RIGID FRAME BRIDGE
FOR RIVER ROAD

Thesis for the Degree of B. S.
MICHIGAN STATE COLLEGE
George H. Cully
1940

GEORGE H. CULLY
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Rigid Frame Bridge
for River Road

A Thesis Submitted to

The Faculty of
Michigan State College
of
Agriculture and Applied Science

by

George H. Gully

Candidate for the Degree of
Bachelor of Science

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THESIS

INTRODUCTION

The River Road is a public thoroughfare parallel to the Red Cedar River. When the college first acquired this land south of the river, it was maintained as a lane for moving cattle and machinery from the farm buildings on campus to the various fields and pastures.

As the college gradually grew, the importance of this river road increased. From a farm lane to a public highway will be its progress in the next two years. The college is now proposing to pave this road eastward from the Gymnasium Bridge to Farm Lane and continue in this direction to the new radio tower. Its importance then will be greatly magnified as a public highway for people interested in the school's beautiful campus and farm lands.

Some three thousand feet east from the Gymnasium bridge is a single track railroad siding that crosses the proposed new pavement in a north and south direction. This track is frequently used to bring coal and other transit materials into the college. The contour elevations around this particular region are very unfavorable to a level crossing due to a high bank on the north side of the river. To maintain a level grade desirable for the railroad it would be necessary to have a "hump" in the roadway or a very undesirable fill on both sides of the track, extending for several hundred feet in an east and west direction. Furthermore, the crossing is at such a point that the line of sight is

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obstructed by a plot of pine trees which are very valuable. To cut these trees and grade the surrounding land to the required level of the road would entail quite an unnecessary expense.

With this in mind, I am suggesting for this location, a rigid frame concrete bridge resembling in general the beautiful Farm Lane bridge. This proposed structure has an attractive design and combines maximum beauty with necessary utility. It has proven to be economical, sturdy and durable yet graceful and artistic in appearance.

Conditions for this bridge specify a clear span of 42 feet, 17 ft. single railroad track, two 6 ft. sidewalks and a clearance of 16 ft. above the crown of the River Road pavement.

Specifications used in working these necessary problems were obtained through the Portland Cement Association.

I wish to acknowledge the assistance given me by Mr. Wiley of the Portland Cement Association; Mr. Miller and Mr. Meyer of the Michigan State College faculty.

Problem 1.

Frame Dimensions. Axis and Coefficients

Let Fig. 1 represent a right, hinged and symmetrical rigid frame with solid concrete deck and end walls. Determine the frame axis and coefficients.

The assumed frame axis are shown as dash and dot lines in Fig. . It is satisfactory to take the axis of the end walls as vertical lines 2 ft. 0 in. behind the faces of the walls. The actual deck axis is placed midway between the extrados and intrados; but the deck axis used in the moment distribution is assumed to be straight between the theoretical corner joints.

The elements of deck and end walls, which will be used in selecting frame coefficients from Charts II and III. The deck element is 42 ft. long, 3 ft. 0 in. deep at the crown and 6 ft. 0 in. deep at the corner joints. It will be assumed that the intrados of the deck is a parabola and that the moments of inertia are proportional to the cube of the depth.

The deck coefficient will be selected from Chart II. Enter the diagram with

$$d' = \frac{6.00 - 3.00}{3.00} = 1.00$$

and find S = 12 and rS = 8. The stiffness of the deck at b, or the moment at b necessary to give be a unit rotation at b when c is fixed, is

$$S \times \frac{I}{L}\text{, or proportional to } 12 \times \frac{3^3}{42} = 2.57$$

