THE APPLICATION OF LINEAR GRAPH THEORY TO URBAN WORK TRIP DISTRIBUTION

Thesis for the Degree of M. S.

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Earl Duane Cubitt

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This is to certify that the

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ABSTRACT

THE APPLICATION OF LINEAR GRAPH THEORY

TO URBAN WORK TRIP DISTRIBUTION

By Earl Duane Cubitt

A linear graph model of trip distribution was developed and applied in an effort to determine the applicability of linear graph theory to urban traffic forecasting.

The model is developed by application of theoretical considerations to a hypothetical three-zone city. It is applied, along with the gravity model, to Flint, Michigan, O and D data to test its effectiveness in distributing work trips. The linear graph model was also applied to a sampling of Philadelphia work trips to see how effective it was on volume data shown to be somewhat dependent on a travel distance measure.

Tests were made on the Flint work trip data which indicated that the volume of traffic between zones depended very little on the travel distance between zones.

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The results indicate that the model warrants further investigation. In all cases the linear graph model reproduced the given distribution better than did the gravity model. Relative error values obtained for the most significant data analyzed were approximately 30 percent.

THE APPLICATION OF LINEAR GRAPH THEORY

TO URBAN WORK TRIP DISTRIBUTION

By

Earl Duane Cubitt

A THESIS

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PREFACE

This study was originally intended to be an extension of Dr. William Grecco's Ph.D. Thesis, "The Application of Systems Engineering Techniques to Urban Traffic Forecasting." The model developed in his thesis and applied to a hypothetical city was to be applied to Origin and Destination data from Flint, Michigan. Analysis of the problem indicated that direct application of Dr. Grecco's model was impractical due to the large number of simultaneous equations which would have to be solved. A simpler model, utilizing Dr. Grecco's basic concepts, was then developed and applied to the Flint O and D data.

I would like to thank Dr. S. M. Breuning for his able counsel and guidance in the early stages of this investigation; Mr. Leo Farman, Chief of the Origin and Destination Study Division of the Michigan State Highway Department, for providing internal work trip data not originally tabulated in the 1950 Flint O and D Study; and Mr. Karl Dieter, Mathematician for the Traffic Research Corporation, for discussing with me his published research on the Propensity Function.

I would also like to thank the Automotive Safety Foundation for sponsoring this research project. Research on this project was also accomplished while I was supported by a summer fellowship from the National Science Foundation. This assistance is also acknowledged.

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CHAPTER I

INTRODUCTION

The purpose of this thesis was to develop a linear graph model of trip distribution and to apply it to Flint O and D data. The model developed was applied to internal work trips taken from a 1950 Flint O and D Study conducted by the Michigan State Highway Department.

Many models for forecasting traffic have been developed and applied with the hope that conventional O and D studies may some day be unnecessary. The large quantity of data which must be analyzed when synthesizing and projecting urban traffic makes planning studies and related research work both difficult and expensive. Even with the advent of high speed computers, progress in the field of traffic forecasting is relatively slow. At least part of this can be attributed to the complexity of the traffic flow system. This "soft system" is very difficult, if not impossible, to accurately synthesize. Nevertheless, many researchers, some working on large urban studies and others working independently, are continually improving techniques and suggesting new models to provide a more accurate synthesis of present flow systems. This, in turn, results in more accurate forecasts. It is hoped that the linear graph model of trip distribution developed and applied in this thesis will also assist in more accurately synthesizing traffic flow systems.

The following chapters include a brief review of trip distribution models, the development of the linear graph model, and the application of the model to Flint data. A gravity model is also applied to the Flint data for comparison purposes. The results of these model applications are presented and analyzed and further tests of the linear graph model and of the Flint data are then made. Conclusions are then presented.

CHAPTER II

REVIEW OF LITERATURE

Much has been written during the past decade about procedures and models for forecasting traffic. In general, a traffic forecasting model usually separates traffic forecasting into three study areas: trip generation, trip distribution, and traffic assignment. All three areas are important to a general forecasting model. This thesis and the subsequent literature review was mainly concerned with trip distribution.

Many theoretical trip distribution models have been proposed. A list of most of these models and references are as follows: Gravity Model (7, 25, 46, 48, 49, 56, 60, 61, 62, 65); Friction Factor Method (14, 19); BPR Model (41); Electrostatic Model (33, 34); Interactance Model (64); Opportunity Model (2, 16, 48, 49); Linear Programming Method (3, 12, 17, 38); Multiple Regression Method (47); Calculus Model (32); Probability Model (57); and the Linear Graph Model (13, 26).

The gravity model has been the most widely accepted procedure for trip distribution. The model is based on

Newton's Law of Gravity, hence the name gravity model.

One form of the gravity model equation (48) is

$$T_{ij} = T_i \frac{M_j / D_{ij}^{x}}{\sum_{j} (M_j / (D_{ij})^{x})}$$

where

T_{ij} = trips from zone i to j

T_i = trips from zone i

M_j = measure of attraction of zone j for certain types of trips

D_{ij} = travel time from zone i to j

x = empirically determined exponent.

Dieter (25) used a different form of the gravity model in distributing work trips in Toronto. The gravity model was stated as

$$T_{ij} = G_i A_j F(r_{ij})$$

where

T_{ij} = flow from zone i to j

G_i = generating intensity of zone i

A_j = absorbing intensity (attraction) of zone j

r_{ij} = description of travel time from zone i to j

$F(r_{ij})$ = Propensity Function, an empirical function that describes the travel time influence on the level of flow from zone i to j.

Using data taken from a 1954 Toronto study, trip propensity function values were calculated and plotted against travel distance. Various analytic expressions were fitted to this plot. Dieter found that an exponential curve of the form

$$F(r_{ij}) = \alpha e^{-\beta r_{ij}}$$

was the function best respresenting the data (Figure 1). The final form of the gravity model was

$$T_{ij} = G_i A_j \alpha e^{-\beta r_{ij}} \qquad (1)$$

Dieter originally assumed that

$$G_i = \sum_j T_{ij} \qquad (2)$$

and

$$A_j = \sum_i T_{ij} \qquad (3)$$

Since the propensity equation only approximates the actual propensity values, the calculated T_{ij} values found by equation 1 will only approximate the actual T_{ij} values. Equations 2 and 3 must still hold true for a rigorous solution, which results in an iteration procedure to adjust the T_{ij} values to satisfy these conditions. Dieter describes a new and different iteration procedure for accomplishing this adjustment.

Dr. William Grecco (13, 26) was the first to apply linear graph theory to urban traffic forecasting, more specifically to urban work trip distribution. He also conducted studies on trip generation as well. From this attempt

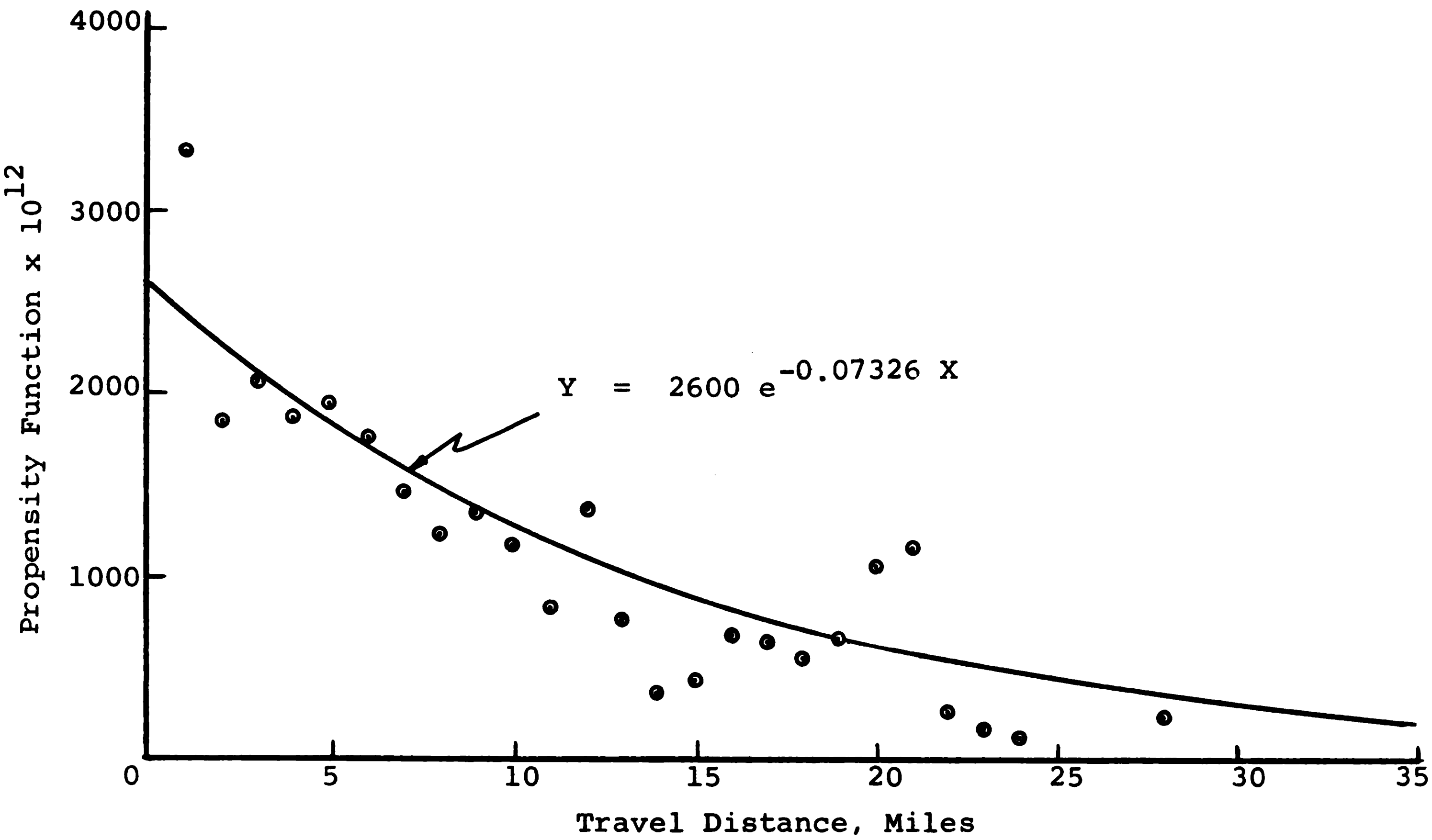

Figure 1. Empirical Propensity Values and Fitted Equation, Gravity Model, Toronto Work Trips.

(Ref. 25)

evolved a linear graph model of traffic forecasting. No attempt was made by Grecco to apply linear graph theory to traffic assignment.

Grecco's original investigations were applied to a hypothetical seven-zone city in distributing home-based work trips. For comparison purposes, the hypothetical system was solved by the electrostatic and gravity models. The linear graph model reproduced the hypothetical flow pattern better than the other two models and no iteration was necessary, since the boundary conditions present in the flow system were automatically taken care of in the model's solution.

It was discovered early in this investigation that solution by Grecco's model, which was a direct solution of the whole distribution system simultaneously via linear graph theory, was nearly impossible when dealing with cities with large numbers of O and D zones. The direct system solution for the city of Flint would have required the solution of approximately 9,000 simultaneous equations. It was necessary, then, to develop a simpler model based on Dr. Grecco's principles.

CHAPTER III

DEVELOPMENT AND APPLICATION OF MODEL

Linear Graph Theory

Original work by Grecco in applying linear graph theory to urban traffic forecasting considered only work trips from residential zones to employment zones. A general model for the distribution of work trips* between zones should also consider work trips to residential areas. When applying the current linear graph model, zones which both generated and attracted work trips were considered as performing both functions.

Before constructing the linear graph of an urban trip distribution system, the components of the system must be defined. Since each residential zone will normally both generate and attract work trips, each must be represented by two components. Non-residential zones not generating from-home work trips need be represented only by an attraction component. The route connects the generation and attraction

*Only the from-home to-work trip is considered as a work trip in this thesis.

components, so that each route must also be represented by a component. All components in this system are assumed to possess only two terminals. The terminal graphs of a generation, an attraction, and a route component properly interconnected represent the traffic flow from a generation zone to an attraction zone via a route. Proper interconnection of the terminal graphs for all components in the system generates the linear graph of the system.

From the above brief discussion of linear graph construction for a trip distribution system, it is quite evident that a linear graph of a large system containing many O and D zones would be very complex and difficult to construct simply from a mechanical point of view. The principles of linear graph theory as applied to the current model do not require an actual graph of the system, since the cutset and circuit matrices (Appendix F) need not be written. It is only necessary to understand the basic principles of linear graph theory as applied to the model, so that conceptually a linear graph of the distribution system under consideration can be generated to theoretically justify the model's application.

The following example is intended to clarify the conceptual framework of the linear graph model. More specifically, it is intended to clarify the component representation in the linear graph, to illustrate the proper interconnection and orientation of all elements in the linear graph of the given system, and to point out two basic requirements of the linear graph and how they are fulfilled.

Suppose a city has three zones, two residential and one industrial (Figure 2). Assume that the industrial zone does not generate any from-home work trips and that all other possible interzonal and intrazonal trip movements are significant. Figure 3 represents the linear graph of the flow system generated by these zones and their interzonal and intrazonal trip movements.

