A COMPARISON OF REINFORCED CONCRETE AND PRESTRESSED CONCRETE SLAB DESIGNS

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Zeki Alemdargil

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THESIS

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A COMPARISON OF REINFORCED CONCRETE AND PRESTRESSED CONCRETE SLAB DESIGNS

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AND

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By

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A THESIS

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INDEX

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Introduction

Application of a virtually unexplored concept--prestressing--to concrete construction makes possible remarkable advances in structural engineering.

Eugene Freyssinet's five flat arch bridges over the Marne near Meaux, for instance which have a depth at the crown of only 37" for a span of 243.

Roebling's 14,000 square foot jointless slab in a Chicago warehouse, which is only 3" thick yet does not show even a hair crack, though it has been subjected to heavy loads and large temperature variations for two and one-half years.

Gustave Magnel's hangars at Brussels airport with 165-foot beams, and Preload Corporation's dome at Kansas City spanning 180 feet--though only 5" thick.

The secret lies in applying forces to the concrete to overcome the tensile stresses caused by loads, volume changes and impact. This creates concrete capable of taking tension without cracking, and makes it an efficient building material with many applications apart from the spectacular ones cited.

The great shortcoming of concrete is that its strength in tension is much smaller than in compression. To overcome this weakness, the usual procedure has been to reinforce the concrete and put in steel rods to take tension. The technique has proved very successful with respect to safety, but it has not prevented concrete from cracking.

In recent years, however, great strides have been made in improving the quality and strength of both concrete and steel. Reliable concrete with a compressive strength of 9000 to 12000 psi and suitable steel with a tensile strength of over 200,000 psi can be attained readily.

However, these materials cannot be used to full advantage in reinforced concrete beams or slabs. In the first place, the added expense of obtaining high-strength concrete is not warranted; secondly, if high-strength steel is used, the strain will be several times that of mild steel and wide cracks will br formed in the beam under working loads.

But full use can be made of the best concrete and highest strength steel obtainable if the concrete is prestressed.

Since the steel is stretched independently to compress the concrete, steel with high tensile strength is an asset. The reasons for this are that large prestress can be applied with a small amount of steel, and the stress losses in the steel, due to plastic flow and shrinkage of the concrete become proportionally smaller as the applied tension is increased. High strength concrete is advantageous because the amount of prestress that can be applied increases with the compressive strength of the concrete. And obviously the greater the prestress the greater the B. M. that a prestressed concrete slab can carry.

Prestressing converts into an elastic material--one for which the stress and deformations can be accurately computed. Since the entire concrete section carries bending moment, a prestressed beam can carry much heavier loads without cracking than a reinforced concrete beam of the same section. Or it can span much greater distances than has so far been possible with conventionally-designed beams. In fact, prestressing places concrete in full economic competition with structural steel.

One of the most impressive characteristics of prestressing is that it enables long-span beams to be constructed of short, precast units. Much like a group of books can be lifted from a shelf by applying longidudinal pressure to the end pair, such a beam is given a monolithe character by the prestress.

Another big advantage is that if heavy loads induce large deflections and cause slight cracks, a prestress beam will recover its original shape and the cracks will be effectively sealed when the loads are removed. Tests were made by the John P. Roeblings Sons Co. in which a 10' long, 1-5/8" thick prestressed concrete slab was deflected as much as 3" or 1/40th span. Despite this large deflection, the slab returned to its original shape on removal of the load, no cracks being apparent.

The Purpose of This Thesis

Prestressed concrete holds the promise of providing a new structural material by making more efficient use of the physical properties of steel and concrete in combination. The purpose of this thesis is to show the design of prestressed concrete and reinforced concrete and thus making a comparison of the two.

Notations

A = cross sectional area of a slab

A_s = cross sectional area of reinforcing steel

b = width of the slab

b = as a subscript denotes the bottom fiber of a slab

c = as a subscript denotes concrete

d = depth of slab (effective)

d = a subscript denotes "dead load"

e = eccentricity of the steel from the neutral axis

E_c = Modulus of elasticity for concrete

E_s = Modulus of elasticity for steel

F_i = Initial prestress force supplied by the steel.

f = denotes a fiber unit stress

f_c = Allowable fiber unit stress for concrete in compression

f_{ct} or f_{tc} = Allowable fiber unit stress for concrete in tension

f_{du} = upper fiber stress due to the dead load

f_{ul} = upper fiber stress due to the live load

f_{dl} - f_{lb} = lower or bottom fiber stress due to the live load

I = moment of inertia

M_t = Moment or total bending moment

M_l = Moment due to the live load on beam

M_d = Moment due to dead load on the beam

η = proportion of F_i that remains permanently, usually has a value of .85

n = ratio of E_s/E_c

p = percentage of steel in the entire cross section

r = Radius of gyration of the concrete section

s = A subscript denoting steel

u = A subscript denoting top fiber

V = Total shear

v = Unit shear stress

Design Theory

Fundamental conditions:

The most important difference between a structural unit of reinforced concrete and one of prestressed concrete is that in the latter, all the concrete in any cross section must be kept in compression.

In general, a prestressed concrete beam subject to bending only must resist two moments. The first is produced while the prestressed is being established and is the result of the moment due to the dead load plus the moment caused by the prestress force. These two moments are in opposition to each other and result in smaller total stress.

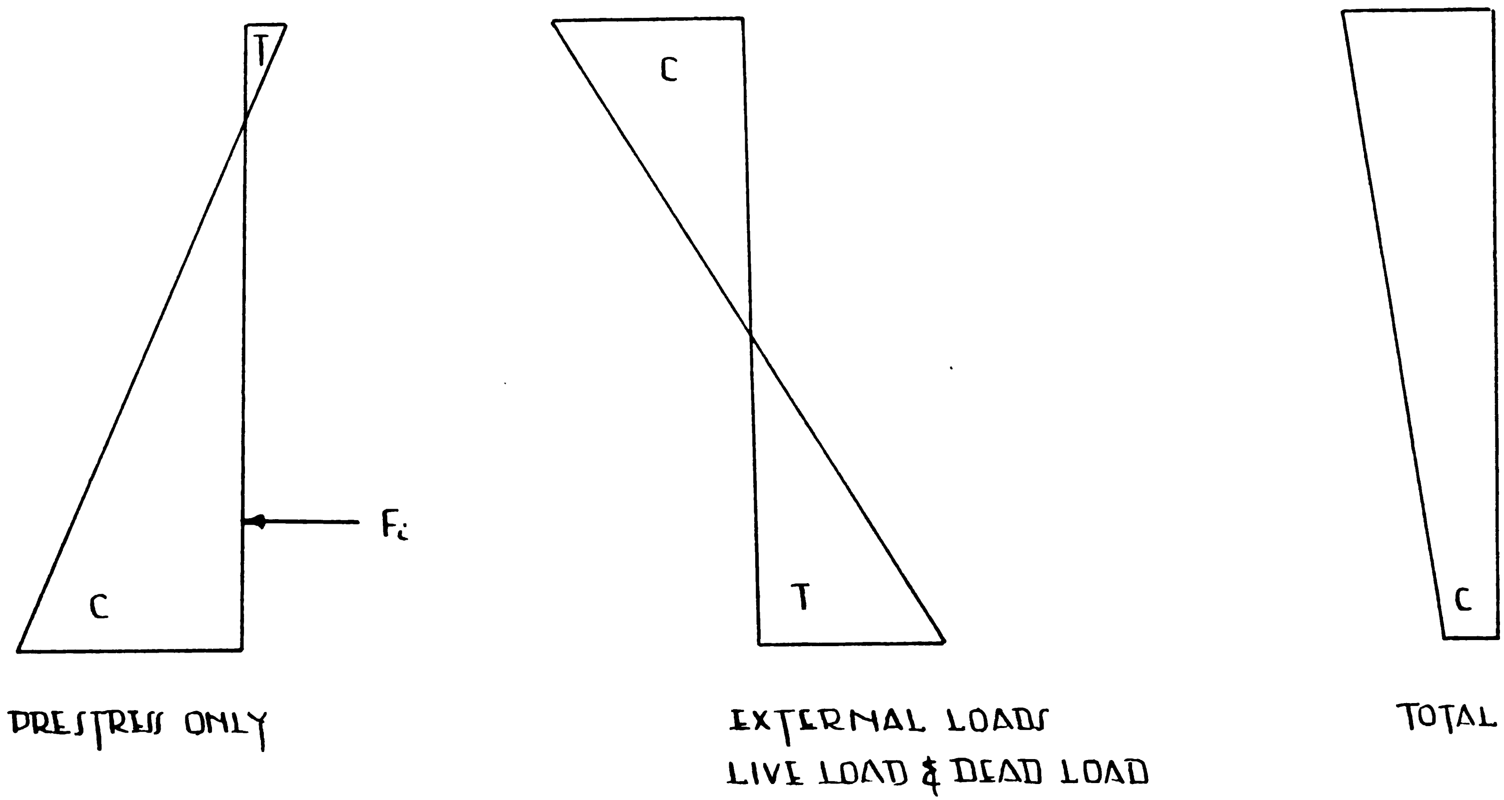
The second moment is produced by loading the beam in actual use. This moment is generally called the live load moment and includes impact.

Next, consider a beam of cross-section A, a steel force F_1, eccentricity of the steel, e, and the distance from the neutral axis to the upper and lower fibers, y_1 and y_2, respectively as shown in fig.

Assume that the bending moments are such that the top fiber is in compression for a positive moment. Then the section of the beam at the point of maximum bending stress must satisfy the following conditions.

In top fiber

1) As soon as the prestress has been established, the combination of stress in the concrete due to the beam weight, W_d, plus the stress due to prestress force, F_1 must not exceed


T
C
F_i
PRESTRESS ONLY
C
T
EXTERNAL LOADS
LIVE LOAD & DEAD LOAD
C
TOTAL

the permissible concrete tensile stress, f_{cf}. f_{cf} is equal to zero as minimum compression value.

2) After a period of time, the beam is acting under W_d, the dead load and W_1, live loads combination. The compressive stress in the concrete must not exceed the permissible concrete stress, f_c.

In the bottom fiber

1) As soon as the pre-stress has been established the stress in the concrete due to combined effects of the prestressed moment and the dead load moment must not exceed the allowable compressive stress in the concrete, f_c.

2) After a period of time under the combined live and dead loads plus the initial prestress, the tensile stresses in the concrete must not exceed f_{tc}.

Using the general fiber stress equation

$$f = F/A \mp Mc/I$$

now it is possible to find the fiber stresses due to prestress acting alone.

