FLOW OF POLYVINYLPYRROLIDONE SOLUTIONS THROUGH PACKED BEDS

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ABSTRACT

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By

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The semi-theoretical Ergun equation is well established for flow of Newtonian fluids through packed beds. This definitive study provides a basis for analysis of effects of non-Newtonian fluid flow through packed beds and is based on data for aqueous solutions of Polyvinylpyrrolidone (PVP), which are viscous non-Newtonian fluids. A number of equations describing flow phenomena in packed beds are derivable for purely viscous non-Newtonian fluids; all of which reduce to the Blake-Kozeny form in the case of Newtonian fluids.

Meter's four parameter model was assumed to characterize the rheological behaviour of aqueous PVP solutions. The Meter's model analog of the Ergun equation was employed to correlate the pressure drop-flow rate data. Both numerical and analytical techniques were employed in this analysis. Packed bed pressure drop versus flow rate data were obtained for aqueous PVP solutions of concentration 0.5, 1.0, 3.0 and 4.0 percent by weight. It was concluded that the modified Ergun equation may be used to describe results for the 0.5 and 1.0 percent by weight PVP solutions. However, wide deviations between the experimental friction factor and those from the Ergun equation were observed for 3.0 and 4.0 percent solutions. It is speculated that this departure from the modified Ergun equation is a result of surface and viscoelastic effects. Modification in the capillary model may be needed to account for such effects. A comparative study between constant flow rate and constant pressure drop experiments is suggested in order to further resolve this matter.

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THROUGH PACKED BEDS

By

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NOMENCLATURE

D_c	= Column diameter
D_p	= Particle diameter
f_{expt}	= Experimental friction factor
f_{calc}	= Calculated friction factor
G_0	= Mass flow rate based on superficial velocity
L	= Bed length
M	= Correction factor for wall effects
$(N_{Re})_{eff}$	= Effective Reynolds number
Q	= Flow rate
R	= Radius
R_h	= Hydraulic radius
T_m	= Shear stress when η drops down to $1/2(\eta_0+\eta_\infty)$
T_{rz}	= Shear stress
T_w^*, T_w	= Wall shear stress
V_z	= Velocity
V_z	= Average velocity
ΔP	= Pressure drop
η	= Viscosity
η_0	= Viscosity at zero shear stress
η_∞	= Viscosity at infinite shear stress
ϵ	= Porosity
ρ	= Density
γ_w	= Wall shear rate

INTRODUCTION

A study of non-Newtonian fluid flow through porous media has wide applications in engineering science. Packed bed equations are very useful in individual problems such as industrial filtration of polymer solutions and slurries, movement of aqueous solutions through sand in secondary oil recovery, and design of packed bed reactors and towers. Furthermore, models proposed in the literature to describe non-Newtonian fluid behaviour have been tested in most cases only in relatively simple flow geometry. A test of these models for highly complex geometry of packed beds, for example, enhances the reliability and confidence in their application.

The most definitive study of flow of Newtonian fluids through porous medium appears to be that of Ergun [1]. Ergun's study provides a basis for similar analysis of non-Newtonian fluid flow. Good discussions of this analysis and extension of Ergun equation to viscous non-Newtonian fluids are provided by Bird et al.[2], Gaitonde et al.[3], Marshall et al.[4], Christopher et al.[5], Sadowski et al.[6], and Park et al.[7]. Different types of generalized methods to study this problem have been proposed. Aside from dimensional analysis, the main approach to this problem may be categorized as (1) generalized Darcy's law approach, (2) the capillary model combined with a particular rheological equation, or (3) the use of Newtonian equations containing an apparent viscosity evaluated at some appropriate average bed shear rate. So far, the second approach has been most preferred. Bird, Sadowski, Marshall, Gaitonde, and Park are a few among those who used this approach selecting Ellis, Power law, Hershel Bulkley, Spriggs and Meter's rheological equations for a variety of polymer solutions.

The capillary model equation for flow in packed bed is a simplification since the actual flow involves fluids passing through irregular interstices between the particles. Deviations might be expected for non-Newtonian fluid flow through packed beds, due to frequent acceleration and deceleration. A literature survey indicated some evidence of viscoelastic effects and surface adsorption phenomena. Sadowski pointed this out and claimed to have observed viscoelastic and surface adsorption effects for aqueous polymer solutions. Marshall and Metzner also observed similar deviations and argued that the viscoelastic effect is the single cause of deviations. On the contrary, Christopher, Middleman, and Gaitonde, in similar studies of polymeric fluids, observed no such deviations, hence reported absence of viscoelastic effects. Christopher correlated Sadowski's data successfully with capillary model equation combined with Power law and surmised that Sadowski's conclusion is an artifact of his modification of Blake-Kozeny equation.

More recently Park et al. reported occurrence of viscoelastic effects for dilute polymeric solutions. In particular for PVP solutions departure from Ergun equation occurred at effective Reynolds number greater than one. It was noted that the deviations were larger for most dilute solution in contrast to the other polymeric solutions. The deviations decreased with an increase in concentration. Thus it was decided to investigate PVP solutions further and extend the data to higher Reynolds numbers.

The following assumptions were made in developing the capillary model equations for packed beds in this analysis:

(1) The fluid is incompressible.

(2) The porous medium is isotropic and of regular geometry.

(3) Inertial terms from the equation of motion are deleted.

(4) The fluid is homogeneous.

PACKED BED THEORY AND EQUATIONS

Analyses of non-Newtonian fluid flow through packed bed generally fall into three categories: (1) generalized scaleup, (2) the capillary model combined with a particular rheological equation, and (3) the use of Newtonian equation containing an apparent viscosity evaluated at some average bed shear rate. Approach (1) is an extension of Darcy's law and does not require a rheological model. Darcy's law simply states that for a given bed and Newtonian fluid, flow rate is proportional to the pressure drop. As discussed by Park et al.[7], the inherent complexities involved in the solution are quite cumbersome. The use of Newtonian equations containing an apparent viscosity evaluated at some appropriate average bed shear rate is not generally an acceptable solution, although it may turn out to be reliable for a particular solution. The second method as the choice made by the earlier workers, is both reasonable and supported by extensive experimental measurements. It is chosen as the basis for analyzing measurements of this study.

In the second method, the packed bed is regarded as a bundle of capillary tubes of complex cross sections and shape. The theory is then developed by applying the results of hydrodynamic analysis of straight tube to the collection of crooked capillaries. The tortuous shape of the capillaries is accommodated by making an appropriate correction to the length by a factor 25/12, as in the Newtonian case. Of course, it becomes necessary to make a suitable choice of rheological equation.

Apparently the choice is between a constitutive equation which describes viscoelastic or normal stress effects and equations describing purely viscous flow behaviour. An appropriate choice would be that one which fits the experimental viscometric data for all ranges of shear rate; at least in the range of average bed shear rate.

It has been shown that Meter's[8] model given below may be used to predict the non-Newtonian properties of the PVP solutions.

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + \left|\frac{T}{T_m}\right|^{\alpha-1}} \tag{1}$$

where η = viscosity,

η_0, η_∞ = viscosities at shear rates approaching zero and infinity,

T_m = shear rate when viscosity drops down to 1/2, and

α = constant exponent.

Park[9] investigated this model for PVP solutions and determined the parameters η_∞, η_0, T_m and α. Table 6 contains the parameters for the relevant solutions. Equation (1) can be rewritten as:

$$\frac{\eta_0}{\eta} = \left[1 + \left|\frac{T}{T_m}\right|^{\alpha-1}\right] \sum_{J=0}^{\infty} \left[- \left|\frac{T}{T_m}\right|^{\alpha-1} \frac{\eta_\infty}{\eta_0}\right]^J \tag{2}$$

The equations derived in Appendix A based on the hydrodynamic analysis of the capillary model of the packed bed for Meter's model may be summarized as follows. The average velocity V_z through the available cross section for flow of the capillary is

$$\langle V_z \rangle = \frac{2R_h}{\eta_0 T_w^3} \int_0^{T_w} T_{rz} \left[\frac{1 + \left[\frac{T_{rz}}{T_m}\right]^{\alpha-1}}{1 + \frac{\eta_\infty}{\eta_0}\left[\frac{T_{rz}}{T_m}\right]^{\alpha-1}}\right] dT_{rz} \tag{3}$$

where $T_w = 2R_h \Delta P/2L$, shear stress at the wall of the capillary,

ΔP = pressure drop, and L = length of the bed.

The effective viscosity η_{eff} is:

$$\frac{1}{\eta_{eff}} = \frac{1}{\eta_0}\left[1 + \frac{2}{\alpha+1}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1} - \frac{\eta_\infty}{\eta_0}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\left\{\frac{4}{\alpha+3} + \frac{2}{\alpha+1}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\right\}\right.$$
$$\left. + \left(\frac{\eta_\infty}{\eta_0}\right)^2\left(\frac{T_w^*}{T_m}\right)^{2\alpha-2}\left\{\frac{2}{\alpha+1} + \frac{4}{3\alpha+1}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\right\} + - - + - - -\right] \tag{4}$$

where $T_w^* = 12\ D_p \Delta P\ /\ 25\ 6M(1-\epsilon)L$, is a measure of wall shear stress in the bed. The effective viscosity so defined is useful only when $(\eta_\infty/\eta_0)^2(T_w^*/T_m)^2 < 1$. When $(\eta_\infty/\eta_0)^2(T_w^*/T_m)^2 \geq 1$, superficial velocity V_0 based on the column diameter, expressed as a product of average velocity and porosity ϵ,

$$V_0 = \epsilon\langle V_z\rangle = \frac{\epsilon 2R_h}{\eta_0 T_w^{*3}}\int_0^{T_w^*} T_{rz}^3\left[\frac{1 + \left(\frac{T_{rz}}{T_m}\right)^{\alpha-1}}{1 + \frac{\eta_\infty}{\eta_0}\left(\frac{T_{rz}}{T_m}\right)^{\alpha-1}}\right]dT_{rz} \tag{5}$$

$$\eta_{eff} = \frac{\epsilon^3 D_p^2 \Delta P}{150\ M^2(1-\epsilon)^2 V_0} \tag{6}$$

and

$$f = \frac{150(1-\epsilon)M\eta_{eff}}{D_p G_0} \tag{7}$$

This result is good when the void fraction is less than 0.5 and is valid only in the region given by $D_p G_0/(1-\epsilon)M\eta_{eff} < 10$, where $G_0 = \rho V_0$. For highly turbulent flow in packed beds the friction factor is a function of roughness only, and remains fairly constant and is,

$$f = 1.75 \tag{8}$$

Hence the modified Ergun equation as in Appendix A is:

$$\left[\frac{\rho\ \Delta P}{G_0^{\ 2} M}\right]\left[\frac{D_p}{L}\right]\left[\frac{\epsilon^3}{1-\epsilon}\right] = \frac{150}{\dfrac{D_p G_0}{(1-\epsilon) M \eta_{eff}}} + 1.75 \qquad (9)$$

If we define

$$f_{expt} = \left[\frac{\rho\ \Delta P}{G_0^{\ 2} M}\right]\left[\frac{D_p}{L}\right]\left[\frac{\epsilon^3}{1-\epsilon}\right]$$

and

$$f\ calc = 150 \Big/ \frac{D_p G_0}{(1-\epsilon) M \eta_{eff}} + 1.75$$

it can be seen that in the low flow regions the logarithmic plot of f_{expt} vs. $(N_{Re})_{eff}$ will be a straight line with a slope of -1.

Equation (5) is numerically solved to obtain effective viscosity. Appendix A contains the details and the techniques involved in the analysis. Hence in principle one can modify the capillary tube approach to account for departure from Newtonian flow. In this study the Meter's model equation for flow through a tube packed with spherical particles was applied.

EXPERIMENTAL PROCEDURE

The schematic diagram (Figure 1) depicts the experimental set up. The equipment consisted of two glass columns of inner diameter 1 inch and 1/2 inch and spherical glass beads of average diameter 0.1621, 0.0597 and 0.0432 centimeters. Segments of glass columns are assembled together with aluminum flanges. Two stainless steel screens at either end contained the glass beads in the column. Extensive care is needed while packing to avoid any possible air entrapment. On either end, at least 6 inches of extra packing above the test section were provided to avoid end effects and foreign particles influencing the flow pattern in the test section. Two pre-calibrated rotameters for low and high flow rates respectively were used. "U" tube mercury manometers served to measure the pressure drop. The packed glass column and the tank containing the solution were immersed in constant temperature water jacket and bath respectively. Extreme care was taken to maintain the temperature at $21 \pm 0.5^{o}C$ in the baths. Water was recirculated for the entire period in the bath and the jacket.