FRAME DIMENSIONS

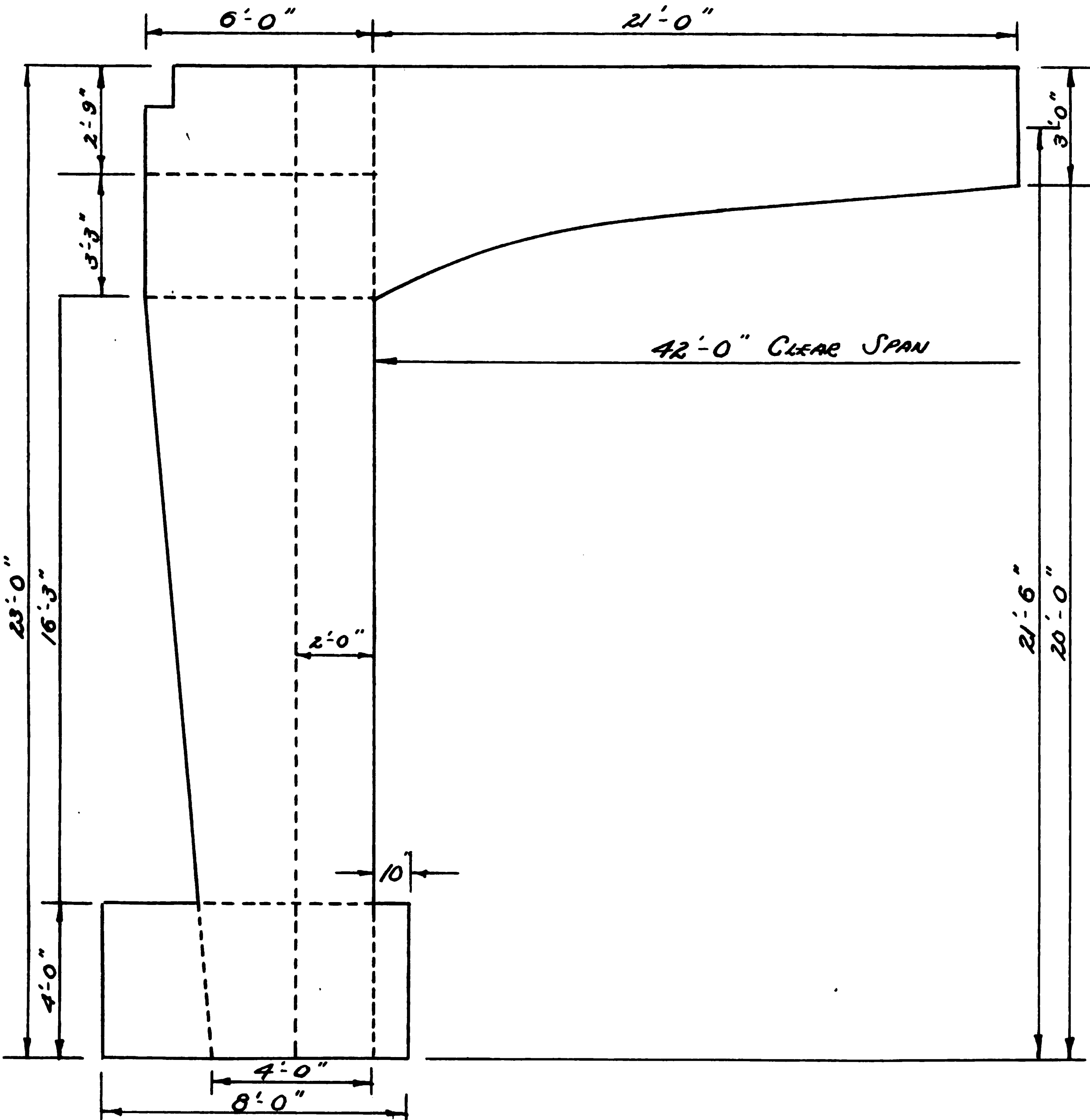


FIG. 1

The carry-over factor, r, equals

$$\frac{rS}{S} = \frac{8}{12} = 0.666$$

The end wall element is a trapezoid with a theoretical height of 20 ft. 3 in. confined by two straight lines 4 ft. 0 in. apart at a and 6 ft. 0 in. apart at b. Enter Chart III with

$$d' = \frac{6.00 - 4.00}{4.00} = 0.50$$

and find $S_b = 10.0$, $S_a = 5.4$, $r_a S_a = r_b S_b = 3.8$. The stiffness at b when a is fixed is $S_b \times I/L$. When a is hinged, as in this frame, it can be shown that the stiffness at b is

$$S_b \times \frac{I}{L}(1 - r_a r_b) = 10 \times (1 - \frac{3.8^2}{10.0 \times 5.4}) \times \frac{I}{L} = 7.4 \times \frac{I}{L} =$$

$$7.4 \times \frac{4.0^3}{20.25} = 23.4$$

The relative stiffness in per cent at b is then

$$\frac{2.57}{2.57 + 23.4} \times 100 = 10 \text{ for the deck and}$$

$$\frac{23.4}{2.57 + 23.4} \times 100 = 90 \text{ for the wall.}$$

Problem 2.

