A STUDY OF THE HANDLING BEHAVIOR OF THE FRONT WHEEL DRIVE AUTOMOBILE

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Fredric C. Aldrich

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This is to certify that the

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ABSTRACT

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DRIVE AUTOMOBILE

by Fredric C. Aldrich

Anyone who has ever driven a front wheel drive automobile probably noticed that if, during a turn, power is applied, an increase in steering wheel angle is required to maintain the desired path. It is the purpose of this study to determine what factors cause this property to exist. The problem is examined through the use of a linear mathematical model and a 1/10 scale working mechanical model.

The mathematical model is developed to represent the yawing and sideslip motions of a rigid, idealized automobile. The equations of motion are linearized and simplified in accordance with generally accepted pneumatic tire and automobile handling theory. Only steady-state solutions of the equations of motion are considered.

The basic steady-state solution of the equations of motion indicates that driving force should have the effect of decreasing the radius of turn upon application. This contradicts the evidence available.

The equations are then modified to consider a linear interaction between driving force and the cornering power of the pneumatic tire. By contrasting the front and rear wheel drive automobiles, it is shown that this factor is of minor importance.

The mechanical model exhibits a behavior somewhat between that of the actual automobile and the mathematical model. Since the model is suspensionless, this would indicate that the problem is partially one of suspension and partially one of pneumatic tire properties.

It is concluded that the behavior of the front wheel drive automobile cannot be attributed entirely to any basic dynamic characteristic of the automobile or to the interaction of cornering power of the tire and driving force. Instead the behavior appears to be caused by a number of minor factors which do not exist or tend to cancel in the rear wheel drive automobile.

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NOTATION

Symbol	Units	
F_{DF}	lbs.	Front wheel driving force.
F_{DR}	lbs.	Rear wheel driving force.
F_F	lbs.	Lumped cornering force of both front tires.
F_R	lbs.	Lumped cornering force of both rear tires.
α_F	rad.	Average slip angle of both front tires measured from the wheel heading to the wheel center velocity vector.
α_R	rad.	Average slip angle of both rear tires measured from the wheel heading to the wheel center velocity vector.
δ	rad.	Average front wheel steering angle.
β	rad.	Heading angle measured from the vehicle centerline to the velocity vector of the vehicle center of gravity.
ϕ_F	rad.	See Figure 1.
ϕ_R	rad.	See Figure 1.
$C_F < 0$	$\frac{\text{lbs.}}{\text{rad.}}$	Lumped front tire cornering coefficient independent of driving force.
$C_F' < 0$	$\frac{\text{lbs.}}{\text{rad.}}$	Lumped, driving force dependent, front tire cornering coefficient.
$C_R < 0$	$\frac{\text{lbs.}}{\text{rad.}}$	Lumped rear tire cornering coefficient independent of driving force.
$C_R' < 0$	$\frac{\text{lbs.}}{\text{rad.}}$	Lumped, driving force dependent, rear tire cornering coefficient.
C_D	1/lbs.	Coupling coefficient between driving force and cornering coefficient.
V	ft/sec.	Velocity of the center of gravity of the vehicle.
V_F	ft/sec.	Velocity of the center of the front axle.
V_R	ft/sec.	Velocity of the center of the rear axle.
r	$\frac{\text{rad.}}{\text{sec.}}$	Angular velocity of the vehicle.

Symbol	Units	
R'	ft.	Distance between the center of gravity of the vehicle and the center of curvature of its path.
R	ft.	Distance between the center of gravity of the vehicle and its instant center of velocity.
$\dot{r}$	$\frac{\text{rad.}}{\text{sec.}^2}$	Time derivative of r.
$\dot{\beta}$	$\frac{\text{rad.}}{\text{sec.}}$	Time derivative of β.
a	ft.	Perpendicular distance between the front axle centerline and the center of gravity of the vehicle.
b	ft.	Perpendicular distance between the rear axle centerline and the center of gravity of the vehicle.
L	ft.	Wheelbase of the vehicle.
M	slug	Mass of the vehicle.
I_Z	slug ft.^2	Polar moment of inertia about a vertical axis through the center of gravity of the vehicle.

I. INTRODUCTION

Anyone who has ever driven a front wheel drive automobile has probably observed that this type of automobile exhibits handling behavior which is in one respect different from that of its rear wheel drive counterpart. This difference in handling behavior appears in the effect of the application of driving force through the driving wheels. In the case of the rear wheel drive automobile, changes in driving force have little effect upon its handling behavior provided a skid is not impending. However, in the case of the front wheel drive automobile, changes in driving force have a very noticeable effect upon its handling behavior. In general it may be said that the response to steering inputs of a front wheel drive automobile is reduced by an increase in driving force applied through the front tires.

The most easily noticed result of this handling property occurs while negotiating a turn. If the driving force through the front tires is changed while negotiating a turn, it must be accompanied by a change in steering wheel angle if the desired path is to be maintained. In general an increase in driving force requires an increase in steering wheel angle.

The purpose of this study is to try to determine the factors which cause this handling property of the front wheel drive automobile. Numerical examples will be limited to automobiles approximately the size and design of the current American compact.

The problem is approached in two ways: through the use of a mathematical model similar to that developed by Millikan and Whitcomb (7) and by means of a 1/10 scale mechanical model.

This presentation will discuss the development of the mathematical model, the design, construction, and operation of the mechanical model, and the results obtained by their application.

The result of this study is that no single factor of those considered accounts for the characteristic behavior. The characteristic behavior appears to be caused by a number of minor factors which either tend to cancel or do not exist in the rear wheel drive automobile. It is concluded that the problem is not one of the basic dynamics of the vehicle but one of details of tire and suspension behavior and possibly other factors yet unknown.

Included in the appendix is a section on the terminology of automobile handling and pneumatic tire behavior. It is suggested that any reader not familiar with this field read this section before continuing further.

II. MATHEMATICAL MODEL

A complete mathematical analysis of the dynamics of the automobile would lead to a set of non-linear differential equations of such complexity as to be of little value. In order that the analysis be useful, a number of simplifying and linearizing assumptions will be made.

It is the purpose of this section to develop and analyze a simplified mathematical model of the handling behavior of the automobile. Special emphasis will be placed on the effect of driving forces on handling behavior.