The average molecular weight of PVP (k-90) used was 360000. Aqueous solutions of PVP of 0.5, 1.0, 3.0, 4.0 percent by weight were prepared in distilled water and filtered to avoid gel.

The solution was forced to flow through the bed by a constant nitrogen pressure in the tank. Prior to recording any data the solution was allowed to flow for some time until the refractive index of the

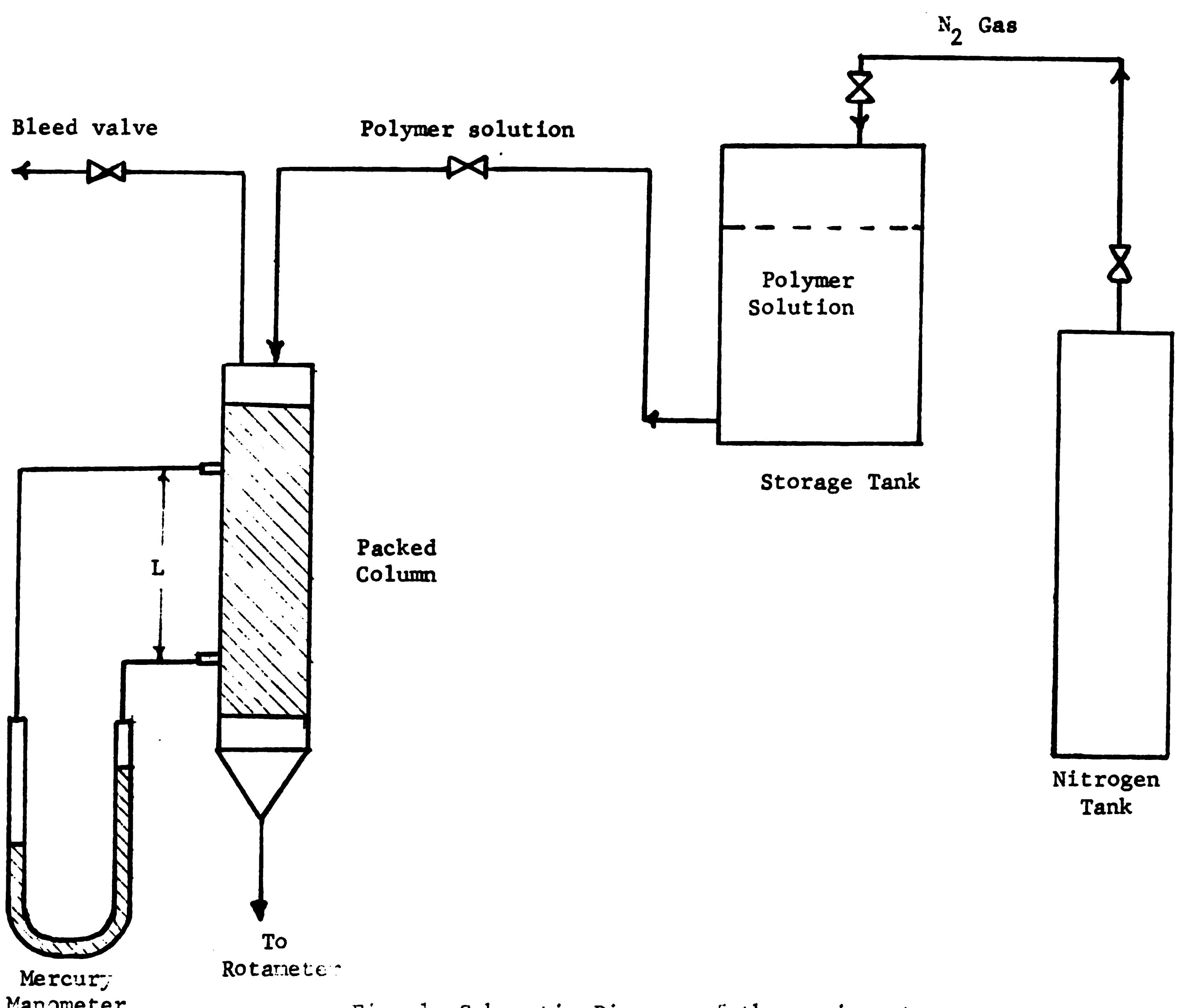

Fig. 1 Schematic Diagram of the equipment.

solution matched with that of the sample. Constant flow rate and pressure drop were the indicators of steady state flow. Pressure drop and flow rate were measured simultaneously under steady state conditions.

The bed porosity was calculated from the packed bed data for distilled water by employing Ergun equation and known particle diameter before each run of solution. Only data for which $(N_{Re})_{eff}$ less than 10 have been used for such bed calibrations. Experimental data were obtained for water in the same range as that of the solutions.

RESULTS AND DISCUSSION

Results of the experiments are summarized in Figures 2 through 6 as a set of plots depicting the f_{expt} and f_{calc} vs. $(N_{Re})_{eff}$ for each aqueous solution. Effective viscosity vs. wall shear rate plots (Figure 7 and Figure 8) are presented to make further comparison and to gauge the usefulness of Ergun equation to predict viscosity of non-Newtonian fluids.

The Newtonian data presented in Figure 6 serves to calibrate the bed and technique. Figures 2 and 3 show the general agreement of 0.5 and 1.0 percent solutions and water with Ergun equation. This agreement is an indication of accuracy and consistency of sets of data. This was felt especially important in view of Park's[9] contrasting conclusions. Having observed greater deviations for 0.5 and 1.0 percent solutions, he concluded that the viscoelastic effects are significant above $(N_{Re})_{eff}$ equal to 1. The check between the Newtonian fluid and PVP solutions confirm no such viscoelastic effects. These results are consistent with those reported by Gaitonde et al. and Christopher et al. even though the flow regions are of considerably higher Reynolds number.

The data for 3.0 and 4.0 percent solutions when plotted (Figure 4 and Figure 5) fall well above the theoretical curve. In fact, the data fall on a line somewhat parallel to the theoretical curve. The ratio of the experimental values of friction factor and those given by the Ergun

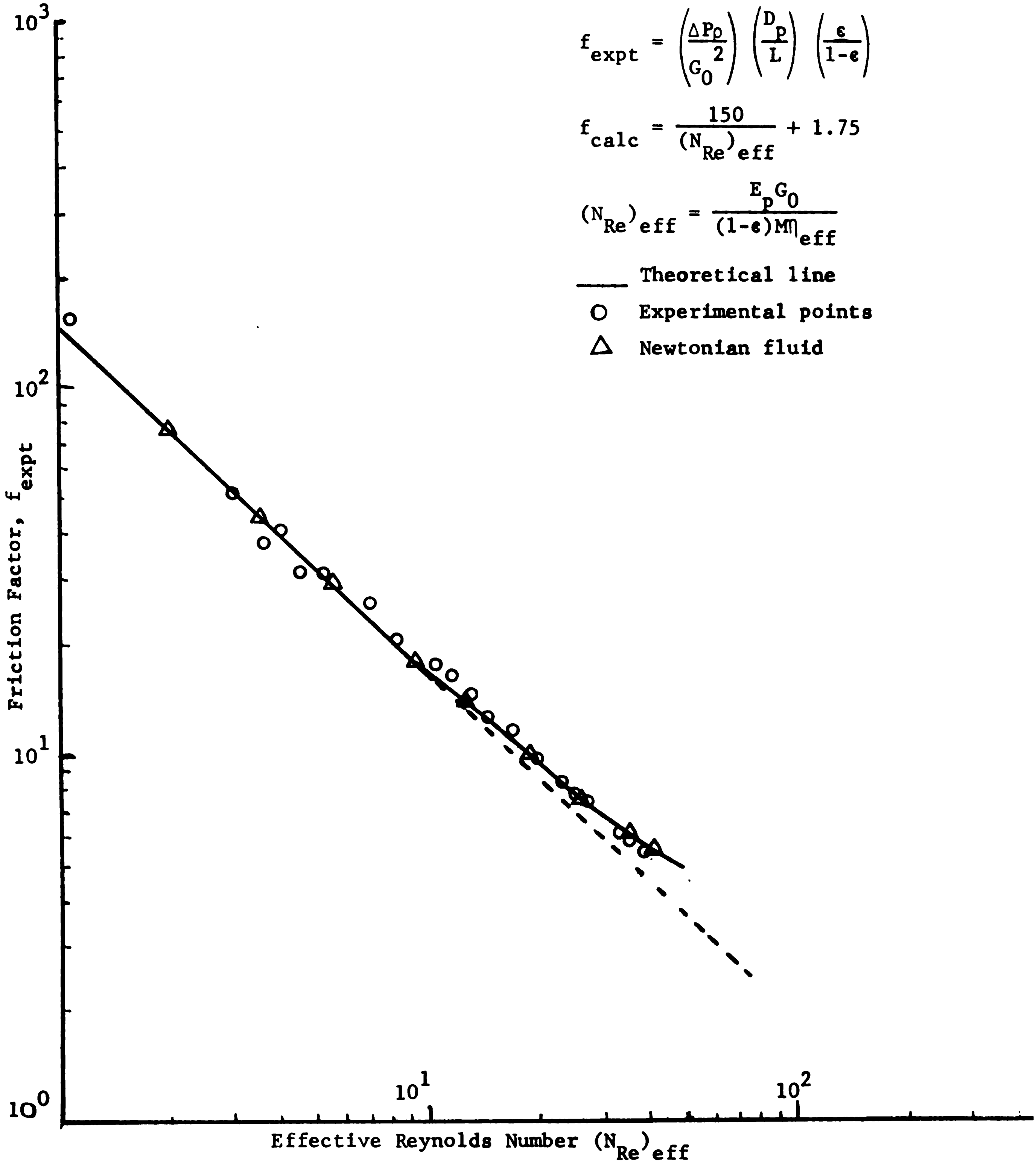

Figure 2. Pressure drop-flow rate correlation for flow of 0.5% PVP solutions through packed beds.

