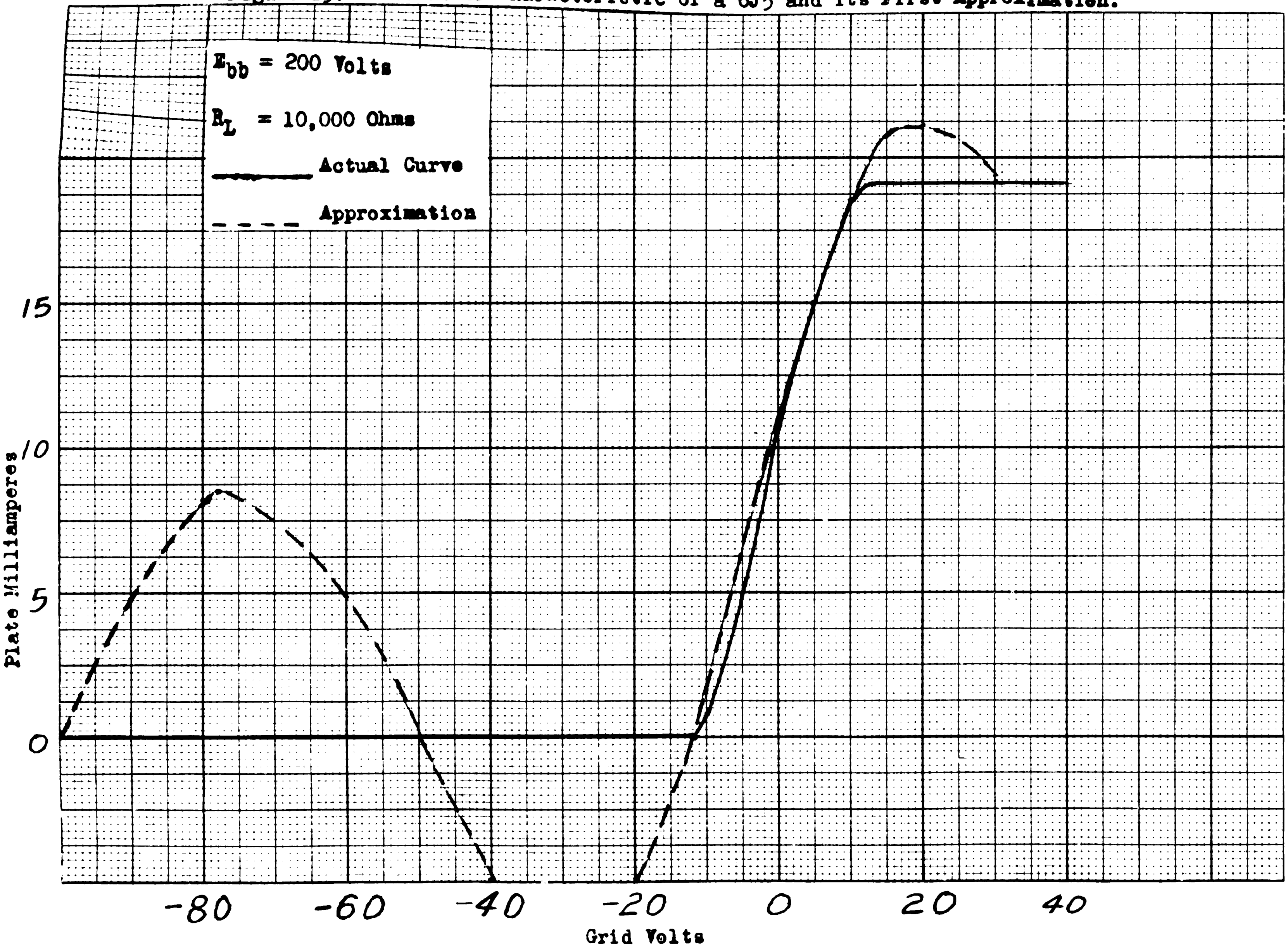


Figure 14. A Transfer Characteristic of a 6J5 and its First Approximation.

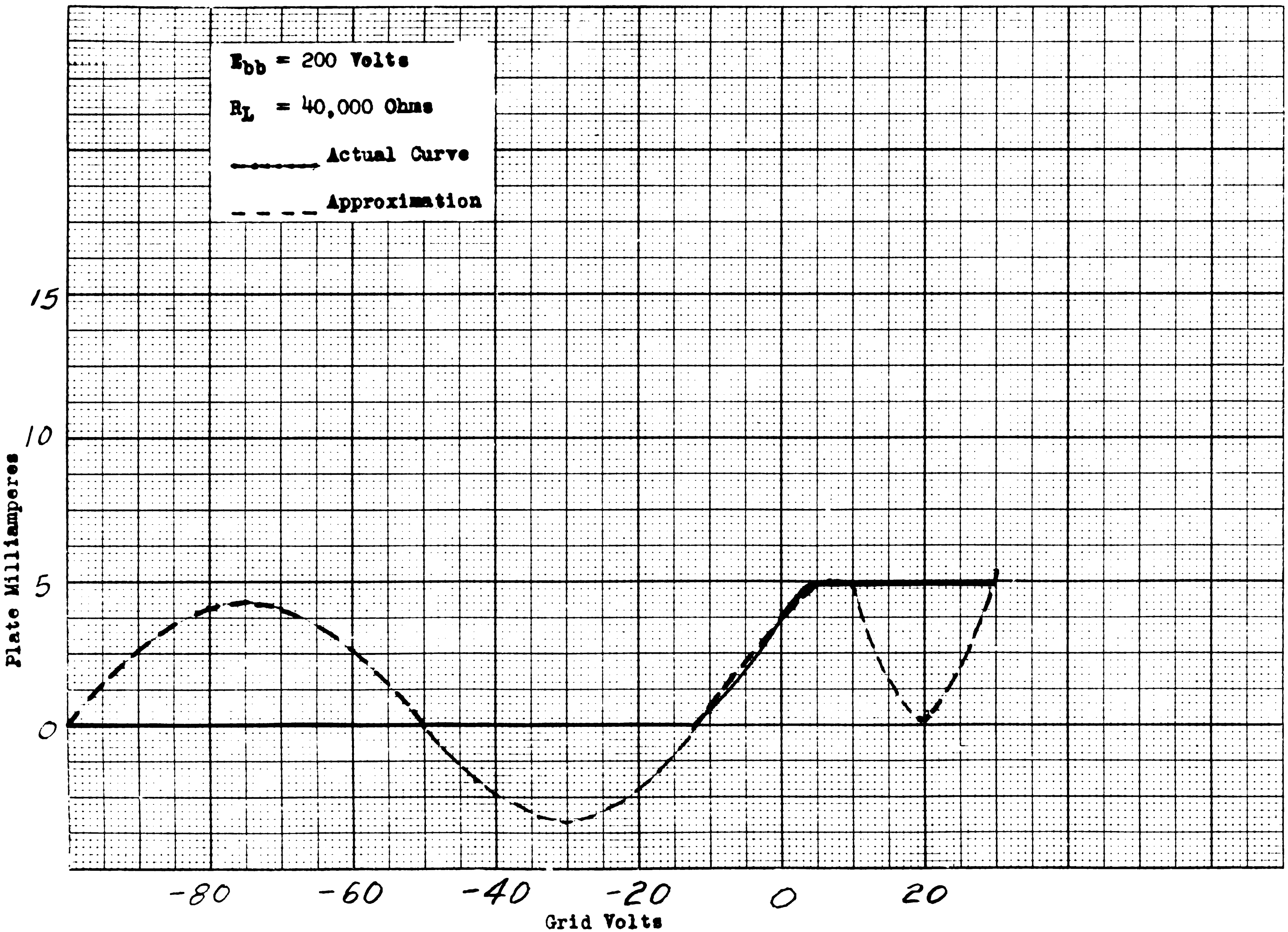


Figure 15. A Transfer Characteristic of a 6J5 and its Second Approximation.

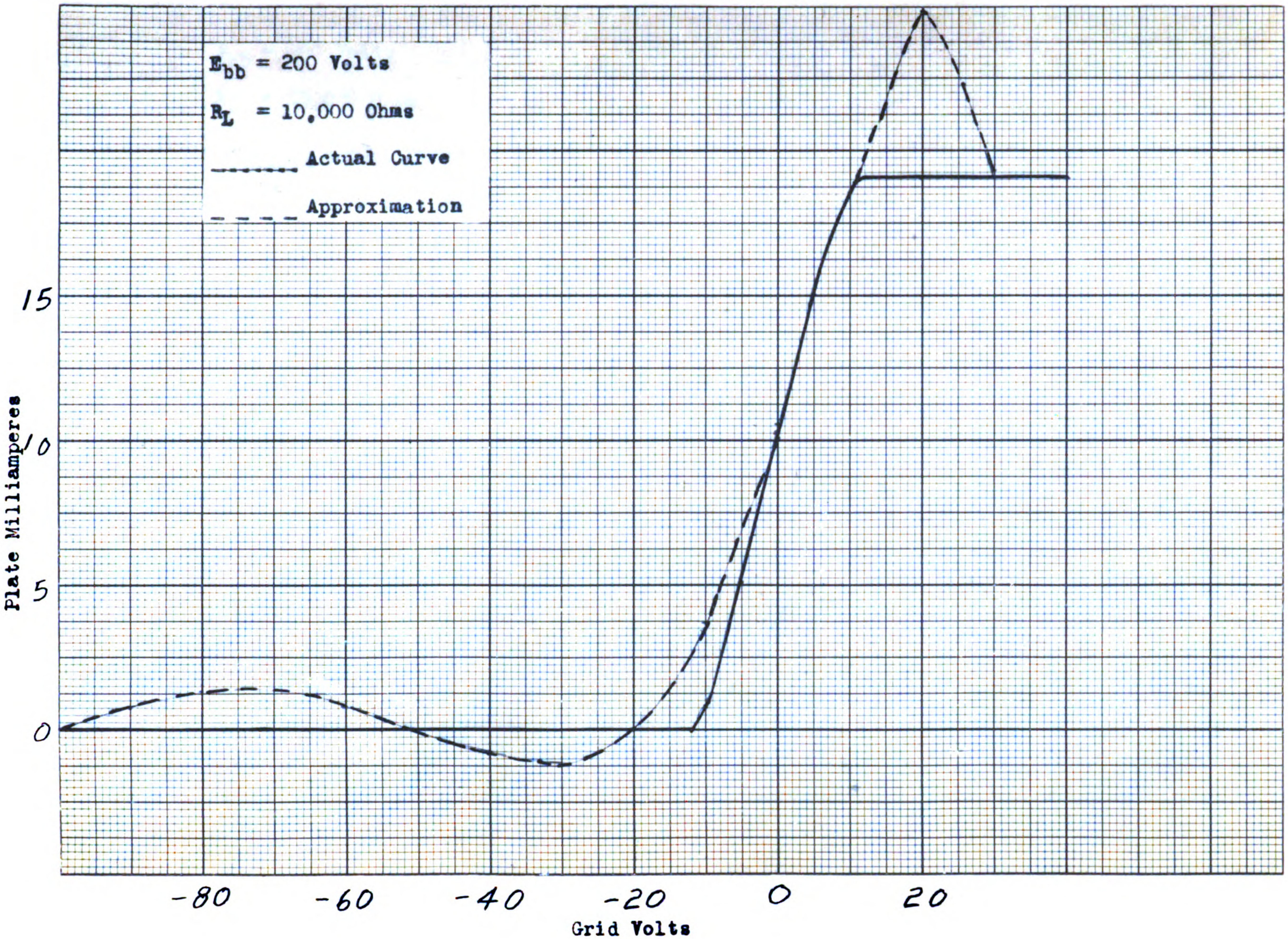
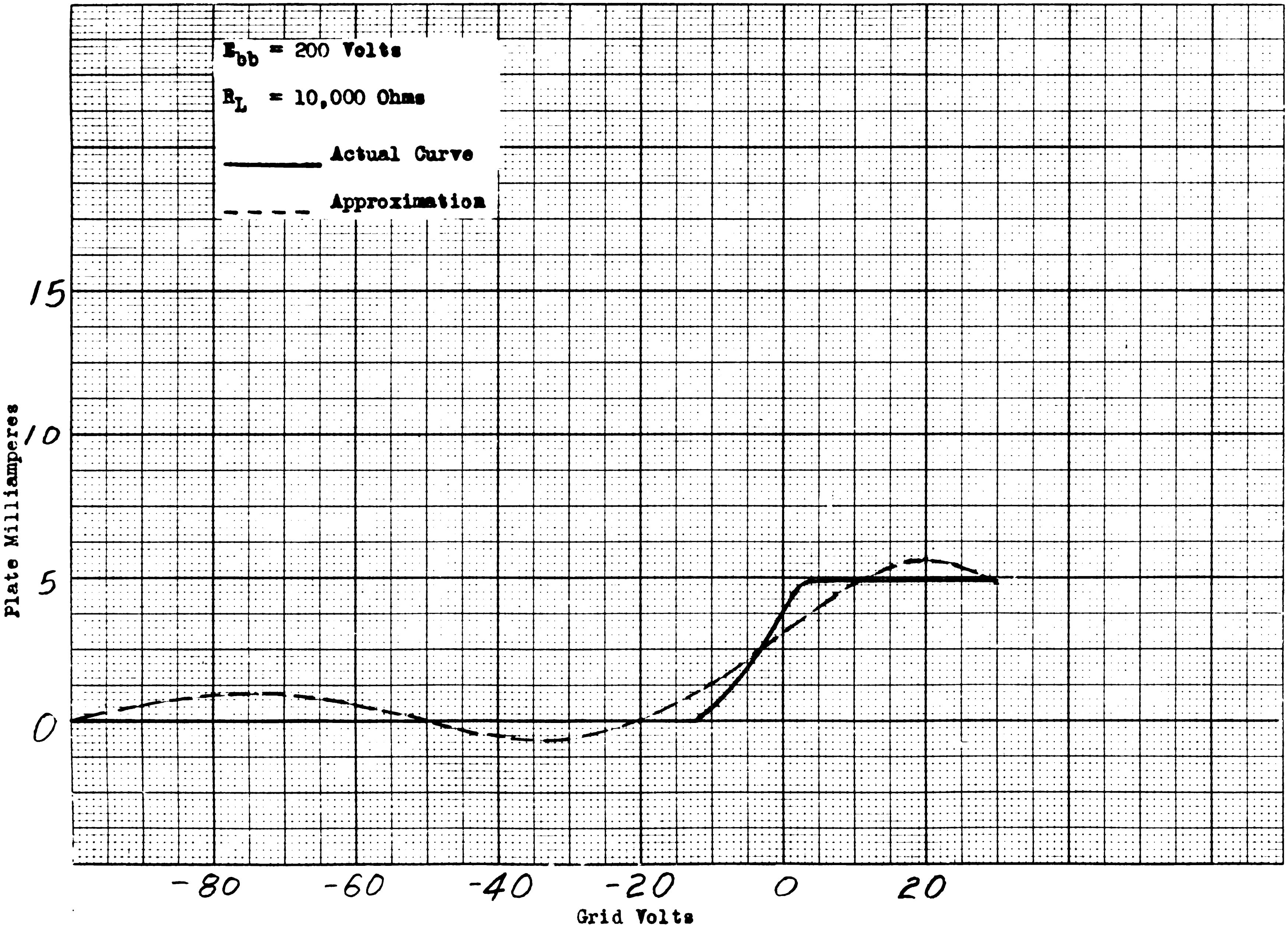


Figure 16. A Transfer Characteristic of a 6J5 and its Second Approximation.



We shall now leave the 6J5 and go on to two more tubes. This pair of tubes are remote cutoff and sharp cutoff pentodes. Only one curve for each tube will be approximated.

For the remote cutoff pentode we will use a 6SG7. The plate voltage will be 200 volts, the screen voltage will be 100 volts and the load resistance will be 40,000 ohms. Our origin will be $e_{c1} = -30$ volts. Since a 6SG7's grid is not supposed to go positive no points with the grid positive were selected. The points used were $e_{c1} = -16$, -9, -5, -2, and 0. The initial matrix and its solution are given in (37) and (38).

$$(37)\quad \begin{bmatrix} 14 & 27.44\text{x}10^{2} & 53.7824\text{x}10^{4} & 1.054\ \text{x}10^{8} & 0.020661\text{x}10^{12} & 0 \\ 21 & 92.61\text{x}10^{2} & 408.4101\text{x}10^{4} & 18.0109\text{x}10^{8} & 0.79428\ \text{x}10^{12} & 0 \\ 25 & 156.25\text{x}10^{2} & 976.5625\text{x}10^{4} & 61.0351\text{x}10^{8} & 3.81470\ \text{x}10^{12} & 1.0 \\ 28 & 219.52\text{x}10^{2} & 1721.0\ \text{x}10^{4} & 134.4929\text{x}10^{8} & 10.57846\ \text{x}10^{12} & 4.0 \\ 30 & 270.00\text{x}10^{2} & 2430.0\ \text{x}10^{4} & 218.7\ \text{x}10^{8} & 19.683\ \text{x}10^{12} & 4.75 \end{bmatrix}$$

$$(38)\quad i_p = -\,5.023856(e_{c1}+30) + 5.078815\text{x}10^{-2}(e_{c1}+30)^3 - 1.685987\text{x}10^{-4}(e_{c1}+30)^5 + 22.5843\text{c}10^{-8}(e_{c1}+30)^7 - 104.56\text{x}10^{-12}(e_{c1}+30).$$

A cursory examination of (38) shows it to be quite obviously a poor approximation (let $e_{c1}+30 = 1$ and see what I mean). As an experiment we will try using only four points for our approximation. They shall be the same four points as the first four points of our first approximation. Our initial matrix and its solution are given in (39) and (40).

$$(39)\quad \begin{bmatrix} 14 & 27.44\text{x}10^{2} & 53.7824\text{x}10^{4} & 1.054\ \text{x}10^{8} & 0 \\ 21 & 92.61\text{x}10^{2} & 408.4101\text{x}10^{4} & 18.0109\text{x}10^{8} & 0 \\ 25 & 156.25\text{x}10^{2} & 976.5625\text{x}10^{4} & 61.0351\text{x}10^{8} & 1.0 \\ 28 & 219.52\text{x}10^{2} & 1721.0\ \text{x}10^{4} & 134.4929\text{x}10^{8} & 4.0 \end{bmatrix}$$

$$(40)\quad i_p = -\,0.028154(e_{c1}+30) + 0.03226\text{x}10^{-2}(e_{c1}+30)^3 - 0.011742\text{x}10^{-4}(e_{c1}+30)^5 + 0.1332(e_{c1}+30)^7.$$

This approximation is plotted in Figure 17.