Basic Approach and Assumptions

There are three different manners in which driving forces could affect handling of the front wheel drive automobile.

Driving forces could produce static deflections in the vehicle suspension and steering systems, which have the effect of steering. Some unpublished data obtained by an American automobile manufacturer indicates that application of driving force through the front tires while turning produces a reduction in the steering angle, apparently caused by an increase in self-aligning torque. The same data also indicates, however, that the behavior of the front wheel drive automobile cannot be entirely explained in this manner. The treatment given here will not consider this deflection, since it is a known factor.

Another factor to be considered is the effect of driving force on the characteristics of the pneumatic tire. As was pointed out above, application of driving force causes an increase in self-aligning torque. It might also be expected that driving force would cause a noticeable reduction in the cornering power of the tire. Data published by Bull (2) and Turner, Hartley, and Joy (5 & 6) indicates, however, that cornering power is little affected by driving force less than 50% of the vertical tire load. A paper by Bergman (4) claims that the behavior of the front wheel drive automobile may be explained on the basis of interaction of driving force and cornering power. However, the data presented by Bergman is essentially the same as that presented by the other investigators mentioned above. For the initial development, it will be assumed that there is no interaction between driving force and cornering power. It will be shown later that this assumption is essentially correct.

The third possible effect would be on the rigid body dynamics of the vehicle. This is possible since, while turning, front and rear wheel driving forces do not have the same line of action. This factor will be considered first.

Considerable work has already been done by the personnel of Cornell Aeronautical Laboratory Inc. on the development and substantiation of a mathematical model of the handling behavior of the automobile. The model to be developed here is based primarily on the work of Millikan and Whitcomb of Cornell. (7)

The six basic degrees of freedom of the automobile are generally divided into three categories: ride, performance, and handling. The handling motions are generally considered to be sideslip, roll, and yaw. In this development only yawing and sideslip motions will be considered. Even though the rolling motion plays an important part in handling behavior its effect is nearly independent of whether the automobile in question has front or rear wheel drive. Since rolling motion will not be considered, it will be convenient to assume that the idealized vehicle to be studies has no width.

Equations of Motion

Using Newton's Second Law and a free body diagram of the idealized automobile (Figure 1) the yawing and sideslip equations of motion may be written as follows.

Sideslip Equation (Forces normal to the velocity V)

1. $$F_R \cos \beta + F_F \cos (\delta - \beta) + F_{DF} \sin (\delta - \beta) - F_{DR} \sin \beta = MV (r + \dot{\beta})$$

Yawing Equation (Moments about the C. G.)

2. $$F_F\, a \cos \delta + F_{DF}\, a \sin \delta - F_R b = I_Z \dot{r}$$

In these equation F_R is the rear tire cornering force, F_F is the front tire cornering force, F_{DF} is the front wheel driving force, F_{DR} is the rear wheel driving force, δ is the steering angle, β is the vehicle sideslip angle, and r is the angular velocity of the vehicle. It should be noted that the equation referred to as the sideslip equation is not the usual sideslip equation since the usual sideslip equation would be obtained by summing forces and accelerations in a direction perpendicular to the longitudinal axis of the vehicle. However, it is more convenient to sum forces and accelerations in a direction normal to the velocity since the resulting equation does not contain components of the air resistance force and the tangential acceleration.

It is a well known property of the pneumatic tire that cornering force is a function of slip angle. To maintain linearity of the mathematical model a linear function will be assumed.

