TEXTURAL ANALYSIS OF OCTAHEDRITE METEORITES BY FOURIER SERIES SHAPE APPROXIMATION

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ABSTRACT

TEXTURAL ANALYSIS OF OCTAHEDRITE METEORITES BY FOURIER SERIES SHAPE APPROXIMATION

By

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Meteorites are generally classified into broad categories according to mineralogical composition, i.e., chondrites, achondrites, hexadrites, octahedrites, ataxites etc. Beyond this gross classification, little work has been done concerning more quantitive aspects of texture. One aspect of texture, shape, is a characteristic which reflects a crystals or lamellae chemical history. Possibly, similar or disimilar groups of crystals or lamellae can be discriminated if the precise shape can be measured.

Recently, a new method of shape discrimination (Ehrlich and Weinberg, 1970) utilizing Fourier Series shape approximation which in essence estimates the crystal or lemellae shape by an expansion of the periphery radius as a function of angle about the crystals center of gravity shows promise in evaluating the importance of shape by precise measurement. The purpose of this study is to determine whether shape evaluation can discriminate between meteorites composed of similar or disimilar crystals or lamellae.

Utilizing this method, four comparisons between kamacite lamellae within octahedrites were made; lamellae from three non-parallel faces of a portion of an octahedrite were compared, lamellae segments and entire lamellae of an octahedrite were compared with lamellae segments and entire lamellae of a second octahedrite, lamellae of two octahedrites thought to be similar were compared, and entire lamellae between a wide range of meteorites were compared.

The results, as shown by this method were:

1. Differences between octahedrites are due to fundamental differences rather than differences due to relative orientation of the cut surface with respect to the Widmanstatten pattern.

2. Where whole crystals are not available, standard segments of lamellae can be used, though, with a slight loss of discriminating power.

3. As an example in the way the method can be used to test a specific hypothesis, an Odessa Octahedrite sample was compared with a Canyon Diablo Octahedrite sample and found to be texturally distinct.

4. Five octahedrites were compared simultaneously and the results showed a pattern of similarities and differences which indicate the strength of the method and suggest a basis for a more refined class for octahedrites.

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BY FOURIER SERIES SHAPE APPROXIMATION

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INTRODUCTION

Meteorites consist of solid matter which have arrived on the earth from some location in outer space. Consequently, meteorites represent samples of the extra-terrestrial universe, and, will represent the major group of such samples not only for the present, but, for a long time in the future.

For these reasons, they have been subjected to intensive study from a wide range of viewpoints.

There are four general objectives of these studies:

(1) The mechanism of meteorite formation.

(2) The place of meteorite formation, i.e., in our solar system or some place beyond, or possibly both.

(3) Determination of the length of time that meteorites have been in space before coming to earth.

(4) How, if at all, were the meteorites transformed when passing through the earths atmosphere.

Practically all data related to these studies are based on detailed determinations of chemical, isotopic, or mineralogical abundances of all or portions of meteorites, and thus give information in terms of equilibrium of various sorts.

Another characteristic of meteorites that has long been observed and appreciated is texture. Texture refers to the geometric aspects of the component particles of a meteorite such as degree of crystallinity, crystal size, crystal shape, and arrangement or geometrical relationships between the constituents (Williams, Turner, and Gilbert, 1954). Textural data carries information concerning kinetics and reaction pathways as well as information concerning the genetic conditions of temperature and pressure which are also expressed mineralogically. If texture does carry such information, then quantitative evaluation of texture should serve as a valuable complement to the above mentioned studies. In addition, subsequent "metamorphic" effects could be manifested as textural modifications even though mineralogy might not change.

One direct application of texture already has been used to roughly classify meteorites texturally after they have been grossly classified into three main groups according to composition i.e., irons, stony irons, and stones.

Prior (1920) developed a classification based on both mineralogical composition, and subordinately, texture, which is presented in summary form in Table 1.

It should be noted that this classification, which is empirical, has apparent relevence to important questions concerning meteorites such as apparently each major type and perhaps subtype, each had a distinct genesis and subsequent history.

TABLE 1.--Classification of the Meteorites*.

	Group	Class	Principal Minerals
Stones	Chondrites	Enstatite	Enstatite, nickel-iron
		Olivine-bronzite	Olivine, bronzite, nickel-iron
		Olivine-hypersthene	Olivine, hypersthene, nickel-iron
		Olive-pigeonite	Olivine, pigeonite
		Carbonaceous	Serpentine
	Achondrites	Aubrites	Enstatite
		Diogenites	Hypersthene
		Chassignite	Olivine
		Ureilites	Olivine, pigeonite, nickel-iron
		Angrite	Augite
		Nakhlites	Diopside, olivine
		Eucrites and howardites	Pyroxene, plagioclase
	Stony-irons	Pallasites	Olivine, nickel-iron
		Siderophyre	Orthopyroxene, nickel-iron
		Lodranite	Orthopyroxene, olivine, nickel-iron
		Mesosiderites	Pyroxene, plagioclase, nickel-iron
	Irons	Hexahedrites	Kamacite
		Octahedrites	Kamacite, taenite
		Ni-rich ataxites	Taenite

These meteorite varieties do not occur proportionately, for example, a study of meteorite falls shows that approximately 85.7% of such falls consist of Stones. Stones consist mainly of silicate minerals, and are further classified as chondrites or achondrites. Chondrites are so named for the presence of small spherical shaped bodies called chondrules. Stones absent in chondrules, approximately 7.1%, are named achondrites.

A smaller percentage of meteorites, approximately 5.7%, consist entirely of metal, mainly nickel iron, which appropriately enough we call irons.

Lastly, the smallest percentage of meteorites, approximately 1.5%, are noted to be a combination of stones and irons and are named stony-irons.

After meteorites are classified into broad compositional categories, as mentioned previously, they are generally subdivided on the basis of texture i.e., stones divided into chondrites and achondrites, irons divided into hexahedrites, octahedrites, ataxites, etc.

Beyond this gross classification, little work has been done concerning more quantitative aspects of texture. If texture does carry information, then quantitative evaluation of texture should serve as a valuable complement to the above mentioned studies. That is, shape variation of phases within groups of meteorites classified alike might provide a detailed insight into solving some of the long unanswered problems.

The study of shape, which until recently, was limited as no precise evaluation techniques were known, now shows promise of being mathematically studied. These studies, hopefully, will reflect important and interesting geological parameters.

Therefore, the object of this thesis is to evaluate the importance of crystal or lamella shape in the solving of meteorite problems by precisely measuring such shape by the Fourier Series shape approximation method.

CHAPTER I

CHARACTERISTICS OF METALLIC TEXTURES

The largest percentage of iron meteorites are internally structured in a unique way and this structure becomes visible when the meteorite is sliced, polished, then etched with a dilute acid (Wood 1968). This structure is called the Widmanstatten structure after Count Alois de Widmanstatten who first observed this pattern in 1808 (Wood 1968). The structure consists of arrays of parallel plate like lamellae of low nickel-iron (low-Ni, 6 to 7%) alloy kamacite which results from the exsolution of an original, homogeneous, taenite crystal. Each meteorite has four such arrays intersecting with each other in such a way that they run parallel to the four planes defined by the faces of an imaginary octahedron and, thus, all iron meteorites which display this pattern are called Octahedrites.

By use of an electron-microprobe it has been found that the spaces between the kamacite crystals contain higher nickel to iron substances; taenite (Wood 1968). Also, troilite FeS, is present in accessory amounts in practically all meteorites. In irons it occurs as comparatively large nodules (Mason 1962).

These major elements in the Widmanstatten pattern, kamacite crystals, generally occur as sub-rectangular lamellae

(Fig. 4, 5, 7, 8, 10, 11, 13, 14) ranging in length from lengths equal to the width up to lengths where the lamellae exceed the length of the meteorite. The irregularity of the kamacite sides is an important aspect of the Widmanstätten pattern in that the general lamellar form of kamacite crystals are a direct result of crystal chemistry and thermodynamic conditions under which they formed. Restricted combinations of crystal chemistry and thermal history determine the Widmanstätten pattern, with its characteristic plate like lamellae. According to Wood (1968):

> Although octahedrites cannot be made in the laboratory (the processes involved take too long), we can understand theoretically how it was done--how the Widmanstätten structure and M-shaped Ni distributions must have been generated in them. Metallurgical experience tells us what would have happened in a mass of molten nickel-iron metal, if it were cooled slowly and steadily. The metal would have crystallized after it cooled beneath (roughly) 1400°C. During hundreds of degrees of cooling thereafter it would have existed in the form of a single homogeneous alloy: taenite. But beneath about 900°C, the situation is not so simple. The phase diagram of the Fe-Ni system (Fig. 1) shows which alloy or alloys should be present, if equilibrium prevails, at various temperatures and assuming various concentrations of Ni in the metal mass. We see at the highest temperatures a large field in which homogeneous taenite is the stable alloy, as already noted. And there is a field to the left, at very low values of Ni content, where homogeneous kamacite is the stable alloy. (The crystal structures of taenite and kamacite are somewhat different.) Between these fields is a third, where equilibrium requires that both taenite and kamacite should be present. The bulk Ni content of the octahedrites (6 to 15%) is such that as they cooled and so passed vertically downward through the phase diagram, they entered this taenite-plus-kamacite field. Here the Widmanstätten structure must have developed