From the linear graph, it is seen that residential zone 1 has 2 two-terminal components, G_1 and A_1, representing it; residential zone 2 has 2 two-terminal components, A_2 and G_2, representing it; and industrial zone 3 has only 1 two-terminal component, A_3, representing it, since zone 3 does not generate any from-home work trips.

The route components are two-terminal components connecting G_1 and G_2 with A_1, A_2, and A_3. Route 11 (M_{11}) and route 22 (M_{22}) represent intrazonal trips while all other route components represent interzonal trips.

Each two-terminal component by linear graph theory has two measurements associated with it: an across variable X and a through variable Y. The Y variable, analogous to the electrical term current, is assumed to be traffic flow in traffic flow problems. The across variable X, analogous to the electrical term voltage, has not as yet been defined in current traffic terminology and is thought to be a pressure differential measurement which in some manner causes traffic flow. A relationship between X and Y must also exist and, as in electrical phenomena, it is assumed to be of the form $X = R\,Y$, where R is a measure of the resistance to flow.

From linear graph theory, all elements whose X values are specified must be made branches of the tree of the linear graph. Since the X values of the generation and attraction components are specified in the current model, the elements representing these components must be made branches of the tree (Figure 3). This results in all branches of the tree of the linear graph having specified X values. Each circuit by definition has only one chord. The rest of the elements in each circuit, in this example and in the linear graph model, consist of branches with specified X values. Since each circuit must satisfy the condition $\sum X = 0$, the X value for the chord in each circuit, which is actually the X value of the route component, can be determined. Orienting the generation elements away from the central node and the attraction elements toward the central node, the route elements can be oriented to make

$$X_m = -(X_g + X_a)$$

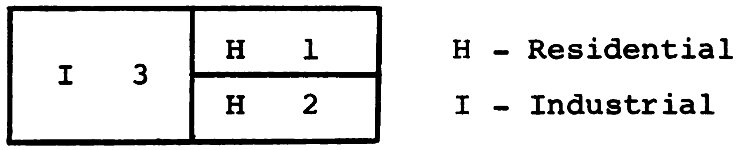

Figure 2. Hypothetical City.

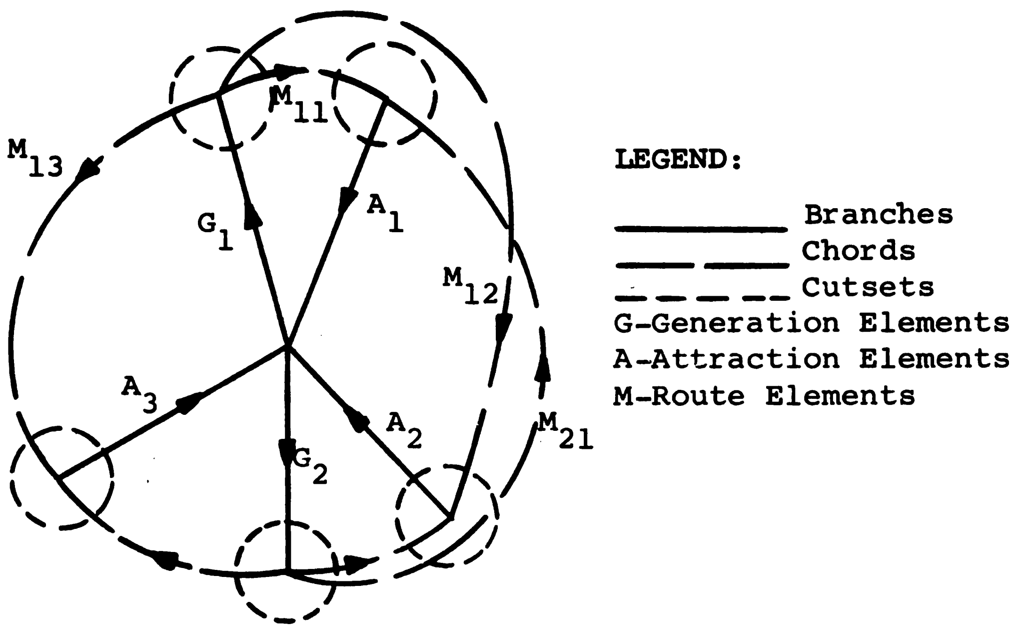

Figure 3. Linear Graph of Hypothetical System.

with the subscripts m, g and a representing the route, generation, and attraction elements respectively. This was the orientation used in Figure 3.

Another requirement of the linear graph is that $\sum Y = 0$ at the nodes. The nodes used in satisfying this condition are those determined by hypothetically cutting the graph into two parts such that only one branch is included in each cutset. This results in as many cutsets, and nodes used to satisfy this requirement, as there are branches. The cutsets are indicated in Figure 3.

When summing the flow or Y values at these nodes, the branch orientation is generally taken as positive. Using this convention in this example, the route X values in the formal solution are all negative, meaning their orientation is actually reversed. This is apparent from the linear graph. Since the positive orientations of the elements representing the generation and attraction zones were assumed, the orientations of the route components will change to compensate for the rigorous conditions of the system. In essence, the X value for each of the route elements in this example and in the linear graph model will be oriented in a direction opposite to that of its Y value. Appendix F contains the formal solution of this sample problem for unknown Y values in terms of known or assumed X and R values.

Application of the above discussion depends on the basic assumption that the X values in the generation and attraction zones (branches) can be specified, so that the route X values (chords) can be determined by $\sum X = 0$. First approximations for these route X's were assumed to be the number of workers residing in generation zones, and the number of jobs available in attraction zones. The $X = RY$ relationship is then dimensionally correct for both zones when using average vehicle occupancy for the R value. For generation and attraction zones

vehicle = trip

jobs = workers

and $X = RY$

when assuming a complete matching of jobs available with workers availabile.

For generation zones

(workers/zone) = (workers/vehicle) (trips/zone)

which when rewritten as

$X/Y = R$

results in the dimensional identity

workers/trip = workers/vehicle

For attraction zones

(jobs/zone) = (workers/vehicle) (trips/zone) which when rewritten as

$$X/Y = R$$

results in the dimensional identity

jobs/trip = workers/vehicle

The preceding analysis is rather idealized with such factors as modal split, pedestrian trips, and unemployment tending to complicate in actuality the assumptions made.

Once the X values for the generation and attraction components are approximated, the X values for the routes are simply a summation of the generation and attraction zone X values for reasons given previously. Travel distance (travel times not available for Flint data) was then selected as the parameter to be used to predict R_{ij}, the factor necessary to convert the route X values to route Y values. The functional relationship for predicting R_{ij} must approximate the relationship between X_{ij} and Y_{ij} given in the equation $X = RY$. An exponential equation with interzonal distance as the independent variable was used, when applying the current model, as the function to predict R_{ij}^{-1} values. R_{ij}^{-1} is called the propensity function.

<u>Propensity Function</u>

The propensity function is an empirical function that describes in what manner travel time, or travel distance, influences the volume of traffic between zones. It is used in this thesis as a measure of the reciprocal of the route resistance factors, R_{ij}^{-1}, for the linear graph model. The investigations for determining the propensity function are based in part on work reported by Karl Dieter, Mathematician for the Traffic Research Corporation (<u>25</u>).

The propensity function is derived (Appendix C) from the basic gravity model formula which can be written in the form

$$T_{ij} = G_i A_j F(r_{ij}) \qquad (4)$$

where

T_{ij} = interzonal volume from zones i to j

G_i = measure of generation for zone i

A_j = measure of attraction for zone j

r_{ij} = travel distance from zones i to j

$F(r_{ij})$ = propensity function.

Arranging the O and D data to be analyzed into distance groups, this equation can be written for all those zone pairs (i, j) in a distance interval group, $r_{ij} = u \pm a$, where u is the midpoint of the interval and 2a is the interval width. Equation 4 can then be rewritten as

$$T_{ij} \cong G_i\, A_j\, F(u)$$

with F(u) representing all those interzonal movements (i, j) whose interzonal distance falls in the interval $u \pm a$.

Arranging interzonal volume data into n distance interval groups results in a set of equations for each distance interval group. The least square solution for each set of these n sets of equations for F(u) is

$$F(u) = \frac{1}{W(u)} \sum_{r_{ij} = u \pm a} T_{ij} G_i\, A_j \qquad (6)$$

in which the weights W(u) are given by

$$W(u) = \sum_{r_{ij} = u \pm a} (G_i\, A_j)^2 \qquad (7)$$

The propensity function value, F(u), for each distance group can then be calculated from the given values of T_{ij}, G_i, and A_j.

To determine the R_{ij} values for the linear graph route components using the above procedure, equation 6 must be modified. It was previously assumed that the X values for the generation zones were the number of workers residing there, and for the attraction zones were the number of jobs available there. Because information on the number of workers and number of jobs was not available for the Flint O and D zones, an approximation of these values was used when applying the model. The workers residing in a generation zone were approximated by the number of work trips originating in that zone, and the jobs available in a destination zone were approximated by the number of work trips destined to that zone. Thus letting

$$G_i = \sum_j T_{ij} \quad \text{(generation zone X)}$$

and

$$A_j = \sum_i T_{ij} \quad \text{(attraction zone X)}$$

and replacing in equations 6 and 7 the product $(G_i A_j)$ by the sum $(G_i + A_j)$, which according to previous discussion represents the X value for the route component, the propensity function calculated by the new equation represents the reciprocal of the route resistance factor in the equation $X = RY$. This follows from

$$T_{ij} = (G_i + A_j) \; F(r_{ij})$$

$$Y_{ij} = R_{ij}^{-1} \; X_{ij}$$

$$X_{ij} = (G_i + A_j)$$

$$Y_{ij} = T_{ij}$$

or

$$R_{ij}^{-1} = F(r_{ij})$$

It follows that knowing the route X_{ij}, $(G_i + A_j)$, and the propensity function, $F(r_{ij})$, for a given interchange, (i, j), the flow, T_{ij}, can be determined for that interchange from

$$Y = R^{-1} X$$

or

$$T_{ij} = F(r_{ij}) \; (G_i + A_j).$$

The least square solution for F(u) in the linear graph model is then

$$F(u) = \frac{1}{W(u)} \sum_{r_{ij} = u \pm a} T_{ij} \; (G_i + A_j) \qquad (8)$$

where

$$W(u) = \sum_{r_{ij} = u \pm a} (G_i + A_j)^2 \qquad (9)$$

The computer program written (Appendix D) to solve for F(u) calculated a series of propensity values for succeedingly larger distance interval groupings. The basic program either determined propensity values for the linear graph model when equation 8 was inserted, or determined propensity values for the gravity model when equation 6 was inserted. This program flexibility allowed for analysis of the Flint data by both the linear graph and gravity models for purposes of comparison. The distance intervals used, 2a, were 0.1, 0.2, 0.5, and 1.0 miles. The propensity values for the 1.0 mile intervals were plotted versus interzonal distance, F(u) vs. u, and a least squares exponential curve (Appendix E) was then fitted to the plot. The equation so fitted was then used to represent $F(r_{ij})$ for the particular model and data classification used in calculating the

F(u) values. The correlation coefficient was also calculated (Appendix E) as a measure of how well the equation fitted the plotted propensity values.

Relative Error

To be able to compare the predictive value of the linear graph model with that of the gravity model, a relative error calculation was made utilizing the computer (Appendix D). The equation solved for the linear graph model error was

$$P = \sqrt{\frac{\sum_{ij} \left[T_{ij} - (G_i + A_j)\ F(r_{ij})\right]^2}{\sum_{ij} T_{ij}^2}} \qquad (10)$$

where $F(r_{ij})$ is represented by an exponential function of the form

$$F(r_{ij}) = \alpha\, e^{-\beta r_{ij}} \qquad (11)$$

The constants α and β were determined by a regression analysis of the F(u) vs. u plots (Appendix D).

The equation solved for the gravity model was

$$P = \sqrt{\frac{\sum_{ij} \left[T_{ij} - G_i A_j\ F(r_{ij})\right]^2}{\sum_{ij} T_{ij}^2}} \qquad (12)$$

with $F(r_{ij})$ again being represented by an exponential function of the form given in equation 11 with the constants α and β determined as above.

CHAPTER IV

ANALYSIS OF FLINT DATA

The linear graph and gravity models were applied to Flint auto driver and total internal work trip data. This information was extracted from original data derived from a 1950 O and D study conducted by the Michigan State Highway Department. This data contained 1312 total work trip interchanges, excluding pedestrian work trips, and 1067 auto driver work trip interchanges out of a possible 9216 internal trip interchanges for the 96 zone study area.

The data was also reduced for further analysis by using only interzonal movements whose work trip volumes were greater or equal to fifty and greater or equal to 100, for both total and auto driver internal work trips. Because the volume data was collected by a 5 percent sampling of dwelling units, using only the larger volumes gave greater significance to the data. This fact has previously been verified by other researchers (50, 57).

The propensity function equation, the calculated relative error, and the correlation coefficient between the propensity equation and the plotted propensity values are given in Table I for both the linear graph and gravity models.

Plots of the propensity values versus distance, F(u) vs. u, and the equations fitted to the data are given in Figures 4 through 7 for all reported total and auto driver internal work trips. Plots for the reduced Flint data are given in Appendix A.

In those cases where two equations were fitted to the data, it was felt that by dropping either the first, last, or both the first and last plotted points, the correlation coefficient could be improved. This was justified because the first and last points, especially the last, in general, represented fewer interchanges than did other plotted points. This was particularly true when working with the reduced data.