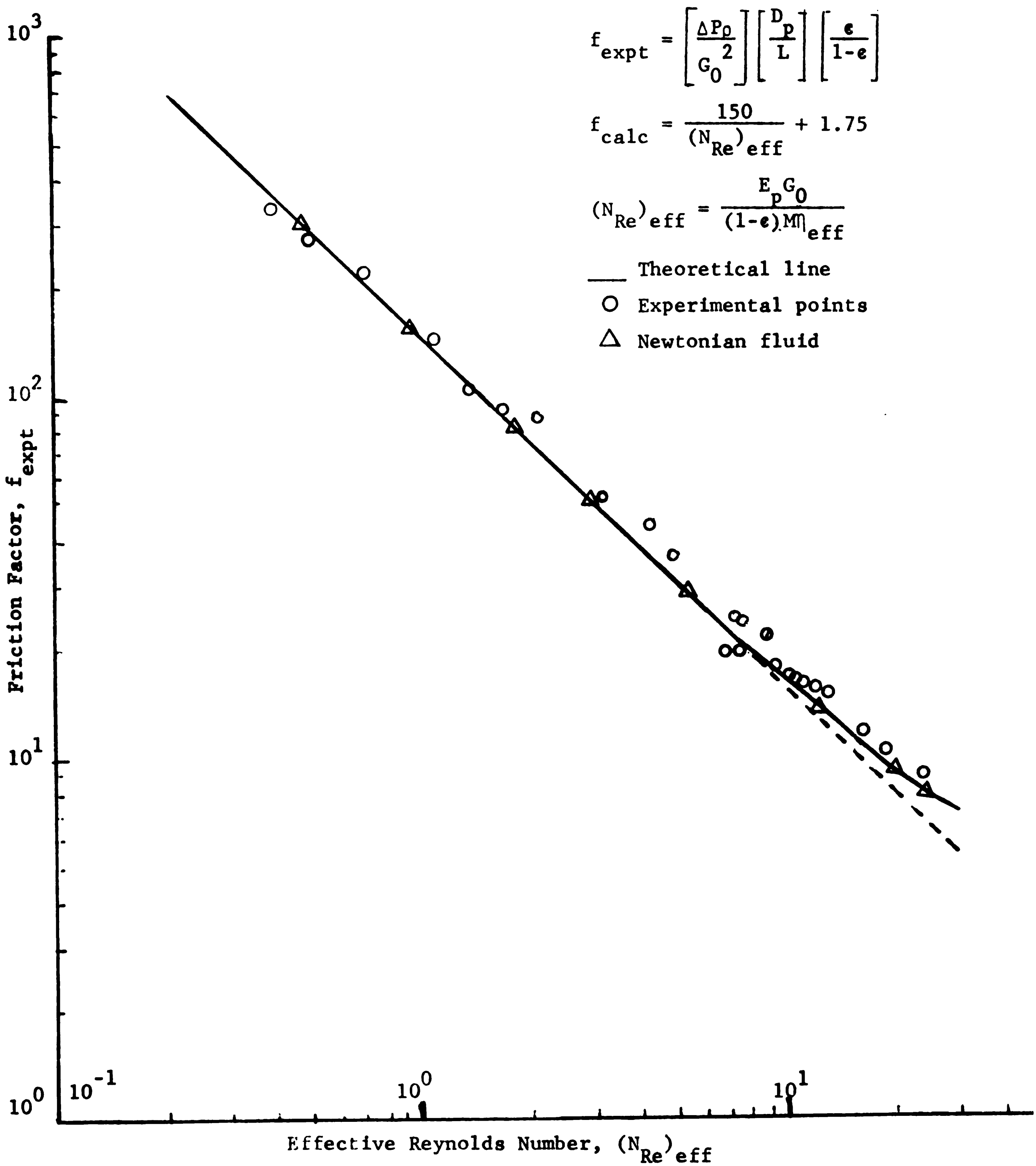

Figure 3. Pressure drop-flow rate correlation for flow of 1.0% PVP solution through packed beds.

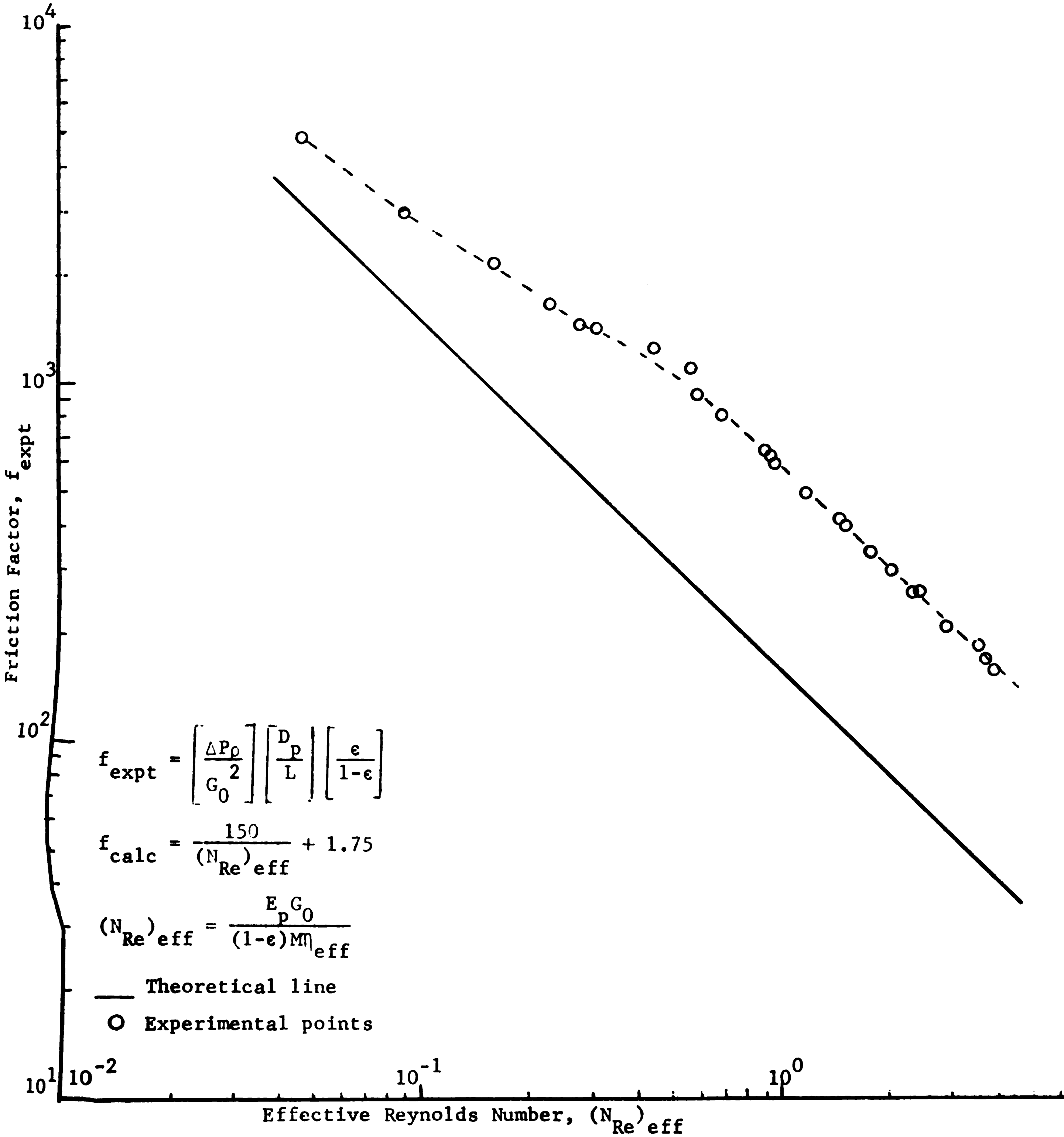

Figure 4. Pressure drop-flow rate correlation for flow of 3.01 PVP solutions through packed beds.

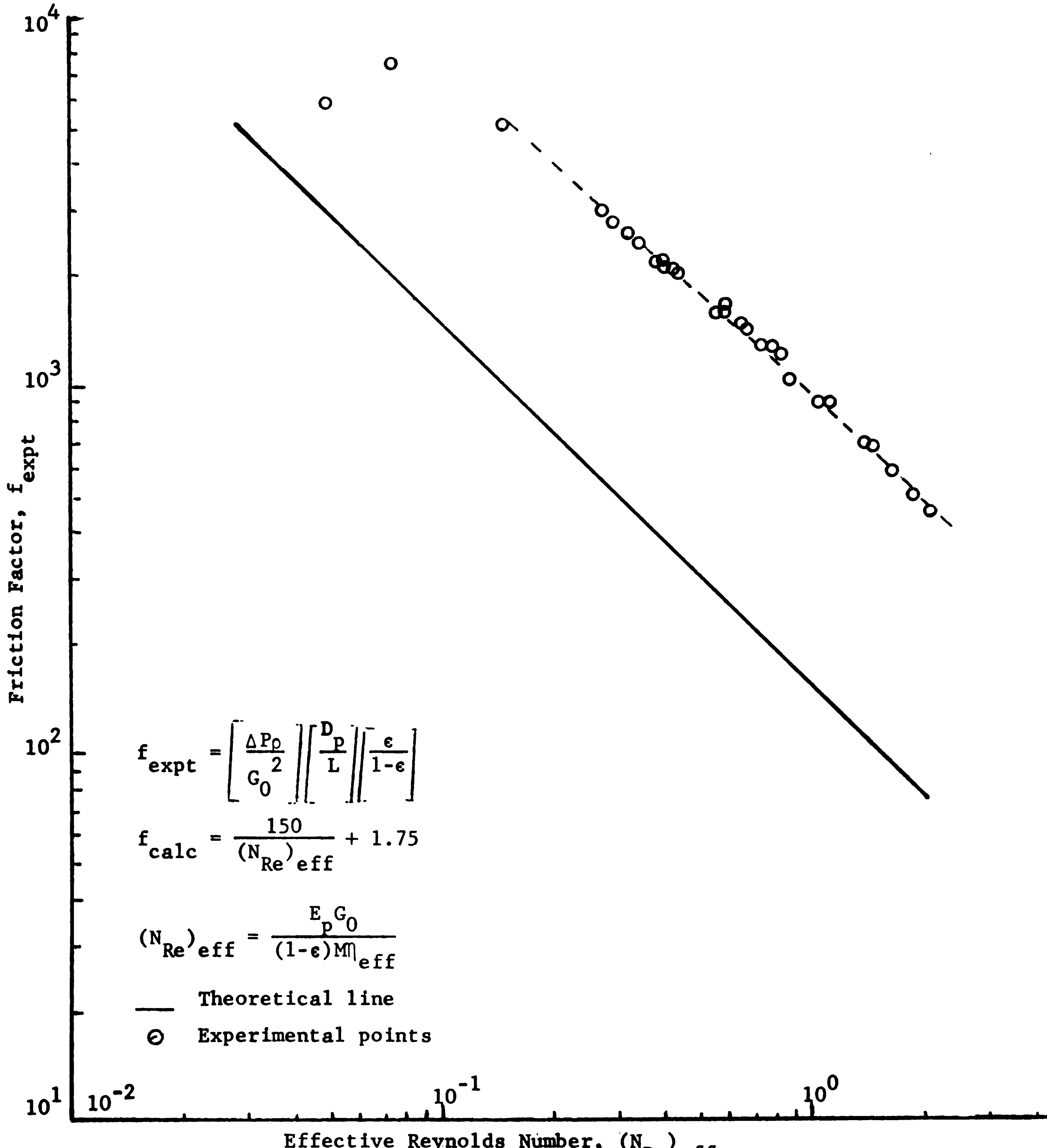

Figure 5. **Pressure drop-flow rate correlation for flow of 4.0% PVP solution through packed beds.**

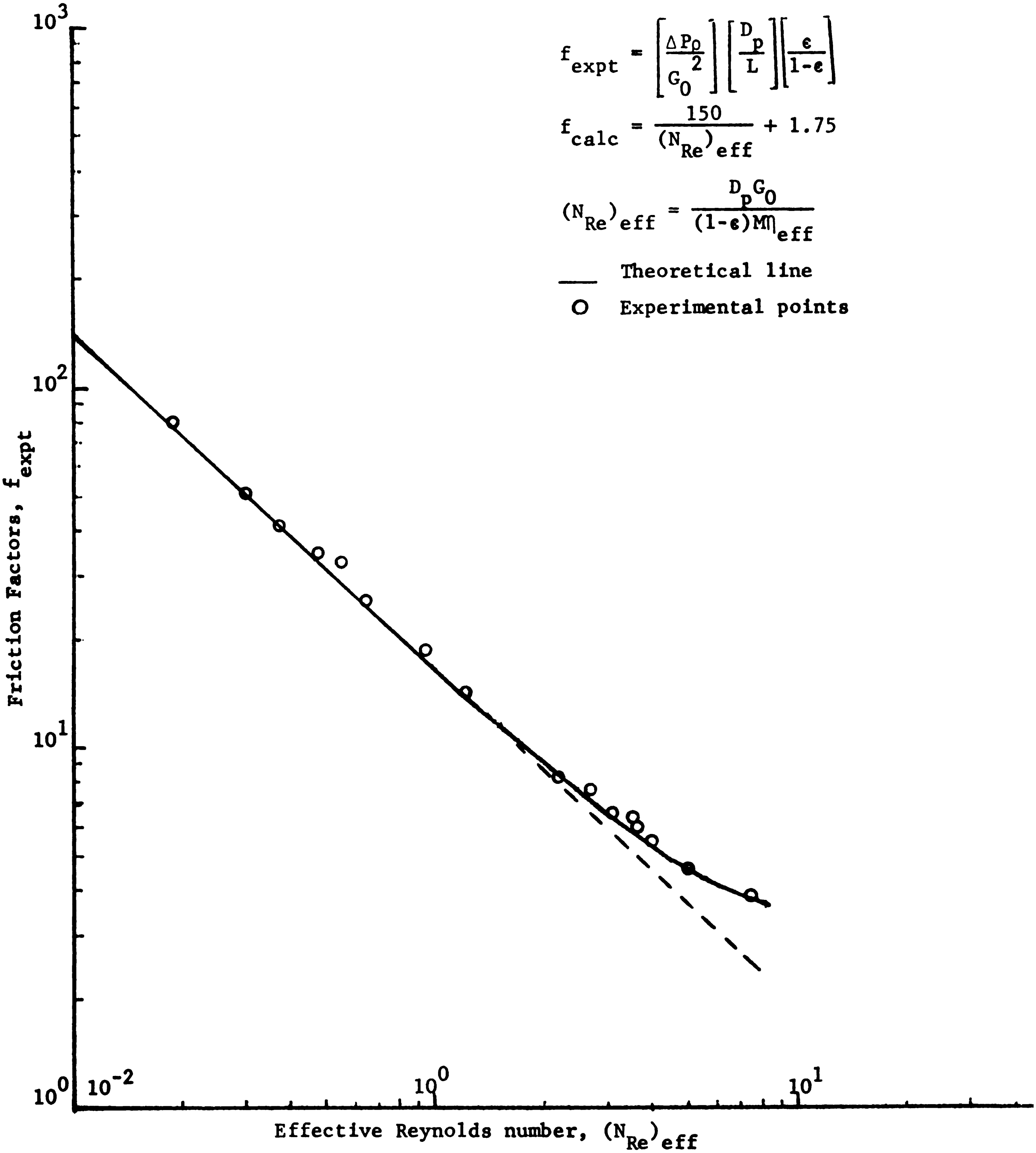

Figure 6. Pressure drop-flow rate correlation for flow of Newtonian fluid (water) through packed beds.

