FLOW OF SHELLED CORN THROUGH ORIFICES IN GRAIN BINS

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by

DALE JOHN EWALT

ABSTRACT

Submitted to the Colleges of Agriculture and Engineering of
Michigan State University of Agriculture and
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MASTER OF SCIENCE
IN
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Department of Agricultural Engineering

Approval F H Buelow

ABSTRACT

The design of materials handling systems could be greatly improved if more data were available that accurately described the flow of grain through an opening. This would allow engineers to better design bins as a part of a system that could be expected to operate independently as desired. Such systems would be a definite aid in grain storage.

A review of literature revealed that some work had been done in the area of semi-fluid flow. Stahl (1950) advanced some equations based on work done with wheat which indicated that flow was proportional to the cube of the diameter of an opening and independent of the head. He also indicated that the flow from a vertical orifice was one-third that from a horizontal opening. Other investigators agreed that a logarithmic relationship would describe grain flow and that flow was proportional to some function of the diameter of the discharge orifice.

The purpose of this work was first to develop a test procedure utilizing suitable equipment so that the flow rates of various grains could be accurately measured. Next, the effect of a sloping bottom in the bin would be studied. Finally, orifices of different sizes and shapes would be used to test their effect on the flow rate of corn, the grain selected for this investigation.

The necessary apparatus was constructed and was basically of two parts: a stationary hopper with sides and bottom that could be easily adjusted and a movable bin and scale. The bin could be elevated so that the corn could be dumped into the hopper. It could then be lowered to receive the grain as it discharged through the various openings placed in

the stationary hopper. The flow rates of the corn were measured by opening the orifice for a certain timed increment and then weighing the discharged corn. An adjustable orifice was used so that an aperture of any size up to six inches square could be obtained. Round orifices could be inserted for testing.

The initial portion of the investigation to test the effect that a sloping bottom had on the flow rate of the corn indicated that the flow rate of the corn increased slightly as the slope of the bottom increased. This was true as long as funnel flow was present.

The remainder of the research endeavor involved the determination of equations describing the flow of grain through horizontal and vertical openings. Orifices were of different sizes and shapes. The corn used in the experiments was shelled, yellow dent with a moisture content ranging from 8.04% db to 12.72% db. The results indicated that the flow rate of corn through a horizontal orifice was faster than that through a vertical opening for a given size. From the data collected, equations were developed to describe the flow of the shelled corn through the circular and the rectangular orifices placed in the bin bottom and side. These equations had a maximum standard error of difference of $\pm$ 3.24% when compared to the experimental data.

The head did not have an effect on the flow rate of the corn. It was noted, however, that the flow rate of the corn decreased as the moisture content of the corn increased. Thus, the moisture content of the corn appeared to influence its flow rate.

No simple correlation could be found between the flow through the horizontal and vertical openings. The contention by Stahl that a certain ratio existed between the flow through the two openings could not be substantiated. In fact, the flow rate through the vertical openings was found to vary from 30% to 50% of that through the horizontal openings. The area of the orifice did not seem to directly determine the flow characteristics for a rectangular opening. This result was also in opposition to Stahl's findings.

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INTRODUCTION

Many engineers today are concerned with the design of adequate handling systems for grain. The thousands of tons of cereal crops that must be moved annually from fields through storage to final milling or feeding operations present a huge problem. This handling must be mechanized at all steps to make any system economical.

Every design is based on calculations using equations and parameters that describe the characteristics of various phases of a system. To date, little information is available which will describe the flow of grain through an opening. The few equations that have been developed are not satisfactory as they are either unreliable or not applicable for many problems.

Shelled corn is one of the most widely used grains in the world. This crop was chosen to be used in this initial phase of the research. Previous investigators had advanced opinions regarding the factors which influenced the flow rate of grain. The variable most often mentioned was the size and shape of the discharge orifice.

The objectives of the research then evolved into the following: first, a method of testing and suitable equipment was to be developed to test the flow characteristics of the corn; next, the effect of a sloping bottom in the bin was to be studied; and finally, orifices of different sizes and shape were to be used to test their effect on the flow rate of the corn. Equations were to be developed to describe the flow rate and were to be kept as simple as possible.

Once the data had been collected, comparisons could be made between the information previously available and that determined through the research. Such a process would aid in determining which lines of endeavor seem to have the most promise toward eventually solving the problem of how grain flows through orifices.

REVIEW OF LITERATURE

An increased interest has developed in the flow of semi-fluids in the past few years. More investigators are attempting to determine the primary factors affecting this type of flow. Such activity has produced a number of relationships describing these controlling factors. All provide a basis upon which to continue the investigation of semi-fluid flow. Grain is classified as a semi-fluid; thus, this prior information can be of considerable aid in determining relationships describing grain flow.

Semi-fluids have flow characteristics which are difficult to predict. Hinchley (1926) made the general comment that the peculiarity of such materials was that their flow from an orifice was proportional to the cube of the area of the orifice and was independent of the head.

As the following information will indicate, there is one characteristic upon which most investigators agree; that the flow is independent of the head. This, in itself, distinguishes the flow of the semi-fluid from that of the fluid.

Deming and Mehring (1929) studied the flow rate of a variety of materials through an inverted truncated cone orifice. Their results indicated that the flow rate varied with a power of the orifice diameter and was influenced by the size and apparent density of the particles, angle of repose of the material, and the cone angle. Their equation was the following:

$$Q = \frac{\mu_1}{D_o^{\,2.5}\, d}\left[0.201 + (0.392 + 2.58 \sin \tfrac{1}{2}\phi)\left(\frac{D_p}{D_o} + 0.130 - 0.161\mu_1\right)\right]$$

Where:

Q = flow rate (hrs/ton)

μ_1 = tan Θ; Θ = static angle of repose (degrees)

D_o = orifice diameter (ft)

D_p = particle diameter (ft)

ϕ = cone angle (degrees)

d = density of the material (lbs/ft^3)

This equation was found applicable for cone angles up to ($\pi - 2\alpha$), where α was the angle of repose. As noted, the relationship contains no reference to head. The equation was developed for various kinds of fertilizer, lead shot, glass beads, and some varieties of seed.

Ketchum (1929) published a book on the design of bins. It was the first comprehensive study devoted primarily to the design of grain storage structures. He and Willis Whited had worked together in 1911 and had shown that the flow of wheat was independent of head and varied as the cube of the orifice diameter. Using glass-sided bins, he advanced the theory that the flow of solids was actually somewhat intermittent. This was caused by the formation and collapse of domes or bridges formed by the mass of grain as it moved through the bin during discharge. With a centrally located bottom orifice, grain from the center of the bin discharged first while grain from the lowest part of the dome discharged last.

Work was also done by Ketchum on the pressures involved in storage bins. Much variation was apparent depending upon different conditions.

In general, the ratio of lateral to vertical pressures varied from 0.3 to 0.6.

Takahasi (1934) studied the flow rate of many types of sand, shot, vegetable seeds and other fine products. He developed an empirical relationship as follows:

$$t = \sqrt{\frac{b}{g}} \quad D_o^{2.5} \left[f(\mu_2) + a\left(\frac{D_p}{D_o}\right) \right]$$

Where:

t = time for a certain flow (min/ft3)

a,b = constants depending on the system of units used

g = gravitational acceleration (ft/sec^2)

$f(\mu_2)$ = Tan Θ ; Θ = kinetic angle of repose (degrees)

D_o = orifice diameter (inches)

D_p = particle diameter (inches)

No comparisons with experimental data were cited.

During the next decade, little work was done on the exploration of semi-fluid flow as it might relate to grain. Newton, et al. (1945), did publish a general equation for semi-fluid flow which had been developed for the flow of catalyst pellets. For pellets 0.1 to 0.2 inches in diameter, the maximum flow rate through a horizontal orifice in a flat-bottomed container was given as:

$$W = 8.50 \quad D^{2.96} \quad H^{0.04}$$

Where:

W = flow rate (lbs/min)

H = ft (head)

D = orifice diameter (inches)

Newton felt the equation was valid as long as the diameter of the orifice was greater than 6 times the particle diameter. Here again, the flow was seen to be very nearly proportional to the cube of the orifice diameter and little affected by head. It was the most simplified relationship developed for semi-fluid flow apparent at the time.

Stahl (1950) developed equations which applied exclusively to grain. His relationships appeared to be based upon experimental data published by Willis Whited in 1901 and subsequently printed in a technical bulletin published by the American Society of Agricultural Engineers (1948). The equations were for the flow of wheat through horizontal openings and were given as follows:

Circular Orifices: $Q = 0.1753\ D^3$

Rectangular Orifices: $Q = 0.2232\ W^2L$

Q = flow rate (bu/min)

D = diameter (inches)

W = width (inches)

L = length (inches) where $L \geq W$

The equation for flow through the rectangular orifice appears to be based solely on the area ratio between a circular and square opening where $D = W$. However, no experimental curves could be found to confirm the rectangular relationship such as had been used to develop the equation for flow through the round orifice. Stahl also stated that the flow through vertical openings was one-third the rate through horizontal apertures. This statement seemed somewhat dubious as it appeared to be based only on the premise that a ratio of approximately one-third exists between the ratio of lateral to vertical pressures in a bin.

During the 1950's the work involving the theoretical explanation of semi-fluid flow seemed to gain impetus. Jenike (1954), at the University of Utah, began formulating a theory regarding the effect that the compaction of the grain had on its flow characteristics. He advanced the following hypothesis: As a hopper opened, the material above the gate began to discharge. Stresses within the grain were immediately redistributed. An arching effect developed much like the dome effect described by Ketchum. If the pressures in the arch were higher than the strength the material had built up between the interlocking particles of grain during filling and after settling, the arch would break down and flow would continue. If the pressures in the arch were lower at some levels, the material would not flow and an obstruction called "doming" would result. Also, as flow occurred, more compact material would appear. Occasionally, only the core would empty out and an obstruction would develop. This phenomenon Jenike called "funnelling." Jenike (1960) is still continuing his work on this theory of compaction.

Barre (1958) further described the process of emptying. The column above the discharge opening leaves first and gradually widens as discharge continues. All grain moves through the core described by Jenike until the hopper has been emptied by literally turning itself inside out. This Barre designated as "funnel flow." In hoppers with very steep sides, the whole mass of grain may flow simultaneously including otherwise stationary material in the hopper. The flowing core increases in diameter until it includes all the grain in the hopper. This is referred to as "mass flow." This explanation by Barre regarding flow was also substantiated by Anderson and Alcock (1954).

Franklin and Johanson (1955) derived a flow equation for a circular orifice using glass beads, lead shot, cracking catalyst, and puffed rice with particle sizes of 0.03 to 0.2 inches, product densities of 7.3 to 676 pounds per cubic foot and orifice diameters of 0.236 to 2.28 inches. Their equation for a horizontal orifice was:

$$W = P_s \frac{D^{2.93}}{(6.288\mu + 23.16)(d + 1.889) - 44.9}$$

Where:

W = flow rate (lbs/min)

P_s = true density of the solid (lbs/ft^3)

μ = tan θ; θ = kinetic angle of friction (degrees)

D = diameter of a circular orifice (inches)

d = average screen size of particles (inches)

An accuracy of $\pm 7\%$ was found by Franklin and Johanson when the theoretical and experimental data was correlated. A relation was also derived for determining the flow rate of an inclined orifice:

$$W_\theta = W_o \frac{\cos\alpha + \cos\theta}{\cos\alpha + 1}$$

Where:

W_θ = discharge rate through an inclined orifice (lbs/min)

θ = inclination of the orifice to the horizontal (degrees)

α = kinetic angle of repose (degrees)

The following factors were found by Franklin to influence the flow rate:

1) A ratio of orifice diameter to particle diameter less than 5.

2) A ratio less than 6 to 3 between the diameter of the cylindrical storage bin and the particle diameter.

3) A grain depth in the cylindrical bin less than one bin diameter.

Brown and Richards (1959) made a study of the flow of glass beads, rounded yellow sand, and sharp grey sand through orifices and developed the following equations for these materials having a specific gravity of approximately 2.5:

Rectangular Orifices: $Q = 2.72\ A\ H^{\frac{1}{2}} \varphi$

Circular Orifices: $Q = 2.24\ D^{2.5} \varphi$

Where:

Q = flow rate (grams/sec)

A = area of orifice (cm^2)

H = perimetral diameter $\left(\frac{4 \times \text{area of orifice}}{\text{perimeter}}\right)$cm

$\varphi = \frac{V}{gH^{\frac{1}{2}}}$ (dimensionless unit)

V = velocity of flow (cm/sec) $= \frac{Q}{\rho A}$

g = acceleration of gravity (cm/sec)

ρ = density of material (gr/cc)

D = diameter of orifice (cm)

If φ was taken as 0.3, the accuracy for the equations was about ±50%. The equations were limited to dry materials of a narrow range of sizes and to values of D/P (D = diameter of a circular orifice in cm, P = mean particle diameter in cm.) of the order of 20 to 30.

Contrary to Jenike's theory, their experiments indicated that the flow rate was independent of the tightness of the initial packing. Flow from narrow vessels was influenced not only by friction at the walls but by the interlocking of particles owing to the proximity of the walls. Other comments made were:

1) Fine particles discharged more rapidly than coarse particles.

2) Spherical particles flowed more quickly than angular particles (larger spheres discharged at about the same rate as the smaller angular particles).

3) For orifices of the same area, a rectangular orifice discharged at about the same rate as an elliptical orifice but both were appreciably slower than the flow from a circular orifice.