3. $$F_F = C_F \alpha_F \qquad F_R = C_R \alpha_F$$

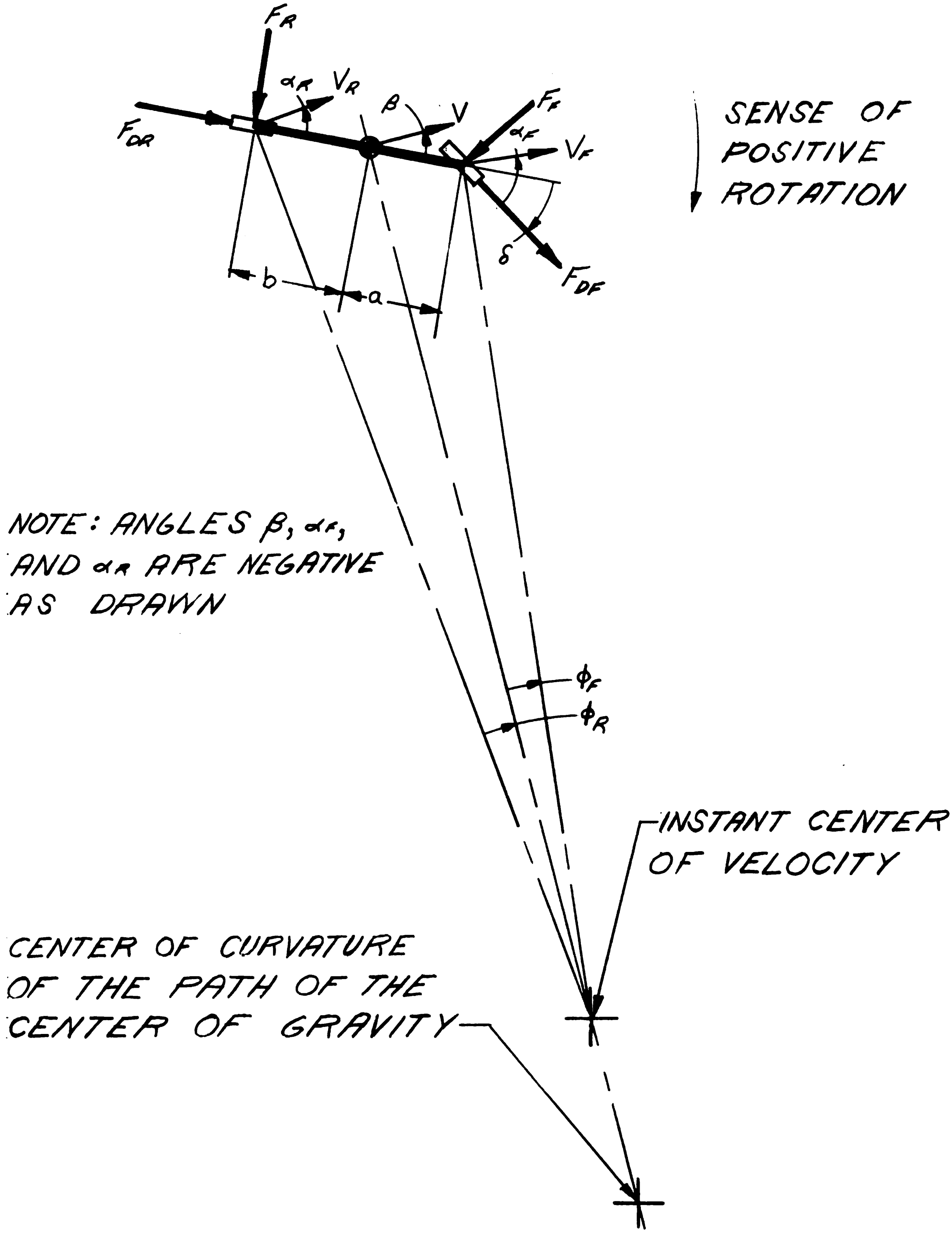

FIGURE 1. FREE BODY DIAGRAM OF THE IDEALIZED AUTOMOBILE

C_F and C_R are called the front and rear tire cornering coefficient and α_F and α_R the front and rear slip angles. Since the slip angle α is defined as the angle measured from the plane of the tire to the velocity vector of the center of the tire, the cornering coefficient C is a negative quantity. The linear function assumed is a good approximation for most tires at slip angles less than 0.1 radian (about 6°). When these relationships are substituted into the equations of motion,the following equations are obtained.

4. $$C_R \alpha_R \cos\beta + C_F \alpha_F \cos(\delta - \beta) + F_{DF} \sin(\delta - \beta) - F_{DR} \sin\beta = MV(r + \dot{\beta})$$

5. $$C_F \alpha_F a \cos\delta + F_{DF} a \sin\delta - C_R \alpha_R b = I_Z \dot{r}$$

Since these equations are non-linear their solution is beyond the scope of this study. If a solution is to be obtained the equations must be linearized by making the assumption of small angles.

Assume

$$\cos\delta = 1 \qquad \sin\delta = \delta$$

$$\cos\beta = 1 \qquad \sin\beta = \beta$$

$$\cos(\delta - \beta) = 1 \qquad \sin(\delta - \beta) = \delta - \beta$$

Substitution of these relationships into the equations of motion yields

6. $$C_R \alpha_R + C_F \alpha_F + F_{DF}(\delta - \beta) - F_{DR} \beta = MV(r + \dot{\beta})$$

7. $$C_F \alpha_F a - C_R \alpha_R b + F_{DF} a \delta = I_Z \dot{r}$$

Now from geometric considerations the following relationships may be written, referring to Figure 1, where $\delta > 0$, $\phi_F > 0$, and $\phi_R > 0$, but $\alpha_F < 0$, $\alpha_R < 0$, and $\beta < 0$ as drawn. V_F, V, and V_R are perpendicular to the three radii drawn from the instant center.

$$\alpha_F = \beta + \phi_F - \delta$$

$$\alpha_R = \beta - \phi_R$$

The angles ϕ_F and ϕ_R may be expressed as follows.

$$\tan \phi_F = \frac{a \cos \beta}{R + a \sin \beta}$$

$$\tan \phi_R = \frac{b \cos \beta}{R - b \sin \beta}$$

Use of the small angle assumption reduces these to the following.

$$\phi_F = \frac{a}{R - a\,\beta} \qquad \phi_R = \frac{b}{R + b\,\beta}$$

For all practical cases the length R will be large compared to a and b if the steering angle δ is restricted to be small. Therefore

$$\phi_F = \frac{a}{R} \qquad \phi_R = \frac{b}{R}$$

It is convenient to make the following substitution.

$$1/R = \frac{r}{V}$$

Now

$$\phi_F = \frac{ar}{V} \qquad \phi_R = \frac{br}{V}$$

$$\alpha_F = \beta + \frac{ar}{V} - \delta \quad \alpha_R = \beta - \frac{br}{V}$$

When these expressions are substituted into the equations of motion, the results are

8. $$C_R\left(\beta - \frac{br}{V}\right) + C_F\left(\beta + \frac{ar}{V} - \delta\right) + F_{DF}(\delta - \beta) - F_{DR}\,\beta = MV(r + \dot{\beta})$$

9. $$C_F a\left(\beta + \frac{ar}{V} - \delta\right) - C_R b\left(\beta - \frac{br}{V}\right) + F_{DF}\, a\, \delta = I_Z \dot{r}$$

or

10. $$\dot{\beta} - \left(\frac{C_R + C_F - F_{DF} - F_{DR}}{MV}\right)\beta - \left(\frac{C_F a - C_R b - MV^2}{MV^2}\right) r = \left(\frac{F_{DF} - C_F}{MV}\right)\delta$$

11. $$\dot{r} - \left(\frac{C_F a - C_R b}{I_Z}\right)\beta - \left(\frac{C_F a^2 + C_R b^2}{I_Z V}\right) r = \left(\frac{F_{DF} - C_F}{I_Z}\right) a\, \delta$$

Consider the steady state solution of these equations.

Assume

$$\dot{\beta} = \dot{r} = 0$$

The equations now become

12. $$(C_F + C_R - F_{DF} - F_{DR})\,\beta + (C_F a - C_R b - MV^2)\,\frac{r}{V} = (C_F - F_{DF})\,\delta$$

13. $$(C_F a - C_R b)\,\beta + (C_F a^2 + C_R b^2)\,\frac{r}{V} = (C_F - F_{DF})\, a\, \delta$$

These may now be solved for the steady state values of the angular velocity r and the vehicle sideslip angle β using Cramer's Rule.