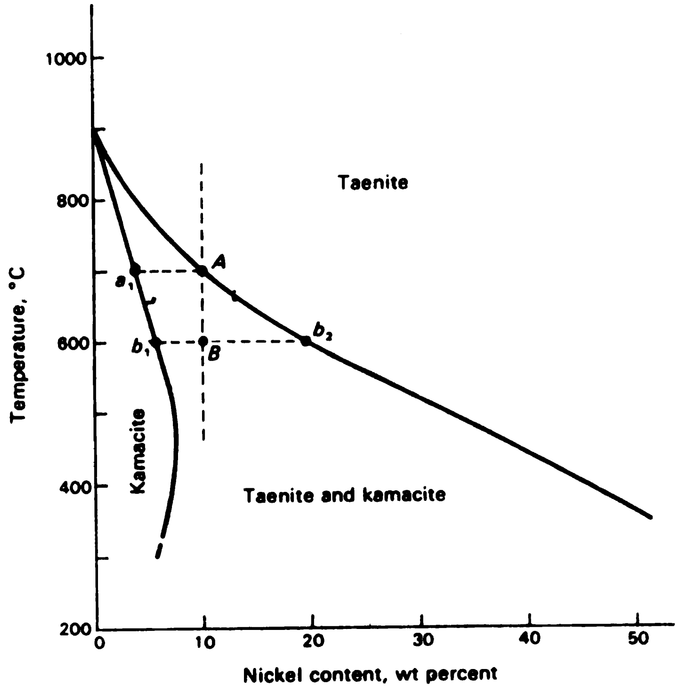

Fig. 1. Phase Diagram of the System Fe-Ni, Beneath 1000°C, at 1 atm Pressure.

Consider a mass with 10% Ni content, cooling along the line AB in Fig. 1. It passes from the taenite field into the taenite-plus-kamacite field at 700°C. At this point kamacite crystals must begin to appear in the mass, if equilibrium is maintained. Apparently when they did appear in the octahedrites, it was in the form of thin sheets or plates that preferred to grow through the original taenite crystal in a few very special directions, namely, parallel to the {111} lattice planes of the taenite. These planes bear an octahedral relationship to one another, so we can see how the peculiar geometry of the octahedrites was established.

The phase diagram tells us what the Ni content of these first-formed kamacite plates will be. If kamacite and taenite are both present and at equilibrium, their compositions will lie on the boundaries of the kamacite-plus-taenite field. Thus at 700°C equilibrium kamacite has composition a_1 in Fig. 1 about 4% Ni, while the taenite still contains ~10% Ni(A).

With further cooling, we can see from the sloping field boundaries that these alloy compositions must change. By the time 600°C is reached, equilibrium kamacite must contain ~5.5% Ni (b_1), taenite

~17% Ni (b_2). The Ni content of taenite tends to increase indefinitely with lowering temperature, while kamacite increases its Ni down to ~500°C but tends to lose Ni below this temperature.

The additional Ni which increases the Ni content of kamacite and taenite comes from the interfaces where kamacite and taenite abut against one another. The additional Ni moves into the alloy crystals by lattice diffusion (Wood 1968). The departure from planer structures of the kamacite are due to differential rates of growth of the kamacite which in turn results from differential rates of diffusion. At high temperatures, the diffusion rate would be high and most likely you would have more planer structures. At lower temperatures, the diffusion rate is decreased and may be a function of degree of imperfection, amount of impurities, volatiles present, etc.

The differences in lamellae textures that we see, especially shape, are basically due to:

1. Different initial bulk compositions (6-15% Ni).
2. Different cooling rates.
3. Different equilibrium temperatures.
4. Metamorphism or heating of the octahedrites.

Thus, each octahedrite with its associated lamellae will have a characteristic shape dependent on the above factors and if we can precisely describe such shape, we can distinguish one group of lamellae from another disimilar group of lamellae.

In choosing the kamacite lamellae as a basis of shape comparison, we find no difficulties in working with the lamellae that have a length not more than three times the width, but the greatly elongated lamellae produce an exaggerated second harmonic, which we will talk about later, that greatly overshadow all other harmonics and depress the other higher harmonics. This problem is overcome by sectioning of the lamellae into shorter, easier to work with shapes.

CHAPTER II

FOURIER SERIES SHAPE APPROXIMATION METHOD

In 1807, the French mathematician Fourier showed that any function which is defined in the interval t=0 to 2π which satifies certain conditions can be expressed as an infinite series of trigonometric functions i.e.,

$$R(\theta) = R_0 + \sum_{n=1}^{\infty} A_n \cos n\theta + \sum_{n=1}^{\infty} B_n \sin n\theta. \quad (1)$$

Mathematically it can be shown that for the best possible approximation to $R(\theta)$, the coefficients R_0, A_n, B_n must be:

$$R_0 = \frac{1}{2\pi} \int_0^{2\pi} R(\theta) d \ , \quad (2a)$$

$$A_n = \frac{1}{\pi} \int_0^{2\pi} R(\theta) \cos n\theta d\theta, \quad (2b)$$

$$B_n = \frac{1}{\pi} \int_0^{2\pi} R(\theta) \sin n\theta d\theta, \quad (2c)$$

n=1,2,3,......

This method is used as a shape approximation which leads to a derivation of a linear equation from which shape per se can be regenerated. For a detailed discussion of the method used, the reader is referred to Ehrlich & Weinberg (1970). The discription which follows is taken largely from that source.

In essence, the lamella shape is estimated by an expansion of the periphery radius as a function of angle about the lamellae center of gravity by the Fourier Series. The radius is therefore given by: (converting for. 1 to polar coordinates)

$$R(\theta) = R_0 + \sum_{n=1}^{\infty} R_n \cos (n\theta - \phi_n), \tag{3}$$

where θ is the polar angle measured from an arbitrary reference line, R_0 is the average radius of the lamella in the plane of interest, n is the harmonic order, R_n is the harmonic amplitude, and ϕ_n is the phase angle. Shape precision is controlled by the number of harmonic orders we compute with precision varying directly as n.

Each term of the expansion, in fact, represents the contribution of a known shape component. The first term R_0 delimits a centered circle with area equal to the area of the figure being documented i.e., the first harmonic is an offset circle, the second harmonic a figure eight, the third harmonic a trefoil, and the fourth harmonic a four-leaf clover.

Shapes represented by various components of the Fourier series together with a regenerated shape from a ten-harmonic series, are shown in Figure 2. Figure 2a represents the combination of the "zeroth" and second harmonics. Note that data is normalized such that R_0 has a value of unity. The coefficient ".18" weights the relative contribution of the second harmonic, and the offset angle ($\alpha = \phi_n/n = 74°$) orients the harmonic with respect to the coordinate system. Figure 2b illustrates the combination of the centered circle and the third harmonic for the same shape; and Figure 2c shows combination of the three orders. The shape produced in Figure 2d indicates exactly the amount of shape information contained in ten harmonics as compared with the initial shape. Centers of the small circles around the outline depict actual points on the perimeter of the original figure.

An improved "fit" between the shape model (in this case a Fourier series of ten harmonics) and the shape of the original figure can be obtained simply by adding more terms to the series. It should be apparent that all shape information can be contained in the shape equation, if desired, since by this method shape may be described mathematically as precisely as desired. Alternatively, a constant level of generalization may be specified through control of the number of harmonics contained in the shape equation.

Raw data required for generation of the Fourier expansion are coordinates for points on the perimeter of the figure. At least twice as many points as the order of the highest desired harmonic must be known. In order to simplify interpretation of the generated model it is convenient to use the center of gravity of the shape as the origin of the radius expansion. If an arbitrary origin of coordinates has been used in data collection, which is likely to be the case if an automatic digitizer is employed, transformation of the coordinates to the center of gravity as origin accompanies transfer of the arbitrary origin to the computer-derived center of gravity.

In the program used for examples cited in this paper, the center of gravity of the figure is calculated from rectangular perimeter coordinates. Subsequently the perimeter points are converted to polar coordinates from which the Fourier terms for each harmonic are calculated. Theory of the procedure is detailed in an earlier paper by Ehrlich and Weinberg (1970). A figure with 40 peripheral points requires about 1.5 seconds of CDC 3600 computer time for generation of the first ten harmonics.

One limitation of the method should be mentioned at this point. A Fourier series is useful in expanding only a single-valued function. Any radius can therefore intersect the perimeter of the figure only once. Irregularities of shape, such as "embayments", which cause multiple intersection of radii and perimeter cannot be accommodated. The perimeters of a few lamellae had to be smoothed somewhat in places on account of embayments or projections causing multiple intersections of lamallae outline and radii. Nevertheless, because such smoothing affects only the high-order harmonics, shape analyses based only on the first few terms of the Fourier expansion should not be severely affected by the limitation.