The fact that four points give a better approximation than five points may seem somewhat surprising at first. However, in this case it must be realized that any finite power series with positive powers tends to go to infinity as the varible goes to infinity. In the five term approximation the curve included a point in saturation and tended to approach a finite value as the varible approached infinity, while the four term approximation did not include the point of saturation and the last point was in the path of increasing i_p with increasing e_{c1}.

For the sharp cutoff pentode we will use a 6SH7. The plate voltage will be 200 volts, screen voltage will be 100 volts while the load resistance will be 40,000 ohms. Our origin will be $e_{c1} = -30$ volts.

Once again we will use only four points. This time they will be $e_{c1} = -12, -4, -2,$ and -1. The initial matrix and its solution are shown below.

$$(41)\quad \begin{bmatrix} 18 & 58.32\text{x}10^2 & 188.9568\text{x}10^4 & 6.1222\text{x}10^8 & 0 \\ 26 & 175.56\text{x}10^2 & 1188.1376\text{x}10^4 & 80.3181\text{x}10^8 & 0 \\ 28 & 219.52\text{x}10^2 & 1721.0\ \text{x}10^4 & 134.4929\text{x}10^8 & 2.05 \\ 29 & 243.89\text{x}10^2 & 2051.1149\text{x}10^4 & 172.4988\text{x}10^8 & 4.5 \end{bmatrix}$$

$$(42)\quad i_p = -\,0.36862(e_{c1}+30) + 0.25778\text{x}10^{-2}(e_{c1}+30)^3 - 0.05782\text{x}10^{-4}(e_{c1}+30)^5 + 0.411\text{x}10^{-8}(e_{c1}+30)^7.$$

This function is plotted in Figure 18.

This concludes our survey of the various tube types and approximations of their transfer curves by power series. So far we have seen

Figure 17. A Transfer Characteristic of a 6SG7 and its Four Term Approximation.

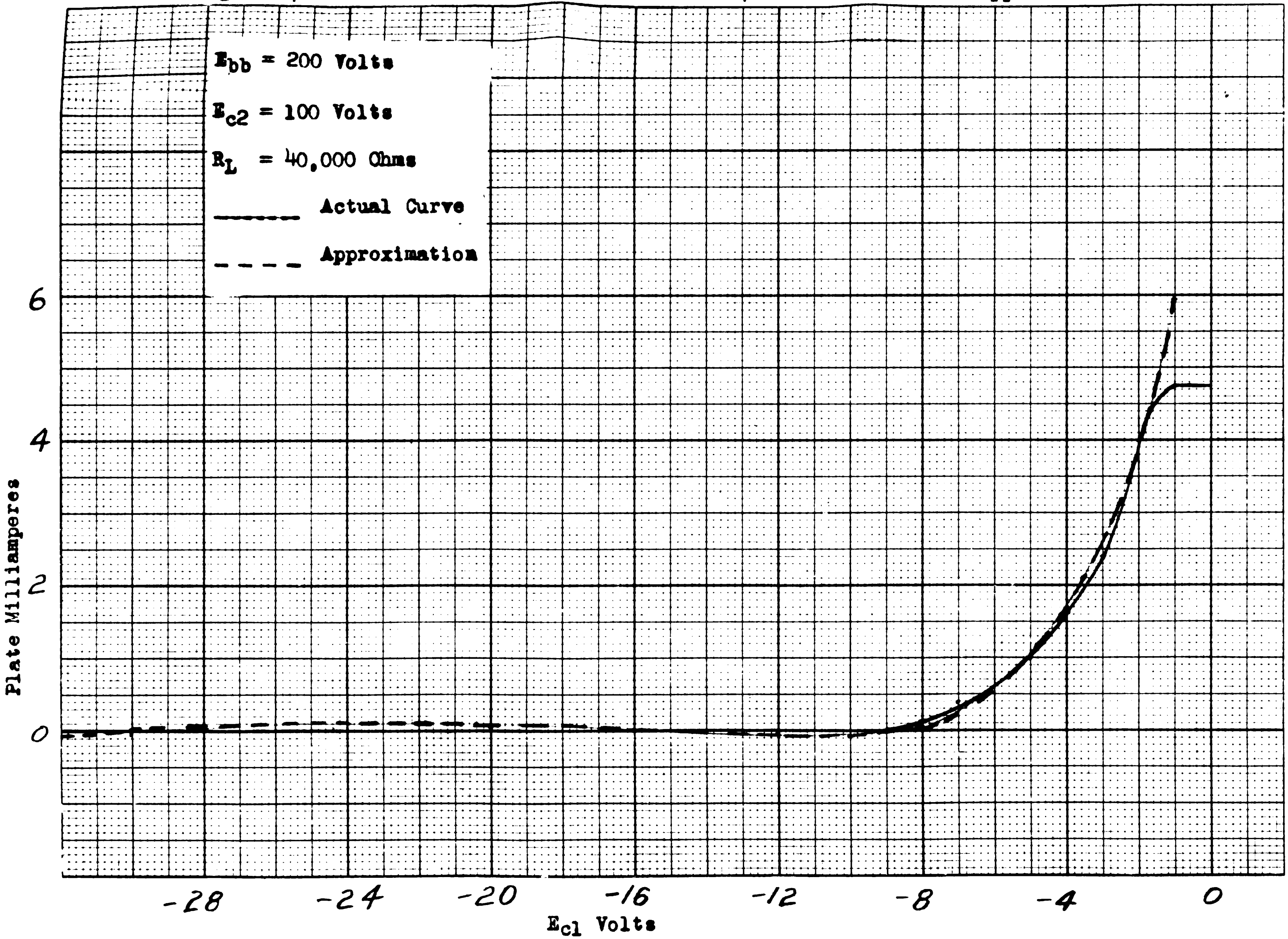
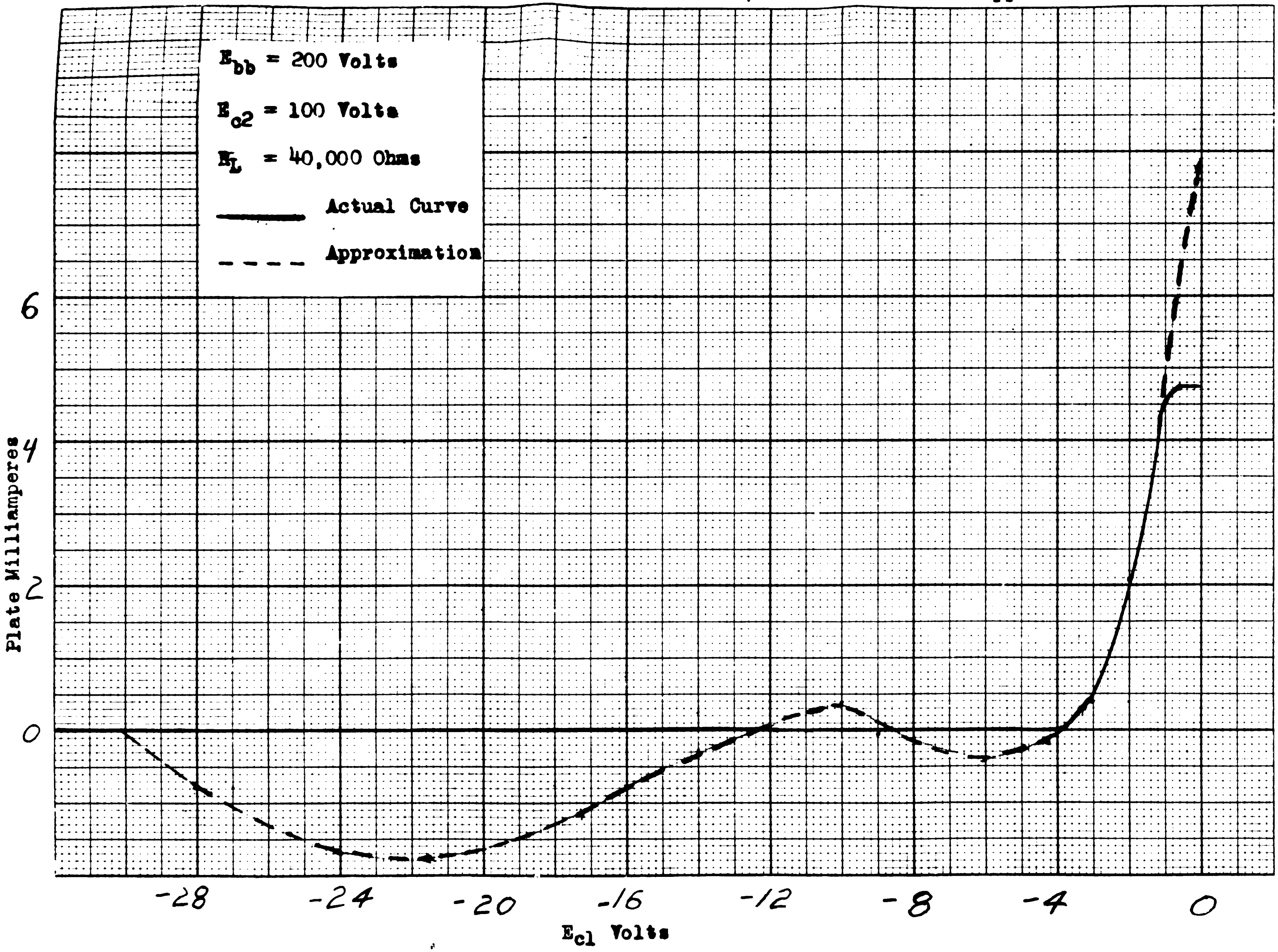


Figure 18. A Transfer Characteristic of a 6SH7 and its Four Term Approximation.



that this method offers a reasonably good approximation for a given plate and screen voltage and a given load resistance over a wide range of grid voltages for most of the common types of amplifying tubes. There are several limitations. First the approximation is not particularly good at saturation. Also the tube must be conducting during some part of the cycle on the grid voltage for the equation to be valid at all. (If this limitation is not accounted for it is entirely possible to predict a plate current when the tube is entirely in cutoff.) At this point we still have to start over if we change either the lead resistance or the plate voltage. Our next step therefore is to try to further reduce these limitations.

CHAPTER II

REDUCTION OF THE LIMITATIONS

The first limitation that we shall take up is that of the poor approximation of the transfer curve when the tube reaches saturation. We will sub-divide the problem into three cases. The first case will be when the tube does not reach saturation, the second case will be when the tube reaches saturation but does not cutoff, and the third case will be when the tube reaches saturation and does cutoff.

The first case is a trivial case. In this case we simply disregard the fact that the curve is poorly approximated at saturation and use it anyway.

The second case presents a far more serious problem. One way of attacking it is to use basically the same method of approximation but choose the origin such that the tube is in saturation and that the variable will take the tube out of saturation. In this case the power series will have a constant term that is equal to the saturation current and the power series except for the constant will be normally negative except when the tube is at saturation and then it should be zero.

For example if the 6V6 tube was to operate from $e_{cl} = 0$ to $e_{cl} = +50$ volts, one could select the origin at $e_{cl} = +40$ volts. One could use the power series of the following form:

(43) $i_p = i_s + a_1(e_{cl}-40) + a_3(e_{cl}-40)^3 + a_5(e_{cl}-40)^5 + \dots$ where i_s is the saturation current of the tube.

In particular if we were to apply 400 volts to the plate and 250 volts to the screen with a load resistance of 2,000 ohms, and the above limitations on the grid voltage we might get an initial matrix as shown in (44).

$$(44)\quad \begin{bmatrix} -10 & -1\times10^3 & -1\times10^5 & -1\times10^7 & 0 \\ -20 & -8\times10^3 & -32\times10^5 & -128\times10^7 & 0 \\ -30 & -27\times10^3 & -243\times10^5 & -2187\times10^7 & -12.5 \\ -40 & -64\times10^3 & -1024\times10^5 & -16384\times10^7 & -65.0 \end{bmatrix}$$

In this case the saturation current was 177.5 milliamperes. The reader should note that the last column is not the plate current but the difference between the plate current and the saturation current. In most cases the problem of the tube saturating does not occur unless the tube also cuts off. This case is only included for completeness and (44) will not be solved.

The third case prsents a difficult problem. If we wish to use the power series, it is obvious that we would need more terms. This would make our task of solving for the coefficients more laborious and the whole problem becomes unreasonable. The only other alternative is to find another way of approximating the curve.

We had earlier mentioned using the hyperbolic tangent. This curve very closely resembles the transfer curve. Our equation might have the form:

$$(45)\quad i_p = a + b\tanh c(e_{c1} + d).$$

This seems to offer us considerable latitude as it appears that we will be able to pick four arbitrary points that we wish to use. Unfortunately this is not the case. The minute we select the saturation current we have chosen a, b, and d. One other point on the slope of the curve will give us c. Thus in our previous example we had a saturation current of 177.5. The tanh varies from -1 to +1. Therefore our b must equal one half of the saturation current or in this case 88.75. Since this point occurs at $e_{cl} + d = 0$, a must equal 88.75. d is also fixed because $i_p = 88.75$ for only one e_{cl}. c is determined by selecting a point on the slope of the curve and putting it into the equation and solving for c. This method is actually easier than that of power series but unless the tube transfer curve is fairly near the shape of the hyperbolic tangent the method will be inexact. (There is also another difficulty, but this will be explained later.) In addition, in order to use the quation we should transform it into a power series that is convergent over the range we desire. This appears not to be too difficult. Before we do let us rewrite the equation in light of what we already know, and let $x = c(e_{cl} + d)$.

(46) $i_p = a(1 + \tanh x)$

or in power series form

(47) $i_p = a(1 + x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \ldots).$

There is, however, a very serious limitation to the power series form. This series converges only in a very limited range. In fact x^2 must be less than $\frac{\pi^2}{4}$. Since the tanh does not approach 1 until x is larger than $\frac{\pi}{2}$ and since we want to go through zero we cannot use the power series form of the tanh. (There is another form of the power series for the tanh x when x is large but this is not valid for small x and thus we gain nothing.) As a result of this limitation we are almost reduced to Fourier analysis in this condition.

The next two limitations were the fact that the present power series approximations were limited to one plate voltage and load resistance. The solution most apparent is to compare the series of the same plate voltage or load resistance and see if some relation can't be worked out with the coefficients of like terms.

It is obvious that we won't be able to vary both the load resistance and the plate voltage. Our choice is to use a fixed load resistance and vary the plate voltage. The reason for this decision is the fact that plate modulation employs a constant load but a varying plate voltage. Actually the method we are going to use would be valid whether we varied plate voltage or load resistance providing the other was held constant. So if the reader desired to keep the plate voltage constant and let the load resistance vary, the same method would work out satisfactorily.

The method is one of comparing coefficients of like terms. In order to make the data more convenient we tabulate it.

For the 6J5 with a load resistance of 20,000 ohms

TABLE I

Summary of Coefficients of $(e_c + 100)$

Plate Volts	(e_c+100)	$-(e_c+100)^3$	$+(e_c+100)^5$	$-(e_c+100)^7$	$+(e_c+100)^9$
200	0.11392	0.07322×10^{-3}	0.01282×10^{-6}	0.7345×10^{-12}	13.3×10^{-18}
300	0.25861	0.17111×10^{-3}	0.03207×10^{-6}	2.1197×10^{-12}	46.8×10^{-18}
400	0.26300	0.17612×10^{-3}	0.03356×10^{-6}	2.1932×10^{-12}	47.4×10^{-18}

From immediate observation it is obvious that there is no linear relationship among the coefficients. Therefore it is necessary that we find some other method to establish the relationship between them.

Let us investigate three methods. First is to plot the points on a graph. The second is to approximate the curve of the graph using the known points and a power series using the odd powers only. The third is a variation of the second. In this case the power series consists of odd and even powers.

The graphical method consists of plotting the value of the coefficients versus plate voltage. A separate curve is needed for each set of coefficients. To find the coefficients of an intermediate voltage one need merely pick them off the curves. Figures 19 through 23 give the set of curves for our particular approximations of the 6J5 transfer curves. For a trial a plate voltage of 250 volts chosen and the transfer curve was approximated from the graphs. The equation of the approximation is shown below.

$$(48)\quad i_p = 0.218(e_c+100) - 0.138\text{x}10^{-3}(e_c+100)^3 + 0.0270\text{x}10^{-6}(e_c+100)^5 - 1.70\text{x}10^{-12}(e_c+100)^7 + 37.3\text{x}10^{-18}(e_c+100)^9.$$

This equation is plotted along with the actual transfer curve in Figure 24.

A more detailed evaluation of the methods will be given later. However, Figure 24 shows quite clearly that (48) is a very poor approximation of that particular curve.