14. $$r = \frac{\begin{vmatrix} C_F + C_R - F_{DF} - F_{DR} & 1 \\ C_F a - C_R b & a \end{vmatrix} (C_F - F_{DF})\,\delta}{\begin{vmatrix} C_F + C_R - F_{DF} - F_{DR} & C_F a - C_R b - MV^2 \\ C_F a - C_R b & C_F a^2 + C_R b^2 \end{vmatrix} 1/V}$$

$$= \frac{\delta\, V\, (C_F - F_{DF})\,[C_R\,(a + b) - (F_{DF} + F_{DR})\, a]}{C_F C_R (a^2 + 2ab + b^2) + MV^2\,(C_F a - C_R b) - (F_{DF} + F_{DR})(C_F a^2 + C_R b^2)}$$

Since $a + b = L$

$$r = \frac{\delta V (C_F - F_{DF}) [C_R L - (F_{DF} + F_{DR}) a]}{C_F C_R L^2 + MV^2 (C_F a - C_R b) - (F_{DF} + F_{DR})(C_F a^2 + C_R b^2)}$$

16.
$$\beta = \frac{\begin{vmatrix} 1 & C_F a - C_R b - MV^2 \\ a & C_F a^2 + C_R b^2 \end{vmatrix} (C_F - F_{DF}) \delta}{C_F C_R L^2 + MV^2 (C_F a - C_R b) - (F_{DF} + F_{DR})(C_F a^2 + C_R b^2)}$$

$$= \frac{(C_R b^2 + C_R ab + MV^2 a) (C_F - F_{DF}) \delta}{C_F C_R L^2 + MV^2 (C_F a - C_R b) - (F_{DF} + F_{DR})(C_F a^2 + C_R b^2)}$$

Now substitute the following relationship into the expression of the angular velocity r, and solve for the radius of turn R. Note that R' is the actual radius of turn but $R' = R$ for the steady-state case

$$\frac{r}{V} = 1/R$$

17. $$1/R = \frac{\delta (C_F - F_{DF}) [C_R L - (F_{DF} + F_{DR}) a]}{C_F C_R L^2 + MV^2 (C_F a - C_R b) - (F_{DF} + F_{DR})(C_F a^2 + C_R b^2)}$$

18. $$R = \frac{C_F C_R L^2 + MV^2 (C_F a - C_R b) - (F_{DF} + F_{DR})(C_F a^2 + C_R b^2)}{\delta (C_F - F_{DF}) [C_R L - (F_{DF} + F_{DR}) a]}$$

Divide numerator and denominator by $C_F C_R L \delta$

19. $$R = \frac{L/\delta + \frac{MV^2}{L\delta} \left(\frac{a}{C_R} - \frac{b}{C_F}\right) - \frac{F_{DF} + F_{DR}}{L\delta} \left(\frac{a^2}{C_R} + \frac{b^2}{C_F}\right)}{1 - \frac{F_{DF}}{C_F} - \frac{(C_F - F_{DF})(F_{DF} + F_{DR}) a}{C_F C_R L}}$$

Consider the front and rear wheel drive cases separately.

Front Wheel Drive

20. $$R = \frac{L/\delta + \frac{MV^2}{L\delta}\left(\frac{a}{C_R} - \frac{b}{C_F}\right) - \frac{F_{DF}}{L\delta}\left(\frac{a^2}{C_R} + \frac{b^2}{C_F}\right)}{1 - \frac{F_{DF}a}{C_R L} + \frac{F_{DF}{}^2 a}{C_F C_R L} - \frac{F_{DF}}{C_F}}$$

Rear Wheel Drive

21. $$R = \frac{L/\delta + \frac{MV^2}{L\delta}\left(\frac{a}{C_R} - \frac{b}{C_F}\right) - \frac{F_{DR}}{L\delta}\left(\frac{a^2}{C_R} + \frac{b^2}{C_F}\right)}{1 - \frac{F_{DR}a}{C_R L}}$$

Observe that if the cornering coefficients C_F and C_R are relatively equal, the equations for the steady state radius of turn R differ only in that the front wheel drive equation contains two more terms in the denominator. Since both these terms are positive (since C_F is negative) these equations indicate that for any given set of initial conditions, an increase in driving force causes the radius of turn of the front wheel drive car to decrease more than that of the rear wheel drive car. This, however, is in direct opposition to the observed effect of driving force upon the real automobile. Since the effect observed in the actual automobile is not a transient one, it must be concluded that the mathematical model as developed does not represent the actual automobile with sufficient accuracy.

Examine again the assumptions made in this development. The most controversial assumption appears to be the one concerning the interaction of driving force and cornering coefficient. Consider now a case where interaction does occur. Assume the following linear interaction between driving force and cornering coefficient.

22. $$C_F' = C_F\,(1 - F_{DF}\,C_D)$$

$$C_R' = C_R\,(1 - F_{DR}\,C_D)$$

Replace the driving-force-independent cornering coefficients in the expression for the steady state radius of turn (Equation 18) with these assumed functions.

Front Wheel Drive

$$23. \quad R = \Big\{C_F C_R L^2 (1 - F_{DF} C_D) + MV^2 [C_F a (1 - F_{DF} C_D) - C_R b] - F_{DF} [C_F a^2 (1 - F_{DR} C_D) - C_R b^2]\Big\} \div$$

$$\delta [C_F (1 - F_{DF} C_D) - F_{DF}] [C_R L - F_{DF} a]$$

Rear Wheel Drive

$$24. \quad R = \Big\{C_F C_R L^2 (1 - F_{DR} C_D) + MV^2 [C_F a - C_R b (1 - F_{DF} C_D)] - F_{DR} [C_F a^2 + C_R b^2 (1 - F_{DR} C_D)]\Big\} \div$$

$$\delta C_F [C_R L (1 - F_{DR} C_D) - F_{DR} a]$$

Figures 2 and 3 show functions 23 and 24 plotted against driving force for various values of the interaction coefficient C_D. The following values were arbitrarily chosen for the various constants in the equation.

$M = 111.9$ slugs $\qquad V = 44$ ft/sec.

$C_F = C_R = -12000$ lbs./rad. $\quad L = 9.333$ ft.

$a = 4.133$ ft. $\qquad b = 5.20$ ft. $\qquad \delta = .1055$ rad.