$f = F/A \mp Mc/I$

$M = F_1 e$

$I = Ar^2$

Then using column analogy method, in the top fiber

$f_{uc} = (F/A) - (Fy_1 e)/I = (F/A)(1 - ey_1/I)$ Tension

In the bottom fiber

$f_{bc} = (F/A) + (Fy_2 e/I) = (F/A)(1 + ey_2/I)$ Compression

So, four conditions described can be expressed mathematically as below:

In top fiber:

1) $-(F_1/A)(ey_1/r^2 - 1) + f_{dt} \leq f_{tc}$

2) $-\eta(F_1/A)(ey_1/r^2 - 1) + f_{dt} + f_{at} \leq f_c$

Bottom fiber:

1) $-(F_1/A)(1 + ey_2/r^2) + f_{db} \leq f_c$

2) $-n(F_1/A)(1 + ey_2/r^2) + f_{db} + f_{ab} \leq f_{tc}$

Depth

Subtracting 1 from 2, we get

$$I/y_1 \geq \frac{M}{f_{tc} + f_c - (1 - n)(F_1/A)(ey_1/r^2 - 1)}$$

An evaluation of the terms from various researches are

$n = .85$

F/A = not larger than $.5f_c$

$ey_1/r^2 = 2$ usually

and that gives

$$I/y_1 \geq M_1/(f_{tc} - .925f_c).$$

Adding 3 to 4

$$I/y_2 \quad \frac{M_1}{f_{tc} + f_c - (1 - n)(F_1/A)(1 - ey_2/r^2)}$$

Evaluating the terms

$$I/y_2 \geq M_1/(f_{tc} + .775f_c)$$

and this gives larger section modulus.

$I/c = bd^2/6$

$I/y_2 = bd^2/6 = M_1/.775f_c$

$d = \sqrt{(m_1 6/bx.775f_c)} = 2.78\sqrt{M_1/bf_e}$

In this equation: M_1 is in inches; b is in inches; f_c is in $\#/in^2$; the depth will then be in inches.

Bond

In prestress concrete, bond is of importance only in cases where the prestress is transferred from the steel to the concrete through the bond.

Where the reinforcement is embedded in the concrete under initial tensile stress, a very strong bond is produced as follows:

Upon release of the force producing tension in the steel, the tension in the steel is reduced due to elastic shortening of the concrete and subsequent volume changes, such as shrinkage. As this happens, the wire reinforcement develops a tendency to enlarge its diameter slightly. As may be readily seen, this action is similar in effect to that of a "forced fit," where considerable resistance to sliding is developed in spite of smooth contact surfaces.

Thus, the bond developed in prestressed concrete is greater than is developed in reinforced concrete due to the "force fit" action.

Rosov develops an equation which he suggests may be used to check the bond stresses. It is:

$$u = V \times Pn(q-k)/C\Sigma_0 d$$

Where:

u = the unit bond stress psi

V = Total shear at any section

p = percentage of steel in the entire cross section

n = ratio of E_s/E_c

C = a constant = $(1/12) - (\frac{1}{2} - k)^2 - (n - 1)p - (k - q)^2$

Σ_o = Total perimeter of the steel

d = depth of a slab or beam

$k = (\frac{1}{2}d/d) = \frac{1}{2}$

<u>Shear and diagonal tension</u>

In conventional reinforced concrete design, diagonal tension is of great importance, and to offset it, an intricate system of special reinforcement must be designed.

Since the longitudinal stress is always compressive for a pre-stressed beam, and since the maximum unit shear stress does not occur at the same place in the beam as the maximum fiber stress, the diagonal compression will never be much in excess of the allowable. However, if shear exists, there will always be a diagonal tensile stress existing.

The diagonal tension at any point on a beam, can be checked by using the equations:

$f_d = v^2/f_1 \qquad f_1 = M_c/I$

$v = VQ/Ib$

From the equations above it is seen that the value for diagonal tension is much smaller than for a conventional reinforced beam where the diagonal tension is computed from $v = 8V/7bd$.

Note: There has not been any attempt made for the reinforced concrete design theory, as everyone is familiar with the theory.

Design Procedures

The time required for making a design is of importance Simplicity is also a major consideration.A structural engineer's work encompasses many kinds of design problems and the design of prestressed concrete is only one subject among many.It is therefore important that the principles of prestressed concrete design be made as easy as possible, to remember and use.Such considiration have influenced the choice and scope of the procedures illustrated in the design.

The slab considered in the design examples is simply supported and has a 32ft. span length 46 ft. span width (consisting of four IO ft.lanes plus 2ft 3 in.side walks and rails, to carry highway loadings H-20,H-I5 and H-IO in current AASHO specifications. The individual slabs are rectangular shaped which is economical shape for prestressed and reinforced concrete.The most suitable and economical layout can not be selected from books nor computed solely by application of equations but must largly based on experience and judgment.

DESIGN OF A HIGHWAY BRIDGE SLAB
WITH REINFORCED CONCRETE

Specifications:

Span Length: 32'00"

Span Width: 46'00" (consisting of four 10' lanes and 2'3" sidewalks and rails)

Span Loading: ASSHO H-20, H-10, H-15

Allow Stresses:

fs = 22000 psi for main reinforcing
16000 psi for stirrup

fc = .45 x 2500 = 1125 psi

v = % 2 x 2500 = 50 without web reinforcing or special anchorage.
% 6 x 2500 = 150 with web reinforcing but without special end anchorage.

u = % 5 x 2500 = 125 psi

n = 10

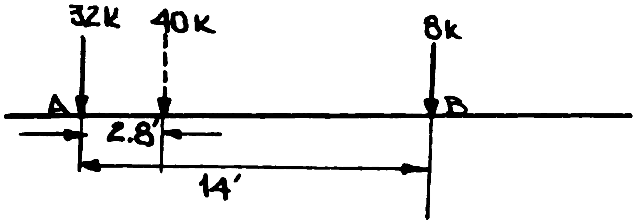
Impact = 50/(L-125) = % 30 but 25% will be sufficient, since concrete stress was somewhat on the safe side.

$$k = \frac{fc}{fc + (fs/n)} = \frac{1125}{1125 + (22000/12)} = .38$$

$$j = 1 - (k/3) = .873$$

$$R = (fc/2)kj = 186.5$$

Design No. 1 - For H-20 Loading



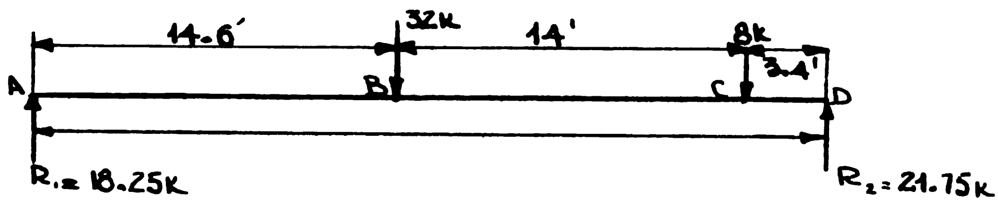
Moment @ A

$8 \times 14 = 40X$

$X = (8 \times 14)/40$

$X = 2.8$

X = the distance from A to the Resultant Force



Maximum Moment and shear at critical sections

Maximum Moment:

M @ B

(14.6)(18.25) = 266.450 ft - kip

Moment per foot = 26.645 ft - #/ft of width

Reactions:

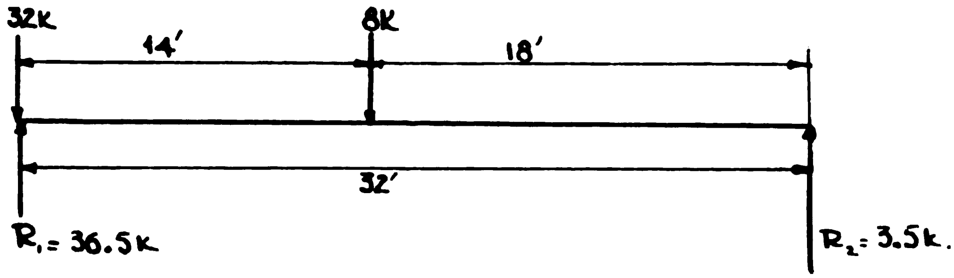
M @ A

$(14.6)(32) + (28.6)(8) = 32 R_2$

R_2 = 21.75k

R_1 = 40 + 21.75 = 18.25k

Maximum Shear



Lane Loading for Maximum Shear

Reactions:

M @ B

$32R_1 + 32x32 + 8 \times 18 = 0$

$32R_1 = 1024 - 144$

$32R_1 = 1168$

$R_1 = 1168/32 = 36.5K$

Maximum shear = 36.5 kips

Shear Diagram

Check Shear

$v = V/bjd$

$v = (3650)/(12x.873x13.35) = 26.3$ psi

$v = 26.3$ psi (Allow 50 psi)

Allow. (50 psi) O.K.

Stirrups are not required.

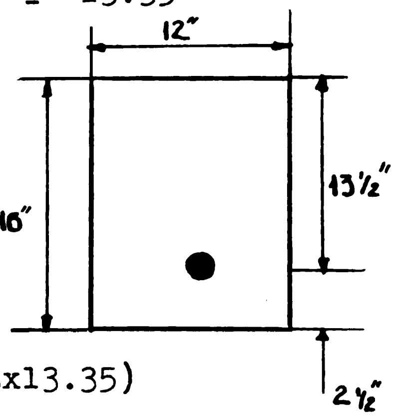
Maximum Live Load Moment:

Live Load Moment + % 25 Impact

26645 + .25(26645) = 33306 ft - #/ft.

<u>Required Depth</u>

$d = \sqrt{M/Rb} = \sqrt{(33306x12)/(186.5x12)} = 13.35''$



Depth of 13½"

Protective cover 2½"

Use an overall depth of 16"

<u>Reinforcement:</u>

$As = M/f_s j d$ = (33306x12)/22000x.873x13.35)

= (33306x6)/(11000x.873x13.35)

= (33306x6)/(9603x13.35)

= 1.555 □"/ft.

As = 1.555 □"/ft.

Try ½"- φ A = .20 □"/ft

Use 8½"- φ A = 1.60 □"/ft.