Our second method consists of approximating the curves of the coefficients versus the plate voltage using power series of the form:

$$(49)\quad a_k = b_1E_{bb} + b_2E_{bb}{}^3 + b_3E_{bb}{}^5.$$

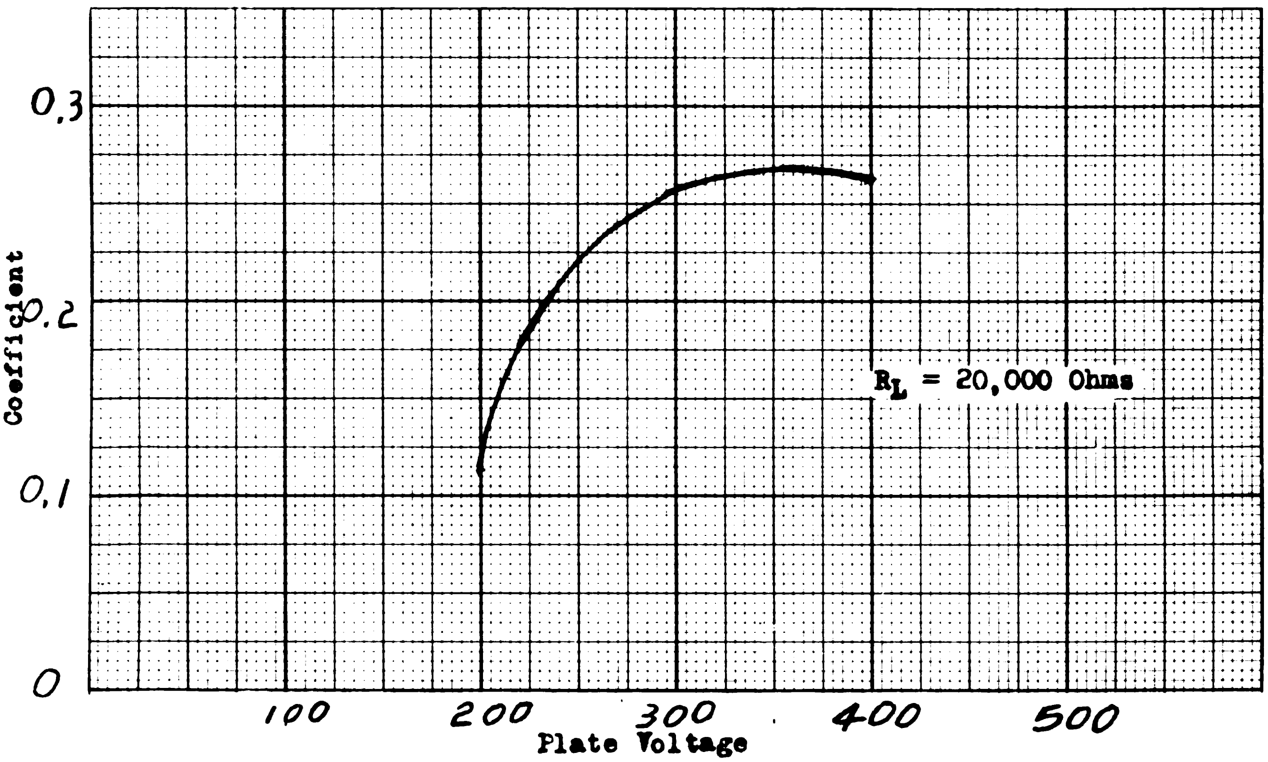


Figure 19. Value of the Coefficient of $(e_c + 100)$ vs Plate Voltage for the 6J5.

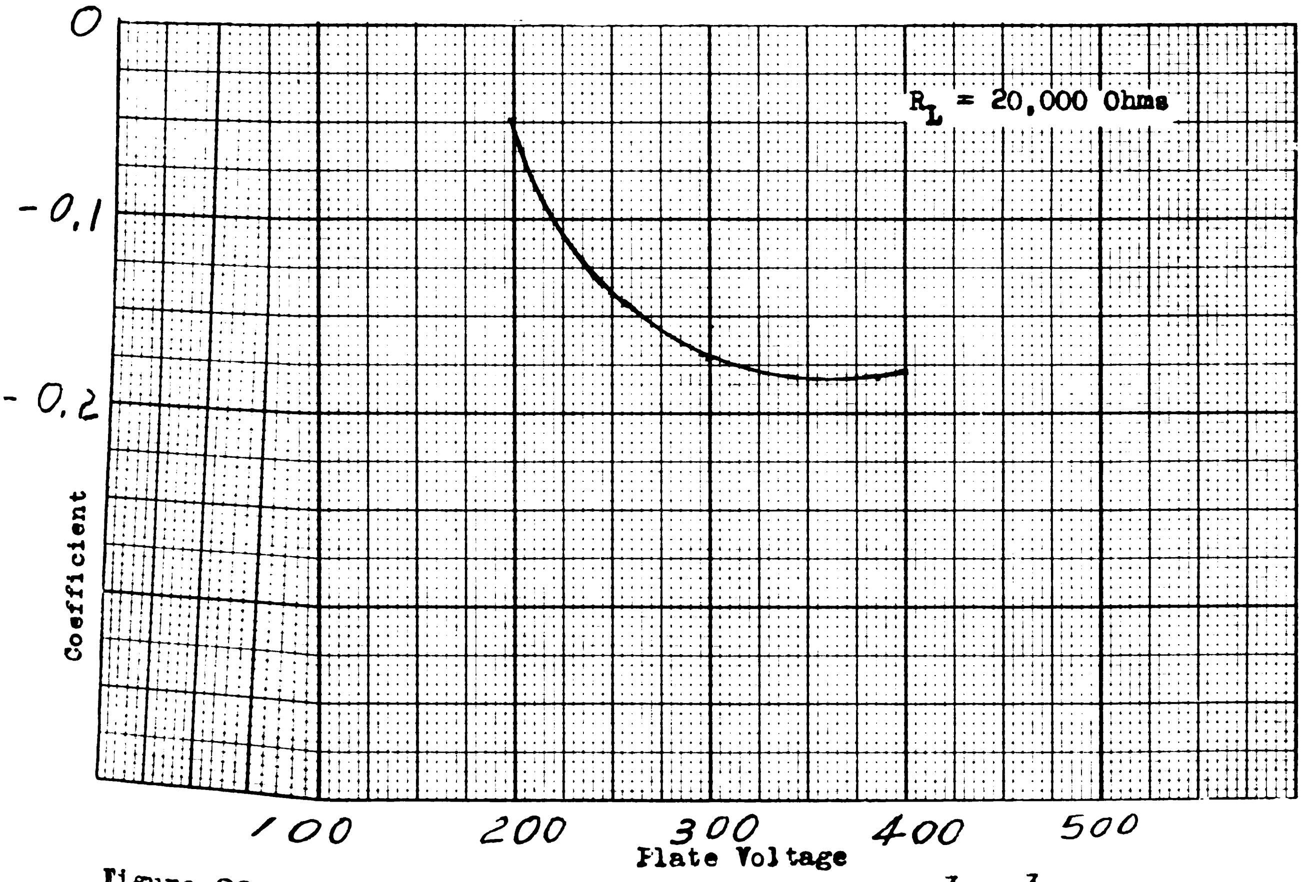


Figure 20. Value of the Coefficient of $(e_c + 100)^3 x 10^{-3}$ vs Plate Voltage.

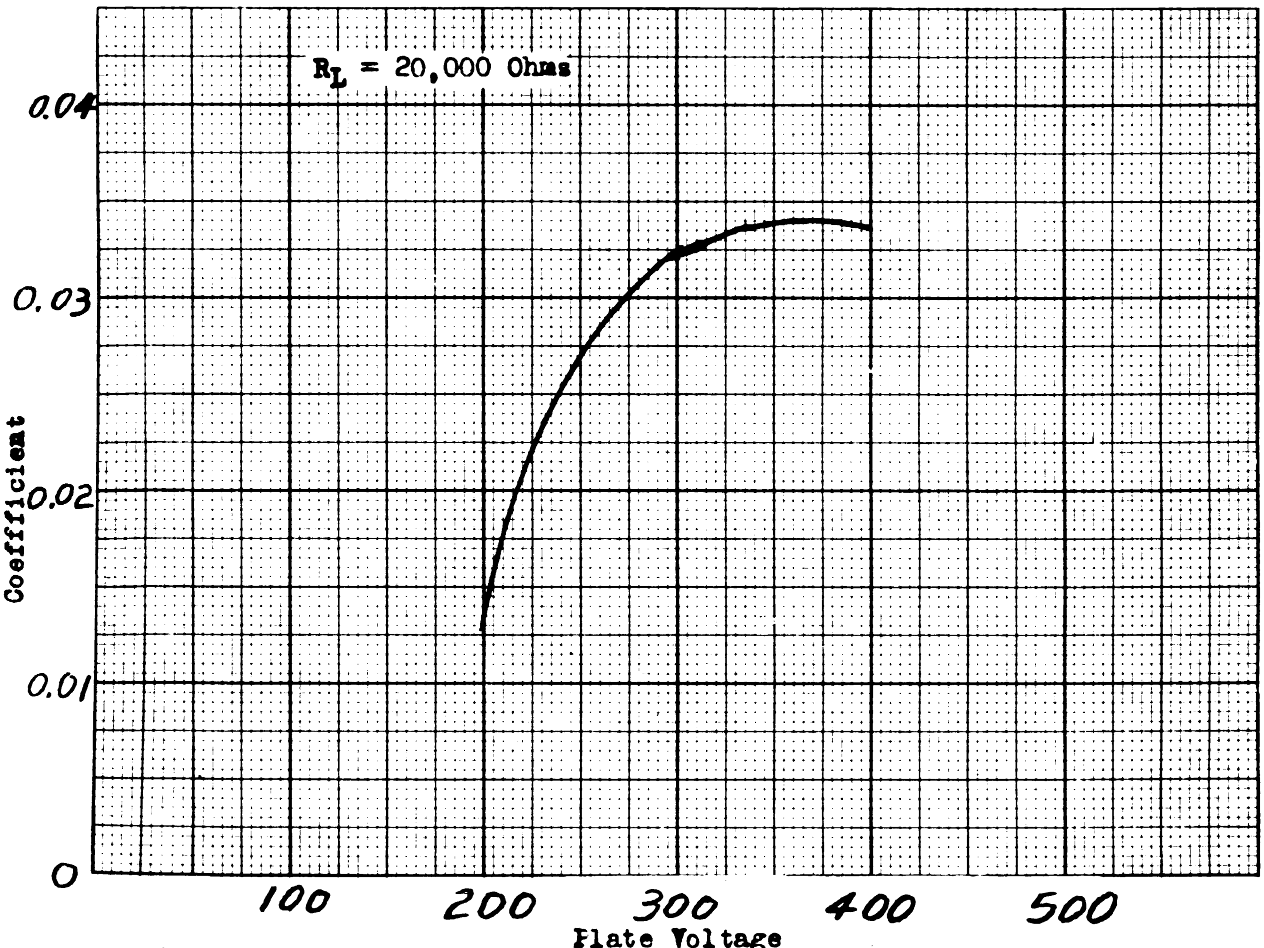


Figure 21. Value of the Coefficient of $(e_c+100)^5 x10^{-6}$ vs Plate Voltage of a 6J5.

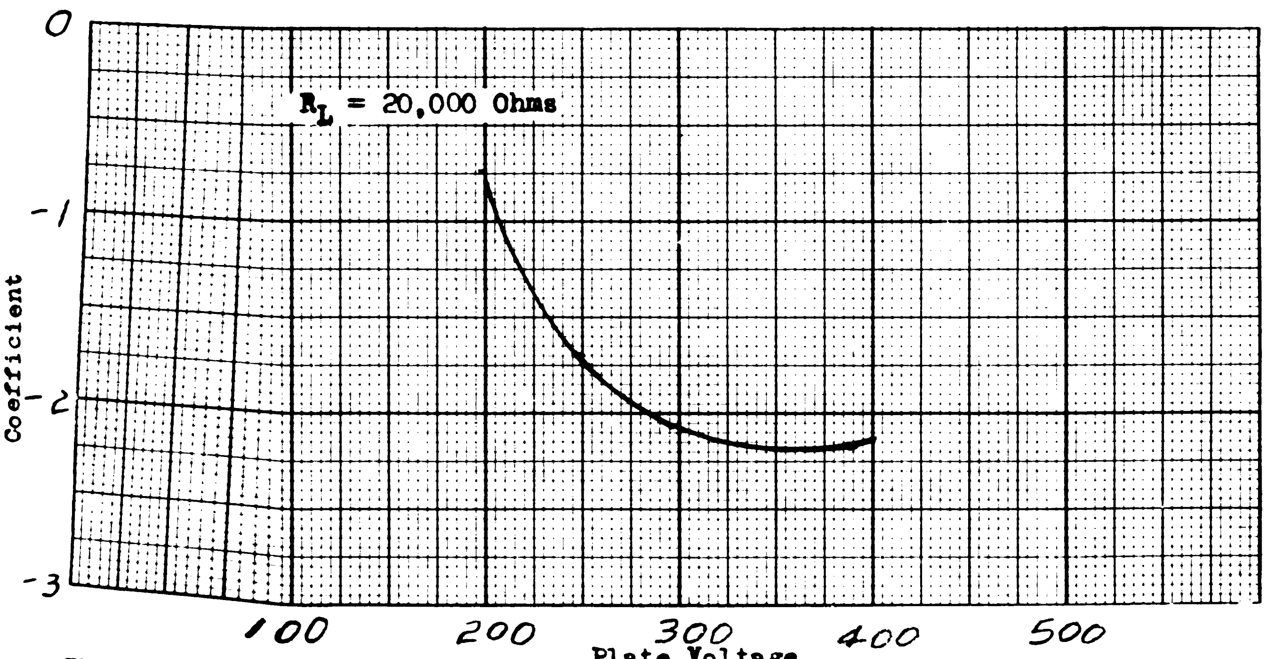
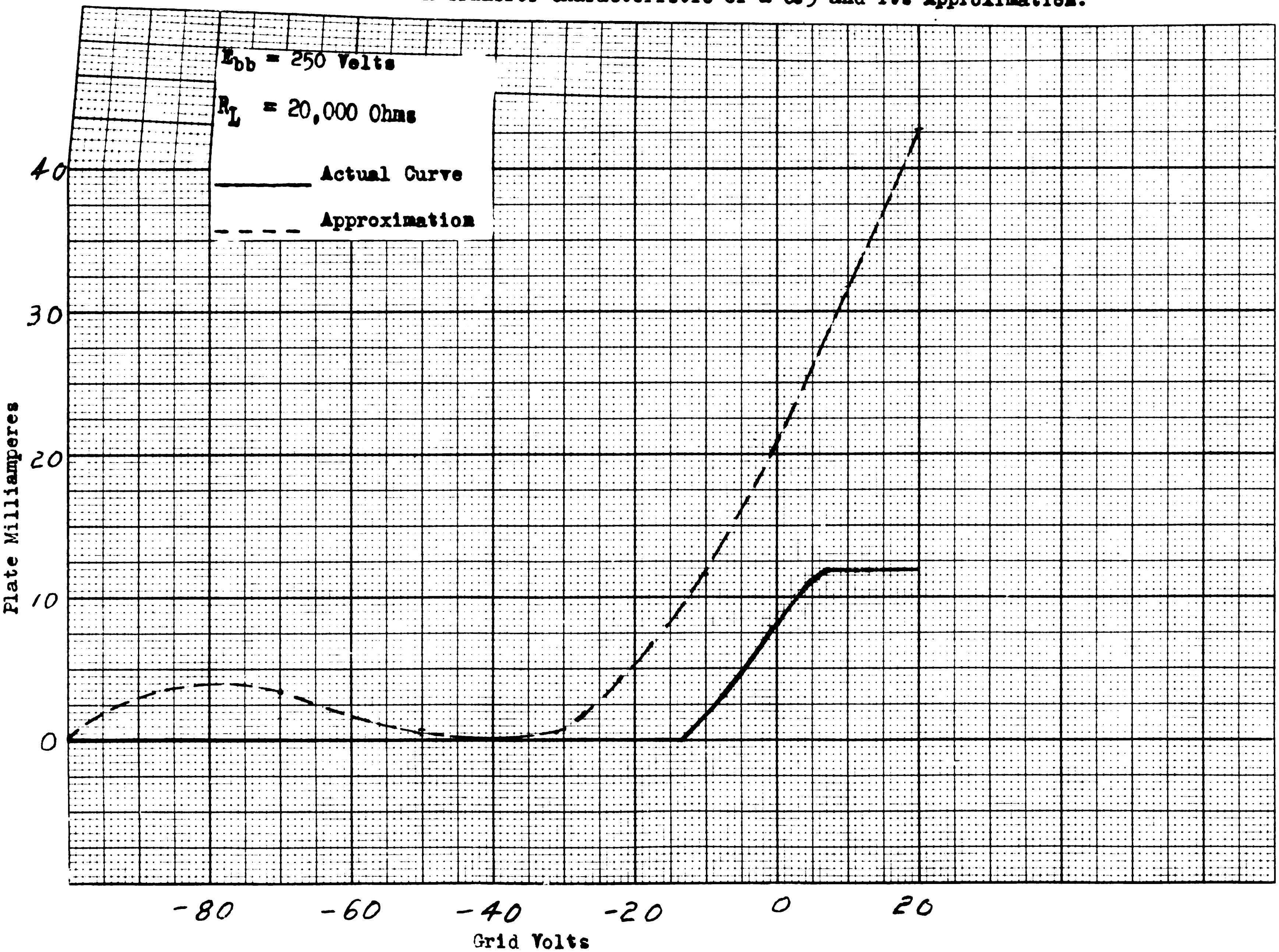


Figure 22. Value of the Coefficient of $(ec+100)^7 x10^{-12}$ vs Plate Voltage of a 6J5.

Figure 23. Value of the Coefficient of $(e_c+100)^9 x10^{-18}$ vs Plate Voltage of a 6J5.

Figure 24. A Transfer Characteristic of a 6J5 and its Approximation.