From Figure 2 and 3 it can be concluded that if the coupling between driving force and cornering coefficient were of a magnitude that would appreciably affect the handling behavior of a front wheel drive automobile, then a similar (but opposite) effect would be noticeable in

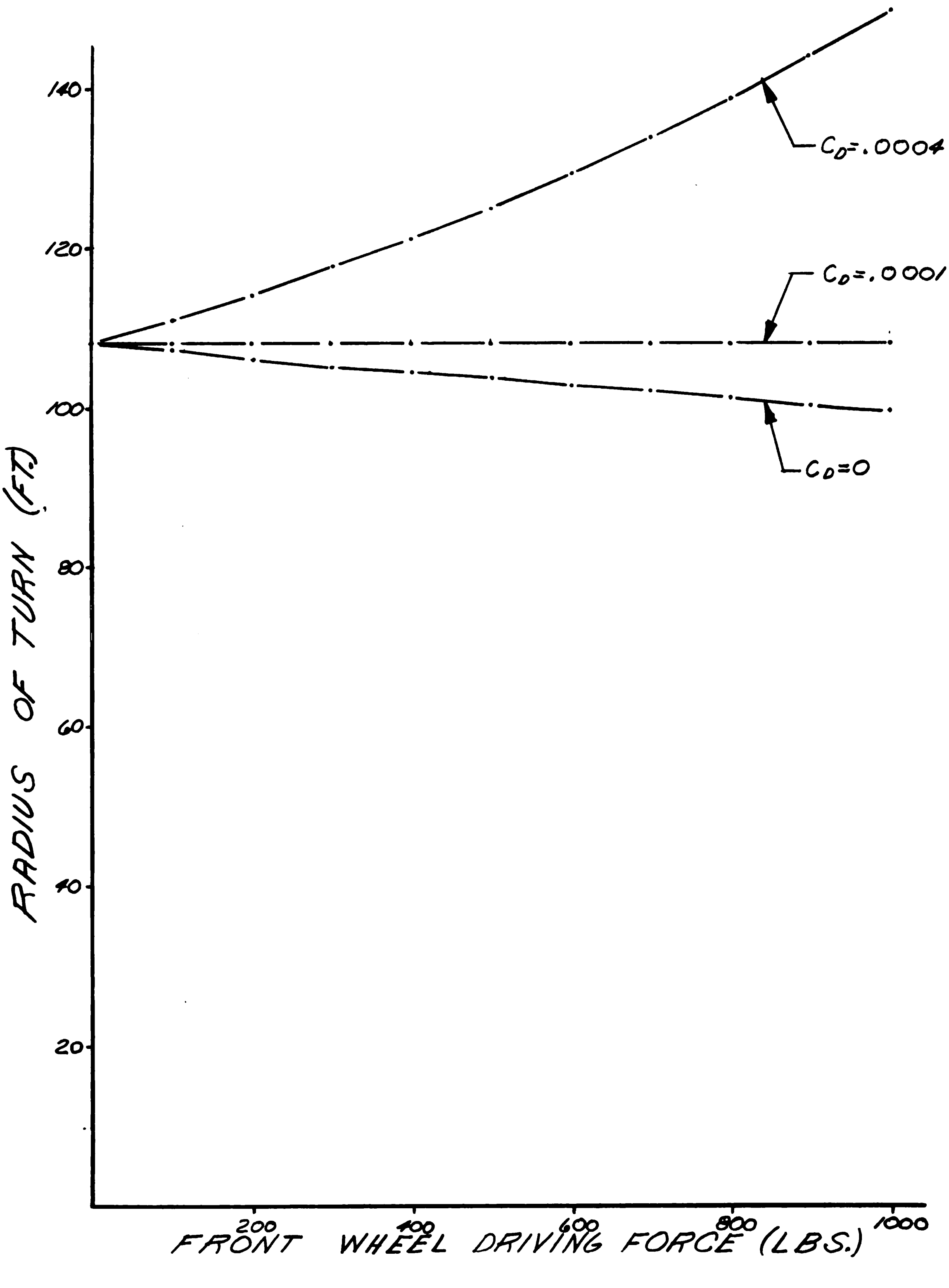

FIGURE 2. PREDICTED EFFECT OF COUPLING COEFFICIENT ON STEADY-STATE RADIUS OF TURN-FRONT WHEEL DRIVE AUTOMOBILE

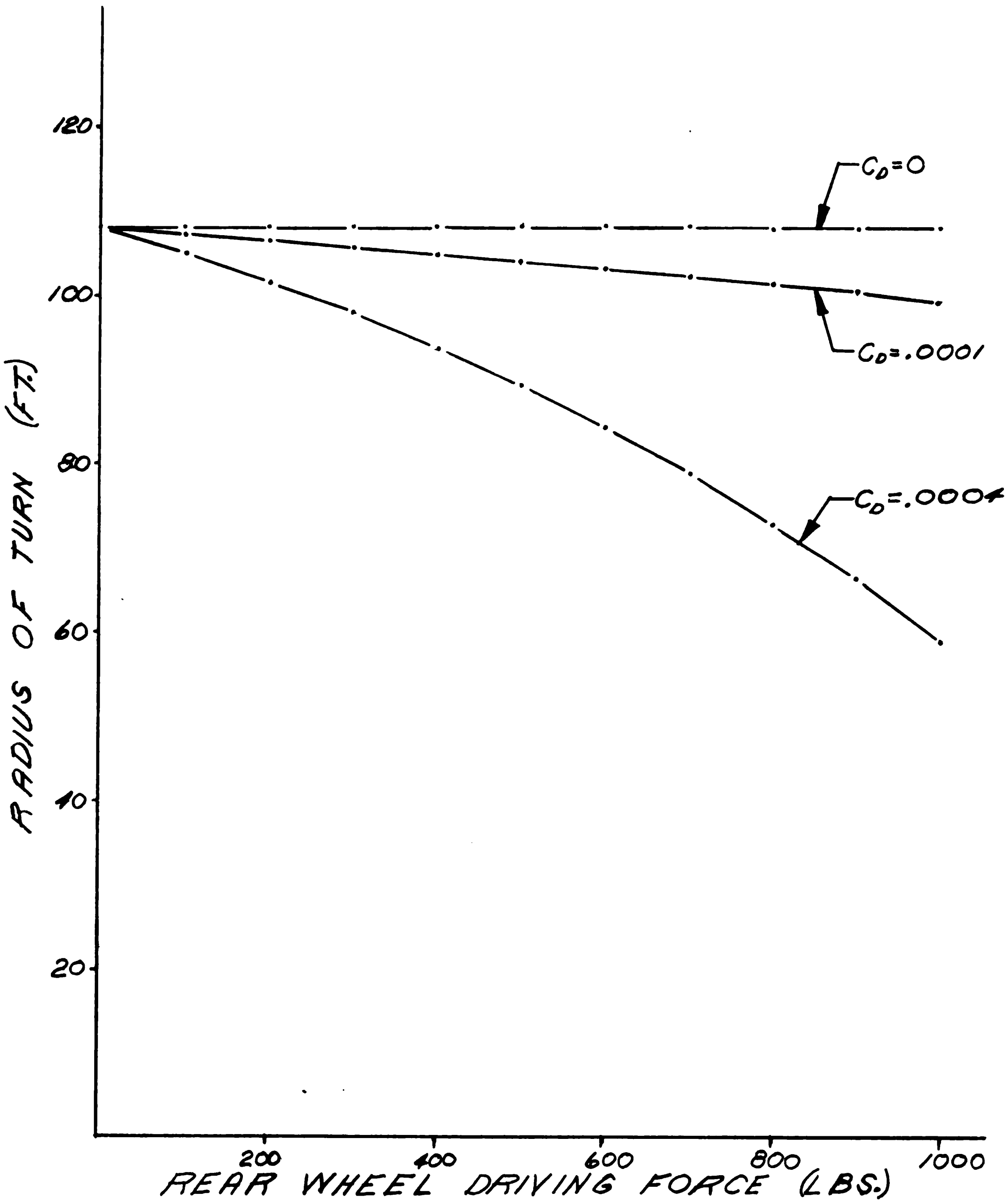

FIGURE 3. PREDICTED EFFECT OF COUPLING COEFFICIENT ON STEADY-STATE RADIUS OF TURN-REAR WHEEL DRIVE AUTOMOBILE

the rear wheel drive automobile. Since such an effect is not noticeable in the rear wheel drive automobile, the original assumption of small interaction between driving force and cornering force must be essentially correct.

Other Considerations

Since it appears that no single factor accounts for the characteristic behavior of the front wheel drive automobile, it must be concluded that this characteristic is caused by a number of small but not negligible effects. In determining these effects the basic criterion used was, "How is the front wheel drive automobile different from the rear wheel drive?". Listed below are four factors which would tend to produce the observed effect.

1. Even though it has been shown that the cornering coefficient is nearly independent of driving force, it is not completely independent. It was mentioned before that application of driving force tends to increase self-aligning torque. In the rear wheel drive automobile an increase in self-aligning torque coupled with a slight decrease in cornering coefficient of the rear tires would have little effect since the two factors tend to cancel. However, if this were to occur in the front tires of a front wheel drive automobile the factors would be additive and would produce a resultant negative yawing moment.

2. It has already been mentioned that increased self-aligning torque due to the application of driving force produces deflection in the steering system which would physically reduce the steering angle.

3. Application of driving force also has a tendency to cause the front tires to toe in. This causes the outside slip angle to be increased while the inside is decreased. Since the plot of cornering force versus slip angle for a pneumatic tire is always at least slightly concave

downward, maximum cornering force is developed when the inside and outside slip angles are equal. Any change from the condition of equal slip angles tends to reduce the front tire cornering force.