Spacing (.20x12)/(1.60) = 1.5"

Use a spacing of 1½"

Check spacing = (2½) (bar diameter)

= 1¼"

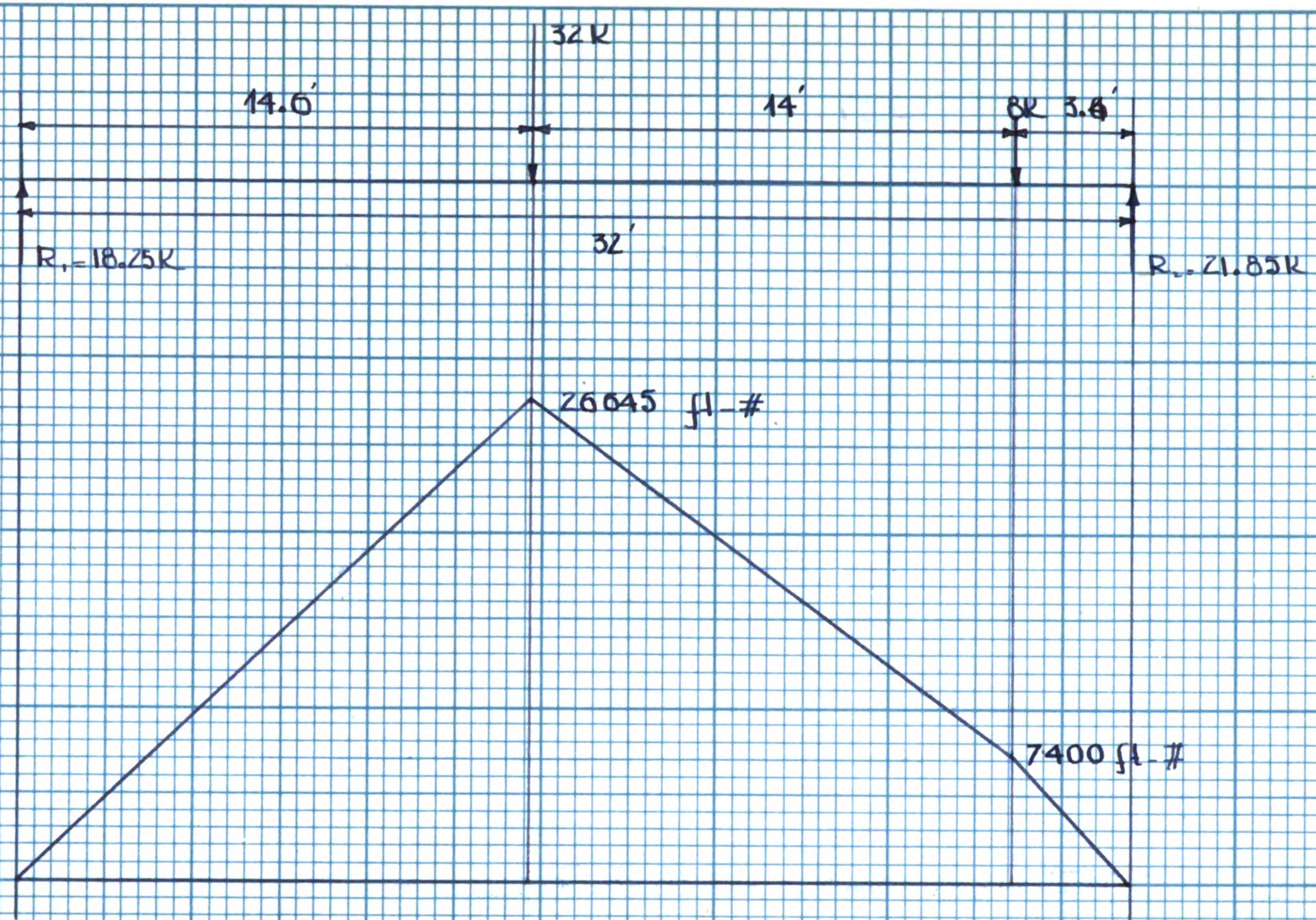
Spacing O.K.

<u>Check bond stress:</u>

$u = V/\Sigma_o jd$

u = (3650)/(12.6x.873x13.35) = 24.8 psi

Bond O.K. (Allow = 125 psi)



Moment Diagram For Max. Condition
H-20 Loading

<u>Temperature and shrinkage reinforcing</u>

A_{ts} = .002(15.5)(12) = .360 □"/ft.

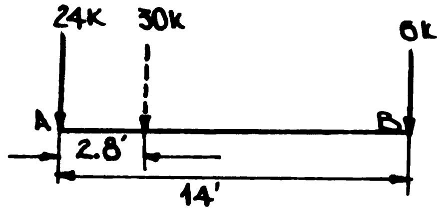
Try 3/8" φ A = .11 □"

Use 3/8" φ 4 - 3/8" c.c. A = .44 □"

s = (12x.11)/(.360) = 3.66

Use 3-3/4" spacing

Design No. 2 for AASHO - H-15 Loading



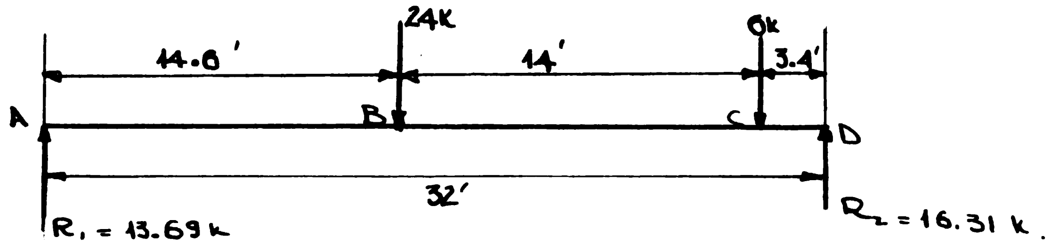
M @ A

6 x 14 = 30x

x = (6x14)/30

x = 2.8

x = the distance from A to the resultant force



Maximum Moment and Shear at Critical Sections

Maximum Moment:

Reactions

$(14.6)(24) + (28.6)(6) = 32R_2$

$R_2 = 16.31k$

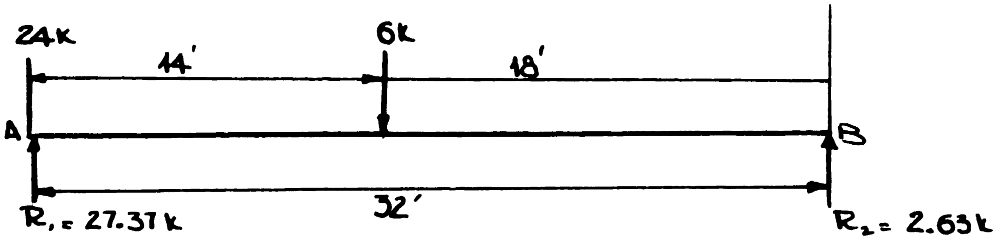
$R_1 = 30 - 16.33 = 13.69k_1$

Maximum Moment

$(14.6)(13.69) = 201{,}000$ ft. - #

Moment per foot = 201,000/10 = 20,100 ft - #/ft.

Maximum Shear:



Reactions:

M @ B

$(24)(32) + (6)(18) = 32R_1$

$32R_1 = 768 + 108$

$32R_1 = 876$

$R_1 = 27.37k = V$

Maximum Shear V = 2737 #/ft.

Maximum Live Load Moment:

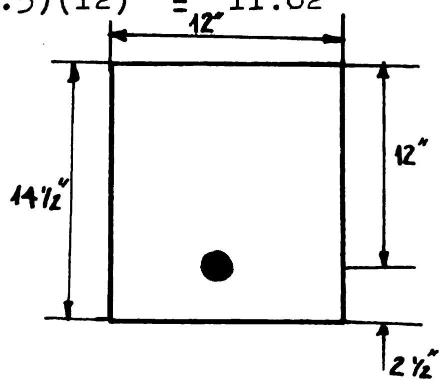
Live Load Moment + %25 Impact

20,100 + (.25)(20,100) = 5025 = 25,125 ft. #/ft.

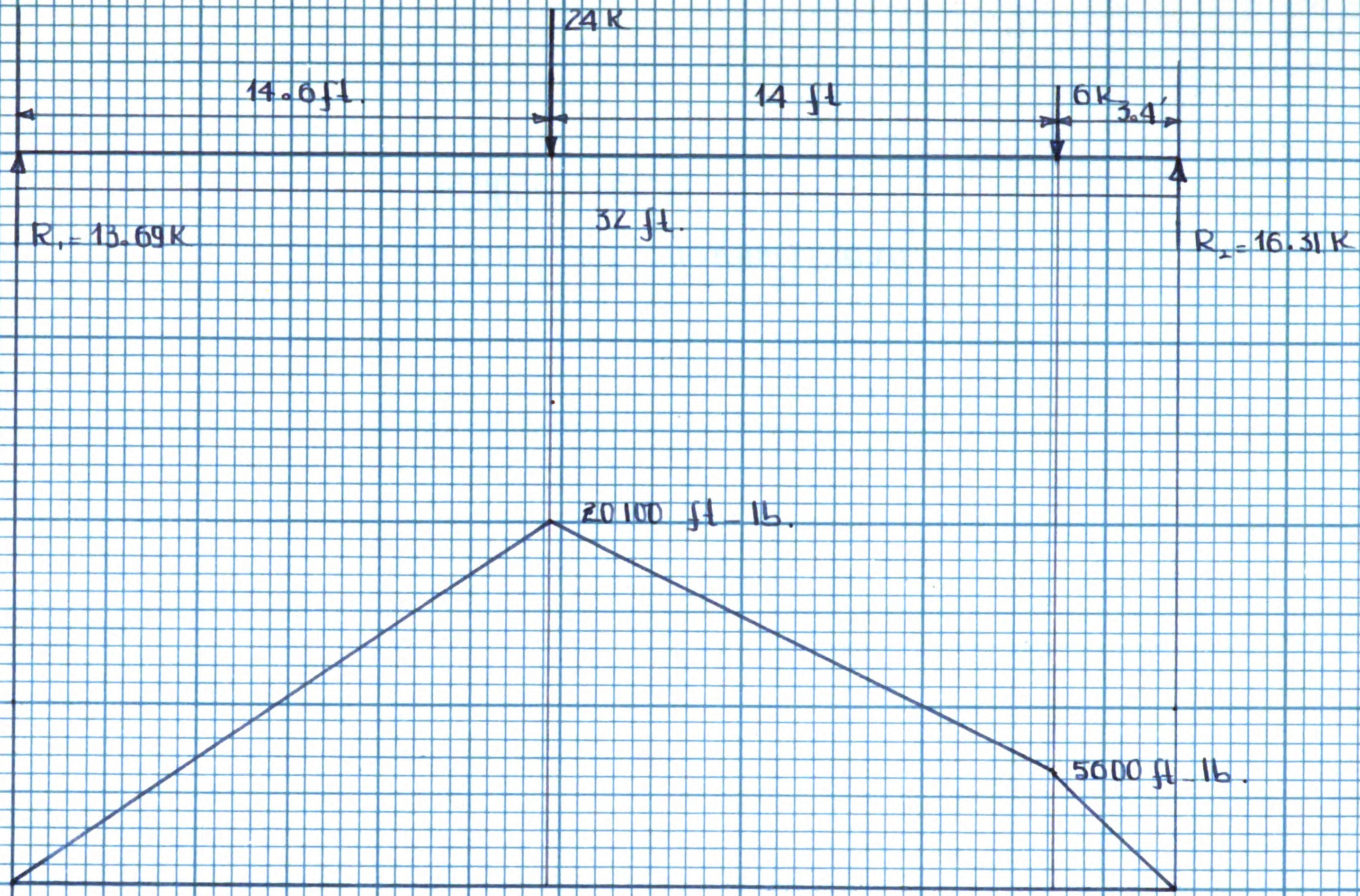
M_{L_1L} = 25,125 ft.-#/ft.

Required Depth.