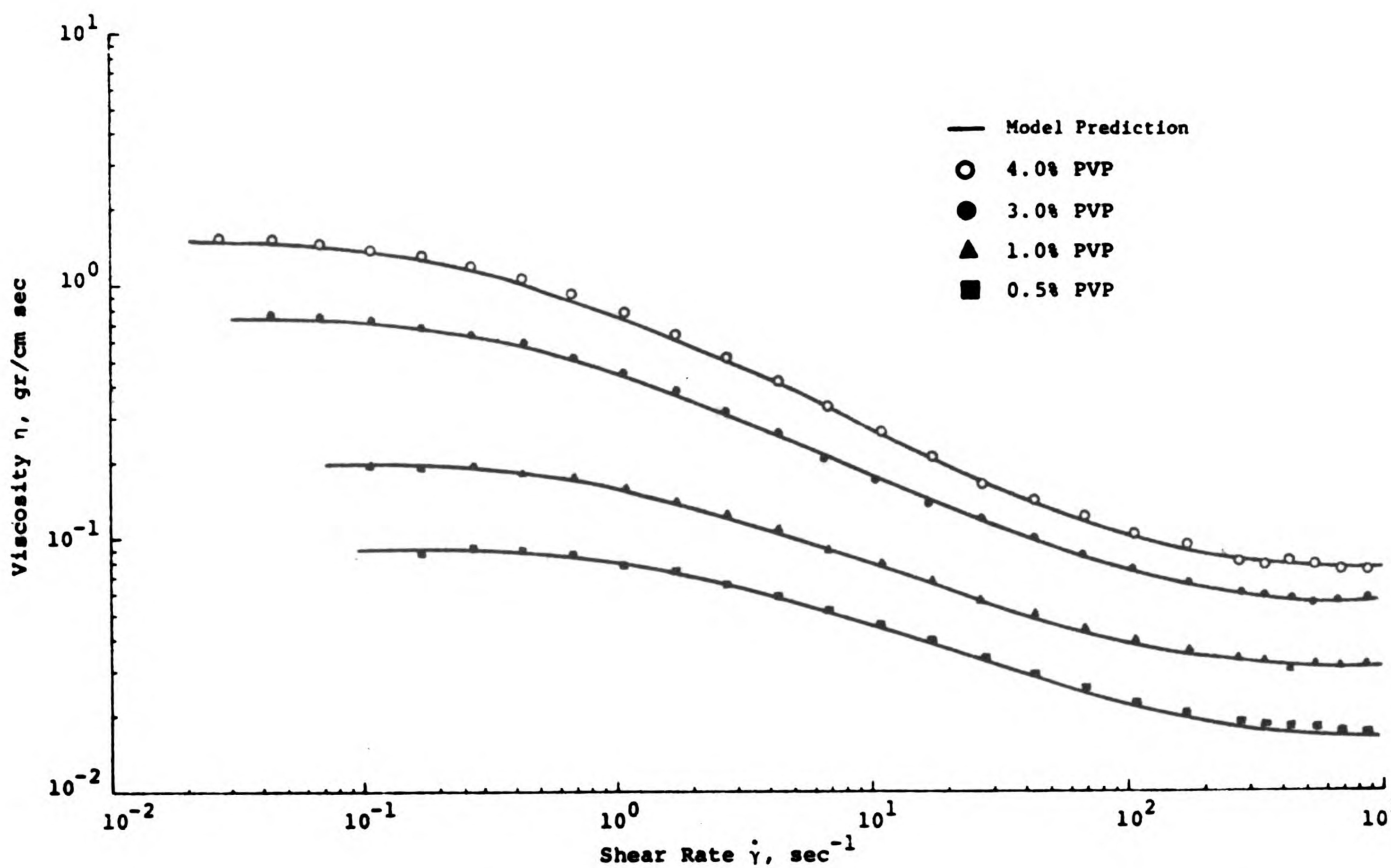

Figure 7 Comparison of Experimental and Calculated Values of the Apparent Viscosity for Aqueous Solutions of PVP

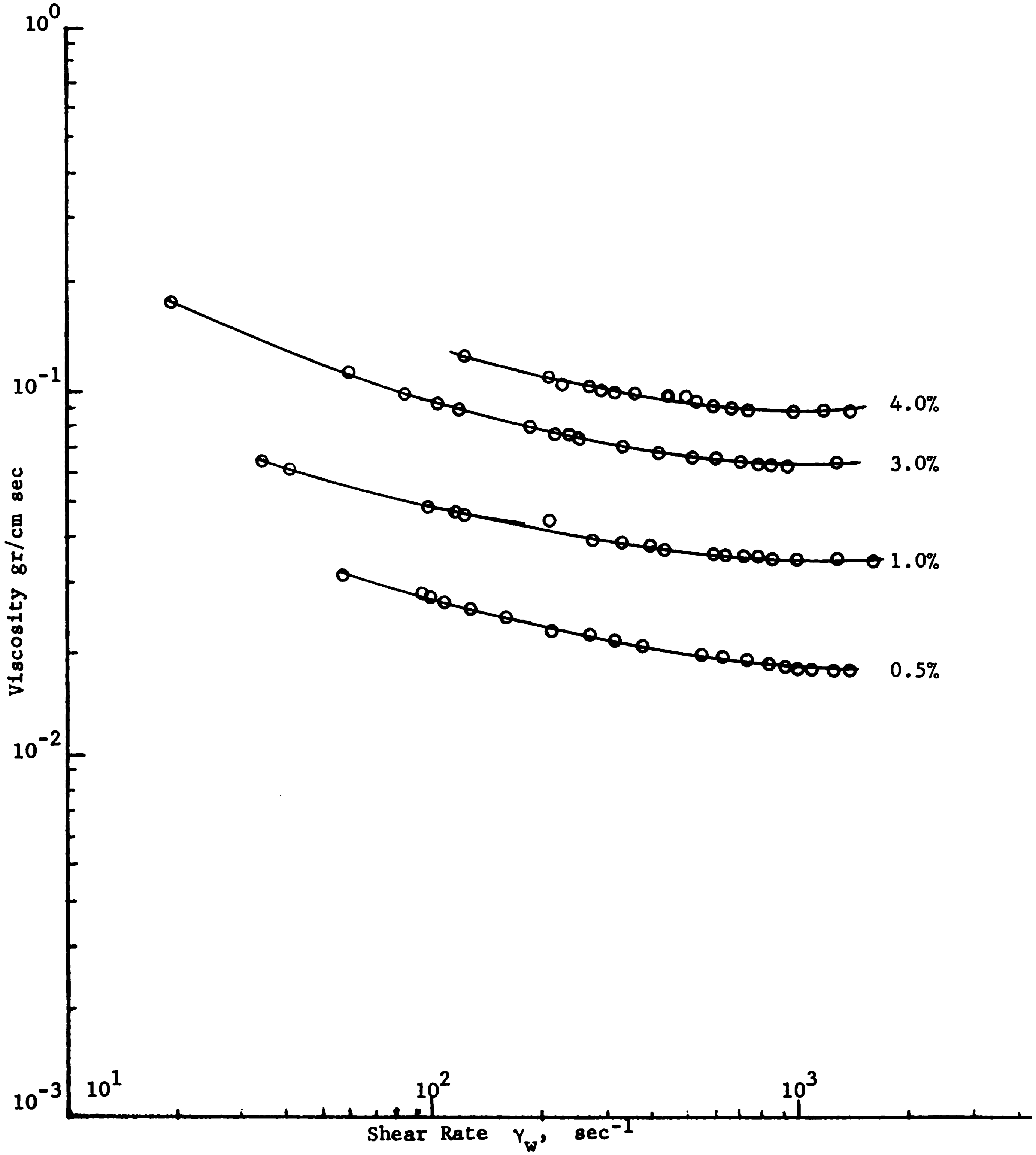

Figure 8. Model predicted values of viscosity and wall shear rate for aqueous solutions of PVP.

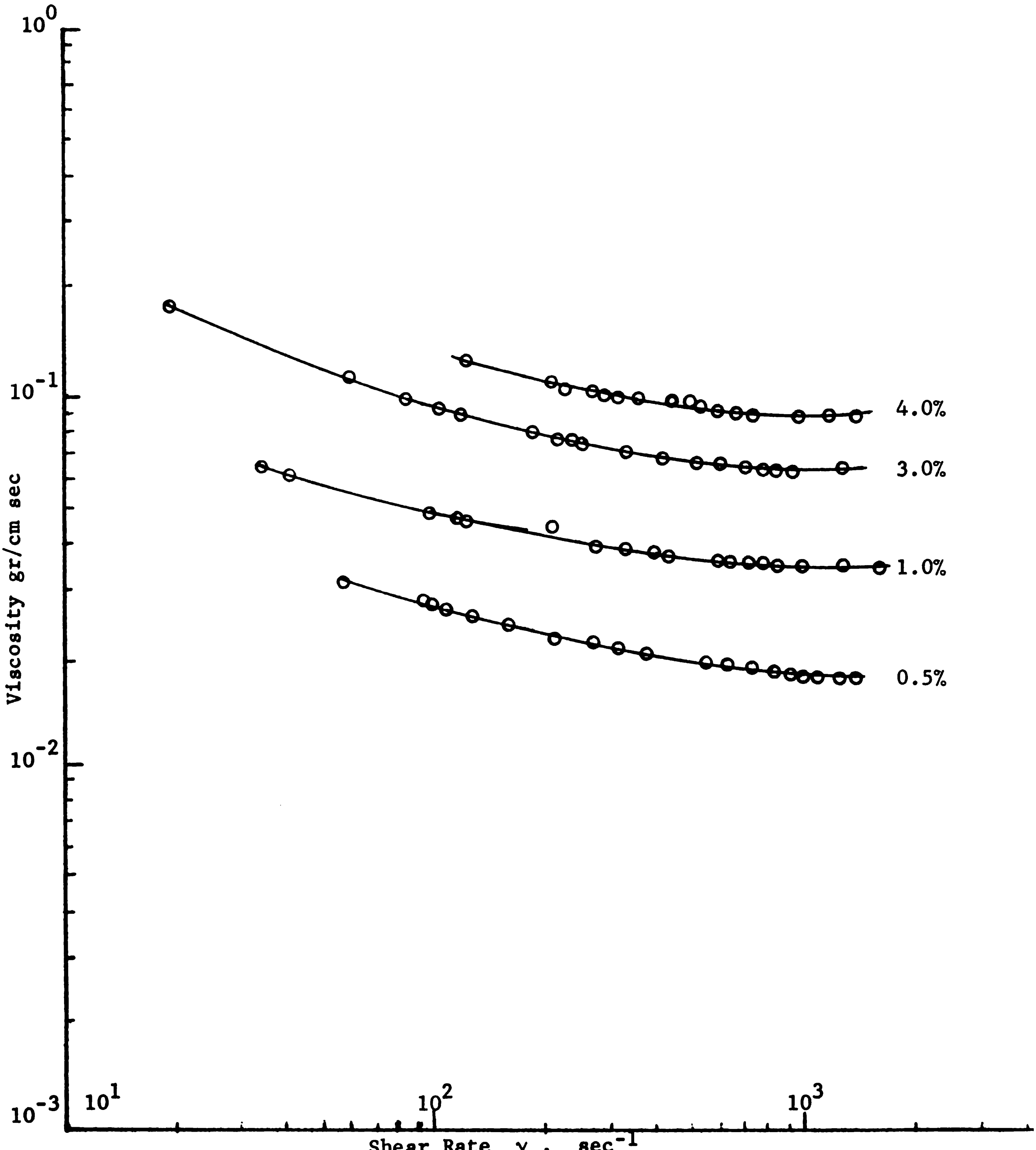

Figure 8. Model predicted values of viscosity and wall shear rate for aqueous solutions of PVP.

equation is as high as 4 for 3.0 percent solution and 6.5 for 4.0 percent solution. Sadowski, Marshall and Metzner also observed major departures from purely viscous fluid flow similar to those reported in Figures 4 and 5 for polymeric solutions. Sadowski, in his analysis of natrasol solutions, contended that the cause of such deviations are viscoelastic and surface adsorption phenomena. However, Christopher et al. correlated Sadowski's original data successfully with power law model equation in place of Ellis model equation used by the latter. Marshal and Metzner argued that the deviation in their case is only due to viscoelastic effect and ruled out surface adsorption.