In making this approximation we shall use only the points we actually know. (Thus in our case we shall use only E_{bb} = 200, 300 and 400 volts.) Once again we will use the matrix method of solution.

$$(50)\quad \begin{bmatrix} 200 & 8\text{x}10^6 & 32\text{x}10^{10} & 0.114 & -0.07322 & 0.01282 & -0.7345 & 13.3 \\ 300 & 27\text{x}10^6 & 243\text{x}10^{10} & 0.259 & -0.17111 & 0.03207 & -2.1197 & 46.8 \\ 400 & 64\text{x}10^6 & 1024\text{x}10^{10} & 0.263 & -0.17612 & 0.03356 & -2.1932 & 47.4 \end{bmatrix}$$

In this matrix the fourth column is the first term coefficients (a_1) of the transfer curve approximations. The fifth column is the third term coefficients (a_3) times 10^3. The sixth is the fifth term coefficients (a_5) times 10^6. The seventh is the seventh term coefficients (a_7) times 10^{12}. The eighth is the ninth term coefficients (a_9) times 10^{18}. (All the even term coefficients are zero.) Solving the matrix gives the following results:

$$(51)\quad a_1 = 0.00706\text{x}10^{-2}E_{bb} + 0.015425\text{x}10^{-6}E_{bb}{}^3 - 0.000735\text{x}10^{-10}E_{bb}{}^5$$

$$(52)\quad a_3\text{x}10^3 = -\ 0.00245\text{x}10^{-2}E_{bb} - 0.01052\text{x}10^{-6}E_{bb}{}^3 + 0.000495\text{x}10^{-10}E_{bb}{}^5$$

$$(53)\quad a_5\text{x}10^6 = -\ 0.000574\text{x}10^{-2}E_{bb} + 0.002142\text{x}10^{-6}E_{bb}{}^3 - 0.000099\text{x}10^{-10}E_{bb}{}^5$$

$$(54)\quad a_7\text{x}10^{12} = 0.175594\text{x}10^{-2}E_{bb} - 0.165867\text{x}10^{-6}E_{bb}{}^3 + 0.007539\text{x}10^{-10}E_{bb}{}^5$$

$$(55)\quad a_9\text{x}10^{18} = -\ 7.485\text{x}10^{-2}E_{bb} + 4.309\text{x}10^{-6}E_{bb}{}^3 - 0.1938\text{x}10^{-10}E_{bb}{}^5$$

To test this approximation we will once again select a plate voltage and compare the calculated current with the current calculated from the published curves. Once again our voltage will be 250 volts. This time our approximation will be

$$(56)\quad i_p = -\ 0.459(e_c+100) + 0.312\text{x}10^{-3}(e_c+100)^3 - 0.064\text{x}10^{-6}(e_c+100)^5 + 5.210\text{x}10^{-12}(e_c+100)^7 - 140.66\text{x}10^{-18}(e_c+100)^9.$$

In order to save space merely a table of the evaluation of the plate

current (actual and calculated) and grid voltage will be given. The portion covered is in the linear region of the tube.

TABLE II

Results of the Approximation Using Odd Power Series for Plate Voltage

Plate voltage	250 volts	250 volts	250 volts
Grid voltage	-10 volts	0 volts	+10 volts
Actual Plate Current	1.7 ma.	8.5 ma.	11.9 ma.
Calculated Plate Current	-0.62 ma.	0.44 ma.	8.01 ma.
Load Resistance	20000 ohms	20000 ohms	20000 ohms

This is a better approximation, but it still is too inaccurate to be of much value.

Since the third method is almost identical with the second method only the matrix and its results will be given. In this matrix also the fourth through eighth columns will be modified as in (50).

$$(57)\quad \begin{bmatrix} 200 & 4x10^4 & 8x10^6 & 0.1139 & -0.07322 & 0.01282 & -0.7345 & 13.3 \\ 300 & 9x10^4 & 27x10^6 & 0.2586 & -0.17111 & 0.03207 & -2.1197 & 46.8 \\ 400 & 16x10^4 & 64x10^6 & 0.2630 & -0.17612 & 0.0ee56 & -2.1932 & 46.4 \end{bmatrix}$$

The results are given below.

$$(58)\quad a_1 = -\ 0.15065x10^{-2}E_{bb} + 0.15350x10^{-4}E_{bb}^{\ 2} - 0.02485x10^{-6}E_{bb}^{\ 3}$$

$$(59)\quad a_3x10^3 = +\ 0.10457x10^{-2}E_{bb} - 0.10403x10^{-4}E_{bb}^{\ 2} + 0.01672x10^{-6}E_{bb}^{\ 3}$$

$$(60)\quad a_5x10^6 = -\ 0.02189x10^{-2}E_{bb} + 0.02073x10^{-4}E_{bb}^{\ 2} - 0.00329x10^{-6}E_{bb}^{\ 3}$$

$$(61)\quad a_7x10^{12} = +\ 1.8042x10^{-2}E_{bb} - 1.5833x10^{-4}E_{bb}^{\ 2} + 0.2488x10^{-6}E_{bb}^{\ 3}$$

$$(62)\quad a_9x10^{18} = -\ 49.35x10^{-2}E_{bb} + 40.70x10^{-4}E_{bb}^{\ 2} - 0.2488x10^{-6}E_{bb}^{\ 3}.$$

Once again a plate voltage was selected and the coefficients were evaluated and an approximation made. This time the approximation was

$$(63)\quad i_p = 0.1945(e_c+100) - 0.1275\times10^{-3}(e_c+100)^3 + 0.02343\times10^{-6}(e_c+100)^5 - 1.4976\times10^{-2}(e_c+100)^7 + 31.78\times10^{-18}(e_c+100)^9.$$

Over the linear region the results of this approximation are tabulated in Table III.

TABLE III

Results of the Approximation Using Odd and Even Power series for Plate Voltage

Plate Voltage	250 volts	250 volts	250 volts
Grid Voltage	-10 volts	0 volts	+10 volts
Actual Plate Current	0.7 ma.	8.5 ma.	11.9 ma.
Calculated Plate Current	3.59 ma.	8.27 ma.	12.13 ma.
Load Resistance	20000 ohms	20000 ohms	20000 ohms

Since this approximation checks fairly well at 250 volts now let us try it at 350 volts. (64) is the approximation.

$$(64)\quad i_p = 0.2877(e_c+100) - 0.1915\times10^{-3}(e_c+100)^3 + 0.03627\times10^{-6}(e_c+100)^5 - 2.413\times10^{-12}(e_c+100)^7 + 53.594\times10^{-18}(e_c+100)^9$$

Table IV gives the results of this approximation.

It is easily seen from Table IV that this is the desired approximation. It is only valid for E_{bb} from 200 to 400 volts but this range could probably be extended by making additional approximations and including those coefficients in the comparison.

TABLE IV

More Results of the Approximation Using Odd and Even Power Series for Plate Voltage

Plate Voltage	350 volts	350 volts	350 volts
Grid Voltage	-10 volts	0 volts	+10 volts
Actual Plate Current	4.95 ma.	12.20 ma.	17.25 ma.
Calculated Plate Current	5.810 ma.	12.26 ma.	17.04 ma.
Load Resistance	20000 ohms	20000 ohms	20000 ohms

In conclusion it should be added that there well could be other methods of finding the intermediate curves than the ones discussed. Also each of the methods discussed may be of value in a particular case. In our case the third way proved to be the best. If we had investigated the 6V6 we might have found the second way the best, or even the first way.

In discussion of the three ways in particular, the second and third ways are essentially the same, and neither offers much advantage over the other except in a particular case. Both of them have the advantage over the graphical method in that they can be calculated to more places. In addition if the plate voltage is continuously varying as in plate modulation the second and third ways can be directly substituted in the equation for the plate current while the graph would again have to be approximated.

CHAPTER III

APPLICATIONS

Now that we have made the approximation of the transfer curve and discussed its limitations it is now time to investigate some of the uses it can be put to. Our first problem will be that of a single tube amplifier using a resistance load with too much bias for operation in the linear region. The tube will be a 6J5, the plate voltage 300 volts, the load resistance 20,000 ohms, and the bias -15 volts. We will apply an alternating current voltage of the form 10cos pt. The circuit diagram is shown in Figure 25.

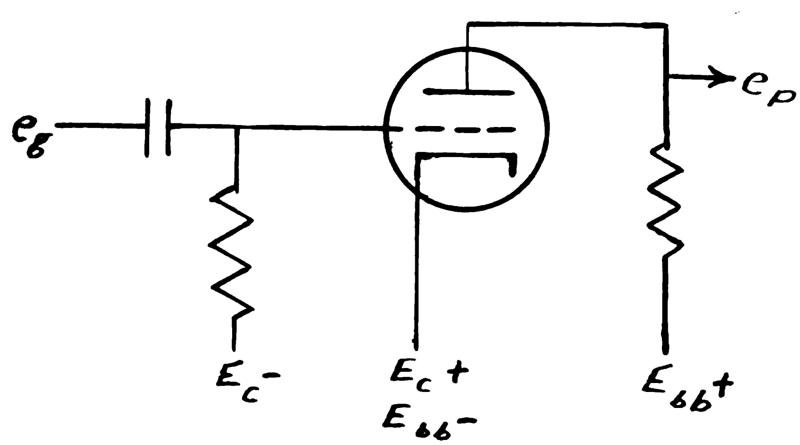


Figure 25. Single-ended Triode Amplifier.

The varible, (e_c + 100), now becomes (85 + 10 cos pt). Putting this into (28) and expanding we get.

$$(65)\quad i_p = 2.078 + 4.718\cos pt + 0.976\cos^2 pt - 0.244\cos^3 pt - 0.0577\cos^4 pt + 0.00183\cos^5 pt + 1.153\text{x}10^{-3}\cos^6 pt + 10.53\text{x}10^{-5}\cos^7 pt + 35.80\text{x}10^{-7}\cos^8 pt + 46.8\text{x}10^{-9}\cos^9 pt.$$

This function in turn may be expanded by the use of trigonometric identities into the more familiar harmonic functions.

$$(66)\quad i_p = 2.544 + 4.536\cos pt + 0.459\cos 2pt - 0.058\cos 3pt - 0.00699\cos 4pt + 0.000126\cos 5pt + 0.0363\text{x}10^{-3}\cos 6pt + 0.165\text{x}10^{-5}\cos 7pt + 0.280\text{x}10^{-7}\cos 8pt + 0.183\text{x}10^{-9}\cos 9pt.$$

To obtain the output voltage it becomes merely necessary to multiply this current by the load resistance. Of course if there is a blocking condenser the direct current component of the voltage will be missing.

Push-pull circuits can also be easily calculated using this approximation. In this case the tubes are figured separately and the plate currents subtracted from each other.

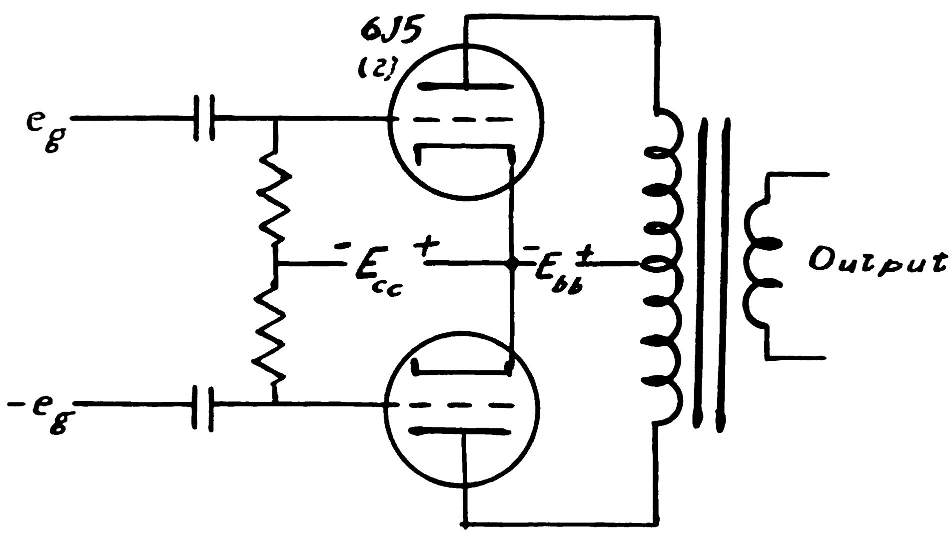


Figure 26. Push-pull Amplifier

For example let us assume that the output transformer appears as a 20,000 ohm load, that $E_{bb} = 300$ volts, that $E_{cc} = 15$ volts and that e_g 10cos pt. This once again means that we use (28). For the top tube the plate current will be identical to (65). The variable for the bottom tube is (85 - 10cos pt). This gives the following results for the bottom tube.

$$(67)\ i_p = 2.078 - 4.718\cos pt + 0.976\cos^2 pt + 0.244\cos^3 pt = 0.0577\cos^4 pt - 0.00183\cos^5 pt + 1.153\times10^{-3}\cos^6 pt - 10.53\times10^{-5}\cos^7 pt + 35.80\times10^{-7}\cos^8 pt - 46.8\times10^{-9}\cos^9 pt.$$

Subtracting (67) from (65) gives us the output current (68)

$$(68)\ i_o = 5.436\cos pt - 0.488\cos^3 pt + 0.00366\cos^5 pt + 21.06\times10^{-5}\cos^7 pt + 93.6\times10^{-9}\cos^9 pt.$$

Or in harmonic form

$$(69)\ i_o = 9.072\cos pt - 0.116\cos 3pt + 0.000252\cos 5pt + 0.330\times10^{-5}\cos 7pt + 0.366\times10^{-9}\cos 9pt.$$

Thus we immediately see that the even harmonics cancel as they should in a balanced push-pull circuit.

One of the most difficult problems to handle analytically as far as vacuum tube circuits are concerned is a plate modulated r.f. amplifier operating in class C conditions. Our approximation offers a solution. It is not an accurate solution inasmuch as several untrue assumptions are made, but the results can be used as a guide on what to expect.

Figure 27 shows a single-ended r. f. amplifier.

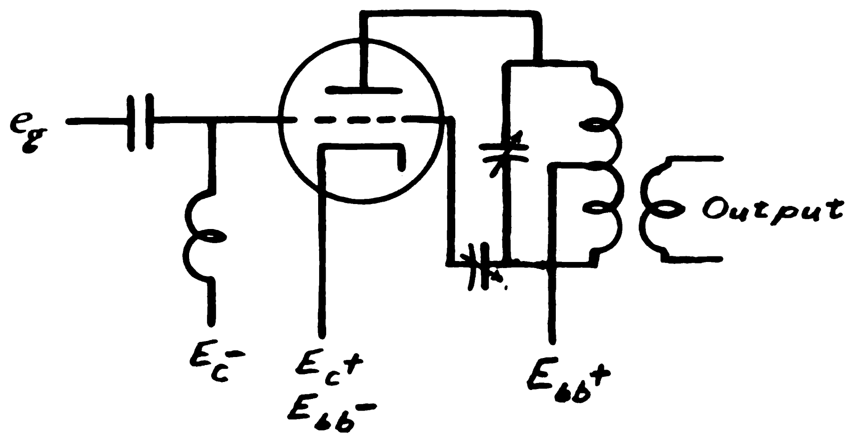


Figure 27. Single-ended Amplifier

The untrue assumptions that we shall make are (1) if grid current is drawn, that it does not affect the bias of the tube, (2) the neutralizing circuit can be ignored in our calculations, and (3) the tank circuit presents a resistive load at all frequencies. This last assumption is particularly objectionable, but it can be justified in that we are interested primarily in the voltage output and not the current. This means that we will find the current by making the assumption of a resistive load and then finding the voltage by multiplying the current by the true load. In such a manner we will be able to guess an approximate solution.

Let us assume that we have such an amplifier. Let $E_{bb} = 300 + 100\cos mt$, $e_c = 40 + 65\cos ct$, and that the tank presents a resistive load of 20,000 ohms at the carrier frequency. The expression for the output current is (70).

$$(70)\quad i_p = [-0.15065\times10^{-2}(300+100\cos mt) + 0.15350\times10^{-4}(300+100\cos mt)^2 - 0.02485\times10^{-6}(300+100\cos mt)^3](40+65\cos ct) + [0.10457\times10^{-2}(300+100\cos mt) - 0.10403\times10^{-4}(300+100\cos mt)^2 + 0.01672\times10^{-6}(300+100\cos mt)^3](40+65\cos ct)^3\times10^{-3} + [-0.02189\times10^{-2}(300+100\cos mt) + 0.02073\times10^{-4}(300+100\cos mt)^2 - 0.00329\times10^{-6}(300+100\cos mt)^3](40+65\cos ct)^5\times10^{-6} + [1.8042\times10^{-2}(300+100\cos mt) - 1.5833\times10^{-4}(300+100\cos mt)^2 + 0.2488\times10^{-6}(300+100\cos mt)^3](40+65\cos ct)^7\times10^{-12} + [-49.35\times10^{-2}(300+100\cos mt) + 40.70\times10^{-4}(300+100\cos mt)^2 - 0.2488\times10^{-6}(300+100\cos mt)^3](40+65\cos ct)^9\times10^{-18}$$

This expression is quite long and its expansion would be of dubious value. Nevertheless we are able to see that the approximation will give us an expression for plate modulation.

There are many more problems that can be solved by this method. Indeed other tubes may be used. Also nothing was done involving wave shaping circuits. However, by this time the reader should have a fairly clear idea of the subject and can apply it to his own particular situation.

In conclusion once again it should be stated that this is only an approximate method as is the case in most non-linear problems. The method can only be as accurate as the approximation. It was found that the calculations required the use of a calculating machine as a slide rule did not offer sufficient accuracy.

It also should be added that the results experimentally may differ from those calculated due to variations from the normal by the tubes themselves.

As far as the results are concerned in this paper, no experimental work was performed to prove or disprove them. However, each of the calculations was checked at least once and most of them were performed by machine.

It is the author's opinion that this method will be of great value when more than one grid voltage is applied to a tube and when the tube goes either into saturation or cut-off (but not both!) For just one particular case the approximation is too much work for the value received. Nevertheless the reader should not forego his own opinion on the subject.

