DESIGN OF SPILLWAYS FOR UNDERPRESSURE

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John Rodger Adams

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This is to certify that the

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ABSTRACT

DESIGN OF SPILLWAYS FOR UNDERPRESSURE

by John Rodger Adams

An experimental investigation of the practicability of basing the design of overflow spillways on the under nappe profile of partially ventilated weir flow was conducted.

A model of a dam spillway was designed to have an underpressure on the spillway surface equal to one-fourth of the head at the maximum, and design, head. A similar model was designed following the current practice of the U. S. Army Corps of Engineers, which permits designing for three-fourths of the probable maximum head, to provide direct comparison.

It was found that the model designed for an underpressure of one-fourth of the maximum head had a shorter curved surface, a higher discharge at the same head, and a smaller magnitude of negative pressure at the maximum head than the model designed for three-fourths of the maximum head.

DESIGN OF SPILLWAYS FOR UNDERPRESSURE

By

JOHN RODGER ADAMS

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LIST OF SYMBOLS

C	Coefficient of discharge
C_d	Coefficient of discharge at design head
F	Froude number
g	Acceleration of gravity
H	Head of dam or weir
H_m	Probable maximum head on dam
H_d	Design head on dam
K	Discharge coefficient for the 60^{o} V-notch weir
L	Crest length of spillway
P	Pressure on spillway surface
Q	Discharge in cubic feet per second
S	Height of dam
V	Velocity
y	Depth of flow

INTRODUCTION

Since the studies by the Bureau of Reclamation for Boulder Dam, the standard criteria for design of an overflow spillway has been based on the under nappe profile of the flow over a fully aerated, vertical, sharp-crested weir. This results in a long curved surface and often requires an overhang on the upstream face of the dam. The nappe curve is approximated by a rising portion of compound curvature, and a descending parabolic arc downstream from the crest. Pressures on the dam are positive at heads below the design head. At the design head, which is considered to be the probable maximum, the pressure on the dam is zero. If the head exceeds the design head, negative pressures will occur on the dam face.

Recently several dams have been designed for a head which was only three-quarters of the probable maximum. This produces a smaller profile and a slightly higher discharge coefficient for a given head. Thus a considerable saving is possible in construction costs. But above the design head--still well below the probable maximum--the pressure on the spillway becomes negative. At the maximum head the pressure distribution is markedly nonuniform with a maximum negative pressure equaling about one-third of the head, as determined in tests at the Waterways Experiment Station (1). Model studies are necessary on each dam of this type since the pressure distribution, discharge coefficient, and danger of cavitation vary with the actual head as well as the ratio of head to design head.

The possibility of using the profile of a partially aerated weir nappe was mentioned by Thorssen (2), who studied the effect of partial aeration on discharge coefficients and on nappe profiles. Such a basis of design might permit shorter crest curves and higher discharge coefficients than those obtained on dams designed by using the nappe profile of fully aerated weir flow.

The present study was undertaken to explore the use of the negative pressure, or underpressure, spillway design. Tests were conducted on two models. One was designed for three-fourths of the maximum head, following the U. S. Corps of Engineers design procedure (1). The other was designed for the full head but with an underpressure equal to one-fourth the head.

The flow over a sharp-crested weir, on which these spillway designs are based, is one of the few flows found in nature which is nearly irrotational, or potential flow. With gravity acting on weir flow with two free surfaces, a general solution by potential theory has not as yet been accomplished. However, there are several approximate methods available, which will produce solutions for specific cases. Flow nets may be constructed graphically, but each is correct for only one ratio of head to weir height and for one pressure difference across the nappe. Flow nets may also be obtained by use of an electric analog or by relaxation of finite difference forms of the differential equations of flow. The electric analogy is possible because the potential flow of water and electricity are described by the same differential equations.

The relaxation of finite difference equations is tedious, but should be adaptable to high speed solution by digital computer. This would then be a rapid way to obtain the flow pattern for any head to weir height ratio and any pressure difference across the flow. An approximation to the lower nappe profile can be obtained by particle mechanics. This results in a parabolic path which is correct if gravity alone acts. Within a distance of one-half the head from the weir the positive pressure in the sharply contracted flow makes this solution inaccurate.

If sufficient ventilation is not provided, pressure less than atmospheric will develop beneath the nappe. The pressure difference across the nappe will depress the flow. The lower pressure will be accompanied by velocities higher than those in the nappe of fully aerated flow. Thus the discharge for a given head is increased. Or, if the discharge is constant, the head will be lowered. The magnitude of this effect has been evaluated by Johnson (3) and Thorssen (2). The following values are taken from the article by Johnson. For a constant discharge, a pressure difference of 0.1H will reduce the head 2% and increase the discharge coefficient 3%; and a pressure difference of 0.3H will reduce the head 7% and increase the discharge coefficient 11%.

The flow pattern will also be modified if a dam overflow spillway is formed by filling the area beneath the lower nappe surface of the weir flow. If a dam were perfectly shaped and if it produced no frictional resistance, that is, if the fluid could slip on the boundary, the flow would not be affected. The Corps of Engineers standard design profile has almost no affect on the pressure at the design head. Extensive tests at the Waterways Experiment Station substantiate this (1). This discussion is concerned only with the conditions at the design head.

The no-slip condition on the dam face may be treated by boundary layer theory. Although this theory will not provide an exact analysis, it will produce qualitatively correct results. In the crest region, the boundary layer has had only a short distance in which to develop and is suppressed in this region of accelerating flow. Except for the small amount of fluid within the boundary layer, the flow is essentially irrotational. Approximate computations of the displacement thickness on the crest indicate an increase in water surface elevation of less than 0.5% of the head. Far down the spillway the boundary layer increases in thickness until eventually the entire flow will come within the boundary layer. This, however, is beyond the interest of the present study.

The boundary layer is a viscous effect. Since the Reynolds number is generally much lower on a model than on a prototype structure, the viscous effects will be larger on the model. Fortunately, in model studies of gravity flows, such as the weir, the Reynolds number is generally high enough to permit neglect of viscous forces. That is, the errors induced by neglecting the viscous forces will be of the same order of magnitude as the experimental errors.

PREPARATION OF MODELS

As in most model studies, the available facilities had a vital affect on the scale of this study. The dams were placed in a section of flume which was 0.67 ft. wide and 2.0 ft. deep. To permit operation at heads above the maximum, a maximum head, $H_m = 0.75$ ft., was used for the hydraulic design of the spillways. Another factor was the available discharge of about one cubic foot per second. Assuming $C = 4.0$ in the equation $Q = CLH^{1.5}$, a solution for L gave a length of 0.38 ft. The crest length of the models was 0.33 ft. The layout of the flume is shown in Fig. 1.

The height, S, of both models was one foot. Since the maximum head of 0.75 ft. was less than twice the height of the dam, the standard profile of the U.S. Army Corps of Engineers could be used (1). The spillway designed for $0.75H_m$ was based on their design equations. The portion from the theoretical weir crest to the highest point on the spillway was formed by a compound curve with radii of $0.2H_d$ and $0.5H_d$. Beyond the crest a parabolic curve, $(x/H_d)^{1.85} = 2(y/H_d)$ was used. A spillway face slope of 60^o was chosen arbitrarily, but this curve did not attain such a slope in the height of the dam. This profile, designated the $0.75H_m$, or under-designed spillway, is shown in Fig. 2.

The design for negative pressure was more complicated. First the Corps of Engineers standard profile for the maximum head was computed. Then the correction factors for the effect of the underpressure were applied as recommended by Thorssen (2). The underpressure of $0.25H_m$ was arbitrarily chosen. Offsets based on Thorssen's data were subtracted from the ordinates of the standard profile to obtain the shape of the underpressure spillway. (See Fig. 3.) The resulting curve cannot be expressed by simple equations as the Corps of Engineers profile can. On this spillway the 60° face slope was tangent to the curve at $x = 1.21H_m$, $y = 0.65H_m$.

The differences between the two profiles were small. To assist in comparing them, the two profiles are superimposed in Fig. 4. Both were $1.33H_m$ high. The base thickness of the dam designed for $0.75H_m$ was $0.11H_m$ larger, and this might result in an overhang on the rear face of a dam. The rise from the beginning of the curve to the crest was $0.095H_m$ for the $0.75H_m$ design and $0.111H_m$ for the underpressure design. The crest was located $0.212H_m$ from the rear of the dam on the $0.75H_m$ design and $0.250H_m$ from the rear on the underpressure design. These slight differences were, however, sufficient to cause decidedly different pressures on the dam surfaces. There was one dimensional difference of significant size. The $0.75H_m$ profile is curved throughout, and would have attained the 60° face slope at $x = 1.80H_m$, while the underpressure profile attains this slope at $x = 1.21H_m$. This difference is even more apparent from the vertical coordinates of the points of tangency. The point of tangency for the $0.75H_m$ curve would have been $1.48H_m$ below the crest, but the point of tangency on the underpressure curve was only $0.65H_m$ below the crest.