Fowler and Glastonbury (1959) used dimensional analysis to produce the following equation for the flow of sand, rape seed, rice, wheat, and sugar through orifices:

$$\frac{W}{\rho_B A \sqrt{2gDh}} = 0.236 \left(\frac{Dh}{\lambda ds} \right)^{0.185}$$

Where:

λ = shape factor

W = weight discharged per unit time (lbs/sec)

ρ_B = bulk density of packing (lbs/ft^3)

g = gravitational constant (ft/sec^2)

Dh = hydraulic or perimetral diameter (inches)

$= 4 \times \frac{\text{area}}{\text{perimeter of orifice}}$

d_s = mean particle size (inches)

Their accuracy was ±10%. They further found the following to be true:

1) The influence of the head was small and could be ignored.

2) There was a slight variation of flow rate with bulk density.

3) Bridging and erratic flow occurred with ratios of D_h/d_s approaching 4 to 6.

4) In large vessels, the influence of the container dimensions was negligible (ratio of diameter of container to spherical diameter of particles).

Dimensional analysis was used again by Rose and Tanaka (1959). Their equation was very complicated and will only be mentioned as existing here. They also produced a curve which could be used to compute discharge rates for openings other than those that were circular. Some soil mechanics theory was utilized in their analysis. Their results indicated that the rate of discharge was independent of the true value of the coefficient of friction of the material but dependent upon the cohesive strength of the materials and proportional to $D^{2.5}$ (D = the orifice diameter). Flow was also independent of the nature of material, provided the particle shapes were substantially similar. Their experiments involved steel balls, silica sand, steel disks, and some fertilizers. Bridging was noted when the minimum dimension of the opening was approximately equal to 3 particle diameters.

Further work on the formation of the flow lines was advanced by O'Callaghan (1960). He contended that the rupture line between the stationary and moving grain in a bin could be represented by a logarithmic spiral. Inclining the hopper sides at the approach to the discharge orifice did not alter the general shape of the rupture line from that which developed in a flat-bottomed container.

Welschof (1961) also developed some equations for the flow of oats, wheat, granular fertilizer, and sand through circular and rectangular orifices. The accuracy of these relationships was dependent on whether the physical properties of the material being measured remained constant. Results were again good except when the ratio of the orifice diameter to the grain diameter became small. In his equation, Welschof used the coefficient of discharge, angle of internal friction, bulk density, and a constant which took into consideration the constriction of the orifice caused by grainstuff passing through. He advanced a set of equations, each of which was dependent upon the shape of the opening. These equations were hard to use, as they required the measurement of physical properties which, in themselves, would be difficult to obtain.

Although the literature review produced some equations for semi-fluid flow, there was considerable discrepancy between the relationships. Many of the references indicated that the flow rate was a function of the diameter of the orifice. Most equations were considered valid only above a certain minimum ratio of D/d_k (D = orifice diameter, d_k = particle diameter).

Basically, the flow equations determined by various investigators are of the form $Q = K\ D^n$ where K is dependent upon the physical properties of the material being tested. The values of n and the limiting values of D/d_k as suggested by these researchers are summarized in Table I. The materials used in each investigator's work are also listed. As noted, no general agreement exists between investigators regarding the effect of the orifice on the flow rate or the limiting ratio of D/d_k.

TABLE I

Summary of previous investigations on granular flow.

Investigator		Materials	n (Power of D)	D/d_k
Deming, Mehring	(1929)	Fertilizer, lead shot, glass beads, seed	2.50	-
Takahasi	(1934)	Sand, shot, and vegetable seeds	2.50	-
Newton, et al.	(1945)	Catalyst pellets	2.96	6
Rausch	(1949)	------------------------	2.80	-
Stahl	(1950)	Wheat	3.00	-
Franklin, Johanson	(1955)	Glass beads, lead shot, cracking catalyst, puffed rice	2.93	5
Brown, Richards	(1959)	Glass beads, sand	2.50	-
Rose, Tanaka	(1959)	Steel balls, sand, fertilizer	2.50	3
Fowler, Glastonbury	(1959)	Sand, rape seed, rice wheat, sugar	----	4-6
Welschof	(1961)	Oats, wheat, fertilizer, sand	----	4

APPARATUS

In constructing the equipment to be used in the experimental studies on grain flow, the primary concern was flexability. Different variables were to be used in the testing, primarily different opening sizes. The apparatus would be most effective by making it as portable and adjustable as possible. With these conditions in mind the equipment in Figure 1 was designed. It was constructed principally for measuring the flow rates of grain through orifices.

The apparatus was composed of two major parts: the stationary hopper in which the grain was retained until the testing was accomplished and a movable bin. The hopper was fastened together with bolts to facilitate making changes. The bottom of the hopper was held in place by clamps and had a hinged door attached to one end. It was possible to elevate one end of this bottom board to provide a certain slope. The hopper was composed of $\frac{1}{2}$-inch plywood; its frame was made of 2"x 4"stock. The hopper had the following interior dimensions: 48 inches high, 12 inches wide, and 24 inches deep.

To facilitate taking weight measurements, the apparatus was constructed so that the hopper was 5 feet off the floor. This allowed a set of scales to stand beneath the hopper. Balance beam Toledo scales mounted on a movable platform and with a capacity of 200 pounds were used in the tests. With these scales, it was possible to read weights to the nearest 1/8 pound.

The height of the hopper also allowed the addition of the movable bin. Arms were attached to the main frame of the equipment and pivoted on the frame at a point 94 inches above the floor. The arms were connected to a winch powered by a $\frac{1}{2}$ horsepower Master gearhead electric motor. This provided a mechanical means of elevating the bin above the hopper as shown in Figure 2, dumping the corn, and then lowering the bin back to the scales. A double-pole, triple throw switch was used to control the motor for raising and lowering the bin.

The hopper held approximately 200 pounds of corn, while the bin held just less than 200 pounds.

Baffles were placed in the corners of the front of the movable bin so that more complete emptying of the bin would occur. A latch on the bin door was operated from the floor by a cord. The moisture content of the corn used in the tests was determined initially by using a Steinlite capacitance type moisture meter. Later, an air oven was used for more accurate moisture content measurements. The orifice was opened and closed manually. All runs were measured with a stop watch.

Fig. 1 - An overall view of the apparatus showing the stationary hopper above the movable bin.

Fig. 2 - The movable bin in position to dump the corn into the hopper.

METHOD OF PROCEDURE

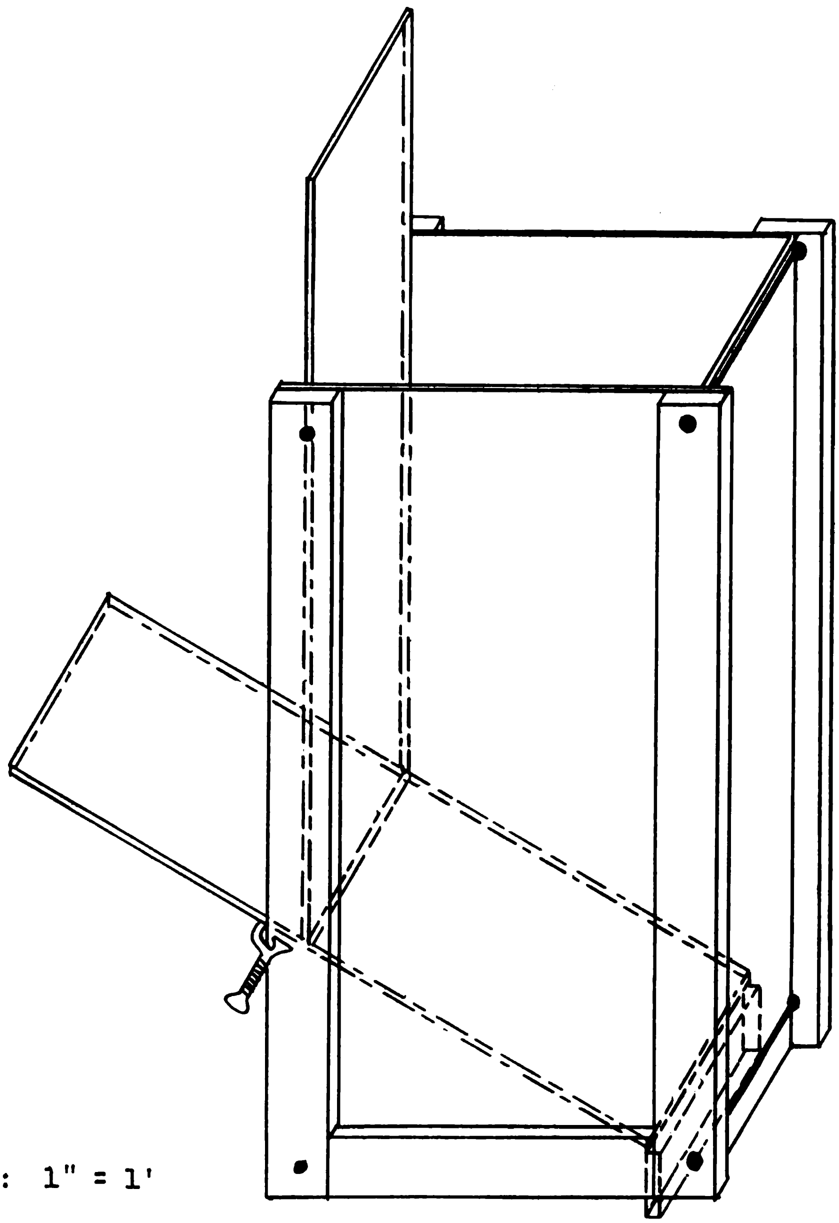
Corn used in the testing was secured from the University farms. The corn had been harvested by a picker-sheller and was a yellow dent variety of moisture content ranging from 8.4% db (dry basis) to 12.72% db. The larger pieces of cob and other foreign material were removed by screening. The corn was quite free of dirt and other foreign materials. Complete cleaning was not accomplished, as it was felt that this would eliminate the essence of actual grain conditions from the testing.

Approximately 100 pounds of the corn was used at a time. It was placed in the moveable bin and elevated above the hopper. The corn was released at this point and allowed to flow into the hopper. The bin was lowered back into position on the scales so that weight measurements could be taken. The elevating arms were disconnected from the bin so that the bin could be weighed on the scales.

Sloping Bottom

The first tests were to find what effects a sloping bottom had on the flow rate of the grain. The bottom of the stationary hopper was not nailed in place for these measurements. It was secured in position by first tightening the bolts on one end of the frame bottom. Different angles were then achieved by moving one end of the bottom up or down and holding it in place with clamps. Figure 3 shows a sketch of the bin with a 30° slope to the bottom.

While testing, the level of the discharge end in the bottom of the hopper was kept in the same position relative to the side of the hopper. While changing the slope of the bottom, this end was not allowed to move up or down. A carpenter's level, graduated so that it would indicate



Scale: 1" = 1'

Fig. 3 - A view of the stationary hopper with the bottom at a 30° angle with the horizontal.

degrees from the horizontal, was used each time the position or slope of the bottom was changed. The level was set at the prescribed angle and the bottom board moved until the bubble on the level was centered. Flow rates were taken at 20°, 30°, 40°, and 52.5°.

The orifice was opened or closed by rapidly dropping or lifting the swinging door. The corn was allowed to empty into the bin and the time for discharge noted for each 10-pound interval up to 75 pounds. This data of weight versus time was plotted for various bottom slopes. All measurements were made with the orifice running full.

Horizontal Openings (openings in the bottom of the bin)

Once the initial tests involving the sloping bottoms were completed, the bottom board was removed. A hole, slightly more than 6 inches square, was cut in the center of the board. A frame was constructed of ¼-inch plywood and bolted over the opening. Four slats made of the same material were placed between the frame and the bottom. Two slats were used to regulate the width and two were to adjust the length. Wing nuts were used on the bolts fastening the frame to the bottom to facilitate loosening the frame to change the aperture size. Two cross members of plywood were placed on the frame to hold the sliding gate. A sketch of the adjustable frame is shown in Figure 4. The gate was constructed of ¼-inch plywood and was moved manually. The bottom board was placed back into position in the hopper and a carpenter's level used to assure that the bottom was horizontal.