APPENDIX A

SOLUTION: VAN DER BIJL'S EQUATION

APPENDIX A

Solution: van der Bijl's Equation

(1) $I_o = \alpha(\gamma E_b + E_c + \epsilon + e\sin pt)^B$

where $E_b = E_{bb} - IR$ and $B = 2$ (R is the value of the load resistance.)

(2) $I_o = \alpha[\gamma(E_{bb} - IR) + E_c + \epsilon + e\sin pt]^2$

Let $V = \gamma E_b + E_c + \epsilon$

Since ϵ is very small compared to $\gamma E_b + E_c$ it may be neglected and

$V = \gamma E_b + E_c$.

Rewriting

(3) $I_o = \alpha(V + e\sin pt)^2$

Expanding the right hand side of (3)

(4) $I_o = \alpha V^2 + 2\alpha Ve\sin pt + \alpha e^2\sin^2 pt$

Substituting the value of V and further expansion

(5) $I_o = \alpha(\gamma^2 E_b^2 + E_c^2 + 2\gamma E_b E_c) + 2\alpha(\gamma E_b + E_c)e\sin pt + \alpha e^2\sin^2 pt$

Substituting $E_{bb} - I_oR$ for E_b, expanding and collecting terms

(6) $\alpha\gamma^2R^2I_o^2 - I_o(1 + 2\alpha\gamma^2E_{bb}R + 2\alpha\gamma RE_c + 2\alpha\gamma Re\sin pt) = -\alpha(V$

$+ e\sin pt)^2$

Rewriting

(7) $I_o^2(\alpha\gamma^2R^2) - I_o(1 + 2\alpha\gamma R[V + e\sin pt]) + \alpha(V + e\sin pt)^2 = 0$

Substituting into the quadratic formula

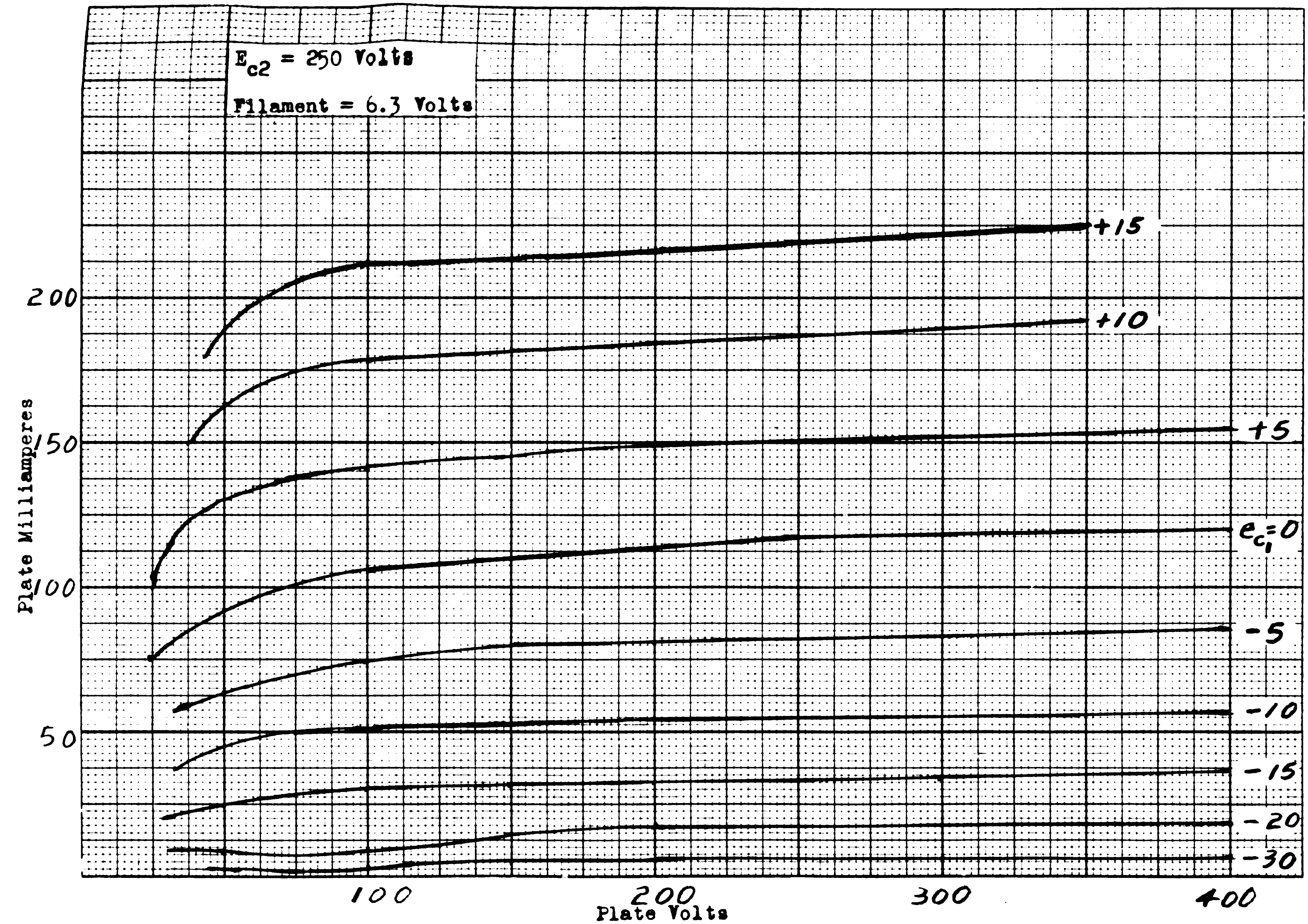
(8) $I_o = \dfrac{1 + 2\alpha\gamma R(V + e\sin pt) - \sqrt{1 + 4\alpha\gamma R(V + e\sin pt)}}{2\alpha\gamma^2R^2}$

APPENDIX B

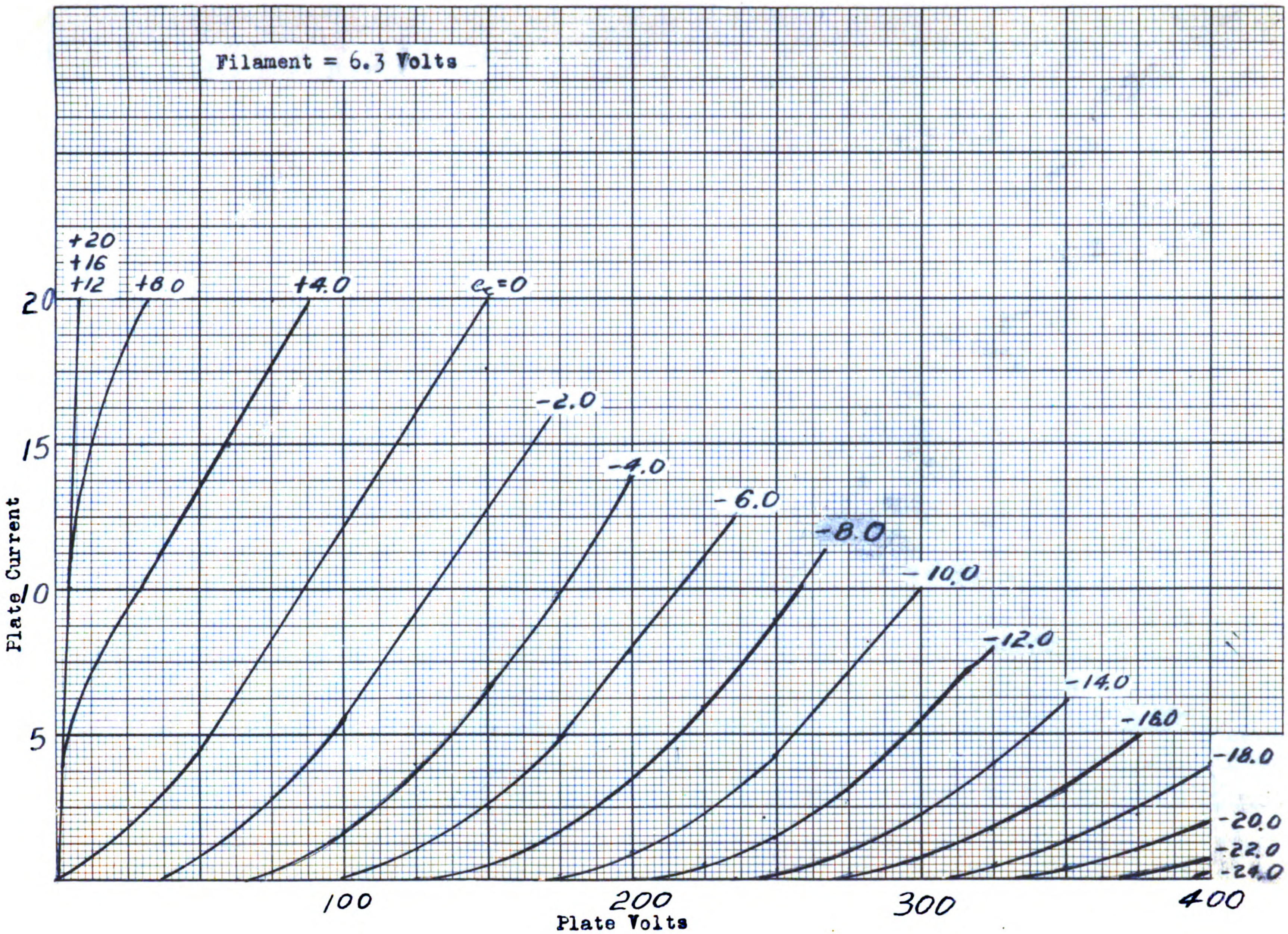
CHARACTERISTICS OF THE TUBES[6]

6 General Electric Company, Electronics Department; Electronic Tubes, Receiving Types; Volume I.

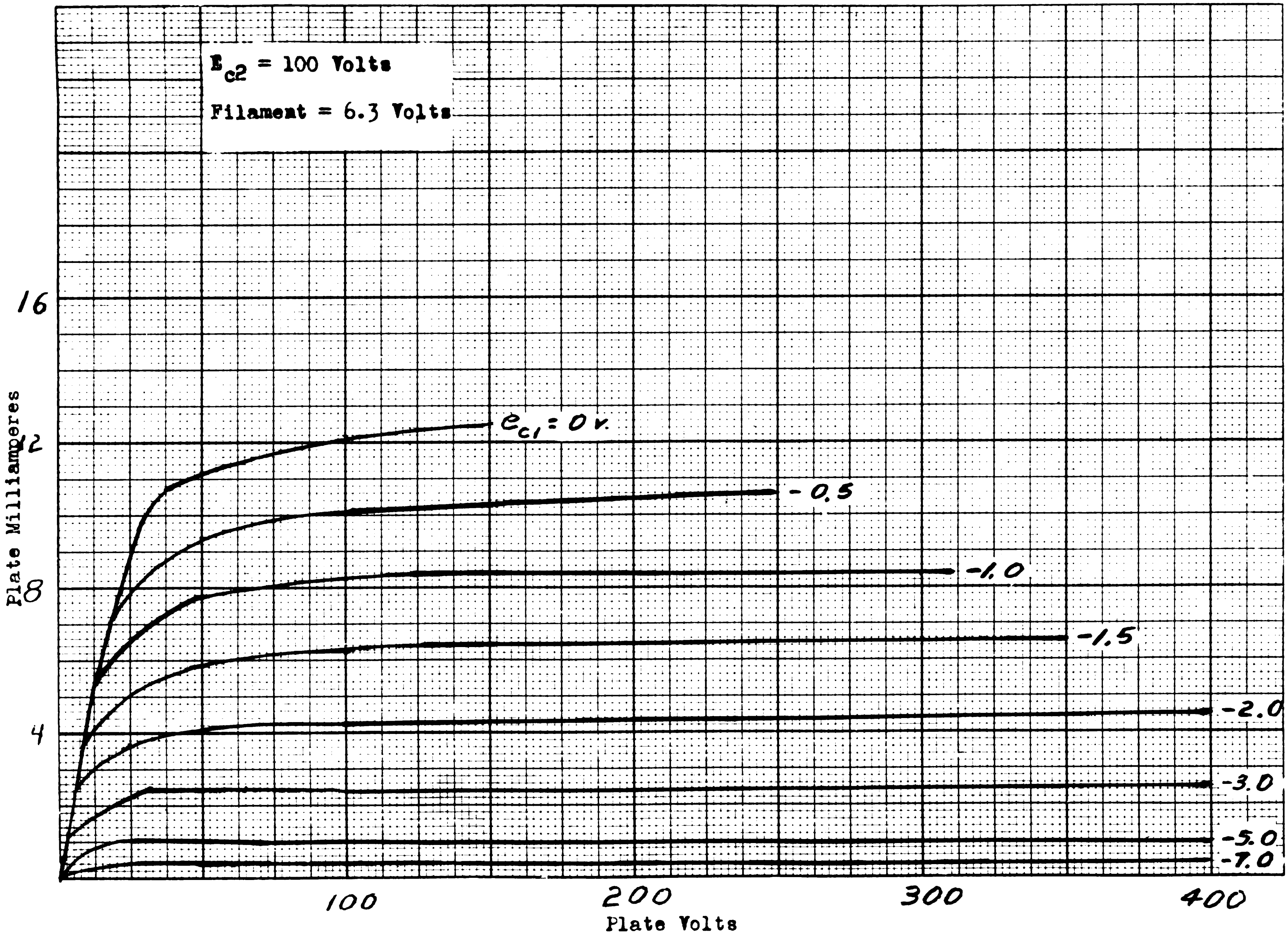
Average Plate Characteristics of a 6V6



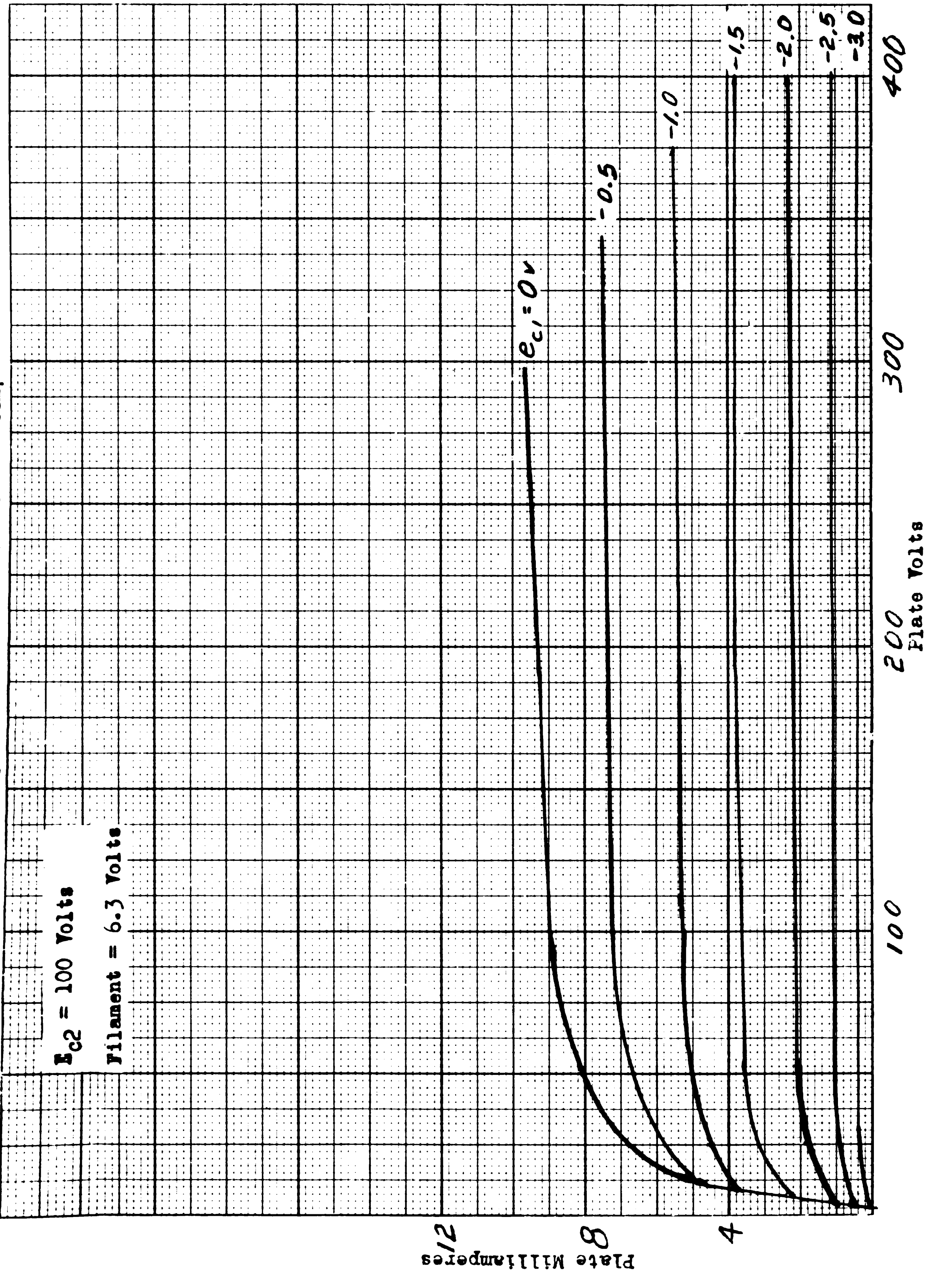
Average Plate Characteristics of a 6J5



Average Plate Characteristics of a 6SG7



Average Plate Characteristics of a 6SH7



APPENDIX C

CALCULATIONS FOR THE APPROXIMATIONS

APPENDIX C

Calculations for the Approximations

Since a step by step process was given in the text only the matrices will be given. The initial matrix is repeated but the final equation will be omitted. Also after the matrix has been half diagonalized just the complete diagonalization will follow it.