The physical construction was quite simple. Two 1/4 brass plates were cut to the shape of each profile. The curves were laid out on the plates by coordinate points with an accuracy of 0. 001 inch, and the plates were finished to shape by hand forming. Then twenty piezometer holes, 1/16 inch in diameter, were drilled in each plate at the locations shown in Figs. 2 and 3. These piezometers were tapped off the side of the plates and connected to a forty tube manometer board by plastic tubing. The two plates were placed symmetrically about the centerline, with the piezometers spaced 0. 146 ft. apart. The plates also served as forms for shaping the concrete and paraffin. The bulk of the dams was filled with concrete, topped with 1/2 inch of sand-cement mortar, and surfaced with paraffin to produce a smooth finish over the entire surface. The tubes from all the piezometers extended through one side of the dam.

Because of the narrowness of the dams a filler wall had to be placed in the flume. This was made of galvanized sheet metal. A smooth curve joined the head tank and the approach channel to the dam. (See Fig. 1.) An opening was provided to permit the plastic tubes from the piezometers to be connected to the manometers. Due to the construction of the flume the approach channel was only two feet long.

The manometers were read by a remote but accurate and efficient method. The board was located so that it could be sighted through an engineer's transit from a distance of 32 ft. The dam could also be seen with the transit, so correlation between the piezometers and manometers was easy. To determine a reading the meniscus was sighted in the transit, the horizontal crosshair superimposed on the bottom of the meniscus; the transit was turned to a steel tape fastened in the center of the board, and the value recorded to 0.001 ft. This value was corrected for horizontal and vertical angles and the zero reading for the particular manometer subtracted from the corrected value to determine the pressure on the spillway at that point.

To obtain the head-discharge relations for the dams a point gage reading to 0.001 ft. was placed above the approach channel 1.7 ft. upstream from the dam. The discharge was measured by a weir placed at the end of the flume. This 60° V-notch weir had been calibrated by time-weight measurements to a discharge of about 1/2 cubic foot per second. The head-discharge rating was extrapolated on a log-log graph and checked by head and coefficient, K, calculations. For discharges over 0.75 cubic foot per second the head-discharge curve deviated slightly from a straight line when calculated from K and H. At flows of one cubic foot per second or more splash and spillage from the weir box reduced the accuracy of the discharge readings. Nevertheless, the discharge should be within 3% of the correct value. The comparative nature of the study makes consistency more important than absolute accuracy in the head-discharge ratings.

TEST RESULTS

The first step in discussing head-discharge relations is to define the head, H, to be used in the equation $Q = CLH^{1.5}$. For this study a measured water surface elevation is used. This is consistent with the head used in designing the spillway profiles.

To compare the results of this investigation with other studies the coefficient of discharge, C, is plotted against H/H_m in Fig. 5. The curve from the Chief Joseph studies is adjusted for velocity of approach, since the velocity head was included in the head used in that report (4). Chief Joseph Dam, on the Columbia River, is designed for a head of 42.1 ft. The probable maximum flow will be passed at a head of 55.4 ft. A 1:33 model was tested in the Bonneville Hydraulic Laboratory of the Corps of Engineers. The Rouse and Reid curve is plotted with $H/1.33H_d$ as the ordinate to permit comparison (5). Their model was 25 centimeters high with a crest design head of 7.09 centimeters. At the design head the under-designed spillway of the current study has a discharge coefficient approximately 2.2% higher than either the Rouse and Reid or the Chief Joseph models. At the maximum head the Rouse and Reid discharge coefficient is 4.24. Chief Joseph's discharge coefficient is 1.7% higher, the present under-designed model's discharge coefficient is 1.9% higher, and the underpressure model's discharge coefficient is 3.3% higher than the Rouse and Reid value. Throughout the range of head the underpressure spillway has, by about 2.5%, the highest discharge coefficient.

The effect of dam height may be seen in Fig. 5, on the right. This graph presents values of discharge coefficient for varying ratios of head to weir height for weir flow with atmospheric pressure below the nappe. The line was adapted from the results of a study of terminal weirs and sills by Kandaswamy and Rouse (6), and is based on the Rehbock equation. Although intended for weirs, this should be approximately correct for dams with zero pressure on the spillway surface. The effect of H/S, which determines the approach velocity, on the discharge coefficient is evident in this graph. The plotted points are for the design head and discharge coefficient of each of the dams previously mentioned. Two points are shown for the underpressure spillway, although this dam does not have zero pressure on its face at any head. The point at $H/S = 0.75$ is at the design head with negative pressure over the entire crest. For the point at $H/S = 0.56$ the average pressure is approximately zero. The only point not within 1.5% of the theoretical value is that for the model designed for $0.75H_m$ in the present study.

The major portion of this study deals with the pressure distribution on spillways. The pressure distribution on the dam designed for $0.75H_m$ was determined to provide direct comparison for the underpressure design.

The pressure profiles for the dam designed for $0.75H_m$ are shown in the upper graphs in Figs. 6 through 11. All pressures are expressed in feet of water divided by the maximum head to make the dimensionless.

The dam profiles are also plotted dimensionlessly.

For comparison with other tests, however, the pressures will be related to the design head. The lowest pressure at the maximum head is $-0.44H_d$, and occurs at $x = 0.074H_d$. This compares with a pressure of $-0.45H_d$ at $x = 0.071H_d$ on Chief Joseph Dam, and a pressure of $-0.48H_d$ at $x = 0.13H_d$ from experiments at the Waterways Experiment Station. At the design head a positive pressure of about $0.05H_d$ existed on the spillway. In an attempt to find the head at which zero pressure would occur, an average pressure of $0.03H_d$ was observed for a head 8% larger than the design head. The change in dam shape caused by such a change in design head is quite small. On the crest the largest difference between the dam designed for 0.562 ft. and one designed for 0.604 ft. would be approximately 0.003 ft. The Waterways Experiment Station tests also indicate slightly positive pressures over nearly the entire spillway at the design head. Tests were conducted at heads greater than the probable maximum, and a negative pressure of $-0.67H_d$ was observed for $H = 1.47H_d = 1.1H_m$.

The second model tested was to operate with a uniform negative pressure of $0.25H_m$ on the spillway at the maximum head. The pressure profiles for this series of tests are presented in the lower portions of Figs. 6 through 11. Downstream at piezometers 17 to 20 (See Fig. 3), the results are quite interesting. At low heads these pressures are quite low, tending to be negative near $x = 0.9H_m$. As the head is increased the peak negative pressure moves upstream to $x = 0.7H_m$

when $H = 0.9H_m$. Note that the pressure at piezometer 20, $x = 1.2H_m$, increases from near zero to approximately twice hydrostatic. This is caused by the tailwater. At low heads the flow is still accelerating and concave down at this point. As the flow increases the water surface downstream rises until it affects the flow as far upstream as $x = 0.7H_m$. Then the last piezometers are in a region of deceleration and streamlines which are concave up. On the under-designed spillway piezometer 20 is 0.159 ft. higher than on the underpressure design and affected only a little by the tailwater at high flows.

At the design head the pressure distribution, though not perfectly uniform, is close to the expected value of $-0.25H_m$. Between $x = 0.1H_m$ and $x = 0.7H_m$ the pressure varies from $-0.19H_m$ to $-0.26H_m$. Beyond this the pressure increases at the tailwater effects become apparent. For a head 10% greater than the design head, the pressure distribution is still fairly uniform with a minimum value of $-0.38H_m$.

The nonuniformity on the crest may be due to inadequacies in the design or errors in construction. In fact, very small changes in the profile could cause substantial changes in the pressure and its distribution. Although not completely analogous, the effect of fabrication errors on the pressures observed on the baffle piers for the Chief Joseph stilling basin, gives some indication of the sensitivity of the critical areas (4). Variation of approximately 0.03 inch from the theoretical curve on the front portion of the sides of the Bluestone type piers on the 1:33 model caused pressure differences as large as 200% between models. Of course these piers are subject to impact and extremely turbulent flow. The negative pressures far down the dam face at low flows may have been caused by inaccuracies in the underpressure offsets or perhaps by piezometer openings slanted downstream, not perpendicular to the surface. At high flows the tailwater effects tend to disguise this.

Starting with low heads and flows, several observations can be made on the pressure distributions of the two models. At heads below $0.5H_m$ the pressures on the crest, that is, between $x = 0$ and $x = 0.4H_m$, are very nearly the same on both dams. The pressure gradients are somewhat steeper on the underpressure design. This is more apparent at $x = 0.75H_m$. At this head, the pressures are the same at $x = 0.2H_m$, but negative pressure develops downstream on the underpressure spillway, while the pressure remains slightly positive over the entire length of the Corps of Engineers standard profile at its design head. If the head is increased to $0.9H_m$, both pressure distributions change markedly. At this head, negative pressure exists over large areas on both spillways. The lowest pressure on the under-designed spillway is $-0.12H_m$ at $x = 0.07H_m$, and the lowest pressure on the underpressure spillway is $-0.18H_m$ at $x = 0.5H_m$. The pressure gradient is adverse on the under-designed spillway, but more important, the piezometeric gradient also has an adverse slope on the crest.