Some lamellae were rejected, for a reason which was first thought to be the result of bi-valueness, but actually

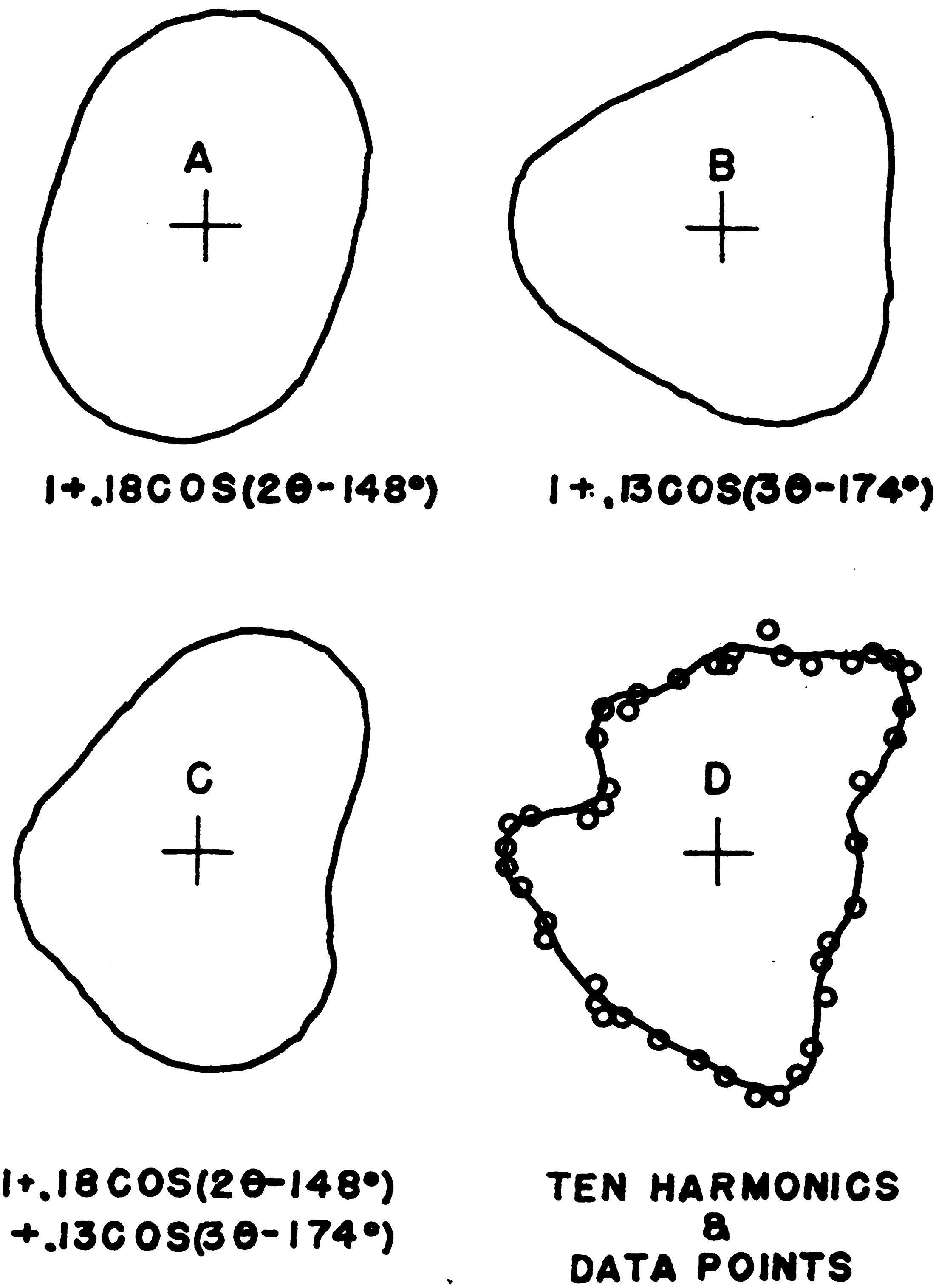

Fig. 2. Harmonic Contribution to Shape
(Ehrlich and Weinberg, 1970).

was the result of rejection by failure to pass another internal test of the program, i.e., the radial angle from the center of gravity to any two adjacent peripheral points should not be more than about six times the average angle. This was implemented as a check in the efficiency of digitization to insure uniform coverage and as a guard against any outright errors of peripheral coordinates. One can easily see that in a roughly circular figure equal angles can be produced by uniform placement of peripheral points, however, if the lamella is very elongate and assymetric where in that one end is thickened, the center of gravity will be shifted to a position where points that subtend equal angles are placed with different unequal spacing around the periphery. This will result in some areas being denser with respect to peripheral points than other areas. Since many lamellae in this study are of this shape and the digitization was performed at equal increments along the periphery, it is not surprising that many lamellae were rejected.

Taking into account the periphery spacing problem along with the above mentioned problems concerning exaggerated second harmonics of the extremely elongate lamellae, the very elongate lamellae were subdivided into relatively equant segments.

In this study two dimensional lamella shapes were quantified in two modes. When the lamella length was less than approximately three times the width, the entire lamella shape was quantified. When lamella length to width ratio was greater than approximately three, the lamellae were subdivided by sectioning them perpendicular to the long axis at intervals equal to the average width. Eliminating the ends, the subdivisions yield quadrate figures with two parallel straight sides that were produced by the perpendicular and the two subparallel sides, each, which are segments of the two long sides of the lamellae. When lamellae are subdivided in this way, comparisons can be made between lamellae in a single meteorite as well as between lamellae from more than one meteorite on a comparitive basis. The only differences between these subdivisions of the original lamellae are the variations of the two boundary segments which as mentioned previously are segments of the two long sides of the lamellae. Therefore, all similarities and unsimilarities found in this study is a reflection of these long sided boundary segments which are in turn a reflection of their parent lamellae.

CHAPTER III

METEORITES SELECTED FOR SHAPE ANALYSIS

This study involves the examination of the shapes of a large number of octahedrite lamellae taken from photographs extracted from various sources (Table 2); some from the literature, some from photographs procured from the Smithsonian Institution, and others from photographs of octahedrite samples at Abrams Planetarium--Michigan State University. The various meteorites fall into a number of sub-classes according to the specific objective in each case.

One group consisting of 39 kamacite lamellae taken from a sample of the Trenton Octahedrite (plates 1,2,3 Table 2), were examined to determine whether observed differences between meteorites could be explained by the differences between sections cutting through three different planer directions of the well known, highly orientated Widmanstatten Pattern.

A second data group consisting of 42 segments from ten Arispe Octahedrite lamellae (plate 4, Table 2) and 44 segments from eight Wiley Octahedrite lamellae* (Table 2) were compared to determine the result of segmentation. Also, 18 entire

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*(Brett and Henderson, 1967).

lamellae were compared with 14 Wiley entire lamellae to determine if the method would be successful in discriminating one crystal from another according to shape.

A third group consisting of 15 lamellae taken from a portion of an Odessa Octahedrite sample (Plate 4, Table 2) were compared with 14 lamellae taken from a portion of a Canyon Diablo sample (Plate 6, Table 2) to determine if any similarities, as suggested by Evans 1961, can be seen using the Fourier Series shape approximation method.

A last data group consisting of 20 Sacramento Mountains Octahedrite lamellae (Plate 7, Table 2), 14 Arispe Octahedrite lamellae, 15 Odessa Octahedrite lamellae, 14 Canyon Diablo Octahedrite lamellae, and 20 Trenton Octahedrite lamellae were compared as a wide range sample.

TABLE 2.--Meteorites Selected for Shape Analysis.

Meteorite	Type	Date and Location of Find*	Source of Textural Photograph
Arispe	Iron-coarse octahedrite	1896-Sonora, Mexico	Specimen borrowed from the U.S. National Museum of the Smithsonion Institution
Canyon Diablo	Iron-medium octahedrite	1936-Coconino County, Arizona, USA	Same
Odessa	Iron-coarse octahedrite	1922-Ector County, Texas, USA	Specimen in Michigan State University collection
Sacramento Mountains	Iron-medium octahedrite	1896-Eddy County, New Mexico, USA	Specimen borrowed from the Museum of Minneapolis Public Library
Trenton	Iron-medium octahedrite	1858-Washington County, Wisconsin, USA	Specimen in Michigan State University collection
Wiley	Iron-microscopic octahedrite	1938-Prowers County, Colorado, USA	Geochimica et Cosmochimica Acta, 1967, Vol. 31, pp. 724-725, Fig. 2.

*(Prior 1953).

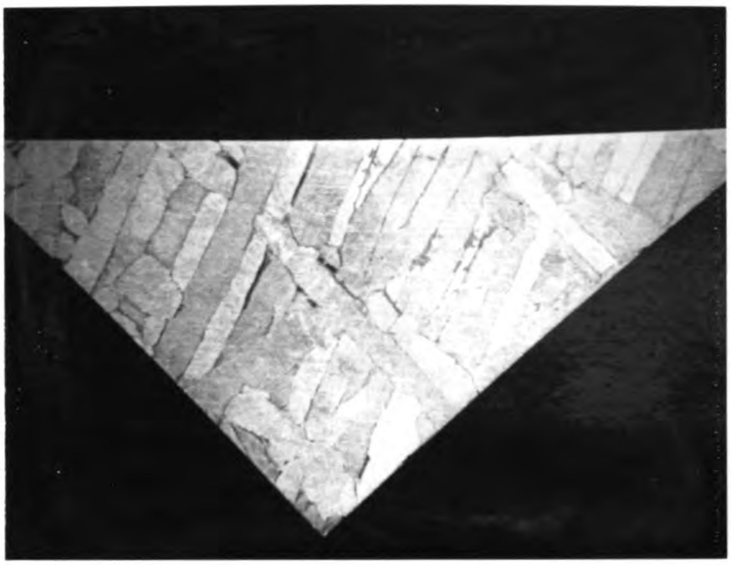

PLATE 1. Trenton Octahedrite--First Face.