4. The front suspension geometry of the modern automobile is such that as the automobile rolls in a turn, the front wheels become cambered toward the outside of the turn thereby producing a camber thrust. Application of driving force may tend to increase this camber thrust. This, however, is pure conjecture since the effect of driving force on camber thrust is not known.

Conclusion

The best that can be concluded from this work is that this approach does not give a positive answer to the problem at hand. It does, however, show that the problem is not one of the basic dynamics of the vehicle but one of the details of tire and suspension behavior and possibly of other factors yet unknown.

III. MECHANICAL MODEL

In any engineering study, it is best to substantiate any theoretical work with some experimental evidence if possible. Since the cost of experimental work on a full size automobile is prohibitive, the experimental work of this study was restricted to the development and study of a 1/10 scale model of a front wheel drive automobile.

Design of the Model

As was the case with the numerical example used before, this model represents an automobile approximately the size and weight of a current American compact. The model is suspensionless, and the front wheels are rigidly mounted with no provision for changing the steering angle. This was done to achieve simplicity and to eliminate the possibility of deflection of the front tires from the design steering angle. Each front wheel is driven separately by a fractional horsepower direct-current electric motor. The motors are connected to the front axle shafts by 4 to 1 nylon gear sets. Separate motors for each front wheel eliminate the necessity of a differential. Power is supplied to the model by an external power supply. An effort was made to keep the center of gravity as low as possible so as to minimize any effects of weight transfer to the outside wheels while turning. Figure 4 is a photograph of the model showing some of the details of construction.

The length scale of the model is 1/10 and the mass scale $(1/10)^3$ or 1/1000. Since the ratio of gravitational force to applied force should be the same in the model as in the full size automobile, the force scale must be the same as the mass scale or 1/1000. It is also desired that the ratio of centripetal acceleration to gravitational