The maximum head is the critical, and presumably the most severe, condition to be expected. At the maximum head the under-designed dam has pressures as low as $-0.34H_m$, while the underpressure spillway, at its design head, has a minimum pressure of $-0.26H_m$. The piezometric slope is adverse on the crest of the under-designed spillway. The underpressure spillway operates as planned, maintaining pressures near $-0.25H_m$ until $x = 0.7H_m$, whereafter the pressure increases under the influence of the tailwater. The minimum pressure is not as far below atmospheric, but an extensive area is subjected to negative pressure near $-0.25H_m$.

For a head 1.1 times the probable maximum, the pressure decreases to values of $-0.50H_m$ and $-0.38H_m$ on the under-designed and underpressure spillways, respectively. Noteworthy is the steep adverse slope of the piezometric line on the crest of the under-designed spillway. Were the crest not smoothly curved and in a region of accelerating flow this would probably result in separation. According to Rouse and Reid (5), separation will not occur until the head becomes 3.5 times the design head. Excepting local irregularities, the piezometric slope on the spillway designed for $0.25H_m$ underpressure is favorable, until affected by the tailwater, for the range of flow to be expected. Consequently separation should be an even more remote possibility on a spillway designed for uniform negative pressure at the probable maximum head, than on one designed for zero pressure at a lesser head.

The variation of minimum pressure with head is presented in Fig. 12. The pressure on the under-designed spillway has a minimum which is slightly positive and nearly constant for heads below the design head. Above the design head the minimum pressure decreases rapidly and almost linearly. The minimum pressure is above atmospheric only for very low heads on the underpressure spillway. The minimum pressure decreases in a smooth curve as the head is increased on this dam. For heads below the probable maximum the pressure is higher on the under-designed spillway. At and above the maximum head the pressure on the underpressure spillway is not as much below atmospheric pressure as that on the under-designed spillway. Although the minimum pressure on the underpressure design is lower than that on the under-designed dam for moderate heads, the pressures are not low enough to be of concern. To provide another comparison the minimum pressure curve from the study by Rouse and Reid is also shown (5). This curve indicates pressures about $0.1H_m$ lower than those observed on the model designed for $0.75H_m$ in the present investigation.

APPLICATION OF MODEL RESULTS TO PROTOTYPE EXAMPLES

Since gravity forces govern the flow over a spillway, the Froude criterion is used for model-prototype scaling. For similarity the model and prototype Froude numbers, $F = V/\sqrt{gy}$, must be equal. For a given length ratio between model and prototype, scale proportions for discharge, velocity, pressure, and any other pertinent quantity may be obtained by equating the model Froude number to the prototype Froude number. The discharge ratio varies as the five-halves power of the length ratio, velocity and time ratios vary as the one-half power of the length ratio, and the pressure ratio varies directly with the length ratio. The scaling of negative pressures from model to prototype is limited by the vapor pressure, at which cavitation must occur. The lowest actual pressure head in both model and prototype that is possible is approximately -33 ft. of water at sea level.

The maximum permissible prototype head may be determined if the lowest safe manometer reading is known. For the following examples a minimum allowable manometer reading of -20 ft. is assumed. Then the maximum head for which a spillway may be designed for $0.75H_m$ is about 60 ft. The alternate use of a spillway designed for an underpressure of $0.33H_m$ is possible for a 60 ft. head. The use of the $0.25H_m$ underpressure would allow heads to 80 ft., or, for a head of 60 ft., would result in negative pressure only 75% of that on a spillway designed for $0.75H_m$.

The prototype discharge per foot of crest length may also be calculated. Using the 60 ft. maximum head from the previous paragraph, the underpressure and under-designed spillways would pass 2040 and 2010 cubic feet per second per foot of crest. At the 45 ft. design head of the under-designed spillway, the discharges per foot of crest would be 1290 and 1260 cubic feet per second over the underpressure and under-designed spillways, respectively.

CONCLUSION

The experimental results presented indicate the feasibility and efficacy of basing spillway profiles on the lower nappe of weir flow with a specified underpressure. The maximum pool and crest elevation are frequently determined by other factors. The permissible underpressure then provides the basis for design of the overflow section.

Several questions may be raised about the current practice of designing spillways for three-fourths of the maximum head. The rather large magnitude of the negative pressure at the maximum head may be an indication of local cavitation, especially if any irregularities exist on the dam surface. Although the probability of separation is quite small, the adverse piezometeric gradient is not entirely desirable. Designing for three-fourths of the maximum head is somewhat arbitrary and overlooks the variation in the magnitude of the minimum pressure with different design heads. Depending on the permissible subatmospheric pressure, the maximum head for which a spillway may be designed for $0.75H_m$ is limited.

The uniform pressure distribution, with a specific value of underpressure at the maximum head provides a more definite evaluation of the danger of cavitation. The generally favorable piezometric gradient should make separation an extremely remote possibility, and perhaps accounts for the larger discharge coefficient on the underpressure spillway. Determining the percent underpressure from the allowable subatmospheric pressure and the maximum head considers explicitly the relation between the minimum pressure and the design head.

Further research is required in several portions of the underpressure design theory. Accurate analytic or experimental determination of the nappe profiles for partially ventilated weir flow is important. Study of the danger of local cavitation, and the tolerances to be permitted in construction, are needed to determine the permissible negative pressure accurately.

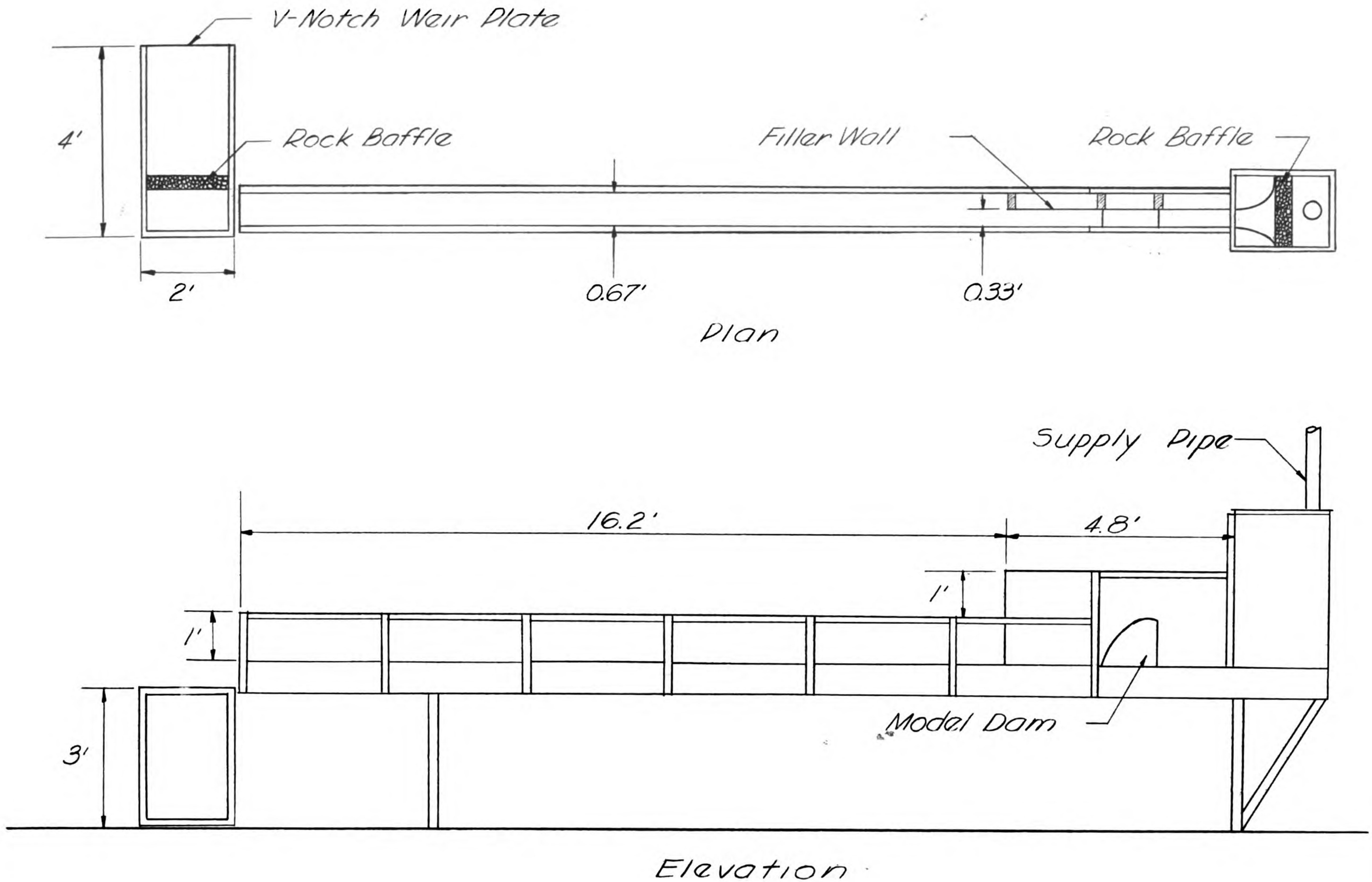
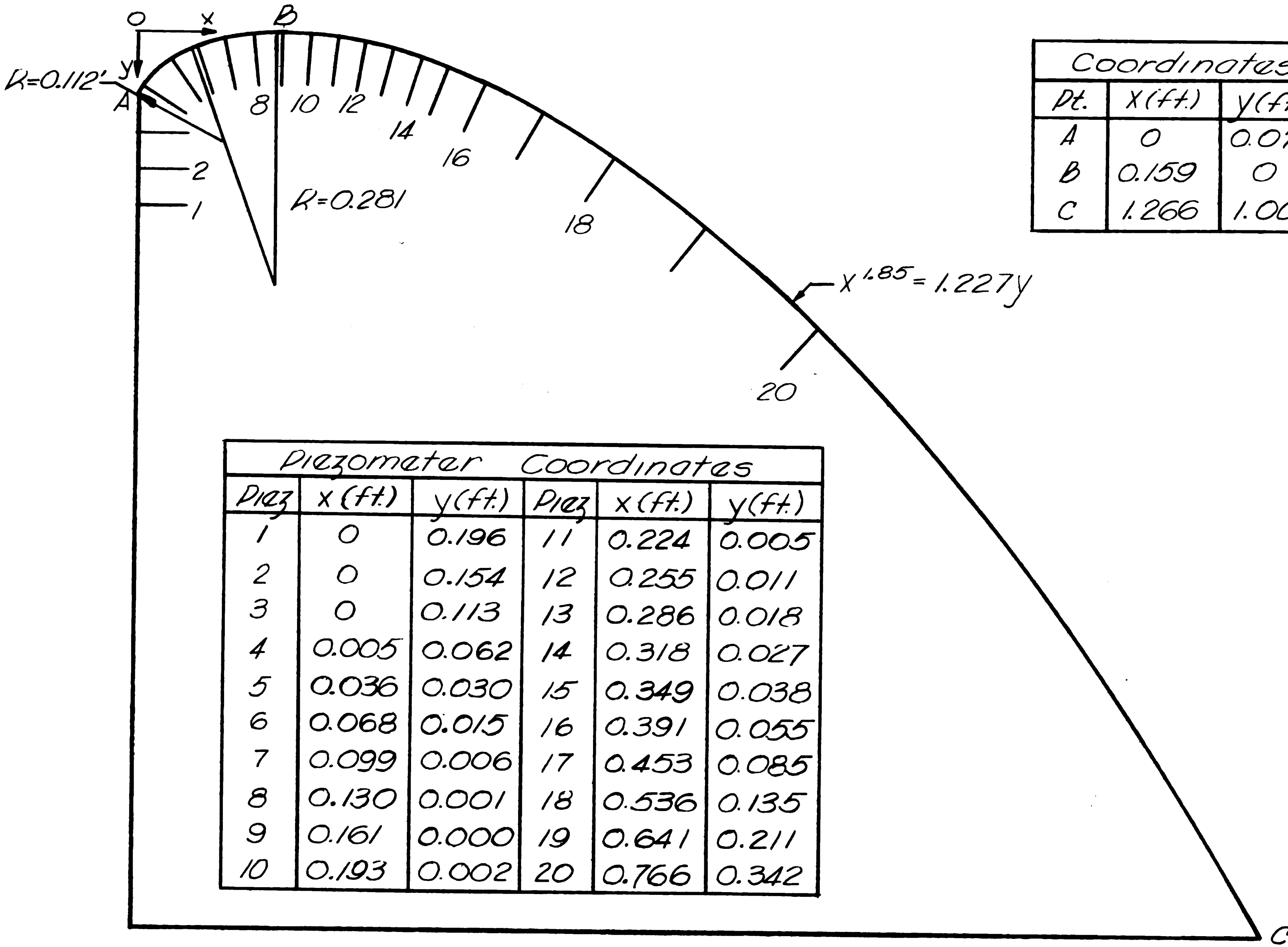