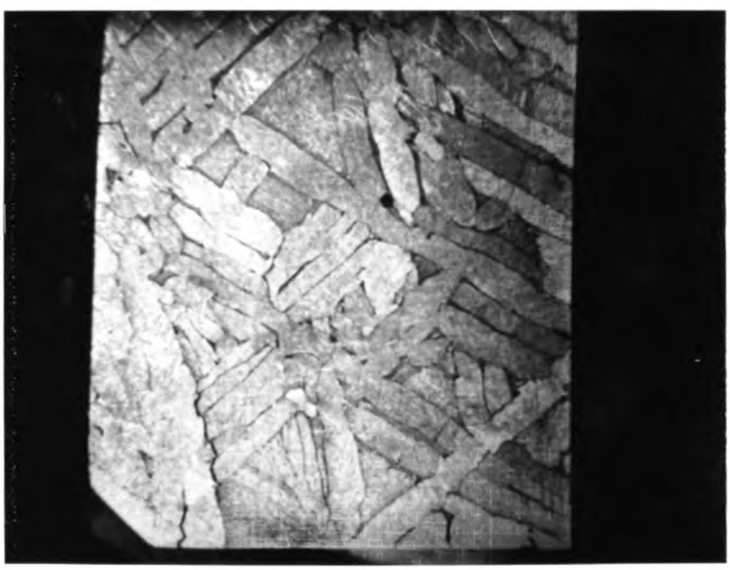

PLATE 2. Trenton Octahedrite--Second Face.

PLATE 3. Trenton Octahedrite--Third Face.

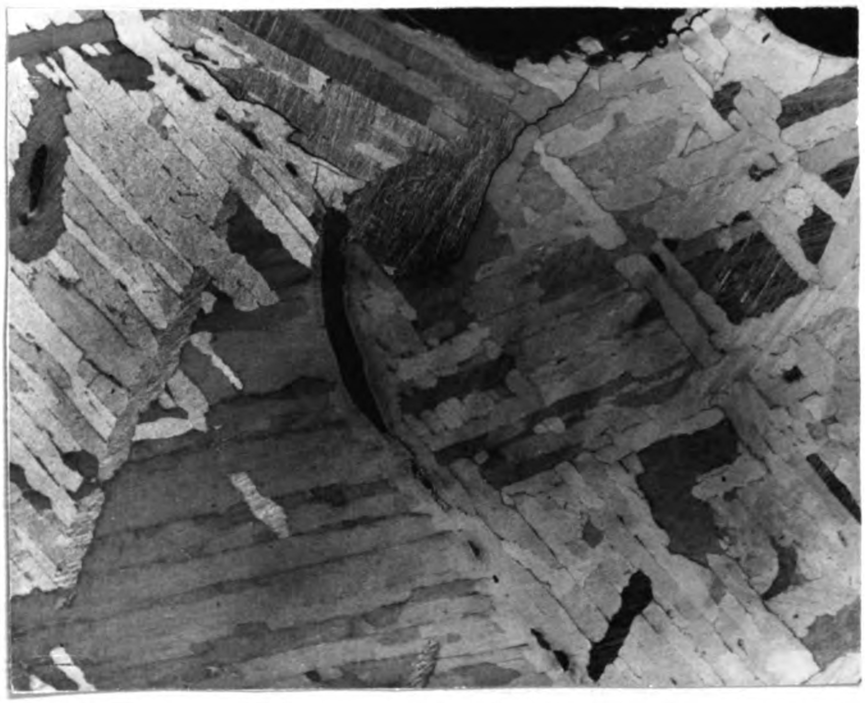

PLATE 4. Arispe Octahedrite.

PLATE 5. Odessa Octahedrite.

PLATE 6. Canyon Diablo Octahedrite.

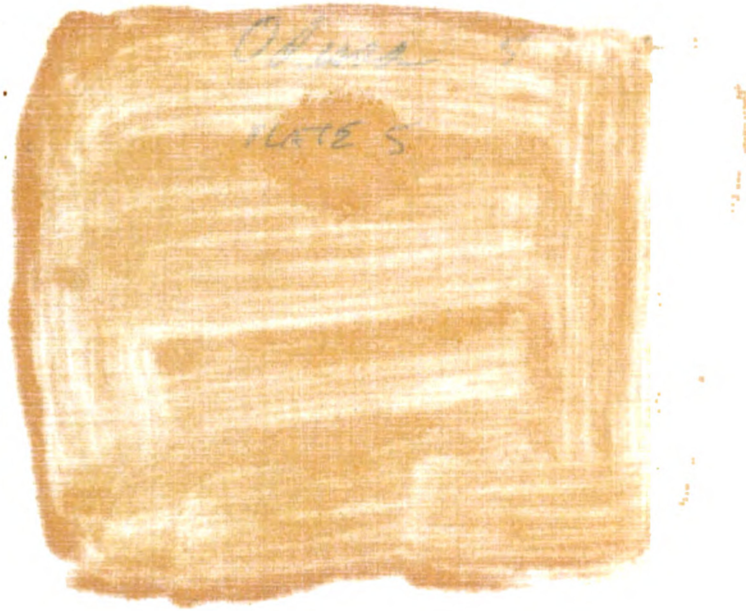
PLATE 5


PLATE 6

PLATE 7. Sacramento Mountains Octahedrite.

CHAPTER IV

RESULTS

Evaluation of the Effect of Orientation on Lamella Shape

Lamellae from three non-parallel faces of a portion cut from the Trenton Octahedrite were compared to determine whether there were differences in the shape of lamellae taken from surfaces cut at different angles through the Widmanstatten Pattern. Fifty seven lamellae were compared; nineteen from one face, twenty from another face, and eighteen from the third face.

The comparison was effected utilizing both ten and five harmonics. Evaluation of results was based on simple inspection of the mean harmonic amplitude spectra (Table 3, Figure 3) and the output of the classification matrix from the discriminate analysis (Table 3). In addition, a statistical index distributed as chi square and related to Maholanohis coefficient of racial distance ("D") was used to evaluate the quality of discrimination.

The results indicate that the orientation of the sampled face to the texture did not produce shape differences between the groups of lamellae from different faces. Inspection of the classification matrices of the discriminate functions show that of 57 lamellae, 22 were correctly classified when utilizing ten harmonics and 21 were correctly classified when utilizing only the first five harmonics. This poor level of discrimination is mirrored by a correspondingly low and non-significant chi square value (Table 3).

The lamellae were evaluated using both the first 5 harmonics and 10 harmonics in order to test the possibility that most of the shape differences were manifested at lower harmonic values, which would indicate that the shape differences were of gross shape. By including higher level harmonics, that had no power of discrimination, the degrees of freedom of chi square would increase without a corresponding increase in the value of the chi square statistic.

A major conclusion from this phase of the study is the orientation of the cut does not affect the shape families of kamacite lamellae determined by the Fourier analysis. Thus, differences between groups of lamellae from two different meteorites can be judged to be due to fundamental differences between meteorites rather than to differences due to relative orientation of the cut surface with respect to the Widmanstatten pattern.

TABLE 3.--Amplitude Spectra and Classification Matrix of Three Faces of Trenton Octahedrite--Ten and Five Harmonics.

Mean Harmonic Amplitude Spectra

Harmonic No.	Meteorite Trenton Face 1	Face 2	Face 3
1	.03831	.03484	.03096
2	.48530	.44411	.45866
3	.04847	.05091	.04785
4	.17167	.16549	.12894
5	.04357	.04771	.03239
6	.07382	.07508	.04911
7	.03165	.03648	.02721
8	.03861	.03584	.02447
9	.02895	.02452	.01818
10	.02549	.02382	.02305

Classification Matrix From Discriminate Analysis

	No. of Lamellae	Meteorite Trenton Face 1	Face 2	Face 3
Face 1	13	5	7	1
Face 2	14	3	9	2
Face 3	12	2	2	8

Mean Equality Degs of Free. 20 Chi Square 28.

TABLE 3.--Continued.

Mean Harmonic Amplitude Spectra

Harmonic No.	Meteorite Trenton Face 1	Face 2	Face 3
1	60.03364	91.40545	*0.26930
2	67.45517	*9.90600	63.99025
3	72.99987	95.48002	35.76653
4	91.26459	*7.11140	98.99962
5	42.61850	69.10987	86.22923

Classification Matrix From Discriminate Analysis

	No. of Lamellae	Meteorite Trenton Face 1	Face 2	Face 3
Face 1	13	8	3	2
Face 2	14	7	7	0
Face 3	12	6	0	6

Mean Equality Degs of Free. 10 Chi Square 18.

Fig. 3. Amplitude Spectra of Lamellae of Three Faces of Trenton Octahedrite.

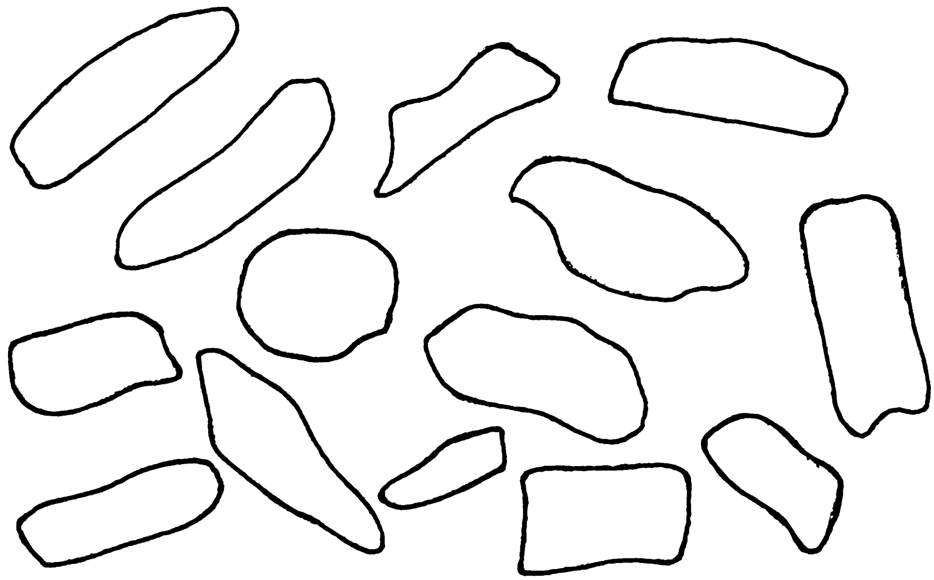

Fig. 4. An Example of Some Trenton Octahedrite Lamellae Outlines.