Distribution of Fixed End Moment

The frame in Problem 1 is subject to a fixed end moment of + 100.00 in the deck at joint b as indicated in Fig. 2. Find the final moments at b and c.

b (90.0)	10.0	0.66	10.0	c (90.0)
0	+100.0	Fixed End Moments	0	0
-90.0	- 10.0	Distributed "	0	0
0	0	Carry Over "	- 6.6	0
0	0	Distributed "	+ .66	+5.94
0	+ 0.435	Carry Over "	0	0
-0.392	-0.043	Distributed "	0	0
0	0	Carry Over "	0. 028	
0	0	Distributed "	+.0028	+.0258
-90.39	+90.39	Total Moments	-5.9658	+5.9658
a				d

Fig. 2.

The computations are recorded in Fig. 2. The first line of figures contains the given moments when b and c are fixed: Zero and + 100.00 at b, zero and zero at c.

When joint b is released, it will be rotated in the clockwise direction by the moment of + 100.00. The rotation induces new moments in ba and bc, moments that counteract the rotation. When the joint comes to rest in its new position, the sum of the newly induced moments in ba and bc equals - 100.00, and the distribution between the two members is in proportion to their stiffness. The distributed moments, - 100.00 x 0.10 = 10 in bc and - 100.00 x 0.80 = -90 in ba, are given in the second line. The total moments after the first cycle of distribution are not recorded but are seen to be: - 90.00 and + 90.00 at b, zero and zero at c.

The second cycle of distribution begins with the carrying over of moment from the joint that was rotated to the joint that remained fixed. Obviously, no moments are carried over to or from joints a and d, which are hinged in this frame. The moment carried over to c from b equals 10 x .66 = 6.6, according to the definition of carry-over factor, and is recorded in the third line. The fourth line gives distributed moments obtained by release and rotation of c, the procedure of distribution moments at c are + 6.6 x 0.10 = + .66 in cb and + 6.6 x 0.90 = + 5.94 in cd. The total moments after the second cycle of distribution are: - 90.00 and + 90.00 at b, - 5.94 and + 5.94 at c.

Two additional cycles of distribution are carried out by a procedure identical with that in cycle 2. The total moments after the fourth cycle are: -90.39 and + 90.39 at b; - 5.96 and + 5.96 at c; these are the final corner moments obtained after distribution of the original F. E. M. of

+ 100.00 in bc at b.

If an F. E. M. of - 100.00 is applied in ba at b(i.e., in the end wall immediately below the corner joint) and the joints are then released, the final corner moments, derived from Fig. 8, become: - 100.00 + 90.00 + .392 = - 9.608 in ba at b; + 10.00 - .435 + .0435 = + 9.608 in bc at b; + 5.966 and - 5.966 at c.

These relative moment values will be useful for subsequent analyses by eliminating repetition of computations.

Problem 3.

Dead Load

The weight of the end walls is carried directly down to the footings and creates no moments. The longitudinal section through the deck is divided into an area 42 ft. long with a constant depth of 3 ft. 0 in., weighing 375 p.s.f. The remaining area is subdivided as shown in Fig. and the areas of the subdivisions, multiplied by 150, are used as concentrated loads for determination of moments.

In order to determine the fixed end moments, enter Chart II with d' = 1.00 (see Problem 1) and select the proper coefficients.