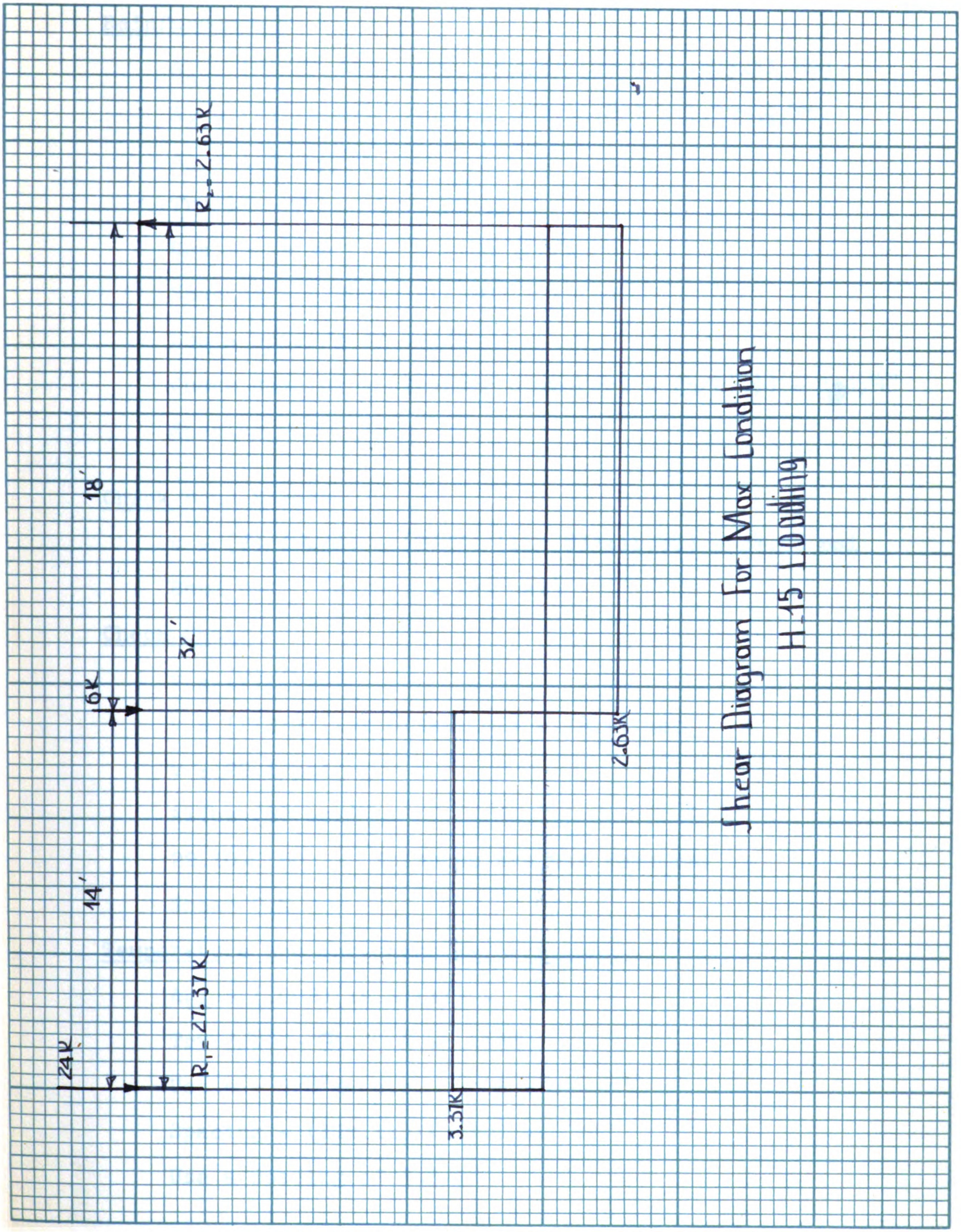
$d = \sqrt{(M/Rb)} = \sqrt{(25,125)(12)/(186.5)(12)} = 11.82"$



Depth	12"
Protective Cover	2½"
Use an overall Depth	14½"



Moment Diagram For Max. Condition
H-15 Loading



Shear Diagram For Max Condition
H-15 Loading

<u>Reinforcement</u>

A_s = (M)/($f_s$jd) = (25,125x12)/(22,000x.873x11.82)

A_s = (25,125x6)/(11,000x.873x11.82) = 1.32 □"/ft.

A_s = 1.32 □"/ft.

Try ½" - Φ A = .20 □"

Use 7 - ½" Φ A = 1.40 □"/ft

Spacing = (.20x12)/(1.40) = 1.715"

Use a spacing of 1-3/4"

Check spacing = (2½)(bar diameter) = (5/2)(½) = 1-1/4"

Spacing O.K.

<u>Check Bond Stress</u>

u = (V)/($\Sigma_o$jd) = (2737)/(11x.873x11.82) = 24.08 psi

Bond O.K. (Allow. = 125 psi)

<u>Check Shear</u>

v = (V)/(bjd)

v = (2737)/(12x.873x11.82) = 22.10 psi

v = 22.1 psi

Allow (50 psi)

$22.10 < 50$ Stirrups are not required

Shear O.K.

<u>Temperature and Shrinkage Reinforcement</u>

A_{ts} = (.002)(14.5)(12) = .348

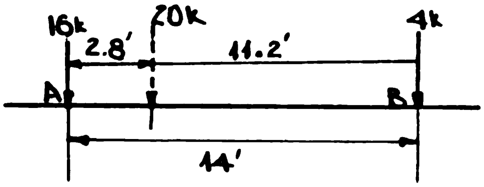
Try 3/8" - Φ A = .11 □"

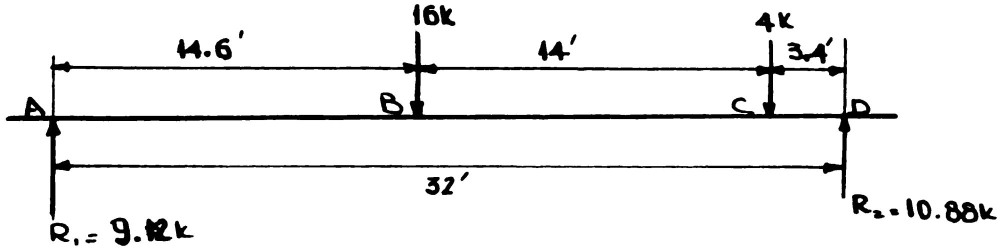
Use 4 - 3/8"Φ c.c. A = .44 □"

Spacing = (12x.11)/(.348) = 3.79"

Use a spacing of 3-3/4"

Design No. 3 - for H-10 Loading





Maximum Moment and Shear at Critical Sections

Maximum Moment:

Reactions:

M @ A

$(14.6 \times 16) + (28.6 \times 4) = 32R_2$

$R_2 = 10.88k$

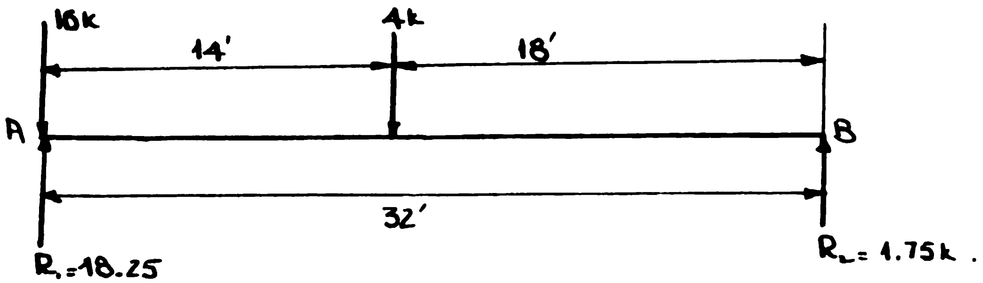
$R_1 = 20 - 10.88 = 9.12$

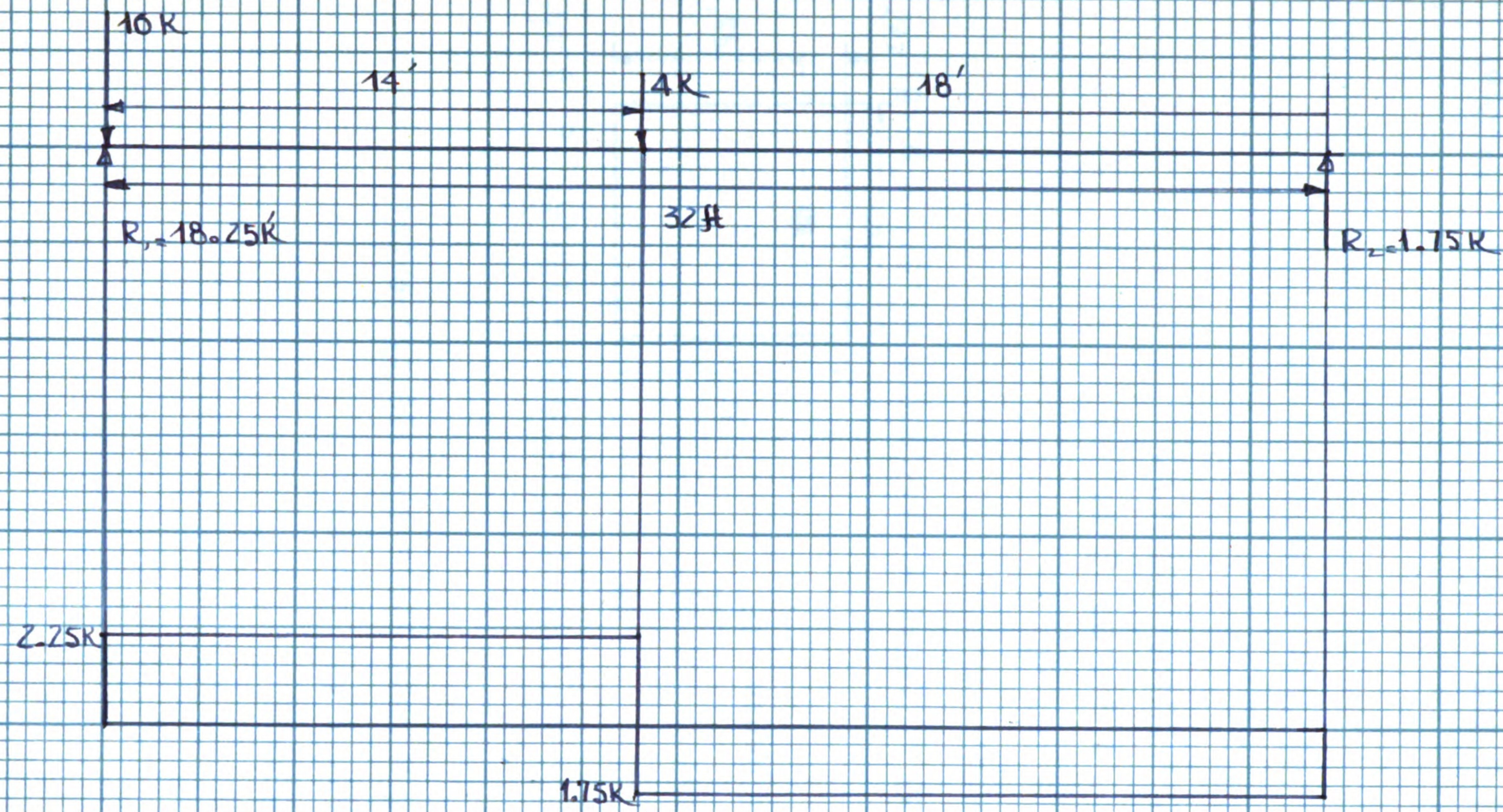
M @ B

$(14.6 \times 9.12) = 133.152$ ft-#

Moment per foot = 133,152 ft-#

Maximum Shear





Shear Diagram For Max. Condition
H-10 Loading

M @ B

$3R_1 = (4 \times 18) + (16 \times 32)$

$R_1 = 18.25k$

$R_2 = 1.75k$

$V = 1825$ #/ft.