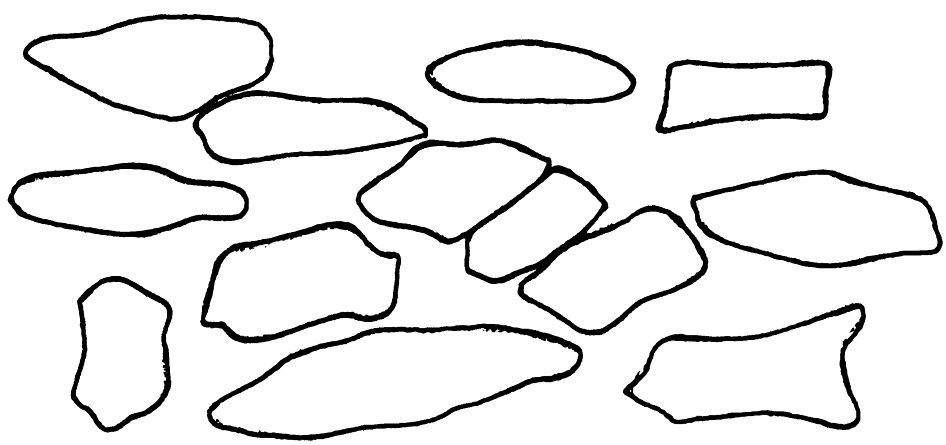

Fig. 5. An Example of Some Sacramento Mountains Octahedrite Lamellae Outlines.

Comparison of Lamellae Within and Between Meteorites

In this study two comparisons were made; lamellae segments are compared (Fig. 6 & 7), and entire lamellae were compared (Fig. 9 & 10). The segments were compared to determine if such sectioning as mentioned above can be used for discrimination between meteorites. The entire lamellae were compared to see whether the method would be successful in discriminating a group of entire lamellae from one meteorite from a similar group taken from another meteorite.

Evaluation of Segments

The segments were compared in two ways; in the first 42 segments from ten Arispe Octahedrite lamellae were compared with 44 segments of eight Wiley Octahedrite lamellae. Inspection of the mean harmonic amplitude spectra (Table 4, Fig. 5) and the classification matrix (Table 4) indicate distinct differences between the two groups. The discriminate function correctly classified 34 of 42 Arispe segments and 28 of 44 Wiley segments. This resulted in a chi square value of 26 with ten degrees of freedom which means that the probability this could occur by chance is less than .01. Thusly, the method can discriminate lamellae segments from one meteorite against lamellae segments of a second meteorite.

The second phase concerns the utilization of the use of segments in meteorite textural analysis to determine the degree that a given segment represents the exact lamellae from which it was taken. To test this hypothesis ten lamellae, six from the Arispe Octahedrite, and four from the Wiley Octahedrite, were chosen for segmentation. Of the ten lamellae, there were 4, 12, 3, 5, 4, 4, 5, 10, 5, and 11 segments respectfully associated with each lammela. The results according to the mean harmonic amplitude spectra (Table 4), and the classification matrix (Table 4) showed that in the case of the first lamella with its associated four segments, three were correctly classified with the parent lamella, four out of twelve correctly classified in the second lamella, one out of three correctly classified in the third lamella, one out of five in the fourth, three out of four in the fifth, two out of four in the sixth, one out of five in the seventh, five out of ten in the eighth, two out of five in the ninth, and nine segments out of eleven were correctly classified in the tenth lamella. When the classification matrix misplaced Arispe segments, it usually grouped the misplaced segments with another Arispe lamella, and when the Wiley segments were misplaced, they were usually grouped with another Wiley lamella. The chi square value of 135 with 90 degrees of freedom indicates the excellant discriminating pattern in grouping the segments with their parent lamella and the probability this could occur by chance is less than .01.

Thus, from the results described above, the segments can be seen to discriminate between meteorites and this power of discrimination probably arises from the character of the

TABLE 4.--Amplitude Spectra and Classification Matrix of Arispe and Wiley Octahedrites--Segments.

Mean Harmonic Amplitude Spectra

Harmonic No.	Meteorite: Arispe	Meteorite: Wiley
1	.00564	.00648
2	.06285	.11841
3	.04393	.04092
4	.12155	.11071
5	.01982	.01693
6	.01877	.03287
7	.01714	.01547
8	.03056	.02690
9	.01095	.01105
10	.00926	.01431

Classification Matrix From Disciminate Analysis

Meteorite	No. of Segments	Meteorite: Arispe	Meteorite: Wiley
Arispe	42	34	8
Wiley	44	16	28

Mean Equality Degs of Free. 10 Chi Square 26.

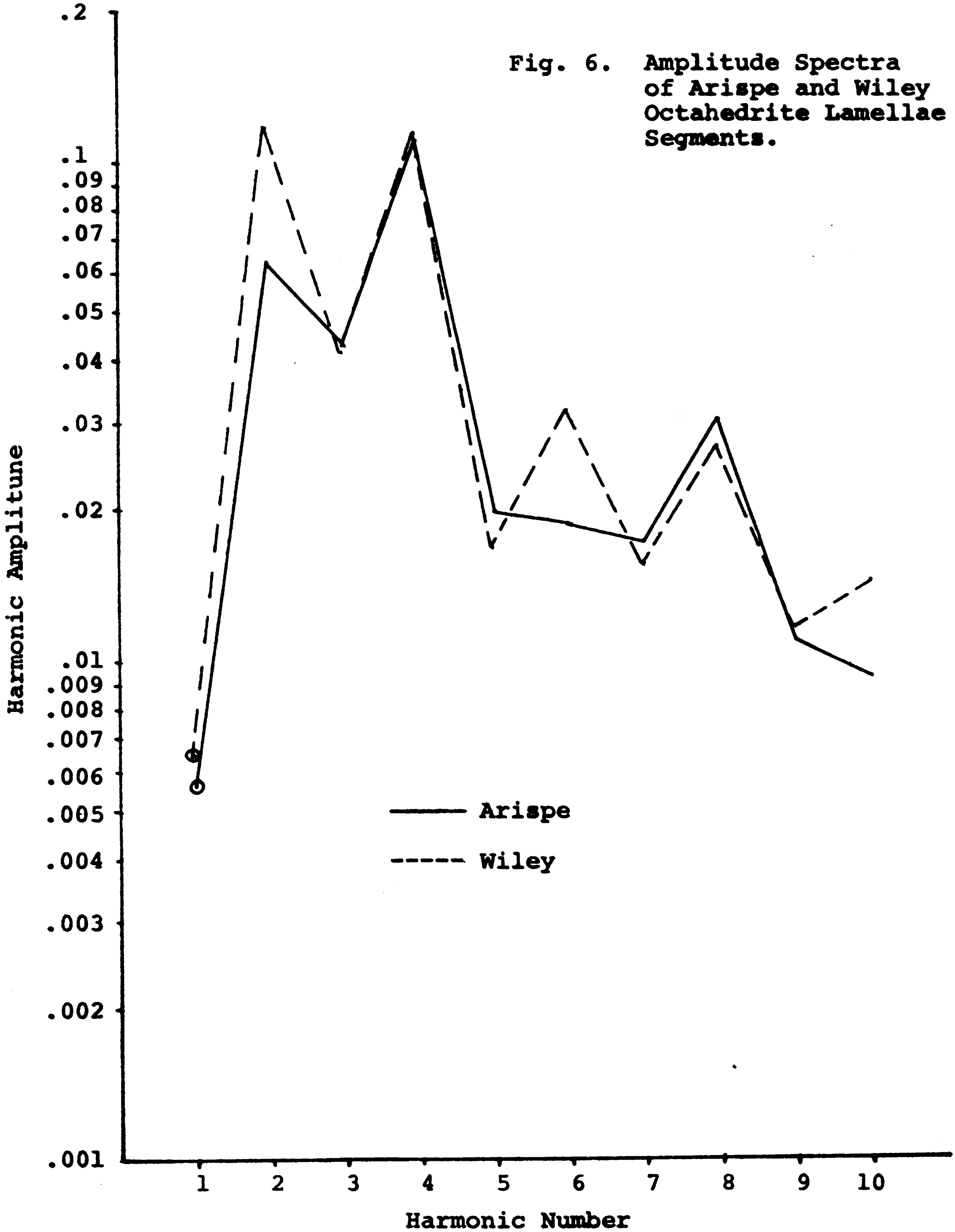

Fig. 6. Amplitude Spectra of Arispe and Wiley Octahedrite Lamellae Segments.

TABLE 5.--Amplitude Spectra and Classification Matrix of Arispe and Wiley Octahedrites--Segments.