Fixed End Moment per foot of width:

Uniform load

$$270 \times 60^2 \times .102 = 67,473 \text{ ft. lb.}$$

$$2470 \times 42 \times 0.83 = 8,610$$
$$875 \times 42 \times 0.182 = 6,688$$
$$174 \times 42 \times 0.195 = 1,425$$
$$174 \times 42 \times 0.12 = 877$$
$$875 \times 42 \times 0.045 = 1,654$$
$$2470 \times 42 \times 0.01 = \underline{104}$$
$$19,358$$

The total moment when the deck is straight is

$$19,358 \times (0.9039 + 0.0596) = 18,400 \text{ ft. lb.}$$

and produces tension in the outside of the corner.

Correcting this moment for curvature of the deck according to Section VIII, the rise of the deck centerline in Fig. 1 being 1 ft. 6 in., the final corner moment is

$$18,400 \times \frac{20.25 + 0.5 \times 1.5}{20.25 + 1.5} = 18,200 \text{ ft. lb.}$$

The crown moment for straight deck centerline can now be found by statics. The total positive moment assuming a simply supported deck is:

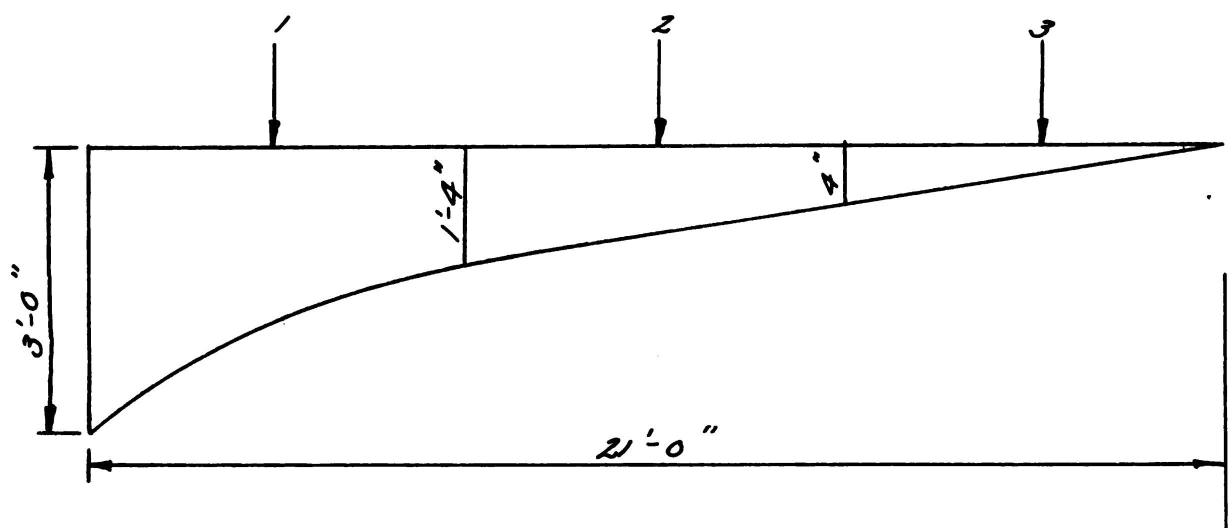
$$2470 \times 42 \times 0.09 = 934$$
$$875 \times 42 \times 0.225 = 8,270$$
$$174 \times 42 \times .419 = 3,070$$
$$1/8 \times 375 \times 42 \times 42 = \underline{82,800}$$
$$95,074 \text{ ft. lb.}$$

The difference between this moment and the negative corner moment created by the same loading,

$$95,074 - 18,400 = 76,674 \text{ ft. lb.},$$

is the moment at the crown of the frame with straight deck.

: LOADING :



	1	2	3
POINT OF APPLICATION	0.91	0.745	0.581
LOAD	2,470#	875#	174#
AREA	16.49	5.83	1.162

: Fig. 3 :

Correct for curvature of deck and determine the final crown moment (tension in bottom of deck): = 74,600.

The total dead load of the frame, one foot wide, is

Wearing surface: 15 x 42	630
Deck: 3 x 42 x 150	18,900
Deck: 0.33 x 3 x 42 x 150	6,300
Corners: 3 x 6.25 x 2 x 150	5,625
	62,392

The vertical reaction on each footing is

0.5 x 62,400 = 31,200 lb.