Table I indicates that even though the correlation coefficient improved by using the above procedure, there was not a general corresponding decrease in the relative error values. This resulted from not altering the data when calculating relative error values with either of the fitted equations. All interchanges used in determining each of the plotted points were also used in calculating the relative error values, including those interchanges represented by the first and last points even though in some cases they were not used in determining the propensity equations. If those interchanges had been eliminated that were represented by the plotted points not used in determining the propensity

TABLE I

MODEL RESULTS - FLINT DATA

LGM - Linear Graph Model
GM - Gravity Model

		$F(r_{ij}) \times 10^5$ LGM	$F(r_{ij}) \times 10^8$ GM	Correlation Coefficient, R LGM	Correlation Coefficient, R GM	Relative Error, P LGM	Relative Error, P GM
Flint Auto Driver Work Trips	All	$Y = 1738\ e^{-.0681\ X}$	$Y = 3164\ e^{-.0698\ X}$	-0.883	-0.927	0.636	0.653
			$Y = 2852\ e^{-.0356\ X}$		-0.548		0.655
	V ≥ 50	$Y = 2911\ e^{-.0846\ X}$	$Y = 3748\ e^{-.0468\ X}$	-0.952	-0.741	0.449	0.544
	V ≥ 100	$Y = 4370\ e^{-.0879\ X}$		-0.921		0.334	
		$Y = 4141\ e^{-.0618\ X}$		-0.832		0.333	
Flint Total Work Trips	All	$Y = 1961\ e^{-.1113\ X}$	$Y = 2021\ e^{-.0990\ X}$	-0.950	-0.965	0.573	0.587
		$Y = 1721\ e^{-.0835\ X}$	$Y = 1693\ e^{-.0624\ X}$	-0.922	-0.853	0.572	0.583
	V ≥ 50	$Y = 2119\ e^{-.0725\ X}$	$Y = 1804\ e^{-.0482\ X}$	-0.909	-0.965	0.480	0.494
			$Y = 1614\ e^{-.0259\ X}$		-0.641		0.493
	V ≥ 100	$Y = 2720\ e^{-.0587\ X}$	$Y = 2091\ e^{-.0552\ X}$	-0.974	-0.881	0.383	0.436
		$Y = 2546\ e^{-.0454\ X}$	$Y = 1773\ e^{-.0170\ X}$	-0.913	-0.324	0.382	0.432

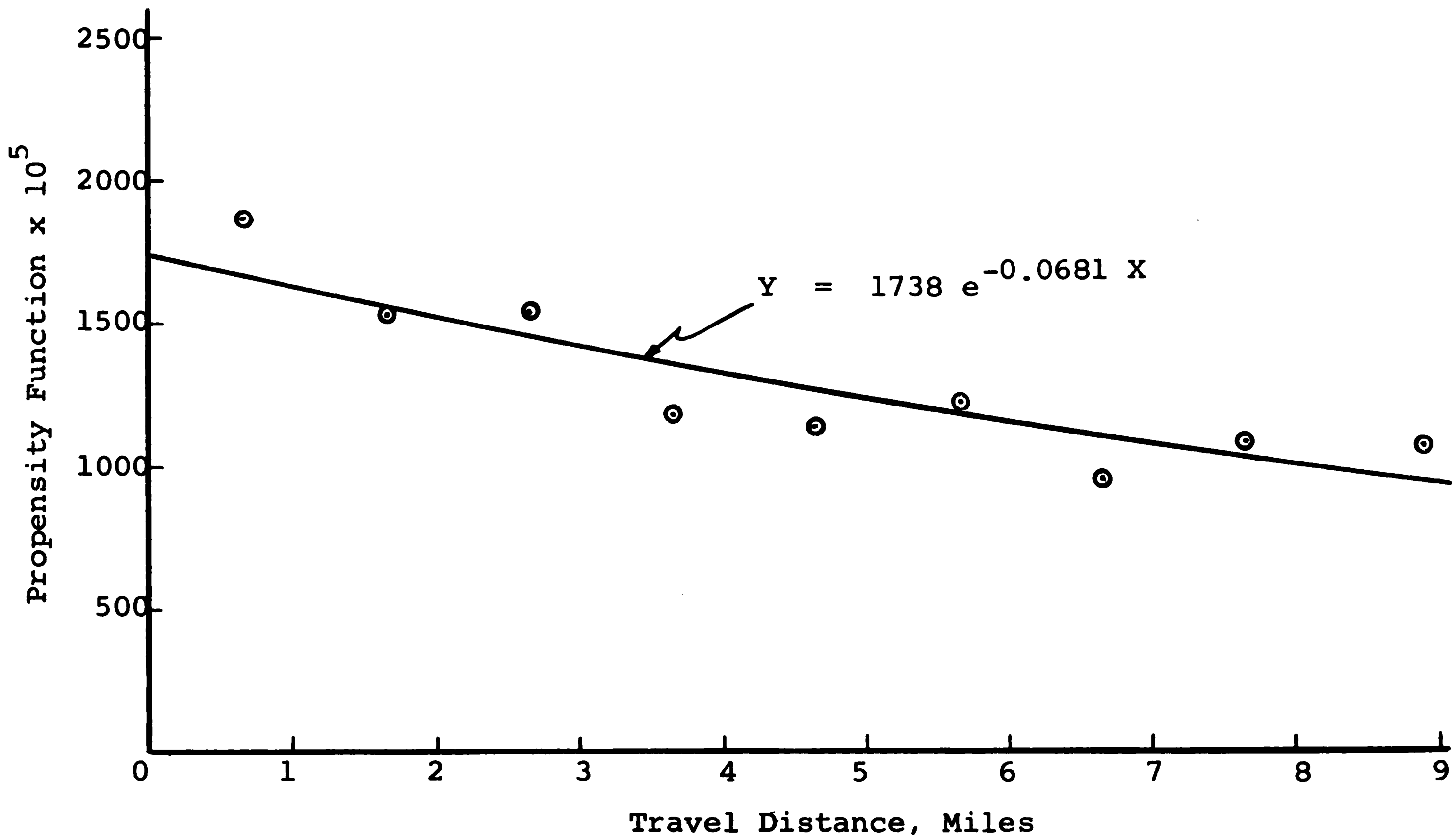

Figure 4. Empirical Propensity Values and Fitted Equation, Linear Graph Model, All Auto Driver Work Trips.

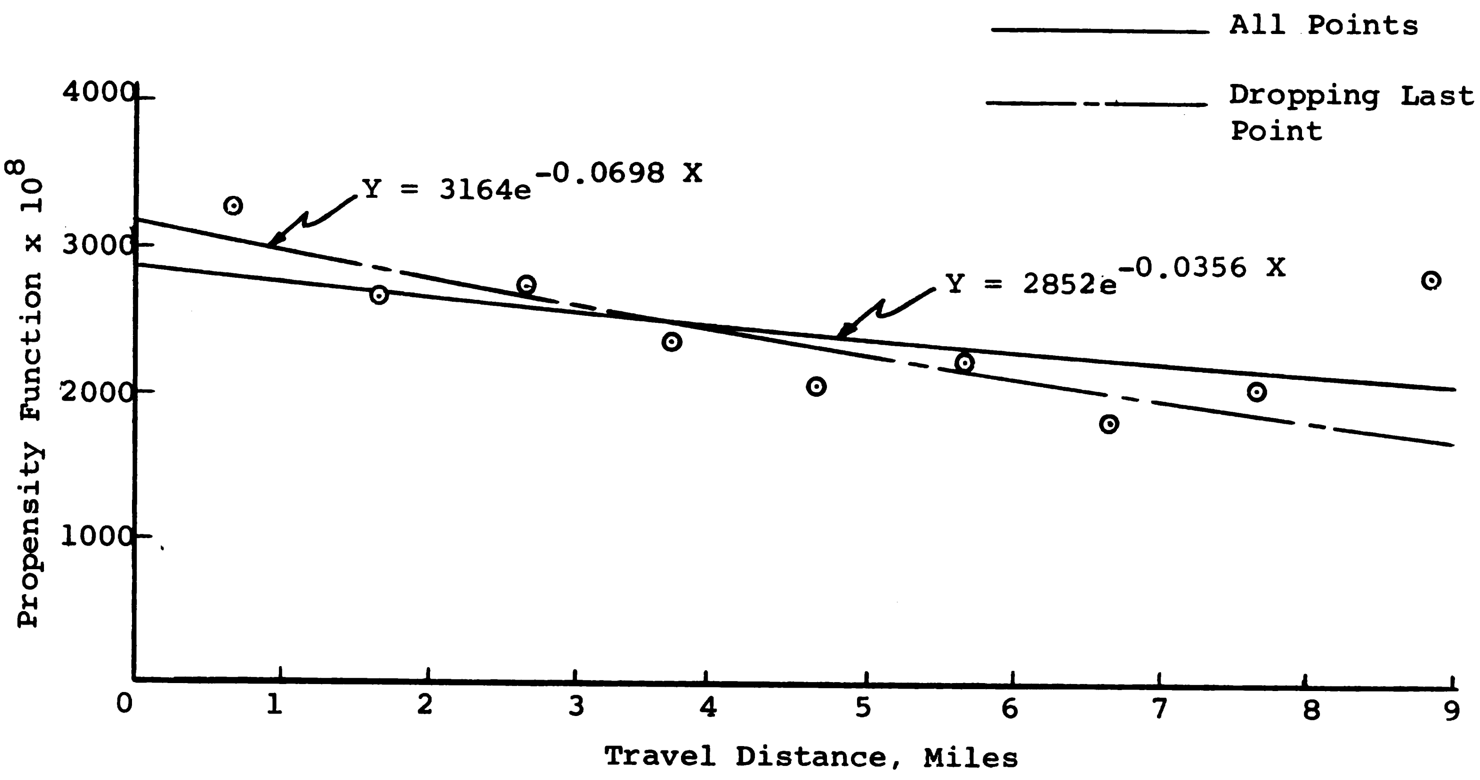

Figure 5. Empirical Propensity Values and Fitted Equations, Gravity Model, All Auto Driver Work Trips.

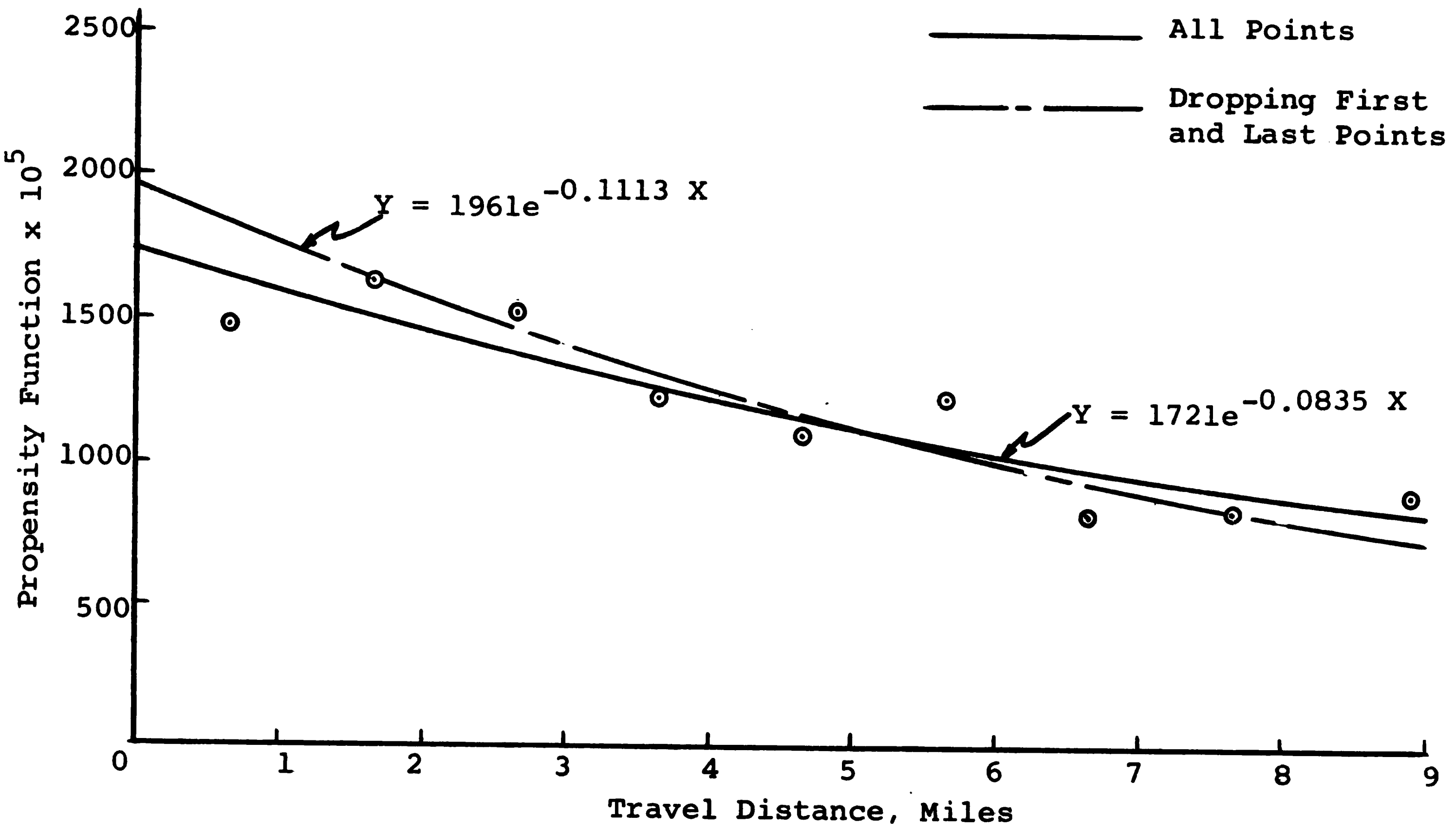

Figure 6. Empirical Propensity Values and Fitted Equations, Linear Graph Model, All Total Work Trips.