The following method was used to find the flow rate for the horizontal openings. The elevation of the bin, filling of the hopper, and lowering of the bin were accomplished as before. The bin was placed on the

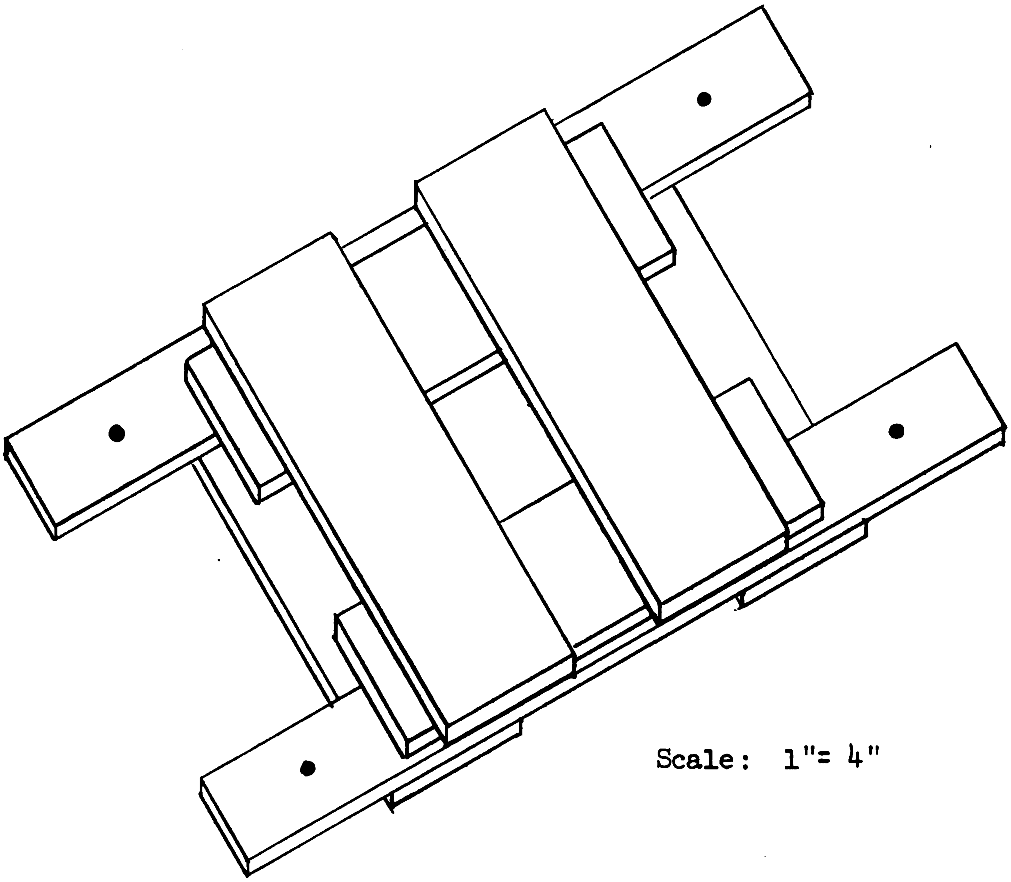


Fig. 4 - A view of the frame showing the four adjustable slats forming a 3-inch long and 1-inch wide orifice.

scales and pushed under the hopper. The bin was then weighed. Figure 5 shows the equipment as it appeared prior to taking tests on flow of corn through a horizontal opening.

The sliding gate was opened for a certain increment of time and the weight of the discharged corn recorded. The time intervals varied from 2½ seconds to 60 seconds. The runs were conducted so that flows were measured at various corn depths. An average of 8 runs was made for each opening size. The average weight recorded in these runs was used in the calculations.

At some opening sizes, it was noted that a very small error in time determination would result in a considerable weight error. It was decided to take these runs for longer time periods to reduce this possible error in time measurement. This procedure materially increased the apparent accuracy of the data.

Runs were made on round, square and rectangular openings. The round orifices were constructed from ¼-inch plywood and were of 8 diameter sizes. The 1½, 2, 2½, 3, and 3½ inch openings were made by using a hole saw. The remaining 4, 4½, and 5 inch diameter holes were cut using an expansion bit on an upright power drill.

The square and rectangular orifices were produced by adjusting the four slats in the frame and then clamping them in place. Runs were made on square holes in sizes differing in ½-inch increments and ranging from 1½ inches up to 5 inches on a side.

Tests were made on the rectangular openings for lengths of 6, 5, 4, and 3 inches. The width was changed in ½-inch increments from a value equal to the length down to 1 inch. The width was always equal to or less than the length. Measurements of the dimensions were made twice to assure accuracy.

Vertical Openings (openings in the side of the bin)

In testing the vertical openings, a 6-inch square was cut in the end of one side board. This made one side of the opening flush with the bottom of the hopper. The same sliding gate arrangement was bolted to the side with a slight modification made to the lower part of the frame to allow for tightening the gate. This outlet side was fixed in position using a carpenter's level to assure a vertical orientation.

Flow rates for vertical openings were determined using the same test procedure and orifice sizes as were described for the horizontal openings. The one exception was that the scales were not moved beneath the hopper, but remained forward under the side opening. Figure 6 shows the equipment prior to taking tests on the flow of corn through a vertical opening.

Fig. 5 - The equipment in position prior to taking measurements on the flow of corn through a horizontal opening.

Fig. 6 - The equipment in position prior to taking measurements on the flow of corn through a vertical opening.

ANALYSIS OF DATA

Results of Tests with Sloping Bin Bottoms

Tests were run to determine the effect a sloping bin bottom would have on an orifice of a given size. Shelled, yellow dent corn of moisture content 11.5% db (dry basis) to 9.2% db was used. The flow measurements were made using a 1" x 12" orifice with a bottom slope of 20°, 30°, 40°, and 52.5° from the horizontal. All tests were made with the entire bottom sloping toward an opening in the bottom side of the bin.

These tests indicated that the flow rate of the corn increased slightly with an increase in the slope of the bottom. This was true as long as the flow was funnel flow. At 52.5° the corn movement was mass flow and, as shown in Figure 7, the rate was slightly below the flow rate for 40°.

An additional series of tests were made to determine the effect of head on flow rate. The same size orifice as before and a bottom slope of 30° were used. The corn was first dumped into the stationary hopper. It was raked back manually on the top so that the depth of grain above the opening was a minimum. The corn was then in funnel flow position. The orifice was opened and flow measurements taken at 10-pound intervals up to a total of 70 pounds. Next, the corn was dumped into the stationary hopper and allowed to remain as it fell. The depth of grain above the opening was greater in the second case than in the first. The orifice was opened and flow measurements were taken at 10-pound intervals up to a total of 70 pounds. An average of 3 tests was taken for each 10-pound interval in both cases.

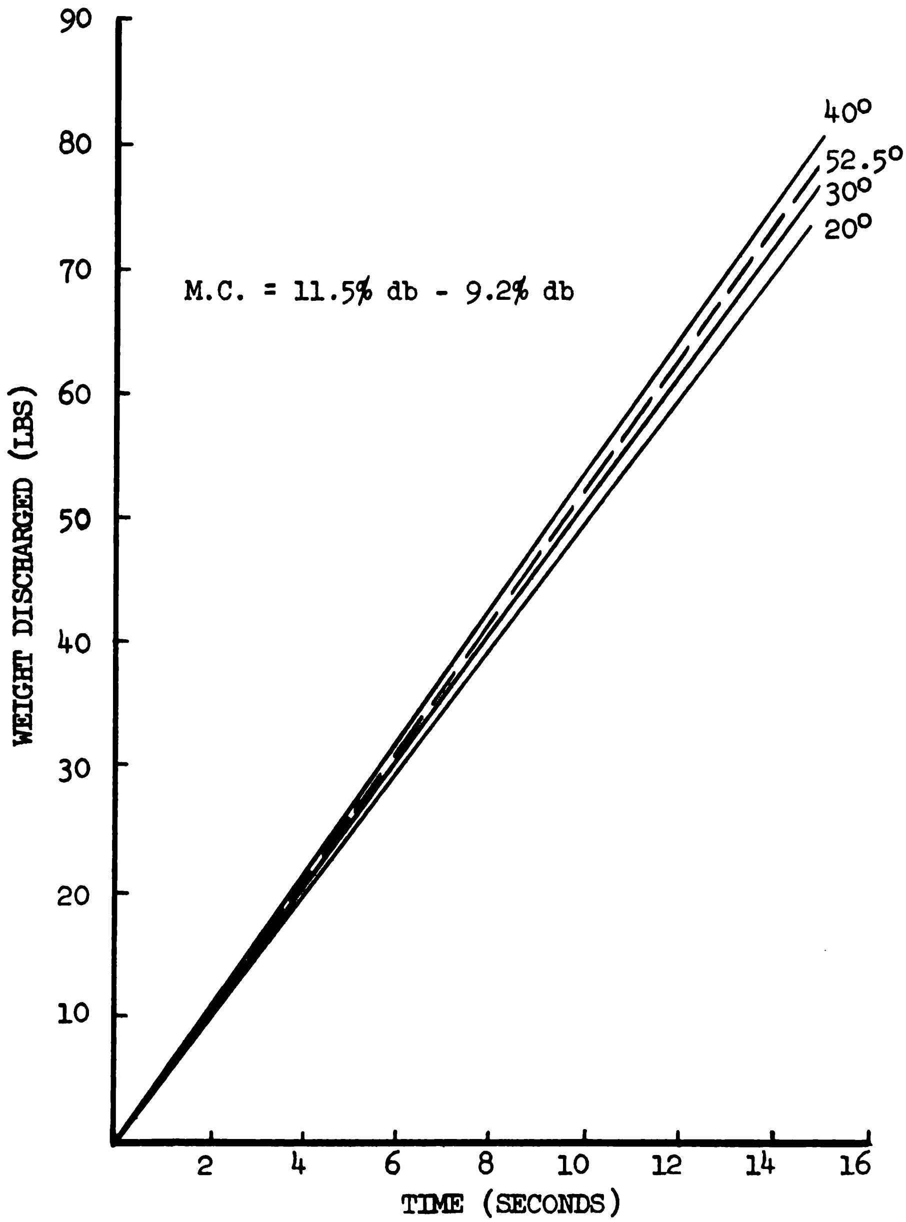


Fig. 7 - Weight discharged versus time for corn flowing through an orifice placed in a bin with a sloping bottom.

Upon comparing the flow measurements for the two situations, no difference was found between the emptying rates. These results substantiate the finding of other researchers that head has no effect on the flow rate of grain.

From the results of this investigation of bins with sloping bottoms, it appeared that the flow rate of corn was slightly larger at greater bottom slopes. Thus, the slope of the bottom of a bin had a slight effect on the discharge rate of corn.

Results of Tests with Horizontal Openings in Bins

The high flow rates of the corn through the horizontal openings affected the accuracy and consistency of the test data. Small errors in either the measurement of time or size of opening caused a large variation in the determination of the corn flow rate through the orifice.

All of the available information on work comparable to the tests conducted indicated that a logarithmic relationship would describe the flow of a semi-fluid through an orifice. The flow also was found to be proportional to some power of the diameter. Since grain is a semi-fluid, these results should have some validity.

The raw data used for deriving the equations for flow through a horizontal opening are found in Tables VII-XI, Appendix A.

Round and Square Orifices

Runs were first made on square and round openings. The data were then plotted on rectangular coordinate paper as shown in Figure 8. Previous investigators had found that a logarithmic relationship would describe the flow of grain through an orifice. Therefore, the data were

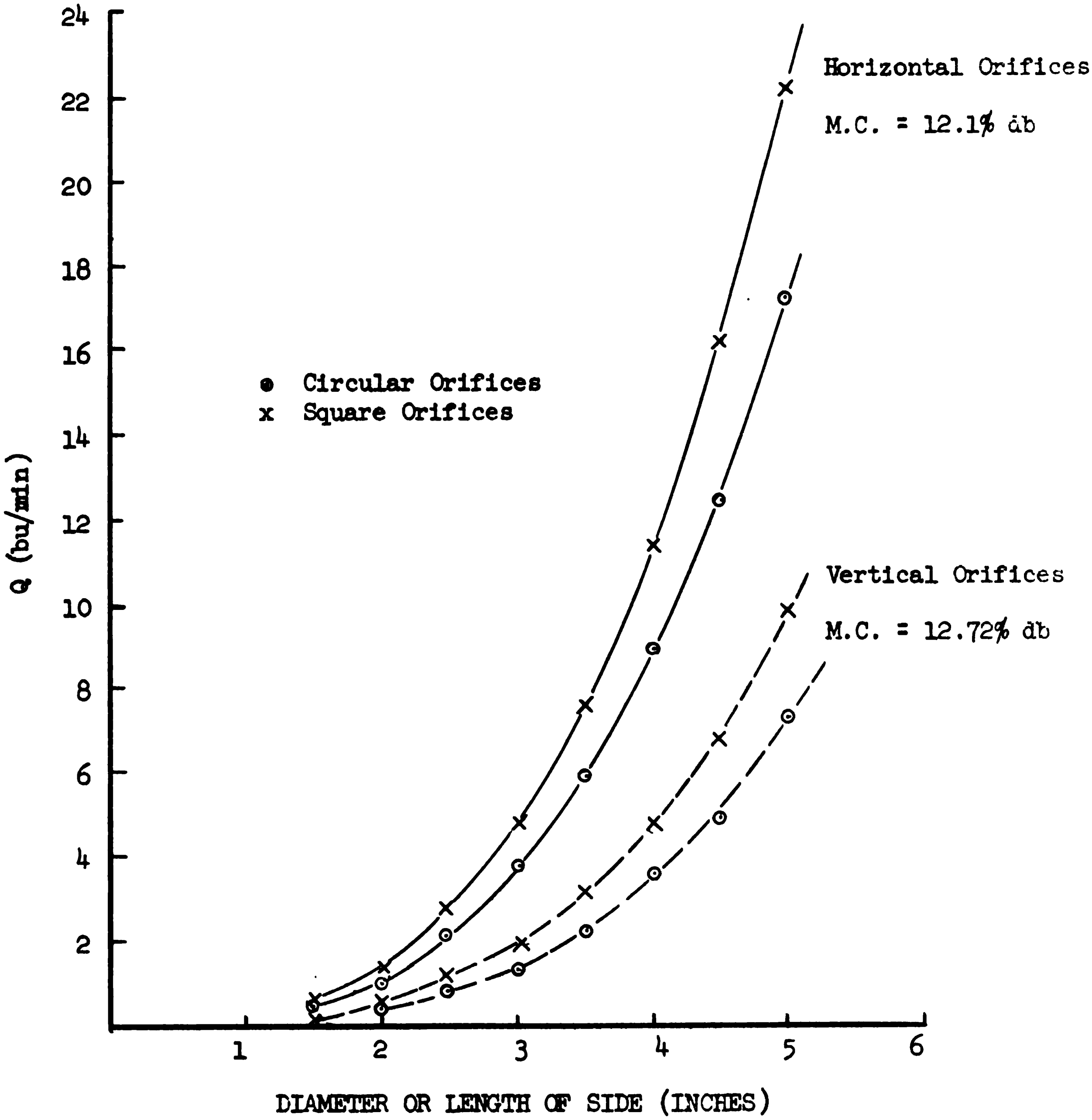


Fig. 8 - Flow rate of corn through a round and a square orifice positioned horizontally and vertically in a bin.

next plotted on log-log paper and a straight line resulted, as shown in Figure 9.