The horizontal thrust at the footing, when the deck is curved, is

$$\frac{18,200}{20.25} = 900.0$$

The crown thrust also equals 900.0 lb., since all the loads are gravity loads.

The moments, thrusts and shears for dead load are shown in Fig. 3 (a), below.

Problem 4.

Live Load

The frame in Problem 1 carries a concentrated live load of 8,750 lb. per foot of width including allowance for impact. Find the maximum moments and the thrusts at the crown and at the corner.

Maximum moment is produced at the crown when the concentrated load is placed at the midpoint and the uniform load covers the entire span. The first step in the analysis is to determine the fixed end moment coefficients by entering Chart II with d' = 1.0

Fixed End Moment per foot of width:

Uniform load: $8750 \times 42^2 \times 0.102 = 1,575,000$ ft. lb.

By using the values from Problem 2, the corner moment is found to be 1,575,00 x (0.9039 x 0.0596) = 1,530,000.

The total positive crown moment assuming a simply supported deck is

$8750 \times 21 \times 0.5 \times 21 = 1,934,000$ ft. lbs.

The difference between this moment and the negative corner moment created by the same loading,

$1,934,000 - 1,530,000 = 404,000$ ft. lbs.,

is the moment at the crown of the frame with straight deck.

Correct for curvature of deck and determine the final crown moment (tension in bottom of deck): = 372,000.

The corner moment when the deck is curved is $1,530,000 \times \frac{21.00}{21.75} = 1,420,000$ ft.lb.

The corresponding horizontal thrust is $\frac{1,420,000}{20.25} = 70,100$

The vertical reaction on each footing is $8750 \times 0.5 \times 42 = 183,750$ lb.

Maximum moment at the corner (point 1.0 in Chart II) is produced with uniform load over the entire span and when the concentrated load is placed at or near point 0.625. This is evident by inspection of Charts I and II. The following values are obtained with the load of 8,750 lbs. at point 0.625.

Fixed End Moment per foot of width

at point 1.0 = $8750 \times 42 \times .20 = 73,500$

at point 0.0 = $8750 \times 42 \times .10 = 36,750$

Corner moment after distribution (according to Fig. 2)

at 1.0 = 73,500 x .749 + 36,750 x .127 = 59,660

at 0.0 = 73,500 x .127 + 36,750 x .749 = 36,825

Maximum corner moment (incl. uniform load) when deck is straight

59,660 + 36,825 x (.749 + .127) = 92,000

Maximum corner moment allowing for curvature of deck

92,000 x .973 = 89,500 lb.

Problem 5.

Change in Length of Deck and Horizontal Displacement

A relative change in length of deck may be either a shortening (temperature drop, shrinkage, outward displacement of footings) or a lengthening (temperature rise, inward displacement of footings).

Assume that the frame in Problem 1 is subject to a deck shortening due to (a) temperature drop of 45° F., (b) shrinkage corresponding to a shrinkage factor of 0.0002, and (c) outward horizontal displacement of the footings equivalent to a contraction coefficient of 0.0002.

The shortening per unit of length is

$$45 \times 0.000006 + 0.0002 + 0.0002 = 0.00067$$

The total shortening in the span of 42 feet is

$$0.00067 \times 42 = 0.04 \text{ ft.} = 0.02$$

This is equivalent to an outward displacement of 0.01 ft. at a and d.

When analyzing the frame by moment distribution, begin by locking the joints b and c. Fig. 4 illustrates the static conditions from which the fixed end moment at b is determined. It can be shown that

$$\text{F. E. M. at } b = \frac{E \times D}{L} \times S_b \times \frac{I_a}{L} \times (1 - r_a r_b) =$$

$$\frac{(3 \times 10^6 \times 12^2) \times 0.01}{20.25} \times 13.3 \times \frac{(1/12 \times 4^3)}{20.25} \times (1 - \frac{3.8^2}{10 \times 5.4} = 394,700 \text{ ft.lbs.}$$

According to Problem 2, the formula for both corner and crown moment - when deck is straight - may be written as

$$\text{Moment} = 394{,}700 \quad (1.000 - 0.900 - .0092) - .0596 \quad = 11{,}841.00$$