Maximum Live Load Moment

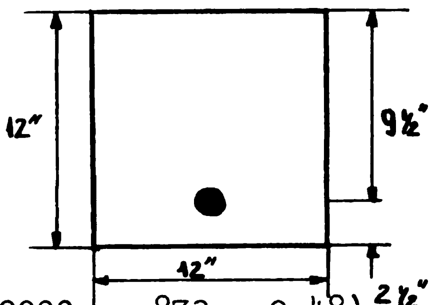
Live Load Moment + %25 Impact

$133{,}152 + (.25 \times 133.152) = 166{,}452$ ft.-#/ft.

Required Depth:

$d = \sqrt{(M)/(Rb)} = \sqrt{(16645.2 \times 12)/(12 \times 186.5)} = 9.48''$

Depth	$9\frac{1}{2}''$
Protective Cover	$2\frac{1}{2}''$
Use an overall depth	$12''$



Reinforcement

$A_s = (M)/(f_s jd) = (16645 \times 12)/(22000 \times .873 \times 9.48)$

$A_s = 1.095$ □"/ft.

Try $\frac{1}{2}''$ ɸ $A = .20$ □"

Use 6 - $\frac{1}{2}''$ ɸ $A = 1.20$ □"/ft.

Spacing $= (.20 \times 12)/(1.20) = 2''$

Use a spacing 2" c.c.

Check spacing $(2\frac{1}{2})$(bar diameter) $= 1\frac{1}{4}''$

Spacing O.K.

Check Bond

$u = (V)/(\Sigma_0 jd) = (1825)/(9.4 \times .873 \times 9.48) = 23.6$psi

Bond O.K. Allow (50 psi)

Check Shear

$v = (V)/(bjd) = (1825)/(12 \times .873 \times 9.48) = 18.4$ psi

$v = 18.4$ psi

Allow. 50 psi O.K.

Stirrups are not required.

Temperature and Shrinkage Steel

$(.002 \times 12 \times 12) = .288$ □"/ft.

Try 3/8" ϕ A = .11 □"

Use 3 - 3/8" ϕ C.C. A = .33 □"/ft

$s = (12 \times .11)/(.288) = 4.6$"

Use 4-3/4" spacing

DESIGN OF A HIGHWAY BRIDGE SLAB
WITH PRE-STRESSED CONCRETE

Specifications:

Span Length - 32'00"

Span Width - 46'00" (consisting of four 10 ft. Lanes and 2'3" sidewalks and rails)

Span Loading ASSHO H-20, H-15, and H-10

Allowable stress in the concrete

Compression, f_c, 1500 psi

Compression ultimate 4500 psi

Tension f_{ct} = 0 psi

n = .85 (Loss in prestress with age)

Allowable steel tension - 120,000 psi

That requires a steel tensile strength of 200,000 psi

Use a wire of .2 inch diameter for reinforcing

Live Load

ASSHO H-20

Impact = (50)/(L - 125) = 50/167 = 30%

But %25 will be sufficient as we are on conservative side.

<u>Design No. 1 - For H-20 Loading</u>

Maximum live load: 33,306 ft.-#

Maximum V = 3650 #/ft.

Computations:

$d = 2.78\sqrt{(M_1)/(bfs)} = 2.78\sqrt{(33306 \times 12)/(12 \times 1500)}$

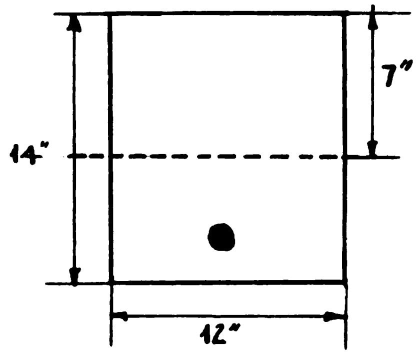
$d = 2.78\sqrt{22.2} = 2.78 \times 4.71 = 13.1"$

Assume a section to satisfy

$I/Y_2 = (M_1)/(.775f_c + f_{tc}) = (33306 \times 12)/(1162.5)$

$I/Y_2 = 342 \text{ in.}^3$

<u>Check the depth</u>



$I/C = (bd^2)/6$

$= (12 \times 14 \times 14)/6$

$= 28 \times 14$

$= 392 \text{ in}^3$

Depth is O.K.

The Dead Load Weight

Weight of concrete $150\#/ft^3$

$Wd = 150(14/12) = 175 \ \#/ft^2$

$Md = 1/8Wl^2 = (175 \times 12 \times 32 \times 32)/8 = 1536 \times 175$

$Md = 268500 \text{ in } \#/ft$

Concrete stresses

$s = (6M)/(bd^2)$

$f_{du} = f_{db} = (268500 \times 6)/(12 \times 14 \times 14) = 686 \ \#/in^2$

$f_{ul} = f_{lb} = (33306 \times 6 \times 12)/(12 \times 14 \times 14) = 1020$

Determination e and F_1

$P/A \mp (Pe/I)y \quad f_{tc} = 0$

$e = r^2/y$

$r^2 = I/A$

$I = (1/12)bd^3 = (1/12)(12)(14)^3$

$I = 196 \times 14 = 2744 \text{ in}^4$

$A = 14 \times 12 = 168 \text{ in}^2$

$r^2 = 2744/168 \doteq 16.3$

$y = 6$

$e = 16.3/7 = 2.33''$ For safety use an $e = 3.25''$

$M_{total} = M_{L.L.} + M_{d.L.}$

$f_{tc} = F_1/A \quad -y(F_{1e} - M_t)/(I) = 0$

$F_1 = (M_t)/(r^2/y + e)$

There is a loss in the concrete due to creep and shrinkage. Therefore a 20% loss is estimated a reasonable reduction in prestress for this problem.

Then F_1 becomes

$F_1 = (M_t)/(.80)(r^2/y + e)$

$M_t = 268500 \quad (33306 \times 12) = 668172$

$F_1 = 668172/(.80)(2.31 + 3.25) = 150000\#$

$F_1 = 150000\ \#$

Steel Area

F_1/Allowable steel stress   150000/120000   1.25"/Ft

Try .2" (#5 American wire size)   A = .0314 □"

No. of wires   1.25/.0314 = 39.9 #

Use 40 wires

Spacing of wires

3 rows of 14 wires each /Ft

Wire spacing   12/14   .85"

Use a spacing of 1" apart

Note: There is a reduction in the concrete area because of the wires. For practical purposes it has been omitted.

Check Concrete Stresses

$F_1 = 150000$ #/ft $\quad e = 3"$

1) $-(F_1/A)(ey_1/r^2 - 1) + f_{dt} \leq f_t = 0$

$-(150000/166.75)(3x6.98/16.4 - 1) + 686 = f_t$

$-900(1.273 - 1) + 686 = f_t$

$-900(.273) + 686 = +441 = f_t \quad f_t = 0 \quad$ O.K.

+ compression

2) $-n(F_1/A)(ey_1/r^2 - 1) + f_{dt} + f_{at} \leq f_c$

$-.85(150000/166.75)(.273) + 686 + 1021 \leq 1500$ psi

$-.85(900)(.273) + 686 + 1021 \leq 1500$ psi

$-208 + 686 + 1021 \leq 1500$ psi

1499 psi $<$ 1500 psi O.K.

3) $-(F_1/A)(1 + ey_2/r^2) + f_{db} \leq f_c$ 1500 psi

$-900(1 + 3(7.02)/16.3) + 686 \quad f_c$

$-2050 + 686 = 1350 < 1500$ psi O.K.

4) $-n(F_1/A)(1 + ey_2/r^2) + f_{db} + f_{ab} \leq f_t \quad f_t = 0$

$-.85(900)(2.282) + 686 + 1030 < 0$

$-1742.5 + 1723 < 0$

$-19.5 = 0$ O.K.

Concrete Stresses OK

Check Bond

$u = V \times Pn(q-b)/c \leq_0 d$

$V = 3650$ #/ft

$P = A_w/A_c = 1.25/168 = .00745$

$q = (1/3)d/14 = 10/14 = .714$

$n = E_s/E_c = 6$

$d = 12''$

$\Sigma_o = 3.14 \times .2 \times 38$

$C = (1/12) - (\frac{1}{2} - k)^2 - (n - 1)P - (k - q_t)^2$

$C = (1/12) - 0 - 5(.0081)(.75 - .5)$

$C = .083 - .006090$

$C = .077$

$u = V \times Pn(q-b)/C \Sigma_o d$

$u = 2737 \times (.0081 \times 6 \times .75-12)/(.077 \times 23.864)$

$u = 71 \#/in^2$

Allow. bond 80 $\quad 71 < 80 \quad$ O.K.

Diagonal Tension

$f_d = v^2/f_1 \qquad f_1 = 1500$ psi

$v = V/bjd = VQ/Ib$

$Q = 12 \times (6 \times 6)/2 = 216\ in^3$

$I = 1728$

$v = (2737 \times 18)/1728 = 28.58$ psi

$f_c = M_c/I = F_1ec/I = (140000 \times 2.75 \times 6)/1728 = 1020$

Use a factor of safety $\frac{1}{2}$

$f_c = 510$ psi

$f_d = (28.58 \times 28.58)/510 = 1.6$ psi

Allow diagonal tension = 18 psi

$1.6 < 18 \quad$ O.K.

Section of the end

$(2/3)d_1 = 9''$

$d_1 = 13.5''$ min. depth

$I = (1/12)(12)(13.5)^3 =$

$r^2 = (13.5)^2/12 = 15.2$

Try a depth of 14"min.