Mean Harmonic Amplitude Spectra

Harmonic No.	Meteorite Arispe	Arispe	Arispe	Arispe	Arispe
1	.00282	.00609	.00818	.00625	.00264
2	.01862	.04870	.05974	.07603	.06087
3	.02150	.04693	.05577	.03685	.03787
4	.12415	.12285	.12389	.12466	.11811
5	.01365	.02029	.02612	.01636	.00802
6	.01037	.01495	.01764	.02080	.02288
7	.01309	.01731	.01716	.01652	.02085
8	.03902	.03143	.03373	.03089	.02643
9	.00620	.01256	.01035	.00726	.00810
10	.00650	.00824	.00972	.01027	.01050

Mean Harmonic Amplitude Spectra

Harmonic No.	Arispe	Wiley	Wiley	Wiley	Wiley
1	.00452	.00401	.00732	.00396	.00834
2	.04730	.06983	.14229	.08141	.17271
3	.04984	.03728	.04056	.05467	.03853
4	.12666	.12360	.11428	.11634	.09885
5	.02357	.01261	.01569	.01067	.02149
6	.01416	.02162	.03383	.01835	.05377
7	.01640	.01791	.01401	.02064	.01391
8	.03176	.03290	.03278	.02853	.01706
9	.01291	.00884	.01101	.00675	.01481
10	.00754	.01074	.01810	.01056	.01957

TABLE 5.--Continued.

Classification Matrix From Discriminate Analysis

Lamella No.		No. of Segments in each Lamella	1	2	3	4	5	6	7	8	9	10
1	Arispe	4	3	1	0	0	0	0	0	0	0	0
2	Arispe	12	0	4	0	1	0	5	0	1	1	0
3	Arispe	3	1	0	1	0	0	0	0	0	1	0
4	Arispe	5	1	0	1	1	1	0	0	0	0	1
5	Arispe	4	0	0	0	0	3	0	0	0	1	0
6	Arispe	4	0	0	0	1	1	2	0	0	0	0
7	Wiley	5	0	1	0	1	1	0	1	1	0	0
8	Wiley	10	3	0	0	1	1	0	0	5	0	0
9	Wiley	5	1	1	0	0	1	0	0	0	2	0
10	Wiley	11	0	0	0	0	0	1	0	1	0	9

Mean Equality Degs of Free. 90 Chi Square 135.

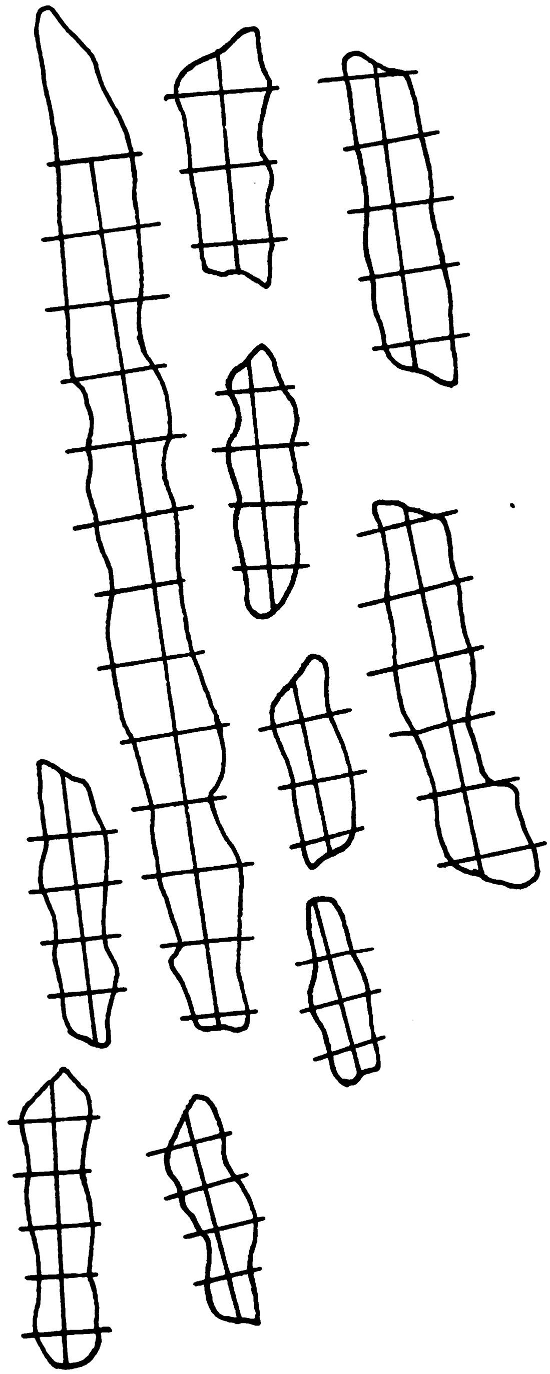

Fig. 7. An Example of Some Segmented Arispe Octahedrite Lamellae.

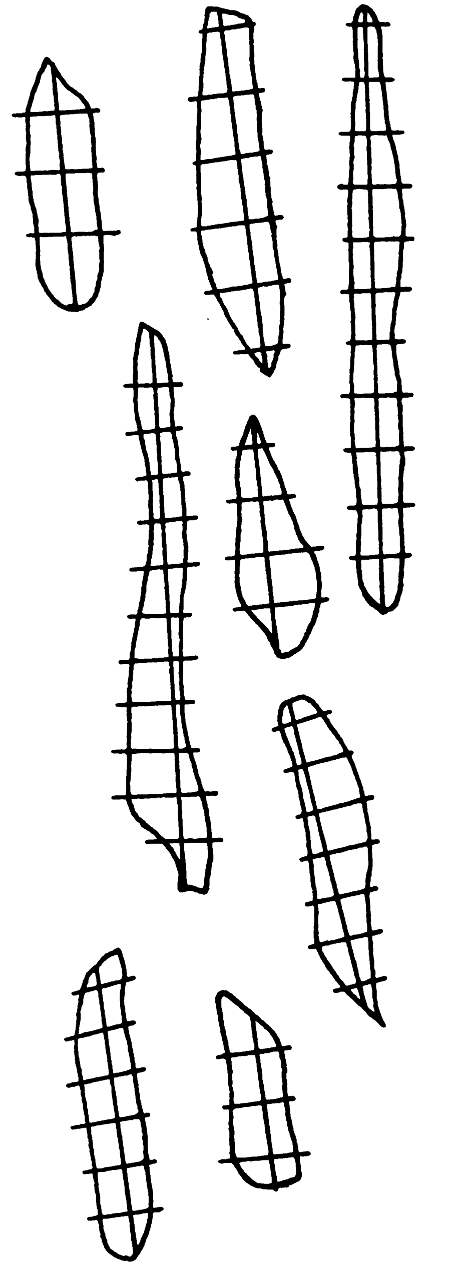

Fig. 8. An Example of Some Segmented Wiley Octahedrite Lamellae.

parent lamella. As will be seen below, the segments do not discriminate as well as entire lamellae, and when relatively short entire lamellae are available, their shapes might be used in preference to segmented lamellae. If only relatively elongate lamellae are available, they can be segmented and compared.

Comparison Between Meteorites Using Groups of Entire Lamellae

Thirty two entire lamellae, 14 Wiley Octahedrite lamellae and 18 Arispe Octahedrite lamellae were compared to determine the spectrum of variation and to see if the method would be successful in discriminating one meteorite from another on the basis of lamellae shape. Inspection of the mean harmonic amplitude spectra (Table 6, Fig. 8), and classification matrix (Table 6) shows that of the 14 Wiley lamellae all were correctly classified, and of 18 Arispe lamellae 15 were correctly classified. This resulted in a chi square value of 28 with ten degrees of freedom which means that the probability this could occur by chance is less than .01. This indicates the higher degree of discrimination in using the entire lamellae as compared to segmented lamellae.

TABLE 6.--Amplitude Spectra and Classification Matrix of Arispe and Wiley Octahedrites--Entire Lamellae.

Mean Harmonic Amplitude Spectra

Harmonic No.	Meteorite Wiley	Meteorite Arispe
1	.02684	.02986
2	.43103	.51347
3	.03586	.03607
4	.16082	.17374
5	.04364	.04102
6	.07603	.06751
7	.03427	.03538
8	.04065	.03867
9	.02268	.02690
10	.02484	.02847

Classification Matrix From Discriminate Analysis

Meteorite	No. of Lamellae	Meteorite Wiley	Meteorite Arispe
Wiley	14	14	0
Arispe	18	3	15

Mean Equality Degs of Free. 10 Chi Square 28.