6V6 First Approximation For

E_{bb} = 300 and 200 volts with R_L = 2000 ohms

$$
(1)\begin{bmatrix}
40 & 64\times10^{3} & 102.4\times10^{6} & 0.16384\times10^{12} & 0.000262\times10^{18} & 0 & 0 \\
65 & 274.625\times10^{3} & 1160.3\times10^{6} & 4.902\times10^{12} & 0.0207119\times10^{18} & 0 & 0 \\
100 & 1000\times10^{3} & 10000\times10^{6} & 100\times10^{12} & 1\times10^{18} & 112.5 & 82.5 \\
115 & 1520.875\times10^{3} & 20113.6\times10^{6} & 266\times10^{12} & 3.518\times10^{18} & 135 & 87.5 \\
150 & 3375\times10^{3} & 75937.9\times10^{6} & 1708.6\times10^{12} & 38.44\times10^{18} & 135 & 87.5
\end{bmatrix}
$$

$$
(2)\begin{bmatrix}
1 & 1.6\times10^{3} & 2.56\times10^{6} & 0.004906\times10^{12} & 0.00000655\times10^{15} & 0 & 0 \\
0 & 170.625\times10^{3} & 993.9\times10^{6} & 4.6358\times10^{12} & 0.020286\times10^{18} & 0 & 0 \\
0 & 840\times10^{3} & 9744\times10^{6} & 99.5904\times10^{12} & 0.999345\times10^{18} & 112.5 & 82.5 \\
0 & 1336.875\times10^{3} & 19818.2\times10^{6} & 265.529\times10^{12} & 3.517247\times10^{18} & 135 & 87.5 \\
0 & 3135\times10^{3} & 75553.9\times10^{6} & 1707.986\times10^{12} & 38.439\times10^{18} & 135 & 87.5
\end{bmatrix}
$$

$$
(3)\begin{bmatrix}
1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 & 0 \\
0 & 1 & 5.825\times10^{3} & 0.02717\times10^{9} & 0.0001189\times10^{15} & 0 & 0 \\
0 & 0 & 4.851\times10^{6} & 76.7676\times10^{9} & 0.899469\times10^{15} & 112.5\times10^{-3} & 82.5\times10^{-3} \\
0 & 0 & 12.032\times10^{6} & 229.206\times10^{9} & 3.358293\times10^{15} & 135\times10^{-3} & 87.5\times10^{-3} \\
0 & 0 & 57.292\times10^{6} & 1622.808\times10^{9} & 38.0662485\times10^{15} & 135\times10^{-3} & 87.5\times10^{-3}
\end{bmatrix}
$$

$$
(4)\quad\begin{bmatrix}
1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 & 0 \\
0 & 1 & 5.825\times10^{3} & 27.17\times10^{6} & 118.9\times10^{9} & 0 & 0 \\
0 & 0 & 1 & 15.835\times10^{3} & 0.1854\times10^{9} & 23.191\times10^{-9} & 17.007\times10^{-9} \\
0 & 0 & 0 & 38.800\times10^{3} & 1.1276\times10^{9} & -144.034\times10^{-9} & -117.128\times10^{-9} \\
0 & 0 & 0 & 716.162\times10^{3} & 27.444\times10^{9} & -1193.659\times10^{-9} & -886.865\times10^{-9}
\end{bmatrix}
$$

$$
(5)\quad\begin{bmatrix}
1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 & 0 \\
0 & 1 & 5.825\times10^{3} & 27.17\times10^{6} & 118.9\times10^{9} & 0 & 0 \\
0 & 0 & 1 & 15.825\times10^{3} & 185.4\times10^{6} & 23.191\times10^{-9} & 17.007\times10^{-9} \\
0 & 0 & 0 & 1 & 0.02906\times10^{6} & -3.712\times10^{-12} & -3.019\times10^{-12} \\
0 & 0 & 0 & 0 & 6.632\times10^{6} & 1464.7\times10^{-12} & 1275.2\times10^{-12}
\end{bmatrix}
$$

$$
(6)\quad\begin{bmatrix}
1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 0 & -1.448\times10^{-3} & -1.258\times10^{-3} \\
0 & 1 & 5.825\times10^{3} & 27.17\times10^{6} & 0 & -26.277\times10^{-6} & -22.829\times10^{-6} \\
0 & 0 & 1 & 15.825\times10^{3} & 0 & -17.782\times10^{-9} & -18.590\times10^{-9} \\
0 & 0 & 0 & 1 & 0 & -10.134\times10^{-12} & -8.599\times10^{-12} \\
0 & 0 & 0 & 0 & 1 & 0.221\times10^{-15} & 0.192\times10^{-15}
\end{bmatrix}
$$

$$
(7)\quad\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 605.48\times10^{-3} & 490.90\times10^{-3} \\
0 & 1 & 0 & 0 & 0 & -581.53\times10^{-6} & -473.57\times10^{-6} \\
0 & 0 & 1 & 0 & 0 & 142.59\times10^{-9} & 117.49\times10^{-9} \\
0 & 0 & 0 & 1 & 0 & -10.134\times10^{-12} & -8.599\times10^{-12} \\
0 & 0 & 0 & 0 & 1 & 0.221\times10^{-15} & 0.192\times10^{-15}
\end{bmatrix}
$$

6V6 Second Approximation for

E_{bb} = 300 and 200 volts with R_L = 2000 ohms

$$(8)\quad \begin{bmatrix} 40 & 64\times10^{3} & 102.4\times10^{6} & 0.16384\times10^{12} & 0.000262\times10^{18} & 0 & 0 \\ 65 & 274.625\times10^{3} & 1160.3\times10^{6} & 4.902\times10^{12} & 0.0207119\times10^{18} & 0 & 0 \\ 90 & 729\times10^{3} & 5904.0\times10^{6} & 47.8297\times10^{12} & 0.38742\times10^{18} & 57.5 & 50 \\ 110 & 1331\times10^{3} & 16105.1\times10^{6} & 194.8717\times10^{12} & 2.3579\times10^{18} & 135 & 87.5 \\ 130 & 2197\times10^{3} & 37129.3\times10^{6} & 627.4852\times10^{12} & 10.6045\times10^{18} & 135 & 87.5 \end{bmatrix}$$

$$(9)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 0.004096\times10^{12} & 0.00000655\times10^{18} & 0 & 0 \\ 0 & 170.625\times10^{3} & 993.9\times10^{6} & 4.6358\times10^{12} & 0.020286\times10^{18} & 0 & 0 \\ 0 & 585\times10^{3} & 5674.5\times10^{6} & 47.461\times10^{12} & 0.38683\times10^{18} & 57.5 & 50 \\ 0 & 1155\times10^{3} & 15823.5\times10^{6} & 194.421\times10^{12} & 2.35718\times10^{18} & 135 & 87.5 \\ 0 & 1989\times10^{3} & 36796.5\times10^{6} & 626.953\times10^{12} & 10.6036\times10^{18} & 135 & 87.5 \end{bmatrix}$$

$$(10)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 & 0 \\ 0 & 1 & 5.825\times10^{3} & 0.02717\times10^{9} & 0.0001189\times10^{15} & 0 & \\ 0 & 0 & 2.266875\times10^{6} & 31.567\times10^{9} & 0.31727\times10^{15} & 57.5\times10^{-3} & 50\times10^{-3} \\ 0 & 0 & 9.095625\times10^{6} & 163.040\times10^{9} & 2.21985\times10^{15} & 135\times10^{-3} & 87.5\times10^{-3} \\ 0 & 0 & 25.210575\times10^{6} & 572.912\times10^{9} & 10.3671\times10^{15} & 135\times10^{-3} & 87.5\times10^{-3} \end{bmatrix}$$

$$(11)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 & 0 \\ 0 & 1 & 5.825\times10^{3} & 27.17\times10^{6} & 118.9\times10^{9} & 0 & 0 \\ 0 & 0 & 1 & 13.925\times10^{3} & 0.13996\times10^{9} & 25.365\times10^{-9} & 22.057\times10^{-9} \\ 0 & 0 & 0 & 36.3834\times10^{3} & 0.94683\times10^{9} & -95.7105\times10^{-9} & -113.122\times10^{-9} \\ 0 & 0 & 0 & 221.8547\times10^{3} & 6.83863\times10^{9} & -504.4662\times10^{-9} & -467.570\times10^{-9} \end{bmatrix}$$

$$
(12)\quad \begin{bmatrix}
1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 & 0 \\
0 & 1 & 5.825\times10^{6} & 27.17\times10^{3} & 118.9\times10^{9} & 0 & 0 \\
0 & 0 & 1 & 13.925\times10^{3} & 139.6\times10^{6} & 25.365\times10^{-9} & 22.057\times10^{-9} \\
0 & 0 & 0 & 1 & 0.02602\times10^{6} & -2.6396\times10^{-12} & -3.109\times10^{-12} \\
0 & 0 & 0 & 0 & 1.06597\times10^{6} & 79.1448\times10^{-12} & 222.176\times10^{-12}
\end{bmatrix}
$$

$$
(13)\quad \begin{bmatrix}
1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 0 & -0.486\times10^{-3} & -1.365\times10^{-3} \\
0 & 1 & 5.825\times10^{6} & 27.17\times10^{6} & 0 & -8.828\times10^{-6} & -24.782\times10^{-6} \\
0 & 0 & 1 & 13.925\times10^{3} & 0 & 15.000\times10^{-9} & -7.039\times10^{-9} \\
0 & 0 & 0 & 1 & 0 & -4.563\times10^{-12} & -8.532\times10^{-12} \\
0 & 0 & 0 & 0 & 1 & 0.074247\times10^{-15} & 0.208426\times10^{-15}
\end{bmatrix}
$$

$$
(14)\quad \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 364.899\times10^{-3} & 457.889\times10^{-3} \\
0 & 1 & 0 & 0 & 0 & -342.348\times10^{-6} & -444.022\times10^{-6} \\
0 & 0 & 1 & 0 & 0 & 78.540\times10^{-9} & 111.769\times10^{-9} \\
0 & 0 & 0 & 1 & 0 & -4.563\times10^{-12} & -8.532\times10^{-12} \\
0 & 0 & 0 & 0 & 1 & 0.074247\times10^{-15} & 0.208426\times10^{-15}
\end{bmatrix}
$$

6V6 First Approximation for

E_{bb} = 200 volts with R_L = 4000 ohms

$$
(15)\quad \begin{bmatrix}
40 & 64\times10^{3} & 102.4\times10^{6} & 0.16384\times10^{12} & 0.000262\times10^{18} & 0 \\
65 & 274.625\times10^{3} & 1160.29\times10^{6} & 4.902\times10^{12} & 0.0207119\times10^{18} & 0 \\
85 & 614.125\times10^{3} & 4437.05\times10^{6} & 32.058\times10^{12} & 0.23162\times10^{18} & 30 \\
95 & 857.375\times10^{3} & 7737.81\times10^{6} & 69.983\times10^{12} & 0.63025\times10^{18} & 45 \\
120 & 1728\times10^{3} & 24883.2\times10^{6} & 358.3181\times10^{12} & 5.515978\times10^{18} & 45
\end{bmatrix}
$$

$$(16)\quad \begin{bmatrix} 1 & 1.6 \times 10^{3} & 2.56 \times 10^{6} & 0.004096 \times 10^{12} & 0.00000655 \times 10^{18} & 0 \\ 0 & 170.625 \times 10^{3} & 993.9 \times 10^{6} & 4.6358 \times 10^{12} & 0.020286 \times 10^{18} & 0 \\ 0 & 478.125 \times 10^{3} & 4219.45 \times 10^{6} & 31.7098 \times 10^{12} & 0.23106 \times 10^{18} & 30 \\ 0 & 705.375 \times 10^{3} & 7494.61 \times 10^{6} & 69.5939 \times 10^{12} & 0.62963 \times 10^{18} & 45 \\ 0 & 1536 \times 10^{3} & 24575.80 \times 10^{6} & 357.8266 \times 10^{12} & 5.51519 \times 10^{18} & 45 \end{bmatrix}$$

$$(17)\quad \begin{bmatrix} 1 & 1.6 \times 10^{3} & 2.56 \times 10^{6} & 4.096 \times 10^{9} & 6.55 \times 10^{12} & 0 \\ 0 & 1 & 5.825 \times 10^{3} & 0.02717 \times 10^{9} & 0.0001189 \times 10^{15} & 0 \\ 0 & 0 & 1.4344 \times 10^{6} & 18.7191 \times 10^{9} & 0.17425 \times 10^{15} & 30 \times 10^{-3} \\ 0 & 0 & 3.3858 \times 10^{6} & 50.4289 \times 10^{9} & 0.54573 \times 10^{15} & 45 \times 10^{-3} \\ 0 & 0 & 15.6286 \times 10^{6} & 316.0935 \times 10^{9} & 5.3326 \times 10^{15} & 45 \times 10^{-3} \end{bmatrix}$$

$$(18)\quad \begin{bmatrix} 1 & 1.6 \times 10^{3} & 2.56 \times 10^{6} & 4.096 \times 10^{9} & 6.55 \times 10^{12} & 0 \\ 0 & 1 & 5.825 \times 10^{3} & 27.17 \times 10^{6} & 118.9 \times 10^{9} & 0 \\ 0 & 0 & 1 & 13.05 \times 10^{3} & 0.12148 \times 10^{9} & 20.915 \times 10^{-9} \\ 0 & 0 & 0 & 6.244 \times 10^{3} & 0.13442 \times 10^{9} & -25.814 \times 10^{-9} \\ 0 & 0 & 0 & 112.140 \times 10^{3} & 3.43403 \times 10^{9} & -281.872 \times 10^{-9} \end{bmatrix}$$

$$(19)\quad \begin{bmatrix} 1 & 1.6 \times 10^{3} & 2.56 \times 10^{6} & 4.096 \times 10^{9} & 6.55 \times 10^{12} & 0 \\ 0 & 1 & 5.825 \times 10^{3} & 27.17 \times 10^{6} & 118.9 \times 10^{9} & 0 \\ 0 & 0 & 1 & 13.05 \times 10^{3} & 121.48 \times 10^{6} & 20.915 \times 10^{-9} \\ 0 & 0 & 0 & 1 & 0.0215 \times 10^{6} & -4.134 \times 10^{-12} \\ 0 & 0 & 0 & 0 & 1.0230 \times 10^{6} & 181.714 \times 10^{-12} \end{bmatrix}$$

$$(20)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 0 & -1.163\times10^{-3} \\ 0 & 1 & 5.825\times10^{3} & 27.17\times10^{6} & 0 & -21.117\times10^{-6} \\ 0 & 0 & 1 & 13.05\times10^{3} & 0 & -0.660\times10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -7.952\times10^{-12} \\ 0 & 0 & 0 & 0 & 1 & 0.1776\times10^{-15} \end{bmatrix}$$

$$(21)\quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 416.54\times10^{-3} \\ 0 & 1 & 0 & 0 & 0 & -405.69\times10^{-6} \\ 0 & 0 & 1 & 0 & 0 & 103.113\times10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -7.952\times10^{-12} \\ 0 & 0 & 0 & 0 & 1 & 0.1776\times10^{-15} \end{bmatrix}$$

6V6 First Approximation for

E_{bb} = 200 volts with R_L = 1000 ohms

$$(22)\quad \begin{bmatrix} 40 & 64\times10^{3} & 102.4\times10^{6} & 0.16384\times10^{12} & 0.000262\times10^{18} & 0 \\ 65 & 274.625\times10^{3} & 1160.3\times10^{6} & 4.902\times10^{12} & 0.0207119\times10^{18} & 0 \\ 100 & 1000\times10^{3} & 10000\times10^{6} & 100\times10^{12} & 1\times10^{18} & 107 \\ 110 & 1331\times10^{3} & 16105.1\times10^{6} & 194.8717\times10^{12} & 2.3579\times10^{18} & 160 \\ 130 & 2197\times10^{3} & 37129.3\times10^{6} & 627.4852\times10^{12} & 10.6045\times10^{18} & 160 \end{bmatrix}$$

$$(23)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 0.004096\times10^{12} & 0.00000655\times10^{18} & 0 \\ 0 & 170.625\times10^{3} & 993.9\times10^{6} & 4.6358\times10^{12} & 0.020286\times10^{18} & 0 \\ 0 & 840\times10^{3} & 9744.\times10^{6} & 99.5904\times10^{12} & 0.999345\times10^{18} & 107 \\ 0 & 1155\times10^{3} & 15823.5\times10^{6} & 194.421\times10^{12} & 2.21985\times10^{18} & 160 \\ 0 & 1989\times10^{3} & 36796\times10^{6} & 626.953\times10^{12} & 10.6036\times10^{18} & 160 \end{bmatrix}$$

$$(24)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 \\ 0 & 1 & 5.825\times10^{3} & 0.02717\times10^{9} & 0.0001189\times10^{15} & 0 \\ 0 & 0 & 4.851\times10^{6} & 76.7676\times10^{9} & 0.999345\times10^{15} & 107\times10^{-3} \\ 0 & 0 & 9.095625\times10^{6} & 163.040\times10^{9} & 2.21985\times10^{15} & 160\times10^{-3} \\ 0 & 0 & 25.210575\times10^{6} & 572.912\times10^{9} & 10.3671\times10^{15} & 160\times10^{-3} \end{bmatrix}$$

$$(25)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 \\ 0 & 1 & 5.825\times10^{3} & 27.17\times10^{6} & 118.9\times10^{9} & 0 \\ 0 & 0 & 1 & 15.825\times10^{3} & 0.1854\times10^{9} & 22.057\times10^{-9} \\ 0 & 0 & 0 & 19.102\times10^{3} & 0.5335\times10^{6} & -40.622\times10^{-9} \\ 0 & 0 & 0 & 173.955\times10^{3} & 5.6931\times10^{6} & -396.070\times10^{-9} \end{bmatrix}$$

$$(26)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 6.55\times10^{12} & 0 \\ 0 & 1 & 5.825\times10^{6} & 27.17\times10^{6} & 118.9\times10^{9} & 0 \\ 0 & 0 & 1 & 15.825\times10^{3} & 185.4\times10^{6} & 22.057\times10^{-9} \\ 0 & 0 & 0 & 1 & 0.027929\times10^{6} & -2.127\times10^{-12} \\ 0 & 0 & 0 & 0 & 0.83471\times10^{6} & -26.068\times10^{-12} \end{bmatrix}$$

$$(27)\quad \begin{bmatrix} 1 & 1.6\times10^{3} & 2.56\times10^{6} & 4.096\times10^{9} & 0 & 0.205\times10^{-3} \\ 0 & 1 & 5.825\times10^{3} & 27.17\times10^{6} & 0 & 3.713\times10^{-6} \\ 0 & 0 & 1 & 15.825\times10^{3} & 0 & 27.847\times10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -1.255\times10^{-12} \\ 0 & 0 & 0 & 0 & 1 & -0.03123\times10^{-15} \end{bmatrix}$$

$$(28)\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 267.347\ \mathrm{x}10^{-3} \\ 0 & 1 & 0 & 0 & 0 & -240.082\ \mathrm{x}10^{-6} \\ 0 & 0 & 1 & 0 & 0 & 47.707\ \mathrm{x}10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -1.255\ \mathrm{x}10^{-12} \\ 0 & 0 & 0 & 0 & 1 & -0.0312\mathrm{x}10^{-15} \end{bmatrix}$$

In order to save space and avoid needless repetition if a matrix, [A], is repeated entirely in another matrix, [B], then

$$[B] = \begin{bmatrix} & b_1 \\ & b_2 \\ & \cdot \\ [A] & \cdot \\ & \cdot \\ & \cdot \\ & b_n \end{bmatrix}$$

where the column of b is the last column(s) of matrix B . Thus for example if $[A] = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ j & k & m & n \end{bmatrix}$ and if $[B] = \begin{bmatrix} a & b & c & d & x \\ e & f & g & h & y \\ j & k & m & n & z \end{bmatrix}$

Then [B] could be written $\begin{bmatrix} & x \\ [A] & y \\ & z \end{bmatrix}$

(Note: This is a particular notation invented by the author for convenience on his part. There is no justification for this in any laws of matrix algebra but just is a manner of notation.)