The characteristic parameters for a fluid, Ellis and Deborah numbers only indicate the threshold value of the friction factor beyond which deviations appear. As compared to the prior investigations, the present study differs in the fact that the range of Reynolds number is very high. Marshall and Metzner showed that above Deborah numbers 0.1 to 1.0 the deviations might appear. The utility of such a number in this range of Reynolds number is not meaningful since either Ellis or Deborah number would be well above 1.

A qualitative discussion is presented below in an attempt to explain the deviations.

The theoretical analysis is based on the solution of viscometric flow problem (i.e., flow in a long straight circular tube). In such a flow no time dependent elastic effects are included or expected. In a porous medium the fluid moves through a tortuous path and it encounters constantly changing cross sections. The complexities in the bed subjects an elemental volume of a solution to continual acceleration and deceleration as it flows through the interstices. If the fluid relaxation time

η/T_m is small with respect to the time required to go through a contraction or expansion V_0/D_p, the fluid will accommodate and no elastic effects would be observed. For example, Newtonian fluids accommodate quickly. On the other hand, if the relaxation time is large with respect to the time to go through a contraction or expansion, the fluid will not accommodate. As a result, the elastic effects cause deviation from the theoretical analysis.

Secondly, the surface effects arise as polymer molecules get adsorbed on a solid surface at multiple points of attachment. The remainder of the molecules extend more or less freely into the solvent and serve as additional points of attachment for an eventual gel formation. In a flow system, at the points of contact between the bed particles, a network of molecules may be formed. At the same time the flowing fluid constantly tends to remove adsorbed molecules from the surface. The cumulative effect is to decrease the bed permeability and a rise in pressure drop would be observed. The experimental friction factor values would be greater than the model predicted values.

Thus the viscoelastic and surface effects are the causes of deviations observed for 3.0 and 4.0 percent PVP solutions. From Figures 4 and 5 it appears as though the adsorption builds up as the flow rate increases (indicated by the increase in the observed friction factor) to a certain extent and tends to remain constant (indicated by the portion of the curve which is fairly parallel to the theoretical line). It is possible that this phenomena is due to constant flow rates. Although Marshall and Metzner ruled out adsorption, the implication to be drawn here is that neither one of the effects should be discarded from use in analysis. Two of the reasons sighted in discarding adsorption are that the flows through the capillary tubes of comparable size revealed no such problems (Ueber 1964), and solution concentrations employed were chosen partly on considerations of optical clarity. The capillaries formed in the bed probably do not compare in the surface adsorption property with the straight capillary tubes of the same size, and the absence of adsorption on the straight capillary tubes for PVP solutions needs to be verified. The optical clarity of the solutions does not have much weight on surface adsorption.

It is hardly possible to distinguish quantitatively the two effects separately. A test for this would be a comparison of constant flow rate experiments with the constant pressure drop experiments. In the constant pressure drop experiments, the formation of gel or adsorption decrease the flow rate. This decrease in flow rate further increases adsorption. In the absence of this phenomena the experiments are identical.

No attempt has been made to conduct constant pressure drop experiments and to correlate the surface effects quantitatively to Ergun equation. Present theoretical analysis does not include such effects and hence the equation fails to explain the data for 3.0 and 4.0 percent solutions. A thorough analysis supported by constant pressure drop and flow rate experiments is needed to accommodate the effects.

CONCLUSIONS

Results of the present investigation may be summarized as follows:

1. Modified Ergun equation may be used for 0.5 and 1.0 percent PVP solutions. No evidence of viscoelastic effects were observed.
2. For higher concentrations, i.e., 3.0 and 4.0 percent solutions, surface and viscoelastic effects were very significant. Large deviations between experimental values of friction factor and those from modified Ergun equation were observed.
3. Capillary model equation for packed bed needs further modification to account for viscoelastic and surface effects. A comparison between constant pressure drop and constant flow rate experiments is desired to account quantitatively the deviations.
4. A thorough investigation at low Reynolds number (below 0.5) may also be made for 3.0 and 4.0 percent solutions to determine the adsorption effects (viscoelastic effects are negligible in that range).

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APPENDICES

APPENDIX A

PACKED BED THEORY AND EQUATIONS

1. Rheological equation for PVP solutions:

Polymers and polymer solutions exhibit the same general behaviour with regard to the non-Newtonian viscosity as a function of shear stress T. In the limit of very small shear stress the viscosity approaches a lower limiting viscosity η_0. With increasing shear stress the viscosity η decreases, and if the shear stress can be increased sufficiently, the viscosity becomes constant at an upper limit η_∞. Hence η_0 and η_∞ are measurable characteristic quantities of the fluid. Another measurable quantity is T_m, the shear stress when η drops down to $1/2(\eta_0 + \eta_\infty)$. Meter related these properties and suggested a model:

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + \left|\frac{T}{T_m}\right|^{\alpha - 1}} \qquad \text{(A-1)}$$

where α = exponent (a constant). α indicates the abruptness of the transition from η_0 to η_∞. η_0/T_m, η_∞/T_m are two characteristic times of the fluid. If η_∞ is much smaller than η_0, Equation (A-1) can be rewritten as

$$\frac{1}{\eta} = \left[1 + \left|\frac{T}{T_m}\right|^{\alpha-1}\right] \sum_{J=0}^{\infty} \left[- \left|\frac{T}{T_m}\right|^{\alpha-1} (\eta_\infty/\eta_0)\right]^J \qquad \text{(A-2)}$$

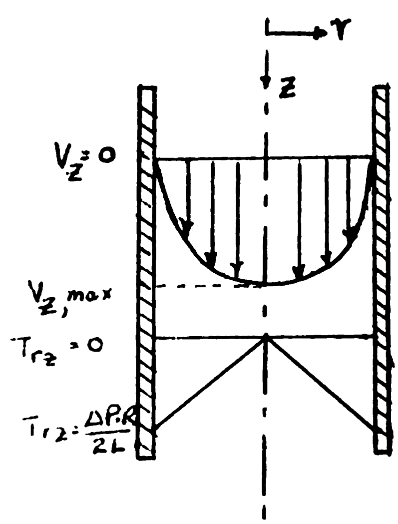
Park [9] studied extensively the rheological behavior of PVP solutions and determined the parameters conforming to Meter's model. The parameters are listed in Table 1.

2. Packed bed equations:

It will be shown that the packed bed equations for non-Newtonian fluids reduces to the Newtonian form of Blake - Kozney and Ergun equations. The following assumptions are made in the derivation of the relevent equations.

1. The fluid is incompressible.
2. The porous medium is isotropic and of regular geometry.
3. Inertial terms from the equation of motion are deleted.
4. Fluid is homogeneous.
5. Temperature is constant.

Consider the flow of non-Newtonian fluid through a circular tube of radius R.



Making a momentum balance over the shell of a thickness dr and length L ; the following differential equation is obtained.

$$\frac{d}{dr}(r\ T_{rz}) = (\frac{\Delta P}{L})\ r \qquad \text{(A-3)}$$

where T_{rz} = shear stress, r = radius of the shell, and ΔP = pressure drop. Integrating,

$$T_{rz} = \frac{\Delta P}{2L} r + \frac{C}{r} \tag{A-4}$$

The constant C must be zero if the momentum flux is not to be infinite at r=0. Hence

$$T_{rz} = \frac{\Delta P}{2L} r \tag{A-5}$$

At the wall,

$$(T_{rz})_{r=R} = T_w = \frac{R\Delta P}{2L} \tag{A-6}$$

$$T_{rz} = T_w (r/R) \tag{A-7}$$

It is assumed that (1) the fluid is in steady state laminar flow, (2) the fluid is time independent, and (3) there is no slip between the fluid and the tube wall. The volumetric flow rate through the cylindrical shell of thickness dr and length L is

$$dQ = V_z \, 2\pi \, r \, dr \tag{A-8}$$

where V_z = velocity at radius r.

$$\begin{aligned} Q &= \pi \int_0^R V_z \, 2r \, dr \\ &= \pi \int_0^R V_z \, d(r^2) \end{aligned} \tag{A-9}$$

Integrating by parts,

$$Q = \pi [V_z R^2 - \int_0^R r^2 \, dV_z]$$

$V_{z_{r=R}} = 0$, hence

$$Q = -\pi \int_0^R r^2 \, dV_z \tag{A-10}$$

From equation (A-7)

$$r^2 = \frac{R^2 T_{rz}^2}{T_w^2} , \quad dr = \frac{R}{T_w} dT_{rz}$$

Making these substitutions into equation (A-10),

$$Q = -\pi \int_0^{T_w} \frac{R^2 T_{rz}^2}{T_w^2} \left(\frac{dV_z}{dr}\right) dT_{rz}$$

$$\langle V_z \rangle = \frac{Q}{\pi R^2} = -\frac{R}{T_w^3} \int_0^{T_w} T_{rz}^2 \left(\frac{dV_z}{dr}\right) dT_{rz} \tag{A-11}$$

where $\langle V_z \rangle$ = average velocity. From Meter's equation,

$$-\frac{dV_z}{dr} = \frac{T_{rz}}{\eta_\infty + \dfrac{\eta_0 - \eta_\infty}{1 + \left(\dfrac{T_{rz}}{T_m}\right)^{\alpha-1}}} = T_{rz}\left[\frac{1 + \dfrac{T_{rz}}{T_m}^{\alpha-1}}{1 + \dfrac{\eta_\infty}{\eta_0}\left(\dfrac{T_{rz}}{T_m}\right)^{\alpha-1}}\right] \tag{A-12}$$

Hence

$$\langle V_z \rangle = \frac{R}{T_w^3} \int_0^{T_w} T_{rz}^3 \left[\frac{1 + \left(\dfrac{T_{rz}}{T_m}\right)^{\alpha-1}}{1 + \dfrac{\eta_\infty}{\eta_0}\left(\dfrac{T_{rz}}{T_m}\right)^{\alpha-1}}\right] dT_{rz} \tag{A-13}$$

If $\eta_0 \gg \eta_\infty$, the denominator can be expressed in terms of powers of $(\eta_\infty/\eta_0)(T_{rz}/T_m)$. Equation (A-13) can be integrated analytically.