Final moments - allowing for curvature of deck - equal

$$\text{Corner: } 11{,}841 \times \frac{20.25}{21.75}^{2} = 10{,}242$$

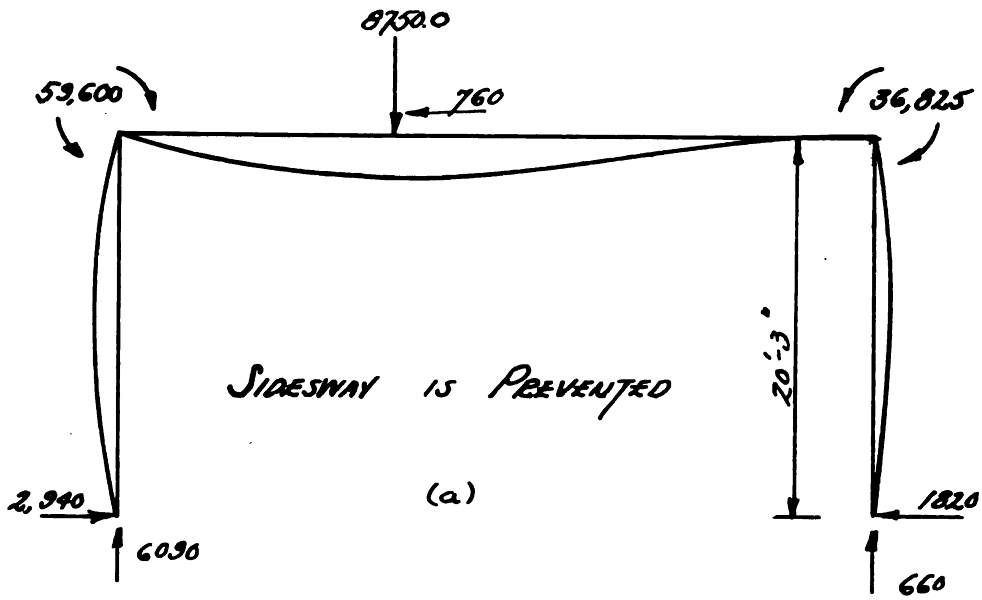
$$\text{Crown: } 11{,}841 \times \frac{20.25}{21.75} = 10{,}071$$

Problem 6.

Earth Pressure

In most rigid frame bridges, the walls are relatively short and stubby. In these cases, earth pressure moments determined by moment distribution are sensitive to changes both in frame coefficients and in the assumed curvature of the deck. Since, in addition, these moments are based upon uncertain assumptions incident to the conventional earth pressure theory, a rather poor degree of accuracy may be expected. Analysis for earth pressure by moment distribution may then be unsatisfactory.

In comparison with the effect of other loads, earth pressure moments are generally of little importance in stress determination. In this example, the corner moment due to earth pressure is less than 2 per cent of that caused by dead plus live load and may therefore be disregarded. At the crown, both dead and live loads create tension at the intrados, while earth pressure creates tension at the extrados; it is therefore more conservative to disregard the earth pressure moment at the crown.