The equations, therefore, were of the following form:

For a square orifice:

$$Q = K W^n$$

Where:

Q = flow rate of corn (bu/min)

W = width of orifice (inches)

K, n = constants

For a round orifice:

$$Q = K D^n$$

Where:

D = diameter of orifice (inches)

The unknowns K and n were determined by simultaneously solving two equations. The resulting equation for the flow of shelled, yellow dent corn of moisture content 8.4% db through a square orifice in the bottom of a bin is:

$$Q = 0.1755\, W^{3.01}$$

The flow of the same corn through a circular orifice was found similarly to be:

$$Q = 0.1196\, D^{3.10}$$

A comparison of the actual data to that determined using the derived flow equation was made for the round orifice. As shown in Figure 10,

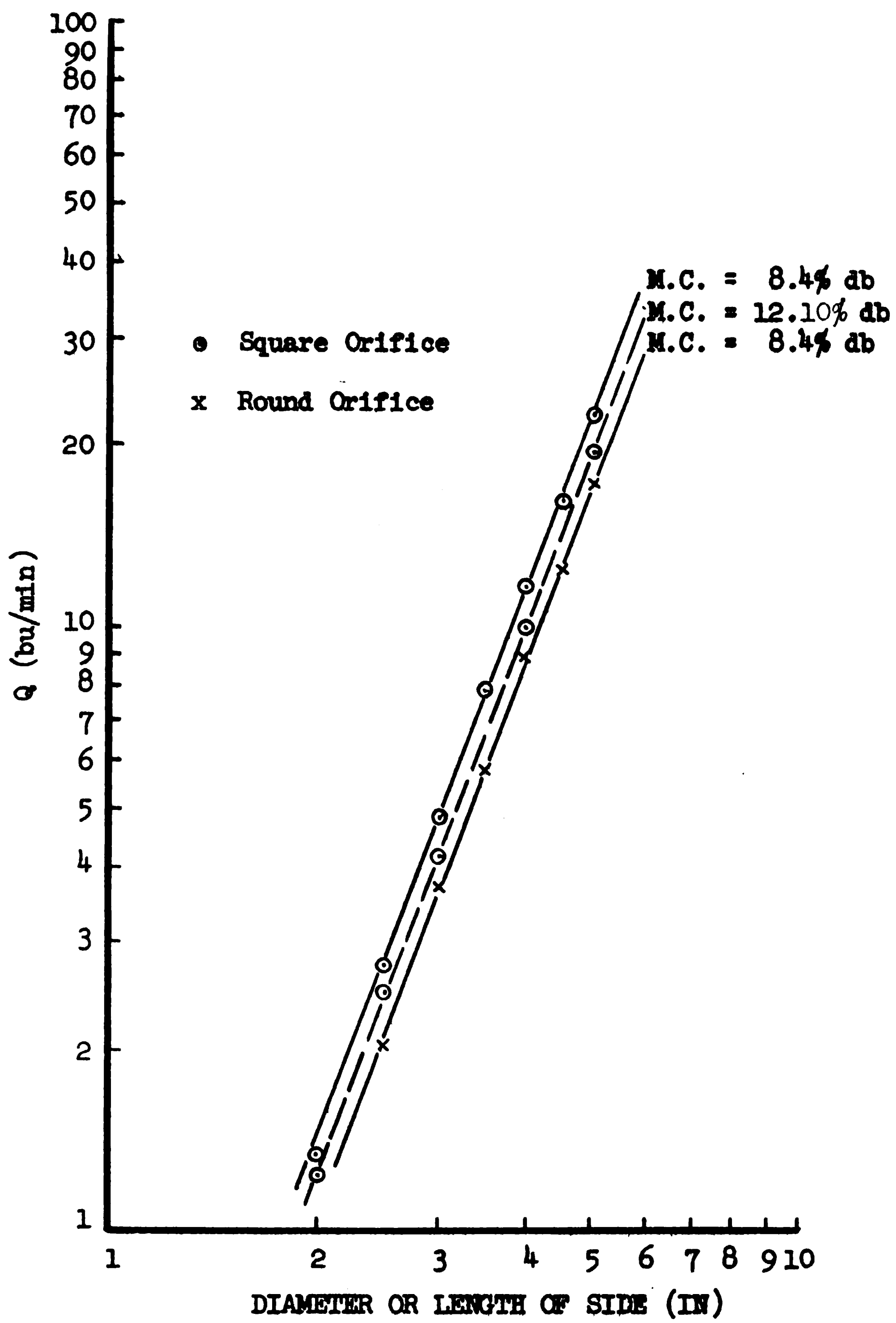


Fig. 9 - Logarithmic plot of the flow of corn through a square and round orifice positioned horizontally in a bin.

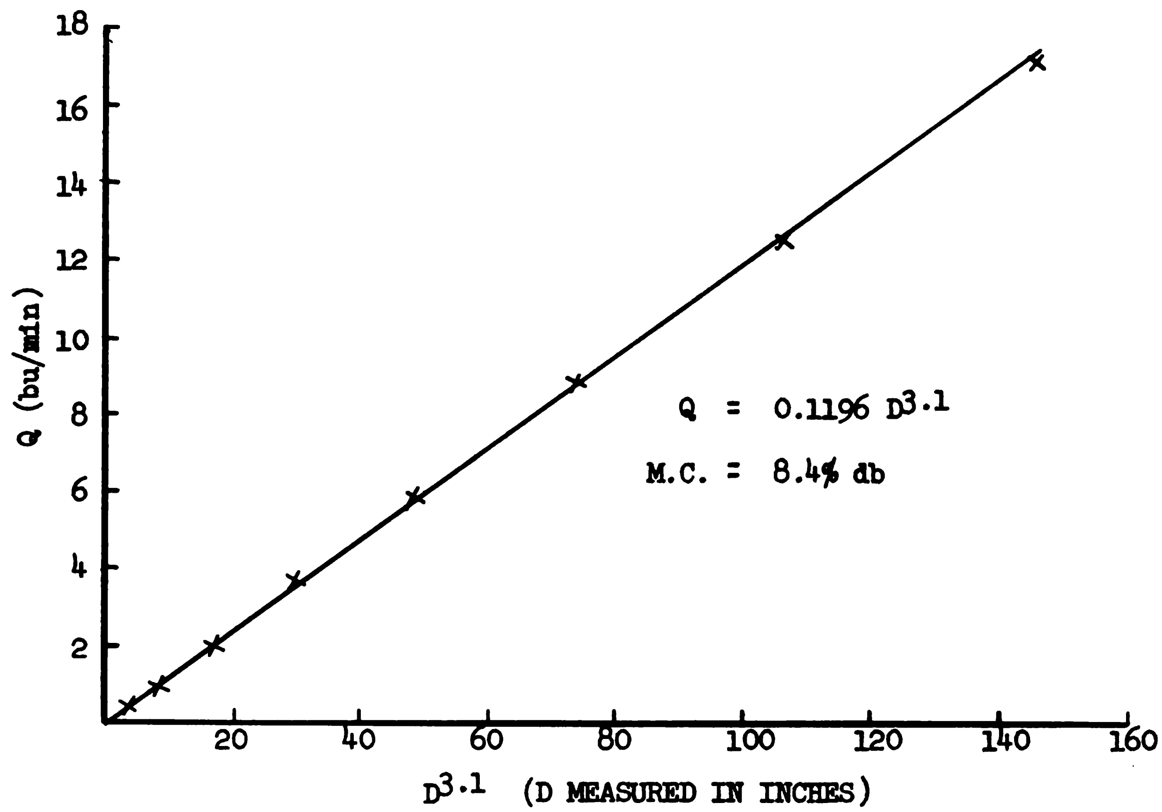


Fig. 10 - Actual flow rate of corn compared to the flow equation for a round orifice positioned horizontally in a bin.

the equation proved quite accurate. Table II shows that the maximum difference between the equation and the data is 4.06% with a standard deviation of difference of ± 2.75%. The equation for the square opening is discussed as part of the development of an equation for the flow of corn through a rectangular orifice.

Rectangular Orifices

The relationship describing the flow through a rectangular orifice was determined by making runs using orifices of lengths 6, 5, 4, and 3 inches. The width of the orifice was varied by ½-inch increments from 1 inch up to a value equal to the length. A plot of these data on rectangular coordinate paper indicated a possible logarithmic relationship. However, in presenting the information on log-log paper by plotting the flow rate versus the width, slightly curved lines resulted.

Several methods were attempted in order to describe the curved lines mathematically. All lacked reasonable accuracy and it appeared that a new approach to a solution was required. It was observed that for rectangular openings with a constant ratio of W/L, the flow rate varied approximately as W^3. According to this relationship, the flow rate then varied as $A^{1.5}$ where A is the area of the orifice (in^2). Therefore, the data were plotted on a graph using $Q/A^{1.5}$ as the ordinate and W/L as the abscissa. The first representation of the data in this fashion gave very inconsistent results. Runs were made on the vertical openings to provide some clue regarding the rectangular relationship. (Vertical openings will be discussed later.) The data for the vertical openings produced a definite family of curves with each curve representing a different orifice length. Since the initial tests on the rectangular orifices positioned

TABLE II

A comparison of the actual flow data to that determined using the derived flow equation for a round orifice positioned horizontally.

Diameter (inches)	Q (From Equation) (bu/min)	Q (Actual) (bu/min)	Per Cent Difference
1.5	0.42	0.44	+ 3.22
2.0	1.03	0.99	- 4.06
2.5	2.06	2.02	- 1.98
3.0	3.58	3.73	+ 4.02
3.5	5.80	5.90	+ 1.70
4.0	8.78	8.95	+ 1.90
4.5	12.68	12.40	- 2.26
5.0	17.45	17.20	- 1.45

horizontally did not give similar results, the tests were re-run. Also, the flow rates were high during the first tests and the timed intervals had been necessarily small, many being only 2.5 seconds in duration. An error of only 0.2 seconds here would result in an 8% error in time measurement. To alleviate this discrepancy during the second tests, some runs were taken over longer periods of time. This new technique greatly improved the accuracy of the data.

In plotting the second set of data (Table XI, Appendix A) using $Q/A^{1.5}$ versus W/L, a smooth family of curves resulted. This indicated that both the width and the length had some non-linear effect upon the flow rate.

As the square is simply a special case of the rectangle, it was evident that any equation describing the flow through a rectangular orifice would also have to be valid for the square orifice. The square aperture had already been described by a logarithmic relationship. Thus, the equation for the rectangular orifices seemed likely to be of the form:

$$Q = K W^m L^n$$

Where:

Q = flow rate of corn (bu/min)

W = width of the orifice (inches)

L = length of the orifice (inches)

K,m,n = constants

$W \leq L$

Furthermore, when the width equalled the length, as in the square, m+n would have to be equal to the coefficient of W as determined for the square orifice. This was necessary, as the equation for rectangular orifices must be valid for the square orifices.

Since a new set of data had been obtained from corn of different moisture content, a new relationship was first obtained for the square orifices. The data were plotted as before on log-log paper, as shown in Figure 9. Points were selected and the necessary mathematics carried out to solve for K, m and n. The resulting equations for shelled, yellow dent corn of moisture content 12.1% db flowing through a square orifice placed in the bottom of a bin follows:

$$Q = 0.1541\ W^{3.01}$$

Three sets of points were next chosen from the data plotted in Figure 11. The sets of data selected were those falling on the curves representing various orifice lengths. The points yielded three equations in three unknowns. These equations were solved simultaneously and the following relationship resulted for rectangular orifices in the bottom of a bin for shelled, yellow dent corn of moisture content 12.1% db:

$$Q = 0.1531\ W^{1.62}\ L^{1.40}$$

The constant K was found to be nearly equal to that for the square orifices and $m + n$ was approximately equal to 3.01. Thus, this relationship can be used to describe the flow through the square openings.

A comparison of the actual data to that determined using the derived flow equation was made for the rectangular orifices. As shown in Figure 12, the equation is quite accurate. Table III shows that the maximum difference between the equation and the data is 4.39% with a standard deviation of difference of ± 1.83% for all widths greater than 1.5 inches.