6J5 First Approximation for

E_{bb} = 300 volts with R_L = 20000 ohms

$$(29)\quad \begin{bmatrix} 50 & 125\times10^{3} & 312.5\times10^{6} & 0.78125\times10^{12} & 0.001953\times10^{18} & 0 \\ 80 & 512\times10^{3} & 3276.8\times10^{6} & 20.97152\times10^{12} & 0.13421\times10^{18} & 0 \\ 98 & 941.192\times10^{3} & 9039.2\times10^{6} & 86.81255\times10^{12} & 0.83375\times10^{18} & 9.2 \\ 110 & 1331\times10^{3} & 16105.1\times10^{6} & 194.87171\times10^{12} & 2.35795\times10^{18} & 14.5 \\ 130 & 2197\times10^{3} & 37129.3\times10^{6} & 627.48517\times10^{12} & 10.60450\times10^{18} & 14.7 \end{bmatrix}$$

$$(30)\quad \begin{bmatrix} 1 & 2.5\times10^{3} & 6.25\times10^{6} & 0.015625\times10^{12} & 0.00003906\times10^{18} & 0 \\ 0 & 312\times10^{3} & 2776.8\times10^{6} & 19.72152\times10^{12} & 0.1310852\times10^{18} & 0 \\ 0 & 696.192\times10^{3} & 8426.7\times10^{6} & 85.28130\times10^{12} & 0.8299221\times10^{18} & 9.2 \\ 0 & 1056\times10^{3} & 15417.6\times10^{6} & 193.15295\times10^{12} & 2.353653\times10^{18} & 14.5 \\ 0 & 1872\times10^{3} & 36316.8\times10^{6} & 625.45392\times10^{12} & 10.59942\times10^{18} & 14.7 \end{bmatrix}$$

$$(31)\quad \begin{bmatrix} 1 & 2.5\times10^{3} & 6.25\times10^{6} & 15.625\times10^{9} & 39.06\times10^{12} & 0 \\ 0 & 1 & 8.9\times10^{3} & 0.06321\times10^{9} & 0.000420\times10^{15} & 0 \\ 0 & 0 & 2.23059\times10^{6} & 41.2768\times10^{9} & 0.53752\times10^{15} & 9.2\times10^{-3} \\ 0 & 0 & 6.0192\times10^{6} & 126.4032\times10^{9} & 1.91013\times10^{15} & 14.5\times10^{-3} \\ 0 & 0 & 19.6560\times10^{6} & 507.1248\times10^{9} & 9.81318\times10^{15} & 14.7\times10^{-3} \end{bmatrix}$$

$$(32)\quad \begin{bmatrix} 1 & 2.5\times10^{3} & 6.25\times10^{6} & 15.625\times10^{9} & 39.06\times10^{12} & 0 \\ 0 & 1 & 8.9\times10^{6} & 63.21\times10^{6} & 420.0\times10^{9} & 0 \\ 0 & 0 & 1 & 18.505\times10^{3} & 0.24098\times10^{9} & 4.1245\times10^{-9} \\ 0 & 0 & 0 & 15.018\times10^{3} & 0.45962\times10^{9} & -10.3262\times10^{-9} \\ 0 & 0 & 0 & 143.391\times10^{3} & 5.07648\times10^{9} & -66.3712\times10^{-9} \end{bmatrix}$$

$$
(33)\begin{bmatrix}
1 & 2.5\text{x}10^{3} & 6.25\text{x}10^{6} & 15.625\text{x}10^{9} & 39.06 & \text{x}10^{12} & 0 \\
0 & 1 & 8.9\ \text{x}10^{6} & 63.21\ \text{x}10^{6} & 420 & \text{x}10^{9} & 0 \\
0 & 0 & 1 & 18.505\text{x}10^{3} & 240.98 & \text{x}10^{6} & 4.1245\text{x}10^{-9} \\
0 & 0 & 0 & 1 & & 0.03060\text{x}10^{6} & -0.6876\text{x}10^{-12} \\
0 & 0 & 0 & 0 & & 0.68872\text{x}10^{6} & 32.225\ \text{x}10^{-12}
\end{bmatrix}
$$

$$
(34)\begin{bmatrix}
1 & 2.5\text{x}10^{3} & 6.25\text{x}10^{6} & 15.625\text{x}10^{9} & 0 & -1.828\ \text{x}10^{-3} \\
0 & 1 & 8.9\ \text{c}10^{3} & 63.21\ \text{x}10^{6} & 0 & -19.656\ \text{x}10^{-6} \\
0 & 0 & 1 & 18.505\text{x}10^{3} & 0 & -7.1534\text{x}10^{-9} \\
0 & 0 & 0 & 1 & 0 & -2.1197\text{x}10^{-12} \\
0 & 0 & 0 & 0 & 1 & 0.0468\text{x}10^{-15}
\end{bmatrix}
$$

$$
(35)\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 258.612\ \text{x}10^{-3} \\
0 & 1 & 0 & 0 & 0 & -171.107\ \text{c}10^{-6} \\
0 & 0 & 1 & 0 & 0 & 32.0716\text{x}10^{-9} \\
0 & 0 & 0 & 1 & 0 & -2.1197\text{x}10^{-12} \\
0 & 0 & 0 & 0 & 1 & 0.0468\text{x}10^{-15}
\end{bmatrix}
$$

6J5 First Approximation for

E_{bb} = 200 volts and 400 volts with R_L = 20000 ohms

$$
(36)\begin{bmatrix}
 & 0 & 0 \\
 & 0 & 1.2 \\
(29) & 5.6 & 12.5 \\
 & 9.7 & 19.3 \\
 & 9.7 & 19.6
\end{bmatrix}
\qquad
(37)\begin{bmatrix}
 & 0 & 0 \\
 & 0 & 1.2 \\
(30) & 5.6 & 12.5 \\
 & 9.7 & 19.3 \\
 & 9.7 & 19.6
\end{bmatrix}
$$

$$(38)\begin{bmatrix} & 0 & 0 \\ & 0 & 0.0038\times10^{-3} \\ (31) & 5.6\times10^{-3} & 9.8\times10^{-3} \\ & 9.7\times10^{-3} & 15.287\times10^{-3} \\ & 9.7\times10^{-3} & 12.486\times10^{-9} \end{bmatrix} \qquad (39)\begin{bmatrix} & 0 & 0 \\ & 0 & 3.8\times10^{-6} \\ (32) & 2.4286\times10^{-9} & 4.393\times10^{-9} \\ & -4.9182\times10^{-9} & -11.155\times10^{-9} \\ & -38.036\times10^{-9} & -73.863\times10^{-9} \end{bmatrix}$$

$$(40)\begin{bmatrix} & 0 & 0 \\ & 0 & 3.8\times10^{-6} \\ (33) & 2.4286\times10^{-9} & 4.393\times10^{-9} \\ & -0.3275\times10^{-12} & -0.7428\times10^{-12} \\ & 8.9245\times10^{-12} & 32.647\times10^{-12} \end{bmatrix} \qquad (41)\begin{bmatrix} & -0.5195\times10^{-3} & -1.851\times10^{-3} \\ & -5.586\times10^{-6} & -16.108\times10^{-6} \\ (34) & -0.7764\times10^{-9} & -7.0295\times10^{-9} \\ & -0.7345\times10^{-12} & -2.1932\times10^{-12} \\ & 0.0133\times10^{-15} & 0.0474\times10^{-15} \end{bmatrix}$$

$$(42)\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 113.924\times10^{-3} & 263.00\times10^{-3} \\ 0 & 1 & 0 & 0 & 0 & -73.229\times10^{-6} & -176.12\times10^{-6} \\ 0 & 0 & 1 & 0 & 0 & 12.817\times10^{-9} & 33.556\times10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -0.7345\times10^{-12} & -2.1932\times10^{-12} \\ 0 & 0 & 0 & 0 & 1 & 0.0133\times10^{-15} & 0.0474\times10^{-15} \end{bmatrix}$$

6J5 First Approximation for

E_{bb} = 200 volts with R_L = 10000 and 40000 ohms

$$(43)\begin{bmatrix} 50 & 125\times10^{3} & 312.5\times10^{6} & 0.78125\times10^{12} & 0.001953\times10^{18} & 0 & 0 \\ 88 & 681.472\times10^{3} & 5277.3\times10^{6} & 40.8678\times10^{12} & 0.31648\times10^{18} & 0 & 0 \\ 100 & 1000\times10^{3} & 10000\times10^{6} & 100\times10^{12} & 1.0\times10^{18} & 10.8 & 3.8 \\ 110 & 1331\times10^{3} & 16105.1\times10^{6} & 194.8717\times10^{12} & 2.35795\times10^{18} & 18.6 & 4.95 \\ 130 & 2197\times10^{3} & 37129.3\times10^{6} & 627.48517\times10^{12} & 10.60450\times10^{18} & 19.1 & 4.95 \end{bmatrix}$$

$$(44)\quad \begin{bmatrix} 1 & 2.5\times10^{3} & 6.25\times10^{6} & 0.015625\times10^{12} & 0.00003906\times10^{18} & 0 & 0 \\ 0 & 461.472\times10^{3} & 4727.3\times10^{6} & 39.4928\times10^{12} & 0.31304272\times10^{18} & 0 & 0 \\ 0 & 750\times10^{3} & 9375\times10^{6} & 98.4375\times10^{12} & 0.996094\times10^{18} & 10.8 & 3.8 \\ 0 & 1056\times10^{3} & 15417.6\times10^{6} & 193.1530\times10^{12} & 2.35365\times10^{18} & 18.6 & 4.95 \\ 0 & 1872\times10^{3} & 36316.8\times10^{6} & 625.45392\times10^{12} & 10.59422\times10^{18} & 19.1 & 4.95 \end{bmatrix}$$

$$(45)\quad \begin{bmatrix} 1 & 2.5\times10^{3} & 6.25\times10^{6} & 15.625\times10^{9} & 39.06\times10^{12} & 0 & 0 \\ 0 & 1 & 10.244\times10^{3} & 0.8558\times10^{9} & 0.0006784\times10^{15} & 0 & 0 \\ 0 & 0 & 1.692\times10^{6} & 34.2525\times10^{9} & 0.487294\times10^{15} & 10.8\times10^{-3} & 3.8\times10^{-3} \\ 0 & 0 & 4.5999\times10^{6} & 102.7895\times10^{9} & 1.637260\times10^{15} & 18.6\times10^{-3} & 4.95\times10^{-3} \\ 0 & 0 & 17.140\times10^{6} & 465.2481\times10^{9} & 9.324255\times10^{15} & 19.1\times10^{-3} & 4.95\times10^{-3} \end{bmatrix}$$

$$(46)\quad \begin{bmatrix} 1 & 2.5\times10^{3} & 6.25\times10^{6} & 15.625\times10^{9} & 39.06\times10^{12} & 0 & 0 \\ 0 & 1 & 10.244\times10^{3} & 85.58\times10^{6} & 678.4\times10^{9} & 0 & 0 \\ 0 & 0 & 1 & 20.244\times10^{3} & 0.2880\times10^{9} & 6.383\times10^{-9} & 2.246\times10^{-9} \\ 0 & 0 & 0 & 9.66\times10^{3} & 0.31249\times10^{9} & -10.761\times10^{-9} & -5.381\times10^{-9} \\ 0 & 0 & 0 & 118.266\times10^{3} & 4.3879\times10^{9} & -90.305\times10^{-9} & -33.546\times10^{-9} \end{bmatrix}$$

$$(47)\quad \begin{bmatrix} 1 & 2.5\times10^{3} & 6.25\times10^{6} & 15.625\times10^{9} & 39.06\times10^{12} & 0 & 0 \\ 0 & 1 & 10.244\times10^{3} & 85.58\times10^{6} & 678.4\times10^{9} & 0 & 0 \\ 0 & 0 & 1 & 20.244\times10^{3} & 288\times10^{6} & 6.383\times10^{-9} & 2.246\times10^{-9} \\ 0 & 0 & 0 & 1 & 0.03235\times10^{6} & -1.114\times10^{-12} & -0.5570\times10^{-12} \\ 0 & 0 & 0 & 0 & 0.5620\times10^{6} & 41.443\times10^{-12} & 32.328\times10^{-12} \end{bmatrix}$$

$$(48)\quad \begin{bmatrix} 1 & 2.5\times10^{3} & 6.25\times10^{6} & 15.625\times10^{9} & 0 & -2.88\times10^{-3} & 2.246\times10^{-3} \\ 0 & 1 & 10.244\times10^{3} & 85.58\times10^{6} & 0 & -50.00\times10^{-6} & -39.008\times10^{-6} \\ 0 & 0 & 1 & 20.244\times10^{3} & 0 & -14.843\times10^{-9} & -14.314\times10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -3.498\times10^{-12} & -2.417\times10^{-12} \\ 0 & 0 & 0 & 0 & 1 & 0.0737\times10^{-15} & 0.0575\times10^{-15} \end{bmatrix}$$

$$(49)\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 511.96\times10^{-3} & 286.088\times10^{-3} \\ 0 & 1 & 0 & 0 & 0 & -324.00\times10^{-6} & -186.767\times10^{-6} \\ 0 & 0 & 1 & 0 & 0 & 55.970\times10^{-9} & 34.616\times10^{-9} \\ 0 & 0 & 0 & 1 & 0 & -3.498\times10^{-12} & -2.417\times10^{-12} \\ 0 & 0 & 0 & 0 & 1 & 0.0737\times10^{-15} & 0.0575\times10^{-15} \end{bmatrix}$$

6J5 Second Approximation for

E_{bb} = 200 volts with R_L = 10000 and 40000 ohms

$$(50)\begin{bmatrix} & 0 & 0 \\ & 0 & 0 \\ (29) & 8.7 & 2.8 \\ & 18.6 & 4.95 \\ & 19.1 & 4.95 \end{bmatrix} \qquad (51)\begin{bmatrix} & 0 & 0 \\ & 0 & 0 \\ (30) & 8.7 & 2.8 \\ & 18.6 & 4.95 \\ & 19.1 & 4.95 \end{bmatrix}$$

$$(52)\begin{bmatrix} & 0 & 0 \\ & 0 & 0 \\ (32) & 8.7\times10^{-3} & 2.8\times10^{-3} \\ & 18.6\times10^{-3} & 4.95\times10^{-3} \\ & 19.1\times10^{-3} & 4.95\times10^{-3} \end{bmatrix} \qquad (53)\begin{bmatrix} & 0 & 0 \\ & 0 & 0 \\ (33) & 3.900\times10^{-9} & 1.255\times10^{-9} \\ & -4.875\times10^{-9} & -2.604\times10^{-9} \\ & -57.558\times10^{-9} & -19.718\times10^{-9} \end{bmatrix}$$

$$(54)\begin{bmatrix} & 0 & 0 \\ & 0 & 0 \\ (34) & 3.900\times10^{-9} & 1.255\times10^{-9} \\ & -0.3246\times10^{-9} & -0.1734\times10^{-12} \\ & -11.0137\times10^{-12} & 5.146\times10^{-12} \end{bmatrix} (55)\begin{bmatrix} & 0.625\times10^{-3} & -0.2918\times10^{-3} \\ & 6.716\times10^{-6} & -3.1374\times10^{-6} \\ (35) & 7.753\times10^{-9} & -0.5451\times10^{-9} \\ & 0.16469\times10^{-12} & -0.4020\times10^{-12} \\ & -0.01599\times10^{-15} & 0.00747\times10^{-15} \end{bmatrix}$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 82.5735\times10^{-3} & 60.6088\times10^{-3} \\
0 & 1 & 0 & 0 & 0 & -45.5722\times10^{-6} & -39.0827\times10^{-6} \\
0 & 0 & 1 & 0 & 0 & 4.7054\times10^{-9} & 6.8939\times10^{-9} \\
0 & 0 & 0 & 1 & 0 & 0.16469\times10^{-12} & -0.4020\times10^{-12} \\
0 & 0 & 0 & 0 & 1 & -0.01599\times10^{-15} & 0.00747\times10^{-15}
\end{bmatrix} \tag{56}
$$