$A = 168$

$r^2 = 2750/168 = 16.4$

$e = 2"$

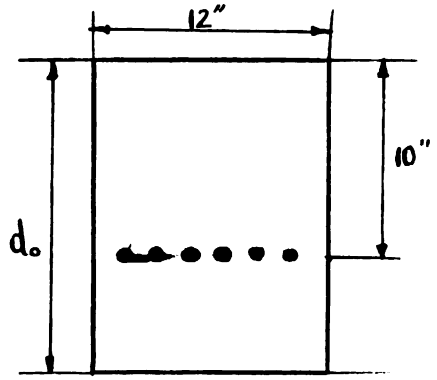
$f_u = -(150000/192)(2x8/21.3 - 1)$

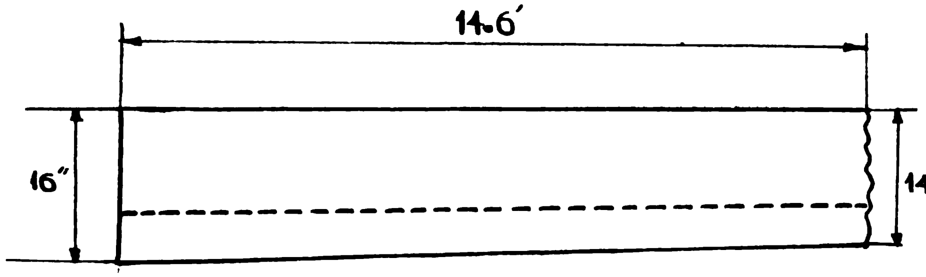
$f_u = -781(.75 - 1)$

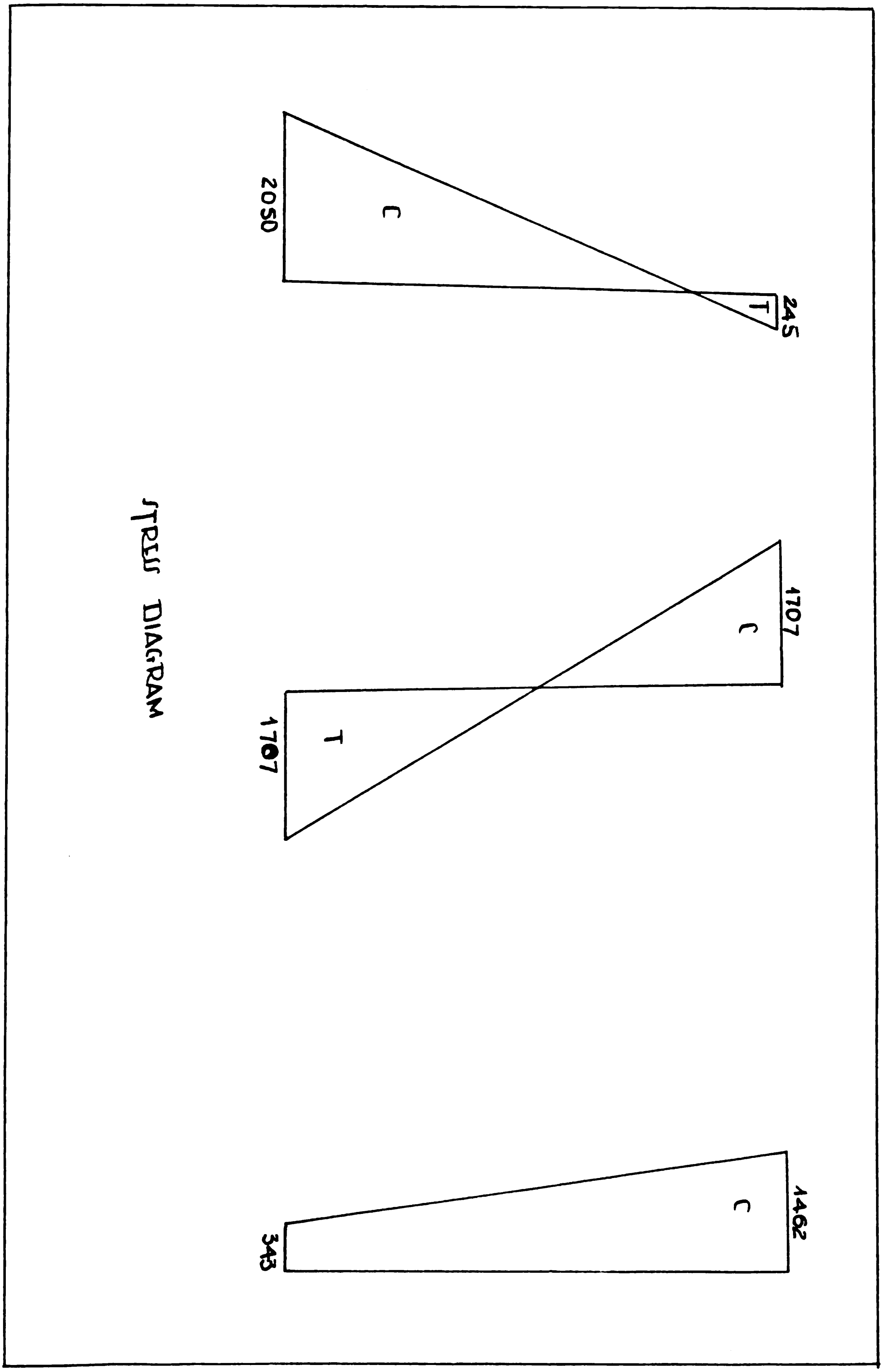
$f_u = +195$ #/in^2 (compression)

$f_b = -781(1.75)$

$f_b = (1365)$ psi $<$ 1500   O.K.






STRESS DIAGRAM
2050
C
T
245
1707
C
T
1707
1462
C
343

Design No. 2 - For H-15 Loading

Maximum Live Load M = 25125 ft-#

Maximum V = 2737 #/ft

a) Reg: depth

$d = 2.78\sqrt{(M/bf_c)} = 2.78\sqrt{(25125 \times 12)/(12 \times 1500)}$

$d = 11.4"$

Assume a depth of 12"

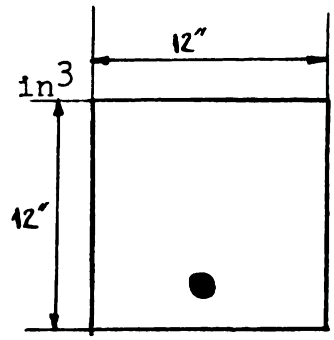
b) Assume a section to satisfy equation

$I/Y = M_1/.775 f_{ct} f_{tc} = (25125 \times 12)/1162.5 = 259 \text{ in}^3$

Check the depth

$I/C = bd^2/6 = (12 \times 12 \times 12)/6 = 288 \text{ in}^3$

Section O.K.



c) Dead Load Weight

The weight of concrete = 150 $\#/ft^3$

$W_d = 150(12/12) = 150$ #/ft

$M_d = (1/8)W\ell^2 = (150 \times 12 \times 32 \times 32)/8 = 230500$ in#/ft

d) Stresses using

$S = 6M/bd^2$

$f_{du} = f_{db} = (6 \times 230500)/(12)(12)^2 = 800$ psi

$f_{du} = 800$ psi

$f_{ul} = (25125 \times 12 \times 6)/(12 \times 12 \times 12) = 1047$ psi

$f_{ul} = 1047$ psi

Determination of e and F_1

$e = r^2/y_t$

$r^2 = I/A = (1/12)(b)(d)^3/bd$

$r^2 = d^2/12 = (12)^2/12 = 12$

$e = 12/6 = 2"$ Use an $e = 2\text{-}3/4"$

$f_{tc} = (F_1/A) \mp (F_{1e}\ M_t/)y = 0$

$F_1 = M_t/(r^2/y + e)$

$F_1 = M_t/(.80)(r^2/y + e)$

$M_t = (25125 \times 12) + 230500 = 531000"\#/Ft$

$F_1 = 531000/(.80)(2 + 2.75) = 140000\ \#$

Compute Wire Area

$A_w = F_1$/Allowable steel stress

$A_2 = 140000/120000 = 1.167\ in^2/Ft$

Try .2" (#5 American size) $A = .0314\ in^2$

No. of bars = 1.167/.0314 37.7

Use 38" wires

Spacing of Wires

2 rows of 19 wires each /Ft

Wire spacing = .12"/19 = .632"

Use a spacing of 3/4" apart

Note: There is a reduction in the concrete area because of the wires. For practical purposes it has been omitted.

Check Concrete Stresses

$F_1 = 140{,}000\ \#$ $\quad e = 2.75''$

1) $-(F_1/A)(ey_1/r^2 - 1) + f_{dt} \leqq f_t = 0$

$-\frac{140{,}000}{143.833}\left\{\frac{2.75 \times 6}{12} - 1\right\} + 800 \leqq f_t$

$-975(1.37 - 1) + 800 \leqq f_t$

$-360 + 800 \leqq f_t$

$+440$ psi $\quad f_t = 0$ + compression O.K.

2) $-\eta(F_1/A)(ey_1/r^2 - 1) + f_{dt} + f_{at} \leqq f_c$ $\quad f_c = 1500$ psi

$(.85)(14000/143.833)(2.75x6/12 - 1) + 800 + 1045 \leqq f_c$

$(.85)(.037)(975) + 800 + 1045 \leqq f_c$

$-306 + 800 + 1045 \leqq 1439$ psi

1439 psi $<$ 1500 psi O.K.

3) $-(F_1/A)(1 - ey_2/r^2) + f_{db} \leqq f_c$ $\quad f_c = 1500$ psi

$-(.85)(140000/143.833)(1 - 1.37) + 800 \leqq f_c$

$(.85)(975)(2.37) + 800 \leqq f_c$

$-1962 + 800 = -1162$

$-1162 < 1500$ O.K.

4) $-\eta(F_1/A)(1 + ey_2/r^2) + f_{db} + f_{ab} \leqq f_t = 0$

$(.85)(140000/143.833)(1 + 1.37) + 800 + 1045$

$-1962 + 800 + 1045 = -117$

$-117 < 0$ O.K.

Concrete stresses are O.K.

Check Bond

$u = V \times Pn(q-b)/C \leqq_0 d$

$P = 1.167/144 = .0081$

$q = 8/12 = .75$

$b = 12''$

$b = 12''$

$C = 1/12 - (\frac{1}{2} - k)^2 - (n - 1)P_t(q_t - k)^2$

$n = E_s/E_c = 6$

$\Sigma_o = d \times$ No. of bars $= 3.14 \times .2 \times 40 = 25.1$ in

$d = 14''$

$k = (d/2)/d = .500$

$C = .083 - 0 - (5 \times .00745 \times .714 - .500)$

$C = .083 - (5 \times .00745 \times .215)$

$C = .083 - (5 \times .0016)$

$C = .091$

$u = 3650 \times (.00745 \times 6 \times -11.286)/(.091 \times 25.1 \times 14)$

$= 3650 \times .0158$

$= 57.6\ \#/in^2$

All. Minimum Bond $= .04\ f'_c = 2000 \times .04 = 80$ psi

Diagonal Tension

$f_d = V^2/f_1 \qquad f_1 = 1500$ psi

$v = V/bjd = VQ/Ib$

$v = VQ/Ib = (3650 \times 294)/(2732.75 \times 12) = 32.6$

$Q = 12 \times (7 \times 7)/2$

$Q = 294\ in^3$

$f_1 = M_c/I = F_{ec}/I = (15000 \times 3 \times 7)/2732 = 1150$

Use a safety factor of $\frac{1}{2}$

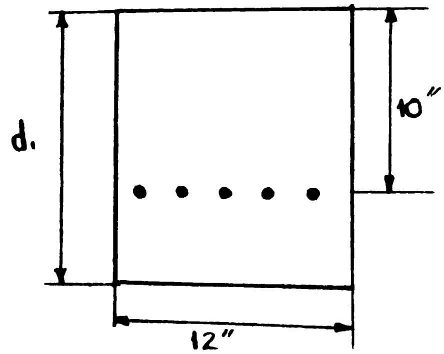
$f_1 = 1150/2 = 575$

$f_d = (32.6 \times 32.6)/575 = 1.85$ psi

Allow. $.012\ f_c = .012 \times 2000 = 24\ \#/in^2$

No condition of diagonal tension exists.