Analytical Technique:

$$\langle v_z \rangle = \frac{R}{T_w^3} \int_0^{T_w} T_{rz}^3 \left[1 + \left(\frac{T_{rz}}{T_m}\right)^{\alpha-1}\right] \left[\sum_{J=0}^{\infty} - \frac{\eta_\infty}{\eta_0} \left(\frac{T_{rz}}{T_m}\right)^{\alpha-1}\right]^J d\,T_{rz} \qquad \text{(A-14)}$$

Integrating term by term,

$$\langle v_z \rangle = \frac{R}{T_w^3} \left[\frac{T_w^4}{4}\left\{1 + \frac{4}{\alpha+3}\left(\frac{T_w}{T_m}\right)^{\alpha-1} - \frac{\eta_\infty}{\eta_0}\left(\frac{T_w}{T_m}\right)^{\alpha-1}\left\{\frac{4}{\alpha+3} + \frac{2}{\alpha+1}\left(\frac{T_w}{T_m}\right)^{\alpha-1}\right\}\right.\right.$$
$$\left.\left. + \left(\frac{\eta_\infty}{\eta_0}\right)^2\left(\frac{T_w}{T_m}\right)^{2\alpha-2}\left\{\frac{2}{\alpha+1} + \frac{4}{3\alpha+1}\left(\frac{T_w}{T_m}\right)^{\alpha-1}\right\} + - + - + - \right\}\right] \qquad \text{(A-15)}$$

Imagine that the packed bed is a bundle of tubes of very complicated cross section with hydraulic radius R_h. The average velocity in the available cross section for flow in a single tube is:

$$\langle v_z \rangle = \frac{2R_h T_w}{4\eta_0}\left[1 + \frac{4}{\alpha+3}\left(\frac{T_w}{T_m}\right)^{\alpha-1} - \frac{\eta_\infty}{\eta_0}\left(\frac{T_w}{T_m}\right)^{\alpha-1}\left\{\frac{4}{\alpha+3} + \frac{2}{\alpha+1}\left(\frac{T_w}{T_m}\right)^{\alpha-1}\right\}\right.$$
$$\left. + \left(\frac{\eta_\infty}{\eta_0}\right)^2\left(\frac{T_w}{T_m}\right)^{2\alpha-2}\left\{\frac{2}{\alpha+1} + \frac{4}{3\alpha+1}\left(\frac{T_w}{T_m}\right)^{\alpha-1}\right\} + - + - \right] \qquad \text{(A-16)}$$

The hydraulic radius may be expressed in terms of the void fraction "ε" and the wetted surface "a" per unit volume of the bed in the following way.

$$R_h = \frac{\text{cross section available for flow}}{\text{wetted perimeter}}$$

$$R_h = \frac{\text{volume available for flow}}{\text{total wetted surface}}$$

$$= \frac{\left(\frac{\text{volume of voids}}{\text{volume of bed}}\right)}{\left(\frac{\text{wetted surface}}{\text{volume of bed}}\right)} = \frac{\varepsilon}{a}$$

where "a" is related to specific surface a_v (the total particle surface/the volume of the particle) by

$$a = a_v(1-\varepsilon)$$

The quantity a_v defined in terms of the mean particle diameter D_p is

$$a_v = 6/D_p$$

$$a = \frac{6(1-\varepsilon)}{D_p} \tag{A-17}$$

$$R_h = \frac{D_p}{6(1-\varepsilon)}$$

Mehta and Hawley [8] modified hydraulic radius for Newtonian fluid flow through packed bed as

$$R_h = \frac{\text{wetted surface of spheres + wetted surface of wall}}{\text{volume of bed}}$$

$$= \frac{\varepsilon}{\frac{6(1-\varepsilon)}{D_p} + \frac{4}{D_c}}$$

$$= \frac{\varepsilon}{\left[1 + \frac{4D_p}{6(1-\varepsilon)}\right] 6(1-\varepsilon)}$$

$$= \frac{\varepsilon D_p}{6(1-\varepsilon)M} \quad \text{where} \quad M = \frac{4D_p}{6D_c(1-\varepsilon)} + 1$$

For packed beds the superficial velocity V_0 is given by

$$V_0 = \langle V_z \rangle \varepsilon$$

substituting for R, T_w in (A-16),

$$V_0 = \epsilon\left[\frac{2\epsilon D_p}{6(1-\epsilon)M}\right]^2 \frac{\Delta P}{2L4\eta_0}\left[1 + \frac{4}{\alpha+3}\left(\frac{2R_h\Delta P}{2LT_m}\right)^{\alpha-1} - \frac{\eta_\infty}{\eta_0}\left(\frac{2R_h\Delta P}{2LT_m}\right)^{\alpha-1}\right.$$

$$\left\{\frac{4}{\alpha+3} + \frac{2}{\alpha+1}\left(\frac{2R_h\Delta P}{2LT_m}\right)^{\alpha-1}\right\} + \left(\frac{\eta_\infty}{\eta_0}\right)^2\left(\frac{2R_h\Delta P}{2LT_m}\right)^{2\alpha-2} \quad \text{(A-18)}$$

$$\left.\left\{\frac{2}{\alpha+1} + \frac{4}{3\alpha+1}\left(\frac{2R_h\Delta P}{2LT_m}\right)\right\} + - - + - - + - - \right]$$

A second assumption implicitly made in the foregoing development is that the path of the fluid going through in the bed is of length L; that is the same as the length of the bed. Actually, of course, the liquid traverses a very tortuous path, the length of which may be half again as long as the length L. For Newtonian fluids experimental measurements indicated the length be changed to $(25/12)$ L. It is quite logical to assume the same value for non-Newtonian fluids. Insertion of this value into Equation (A-18) gives

$$V_0 = \frac{\epsilon^3 D_p^2 \Delta P}{150\ M^2(1-\epsilon)^2 L\eta_0}\left[1 + \frac{4}{\alpha+3}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1} - \frac{\eta_\infty}{\eta_0}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\left\{\frac{4}{\alpha+3} + \frac{2}{\alpha+1}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\right\}\right.$$

$$\left. + \left(\frac{\eta_\infty}{\eta_0}\right)^2\left(\frac{T_w^*}{T_m}\right)^{2\alpha-2}\left\{\frac{2}{\alpha+1} + \frac{4}{3\alpha+1}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\right\} + - - \right] \quad \text{(A-19)}$$

where $T_w^* = 12\ D_p\ \epsilon\ \Delta P/25\ 6M(1-\epsilon)L$. Equation (A-19) is Meter's model analog of Blake-Kozeny equation for Newtonian fluids; i.e.,

$$V_0 = \frac{\epsilon^3 D_p^2 \Delta P}{150\ M^2 L(1-\epsilon)^2 \eta_{eff}} \quad \text{(A-20)}$$

where

$$\frac{1}{\eta_{eff}} = \frac{1}{\eta_0}\left[1 + \frac{4}{\alpha+3}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1} - \frac{\eta_\infty}{\eta_0}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\left\{\frac{4}{\alpha+3} + \frac{2}{\alpha+1}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\right\} + \left(\frac{\eta_\infty}{\eta_0}\right)^2\left(\frac{T_w^*}{T_m}\right)^{2\alpha-2}\left\{\frac{2}{\alpha+1} + \frac{4}{3\alpha+1}\left(\frac{T_w^*}{T_m}\right)^{\alpha-1}\right\} + \text{-----}\right] . \qquad \text{(A-21)}$$

However Equation (A-19, (A-21) are applicable when

$$\left(\frac{\eta_\infty}{\eta_0}\right)^2\left(\frac{T_w^*}{T_m}\right)^2 \leq 1$$

Numerical technique discussed later in this section solves this problem and has no bounds as above.

For highly turbulent flow, friction factor is only a function of roughness. For the flow of fluid through a bed of spheres, the pressure drop ΔP is given by

$$\Delta P = F/A \qquad \text{(A-22)}$$

where F is the force exerted on the solid surface and A is the cross sectional area. The friction factor f, a dimensionless quantity, is also called a drag coefficient. It is approximately a constant at higher Reynolds numbers.

Consider the fluid flowing through a cylindrical tube as before. The fluid will exert a force F on the solid surface which is equal to:

$$F = A' K f$$

where A' is the surface area of the column or the wetted surface, K is the kinetic energy per unit volume, and f is the friction factor; therefore,

$$K = \frac{1}{2} \rho \langle v_z \rangle^2 \quad ,$$

and $A' = 2\pi R\,L$

$$F = (2\pi R\,L)\left(\frac{1}{2}\rho\,\langle V_z\rangle^2\right) f \qquad \text{(A-23)}$$

substituting for R in terms of hydraulic radius,

$$f = \frac{2R_h \Delta P}{2L(\frac{1}{2}\rho\,\langle V_z\rangle^2)}$$

Experimental data for Newtonian fluids indicated that:

$$6\,f = 3.5$$

Hence

$$\frac{\Delta P}{L} = \frac{1.75\,\rho(1-\epsilon)V_0^2}{\epsilon^3 D_p} \qquad \text{(A-24)}$$

Combining Equation (A-20 and Equation (A-24),

$$\frac{\Delta P}{L} = \frac{150\,\eta_{eff} M^2 V_0 (1-\epsilon)^2}{D_p^2 \epsilon^3} + \frac{1.75\,\rho(1-\epsilon)V_0^2 M}{\epsilon^3 D_p} \qquad \text{(A-25)}$$

which is Ergun equation for non-Newtonian fluids. Rewriting in terms of G_0, mass flow rate, and in the dimensionless groups:

$$\frac{\Delta P\,\rho}{M G_0^2}\;\frac{D_p}{L}\;\frac{\epsilon^3}{1-\epsilon} = 150\,\frac{M(1-\epsilon)}{\frac{D_p G_0}{\eta_{eff}}} + 1.75 \qquad \text{(A-26)}$$

and effective Reynolds number is:

$$(N_{Re})_{eff} = \frac{D_p G_0}{(1-\epsilon)M\eta_{eff}}\;. \qquad \text{(A-27)}$$

Numerical Technique:

The Ergun equation so derived is only applicable when

$$(\eta_\infty/\eta_0)^2 (T_w^*/T_m)^2 \leq 1$$

A numerical integration of Equation (A-13) does not impose any such bounds on Ergun equation.

With proper substitution for R, T_w in Equation (A-13) gives

$$V_z = \frac{2\epsilon D_r}{6M(1-\epsilon)\, T_w^{*3}} \int_0^{T_w^*} T_{rz}^3 \left[\frac{1 + \left(\frac{T_{rz}}{T_m}\right)^{\alpha-1}}{1 + \frac{\eta_\infty}{\eta_0}\left(\frac{T_{rz}}{T_m}\right)^{\alpha-1}} \right] dT_{rz} \qquad \text{(A-28)}$$

From Equation (A-20) and (A-28)

$$\eta_{eff} = \frac{\epsilon^3 D_p^2 \Delta P}{150\, M^2 (1-\epsilon)^2 [\,\langle V_z \rangle\ \epsilon]} \qquad \text{(A-29)}$$

$$(N_{Re})_{eff} = \frac{D_p G_0}{M(1-\epsilon)\eta_{eff}} \qquad \text{(A-30)}$$

With these two definitions Ergun equation is versatile and is applicable at all ranges.

The method involves application of Simpson's three point integration formula. The salient feature of the computer program is the Subroutine Sizsimp which divides the intervals 'n' number of times and applies Simpson's rule over each interval. This is repeated until Abs. $(\langle V_z \rangle \big|_n - V_z\ {}_{n-1}) < 10^{-10}$.