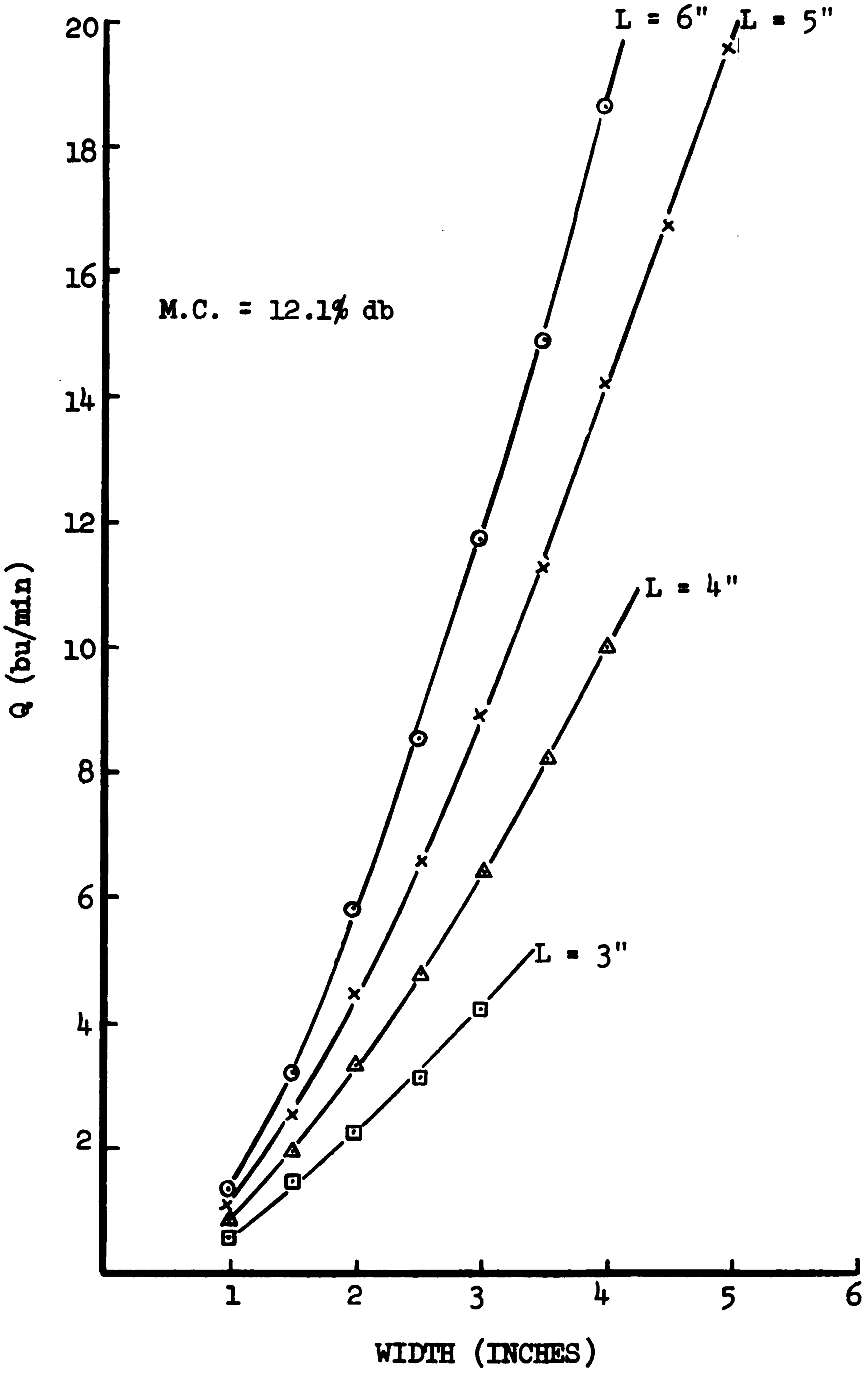


Fig. 11 - Flow of corn through a rectangular orifice of a given length and varying width with the orifice positioned horizontally in a bin.

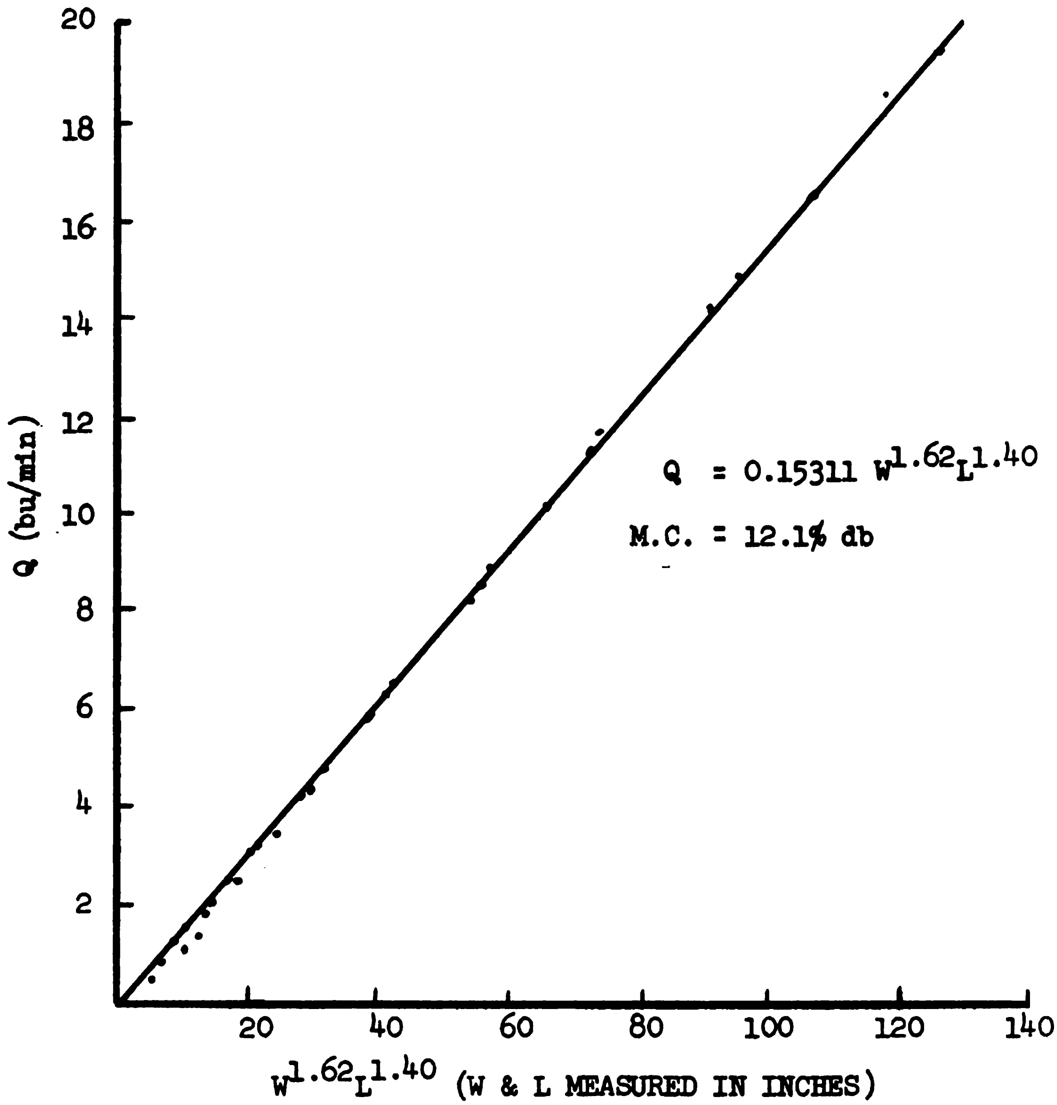


Fig. 12 - Actual flow rate of corn compared to the flow equation for a rectangular orifice positioned horizontally in a bin.

TABLE III

A comparison of the actual flow data to that determined using the derived flow equation for a rectangular orifice positioned horizontally.

Size L x W (inches)	Q (Flow Equation) (bu/min)	Q (Actual) (bu/min)	Per Cent Difference
6 x 1.0	1.88	1.31	- 43.30
1.5	3.64	3.22	+ 12.30
2.0	5.80	5.74	- 1.05
2.5	8.34	8.56	+ 2.57
3.0	11.20	11.72	+ 4.39
3.5	14.34	14.84	+ 3.36
4.0	17.81	18.61	+ 4.29
5 x 1.0	1.46	1.01	- 45.00
1.5	2.82	2.46	- 14.62
2.0	4.48	4.43	- 1.13
2.5	6.45	6.61	+ 2.42
3.0	8.66	8.87	+ 2.36
3.5	11.11	11.22	+ 0.98
4.0	13.80	14.14	+ 2.40
4.5	16.70	16.75	+ 0.30
5.0	19.80	19.61	- 0.97
4 x 1.0	1.06	0.80	- 32.50
1.5	2.06	1.94	- 6.20
2.0	3.28	3.23	- 1.55
2.5	4.72	4.76	+ 0.84
3.0	6.35	6.37	+ 0.31
3.5	8.11	8.14	+ 0.37
4.0	10.08	10.00	- 0.80
3 x 1.0	0.71	0.58	- 21.60
1.5	1.38	1.40	+ 1.43
2.0	2.19	2.18	- 0.46
2.5	3.17	3.13	- 1.27
3.0	4.22	4.20	- 0.48

The accuracy found using the computed equations describing the flow of corn through round and rectangular orifices seems very good, considering the equipment used. Also, the runs were made over a period of days. During this time, temperatures fluctuated, the moisture content of the corn varied, and the relative humidity of the ambient air changed. All these factors could conceivably have an effect on the flow rate of the corn.

An indication of the effect the moisture content of the corn can have is seen in the comparison of the two equations developed for the square orifices. They were as follows:

$$Q = 0.1755\ W^{3.01} \qquad M.C. = 8.4\%\ db$$

$$Q = 0.1541\ W^{3.01} \qquad M.C. = 12.1\%\ db$$

The moisture contents were computed using the air-oven procedure of drying. Figure 9 indicates that a change in moisture content will not affect the slope of the flow equation, but will simply displace the relationship by changing the constant. In this case, an increase in the moisture content of the corn of approximately 4% decreased the flow rate by 12%.

In another comparison of the effect a change in moisture content has on flow, runs were made using an opening of 3/4" x 12" and a bottom slope of 20°. The moisture contents of the corn samples tested were 7.07% and 9.63% as measured on a Steinlite moisture meter. Here, the decrease in flow rate was 8.7% for an increase in corn moisture content of approximately 2.56%. These two comparisons indicated an approximate decrease in flow rate of 3% for every 1% increase in moisture content. It could not be definitely concluded that a linear relationship existed between the decrease in product moisture content and the corresponding increase in flow rate, however.

The achieved accuracy was quoted for widths greater than 1.5 inches. An accuracy of 1.43% was obtained for an opening 3" x 1.5". This might indicate that a certain ratio of W/L is actually a more proper limiting value for the derived equations.

Results of Tests with Vertical Openings in Bins

The same opening sizes and shapes as used for the horizontal openings were used in testing orifices placed in the side of the bin. The length of the orifice was made parallel to the bottom of the hopper. The hopper bottom was flat throughout the testing.

Little information was available regarding the flow of corn through vertical openings. Stahl (1950) did state that the flow of wheat through a vertical opening was 1/3 the flow through a horizontal opening. Other than this one individual, few investigators mentioned vertical orifices.

Square and Round Orifices

Runs were made initially on the square and round orifices. The resulting data are shown on rectangular coordinate paper in Figure 8 comparing the flow rate versus the diameter and the length of a side. As in the case of the horizontal openings, a logarithmic relationship was assumed to describe the data. Upon plotting the information on log-log paper, a straight line resulted for both shapes as shown in Figure 13. The equations were of the following form:

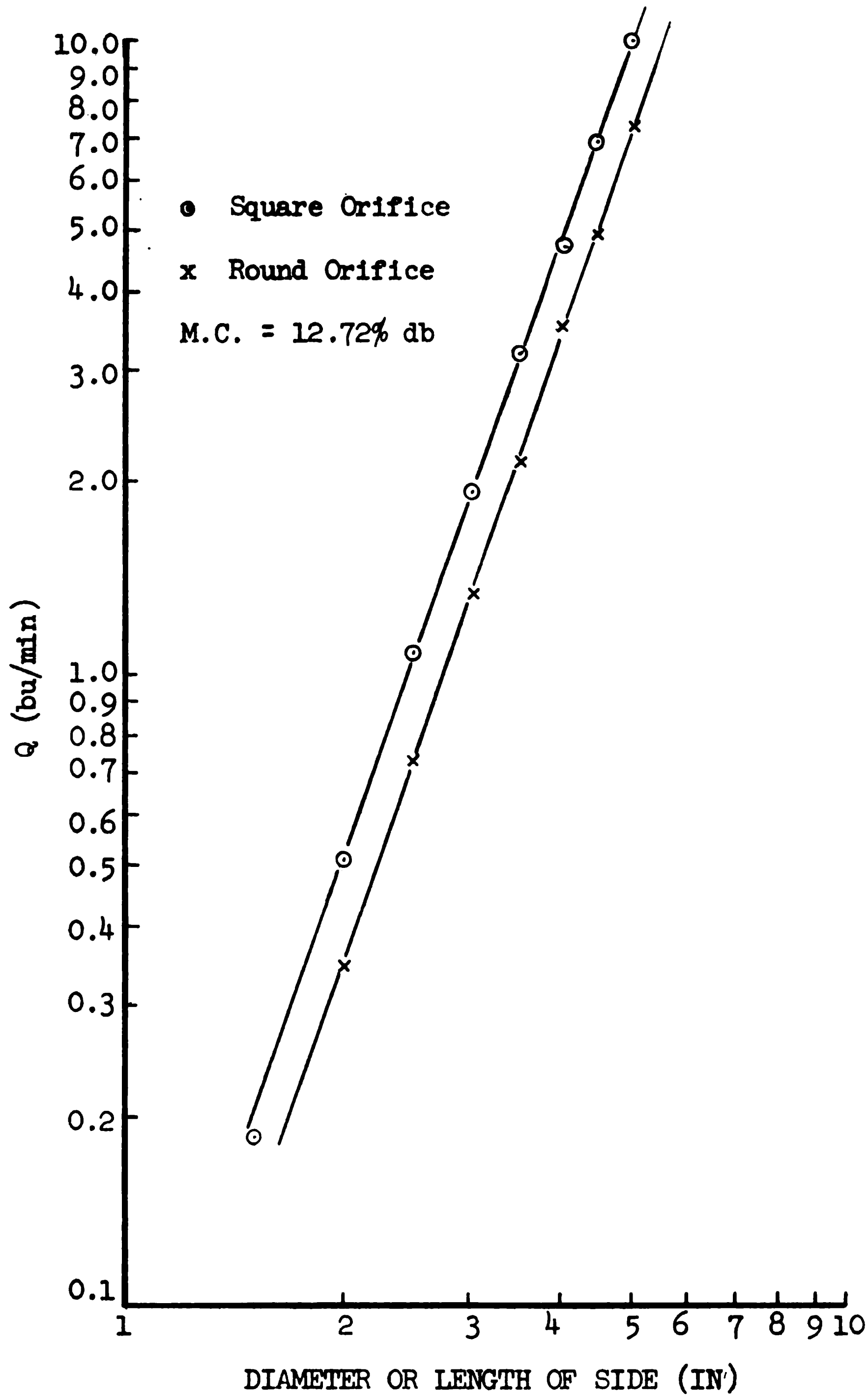


Fig. 13 - Logarithmic plot of the flow of corn through a round and square orifice positioned vertically in a bin.