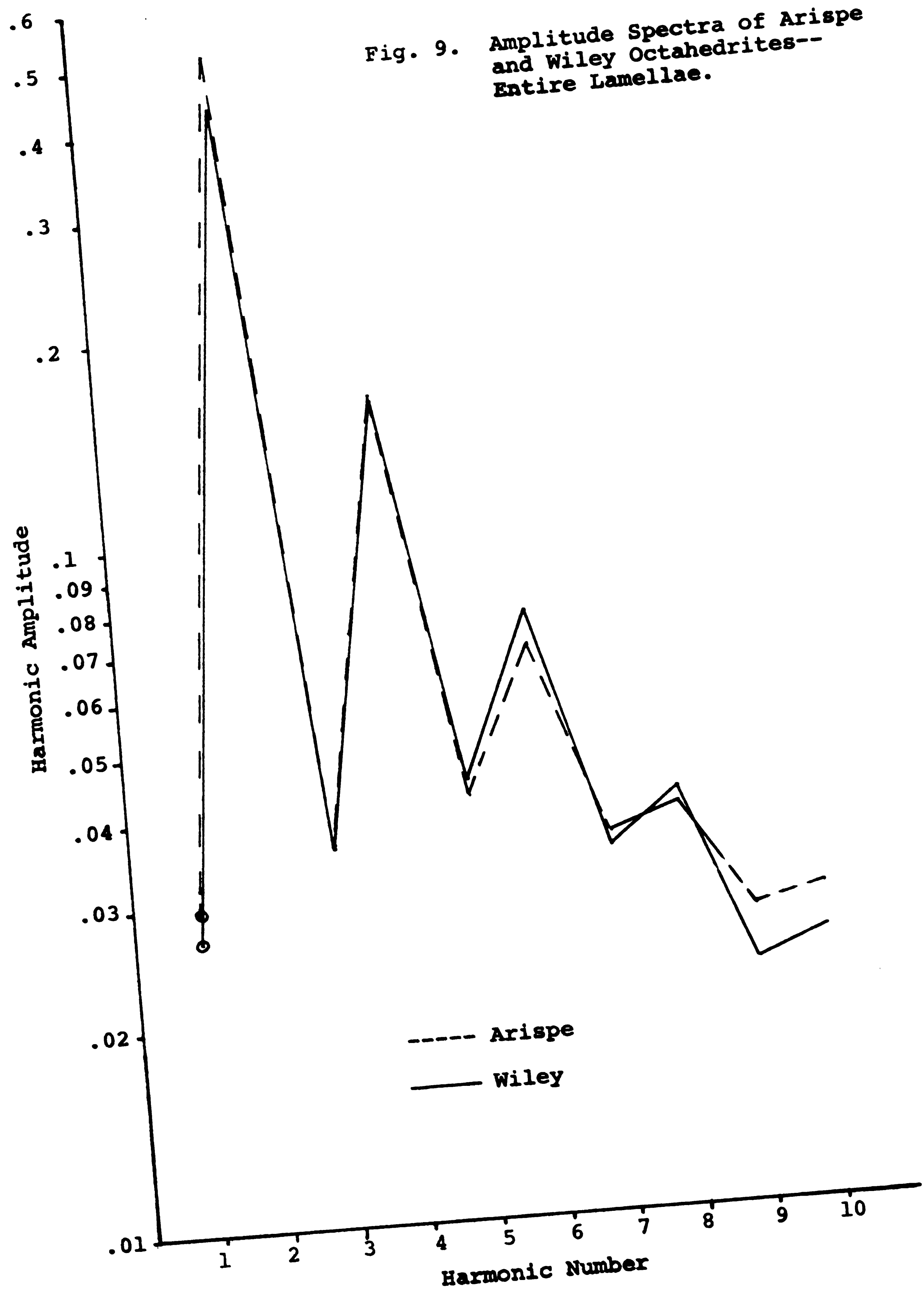
Fig. 9. Amplitude Spectra of Arispe and Wiley Octahedrites--Entire Lamellae.
Harmonic Amplitude
.6
.5
.4
.3
.2
.1
.09
.08
.07
.06
.05
.04
.03
.02
.01
Arispe
Wiley
1
2
3
4
5
6
7
8
9
10
Harmonic Number

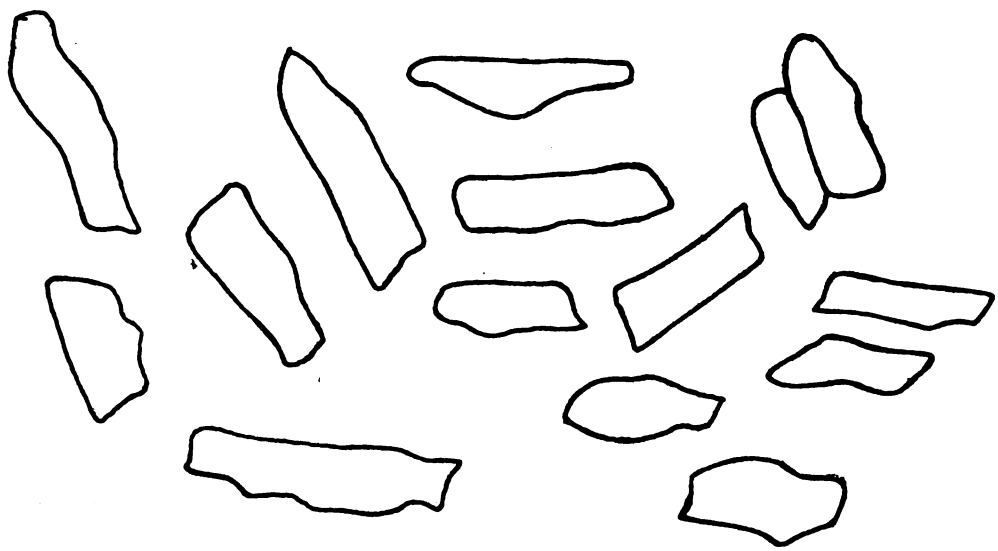

Fig. 10. An Example of Some Arispe Octahedrite Entire Lamellae.

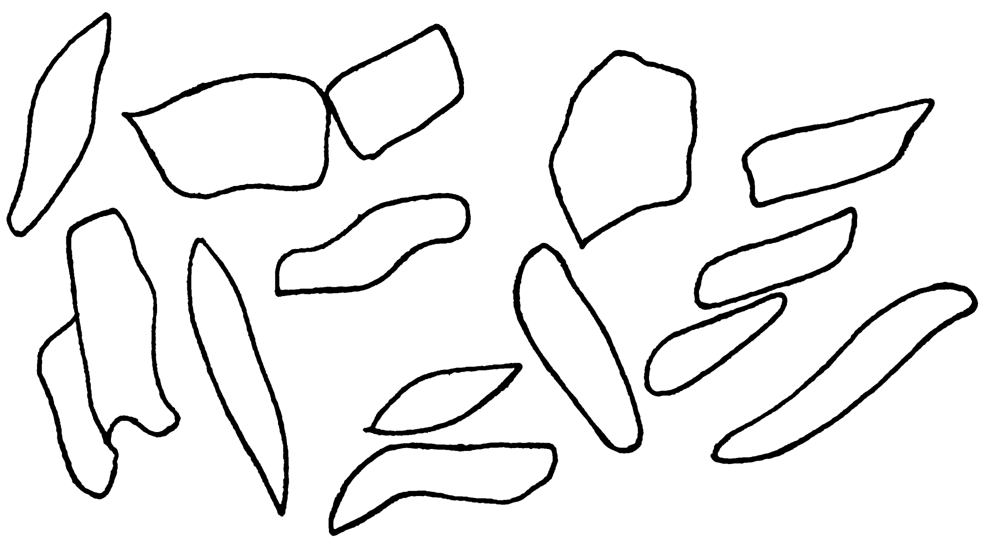

Fig. 11. An Example of Some Wiley Octahedrite Entire Lamellae.

Comparison of Two Meteorites With a Possible Common Genesis

Evans (1961) pointed out the close compositional similarities between the Odessa Meteorites and those found at Meteor Crater in Arizona, i.e., Canyon Diablo Meteorites. In addition to these similarities, the two craters are relatively close to each other. Evans felt that the compositional similarities and the closeness of the craters might not be fortuitous. He implies that both the Odessa Meteroites and the Canyon Diablo group may have had their origin in the same group of meteoroids.

The purpose of this part of the study was to pursue Evans' argument a step further, and to compare the lamellae shape between an Odessa Octahedrite sample and a Canyon Diablo Octahedrite sample. Entire lamellae were used in the Fourier Series shape approximation for this comparison.

Inspection of the mean harmonic amplitude spectra (Table 7, Fig. 12) and the classification matrix (Table 7) indicate distinct differences between the two groups. The discriminate function misclassified only one of 15 Odessa lamellae and none of 14 Canyon Diablo lamellae. This resulted in a chi square value of 26 with ten degrees of freedom which means that the probability this could occur by chance is less than .01.

Thus, the compositional and geographic similarities observed by Evans do not include textural similarities.

Therefore, if the Odessa Octahedrite sample and Canyon Diablo Octahedrite sample that were examined in this study actually were part of a cluster of meteorites with a common orbit, then, the meteorites in that cluster must be heterogeneous texturally. On the other hand, it is possible that perhaps the two meteorites may have had totally independent histories.

Comparison of a Wide Range of Meteorites

In addition to testing a simple hypothesis as was done in the preceeding section, it is of interest to compare texturally a wide range of meteorites which in all probability have little common history. Such a comparison can, at the same time, test the discriminating power of this method and also indicate a means towards refining the textural classification of octahedrites.

Twenty Trenton, 20 Sacramento Mountains, 14 Arispe, 15 Odessa, and 14 Canyon Diablo lamellae were compared.

Inspection of the mean harmonic amplitude spectra (Table 8) and the classification matrix (Table 8) indicate distinct differences between the two groups. The discriminate function correctly classified 10 of 20 Trenton lamellae, 7 of 20 Sacramento Mountains lamellae, 7 of 14 Arispe lamellae, 5 of 15 Odessa lamellae, and 9 of 14 Canyon Diablo lamellae. This resulted in a chi square value of 66 with 40 degrees of freedom which means that the probability this could occur by chance is less than .01.

TABLE 7.--Amplitude Spectra and Classification Matrix of Odessa and Canyon Diablo Octahedrites--Entire Lamellae.

Mean Harmonic Amplitude

Harmonic No.	Meteorite: Odessa	Meteorite: Canyon Diablo
1	.03076	.02705
2	.47804	.53255
3	.03988	.03327
4	.15693	.17614
5	.03684	.03945
6	.07409	.05612
7	.02331	.03000
8	.03819	.03830
9	.01959	.01662
10	.02083	.03015

Classification Matrix From Discriminate Analysis

Meteorite	No. of Lamellae	Meteorite: Odessa	Meteorite: Canyon Diablo
Odessa	15	14	1
Canyon Diablo	14	0	14

Mean Equality Degs of Free. 10 Chi Square 44.