Section at the end of beam



$(2/3)d_1 = 10"$

$d_1 = (10 \times 3)/2 = 15"$ min.

$I = (1/12)bd^3$

$I = (15)^3 = 3380$ in

$r^2 = 3380/180 = 18.8$

Use $16" = d_1$

$A = 192$

$r^2 = 21.3$

$e = 2"$

$f_u = -(F_1/A)(ey/r^2 - 1)$

$f_u = -(140000/168)(2x7/16.4 - 1)$

$f_u = (140000/168)(.855 - 1) = 121$ psi compression

$f_b = -(140000/168)(2x7/16.4 + 1)$

$-(140000/168)(1.855) = 1547$ psi

$1547 > 1500$

So try a section 15"

$I = (1/12)bd^3 = (1/12)(12)(15)^3$

$I = 3380$

$r^2 = 3380/180 = 18.8$

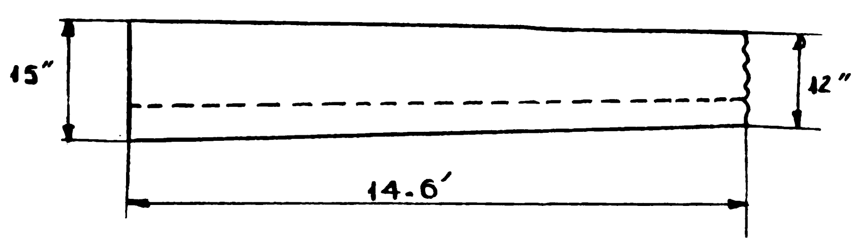
$e = 2$

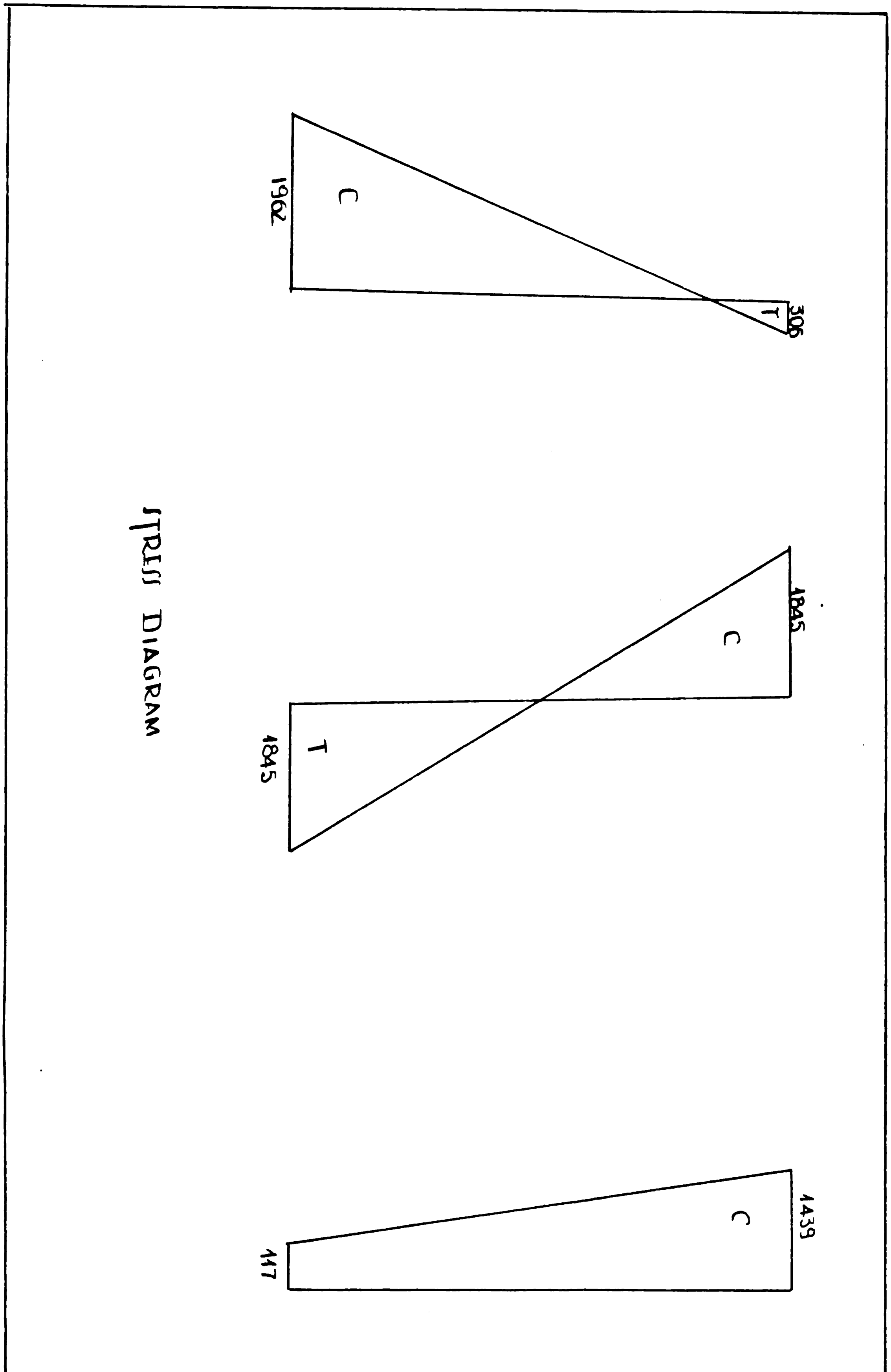
$A = 180$

$f_u = -(140000/180)(.78 - 1) = 171.0$ compression

$f_b = (140000/180)(2x7.5/18.8 + 1) = 1385$ psi

$1385 > 1500$ O.K.




STRESS DIAGRAM
1962
C
T
306
1845
C
T
1845
1439
C
117

Design No. 3 - For H-10 Loading

Maximum Live Load M = 16645 ft-#

Maximum V = 1825 #/ft.

a) Reg. depth

$d = 2.78\sqrt{(M/bfc)} = 2.78\sqrt{(16645 \times 12)/(12 \times 1500)}$

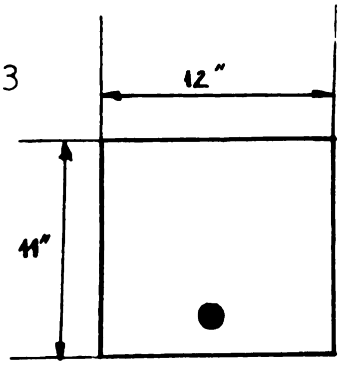
$d = 10.61"$

b) Assume a section to satisfy equation

$I/Y_2 = M_1/(.775f_c - f_{tc}) = (16645 \times 12)/1162.5 = 143 \text{ in}^3$

Check the depth

$I/C = bd^2/6 = (12 \times 11^2)/6 = 242 \text{ in}^3$



Section O.K.

c) Dead Load Weight

Weight of concrete = $150 \#/ft^3$

$W_d = 150 \times (11/12) = 137.5 \#/ft.$

$M_d = (1/8)W_d l^2 = (1/8)(137.5)(32)^2 = 176500$ in-#/ft.

d) Concrete stresses

$S = 6M/bd^2$

$f_{du} = (176500 \times 6)/12 \times 11^2) = 176500/242 = 730$ psi

$f_{ul} = (16645 \times 6 \times 12)/(12 \times 11^2) = 825$ psi

e) Determination of e and F_1

$F/A - (P_e/I)y = 0$

$e = r^2/y$

$r^2 = I/A = (1/12 \times 12 \times 11^3)/(12 \times 11)$

$r^2 = 10.08"$

$e = 22/12 = 1.831"$ Use an $e = 2.25"$

$f_{tc} = F_1/A - y(F_1e - M_t)/I = 0$

$F_1 = (M_t/(r^2/y + e)$

M_{total} = 176,500 + 199742 = 376242 ft-#

F_1 = (376242)/(.80)(1.831 + 2.25) = 115200 #

F_1 = 115200 #

Compute Wire Area

(F_1)/(Allow. Steel Stress) = 115200/120000 = .962 "/ft.

Try .2" (#/5 American Size) A = .0314 □"

No. of wires = .962/.0314 = 30.6"

Use 32 wires

Spacing of Wires

2 rows of 16 wires each/ft.

Wire spacing = 16/12 = 1.32"

Use a spacing of 1½" apart

Stresses due to Loads

f_{dt} = (176500 x 5.5)/1331 = 728 psi

$f_{db} = f_{dt}$

$I = (1/12)bd^3$

$I = (1/12)(12)(11)^3$

$I = 1331\ in^4$

f_{at} = (16645 x 12 x 5.5)/1331 = 825 psi

$f_{at} = f_{ab}$

Check Concrete Stresses

F_1 = 115200# e = 2.25"

1) $-(F_1/A)(ey_1/r^2 - 1) + f_{dt} \leq f_t \qquad f_t = 0$

$-(115200/132)(2.25x5.5/10.08 - 1) + 730 \leq f_t$

-(875 x .23) + 730

+204 - 730 = -526 compression

O.K.

2) $-\eta(F_1/A)(ey_1/r^2 - 1) + f_{dt} + f_{at} \leq f_c$

$-(.85)(115200/132)(2.25x5.5/10.8 - 1) + 730 + 825 \leq f_c$

$-(.85)(875)(.230) + 730 + 825 = 1383$ psi

$1383 > 1500$ O.K.

3) $-(F_1/A)(1 - ey_2/r^2) + f_{db} \leq f_c$

$-(115200/132)(1 - 2.25x5.5/10.8) + 730 \quad f_c$

$875(2.23) + 730$

$-1950 + 730 = 1220 \leq 1500$ O.K.

4) $-\eta(F_1/A)(1 + ey_2/r^2) + f_{db} + f_{ab} \leq f_t$

$.85(115200/132)(1 - 2.25x5.5/10.8) + 825 + 730 \leq f_t$

-111 0 O.K.