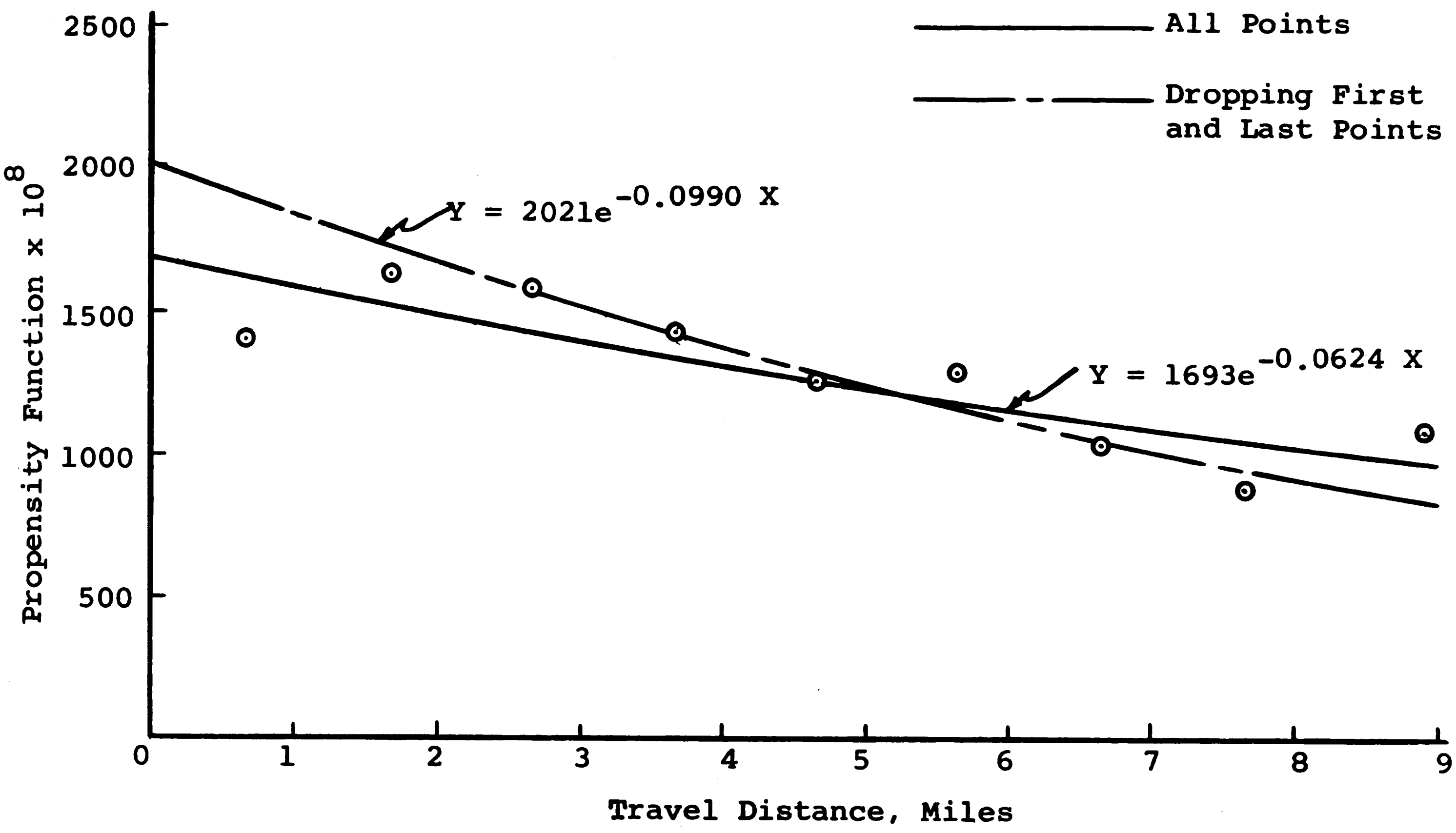

Figure 7. Empirical Propensity Values and Fitted Equations, Gravity Model, All Total Work Trips.

equations, the relative error values would have improved if the correlation coefficient improved.

The relative error values given in Table I show that in every data group, the linear graph model reproduced the given trip distribution better than did the gravity model. The best error value was obtained for auto driver work trips when considering only volumes greater or equal to 100. This is probably the result of two factors: (1) these volumes are more significant because of their size, and (2) auto driver trips are usually more time dependent than are total work trips. The inclusion of mass transit trips in total work trips is the reason the second factor is usually true. Volume dependence on travel time, or travel distance, is the basis of propensity function determinations and is basic, in one form or another, to all successful distribution models.

Had the Flint data been more significant (see Chapter V), further inferences, such as how the constants varied in the different propensity equations, could have been made from Table I. The lack of significance in the data makes further analysis of the model results not very meaningful.

CHAPTER V

ADDITIONAL DATA AND MODEL TESTS

Flint Data

Test Procedure. This analysis was an attempt to determine if interzonal volumes for the Flint work trip data depended on distance. The analysis followed procedures reported by Lapin (40).

The basic procedure is to first determine the probable interchange between all zones of importance. The probable interchange is the calculated work trip volume between zones considering the attraction of the destination zone as the only influencing factor on the volume of trips between zones. The percentage of the total work trips destined to a particular zone is used as the measure of attractiveness of that particular destination zone. The probable interchange calculations assume that if 20 percent of all work trip destinations are to zone M, then 20 percent of the work trips originating in zone N are destined to zone M. The equation for the probable interchange can be written as

$$T'_{ij} = G_i \, (A_j / \sum A_j)$$

where

T'_{ij} = probable interchange

G_i = work trips originating in zone i

A_j = work trips destined to zone j

$\sum A_j$ = total work trips destined to all zones.

According to current literature, it is generally assumed, and not unreasonably so, that the number of trips between two zones decreases as some function of travel time or travel distance. If travel time is not considered, those zone interchanges with small travel times will be under-assigned and those with large travel times will be over-assigned.

From O and D data, the actual interzonal volumes, T_{ij}, are known. By taking a ratio of the actual interchanges to the probable interchanges, with the resulting ratio being called the probability interchange, a measure of this over-assignment and under-assignment is obtained. This can be stated as

$$M_{ij} = \frac{T_{ij}}{T'_{ij}}$$

where

M_{ij} = probability interchange

T_{ij} = actual interchange

T'_{ij} = probable interchange.

Too large a value of this ratio indicates an under-assignment which should occur for interzonal movements with small values of travel time, and too small a value of this ratio would indicate an over-assignment which should occur for interzonal movements with large values of travel time.

By plotting the reciprocal of the probability interchange, M_{ij}^{-1}, versus travel time or travel distance, r_{ij}, a measure of the effect which travel time or travel distance has on interzonal volumes is obtained. For a perfect relationship, assuming as Lapin did that the M_{ij} vs. r_{ij} plot should result in a hyperbola, the plot of M_{ij}^{-1} vs. r_{ij} should result in a straight line.

Test Application. The preceding analysis was applied to a sampling of total work trips from the Flint work trip data. The probability interchange values were computed and the reciprocals of these values were plotted versus travel distance. The resulting plot is given in Figure 8. Since there was a large amount of scatter in the resulting plot, statistical tests were applied to determine if the expected linear relationship could be rejected.

The linear regression equation calculated for this plot was

$$Y = 1.013 + 0.0757 X$$

The slope of this line is very small, indicating that travel distance for this data may have little effect on the volume between zones.

A linearity test was made on the data. The F ratio for the linearity test was 1.6195 and the critical tabulated F value at the 5 percent level of significance was 1.82. Therefore, on the basis of this test at the 5 percent level of significance, the linearity hypothesis cannot be rejected.

The correlation coefficient between the two variables, M_{ij}^{-1} and r_{ij}, was also calculated and compared with a tabulated value to test if the population correlation coefficient was zero. If it is zero, this means in normal populations, that the two variables are independent and thus the linear relationship between the two variables does not exist.

The calculated correlation coefficient was 0.1476 and the tabulated critical value at the 5 percent level of significance was 0.131. Thus, at the 5 percent level of significance, the hypothesis that the population correlation coefficient is zero must be rejected. This means that we cannot reject the linear relationship between the two variables on the basis of this test.

Although we cannot statistically reject the linear relationship, the calculations indicate that a very weak

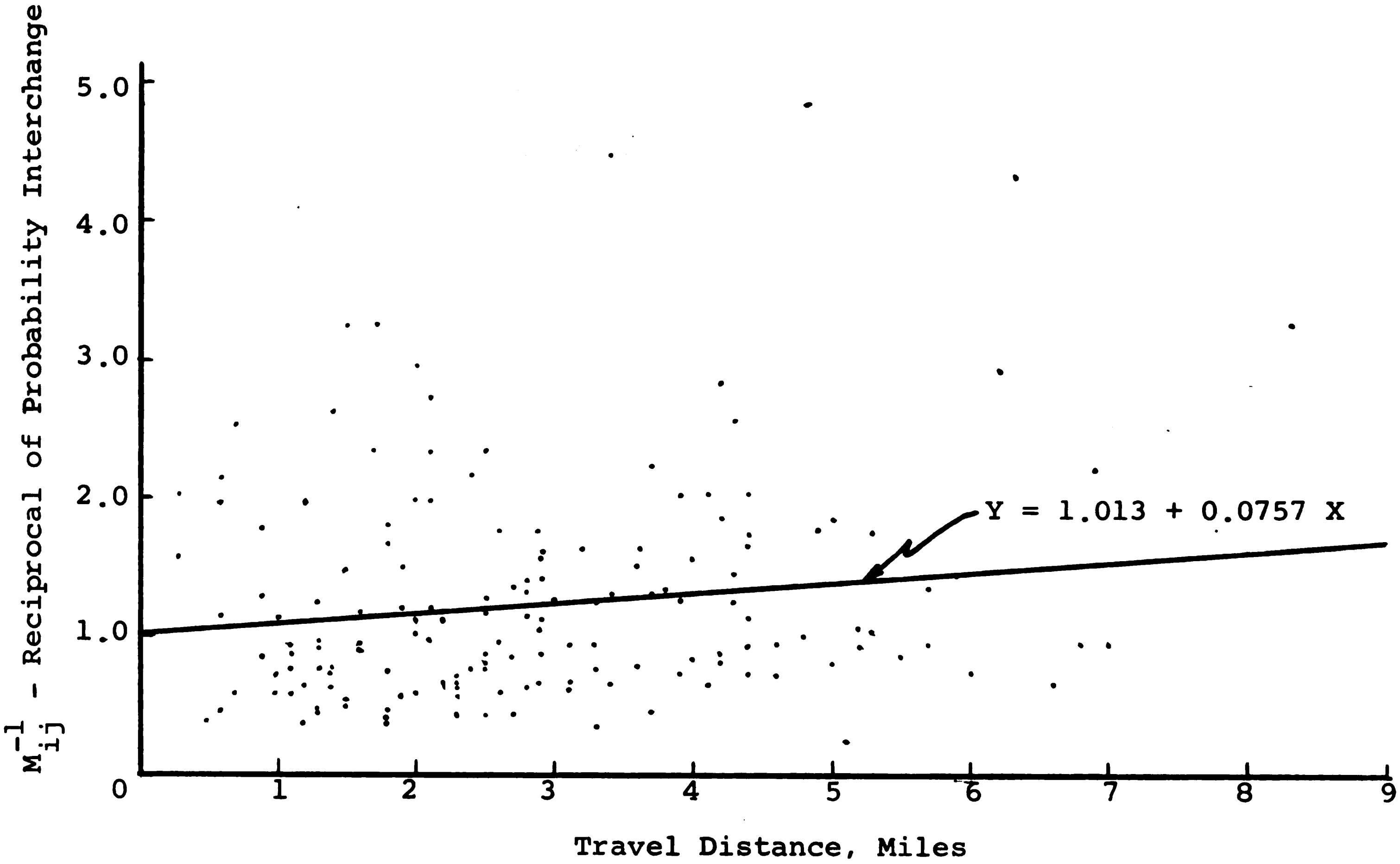

Figure 8. Dependence of Interzonal Volume on Distance, Flint Total Work Trips.

relationship exists. Since the resistance factors, R_{ij}, for the route components are based on this distance dependence, the Flint data cannot be expected to give very conclusive results.

An M_{ij}^{-1} vs. r_{ij} plot for a sample of auto driver work trips from the Flint data was also made. Although not analyzed statistically, the plot indicated similar results to that of the total work trip plot.

Linear Graph Model Applied to Philadelphia Work Trips

To further test the linear graph model, it was applied to Philadelphia work trip data used by Lapin (40) in constructing the probability interchange versus travel time plots. Since these trip volumes did show a degree of travel time dependence in Lapin's analysis, it was felt that the results of this test would be more conclusive than the results from the Flint data. The results from this analysis are given in Table II and the F(u) vs. u plot for this analysis is given in Figure 9.