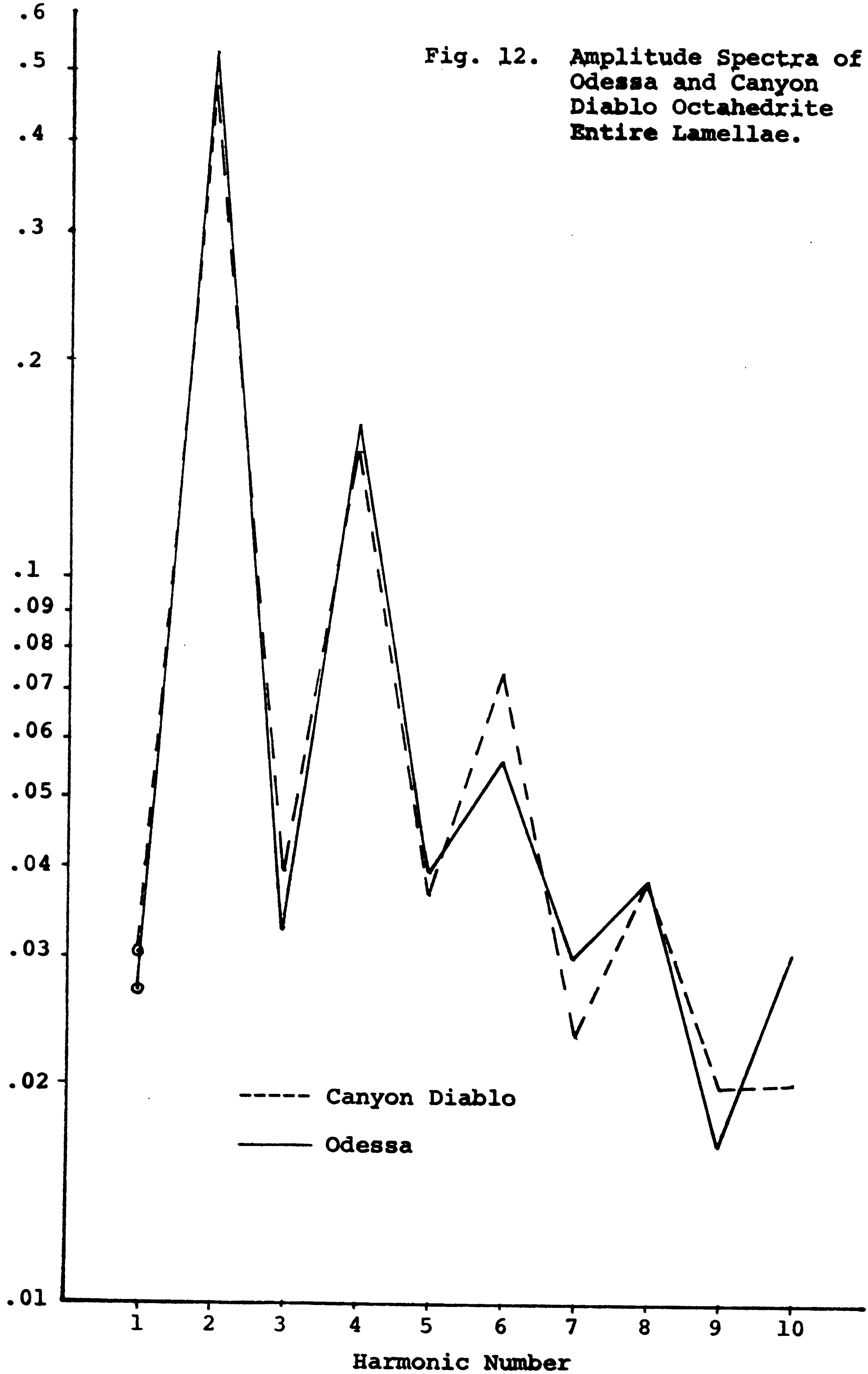

Fig. 12. **Amplitude Spectra of Odessa and Canyon Diablo Octahedrite Entire Lamellae.**

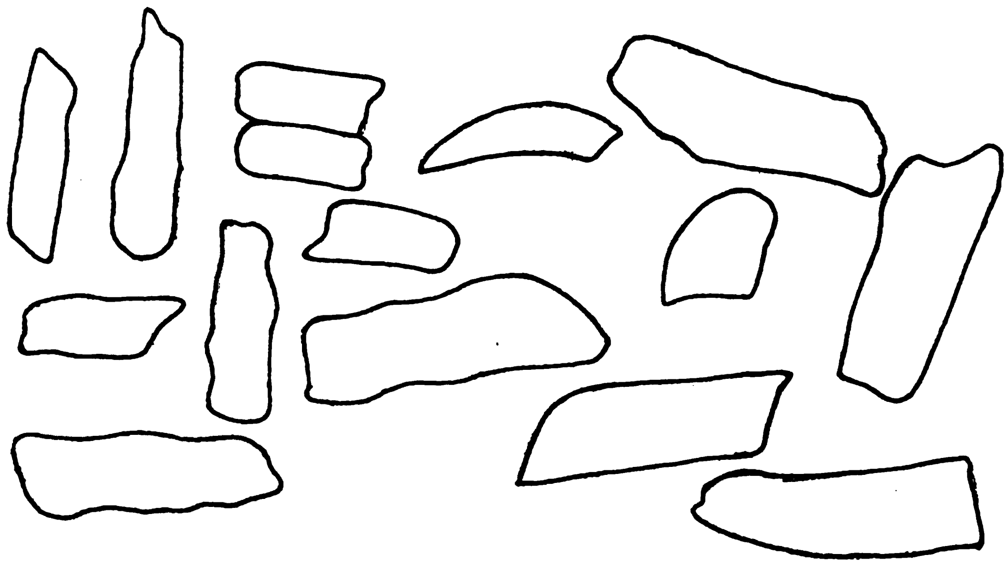

Fig. 13. An Example of Some Canyon Diablo Octahedrite Entire Lamellae.

Fig. 14. An Example of Some Odessa Octahedrite Entire Lamellae.

TABLE 8.--Amplitude Spectra and Classification Matrix of Trenton, Sacramento Mountains, Arispe, Odessa, and Canyon Diablo Octahedrites--Entire Lamellae.

Mean Harmonic Amplitude Spectra

Harmonic No.	Trenton	Sacramento Mountains	Arispe	Odessa	Canyon Diablo
1	.02413	.06559	.03953	.03076	.02705
2	.39934	.44079	.52841	.47804	.53255
3	.04366	.05398	.04516	.03988	.03327
4	.11360	.14878	.19332	.15693	.17614
5	.03823	.04772	.04781	.03684	.03945
6	.05456	.06692	.08383	.07409	.05612
7	.02842	.03467	.03019	.02331	.03000
8	.02895	.03753	.03845	.03819	.03830
9	.01823	.02144	.02353	.01950	.01662
10	.01629	.02558	.02353	.02083	.03015

Classification Matrix From Discriminate Analysis

Meteorite	No. of Lamellae	Trenton	Sac. Mts.	Arispe	Odessa	Canyon Diablo
Trenton	20	10	4	1	4	1
Sac. Mts.	20	4	7	5	1	3
Arispe	14	1	1	7	3	2
Odessa	15	2	2	2	5	4
Canyon Diablo	14	2	1	2	0	9

Mean Equality Degs of Free. 40 Chi Square 66.

The classification matrix can be inspected for the occurence of regularity and misclassification of lamellae, although, data in this case are too scanty for any definitive conclusions, they will serve as an example of determining textural families within the octahedrite group.

The only two meteorites that carry large numbers of lamellae mutually misclassified within each other are Trenton and Sacramento Mountains. Inspection of the invalid assignment of lamellae through the classification matrix shows that the Odessa stands out as a center for the greatest number of misclassifications. The reasons for the similarities and differences should be investigated further. More detailed work along these lines might result in a more informative classification of octahedrites.

CHAPTER V

SUMMARY AND CONCLUSIONS

The Fourier Series shape approximation method is successful in that shapes of kamacite lamellae can be used to discriminate between meteorites. Four comparisons were made: lamellae from non-parallel faces of the same octahedrite sample were compared to evaluate the effect of orientation on lamella shape; segments of lamellae and entire lamellae, of different meteorites, were compared to test for discrimination; lamellae of two meteorites thought to be similar were compared; and lamellae between a wide range of meteorites were compared.

It was shown in this study that: 1. Shape famalies of kamacite are independent of the plane of observation and, therefore, random sections through meteorites could be used. This shows that differences between meteorites are due to fundamental differences rather than differences due to relative orientation of the cut surface with respect to the Widmanstatten pattern.

2. Where entire lamellae are not available, standard segments of lamellae can be used, though, with a slight loss of discriminating power.

3. As an example in the way the method can be used to test a specific hypothesis, an Odessa Octahedrite sample was compared with a Canyon Diablo Octahedrite sample and found to be texturally distinct.

4. Five meteorites were compared simultaneously and the results showed a pattern of similarities and differences which indicate the strength of the method and suggest a basis for a more refined class for octahedrites.

Since significant differences in kamacite lamellae shape were noted from meteorite to meteorite, it must be inferred that these differences must be due to differences in one or more factors which control their thermal history. Therefore, further development of this method coupled, perhaps, with data obtained experimentally can lend to a more complete and understanding of the thermal history of octahedrites.

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