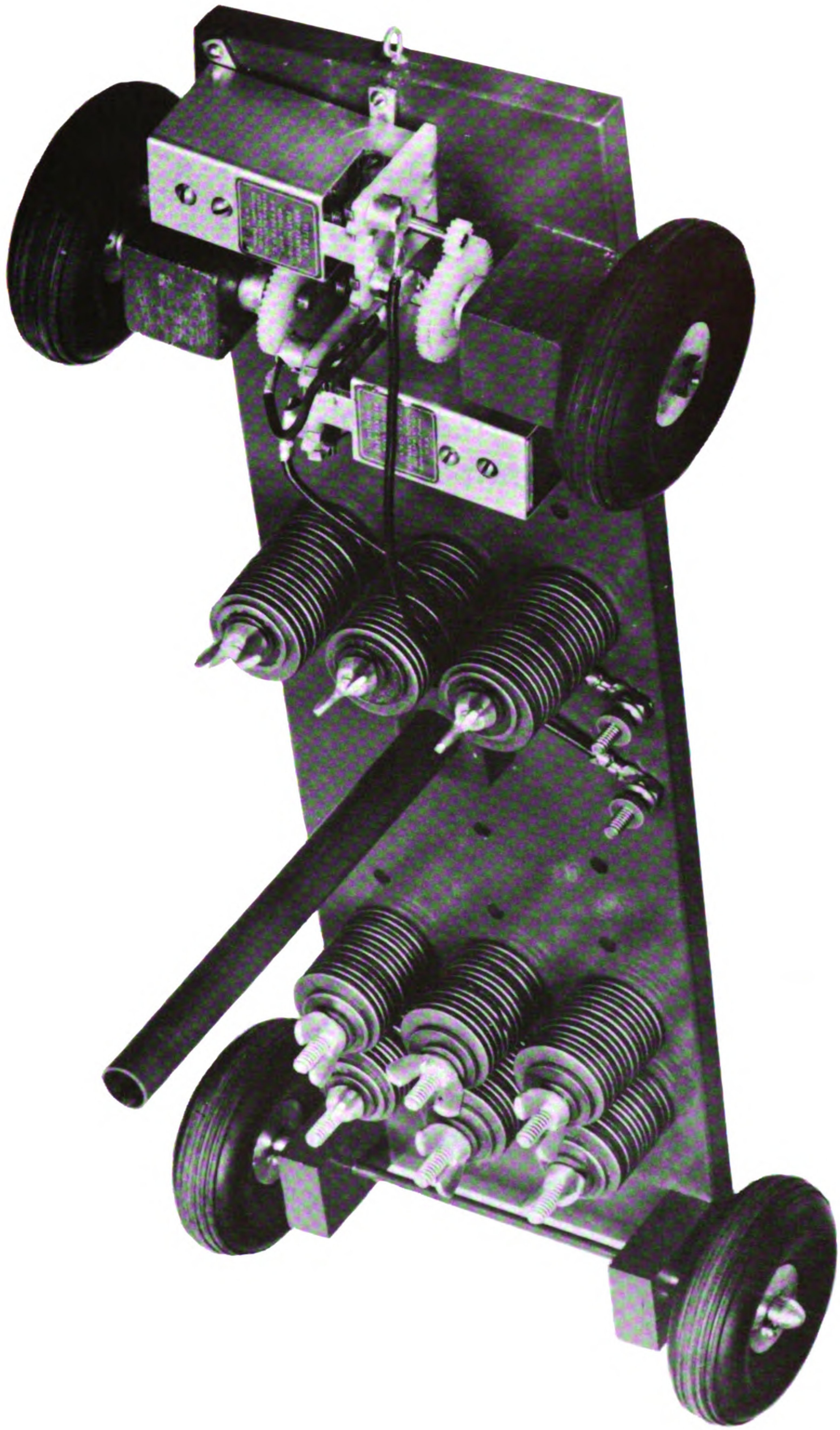

FIGURE 4. MECHANICAL MODEL

acceleration (Froude number $= \frac{V^2}{Rg}$) be the same in the model and full size automobile. To obtain this, the velocity scale must be $1/\sqrt{10}$. The dimensions of the model are

M = .112 slugs (3.6 lbs.)	L = 11.2 in.
a = 5.04 in.	b = 6.16 in.
tread = 5.6 in.	tire diameter = 2.25 in.

In order that the space required to operate the model be reasonable the steering angle is fixed at 0.118 rad. (6.8°). This gives a static radius ($R_{static} = \frac{L}{\sin \delta}$) of 7.90 ft.

A major problem in the design of this model was how it was to be instrumented. Because of cost, electronic apparatus was not considered. Since the emphasis has been on steady-state conditions, the only information necessary is the steady-state radius of turn and the average velocity. The radius of turn can be obtained if the path of the model is known. To plot this path, a piece of chalk is mounted in a vertical tube located at the center of gravity of the model. As the model moves, the chalk marks its path on the running surface. By measuring the time required for one circuit and the diameter of the circle marked by the chalk, the steady-state radius of turn and average velocity may be found.

There is only one magnitude of driving force which gives a steady-state condition for given values of steering angle and velocity, provided the drag force remains constant. Since it is advantageous to conduct all testing at a fixed velocity, some method must be devised for varying the drive force while maintaining steady-state conditions. This is done by increasing the drag force through the application of various size weights to the chalk mentioned previously. This, however, adds mass to the model which was accounted for in the theoretical calculations described later.

Another problem was that of tires. Ideally, the tires should be scale model pneumatic tires. However, such tires are not available. The tires used were semi-pneumatic tires intended for use as model airplane tires. They left a great deal to be desired in that they were of generally poor quality and exhibited a non-linear cornering force to slip angle relationship for slip angles greater than 3°. The cornering coefficient of these tires was estimated to be approximately - 4 lbs./rad.

Operating Procedure

Data was obtained by operating the model at a constant speed of 6.9 ft./sec. and measuring the steady-state radius of turn for five different values of driving force. This data is plotted in Figure 5. Each point on the curve represents the average of two runs.

Calculating the theoretical values of radius of turn for each of the values of driving force presented some problems since no exact values for the cornering coefficients of the tires of the model were known. This was solved by using the data from the model for the smaller values of driving force as a guide point. Using this data, $C_R = -8$ lb./rad., and Equation 20, a value of $C_F = -10.82$ lbs./rad. was calculated. Equation 20 is plotted in Figure 5 using the above stated values of the cornering coefficients and the appropriate values of adjusted mass and driving force.