The resulting error value when using the exponential curve to approximate the propensity values is 0.296. This is an improvement over the Flint error values and compares very favorably with the error value of 0.3191 obtained by Dieter when using the gravity model (25) to distribute

Toronto work trips. When using the hyperbola curve to approximate the Philadelphia propensity values, the error value is slightly higher at 0.360.

This analysis indicated that when using data known to be somewhat dependent on a travel distance measure, the error values, determined by equation 10, improve. The improved error values indicate an increase in the effectiveness of the linear graph model in reconstructing the given work trip distribution.

TABLE II

LINEAR GRAPH MODEL RESULTS

PHILADELPHIA DATA

$F(r_{ij}) \times 10^5$	Correlation Coefficient, R	Relative Error, P
$Y = 1108\ e^{-0.0318\ X}$	-0.660	0.296
$\frac{1}{Y} = 0.6402\ X + 7.05$	-0.690	0.360

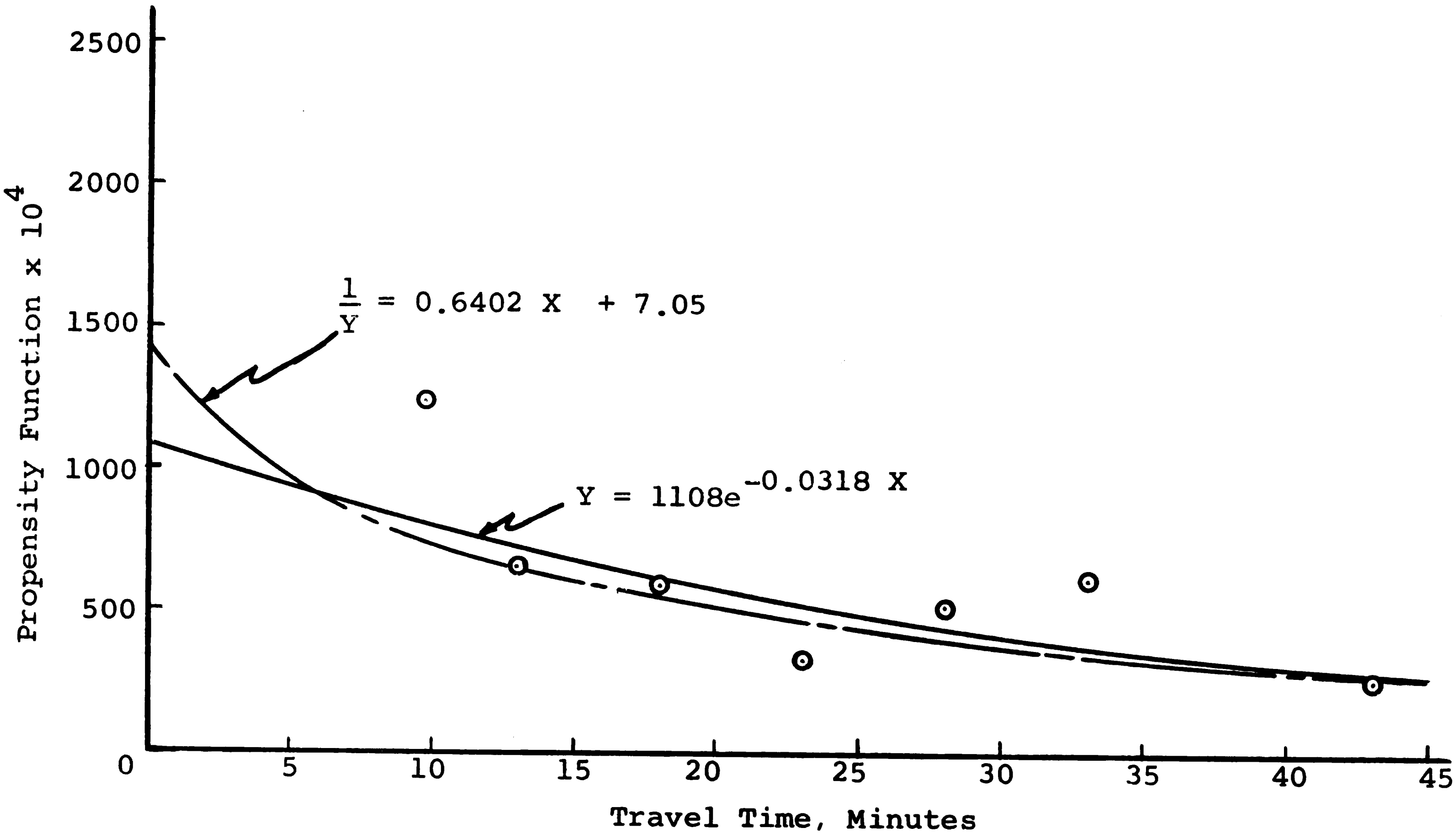

Figure 9. Empirical Propensity Values and Fitted Equations, Philadelphia Work Trips, Linear Graph Model.

CHAPTER VI

CONCLUSIONS

The linear graph model does a better job of reproducing the work trip distributions given in the Flint data than does the conventional gravity model. Although the relative error values were rather large in some instances, the reduced data values compare favorably with values obtained by Dieter (25). When applying the linear graph model to Philadelphia work trip data known to be somewhat dependent on travel time, the error value, when using an exponential equation for the propensity function, is the best value obtained in the investigation.

Further refinements in the linear graph model can be expected to improve these error values. Much room for improvement is present in the X value determinations for the generation and attraction zones. Now that a procedure is established, for given data, various parameters or combinations of parameters can be used for generation and attraction X values to determine a propensity function equation. This equation can, in turn, be used to calculate a relative error value. Those parameters resulting in the best relative error values can then aid in further investigations into what constitutes the X values in traffic flow.

Finally, the size of Flint (200,000 in study area) and the large number of O and D zones (96) resulted in very small interzonal volumes when considering only interzonal work trip data. The signficiance of these volumes in some cases was dubious, and it was shown that the size of the interzonal volumes depended little on travel distance. It is expected that the error values will improve when the model is applied to more significant data. Using travel times rather than travel distances should also increase the effectiveness of the linear graph model.

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APPENDIX A

Propensity Plots for the Reduced Flint Data

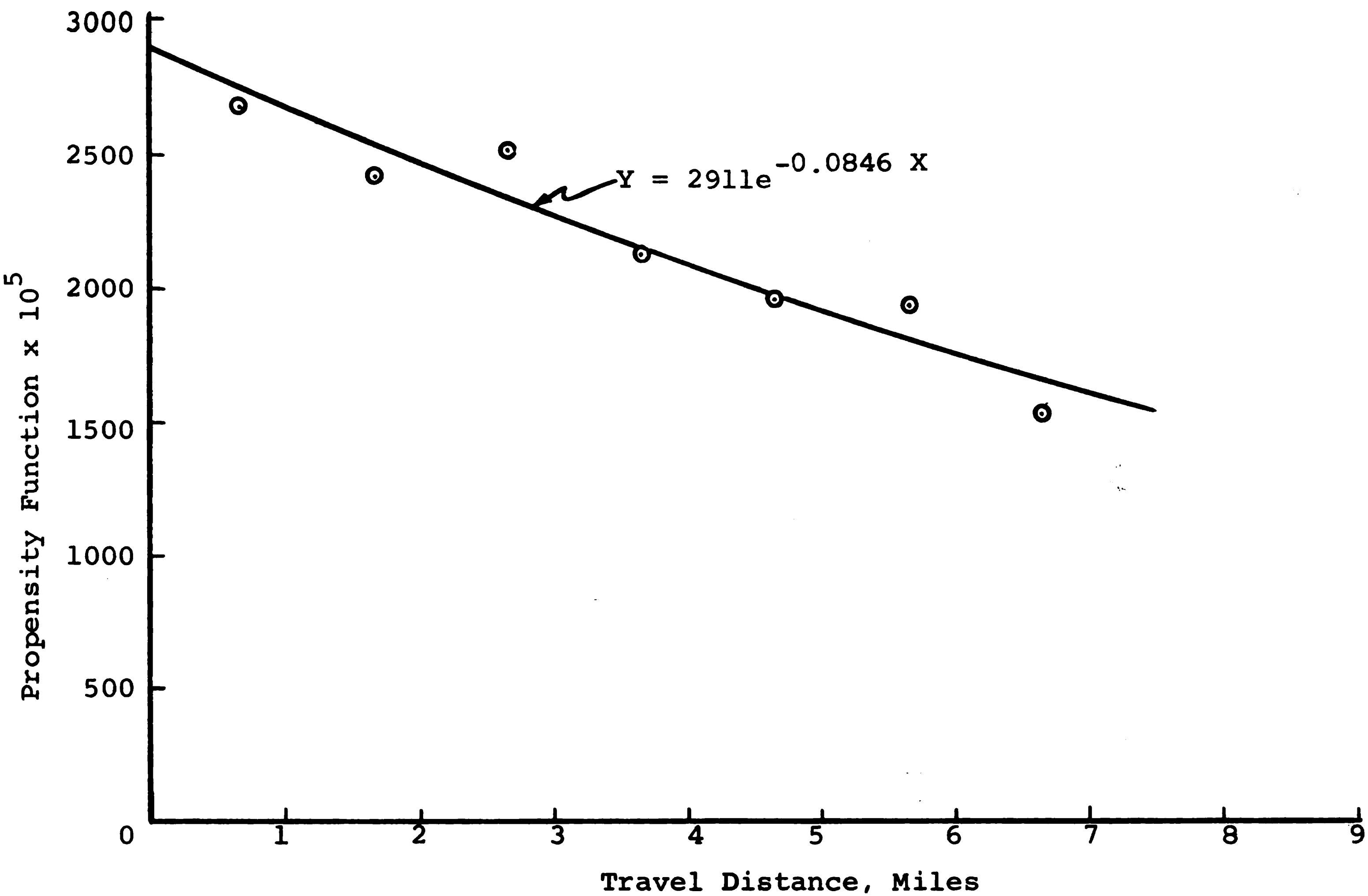


Figure 10. Empirical Propensity Values and Fitted Equation, Linear Graph Model, Auto Driver Work Trips, Volumes $\geq$ 50.

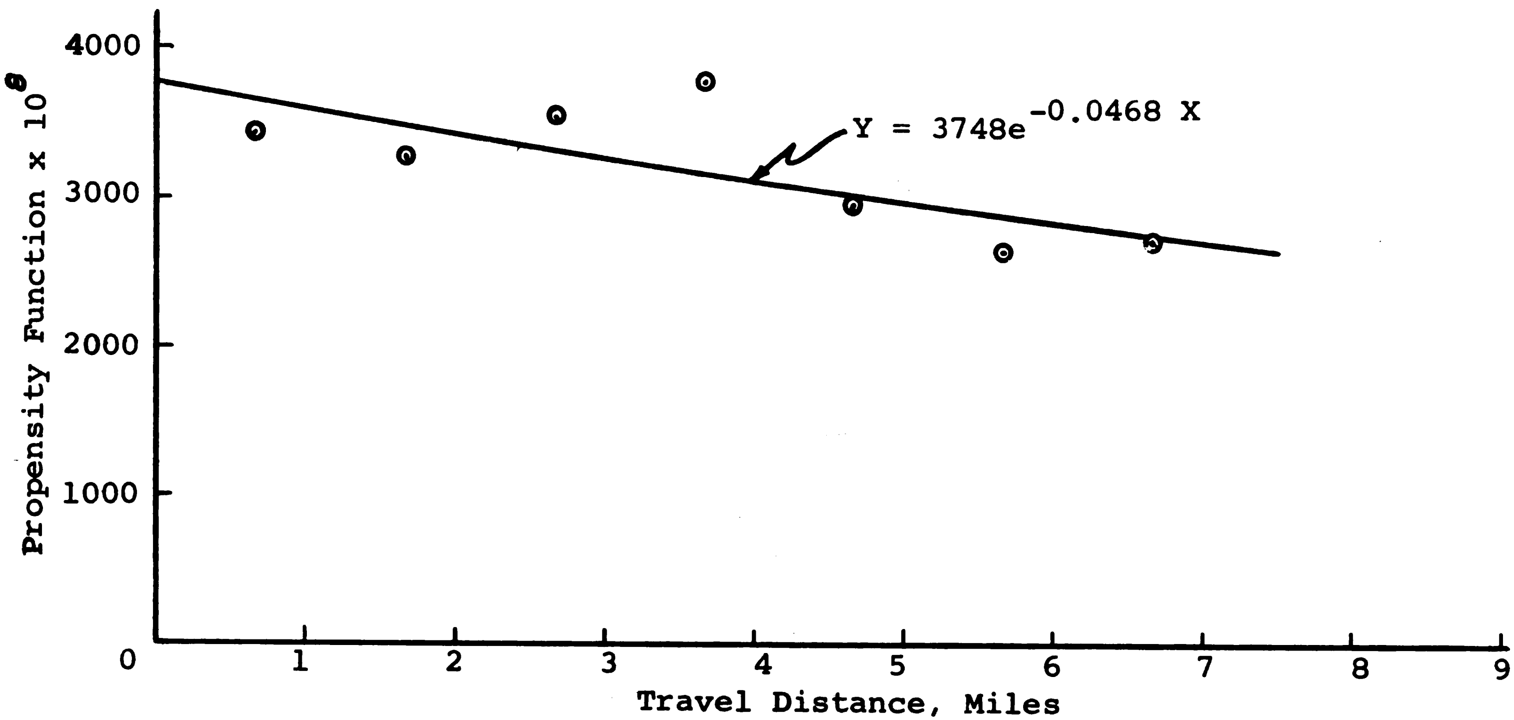


Figure 11. Empirical Propensity Values and Fitted Equation, Gravity Model, Auto Driver Work Trips, Volumes ≥ 50.