6SG7 Five Term Approximation for

E_{bb} = 200 volts with R_L = 40000 ohms

$$
\begin{bmatrix}
14 & 27.44\times10^{2} & 53.7824\times10^{4} & 1.054\times10^{8} & 0.020661\times10^{12} & 0 \\
21 & 92.61\times10^{2} & 408.4101\times10^{4} & 18.0109\times10^{8} & 0.79428\times10^{12} & 0 \\
25 & 156.25\times10^{2} & 976.5625\times10^{4} & 61.0351\times10^{8} & 3.81470\times10^{12} & 1.0 \\
28 & 219.52\times10^{2} & 1721.0\times10^{4} & 134.4929\times10^{8} & 10.57846\times10^{12} & 4.0 \\
30 & 270.00\times10^{2} & 2430.0\times10^{4} & 218.700\times10^{8} & 19.6830\times10^{12} & 4.75
\end{bmatrix} \tag{57}
$$

$$
\begin{bmatrix}
1 & 1.96\times10^{2} & 3.8416\times10^{4} & 0.07530\times10^{8} & 0.001476\times10^{12} & 0 \\
0 & 51.45\times10^{2} & 327.7365\times10^{4} & 16.430\times10^{8} & 0.76328\times10^{12} & 0 \\
0 & 107.25\times10^{2} & 880.522\times10^{4} & 59.153\times10^{8} & 3.7778\times10^{12} & 1.0 \\
0 & 164.64\times10^{2} & 1613.435\times10^{4} & 132.3845\times10^{8} & 10.5371\times10^{12} & 4.0 \\
0 & 211.20\times10^{2} & 2314.752\times10^{4} & 216.441\times10^{8} & 19.6387\times10^{12} & 4.75
\end{bmatrix} \tag{58}
$$

$$
\begin{bmatrix}
1 & 1.96\times10^{2} & 3.8416\times10^{4} & 7.530\times10^{6} & 14.76\times10^{8} & 0 \\
0 & 1 & 6.3700\times10^{2} & 0.31934\times10^{6} & 0.014835\times10^{10} & 0 \\
0 & 0 & 1.9734\times10^{4} & 24.9038\times10^{6} & 2.186746\times10^{10} & 1.0\times10^{-2} \\
0 & 0 & 5.64678\times10^{4} & 79.8084\times10^{6} & 8.094666\times10^{10} & 4.0\times10^{-2} \\
0 & 0 & 9.69408\times10^{4} & 148.9964\times10^{6} & 16.505548\times10^{10} & 4.75\times10^{-2}
\end{bmatrix} \tag{59}
$$

$$(60)\quad \begin{bmatrix} 1 & 1.96\times10^{2} & 3.8416\times10^{4} & 7.530\times10^{6} & 14.76\times10^{8} & 0 \\ 0 & 1 & 6.37\times10^{2} & 31.934\times10^{4} & 148.35\times10^{6} & 0 \\ 0 & 0 & 1 & 12.6197\times10^{2} & 1.10847\times10^{6} & 0.5067\times10^{-6} \\ 0 & 0 & 0 & 8.5477\times10^{2} & 1.83538\times10^{6} & 1.1388\times10^{-6} \\ 0 & 0 & 0 & 26.6600\times10^{2} & 5.75995\times10^{6} & -0.162\times10^{-6} \end{bmatrix}$$

$$(61)\quad \begin{bmatrix} 1 & 1.96\times10^{2} & 3.8416\times10^{4} & 7.530\times10^{6} & 14.76\times10^{8} & 0 \\ 0 & 1 & 6.3700\times10^{4} & 31.934\times10^{4} & 148.35\times10^{6} & 0 \\ 0 & 0 & 1 & 12.6197\times10^{2} & 110.847\times10^{4} & 0.5067\times10^{-6} \\ 0 & 0 & 0 & 1 & 21.472\times10^{2} & 0.1332\times10^{-8} \\ 0 & 0 & 0 & 0 & 0.03551\times10^{4} & -3.713\times10^{-8} \end{bmatrix}$$

$$(62)\quad \begin{bmatrix} 1 & 1.96\times10^{2} & 3.8416\times10^{4} & 7.530\times10^{6} & 0 & 15.4331\times10^{-2} \\ 0 & 1 & 6.3700\times10^{4} & 31.934\times10^{4} & 0 & 155.1148\times10^{-4} \\ 0 & 0 & 1 & 12.6197\times10^{2} & 0 & 116.4083\times10^{-6} \\ 0 & 0 & 0 & 1 & 0 & 22.5843\times10^{-8} \\ 0 & 0 & 0 & 0 & 1 & -1.0456\times10^{-10} \end{bmatrix}$$

$$(63)\quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -502.3856\times10^{-2} \\ 0 & 1 & 0 & 0 & 0 & 507.8815\times10^{-4} \\ 0 & 0 & 1 & 0 & 0 & -168.5987\times10^{-6} \\ 0 & 0 & 0 & 1 & 0 & 22.5843\times10^{-8} \\ 0 & 0 & 0 & 0 & 1 & -1.0456\times10^{-10} \end{bmatrix}$$

6SG7 Four Term Approximation for

E_{bb} = 200 volts with R_L = 40000 ohms

The four term solution is identical with the five term solution from (57) to (60) with the fifth row and the fifth column omitted.

(64) is the matrix after (60).

$$(64)\begin{bmatrix} 1 & 1.96\times10^{2} & 3.8416\times10^{4} & 0 & -1.003\times10^{-2} \\ 0 & 1 & 6.37\times10^{2} & 0 & -4.2536\times10^{-4} \\ 0 & 0 & 1 & 0 & -1.1742\times10^{-6} \\ 0 & 0 & 0 & 1 & 0.1332\times10^{-8} \end{bmatrix}$$

$$(65)\begin{bmatrix} 1 & 0 & 0 & 0 & -2.8154\times10^{-2} \\ 0 & 1 & 0 & 0 & 3.2261\times10^{-4} \\ 0 & 0 & 1 & 0 & -1.1742\times10^{-6} \\ 0 & 0 & 0 & 1 & 0.1332\times10^{-8} \end{bmatrix}$$

6SH7 First Approximation for

E_{bb} = 200 volts with R_L = 40000 ohms

$$(66)\begin{bmatrix} 18 & 58.32\times10^{2} & 188.9568\times10^{4} & 6.1222\times10^{8} & 0 \\ 26 & 175.56\times10^{2} & 1188.1376\times10^{4} & 80.3181\times10^{8} & 0 \\ 28 & 219.52\times10^{2} & 1721.0\times10^{4} & 134.4929\times10^{8} & 2.05 \\ 29 & 243.89\times10^{2} & 2051.1149\times10^{4} & 172.4988\times10^{8} & 4.5 \end{bmatrix}$$

$$(67)\begin{bmatrix} 1 & 3.24\times10^{2} & 10.4976\times10^{4} & 0.3401\times10^{8} & 0 \\ 0 & 91.32\times10^{2} & 915.200\times10^{4} & 71.4755\times10^{8} & 0 \\ 0 & 128.80\times10^{2} & 1427.0672\times10^{4} & 124.9701\times10^{8} & 2.05 \\ 0 & 149.93\times10^{2} & 1746.6845\times10^{4} & 162.6359\times10^{8} & 4.5 \end{bmatrix}$$

$$(68)\begin{bmatrix} 1 & 3.24\times10^{2} & 10.4976\times10^{4} & 34.01\times10^{6} & 0 \\ 0 & 1 & 10.0219\times10^{2} & 0.7827\times10^{6} & 0 \\ 0 & 0 & 1.362465\times10^{4} & 24.1583\times10^{6} & 2.05\times10^{-2} \\ 0 & 0 & 2.441010\times10^{4} & 45.2857\times10^{6} & 4.5\times10^{-2} \end{bmatrix}$$

$$(69)\quad \begin{bmatrix} 1 & 3.24\text{x}10^{2} & 10.4976\text{x}10^{4} & 34.01\text{x}10^{6} & 0 \\ 0 & 1 & 10.0219\text{x}10^{2} & 78.27\text{x}10^{4} & 0 \\ 0 & 0 & 1 & 17.73\text{x}10^{2} & 1.505\text{x}10^{-6} \\ 0 & 0 & 0 & 2.01\text{x}10^{2} & 0.826\text{x}10^{-6} \end{bmatrix}$$

$$(70)\quad \begin{bmatrix} 1 & 3.24\text{x}10^{2} & 10.4976\text{x}10^{4} & 0 & -13.978\text{x}10^{-2} \\ 0 & 1 & 10.0219\text{x}10^{2} & 0 & -32.169\text{x}10^{-4} \\ 0 & 0 & 1 & 0 & -5.782\text{x}10^{-6} \\ 0 & 0 & 0 & 1 & 0.411\text{x}10^{-8} \end{bmatrix}$$

$$(71)\quad \begin{bmatrix} 1 & 0 & 0 & 0 & -36.802\text{x}10^{-2} \\ 0 & 1 & 0 & 0 & 25.778\text{x}10^{-4} \\ 0 & 0 & 1 & 0 & -5.782\text{x}10^{-6} \\ 0 & 0 & 0 & 1 & 0.411\text{x}10^{-8} \end{bmatrix}$$

APPENDIX D

CALCULATIONS FOR THE COMPARISON OF THE COEFFICIENTS

APPENDIX D

Calculations for the Comparison of the Coefficients

Method Using Odd Powers Only

$$(1)\quad \begin{bmatrix} 2\times10^{2} & 8\times10^{6} & 32\times10^{10} & 0.114 & -0.07322 & 0.01282 & -0.7345 & 13.3 \\ 3\times10^{2} & 27\times10^{6} & 243\times10^{10} & 0.259 & -0.17111 & 0.03207 & -2.1197 & 46.8 \\ 4\times10^{2} & 64\times10^{6} & 1024\times10^{10} & 0.263 & -0.17612 & 0.03356 & -2.1932 & 47.4 \end{bmatrix}$$

$$(2)\quad \begin{bmatrix} 1 & 4\times10^{4} & 16\times10^{8} & 0.057\times10^{-2} & -0.03661\times10^{-2} & 0.00641\times10^{-2} & -0.36725\times10^{-2} & 6.65\times10^{-2} \\ 0 & 15\times10^{4} & 195\times10^{8} & 0.088\times10^{-2} & -0.06127\times10^{-2} & 0.01282\times10^{-2} & -1.01795\times10^{-2} & 26.85\times10^{-2} \\ 0 & 48\times10^{4} & 960\times10^{8} & 0.035\times10^{-2} & -0.02968\times10^{-2} & 0.00792\times10^{-2} & -0.7242\times10^{-2} & 20.80\times10^{-2} \end{bmatrix}$$

$$(3)\quad \begin{bmatrix} 1 & 4\times10^{4} & 16\times10^{8} & 0.057\times10^{-2} & -0.03661\times10^{-2} & 0.00641\times10^{-2} & -0.36725\times10^{-2} & 6.65\times10^{-2} \\ 0 & 1 & 13\times10^{4} & 0.00587\times10^{-6} & -0.004085\times10^{-6} & 0.000855\times10^{-6} & -0.06786\times10^{-6} & 1.79\times10^{-6} \\ 0 & 0 & 336\times10^{4} & -0.2468\times10^{-6} & +0.1664\times10^{-6} & -0.03312\times10^{-6} & +2.5331\times10^{-6} & -65.12\times10^{-6} \end{bmatrix}$$

$$(4)\quad \begin{bmatrix} 1 & 4\times10^{4} & 0 & 0.06876\times10^{-2} & -0.04453\times10^{-2} & 0.007994\times10^{-2} & -0.487874\times10^{-2} & 9.751\times10^{-2} \\ 0 & 1 & 0 & 0.015425\times10^{-6} & -0.01052\times10^{-6} & 0.002142\times10^{-6} & -0.165867\times10^{-6} & 4.309\times10^{-6} \\ 0 & 0 & 1 & -0.000735\times10^{-10} & +0.000495\times10^{-10} & -0.000099\times10^{-10} & +0.007539\times10^{-10} & -0.1938\times10^{-10} \end{bmatrix}$$

$$(5)\quad \begin{bmatrix} 1 & 0 & 0 & 0.00706\text{x}10^{-2} & -0.00245\text{x}10^{-2} & -0.000574\text{x}10^{-2} & +0.175594\text{x}10^{-2} & -7.485\text{x}10^{-2} \\ 0 & 1 & 0 & 0.015425\text{x}10^{-6} & -0.01052\text{x}10^{-6} & 0.002142\text{x}10^{-6} & -0.165867\text{x}10^{-6} & 4.309\text{x}10^{-6} \\ 0 & 0 & 1 & -0.000735\text{x}10^{-10} & +0.000495\text{x}10^{-10} & -0.000099\text{x}10^{-10} & +0.007539\text{x}10^{-10} & -0.1938\text{x}10^{-10} \end{bmatrix}$$

Method Using Odd and Even Powers

$$(6)\quad \begin{bmatrix} 2\text{x}10^{2} & 4\text{x}10^{4} & 8\text{x}10^{6} & 0.1139 & -0.07322 & 0.01282 & -0.7345 & 13.3 \\ 3\text{x}10^{2} & 9\text{x}10^{4} & 27\text{x}10^{6} & 0.2586 & -0.17111 & 0.03207 & -2.1197 & 46.8 \\ 4\text{x}10^{2} & 16\text{x}10^{4} & 64\text{x}10^{6} & 0.2630 & -0.17612 & 0.03356 & -2.1932 & 47.4 \end{bmatrix}$$

$$(7)\quad \begin{bmatrix} 1 & 2\text{x}10^{2} & 4\text{x}10^{4} & 0.05695\text{x}10^{-2} & -0.03661\text{x}10^{-2} & 0.00641\text{x}10^{-2} & -0.36725\text{x}10^{-2} & 6.65\text{x}10^{-2} \\ 0 & 3\text{x}10^{2} & 15\text{x}10^{4} & 0.08775\text{x}10^{-2} & -0.06128\text{x}10^{-2} & 0.01284\text{x}10^{-2} & -1.01795\text{x}10^{-2} & 26.85\text{x}10^{-2} \\ 0 & 8\text{x}10^{2} & 48\text{x}10^{4} & 0.0352\text{x}10^{-2} & -0.02968\text{x}10^{-2} & 0.00792\text{x}10^{-2} & -0.72420\text{x}10^{-2} & 20.80\text{x}10^{-2} \end{bmatrix}$$

$$(8)\quad \begin{bmatrix} 1 & 2\text{x}10^{2} & 4\text{x}10^{4} & 0.05695\text{x}10^{-2} & -0.03661\text{x}10^{-2} & 0.00641\text{x}10^{-2} & -0.36725\text{x}10^{-2} & 6.65\text{x}10^{-2} \\ 0 & 1 & 5\text{x}10^{2} & 0.02925\text{x}10^{-4} & -0.02043\text{x}10^{-4} & 0.00428\text{x}10^{-4} & -0.33932\text{x}10^{-4} & 8.95\text{x}10^{-4} \\ 0 & 0 & 8\text{x}10^{2} & -0.1988\text{x}10^{-4} & 0.13376\text{x}10^{-4} & -0.02632\text{x}10^{-4} & 1.9904\text{x}10^{-4} & -50.8\text{x}10^{-4} \end{bmatrix}$$

$$(9)\quad \begin{bmatrix} 1 & 2\text{x}10^{2} & 0 & 0.15635\text{x}10^{-2} & -0.10349\text{x}10^{-2} & 0.01957\text{x}10^{-2} & -1.3624\text{x}10^{-2} & 32.05\text{x}10^{-2} \\ 0 & 1 & 0 & 0.15350\text{x}10^{-4} & -0.10403\text{x}10^{-4} & 0.02073\text{x}10^{-4} & -1.5833\text{x}10^{-4} & 40.70\text{x}10^{-4} \\ 0 & 0 & 1 & -0.02485\text{x}10^{-6} & 0.00329\text{x}10^{-6} & -0.00329\text{x}10^{-6} & 0.2488\text{x}10^{-6} & -6.35\text{x}10^{-6} \end{bmatrix}$$

$$(10)\begin{bmatrix} 1 & 0 & 0 & -0.15065\times10^{-2} & 0.10457\times10^{-2} & -0.02189\times10^{-2} & 1.8042\times10^{-2} & -49.35\times10^{-2} \\ 0 & 1 & 0 & 0.15350\times10^{-4} & -0.10403\times10^{-4} & 0.02073\times10^{-4} & -1.5833\times10^{-4} & 40.70\times10^{-4} \\ 0 & 0 & 1 & -0.02485\times10^{-6} & +0.01672\times10^{-6} & -0.00329\times10^{-6} & +0.2488\times10^{-6} & -6.35\times10^{-6} \end{bmatrix}$$

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   6V6 Sept. 1951

   6J5 June 1952

   6SG7 Oct. 1952

   6SH7 May 1946

ERRATA

Page 1 - Line 1 several instead of serveral

Page 7 - Line 2 subtract instead of subtact

Page 9 - Line 3 tetrode instead of tretode

Page 9 - Figure 2 tetrode instead of tretode

Page 9 - Footnote ** tetrode instead of tretode

Page 10 - Line 3 entirety instead of entirity

Page 13 - Line 25 actually instead of acutally

Page 36 - Line 7 variable instead of varible

Page 40 - Lines 11-12 variable instead of varible

Page 42 - Line 12 equation instead of quation

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