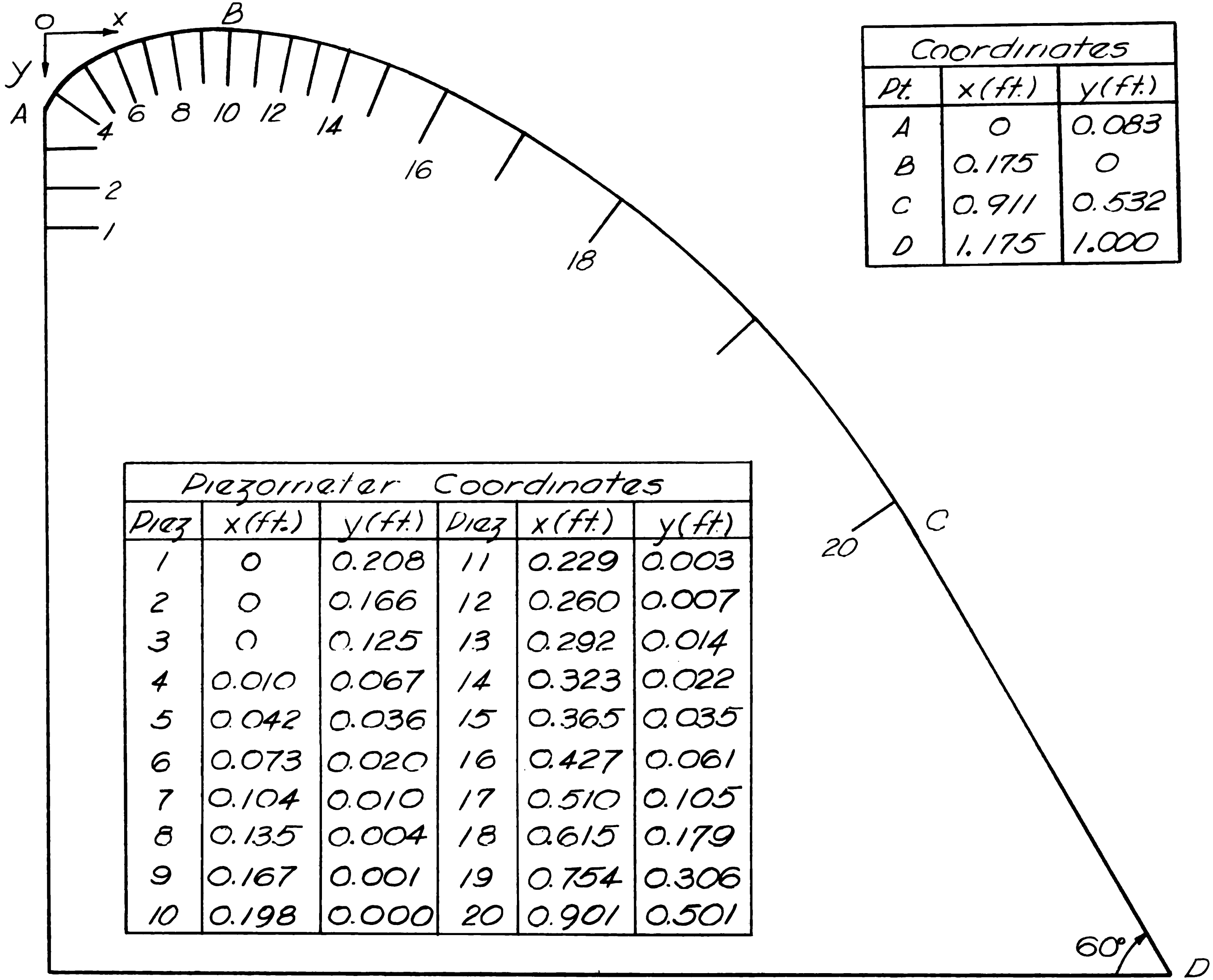
Fig. 1 Test Flume



Coordinates		
Pt.	X(ft.)	y(ft.)
A	0	0.071
B	0.159	0
C	1.266	1.000

Piezometer Coordinates					
Piez	x (ft.)	y(ft.)	Piez	x (ft.)	y(ft.)
1	0	0.196	11	0.224	0.005
2	0	0.154	12	0.255	0.011
3	0	0.113	13	0.286	0.018
4	0.005	0.062	14	0.318	0.027
5	0.036	0.030	15	0.349	0.038
6	0.068	0.015	16	0.391	0.055
7	0.099	0.006	17	0.453	0.085
8	0.130	0.001	18	0.536	0.135
9	0.161	0.000	19	0.641	0.211
10	0.193	0.002	20	0.766	0.342

Fig. 2 Under-Designed Spillway



Coordinates		
Pt.	x(ft.)	y(ft.)
A	0	0.083
B	0.175	0
C	0.911	0.532
D	1.175	1.000

Piezometer Coordinates					
Piez	x(ft.)	y(ft.)	Piez	x(ft.)	y(ft.)
1	0	0.208	11	0.229	0.003
2	0	0.166	12	0.260	0.007
3	0	0.125	13	0.292	0.014
4	0.010	0.067	14	0.323	0.022
5	0.042	0.036	15	0.365	0.035
6	0.073	0.020	16	0.427	0.061
7	0.104	0.010	17	0.510	0.105
8	0.135	0.004	18	0.615	0.179
9	0.167	0.001	19	0.754	0.306
10	0.198	0.000	20	0.901	0.501