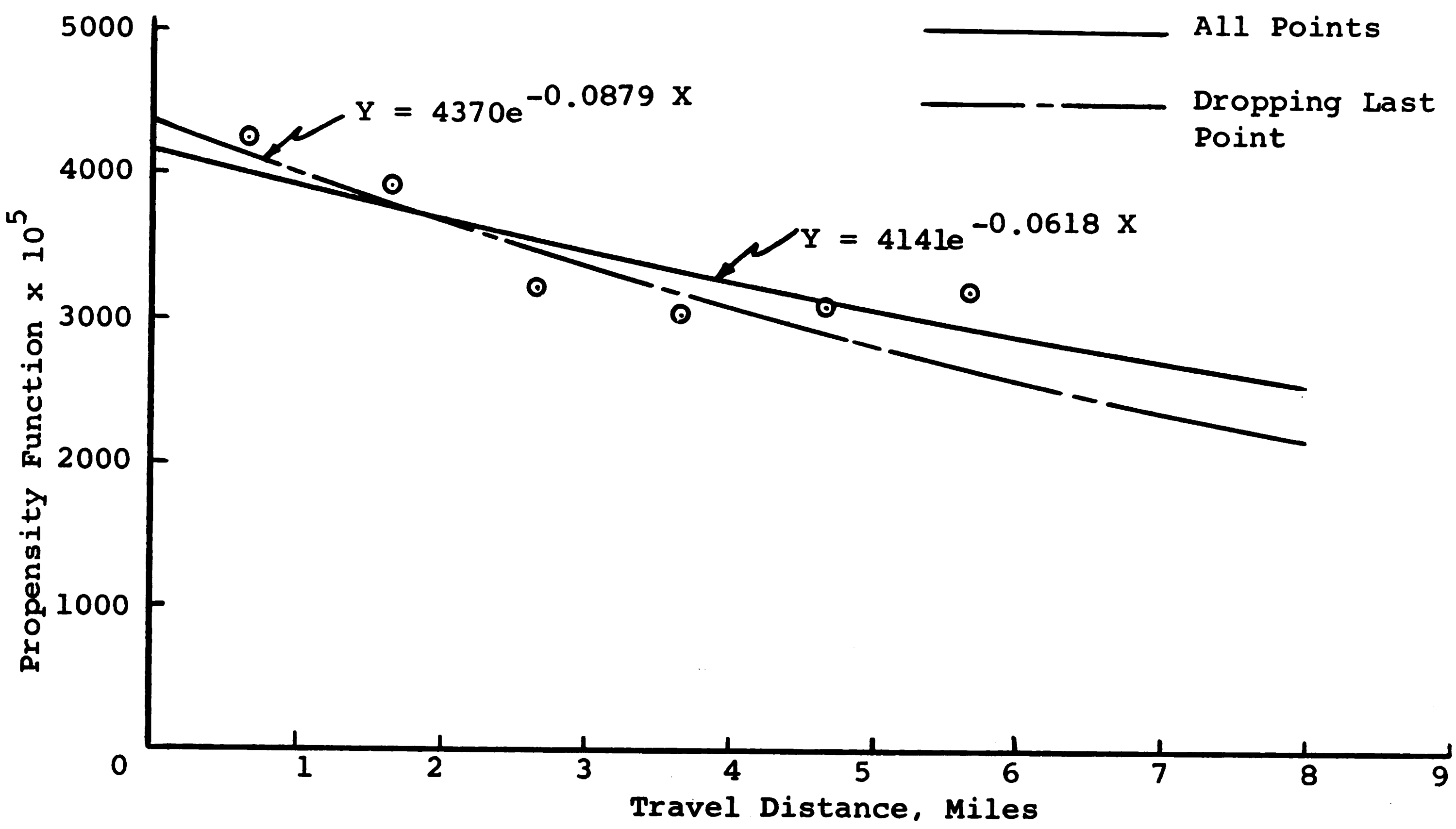


Figure 12. Empirical Propensity Values and Fitted Equations, Linear Graph Model, Auto Driver Work Trips, Volumes $\geq$ 100.

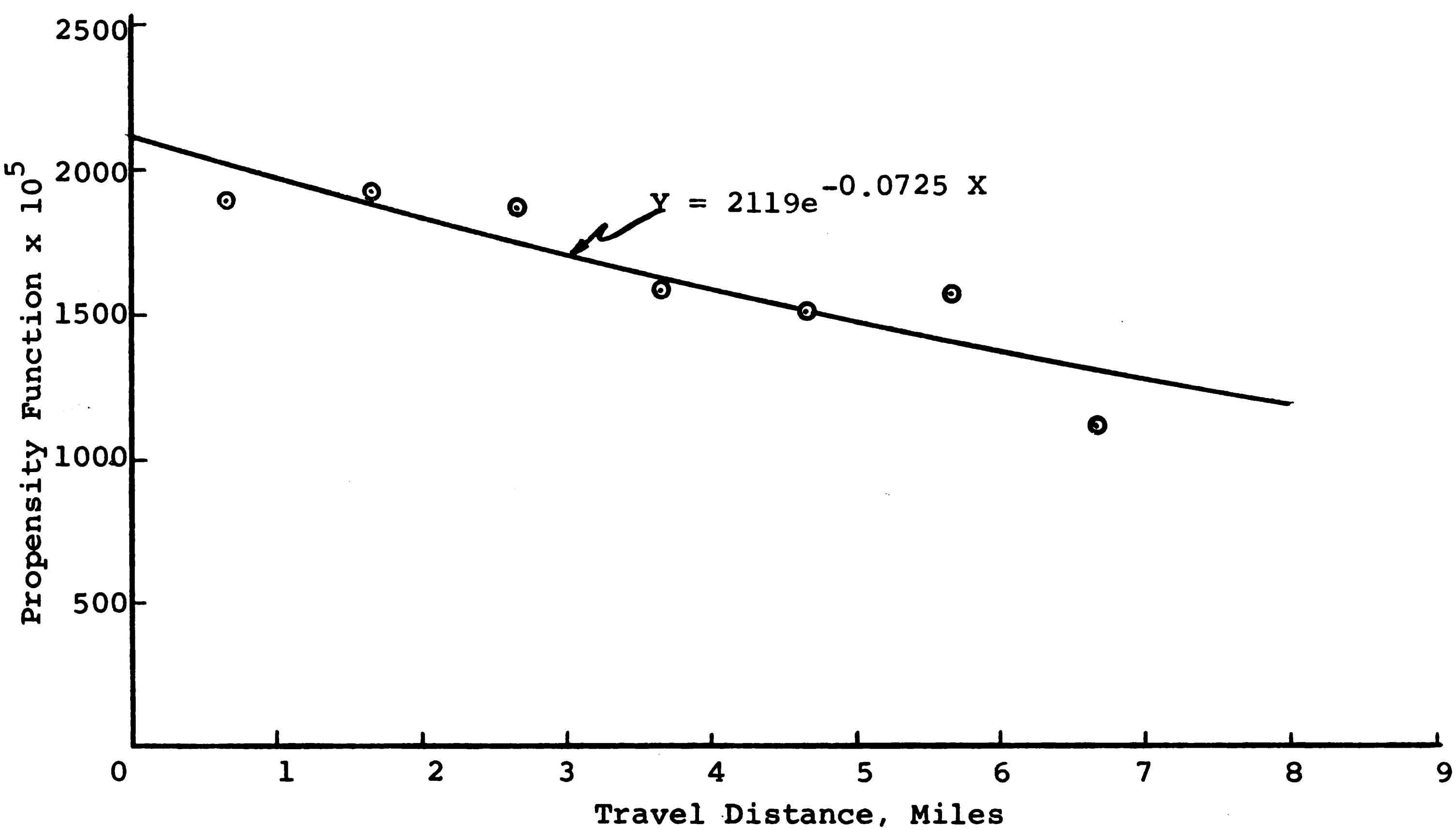


Figure 13. Empirical Propensity Values and Fitted Equation, Linear Graph Model, Total Work Trips, Volumes ≥ 50.

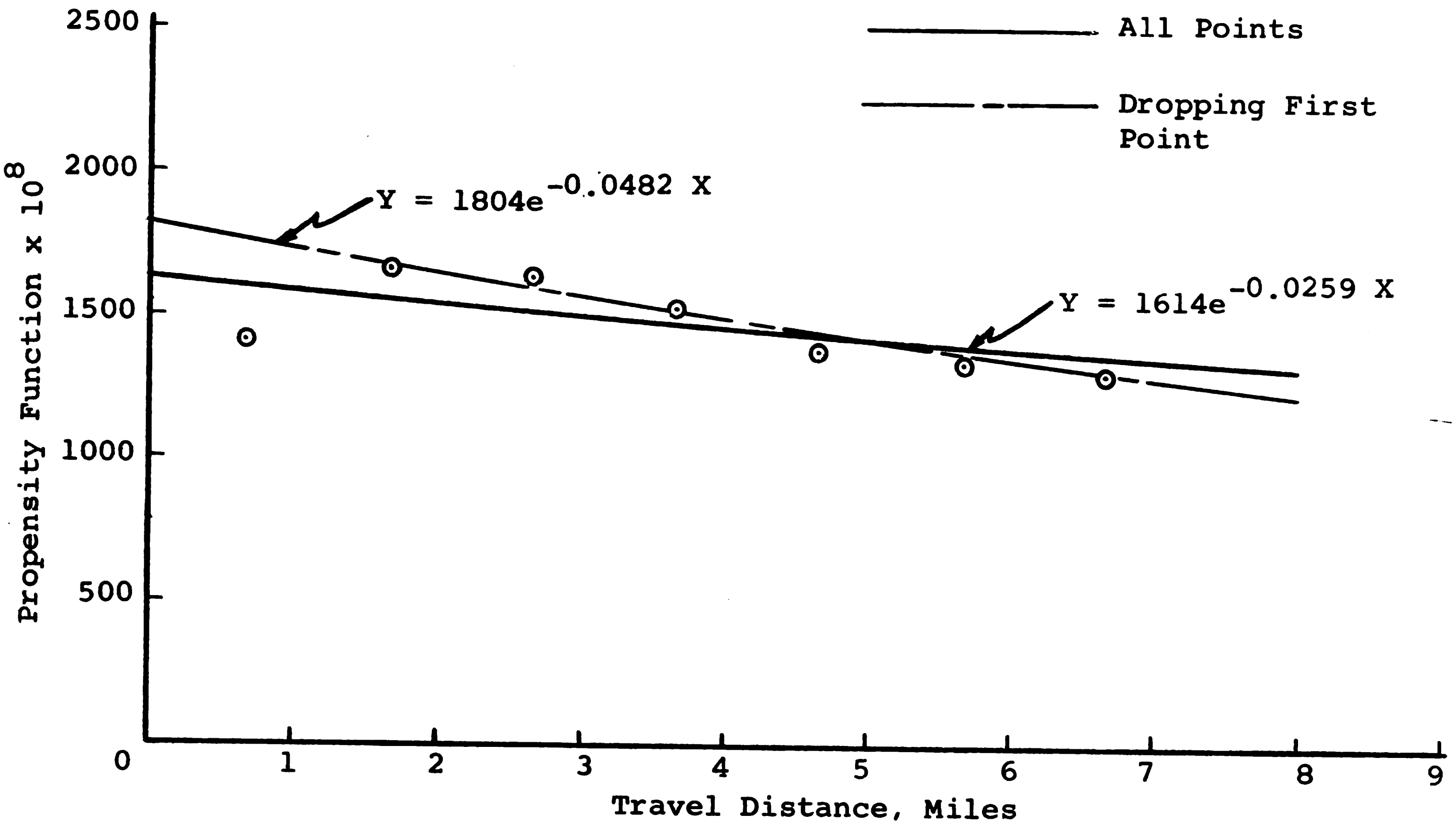


Figure 14. Empirical Propensity Values and Fitted Equations, Gravity Model, Total Work Trips, Volumes $\geq$ 50.