For a square orifice:

$$Q = K W^n$$

Where:

Q = flow rate of corn (bu/min)

W = width of orifice (inches)

K, n = constants

For a circular orifice:

$$Q = K D^n$$

Where:

D = diameter of orifice (inches)

Two sets of points were chosen from each line and the equations solved simultaneously for K and n. The following equation resulted for the flow of shelled, yellow dent corn of moisture content 12.72% db through a square orifice placed vertically in a bin:

$$Q = 0.0523 \; W^{3.26}$$

The equation developed for the same corn flowing through a circular orifice positioned vertically was:

$$Q = 0.0351 \; D^{3.30}$$

A comparison was made between the actual flow rates of the corn and the rates determined using the derived equation for flow through a circular orifice. As shown in Figure 14, the relationship was found quite valid. Table IV shows that the maximum difference between the equation and the data is 4.40% with a standard deviation of difference of ± 2.31%.

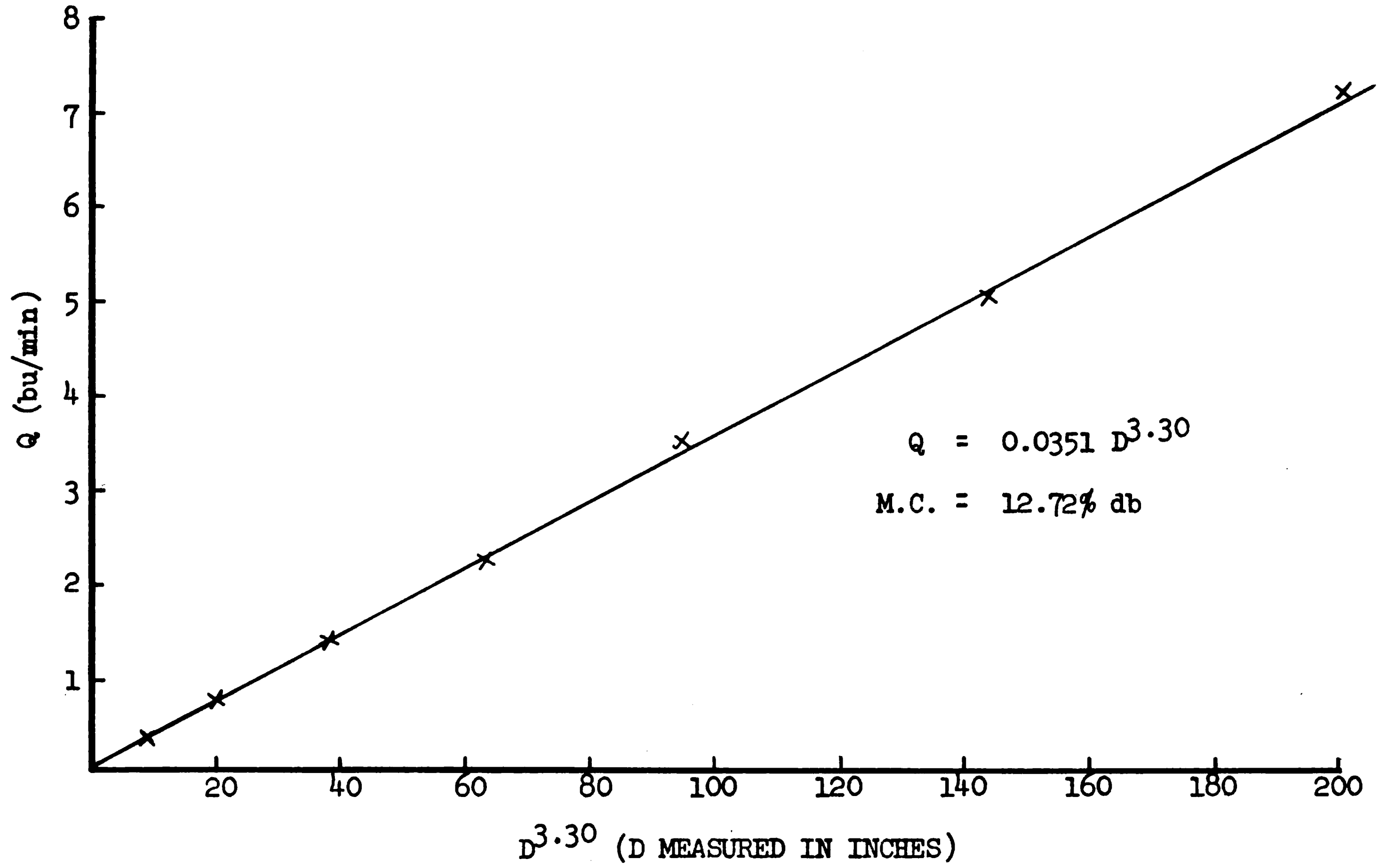


Fig. 14 - Actual flow rate of corn compared to the flow equation for a round orifice positioned vertically in a bin.

TABLE IV

A comparison of the actual flow data to that determined using the derived flow equation for a round orifice positioned vertically.

Diameter (inches)	Q (Flow Equation) (bu/min)	Q (Actual) (bu/min)	Per Cent Difference
1.5	Bridging Occurred		
2.0	0.35	0.34	- 0.87
2.5	0.72	0.72	+ 0.00
3.0	1.32	1.31	- 0.76
3.5	2.18	2.14	- 1.83
4.0	3.41	3.56	+ 4.40
4.5	5.02	4.91	- 2.19
5.0	7.05	7.26	+ 2.98

The equation for the square opening is discussed as part of the development of an equation for the flow of corn through a rectangular orifice.

Rectangular Orifices

The same procedure used in testing the flow rate through the horizontally positioned rectangular orifices was used for testing the vertical openings. All sizes were identical to those used in prior tests. The information obtained produced a family of curves as shown in Figure 15. These curves were similar in appearance to those found for the orifices positioned horizontally and were, therefore, of the form:

$$Q = K W^m L^n$$

Where:

Q = flow rate of corn (bu/min)

W = width of orifice (inches)

L = length of orifice (inches)

K,m,n = constants

$W \leq L$

Three sets of points were chosen from the curves of Figure 15. The resulting three equations were solved simultaneously for the unknowns K, m and n. The final equation developed for the flow of yellow dent corn of moisture content 12.72% db to 12.4% db through a rectangular orifice placed in the side of a bin is then:

$$Q = 0.0528 W^{1.75} L^{1.50}$$

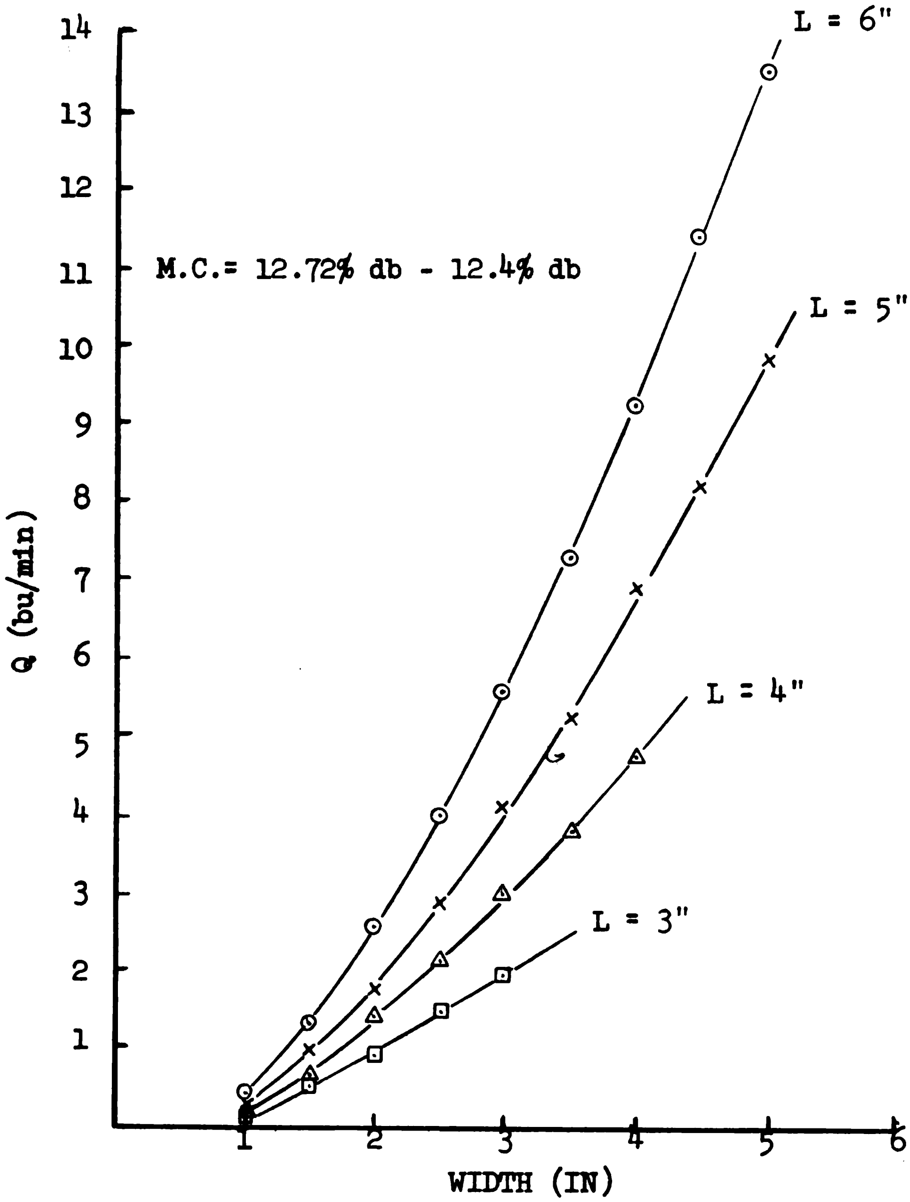


Fig. 15 - Flow of corn through a rectangular orifice of a given length and varying width with the orifice positioned vertically in a bin.

The constant is nearly equal to that found for the equation for square orifices and m + n is approximately equal to 3.26. Thus, the equation is applicable to square openings.

A comparison was made between the actual flow rates of the corn and those determined using the derived equation for the rectangular orifices. As shown in Figure 16, the equation was quite valid. Table V shows that the maximum difference between the equation and the data is 4.92% with a standard deviation of difference of ± 3.24% for all widths above 1.5 inches with the exception of two cases. These two sizes were both found to be outside the best curves determined by plotting the original data. Thus, it would seem that the experimental data for these two cases could be in error.

In the derived equations for flow through vertical openings, the powers of the width of the square orifice and the diameter of the circular orifice were quite similar. There was not much difference between these same exponents for the relationships developed for the horizontal openings. This would indicate a possibility that, for a ratio of W/L equal to 1, the flow rate was dependent upon area. However, when a ratio of areas was taken for a circle and square with the diameter of the circle equal to the side of the square, the following results were obtained:

$$\text{Ratio of } \frac{\text{square area}}{\text{circular area}} = \frac{W^2}{\pi W^2/4} = 1.272$$

The ratio of the constants for the derived equations representing the round and square orifices was $\frac{0.0523}{0.0351} = 1.490$. Thus, it appeared that a factor other than area determined the flow characteristics of the opening.