APPENDIX B

FLOW RATE-PRESSURE DROP DATA FROM EXPERIMENTS

AND

COMPUTER PROGRAM

TABLE - 1 . CONSTANT FLOW RATE EXPERIMENTAL DATA FOR WATER AT 21° C

BED AND FLUID PROPERTIES

PARTICLE DIAMETER	D_p	=	.1621
COLUMN DIAMETER	D_e	=	2.54
BED LENGTH	L	=	45.72
BED POROSITY		=	.4058
DENSITY OF WATER	ρ	=	1.0210
VISCOSITY OF WATER	η	=	.009810

ΔP, mm Hg	Q_0, $\frac{cc}{min}$	G_0 $\frac{gr}{cm^2 sec}$	ΔP, Psia	$N_{Re,eff}$	f_{expt}	f_{calc}
2.9[illegible]	55.00	.1809	.0518	4.6947	41.501	33.701
3.2[illegible]	65.00	.2138	.0572	5.5483	32.787	28.785
3.8[illegible]	75.00	.2467	.0679	6.4019	29.245	25.181
5.[illegible]	110.00	.3618	.0894	9.3894	17.888	17.725
6.9[illegible]	146.00	.4802	.1234	12.4623	14.013	13.786
13.5[illegible]	267.00	.8782	.2414	22.7907	8.198	8.332
15.7[illegible]	302.50	.9950	.2807	25.8209	7.427	7.559
16.6[illegible]	307.00	1.0098	.2968	26.2050	7.625	7.474
16.9[illegible]	317.00	1.0427	.3022	27.0586	7.280	7.294
18.[illegible]	334.00	1.0986	.3218	28.5097	6.985	7.011
19.7[illegible]	359.50	1.1825	.3522	30.6863	6.599	6.638
20.4[illegible]	366.00	1.2039	.3647	31.2412	6.593	6.551
25.[illegible]	405.20	1.3328	.4470	34.5872	6.592	6.087
24.[illegible]	417.00	1.3716	.4291	35.5944	5.975	5.964
27.3[illegible]	466.00	1.5328	.4881	39.7770	5.442	5.521
34.25	586.00	1.9275	.6124	50.0200	4.318	4.749
66.4[illegible]	867.50	2.8534	1.1872	74.0484	3.820	3.776

TABLE - 2 . CONSTANT FLOW RATE EXPERIMENTAL DATA FOR .50 PERCENT PVP SOLUTIONS AT 21° C

BED PROPERTIES AND MODEL PARAMETERS

PARTICLE DIAMETER D_c = .1621
COLUMN DIAMETER D_P = 2.5400
LENGTH OF BED L = 45.7200
BED POROSITY ϵ = .3410
SOLUTION DENSITY ρ = .9976

η_∞ = .0138
η_o = .0942
α = 2.0714
Γ_m = .2728
PERCENT CONC. = .50

$Q, \frac{cc}{min}$	$G_0 \frac{gr}{cm^2 sec}$	ΔP, Psia	$\eta \frac{cm\ sec}{gr}$	$\eta_{eff} \frac{cm\ sec}{gr}$	$N_{Re,eff}$	f_{expt}	f_{calc}	γ_w, sec^{-1}	$T_w, \frac{gm}{cm\ sec^2}$
46.00	.1509	.1075	.0260	.0341	1.1927	105.5709	137.7851	46.9716	1.2614
89.00	.2920	.1970	.0242	.0265	2.7459	51.7037	56.3762	109.1235	2.3127
112.50	.3691	.2239	.0214	.0253	3.6357	36.7717	43.0072	124.3092	2.6280
116.00	.3806	.2669	.0244	.0238	3.9774	41.2267	39.4630	162.4721	3.1326
133.50	.4381	.2687	.0215	.0238	4.5873	31.3356	34.4490	163.8726	3.1536
150.00	.4922	.3385	.0241	.0222	5.5248	31.2743	28.4903	219.3608	3.9736
180.00	.5906	.4048	.0237	.0211	6.4512	25.9700	23.3249	273.1376	4.7515
212.00	.6956	.4496	.0220	.0206	8.3966	20.7927	19.6144	309.8889	5.2771
250.00	.8209	.5320	.0217	.0209	10.0628	17.6923	16.1564	337.3067	5.6664
350.00	1.1485	.7362	.0205	.0192	14.9908	12.4915	11.8166	468.1422	7.5094
387.50	1.2715	.8186	.0202	.0189	16.7567	11.3313	10.7017	514.0409	8.2211
397.00	1.3027	.8598	.0207	.0187	17.3056	11.3388	10.4177	546.7651	8.6358
453.00	1.4864	.9628	.0197	.0185	20.0081	9.7519	9.2470	603.9967	9.4050
523.00	1.7161	1.0963	.0187	.0182	23.4151	8.3302	8.1561	671.2869	10.3402
529.00	1.7358	1.1492	.0195	.0181	23.4496	8.5282	8.0344	711.0153	10.8916
535.00	1.7555	1.2181	.0207	.0180	24.3288	8.8452	7.9155	765.8984	11.6526
560.00	1.8375	1.1858	.0186	.0181	25.2863	7.8544	7.6821	725.3642	11.0213
600.00	1.9688	1.2718	.0183	.0180	27.2644	7.3428	7.2506	762.2389	11.6019
667.00	2.1886	1.4151	.0176	.0174	30.5883	6.6113	6.6538	827.0544	12.4447
731.00	2.3986	1.5444	.0170	.0177	33.7458	6.0268	6.1450	881.2710	13.2501
752.00	2.4675	1.5906	.0168	.0177	34.7713	5.8464	6.0634	895.4304	13.4460
825.00	2.7071	1.7429	.0160	.0176	38.3605	5.3225	5.6603	947.7583	14.1697

TABLE - 3 . CONSTANT FLOW RATE EXPERIMENTAL DATA FOR 1.00 PERCENT PVP SOLUTIONS AT 21°C

BED PROPERTIES AND MODEL PARAMETERS

PARTICLE DIAMETER D_p = .1621
COLUMN DIAMETER D_c = 2.5400
LENGTH OF BED L = 45.7200
BED POROSITY ε = .4212
SOLUTION DENSITY ρ = .9986

η_∞ = .0288
η_0 = .2053
α = 2.1653
λ_m = .3342
PERCENT CONC. = 1.00

$Q, \frac{cc}{min}$	$G_0 \frac{gr}{cm^2 sec}$	ΔP, Psia	$\eta \frac{cm\ sec}{gr}$	$\eta_{eff} \frac{cm\ sec}{gr}$	$N_{Re,eff}$	f_{expt}	f_{calc}	γ_w, sec^{-1}	$\tau_w, \frac{gm}{cm\ sec^2}$
37.00	.1215	.1612	.0546	.0619	.4465	268.3372	303.8696	40.5810	1.4723
38.00	.1051	.1451	.0569	.0650	.4040	322.6694	368.5269	34.8477	1.7750
81.28	.2670	.3399	.0473	.0479	1.4094	106.8859	108.1784	98.1283	3.7911
93.50	.3071	.3493	.0462	.0461	1.6833	91.0444	90.8620	114.2074	4.2732
102.00	.3350	.3672	.0444	.0454	1.8637	80.4258	92.2361	121.5760	4.4923
169.00	.5223	.5785	.0444	.0406	3.2496	52.1495	47.9091	210.0877	7.0782
165.00	.5420	.6376	.0471	.0399	3.4373	53.3733	45.3887	235.1193	7.8013
174.80	.5732	.5051	.0348	.0418	3.4631	37.8007	45.0637	179.1076	6.1797
192.80	.6323	.7234	.0457	.0384	4.1034	44.6103	38.3053	272.3912	8.8751
200.00	.6569	.7379	.0446	.0388	4.2755	42.0416	36.8340	277.7232	9.0285
230.00	.7555	.8490	.0443	.0380	5.0251	36.5733	31.6002	325.0049	10.3872
245.00	.8047	.9027	.0441	.0377	5.3495	34.2721	29.5303	347.9094	11.0446
282.00	.9263	1.0156	.0427	.0371	6.3109	29.1023	25.5184	396.0423	12.4252
296.59	.9742	1.0926	.0436	.0368	6.6953	28.3038	24.1537	428.9114	13.3675
313.00	1.0281	.8257	.0302	.0382	6.8105	19.2068	23.7748	315.0841	10.1023
335.00	1.1003	1.1083	.0419	.0364	7.6391	24.3429	21.3459	474.0227	14.6604
344.00	1.1299	1.0299	.0344	.0370	7.7116	19.8332	21.2013	402.1567	12.6005
320.00	1.0511	1.1159	.0409	.0367	7.2410	24.8330	22.4655	438.8503	13.6524
374.00	1.2284	1.3729	.0427	.0359	8.6424	22.3676	19.1063	548.5827	16.7970
400.30	1.3148	1.5135	.0438	.0356	9.3288	21.5242	17.8292	608.6123	18.5172
452.00	1.4846	1.4884	.0372	.0360	10.4230	16.6022	16.1413	535.3661	16.4183
470.00	1.5438	1.5780	.0378	.0358	10.8935	16.2788	15.5197	568.5222	17.3684
490.00	1.6095	1.7822	.0411	.0354	11.4758	16.9150	14.8210	649.8176	19.6981
527.00	1.7810	1.9255	.0416	.0353	12.4068	15.7490	13.9402	699.4659	21.1212
556.00	1.8262	2.1099	.0425	.0350	13.1704	15.5538	13.1342	768.5920	23.1028
532.00	1.7474	1.9523	.0411	.0352	12.5364	15.7147	13.7152	709.3130	21.4034
691.00	2.2697	2.4538	.0381	.0348	16.4816	11.7113	10.8511	863.8263	25.8335
805.00	2.6441	2.8098	.0376	.0345	19.3490	10.1476	9.5024	1001.9024	29.7940
1005.00	3.3016	3.9584	.0399	.0341	24.4523	8.9310	7.8844	1342.5034	39.5697

TABLE - 4 . CONSTANT FLOW RATE EXPERIMENTAL DATA FOR 3.00 PERCENT PVP SOLUTIONS AT 21°C

BED PROPERTIES AND MODEL PARAMETERS

PARTICLE DIAMETER D_p = .1621
COLUMN DIAMETER D_c = 2.5400
LENGTH OF BED L = 45.7200
BED POROSITY ϵ = .4058
SOLUTION DENSITY ρ = 1.0027

η_∞ = .0526
η_0 = .7824
α = 2.3132
T_m = .5700
PERCENT CONC. = 3.00

Q, $\frac{cc}{min}$	G_0 $\frac{gr}{cm^2 sec}$	ΔP, Psia	η $\frac{cm\ sec}{gr}$	η_{eff} $\frac{cm\ sec}{gr}$	$N_{Re,eff}$	f_{expt}	f_{calc}	γ_{w}, sec^{-1}	$T_w, \frac{gm}{cm\ sec^2}$
10.03	.0331	.2041	.2693	.1919	.0439	4798.7061	3420.5057	17.8794	2.5440
21.80	.0703	.4172	.2590	.1186	.1508	2173.7721	996.4642	57.4234	5.1997
27.43	.0905	.5282	.2546	.1043	.2208	1659.7140	681.1266	81.4409	6.5833
30.85	.1017	.5998	.2571	.0981	.2641	1490.5812	569.7016	97.5567	7.4759
33.27	.1097	.6732	.2675	.0931	.2949	1438.7481	501.4356	114.4001	8.3909
42.27	.1394	.9472	.2462	.0819	.4332	1253.7540	347.4751	178.8989	11.8053
53.09	.1751	1.0779	.2683	.0787	.5662	904.4659	266.0755	210.1345	13.4343
50.34	.1660	1.1746	.3084	.0769	.5499	1046.2222	274.5419	233.3347	14.6394
60.41	.1992	1.2355	.2702	.0759	.6686	800.6682	225.0887	247.9728	15.3982
75.95	.2505	1.5199	.2677	.0721	.8842	631.3411	171.4009	321.2669	19.1919
78.14	.2577	1.5846	.2577	.0717	.9149	613.8183	165.6990	332.0775	19.7498
80.00	.2638	1.5954	.2632	.0716	.9380	589.5476	161.6688	334.6674	19.8837
95.40	.3146	1.8819	.2602	.0695	1.1526	489.0206	131.8948	403.7236	23.4543
116.00	.3826	2.3241	.2641	.0674	1.4460	408.4885	105.4832	510.2204	28.9664
119.34	.3936	2.3474	.2592	.0673	1.4896	389.8089	102.4484	515.8210	29.2565
137.00	.4518	2.6599	.2547	.0663	1.7355	333.9195	88.1822	588.5755	33.0279
157.52	.5195	3.0582	.2555	.0653	2.0256	291.4920	75.6024	665.5950	38.1160
157.50	.5194	3.1137	.2602	.0652	2.0288	296.8654	75.6859	699.9111	38.8076
179.50	.5921	3.4796	.2548	.0645	2.3352	255.3046	65.9834	787.4788	43.3603
182.48	.6018	3.6187	.2607	.0643	2.3813	257.0141	64.7415	820.9347	45.1010
204.33	.6739	4.0872	.2576	.0639	2.6867	220.4954	57.5807	913.9435	49.9436
220.99	.7289	4.2194	.2596	.0636	2.9160	204.3316	53.1912	964.0962	52.5880
222.30	.7332	4.3814	.2587	.0635	2.9403	209.6843	52.7650	1003.4404	54.6077
274.00	.9037	5.3340	.2550	.0628	3.6641	168.0314	42.6877	1230.9540	66.4799
264.20	.8714	5.3931	.2676	.0628	3.5350	182.7303	44.1833	1245.0548	67.2163
285.66	.9428	5.4468	.2494	.0627	3.8266	157.6426	40.9493	1257.8728	67.8858