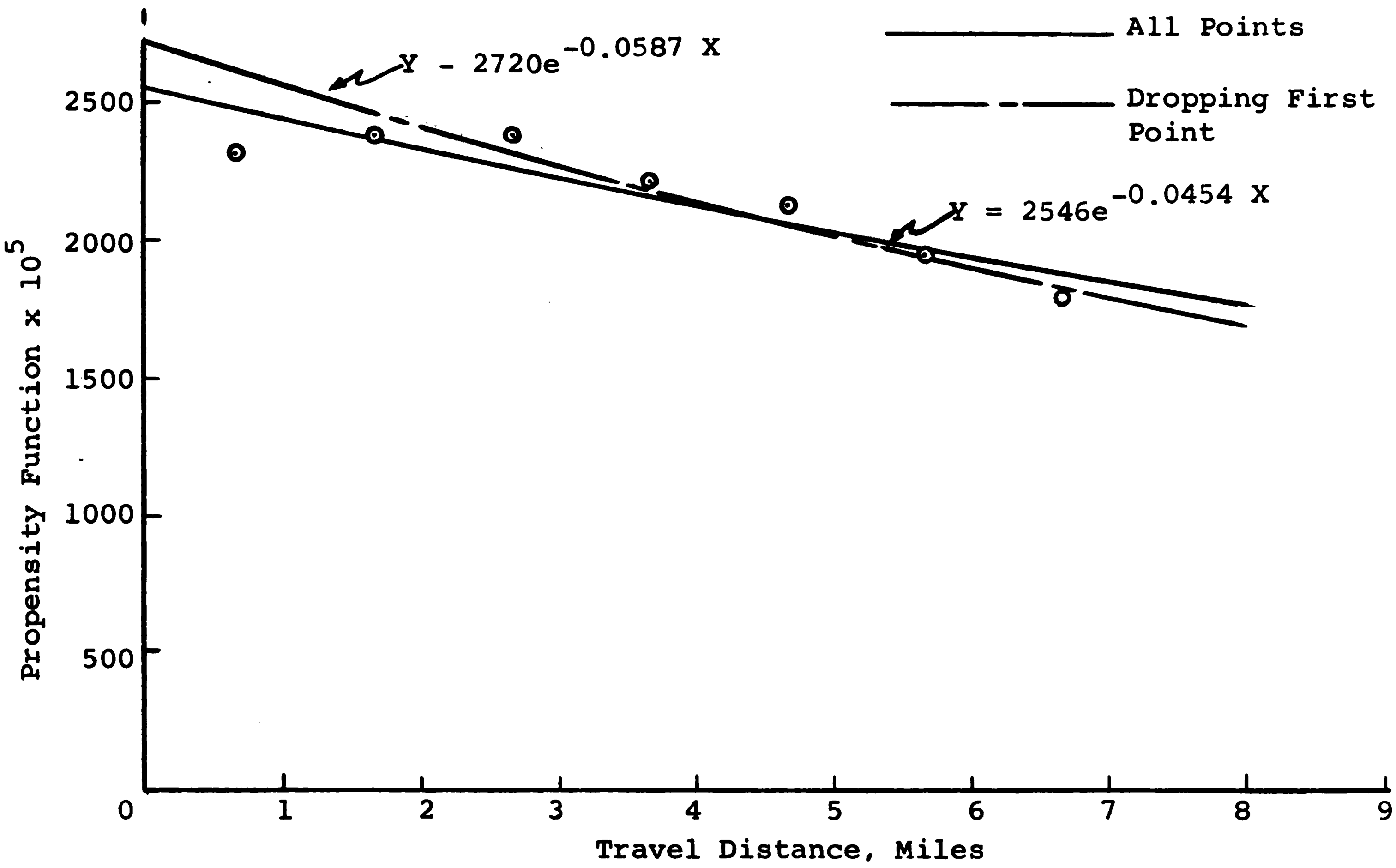


Figure 15. Empirical Propensity Values and Fitted Equations, Linear Graph Model, Total Work Trips, Volumes $\geq$ 100.

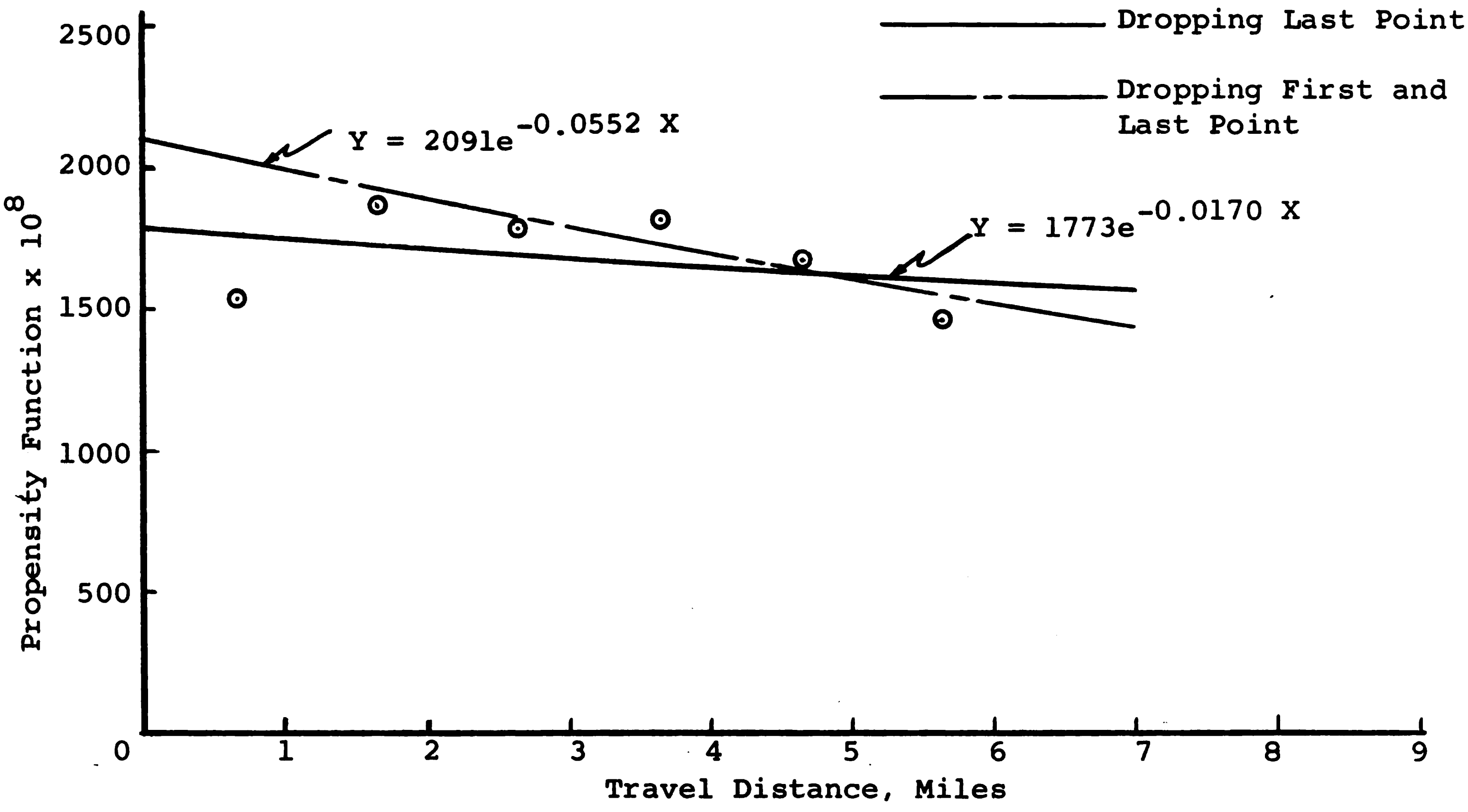


Figure 16. Empirical Propensity Values and Fitted Equations, Gravity Model, Total Work Trips, Volumes ≥ 100.

APPENDIX B

Travel Distance Determinations

TRAVEL DISTANCE DETERMINATIONS

Introduction

The over-the-road distances used in this study were tabulated by the Urban Research Laboratory of Wayne State University and purchased by this project. The following is a discussion of the manner in which these measurements were computed (15).

All of the traffic analysis zones in the study area were examined in order to locate the traffic generation centroids or "zone centers" of each of the zones for the purpose of traffic assignment.

The approach to the problem of freeway assignment followed methodologies developed by the Detroit Metropolitan Area Traffic Study in 1954 (20), so that the speed and distance ratios used in the predictive curves for assignment could be calculated using only distance measurements and an assumption of a constant ratio of speeds on freeways and urban arterials. Basically this involved the trip distance measurement for the shortest and quickest route or path for each zone to zone movement, and the trip distance measurement for the shortest and quickest freeway route or path for each zone movement, so that a speed and distance comparison of the best competing routes could be made.

Measurement of Alternate Trip Distances via Competing Arterials

It is know that there exists a predictable relationship between airline distances between two points and the actual over-the-road distance (9). Therefore, to determine the mathematical relationships, regression lines were constructed from the DMATS curve to compute the ratio factor or multiplier for conversion of airline distance to actual over-the-road distance. The equations for these regression lines are as follows:

> Where X is the airline distance between zones; when X is greater than zero and less than 17.5, the multiplier factor is $Y = -0.01X + 1.275$; when X is greater than 17.5, the multiplier factor is $Y = 1.1$.

Although these relationships of airline distance and over-the-road distance were compiled from a sampling of the Detroit street system, it was felt that the proportion of diagonal and radial streets to simple grid streets in the Flint street system were closely enough representative of the proportion in Detroit. This is not to say that the street systems were necessarily similar, but that the proportion of diagonals to grids was approximately comparable. Thus, actual distances would be slightly less than the right angle distances of a simple grid system. It was concluded, then to base the multiplier factor upon two simple regression lines drawn to fit the curve compiled by DMATS. This gave a close approximation of average alternate urban arterial distances.

Actual individual travel time in minutes between zones was not computed. Over-the-road distances were computed from airline distance measurements for either urban or rural roads. The ratio of competing arterials and freeway routes was the information used to perform a computer traffic assignment.

These values for over-the-road distances between zones were the values used in this project as the basis for the route resistance factor.

APPENDIX C

Derivation of Propensity Function Equations

DERIVATION OF PROPENSITY FUNCTION EQUATIONS

The linear graph model is written as

$$T_{ij} = (G_i + A_j)\ F(r_{ij}) \qquad (13)$$

with each term defined as previously given in the report.

For each distance class, the equation can be rewritten as

$$T_{ij} \approx (G_i + A_j)\ F(u) \qquad (14)$$

where u is the midpoint of the class interval and represents the travel distance for all members of each distance class.

Ideally, each movement in a class, when equation 14 is solved for F(u)

$$F(u) \approx \frac{T_{ij}}{(G_i + A_j)}$$

would give the same value for the constant, F(u). Since it is most likely that a scattering of values would result, solution for the best F(u) value to represent each class is desirable.

Since $[(G_i + A_j)\ F(u)]$ predicts T_{ij}, the sum of squares of the residuals, $[(T_{ij} - (G_i + A_j)\ F(u)]$, will be a minimum in each class if F(u) is determined by least square techniques.

Letting F(u) = k, it follows that

$$\sum_{i,j} (T_{ij} - k\ (G_i + A_j))^2 = \text{minimum}$$

when $$\frac{\partial}{\partial k} \left(\sum_{i,j} (T_{ij} - k\ (G_i + A_j))^2\right) = 0$$

Taking the partial derivative indicated results in

$$2 \sum_{i,j} (G_i + A_j)\ (T_{ij} - k\ (G_i + A_j)) = 0$$

or $$\sum_{i,j} ((G_i + A_j)\ (T_{ij}) - k\ (G_i + A_j)^2) = 0$$

Solving for k results in

$$k = \frac{\sum_{i,j} (G_i + A_j)\ T_{ij}}{\sum_{i,j} (G_i + A_j)^2}$$

This results in, since k = F(u),

$$F(u) = \frac{1}{W} \sum_{i,j} (G_i + A_j)\ T_{ij}$$

where $$W = \sum_{i,j} (G_i + A_j)^2$$

APPENDIX D

Propensity and Error Computer Programs

PROPENSITY AND ERROR COMPUTER PROGRAMS

Comments on Programs

Program listings for calculating the propensity function for total work trips, volumes greater or equal to fifty, and the gravity model and for calculting the relative error for the same data and model are presented for inspection.

Three alternatives were used in both the propensity (program main) and error (program error) programs. These were changing to or from total work trips and auto driver work trips, sorting the volumes, and changing to or from the gravity model and linear graph model.

To change from total work trips to auto driver work trips, the following card was inserted in place of statement labeled number 1 in the given program listing:

```
2 Format  (I1, I2, I1, I2, 4X, I4)
```

To sort the volumes, card 100 in program error and 103 in program main are either

omitted when using all trips on data cards
or left as they appear for volumes $\geq$ 50
or the 50 changed to 100 for volumes $\geq$ 100.

To change from the gravity model to the linear graph model, the asterisk in statements 101 and 102 in program error between EATRAC and AULT1 and EATRAC and AULT2 was changed to a plus sign. For program main, the asterick in statement 15 between INERAT and JATRAC was changed to a plus sign.

Statements 500 and 600 in program error vary with each model and each data classification. These statements tell the computer the propensity equation to be used to determine the propensity values used to calculate the respective interchanges.

Data Card Organization

Zone Data Cards. These cards contain the zone number in columns 1, 2, and 3; the total number of work trips generated by that zone in columns 4, 5, 6, and 7; and the total number of work trips attracted to that zone in columns 8, 9, 10, and 11. One zone data deck is used for total work trips and another zone data deck is used for auto driver work trips.

Class Cards. These cards contain the common distance between zones for the members of that distance group in columns 3 and 4. The decimal belongs between columns 3 and 4 and represents the miles, to the nearest tenth of a mile, that the trips in that class travel between zones. Columns 6 and 7 contain the number of interzonal movements represented in that distance class.