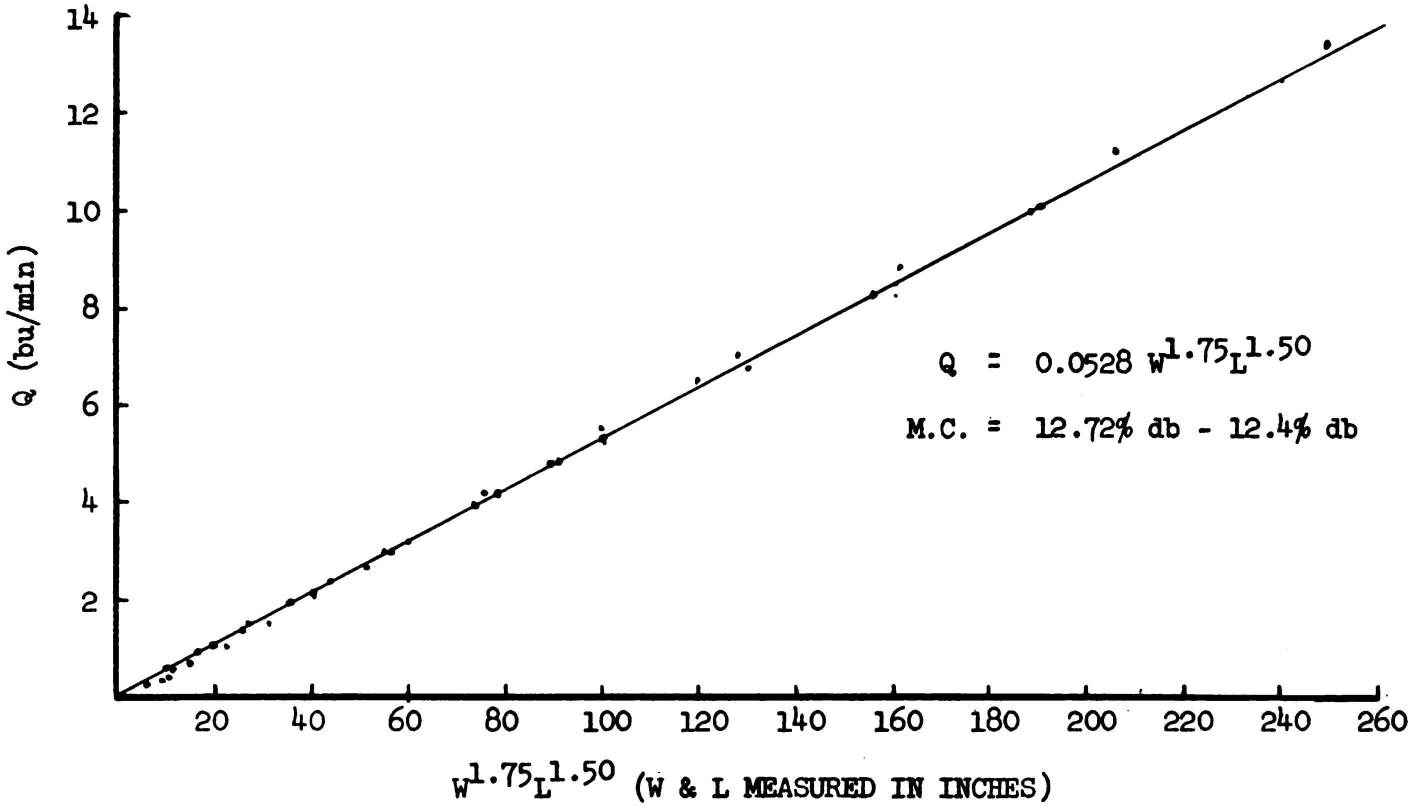


Fig. 16 - Actual flow rate of corn compared to the flow equation for a rectangular orifice positioned vertically in a bin.

TABLE V

A comparison of the actual flow data to that determined using the derived flow equation for a rectangular orifice positioned vertically.

Size L x W (inches)	Q (Flow Equation) (bu/min)	Q (Actual) (bu/min)	Per Cent Difference
6 x 1.0	0.77	0.40	- 92.50
1.5	1.57	1.34	- 17.20
2.0	2.61	2.57	- 1.55
2.5	3.86	4.01	+ 3.74
3.0	5.30	5.57	+ 4.84
3.5	6.95	7.31	+ 4.92
4.0	8.80	9.22	+ 4.55
4.5	10.80	11.38	+ 5.10
5.0	13.10	13.50	+ 2.96
5 x 1.0	0.59	0.31	- 90.20
1.5	1.20	0.98	- 22.40
2.0	1.98	1.82	- 8.80
2.5	2.93	2.92	- 0.34
3.0	4.04	4.09	+ 1.22
3.5	5.28	5.26	- 0.38
4.0	6.70	6.90	+ 2.90
4.5	8.20	8.15	- 0.61
5.0	9.86	9.85	- 0.11
4 x 1.0	0.42	0.22	- 90.80
1.5	0.86	0.71	- 21.10
2.0	1.42	1.36	- 3.63
2.5	2.10	2.11	+ 0.47
3.0	2.88	2.93	+ 1.70
3.5	3.78	3.80	+ 0.53
4.0	4.78	4.72	- 1.27
3 x 1.0	0.27	0.16	- 68.70
1.5	0.56	0.51	- 9.90
2.0	0.92	0.91	- 1.32
2.5	1.36	1.43	+ 4.18
3.0	1.87	1.93	+ 3.11

Stahl (1950) stated that a ratio of 1/3 existed between the flow through a vertical orifice to that through a horizontal opening. It was apparent from the relationships determined during these tests that no similar ratio exists for corn. There is no direct correlation between the powers describing the effect the length and width of the opening have on the flow rates through the vertical and horizontal openings. Thus, a ratio cannot be given which would allow the equation for the horizontal openings to be used to calculate flow through the vertical openings. Indeed, when the flow rates were compared for specifically sized orifices, the ratio of vertical to horizontal flow ranged from approximately 0.3 to 0.5.

The equations derived for the vertical openings were found for orifices whose length was parallel to the bottom of the bin. Tests were not made on the possible effect on flow that would result from making the length perpendicular to the bottom of the hopper.

No definite tests were conducted to determine the ratio of the orifice diameter to the mean particle diameter that would cause bridging. It was noted, however, that bridging would occur when the minimum dimension of the orifice being tested was smaller than approximately 3 particle diameters. Since corn is not spherical, an actual dimension cannot be given.

SUMMARY

The design of materials handling systems could be greatly improved if more data were available to accurately describe the flow of grain through an opening. This would allow the design of bins that could be placed into a system and be expected to operate independently and as desired. Such systems would be a definite aid in grain storage.

A review of literature revealed that some work had been done in the area of semi-fluid flow pertaining to grain. Stahl (1950) produced some equations based on work by Willis Whited indicating that flow was proportional to the cube of the diameter of an opening and independent of the head. He also indicated that the flow from a vertical orifice was one-third that from a horizontal opening. Other investigators agreed that a logarithmic relationship would describe grain flow and that flow was proportional to the diameter of the discharge orifice. These derived equations for semi-fluid flow were very general in nature and required for their solution a knowledge of certain physical properties difficult to evaluate for grain.

The purpose of this work was to attempt to determine the relationship between the flow rate of grain and the orifice size. The resulting equations were to be of a simplified nature that would facilitate their use. The findings of other investigators, principally Stahl, would be evaluated as to their applicability to corn. The effect of a sloping bottom on the flow rate was also studied.

Apparatus was used that had been constructed principally for this work. The equipment was basically of two parts: a stationary hopper with sides and bottom that could be easily adjusted and a movable bin.

The bin elevated the corn so that it could be dumped into the hopper. It was then lowered to receive the grain as it discharged through the various openings placed in the stationary hopper. The flow rates of grain were measured by opening the orifice for a certain timed increment and weighing the discharged corn. An adjustable aperture was used so that an orifice of any size up to six inches square could be obtained. Round openings could also be inserted.

The effect of a sloping bottom on the flow rate was investigated initially. The results indicated that the flow rate of the corn increased slightly as the slope of the bottom increased. This was true as long as funnel flow was present.

The majority of the work involved the determination of equations describing the flow of grain through horizontal and vertical openings. Orifices were of different sizes and shapes. The corn used in the experiments was shelled, yellow dent with a moisture content ranging from 12.72% db to 8.04% db.

The results indicated that the flow rate through a horizontal orifice was faster than that through a vertical opening for a given size. Equations were developed to describe the flow of shelled corn through circular and rectangular orifices in bin bottoms and sides.

These equations had a maximum standard error of difference of ±3.24% when compared to the experimental data. The head did not have an effect on the flow rate of the grain. It was noted, however, that the flow rate of the corn decreased as the corn moisture content increased. Thus, the moisture content of the corn appeared to influence its flow rate.

No correlation could be made between the flow through the horizontal and vertical openings. Thus, the contention by Stahl that a certain ratio existed between the flow through the two openings could not be substantiated. In fact, the flow rate through the vertical openings was found to vary from 30% to 50% of that through the horizontal openings. The area of the orifice did not seem to affect the flow characteristics for a rectangular opening. This result was also in opposition to a finding of Stahl's.

CONCLUSIONS

The results of this work have produced some equations that are unique for corn. The derived relationships have been found to have a maximum standard deviation of difference of $\pm$ 3.24% when compared with experimental measurements for widths greater than 1.5 inches. The equations formulated for yellow dent, shelled corn of the moisture content listed follow:

Horizontal Openings:

Round Orifices:	$Q=0.1196\ D^{3.10}$;	M.C.= 8.4% db
Rectangular Orifices:	$Q=0.1531\ W^{1.62}L^{1.40}$;	M.C.=12.1% db

Vertical Openings:

Round Orifices:	$Q=0.0351\ D^{3.30}$;	M.C.=12.72% db
Rectangular Orifices:	$Q=0.0573\ W^{1.75}L^{1.50}$;	M.C.=12.72% db-12.4% db

Since none of the equations have exponents of the same power, a fixed ratio cannot be determined so that the calculation of flow through vertical openings can be accomplished by using the equations for horizontal orifices. Thus, the contention by Stahl (1950) that this can be done does not appear valid for corn. In fact, actual comparisons of flow rates for horizontal and vertical orifices revealed a ratio of 0.3 to 0.5 between the two. This ratio was found to increase as the ratio of the width to the length (W/L) increased.

Stahl's equations also indicated that area had an effect on the flow rate. If his equations are used to calculate the flow of wheat through a square and a circular opening with the diameter of the circle equal to the length of the side of the square, the ratio between flow rates is exactly the ratio between the areas. This can be shown by the following: as determined previously, the ratio of areas of the aforementioned square and circle is 1.272. The ratio of the constants for Stahl's equations with $D^3 = W^2L$ is 0.2232/0.1753 which is equal to 1.272. The contention that the area affects the flow rate appears invalid in the light of the results found in this research. The effect of both the width and length seems more important.

An increase in the slope of the bottom of the hopper increased the flow rate of the grain slightly. This was true as long as the flow remained funnel flow. At 52.5° mass flow occurred and the flow rate decreased below that recorded at 40°. This result may not occur for all opening sizes. With a larger discharge area, there may be no effect on the flow rate.

The moisture content appeared to have a bearing upon the flow rate of the grain. As the moisture content increased, the flow rate tended to decrease. This may have resulted from the fact that as the moisture content of the corn increased, the friction coefficient of corn on corn increased. Thus, the flow rate of the corn decreased.

Two tests were conducted to evaluate the effect of head on the flow of grain. In both instances, flow rates were measured at different depths in the bin for given openings. Little, if any, variation occurred in the flow rate at the different grain depths. Thus, the head clearly had no effect.

No definite tests were conducted to determine the ratio of the orifice diameter to the mean particle size that would cause bridging. It was noted, however, that bridging occurred when the minimum dimension of the orifice was less than approximately 3 particle diameters.

While the results appeared good regarding correlation between theoretical determinations and experimental data, it was evident that many factors affected the flow rate. Continued investigation is required to develop more adequate equations that will cover grains under different conditions.

RECOMMENDATIONS FOR FURTHER STUDY

1. Tests should be conducted under conditions that maintain a constant moisture content for the grain so that the effect of this variable on flow rate through orifices can be evaluated.
2. The effect that an increase in fine material has on the flow rate should be examined.
3. The effect of the temperature and relative humidity of the ambient conditions upon flow rate should be studied.
4. Different construction materials should be used in the hoppers so that the effect of a change in the coefficient of friction between the grain and the hopper can be measured.
5. A study should be made of the effects on the flow rate that the presence of foreign matter, insect droppings, etc., produces.
6. Further work should be done on the effect a sloping bin bottom has on the flow rate of grain.

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APPENDIX A

EXPERIMENTAL RESULTS

TABLE VI

Flow of corn through a bin with a sloping bottom.

Opening Size (inches) W x L	Observed Type of Flow	Bottom Slope	Weight Discharged (pounds)	Discharge Time (seconds)			Average Time of Discharge (seconds)
1 x 12	Funnel	20°	10	1.7	1.7	1.7	1.7
			20	3.6	3.6	3.6	3.6
			30	5.9	5.8	6.0	5.9
			40	8.0	8.0	8.0	8.0
			50	9.8	9.7	9.7	9.7
			60	12.0	11.8	12.0	11.9
	Funnel	30°	10	1.7	1.8	1.8	1.8
			20	3.6	3.8	3.6	3.7
			30	5.6	5.6		5.6
			40	7.6	7.6		7.6
			50	9.5	9.6	9.5	9.5
			60	11.6	11.6		11.6
			70	13.6	13.6		13.6
			75	14.6	14.6		14.6
	Funnel	40°	10	1.7	1.7	1.8	1.7
			20	3.8	3.6	3.6	3.7
			30	5.7	5.5	5.5	5.6
			40	7.6	7.4	7.4	7.5
			50	9.3	9.2	9.2	9.2
			60	11.2	11.2	11.2	11.2
			70	13.0	12.9	13.1	13.0
			75	14.0	13.8	14.2	14.0
	Mass	52.5°	10	1.6	1.7	1.7	1.7
			20	3.6	3.6	3.8	3.7
			30	5.4	5.6	5.6	5.5
			40	7.2	7.6	7.4	7.4
			50	9.2	9.1	9.2	9.2
			60	11.4	11.2	11.4	11.3
			70	13.2	13.4	13.4	13.3
			75	14.2	14.6	14.5	14.5

TABLE VII

Flow of corn through square apertures placed in the bottom of a hopper.