Check Bond

$u = V \times np(q-b)/c \, {}_od$

$V = 1825$ #

$p = .962/132 = .0072$

$q = 8/11 = .728$

$k = 5.5/11 = .5$

$\Sigma_0 = 3.14 \times .2 \times 32 = 20.01$

$C = .083 - 6(.0072(.728 - .5)$

$C = .083 - .00985$

$C = .07318$

$u = 1825\ (6 \times .0072 \times .728-12)/(.0731 \times 20.01 \times 11) = 52$ psi

u = 52 psi 52 80 O.K.

Allow min. bond = $.04f'c = 2000 \times .04 = 80$ psi

Diagonal Tension

$f_d = v^2/f_1 \qquad f_1 = 1500$ psi

$v = V/bjd = VQ/Ib \qquad Q = 6 \times 5.5 \times 5.5 = 181.5\ in^3$

$v = (1825 \times 181.5)/(1331 \times 12) = 20.6\ \#/in^2$

$f_1 = Mc/I = F_{ec}/I = (115200 \times 2.25 \times 5.5)/1331$

$f_1 = 1071$ Safety factor $\frac{1}{2}$

$f_1 = 537$

$f_d = (20.6)^2/537 = .795$ psi

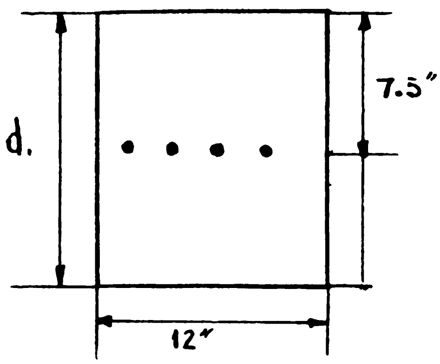
Allow. 24 O.K.

<u>Section at the End</u>

$(2/3)d_1 = 7.5$

$d_1 = 11.2"$ min.

Use $d_1 = 12"$



$A = 144 \text{ in}^2$

$r^2 = 12$

$e = 2"$

$f_u = (F_1/A)(ey_1/r^2 - 1)$

$= (115200/144)(2x6/12 - 1)$

$= (115200/144)(0) = 0$ O.K.

$f_b = -(115200/144)(2x6/12 + 1)$

$= -(115200/144)(2)$

$= -1600$

1600 1500 Not O.K.

Try a section 13"

$A = 169 \text{ in}^2$

$r^2 = 13$

$e = 2"$

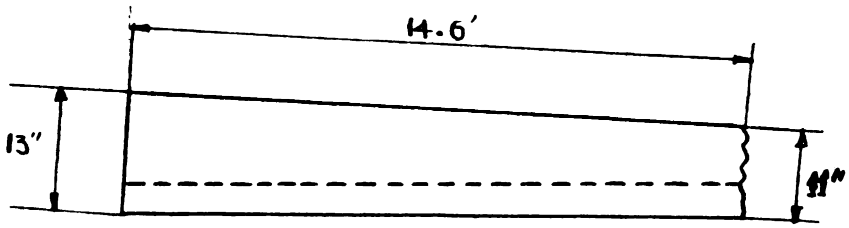
$$f_u = (115200/169)(2x6.5/13 - 1)$$

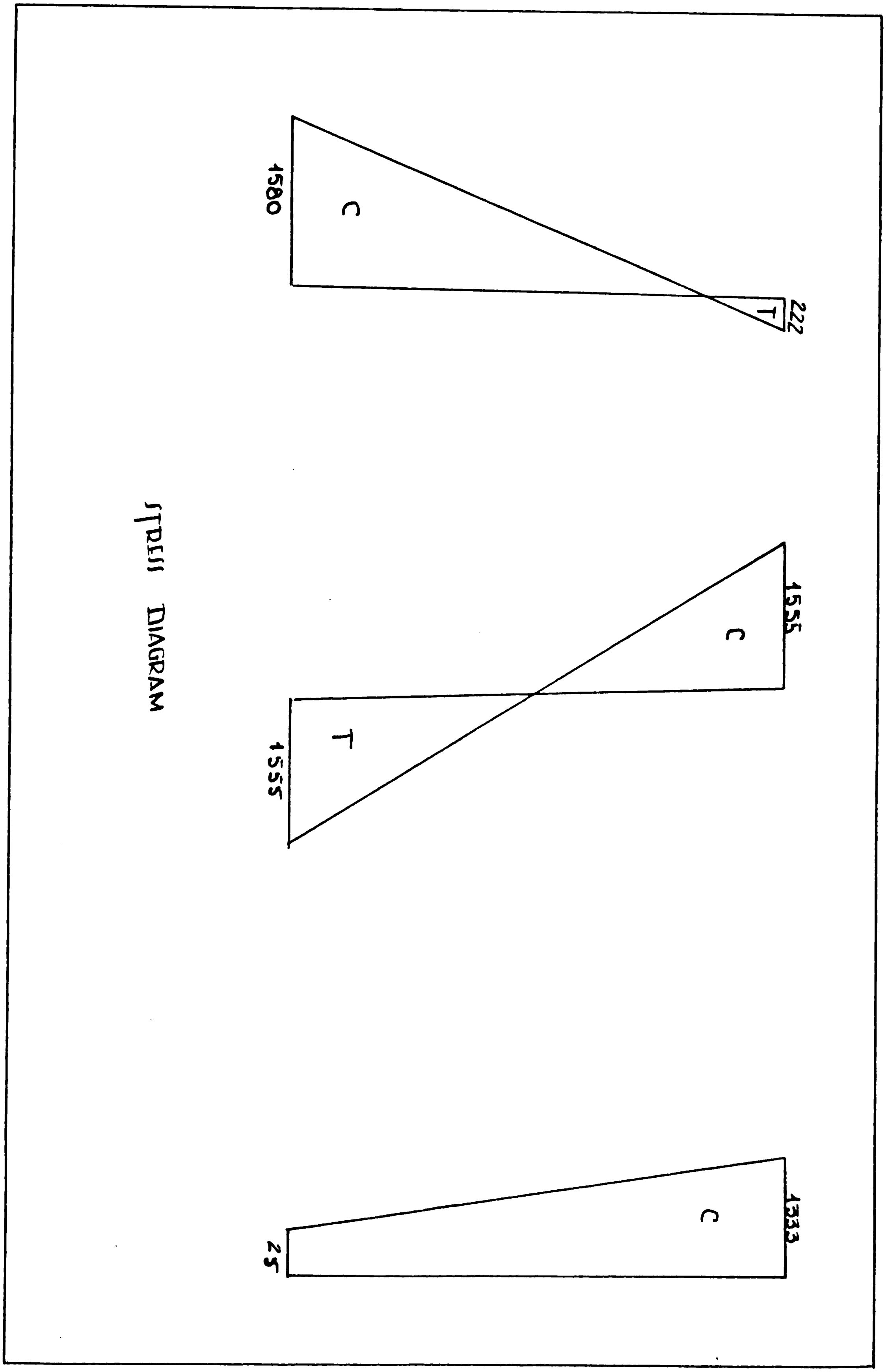
$$= 115200/169 \times .081$$

$$= 55.2 \text{ psi} \quad \text{O.K.}$$

$$f_b = (115200/169)(2x6.5/13 + 1)$$

$$= 1420 \text{ psi} \qquad 1420 > 1500 \qquad \text{O.K.}$$




STRESS DIAGRAM
1580
C
222
T
1555
C
1555
T
1333
C
25

CONCLUSION

Comparative Table For The Designed Slabs:

Design	Max.L.L.Mom. Ft-lb	V_{Max} lb	Depth in	Steel Area in^2
No:I				
Reinforced	33 306	3650	16	1.555
Prestressed	33 306	3650	14	1.250
No:II				
Reinforced	25125	2737	14.5	1.320
Prestressed	25125	2737	12	1.167
No:III				
Reinforced	16 645	1825	12	1.095
Prestressed	16 645	1825	11	0.962

As can be seen from the above table the major difference between the two slabs are in materials. In prestressed concrete economical use of both materials "Concrete and Steel" have been obtained since the total crossectional area of concrete is effective in comression. That is prestressed concrete slab, requires less concrete , less steel at the site. This is an important factor in that reduction of materials is always a desirable point. Purpose of all designs is to construct at the lowest cost with the use of minimum amount of materials.

After careful consideration of the above facts and data presented I came to the conclusion that the prestressed concrete slab would be most economical and best to use for this designs.

Bibliography

1. Sutherland and Reese, "Reinforced Concrete"
2. Portland Cement Assoc.,"Modern Developments in Reinforced Concrete" No:25
3. Schorer,H., "Prestressed Concrete, Design Principles and Reinforcing Units"; American Concrete Institute-Prociding v.14,p.493-528,June 1943
4. Abeles,P.W., "Beams in Prestressed Reinforced Concrete I II and III; Concrete and Constructional Engineering,v.41, pp.147,191,225;1946.
5. Anon.,"A Comparison of Partially and Fully Prestressed Concrete Beams"; Concrete and Constructional Engineering, v.42,p.11-15, January 1947.
6. Anon.,"Prestressed Reinforcement of Fine Wires Gives Spring to Precast Beams",Concrete, v.47,p.8,November, 1939.
7. Abeles,P.W., "Saving Reinforcement by Prestressing",Concrete and Constructional Engineering,v.35,p.328-333, July 1940.
8. Gueritte,T.J., "Recent Developments of Prestressed Concrete Construction With Resulting Economy in the Use of Steel"; Structural Engineer,v.18,p.626,July 1940.
9. Magnel,G., "Prestressed Concrete",London 1948.

10. Magnel,G., "Creep of Steel and Concrete in Relation to Prestressed Concrete"; American Concrete Institute Procidings, v.44,p.485-500, February 1948.

11. Mautner,K.W.,"Rebuttal Saving Reinforcement by Prestressing" ; Concrete and Constructional Engineering, v.36,p.73-95, February 1941.

12. Evans,R.H.,"Relative Merits of Wire and Bar Reinforcement in Prestressed Concrete Beams", Institution of Civil engineers of London Journal,v.18,p.315-329,February 1942.

13. Evans,R.H. and Wilson,G.,"Influence of Prestressing Reinforced Concrete Beams on Their Resistance to Shear", Structural Engineer, v.20,p.109-122, August 1942.

14. M'Ilmoyle,R.H.,"Prestressed Concrete Bridge Beams Being Tested in England",Railway Age,v.123,p.478, Sept.1947.

15. Schorer,H.,"Analysis and Design of Elementary Prestressed Concrete Members",American Concrete Institute Procidings v.43,p.49-87, September 1946.

16. Abeles,P.W.,"Formulae for the Design of Prestressed concrete Beams",Concrete and Constructional Engineering,v. 43,p.115-122 ,April 1948.

17. Abeles,P.W.,"Examples of the Design of Prestressed Concrete Beams",Concrete and Constructional Engineering, v. 43 ,p.149-1156,May 1948.

18. Coff,L.,"Prestressed Concrete , a New Frontier", Engineering News Record,v.143,p.187-190,September 1949.

19. Peabody,"Reinforced Concrete Design" .