Fig. 3 $0.25H_m$ Underpressure Spillway

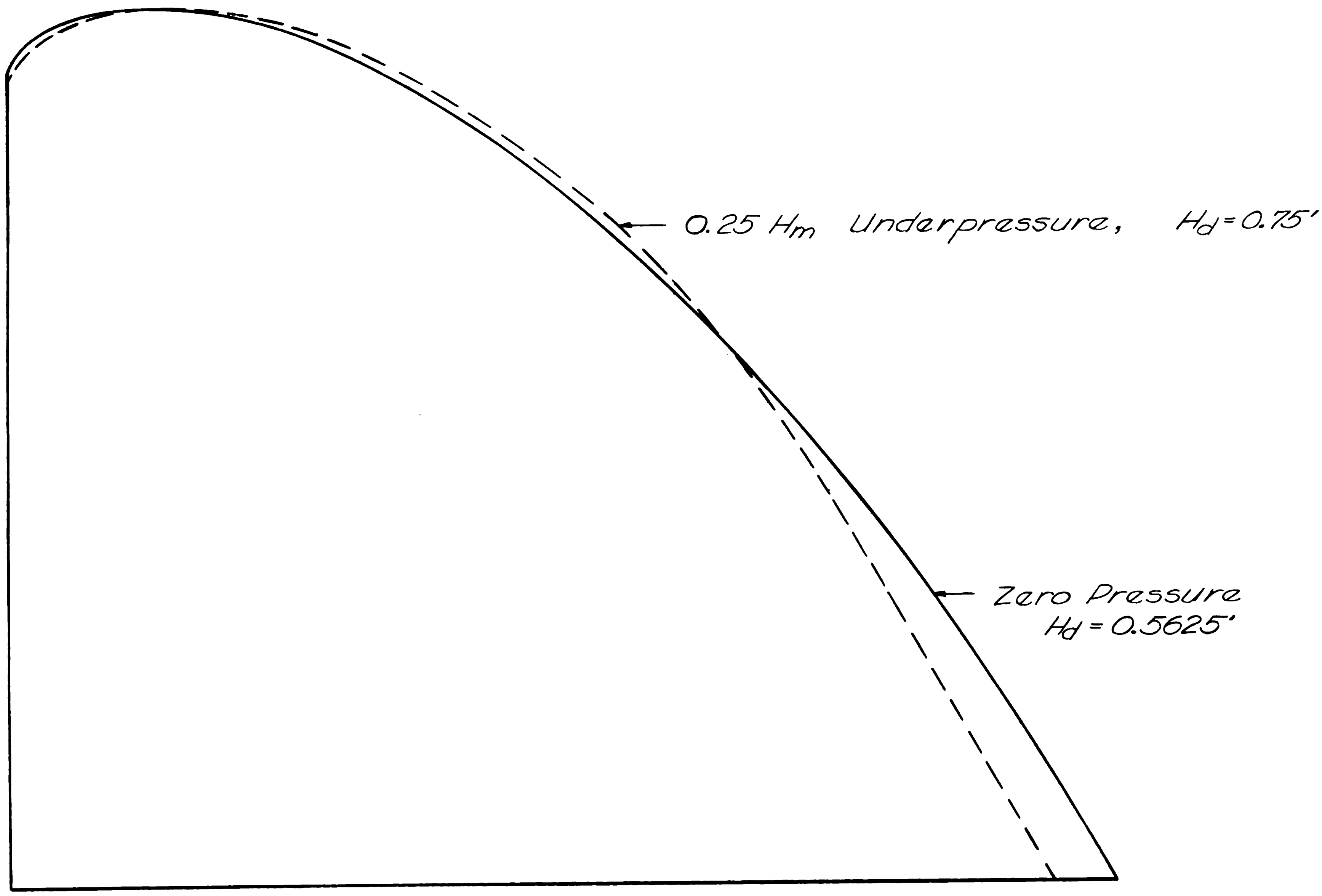


Fig. 4 Comparison of Spillway Designs

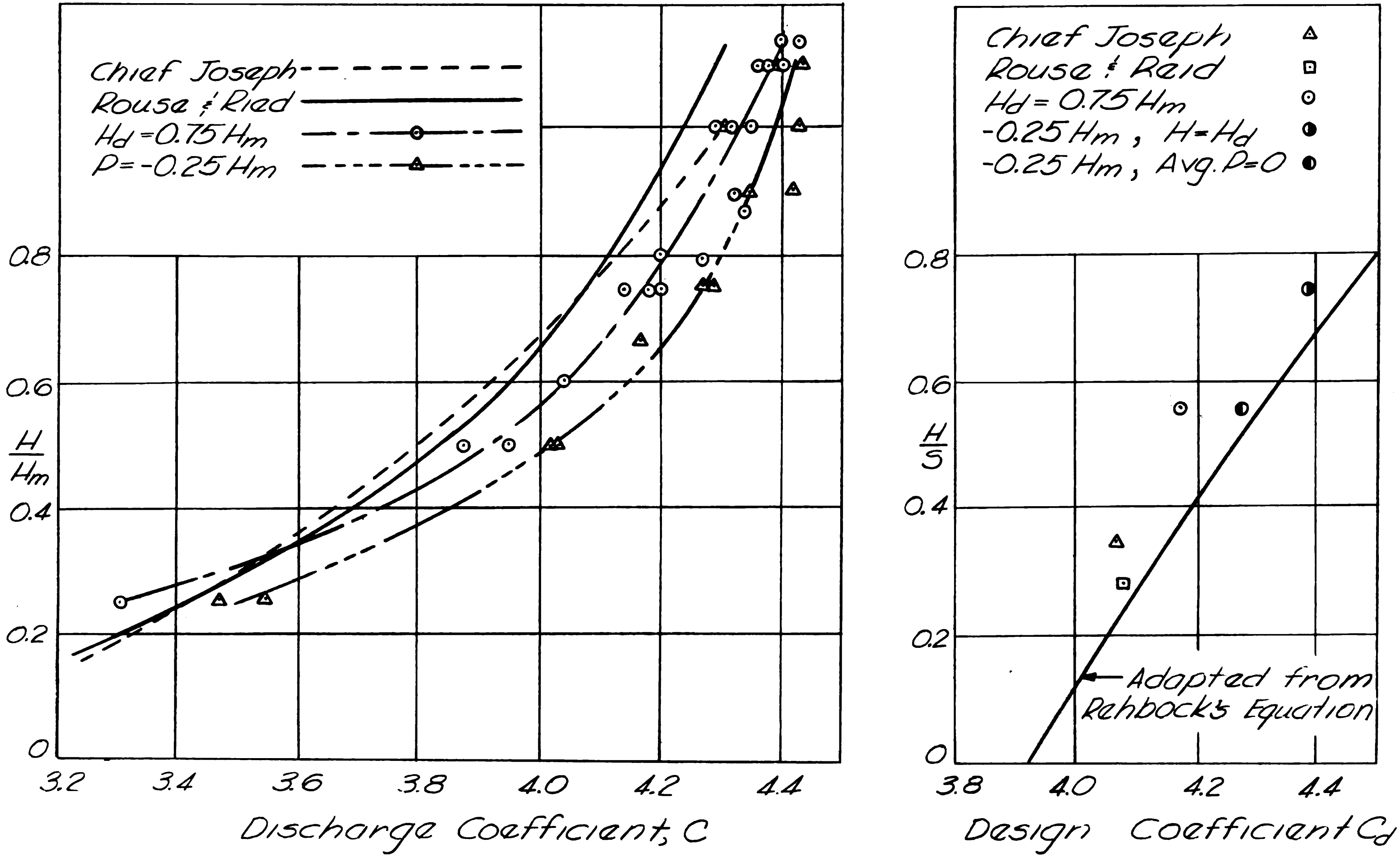


Fig. 5 Head-Discharge Characteristics

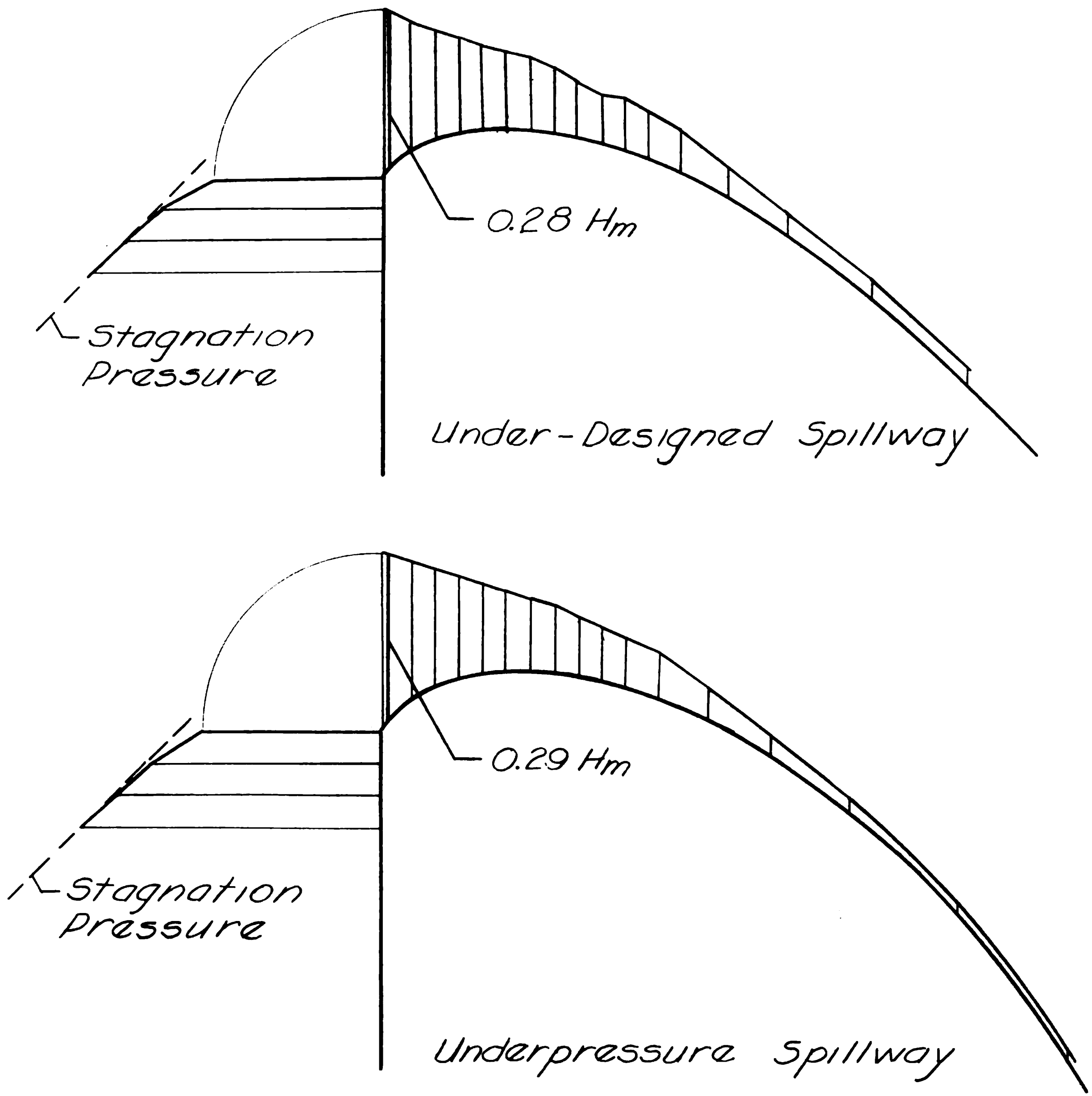


Fig. 6 Spillway Pressures, H= 0.25 Hm

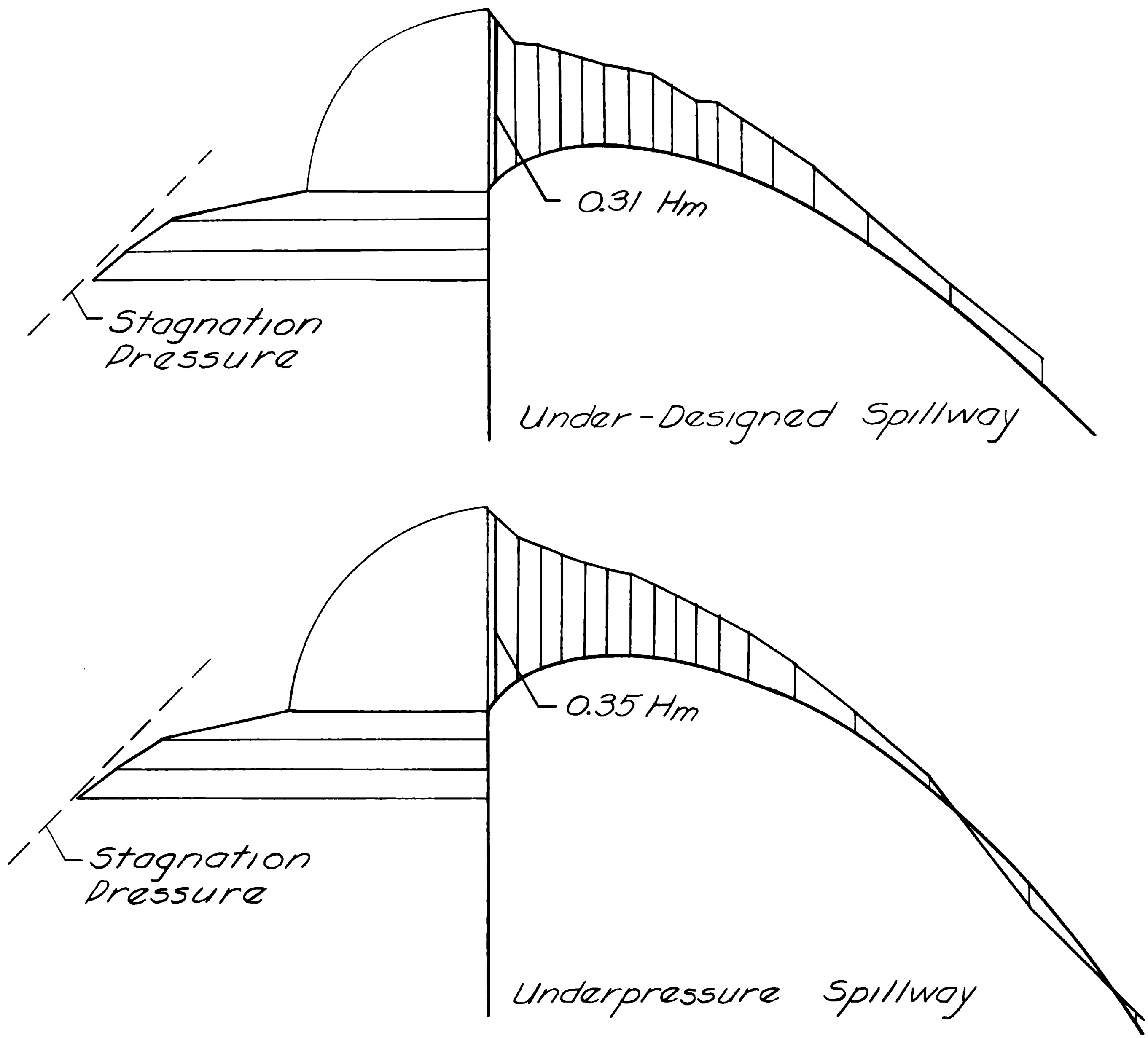


Fig. 7 Spillway Pressures, H - 0.50 Hm