Side Dimension (inches)	Time of Discharge (seconds)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
1.0	10.0	Bridging Occurred							
1.5	10.0	5.2	5.1	5.3	5.0	5.0	5.1	5.2	5.1
2.0	10.0	12.4	12.6	12.4	12.6	12.4			12.5
2.5	5.0	12.8	12.8	12.7	13.0	12.7			12.8
3.0	5.0	21.5	21.9	21.4					22.6
3.5	2.5	18.2	18.5	18.5	18.8				18.5
4.0	2.5	27.1	27.1	27.5	26.0				27.2
4.5	2.5	36.8	37.0	38.1	38.0				37.5
5.0	2.5	54.1	51.0	51.6					52.2

TABLE VIII

Flow of corn through round apertures placed in the bottom of a hopper.

Diameter (inches)	Time of Discharge (seconds)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
1.5	10.0	4.0	4.2	4.1	3.9	4.0	4.4	4.2	4.06
		4.0	4.0	4.1	4.0	4.1	3.9	4.0	
2.0	10.0	9.1	9.2	9.1	9.3				9.20
2.5	5.0	9.5	9.4	9.4	9.3	9.2	9.3		9.35
3.0	2.5	8.9	8.9	8.5	8.7	8.8	8.6		8.70
3.5	2.5	13.5	13.7	13.9	13.6	14.0			13.74
4.0	2.5	21.2	21.3	21.5	21.1				21.30
4.5	2.5	29.9	29.7	29.0	29.0	29.2	29.6		29.40
5.0	2.5	41.1	41.3	41.1					41.20

TABLE IX

Flow of corn through rectangular apertures placed in the bottom of a hopper.

Opening Size (inches) L x W	Time of Discharge (seconds)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
6 x 1.00	10.0	12.6	12.8	12.5	12.6				12.6
1.25	5.0	10.6	10.5	10.5					10.5
1.50	5.0	14.5	14.5	14.9	15.0	14.8			14.7
1.75	5.0	20.6	20.5	20.7					20.6
2.00	2.5	14.0	14.0	13.9	13.9	14.0			14.0
2.50	2.5	21.9	21.4	21.6	21.8				21.7
3.00	2.5	29.5	29.0	28.5	30.0	29.4			29.3
3.50	2.5	39.5	37.6	39.8					39.0
4.00	2.5	48.0	46.0	46.5					46.8
5 x 1.00	10.0	10.0	10.3	10.0	10.0	9.9			10.0
1.50	5.0	12.6	12.6	12.2	12.6	12.5			12.6
2.00	2.5	11.3	11.5	11.2					11.3
2.50	2.5	17.3	17.2	17.0					17.2
3.00	2.5	22.9	23.5	23.4	23.5				23.3
3.50	2.5	30.0	30.9	31.2	31.8				31.0
4.00	2.5	37.6	39.7	37.8	37.1				38.0
4 x 1.00	10.0	8.1	8.0	8.4	7.9	7.7	8.1	8.1	8.0
1.50	5.0	10.0	10.0	10.0	10.1				10.0
2.00	2.5	9.0	9.1	9.1	9.0	9.0			9.0
2.50	2.5	13.7	13.5	13.3	14.0				13.6
3.00	2.5	18.1	18.0	18.0	18.1	17.9			18.0
3.50	2.5	22.5	22.4	22.4	22.5	22.5			22.5
3 x 1.00	10.0	6.0	6.0	5.8	5.9	6.1	5.7	5.9	5.9
1.50	10.0	13.5	13.4	13.3	13.2				13.4
2.00	5.0	11.9	12.2	11.8	11.7				11.9
2.50	2.5	9.1	9.5	9.4	9.4				9.4

TABLE X

Flow of corn through square apertures placed in the bottom of a hopper.

Major Dimen. (in.)	Time of Disch. (sec.)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
2.0	10.0	11.75	11.50	11.50	11.50	11.50	11.50	11.50	11.50
2.5	5.0	12.00	11.75	11.50	11.50	11.50	11.75	11.25	11.61
3.0	10.0	39.38	39.25						39.31
4.0	7.5	70.00	70.00						70.00
5.0	2.5	46.00	45.75	46.75	44.75	47.25	44.00		45.75

TABLE XI

Flow of corn through rectangular apertures placed in the bottom of a hopper.

Size (in.) L x W	Time of Disch. (sec.)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
6x1.0	5.0	6.63	6.00	6.13	6.13	5.88	6.13	6.13	6.11
		6.00	6.00	6.00	6.00	6.25			
1.5	2.5	7.88	7.63	7.38	7.38	7.25	7.75	7.63	7.53
		7.63	7.25	7.50					
2.0	2.5	13.75	13.50	12.50	12.88	13.88	13.25	14.00	13.40
		13.50	13.38	13.00	13.13	13.75			
2.5	2.5	20.25	19.50	19.80	20.40	19.50	20.00	20.00	19.98
		19.30	20.25	20.90	20.00				
3.0	2.5	27.50	27.00	27.30	27.50	27.00	27.80		27.35
3.5	2.5	35.00	34.50	34.38	34.88				24.63
4.0	2.5	43.00	41.10	42.80	43.20	44.70	46.00	42.80	43.10
		42.80	41.50						
5x1.0	10.0	9.50	9.75	9.25	9.25	9.25	9.50	9.50	9.44
		9.50							
1.5	5.0	11.50	11.50	11.50	11.50	11.50			11.50
2.0	2.5	10.00	10.50	10.25	10.50	10.50	10.25	10.38	10.34
2.5	2.5	16.00	15.25	15.25	15.63	15.75	15.00	15.88	15.42
		15.00	15.00						

TABLE XI (cont.)

Flow of corn through rectangular apertures placed in the bottom of a hopper.

Size (in.) L x W	Time of Disch. (sec.)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
5x3.0	2.5	20.50	21.50	21.25	21.00	20.50	20.00	20.00	20.69
		20.75							
3.5	2.5	26.50	26.50	25.25	27.25	25.50	26.00		26.17
4.0	2.5	33.00	33.00	33.50	33.50	32.75			33.10
4.5	2.5	40.25	38.50	38.25	39.25	38.50	34.75		39.08
5.0	2.5	46.00	45.75	46.75	44.75	47.25	44.00		45.75
4x1.0	10.0	7.63	7.63	7.25	7.50	7.63	7.00	7.50	7.50
		7.50	7.50	7.88					
1.5	5.0	9.25	8.75	9.00	8.75	9.13	9.13	9.00	9.03
		9.25							
2.0	2.5	7.75	7.50	7.50	7.50	7.50	7.50	7.38	7.53
		7.63							
2.5	2.5	11.50	11.00	10.75	10.75	11.75	11.00	11.00	11.11
3.0	10.0	59.75	59.75	59.00					59.50
3.5	7.5	56.75	57.25						57.00
4.0	7.5	70.00	70.00						70.00
3x1.0	10.0	5.50	5.50	5.75	5.25	5.25	5.25	5.50	5.43
		5.50	5.50	5.25					
1.5	5.0	6.50	6.50	6.50	6.75	6.75	6.50	6.25	6.53
		6.50	6.50	6.50					
2.0	15.0	30.75	30.50						30.63
2.5	10.0	29.50	29.00						29.25
3.0	10.0	39.38	39.25						39.32

TABLE XII

Flow of corn through round apertures placed in the side of a hopper.

Diameter (inches)	Time of Discharge (seconds)	Weight Discharged (pounds)						Average Weight Discharged (pounds)
1.5	60.0	Bridging occurred periodically						
2.0	20.0	6.2	6.5	6.6	6.3	6.4	6.2	6.40
		6.3	6.5					
2.5	10.0	6.7	6.9	6.8	6.6	6.7	6.8	6.75
		6.7	6.8	6.7				
3.0	5.0	6.0	6.4	6.1	6.1	6.1	6.0	6.13
		6.1	6.1	6.0	6.2			
3.5	5.0	10.1	9.9	9.9	10.1	9.8	10.2	10.00
		10.0						
4.0	2.5	8.3	8.5	8.2	8.5	8.2	7.9	8.32
		8.4	8.3	8.4				
4.5	2.5	11.8	11.0	11.5	11.4	11.5	11.5	11.44
5.0	2.5	17.6	16.9	17.0	16.9	16.8	16.0	16.90
		17.0						

TABLE XIII

Flow of corn through square apertures placed in the side of a bin.

Major Dimen. (inches)	Time of Discharge (seconds)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
1.5	60.0	10.4	10.5	10.4					10.43
2.0	20.0	9.5	9.7	9.5	9.3	9.2	9.3	9.6	9.45
2.5	10.0	10.0	10.0	10.0	10.0	10.2	9.8		10.00
3.0	5.0	8.9	9.2	8.8	8.9	8.9	8.9	8.9	8.90
		8.6							
3.5	2.5	7.5	7.6	7.6	7.9	7.6	7.3	7.1	7.46
		7.1	7.4						
4.0	2.5	11.1	11.1	11.2	11.0	11.1			11.10
4.5	2.5	16.2	16.8	15.6	15.4	15.8	16.0	16.0	16.01
		16.4	15.9						
5.0	2.5	23.3	23.0	22.8	23.0	22.9	23.1		23.0

TABLE XIV

Flow of corn through a rectangular opening placed in the side of a hopper.

Opening Size (inches) L x W	Time of Discharge (seconds)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
6 x 1.0	10.0	3.4	3.8	3.6	3.5	3.7	3.8	3.7	3.70
		3.8	3.8						
1.5	5.0	6.1	6.3	6.0	6.5				6.20
2.0	5.0	11.9	12.1	11.5	11.8	12.0			11.90
2.5	2.5	9.1	9.4	9.5	9.5	9.2	9.8	9.4	9.38
		9.1							
3.0	2.5	12.7	13.2	13.3	12.8				13.00
3.5	2.5	17.1	17.1	17.0	17.0				17.06
4.0	2.5	21.2	21.3	21.9	21.6	21.4			21.50
4.5	2.5	26.5	26.7	27.3	26.0	26.0			26.50
5.0	2.5	31.5	32.0	31.1	31.6	31.4			31.50
5.5	2.5	34.5	34.0	32.9	32.5	33.1			33.40
5 x 1.0	10.0	3.0	3.0	2.9	3.0	2.6	2.9	2.7	2.87
		2.9	2.8						
1.5	10.0	9.1	9.0	8.8	8.6	8.7	9.0	9.1	9.13
		9.1	8.9						
2.0	5.0	8.7	8.5	8.6	8.5	8.3	8.4	8.6	8.50
		8.4	8.5						
2.5	2.5	7.0	7.0	6.9	6.6	6.9	6.7	6.6	6.81
3.0	2.5	9.6	9.7	9.4	9.5	9.5	9.5		9.54
3.5	2.5	12.5	12.4	12.4	11.5	12.0	13.0		12.30
4.0	2.5	16.5	16.1	15.8	16.0	16.5	16.0	16.0	16.10
4.5	2.5	19.0	19.5	18.5	19.0	19.5	18.8	18.6	19.01
		19.2							
5.0	2.5	23.3	23.0	22.8	23.0	22.9	23.1		23.00
4 x 1.0	20.0	3.8	4.2	4.2	4.4	4.3	4.1		4.17
1.5	10.0	6.5	6.7	6.7	6.7	6.5	6.7	6.7	6.62
2.0	10.0	12.3	12.0	12.0	12.0	12.4			12.14
	5.0	6.7	6.8	6.7	6.6	6.6	6.5	6.6	6.65
		6.6	6.6	6.7					
2.5	5.0	10.0	9.0	9.7	9.5	9.1	9.7	9.7	9.85
		10.0	10.0	10.0	9.9	10.1	10.2	10.3	
		10.4	10.0						
3.0	2.5	6.6	6.9	6.9	6.9	6.6	7.1	6.8	6.85
		7.0							
3.5	2.5	8.9	8.7	8.1	8.9	9.2	8.9	9.2	8.89
4.0	2.5	11.3	10.8	10.8	11.0	11.2	11.0	11.0	11.00
		10.5	11.3						

TABLE XIV (cont.)

Flow of corn through a rectangular opening placed in the side of a hopper.

Opening Size (inches) L x W	Time of Discharge (seconds)	Weight Discharged (pounds)							Average Weight Discharged (pounds)
4 x 3.0	2.5	6.6	6.9	6.9	6.9	6.6	7.1	6.8	6.85
		7.0							
3.5	2.5	8.9	8.7	8.1	8.9	9.2	8.9	9.2	8.89
4.0	2.5	11.3	10.8	10.8	11.0	11.2	11.0	11.0	11.00
		10.5	11.3						
3 x 1.0	30.0	Bridging Occurs							
1.5	20.0	9.5	9.0	9.4	9.4	9.5	9.5	9.6	9.42
		9.5							
2.0	10.0	8.5	8.5	8.4	8.5	8.4	8.6	8.5	8.50
		8.6							
2.5	5.0	6.6	6.6	6.7	6.5	6.8	6.5	6.8	6.69
		6.9	6.9	6.6					
3.0	5.0	8.9	8.7	8.8	8.9	9.2	8.9	9.0	9.00
		9.4	9.0						

GLOSSARY

Apparent particle density - the mass per unit volume of a material dependent upon the physical form and heterogenity of the size of the particle composing the mass.

Cone angle ϕ - the angle formed by the sides of a hopper.

Time of flow - the length of time required for a certain amount of material to discharge through an orifice.

Static angle of repose - the maximum angular deviation from the horizontal of the surface of a static pile of granular material.

Kinetic angle of repose - the slope of a continuously moving surface of a pile of granular material.