<u>Propensity Program</u>

```
* 521072 CUBITT, E.D.
      PROGRAM MAIN
      PRINT 6
    6 FORMAT (1H1, 40HE D  CUBITT, FLINT PROPENSITY FUNCTION )
C READ ZONE DATA CARDS IN
      DIMENSION INERAT (9,15), JATRAC (9,15), ENUM (90), DENOM (90)
      DO 301 I = 1,9
      DO 301 J = 1,15
  300 FORMAT (3X,I4,I4)
  301 READ 300, INERAT (I,J), JATRAC (I,J)
      PRINT 5
    5 FORMAT (/10X,67HDISTANCE   PROPENSITY VALUE   NUMBER IN CLASS
     5SUBTOTAL OF LINKS)
      JUM = 0
      DO 7 K = 1, 70
C READ CLASS CARD
      READ 4, DISTAN, LASS
    4 FORMAT (F4.1, 13)
      JUM = JUM + LAS
C ADD EACH CLASS DATA CARD IN ONE CLASS
      ENUM (K) = 0.
      DENOM (K) = 0.
      DO 2 L = 1, LASS
      READ 1, I1, J1, I2, J2, INTERC
    1 FORMAT (I1, I2, I1, I2, I4)
  103 IF  (INTERC - 50)  2, 15, 15
   15 IUM = INERAT (I1, J1)  *  JATRAC  (I2,J2)
C COMPUTE  CLASS PROPENSITY VALUE
      ENTERC = INTERC
      SUM = IUM
      ENUME = ENTERC  *  SUM
      DENOME = SUM  ** 2.
      ENUM (K) = ENUM (K) + ENUME
      DENOM (K) = DENOM (K) + DENOME
    2 CONTINUE
      PROP = ENUM (K)  /  DENOM (K)
    7 PRINT 3, DISTAN, PROP, LASS, JUM
    3 FORMAT (12X, F4.1,  9X, E14.8, 14X, I3, 18X, I6)
      PRINT 9
    9 FORMAT (/11X, 4HDIST, 6X, 10HPROPENSITY)
      DO 10 K = 1, 70, 2
      ENUMD = ENUM (K) + ENUM (K+1)
      DENOMD = DENOM (K) + DENOM (K+1)
      PROP = ENUMD / DENOMD
      KAPPA = K+1
   10 PRINT 11, KAPPA, PROP
```

Propensity Program - (contd)

```
 11 FORMAT (12X, I2, 7X, E14.8)
    PRINT 200
200 FORMAT (1H2)
    DO 12 K = 1, 70, 5
100 ENUMD = ENUM (K) + ENUM (K+1)  +  ENUM (K+2)  +  ENUM (K+3) +
   5ENUM ( K+4)
    DENOMD = DENOM (K)  +  DENOM (K+1)  +  DENOM (K+2)  +  DENOM
   5(K+3)  +  DEN 5OM (K+4)
    PROP = ENUMD / DENOMD
    KAPPA = K+2
 12 PRINT 11, KAPPA, PROP
    PRINT 200
    DO 13 K = 1, 70, 10
    ENUMD = 0.
    DENOMD = 0.
    DO 14 M = 1, 10
    M1 = K + M - 1
    ENUMD = ENUMD + ENUM (M1)
 14 DENOMD = DENOMD + DENOM (M1)
    PROP = ENUMD / DENOMD
    KAPPA = K+5
 13 PRINT 11, KAPPA, PROP
105 STOP
    END
    END
```

<u>Error Program</u>

```
* 521072 CUBITT, E.D.
      PROGRAM ERROR
    6 FORMAT (1H1, 43HE D  CUBITT, FLINT PROPENSITY FUNCTION ERROR,/1H0)
      PRINT 6
C READ ZONE DATA CARDS IN
      DIMENSION INERAT (9,15), JATRAC (9,15)
      DIMENSION ENERAT (9,15), EATRAC (9,15)
      DO 301 I = 1,9
      DO 301 J = 1,15
  300 FORMAT (3X,I4,I4)
  301 READ 300, INERAT (I,J), JATRAC (I,J)
      PRINT 22
   22 FORMAT (16X,2HP1, 17X,2HP2,16X,5HMULT1,15X,5HMULT2)
      SUMP1 = 0.
      SUMP2 = 0.
      DO 7 K = 1, 70
C READ CLASS CARD
      READ 4, R, LASS
    4 FORMAT (F4.1,I3)
C ADD EACH CLASS DATA CARD IN ONE CLASS
      SUMT1 = 0.
      SUMB = 0.
      SUMT2 = 0.
  500 AULT1 = .00001804* (EXPF (-.04823 * R))
  600 AULT2 = .00001614* (EXPF (-.02590 * R))
      DO 20 L = 1, LASS
      READ 1, I1, J1, I2, J2, INTERC
    1 FORMAT (I1, I2, I1, I2, I4)
  100 IF (INTERC - 50) 20, 24, 24
   24 ENTERC = INTERC
      ENERAT (I1,J1) = INERAT (I1,J1)
      EATRAC (I2,J2) = JATRAC (I2,J2)
  101 SUMT1 = SUMT1 + (ABSF (ENTERC - (ENERAT (I1,J1)*EATRAC (I2,J2))*AULT1
     1 ))**2.
  102 SUMT2 = SUMT2 + (ABSF (ENTERC - (ENERAT (I1,J1)*EATRAC (I2,J2))*AULT2
     1 ))**2.
      SUMB = SUMB + ENTERC*ENTERC
   20 CONTINUE
      P1 = SQRTF (SUMT1/SUMB)
      P2 = SQRTF (SUMT2/SUMB)
      PRINT 21, P1, P2, AULT1, AULT2
   21 FORMAT (10X,4 (E14.8, 5X))
      TSUMT1 = TSUMT1 + SUMT1
      TSUMT2 = TSUMT2 + SUMT2
    7 TSUMB = TSUMB + SUMB
      PRINT 23
```

<u>Error Program</u> - (contd)

```
23 FORMAT (1H0, 15X, 5HSUMPL, 14X, 5HSUMP2)
   SUMP1 = SQRTF (TSUMT1/TSUMB)
   SUMP2 = SQRTF (TSUMT2/TSUMB)
   PRINT 34, SUMP1, SUMP2
34 FORMAT (8X, E14.8, 5XE14.8)
   STOP
   END
   END
```

Interchange Cards. These cards contain the zone of origin in columns 1, 2, and 3 and the zone of destination in columns 4, 5, and 6. Columns 7, 8, 9, and 10 contain the volume of total work trips between the two zones and columns 11, 12, 13, and 14 contain the volume of auto driver work trips between the two zones.

The deck submitted with a program and data should contain a program, zone data cards for all zones, and then a class card followed by all interchange cards in that class, the next class card followed by all interchange cards in that class and so on until all classes are represented. The class cards are ordered so that the interzonal distances progress from the smallest to the largest values.

APPENDIX E

Exponential Curve Fitting by Least Squares

EXPONENTIAL CURVE FITTING BY LEAST SQUARES

The 1 mile interval propensity function values, calculated by the program described in Appendix D, were used to fit an exponential function by the method of least squares to the F(u) vs. u plots.

This curve fitting procedure is fairly straight foreward and an example is given using the following set of data:

X_o	Y_o	$U_o=\log_{10}Y_o$	Y_o^2	X_o^2	X_oU_o
0.65	1869	3.26161			
1.65	1535	3.18611			
2.65	1545	3.18893			
3.65	1186	3.07408			
4.65	1145	3.05881			
5.65	1223	3.08743			
6.65	968	2.98579			
7.65	1081	3.03383			
8.90	1070	3.02938			
42.10		27.90600	86.59339	258.9900	128.7415

$\bar{X}_o = 42.10/9 = 4.6777$

$\bar{U}_o = 27.90600/9 = 3.10067$

$\bar{X}_o^2 = 21.8809$

$\bar{X}_o \, \bar{U}_o = 14.5040$

$$m = (\sum X_o U_o - N(X_o U_o)/(\sum X_o^2 - N(X_o^2)) = -1.7945/62.0619$$

$$m = -0.02891$$

$$U - U_o = m\ (X - X_o)$$

$$U - 3.10067 = -0.02891\ (X - 4.6777)$$

$$U - 3.2359 = -0.02891\ X$$

Since $\log_{10} 1721.5 = 3.2359$ and $U = \log_{10} Y$

$$\log_{10} \frac{Y}{1721.5} = -0.02891\ X$$

Changing to natural logarithms

$$\log_e \frac{Y}{1721.5} = -\frac{0.02891\ X}{\log_{10} e} = -\frac{0.02891\ X}{0.43429} = -0.06657\ X$$

or $Y/1721.5 = e^{-0.06657\ X}$

or $Y = 1721.5\ e^{-0.06657\ X}$

The correlation coefficient between the plotted points and the curve is given by

$$R = \frac{N\ (\sum X_o U_o) - \sum X_o \sum U_o}{\sqrt{[N(\sum X_o^2) - (\sum X_o)^2]\ [N(\sum U_o^2) - (\sum U_o)^2]}}$$

$$R = \frac{9(128.7415) - 42.10\ (27.9060)}{\sqrt{[9(258.99)-(42.10)^2]\ [9(86.5934)-(27.9060)^2]}}$$

$$R = 0.8864$$

The computer solution using the above procedure resulted in a slightly different answer, because the computer carries calculations to more than 5 decimal places. The equation fitted to this data by the computer was

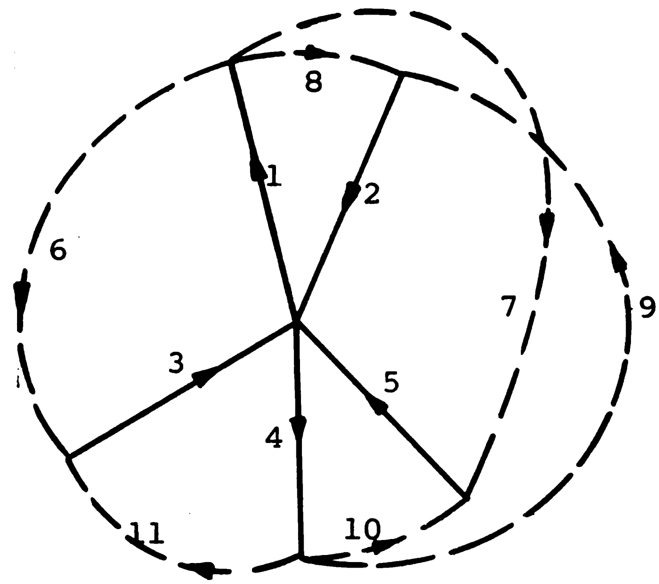
$$Y = 1738\, e^{-0.06814\, X}$$

and the correlation coefficient calculated by the computer for this data and equation was 0.8834.

APPENDIX F

Formal Solution of Sample Linear Graph Problem

FORMAL SOLUTION OF SAMPLE LINEAR GRAPH PROBLEM



Branches

b_1 1, 2, 3, 4, 5

b_2 none

Chords

c_1 6, 7, 8, 9, 10, 11

c_2 none

Figure 17. Linear Graph of Hypothetical System.

Terminal Equation:

$$\begin{bmatrix} X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{10} \\ X_{11} \end{bmatrix} = \begin{bmatrix} R_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{11} \end{bmatrix} \begin{bmatrix} Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \\ Y_{10} \\ Y_{11} \end{bmatrix} \quad (15)$$

Cutset Equation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \\ Y_{10} \\ Y_{11} \end{bmatrix} = 0 \qquad (16)$$

Circuit Equations:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{10} \\ X_{11} \end{bmatrix} = 0 \qquad (17)$$

Rewriting the Circuit Equation:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{10} \\ X_{11} \end{bmatrix} = 0 \qquad (18)$$

Rearranging the Cutset Equations:

$$\begin{bmatrix} Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \\ Y_{10} \\ Y_{11} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \\ Y_{10} \\ Y_{11} \end{bmatrix} \qquad (19)$$

Substituting equations 19 into equations 15:

$$\begin{bmatrix} X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{10} \\ X_{11} \end{bmatrix} = \begin{bmatrix} R_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{11} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \\ Y_{10} \\ Y_{11} \end{bmatrix} \qquad (20)$$

Substituting equations 20 into equations 18:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} + \begin{bmatrix} U \end{bmatrix} \begin{bmatrix} R_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{11} \end{bmatrix} \begin{bmatrix} U \end{bmatrix} \begin{bmatrix} Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \\ Y_{10} \\ Y_{11} \end{bmatrix} = 0 \quad (21)$$

Equations 21 can be written as

$$X_1 + X_3 + R_6Y_6 = 0$$

$$X_1 + X_5 + R_7Y_7 = 0$$

$$X_1 + X_2 + R_8Y_8 = 0$$

$$X_2 + X_4 + R_9Y_9 = 0$$

$$X_4 + X_5 + R_{10}Y_{10} = 0$$

$$X_3 + X_4 + R_{11}Y_{11} = 0$$

Solving any of the above equations for Y results in an equation, since $(X_a + X_b)$ is negative, of the form

$$Y_c = R_c^{-1} (X_a + X_b).$$

Reviewing the graph construction (Figure 3), Figure 17 elements 1 and 4 represent generation zones; elements 2, 3, and 5 represent attraction zones; and elements 6, 7, 8, 9, 10, and 11 represent routes. The above form suggests that the flow Y for a particular route can be determined by summing the X values of the generation and attraction zones corresponding to that route and multiplying this sum by the reciprocal of the resistance factor for that route. This is exactly what the linear graph model does when applied to a distribution system. The R^{-1} values are, of course, the propensity function values.