TABLE - 5 . CONSTANT FLOW RATE EXPERIMENTAL DATA FOR 4.00 PERCENT PVP SOLUTIONS AT 21° C

BED PROPERTIES AND MODEL PARAMETERS

PARTICLE DIAMETER D_p = .1621
COLUMN DIAMETER D_e = 2.5400
LENGTH OF BED L = 45.7200
BED POROSITY ϵ = .4125
SOLUTION DENSITY ρ = 1.0047

η_∞ = .0724
η_0 = 1.5736
α = 2.4712
T_m = .7434
PERCENT CONC. = 4.00

Q, $\frac{cc}{min}$	G_0 $\frac{gr}{cm^2 sec}$	ΔP, Psia	η $\frac{cm\ sec}{gr}$	η_{eff} $\frac{cm\ sec}{gr}$	$N_{Re,eff}$	f_{expt}	f_{calc}	γ_{w}, sec^{-1}	$T_{w}, \frac{gm}{cm\ sec^2}$
13.01	.0430	.3867	.4217	.2186	.0506	5719.1835	2965.8120	31.0719	4.9504
14.15	.0463	.5944	.5960	.1596	.0754	7431.9523	1990.8207	63.4765	7.6089
21.73	.0718	.9488	.6196	.1236	.1495	5032.4363	1004.8776	125.6972	12.1468
35.02	.1157	1.4143	.5730	.1064	.2798	2888.3954	537.9205	210.6068	18.1056
37.05	.1224	1.5253	.5841	.1041	.3026	2783.0757	497.4850	230.9389	19.5266
40.00	.1322	1.6273	.5772	.1023	.3324	2547.4504	452.9487	249.6256	20.8329
42.96	.1410	1.7598	.5852	.1003	.3616	2421.6226	416.5866	273.8674	22.5289
45.79	.1513	1.8798	.5824	.0988	.3940	2245.6796	382.4508	295.7926	24.0644
46.59	.1546	1.9227	.5855	.0983	.4029	2218.9897	374.0834	303.6404	24.6145
50.00	.1652	1.9961	.5663	.0975	.4358	1999.8467	345.9244	317.0391	25.5541
51.20	.1692	2.2038	.6105	.0957	.4550	2105.6199	331.4295	354.8932	28.2127
52.75	.1743	2.2414	.6027	.0954	.4702	2017.5358	320.7588	361.7375	28.6940
62.86	.2277	2.3676	.5341	.0945	.5657	1500.7514	266.9078	384.6955	30.3097
55.08	.1822	2.4330	.6265	.0949	.4979	2008.7458	303.0044	396.5702	31.1463
58.00	.1917	2.4670	.6032	.0938	.5255	1836.7712	287.1909	402.7485	31.5817
65.23	.2156	2.7212	.5916	.0925	.5948	1601.8088	251.8245	446.8615	34.6361
74.37	.2458	3.0434	.5802	.0911	.6941	1378.2094	217.8673	507.1696	38.9615
65.00	.2148	2.6496	.5780	.0928	.5954	1570.7130	253.6767	435.8827	33.9194
70.91	.2343	2.9432	.5885	.0915	.6590	1466.0910	229.3726	484.0454	37.6780
80.00	.2644	3.1884	.5650	.0906	.7508	1247.8023	201.5438	533.3609	40.8179
81.99	.2709	3.3567	.5804	.0901	.7739	1250.6660	195.5731	553.7225	42.9722
84.30	.2786	3.5536	.5976	.0895	.8005	1252.4696	189.1418	544.2102	45.4932
90.60	.2994	3.9260	.6143	.0887	.8684	1197.9649	174.4794	666.2049	50.2603
94.41	.3120	3.8025	.5709	.0890	.9023	1068.6140	167.9950	643.9955	48.6789
112.11	.3705	4.4613	.5638	.0878	1.0858	888.9604	139.8922	752.2986	57.1129
117.09	.3869	4.8285	.5843	.0873	1.1404	882.1180	133.2879	826.0957	61.8135
145.70	.4815	5.7879	.5625	.0864	1.4343	682.9239	106.3330	999.6638	74.0955
151.00	.4990	6.1406	.5758	.0861	1.4909	674.5312	102.3613	1062.6309	78.6105
173.20	.5724	6.8405	.5590	.0857	1.7183	571.1407	89.0444	1187.4792	87.5716
195.40	.6457	7.6032	.5505	.0853	1.9464	498.7741	78.8142	1323.3456	97.3349
216.66	.7160	8.4375	.5507	.0851	2.1657	450.1962	71.0124	1471.8211	108.0149

TABLE 6

METER MODEL PARAMETERS FOR PVP SOLUTIONS AS OBTAINED FROM A WEISSENBERG RHEOGONIOMETER AT 21°C

Solution Concentration (wt. %)	(gr/cc)	η_0 (gr/cm sec)	η_∞ (gr/cm sec)	α (Dimensionless)	τ_m (gr/cm sec)	Experimental Range of Shear Rate (sec^{-1})	% Error in η_{cal}
4.0	1.0047	1.5736	0.0724	2.4712	0.7434	0.0268-1076	2.75
3.0	1.0027	0.7824	0.0526	2.3132	0.5124	0.0427-1076	1.78
1.0	0.9986	0.2053	0.0288	2.1653	0.3342	0.1076-1076	1.88
0.5	0.9976	0.0942	0.0138	2.0714	0.2728	0.1697-1076	2.28

```
      PROGRAM PVP(INPUT,OUTPUT)
      DIMENSION FEXPT(4) , CALF(4), Y(10)   , ZERO ( 4), PO(4) , OD(4)
      REAL M
C     THE PROGRAM COMPUTES FRICTION FACTOR USING EXPERIMENTAL DATA AND
C     FROM THE METER*S MODEL ANALOG OF ERGUN EQUATION.
C     INPUT DATA TO THIS PROGRAM IS FLOW RATE - CUBIC CENTIMETERS PER
C     MIN. PRESSURE DROP - MM. HG. COLUMN DIAMETER - CMS.PARTICLE DIA
C     METER - CMS.SOLUTION DENSITY - GRAMS PER CUBIC CMS. BED POROSITY
C     AND METERS MODEL PARAMETERS.
      K = 0  $  INO = 1     $ IN=0
      N = 4
      PI = 3.141592654
      IN = 1
    8 READ 1, DP ,DC , AL , EPS
    1 FORMAT (4F10.0)
      READ 2,E1,E0,A,TM,RHO,CONC
    2 FORMAT (6F10.0)
      IN = IN +1
      Y(1) = DP $ Y(2) = E1 $ Y(3) = DC $ Y(4) = E0 $ Y(5) = AL
      Y(6) = A $ Y(7) = EPS $ Y(8) = TM $ Y(9) = RHO $ Y(10) = CONC
      PRINT 12 , IN, CONC
   12 FORMAT (*1*,/////////////////////////,15X,* TABLE - *,I1,* , CONSTA
     1NT FLOW RATE EXPERIMENTAL DATA FOR *,F4.2,* PERCENT PVP SOLUTIONS
     2AT 21 C *,///,15X,* BED PROPERTIES AND MODEL PARAMETERS *,//)
      PRINT 3, ( Y(I),I=1,10)
    3 FORMAT(15X,* PARTICLE DIAMETER D   = *,F7.4,40X,* = *,F6.4/,15X,
     1* COLUMN DIAMETER    D  = *,F7.4,40X,* = *,F6.4/,15X,
     2* LENGTH OF BED      L  = *,F7.4,40X,* = *,F6.4/,15X,
     3* BED POROSITY          = *,F7.4,37X,* T  = *,F6.4/,15X,
     4* SOLUTION DENSITY      = *,F7.4,26X,* PERCENT CONC. = *,F4.2)
      PRINT 13
   13 FORMAT (////////)
      AREA = PI*DC*DC/4.0
      FAC = ( 13.5434 - RHO)/13.5434
      M = 4.0*DP/(6.0*DC*(1.0-EPS)) +1.0
      RH = EPS * DP / 6./M/(1.0-EPS)
```

```
    7 READ 4,PD,QO
    4 FORMAT (2F10.0)
      IF(PD.LE.0.0)    GO  TO 500
      DIGIT = 0
      IF(PD.GT.1603.) GO TO 8
      GO = QO/AREA/60.0*RHO
      PDP = PD*0.019337*FAC
      PD = PD* 1333.2*FAC
C     COMPUTE VISCOSITY AND FRICTION FACTOR FROM ERGAN EQUATION
      F = PD*RHO*DP*EPS**3/(GO*GO*AL*(1.0-EPS)*M)
      RELRGN = 150. / (F-1.75)
      VISERGN = DP* GO/REERGN/(1.-EPS)/M
C     COMPUTE EFFECTIVE VISCOSITY,REYNOLDS NUMBER AND FRICTION FACTOR
C     FROM METERS MODEL.
      SPD = PD
      PDR= PD - 1.75*RHO*(GO/RHO)**2*(1.-EPS)*AL/DP/EPS**3
   11 PD = SPD       $    DIGIT = 101.0
   10 CONTINUE
      IF(REEFF.GE.10.0)  PD = PDR
      IF (REEFF.GE.10.0) DIGIT =199.0
      CALL MMODEL(PD,EPS,DP,M,AL,E1,E0,A,TM,RH,DVZDR,ETA,TW,INDEX,RHO,
     1VZA)
      REEFF = DP* GO/M/(1.-EPS)/ETA
      FCALC = 150./REEFF + 1.75
      ERROR = (FCALC - F)/ FCALC*100.
      QCALC = EPS * VZA * AREA * 60.0
      IF(REEFF.LT.9.9.AND.PD.EQ.PDR) GO TO 11
      IF( REEFF.GE.10.0.AND.DIGIT.LT.199.0)   GO TO 10
      PRINT 5,QO,GO,PDP,VISERGN,ETA,REEFF,F,FCALC,DVZDR,TW
    5 FORMAT(19X,F8.2,2(5X,F6.4),2(4X,F6.4),3X,F7.4,3(3X,F9.4),3X,F8.4)
      GO TO 7
  500 CONTINUE
      PRINT 14
   14 FORMAT (*1*)
      END
```

```
      SUBROUTINE MMODEL(PD,EPS,DP,M,AL,E1,E0,A,TM,RH,DVZDR,ETA,TW,INDEX,
     1RHO,VZA)
      REAL M
      INDEX = 0
      TW =(12.0/25.0)*(DP/6./M)*(EPS/(1.-EPS))*(PD/AL)
      D = E1/E)
      C = (TW/TM)**(A-1.)
      DVZDR = FTW(TW,A,TM,D,E0)/TW**2
      CD = C*D
    2 CONTINUE
      INDEX = 2
      TW1=0.0
      CALL SIZSIMP(TW1,TW,A,TM,D,E0,VZA)
      VZA = 2.0*RH/TW**3*VZA
      ETA = PD*DP*DP*EPS**3/(150.*AL*(1.-EPS)**2*VZA*EPS)
      RETURN
      FUNCTION FTW(TW,A,TM,D,E0)
      C = (TW/TM)**(A-1.)
      FTW = (TW**3*(1.+C))/E0/(1.+D*C)
      RETURN
      END
      SUBROUTINE SIZSIMP(S,S1,S2,S3,S4,S5,ANS)
      ACC = 1.E-10
      SUM1=0.0 $ SUM2=0.0 $ ANS=0.0 $ ANS1=0.0  $  N=1
   16 H2 = (S1- S)/N
      S11 = S+ H2
      H = H2/3.
      DO 4 K=1,N
      DO  5  I=1,3,2
      X = S+ H*(I-1)
      FX = FTW(X,S2,S3,S4,S5)
      SUM1 = SUM1 + FX*I
    5 CONTINUE
      DO 6 J=2,4,2
      X = S + H*(J-1)
      FX = FTW(X,S2,S3,S4,S5)
      SUM2 = SUM2 + FX*(4-J+1)
    6 CONTINUE
C     APPLY SIMPSON ALGORITHM.
      ANS = 3.*H/8.*( SUM1 + SUM2 ) + ANS
      S = S11  $    S11 = S11+H2
    4 CONTINUE
      IF(ABS(ANS-ANS1).LE.ACC)GO TO 18
      ANS1 = ANS
      N = N+1
      GO TO 16
   18 CONTINUE
      RETURN
      END
```