8750.0
59,600
760
36,825
SIDESWAY IS PREVENTED
20'-3"
(a)
2,940
6090
1820
660

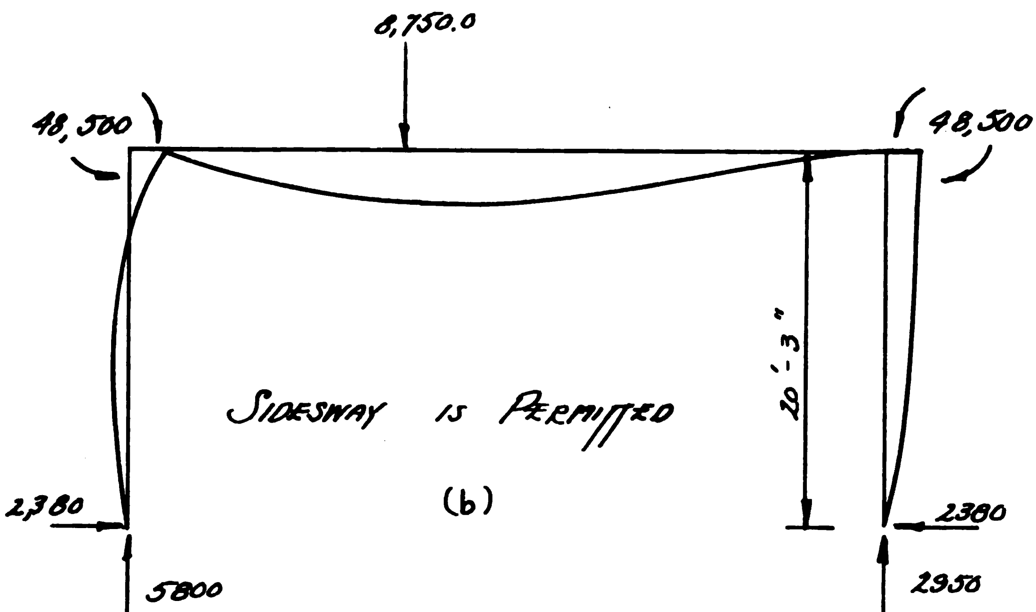

8,750.0
48,500
48,500
SIDESWAY IS PERMITTED
20'-3"
(b)
2,380
5800
2380
2950

Fig. 4

Problem 7.

Dissymmetry and Sidesway

If the frame or loading is unsymmetrical, the moment distribution method as discussed and applied in the foregoing gives horizontal thrusts that apparently do not satisfy the statical requirements for equilibrium.

For illustration, take the frame in Fig.   loaded with 8,750 lb. at point 0.625. The corner moments, determined in Problem 4 for straight deck, are

at point 1.0: 59,600

at point 0.0: 36,825

The corresponding horizontal thrusts are

$$\frac{59,600}{20.25} = 2940 \text{ at a and}$$

$$\frac{36,825}{20.25} = 1820$$

The algebraic sum of the horizontal forces is 2,940 - 1,820 = 1120 lb., but it should be zero to satisfy the static requirement that the sum of the projections of all external forces on any line must be zero. This apparent discrepancy will be clarified by the discussion of sidesway which follows.

It is evident that the deck bc in Fig.   (a) will tend to move sidewise relative to a and d whenever the frame or the loading is unsymmetrical, and also that a lateral displacement of bc will set up moments at the corners. Refer to Problems 2 and 4 and observe that no fixed end moment due to displacement of bc was included in the analysis. The significance of this omission is that points b and c have been kept in their original position vertically above points a and d; or, as it is called, sidesway of the frame has been prevented.

It is obvious that an external force must be added in the line bc when horizontal displacement of bc is to be prevented. The laws of equilibrium require that the force equal 1120 lb. The loads, reactions and deflected axis for the frame in which sidesway is prevented are shown in Fig. (a). The force of 1120 lb. in bc increases the vertical reaction at a (and decreases the vertical reaction at d), thereby making it equal to

$$8,750 \times \frac{26.3}{42} + 1120 \times \frac{20.25}{42} = 6,090 \text{ lb.}$$

The tendency of a strip of frame to move horizontally as indicated may be counteracted by either the adjacent strips of the frame, by the backfill or by the approach slab. The assumption that no sway takes place is therefore preferable, especially since it gives the greater corner moment. It shall be illustrated, however, how readily results obtained by moment distribution may be adjusted to allow for the assumption that sidesway is permitted. Consider, for example, the frame is, analysed by moment distribution, in which a force of 1120 lb. is required to prevent sidesway. Eliminate this force by adding another equal but opposite force in bc, simultaneously displacing the deck horizontally in the direction from b to c. Determine the F. E. M. and then by moment distribution - in a manner similar to that in Problem 5 - the final corner moments. This general procedure can often be simplified. In the frame for example, the added force of 1120 lb. obviously creates the same horizontal thrust at each footing, namely, 560 lb. The total horizontal thrust at both footings when sideway is permitted must therefore be

$$2{,}940 - 560 = 1{,}820 + 560 = 2{,}380 \text{ lb.},$$

and the corner moments at joints a and b are

$$2{,}380 \times 20.25 = 48{,}250 \text{ ft. lb., say } 48{,}500 \text{ ft. lb.}$$

The corresponding maximum negative corner moment is 59,000 when sidesway is prevented.