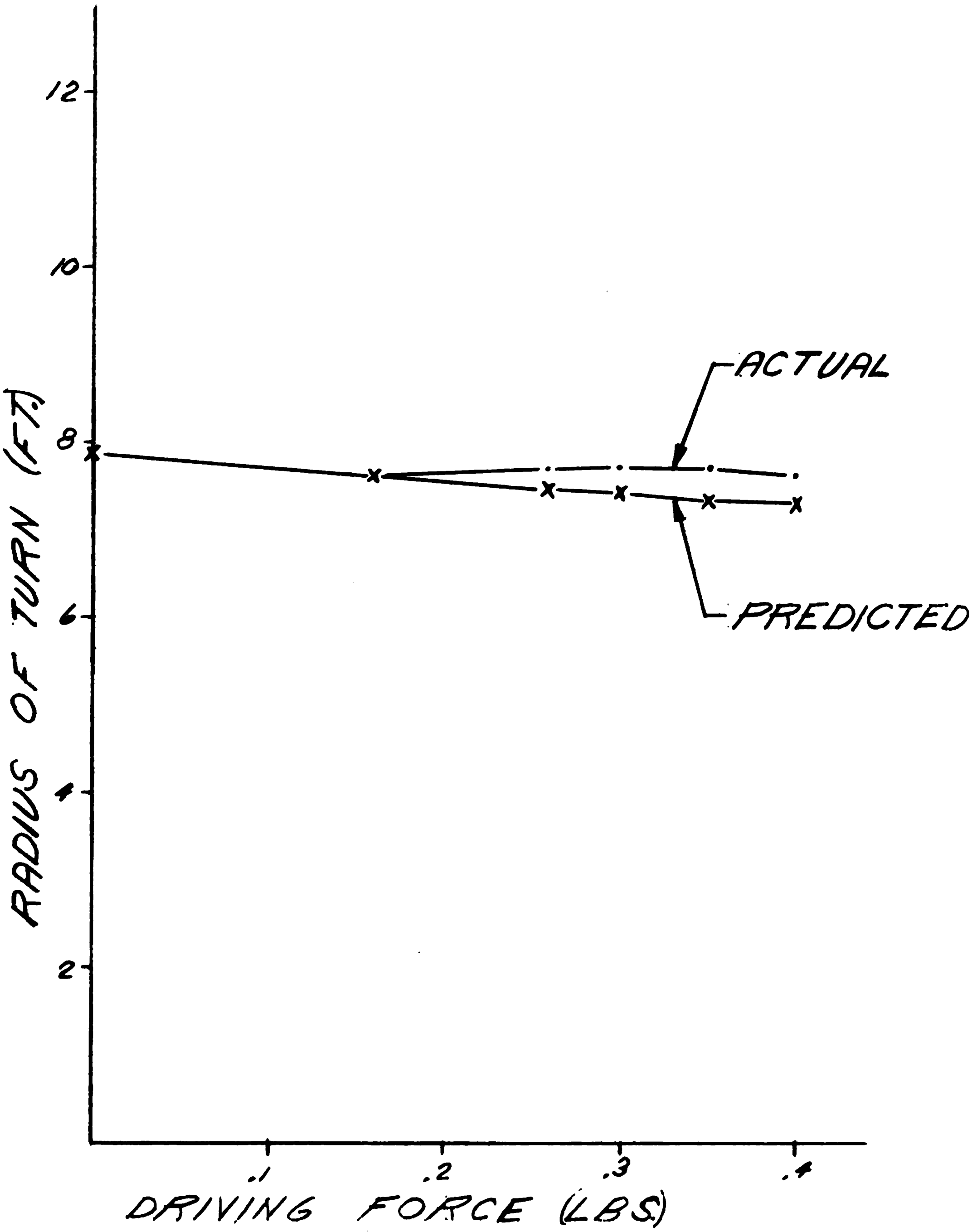

FIGURE 5. PREDICTED AND ACTUAL STEADY-STATE
RADIUS OF TURN OF THE MECHANICAL MODEL

Results and Conclusions

The actual and theoretical curves of Figure 5 do not compare too favorably in that the radius of turn of the model stayed nearly constant with changes in driving force while Equation 20 predicts a decrease in radius of turn with an increase in driving force. This indicates that the model exhibits a behavior somewhere between that of the actual automobile and that of the mathematical model. This would tend to substantiate the conclusion of the previous section that the behavior of the front wheel drive automobile is caused by a combination of pneumatic tire effects and suspension deflection effects.

IV. CONCLUSION

The best that can be concluded from this study is that the behavior of the front wheel drive automobile cannot be attributed entirely to any basic dynamic property of the vehicle or to the interaction of driving force and cornering force of the front tires. The behavior appears to be caused by several minor effects which are either not present or tend to cancel in the rear wheel drive car. A number of these minor effects are listed on page 15 and will not be repeated here.

APPENDIX

TIRE PERFORMANCE AND VEHICLE HANDLING

The pneumatic tire used on the modern automobile must support a number of different types of loading. It must support the weight of the automobile, transmit driving and braking forces, and support side loads while the vehicle is rounding a turn. These side loads are called cornering forces and are generally considered to be perpendicular to the plane of the tire. When a pneumatic tire is exerting a cornering force,the plane of the tire is at a slight angle to the velocity vector of the center of the tire. This angle is called the slip angle of the tire. The pneumatic tire differs from the solid wheel in that the solid wheel develops maximum cornering for a slip angle of about 2° where the pneumatic tire may operate at a slip angle of 12° to 15° before maximum cornering force is developed. In general it may be said that for most tires the slip angle and the cornering force are proportional for slip angles less than 6°.

The cornering force of a pneumatic tire is developed by elastic deflection of the tire tread material in the area of contact of the tire with the road. This area of contact is called the contact patch of the tire. Any given point on the tread of a tire operating at a slip angle enters the contact patch undeflected. Since the plane of rotation of the tire and the velocity of the tire are not aligned, the point in question is deflected from its equilibrium position thereby exerting a cornering force on the automobile. Since the deflection of any given point is larger at the rear of the contact patch than at the front, the resultant cornering force acts to the rear of the vertical axis of the tire and exerts a self-aligning torque on the tire. This torque tends to give the tire an inherent stability.

Automobiles may in general be classified into two types: those which understeer and those which oversteer. If while rounding a turn the rear slip angle is greater than the front, the automobile oversteers. If the opposite is true, the automobile understeers. (If the slip angles are equal, the automobile has neutral steering.) The understeering automobile is stable and its response to steering inputs decreases with increasing velocity. The oversteering vehicle is unstable and its response to steering inputs increases with velocity and becomes infinite at some critical velocity.

Whether a given automobile understeers or oversteers depends basically upon the type of tires used front and rear and the distribution of the weight between front and rear tires. This may also be modified by suspension and aerodynamic properties of the vehicle.

This and additional information on this subject may be found in "Road Manners of the Modern Car," by Olley. (8)

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