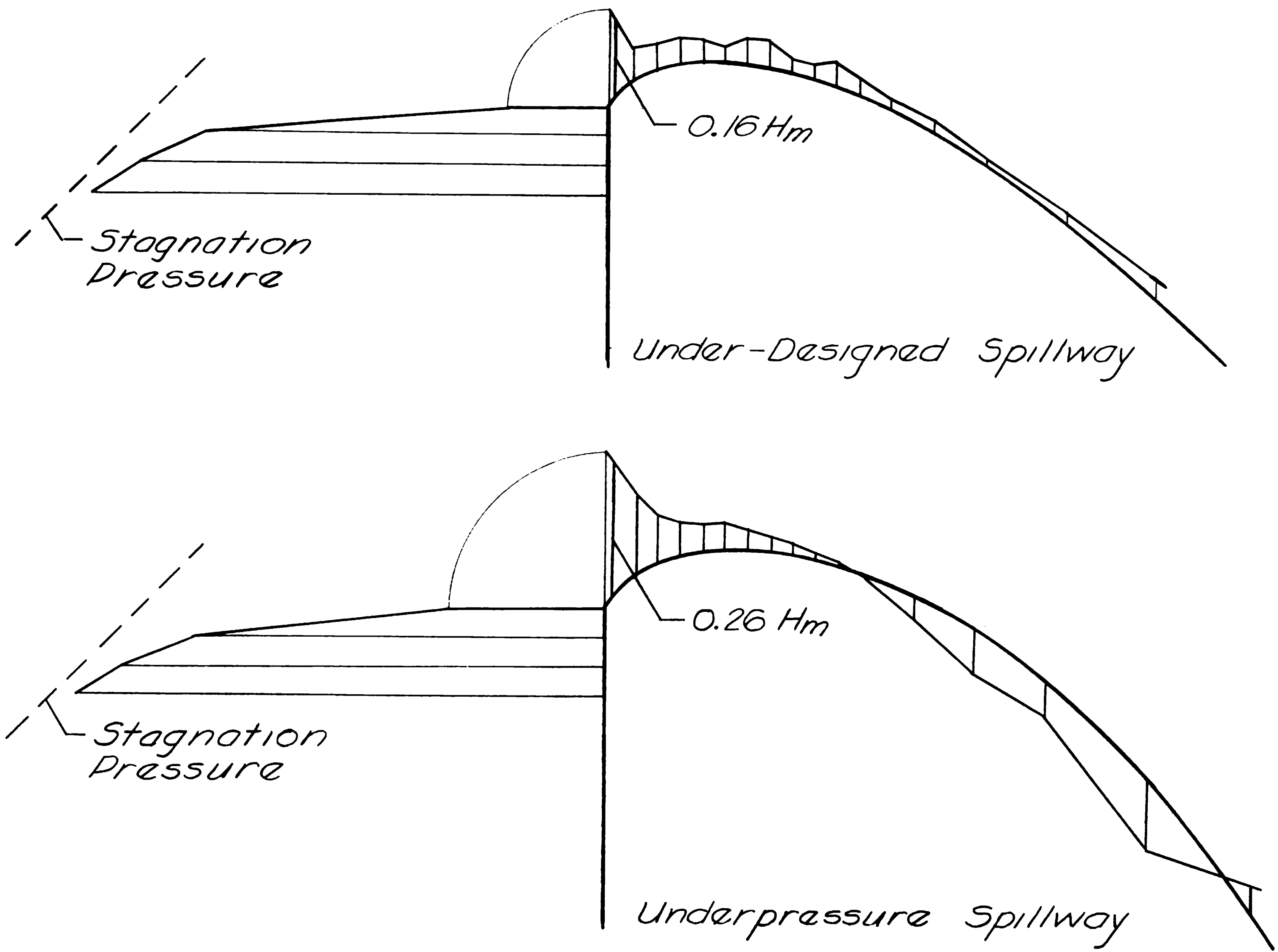


Fig. 8 Spillway Pressures, $H = 0.75 H_m$

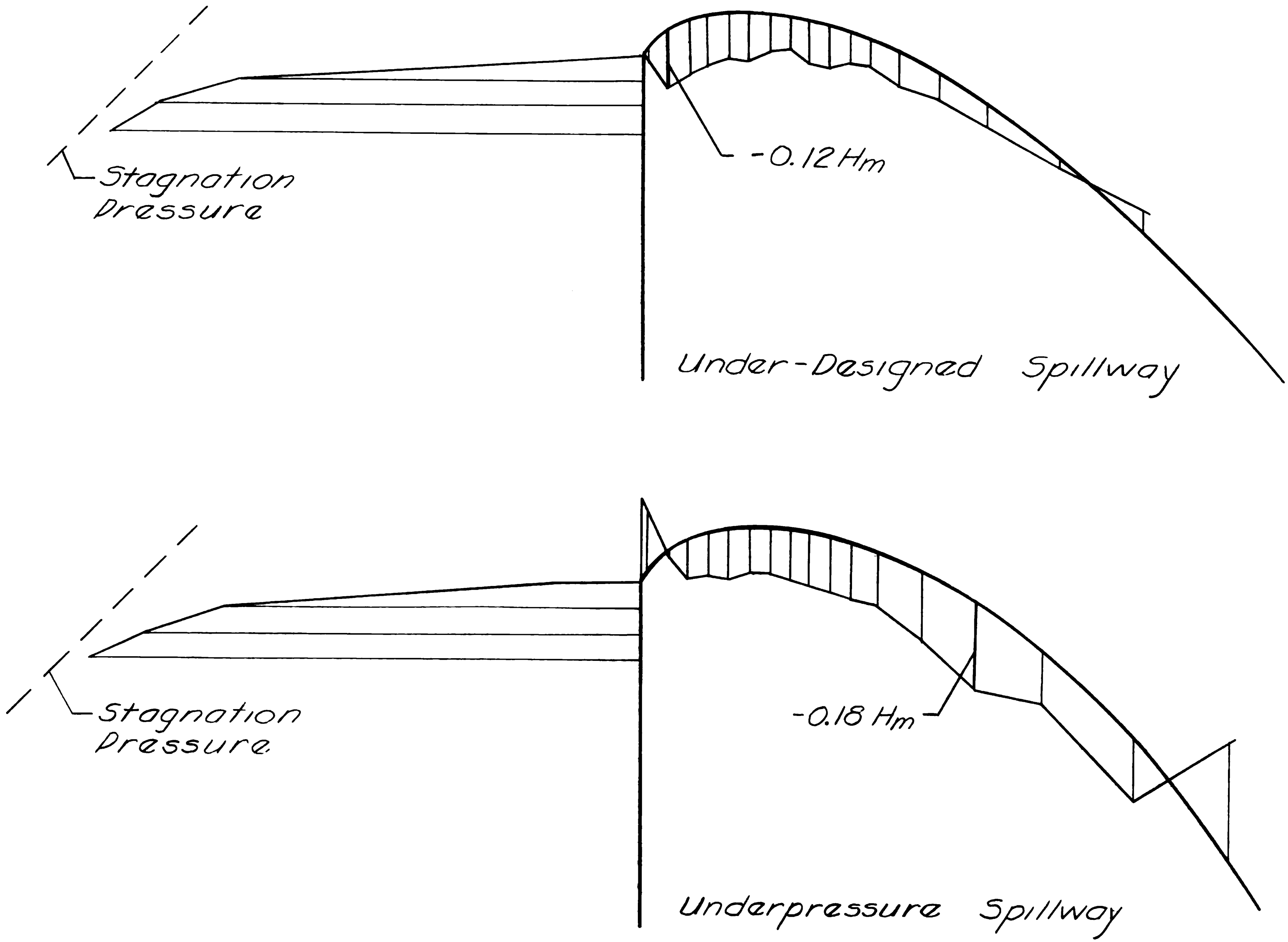


Fig. 9 Spillway Pressures, $H = 0.90\,H_m$

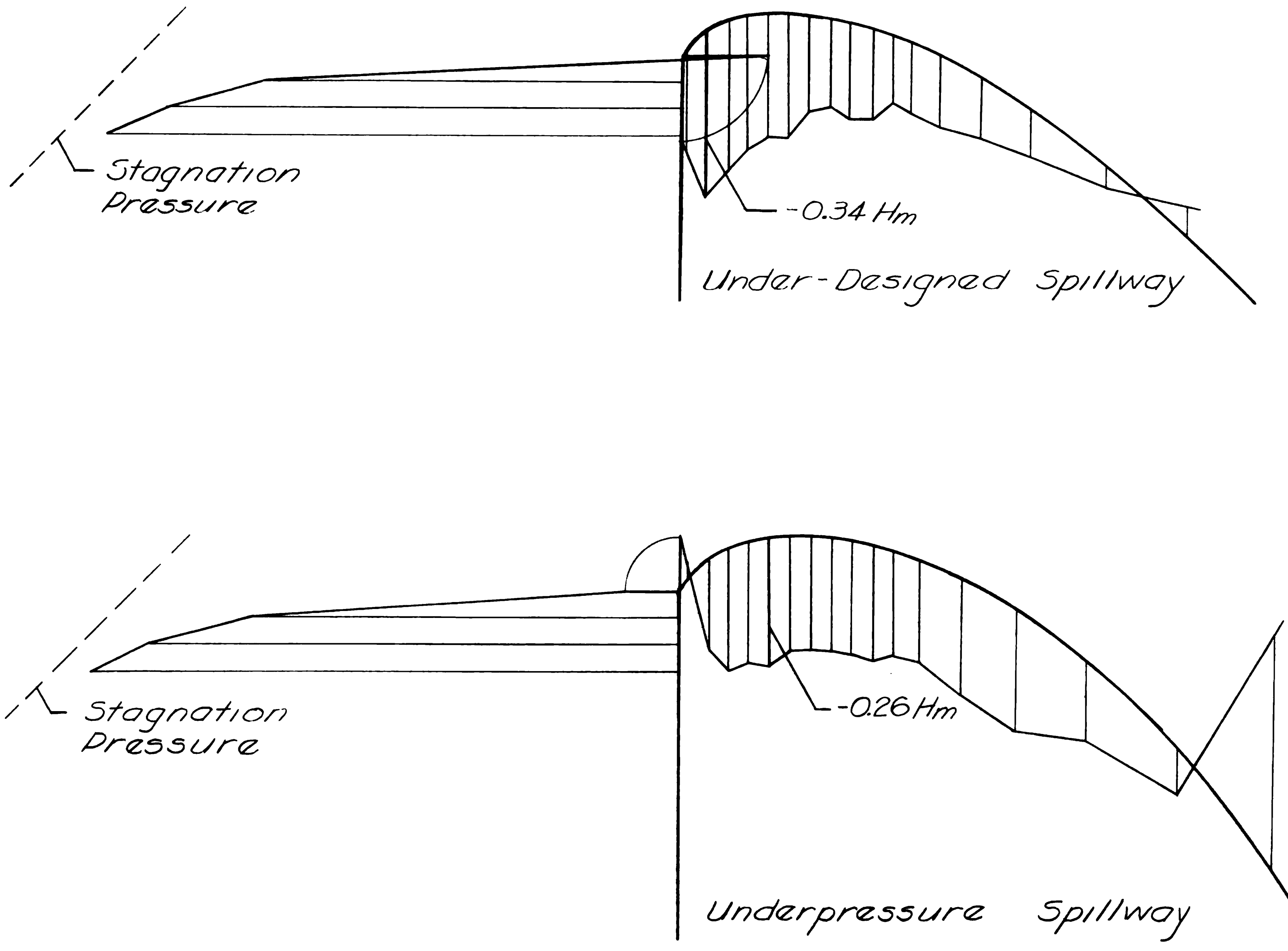


Fig. 10 Spillway Pressures, $H = 1.0 H_m$

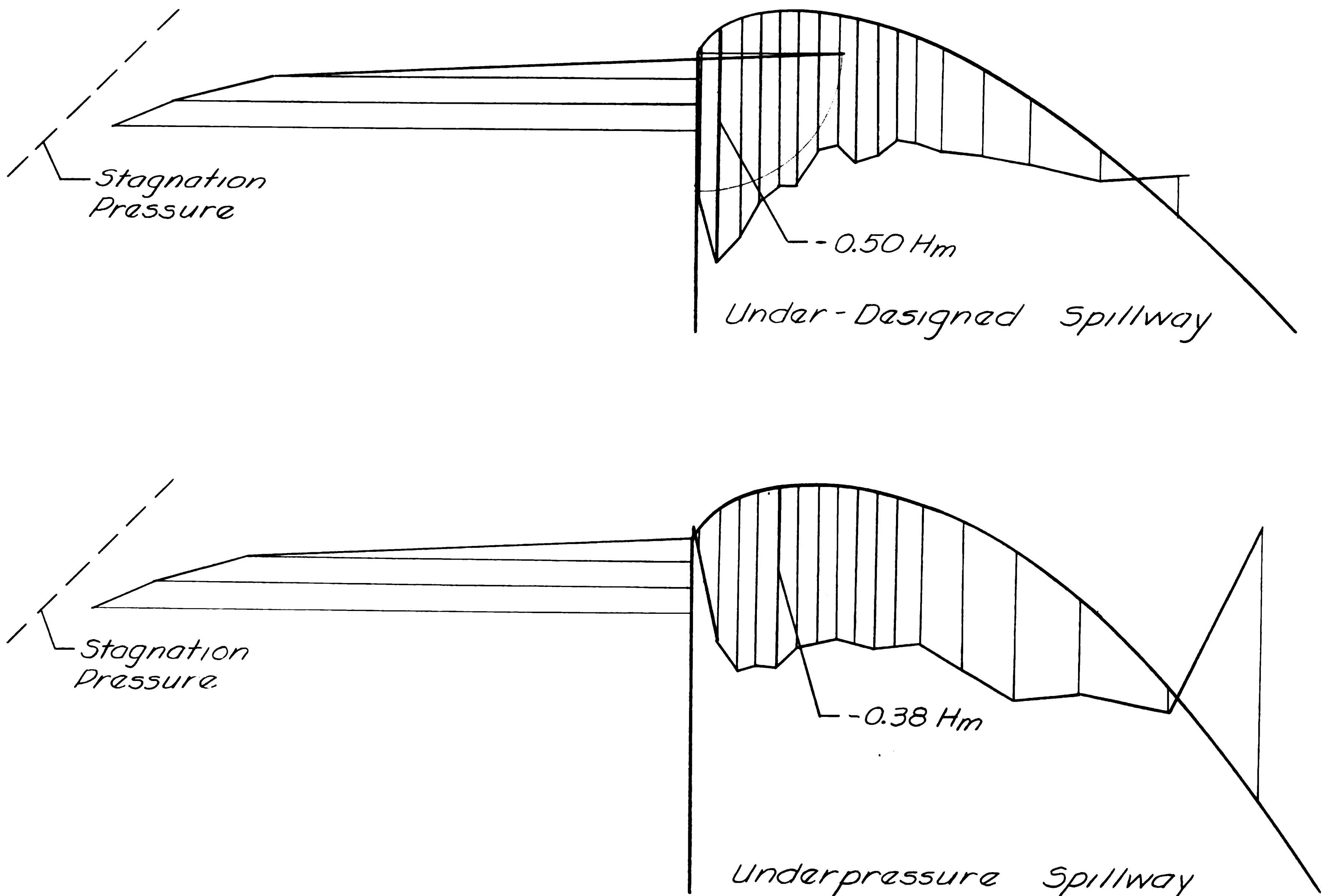


Fig. 11 Spillway Pressures, $H = 1.1\ H_m$

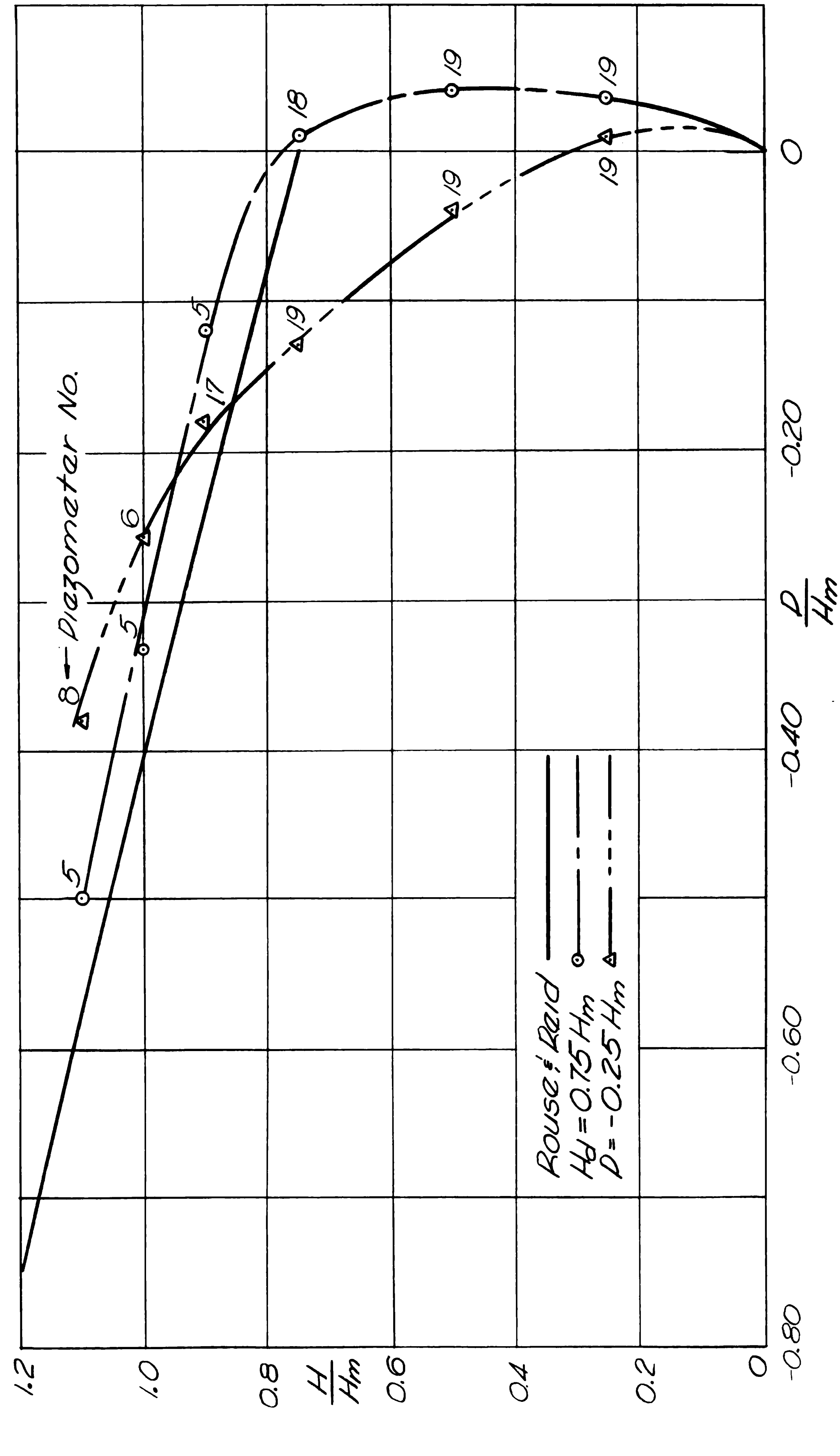


Fig. 12 Minimum Pressure

BIBLIOGRAPHY

1. Hydraulic Design, Spillways, Chapter 3, Part CXVI, Engineering Manual, Civil Works Construction, Department of the Army, Corps of Engineers.

2. Thorssen, L. A., "Effect of Vacuum on a Free Nappe," M. S. Thesis, State University of Iowa, 1948.

3. Johnson, J. W., "The Aeration of Sharp-Crested Weirs," Civil Engineering, March, 1935.

4. Report No. 34-1, Spillway and Stilling Basin for Chief Joseph Dam, Bonneville Hydraulic Laboratory, U. S. Army Corps of Engineers.

5. Rouse, H., and Reid, L., "Model Research on Spillway Crests," Civil Engineering, January, 1935.

6. Kandaswamy, P. K., and Rouse, H., "Characteristics of Flow over Terminal Weirs and Sills," Journal of the Hydraulics Division, ASCE, No. HY4, August, 1957.