Problem 8.

Shears

The frame in Problem 1 is subject to dead and live load, deck shortening and earth pressure. The loads are the same as in Problems 3, 4, 5 and 6 except that the concentrated live load for shear calculations will be taken as 10,000 lb. instead of the 8,750 lb. used for moment calculations. Find the maximum total shear and unit shearing stresses at (a) the crown, (b) the corner, and (c) the top of the footing.

(a) Crown. The shear is zero due to dead load, deck shortening and symmetrical earth pressure. The corresponding unit shearing stress is

$$\frac{5000}{12 \times 7/8 \times 21} = 30 \text{ p. s. i.}$$

Further investigation of the shear based upon the assumption that sideway is prevented (see discussion Problem 7) is unwarranted.

(b) Corner. The total dead load from face to face of end walls is

Wearing Surface: $15 \times 42 =$	630
Deck: $3 \times 42 \times 150 =$	18,900
Deck: $0.33 \times 3 \times 42 \times 150 =$	6,300
	25,830

The maximum shear due to dead load is

$$\frac{25,830}{2} = 12,900 \text{ lb.}$$

The corresponding unit shearing stress is

$$\frac{12,900}{12 \times 7/8 \times 50} = 25 \text{ p. s. i.}$$

(c) Top of Footing. The maximum shear equals the horizontal thrust at the support. The dead load thrust is $\frac{18,200}{20.25} = 900$ lb., see Fig. (a). The maximum horizontal thrust due to live load is produced by the same load arrangement that causes maximum corner moment. The maximum corner moment, derived from the analysis in Problem 4, is

$$\frac{10,000}{8,750} \times 59,600 + 32,200 = 64,000 + 32,200 = 96,200 \text{ ft. lb.}$$

with straight deck; but allowing for curvature of deck it equals

$$96,200 \times \frac{21}{21.75} = 93,000$$

The horizontal thrust is

$$\frac{93,000}{21} = 4,430 \text{ lb.}$$

The horizontal thrusts produced by earth pressure and deck shortening counteract the thrusts due to dead and live load. It is therefore on the safe side to disregard earth pressure and deck shortening and to take the maximum shear as

$$900 + 4,430 = 5,330 \text{ lb.}$$

The corresponding unit shearing stress is

$$\frac{5,330}{12 \times 7/8 \times 48} = 15 \text{ p. s. i.}$$

Problem 9.

Stresses at Crown and Corner

The frame in Problem 1 is subject to dead load, live load and change in length of deck. A summary of these loads and the moments and thrusts they create. Choose tensile and compressive reinforcement and ascertain that the corresponding unit stresses do not exceed the allowable working stresses, which will be chosen conservatively as $f_s = 18{,}000$ and $f_c = 1{,}000$ p. s. i.

Moments and thrusts at the midpoint of the deck are

	Moment	
Dead Load	+ 74,600	+ 900
Live Load	+372,000	+70,100
Change in Deck Length	+ 10,070	- 500
Total	+456,670	+70,500

Eccentricity with respect to centerline: $\frac{456{,}670}{70{,}500} = 6.5 = 78$ in.

The tensile steel area for this moment and thrust must be somewhat less than that required when the axial thrust is disregarded; namely

$$\frac{456{,}670 \times 12}{18{,}000 \times 7/8 \times 36} = 8.1 \text{ sq. in.}$$

With a required steel area of 8.1 further computations prove that this type of beam is not sufficient for such a great moment. A 36 in. beam was used as a limit for workability, expense and strength value but this bridge would require a beam twice that size.

Reducing the impact factor to 25% would not result in a bending moment sufficiently small to produce a beam within the 36" limits.

As a result of these final calculations, I feel justified to say that a Rigid Frame Bridge of this span, is not practical enough to support an E-60 Cooper Engine Loading.

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