AN ANALYTICAL SOLUTION OF SUBSEQUENT FAULTS IN THREE-PHASE POWER SYSTEMS

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WILLARD DALE FRIEDLE

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J A Strelzoff
Major professor

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AN ANALYTICAL SOLUTION OF SUBSEQUENT FAULTS IN THREE-PHASE POWER SYSTEMS

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WILLARD DALE FRIEDLE

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PREFACE

It is the purpose of this thesis to present in an organized manner the conditions, data, and computations for an analytical solution of the voltages and currents in a power system involving a simultaneous fault. The problem will be solved by using the method of symmetrical components.

This paper can be used as a guide for solving fault problems of a similar nature. The problem is worked for a line-to-line fault at one point in the system with the other line grounded at another point in the system.

The author wishes to express his sincere thanks to Dr. J. A. Strelzoff for suggesting the topic of this thesis and for his assistance in its preparation.

The analysis of a three-phase circuit in which phase voltages and currents are balanced, and in which all circuit elements in each phase are balanced and symmetrical, is relatively simple since the treatment of a single-phase leads directly to the three-phase solution. The analysis by Kirchoff's laws is much more difficult, however, when the circuit is not symmetrical, as the result of unbalanced loads or faults that are not symmetrical in the three phases. Symmetrical components is the method now generally adopted for calculating such circuits.

This method of analysis makes possible the prediction, readily and accurately, of the behavior of a power system during unbalanced short-circuit or unbalanced load conditions. The engineer's knowledge of such phenomena has been greatly increased and rapidly developed since its introduction. Modern ideas of protective relaying and fault protection grew from an understanding of the symmetrical component methods.

The extensive use of the network calculator for the determination of short-circuit currents and voltages has been furthered by the development of symmetrical components, since each sequence network may be set up independently as a single-phase system. It is in this connection that the calculator has become indispensable in the analysis of power system performance and design.

Not only has the method of symmetrical components been a valuable tool in system analysis, but also, by providing new and simpler concepts, the understanding of power system behavior has been clarified. The method is responsible for an entirely different procedure in the approach to predicting and analyzing power-system performance.

The material to be presented here involves the solution, by the method of symmetrical components, of a problem involving a simultaneous fault condition. The solution of such problems by network analyzers has been by now fully developed. Occasionally it is desired to solve such a problem by analytical methods only. The object of this thesis is to develop an analytical procedure for solving problems involving subsequent or simultaneous faults.

The term fault will be defined as any condition occurring which is a departure from normal operation. A short circuit or an open circuit are two of the more common conditions which constitute a fault. A simultaneous fault may consist of two or more short circuits on the same or different circuits, open conductors, or any combination of short circuits and open conductors.

Occasionally two or more faults will occur simultaneously at points which are widely separated in a three-phase power system. A combination of an unsymmetrical short circuit with an open circuit may occur, for example, when the short circuit is partially cleared by the opening of a fuse or a single pole circuit breaker. This condition may be described as a subsequent fault, one condition being the result of the other.

These faults, although not occurring too frequently, are important from the standpoint of protection. Relay systems which give satisfactory operation for a fault at a single location may fail to isolate simultaneous faults.

In order to simplify later derivations, a brief discussion of symmetrical components and the equations expressing the voltages and currents in terms of these components will be given at this time.

The fundamental principle of symmetrical components, as applied to three-phase circuits, is that an unbalanced group of three related vectors, for example, voltage or current, can be resolved into three sets of vectors. The three vectors of each set are of equal magnitude and spaced either zero or 120 degrees apart. Each set is a "symmetrical component" of the original unbalanced vectors.

Another way to express this fundamental principle is, any three co-planer vectors V_a, V_b, and V_c can be expressed in terms of three new vectors V_1, V_2, and V_3 by three simultaneous linear equations with constant coefficients, where the choice of coefficients is arbitrary except for the restriction that the determinant made up of the coefficients must not be zero. However, there is only one choice for these coefficients so that the system of three vectors can be replaced by three systems, each consisting of three symmetrical vectors. A system of three symmetrical vectors is one in which the three vectors are equal in magnitude and displaced from each other by equal angles.

For convenience in notation and manipulation, a vector operator is introduced. Through usage it has come to be known as vector a. It is a vector of unit length and is oriented 120 degrees in a positive (counter-clockwise) direction from the reference axis or $a = \epsilon^{j\frac{2\pi}{3}}$. A vector multiplied by it is not changed in magnitude but is simply rotated in position 120 degrees in a forward direction.

In two of the systems of revolving vectors, there is a sequence of phases, while in the third system there is none, the three vectors being in phase.

The voltages and currents over an entire system are then expressed in terms of their components, all referred to the components of the reference phase. The choice of which phase to use as reference is entirely arbitrary, but once it is selected, this phase must be kept as the reference for voltages and currents throughout the system and its analysis. It has become customary is symmetrical component notation to denote the reference phase as "phase a."

The double subscript notation will be used with the small letters a, b, and c to indicate the phases and the numerals 0, 1, and 2 to designate the zero, positive, and negative sequence respectively.

Now using "phase a" as the reference phase and making use of the operator a, the following relations exist:

For positive sequence vectors,

$$V_{b1} = a^2V_{a1}; \; V_{c1} = aV_{a1}$$

For negative sequence vectors,

$$V_{b2} = aV_{a2}; \; V_{c2} = a^2V_{a2}$$

For zero sequence vectors,

$$V_{b0} = V_{a0}; \; V_{c0} = V_{a0}$$

Using these values for the positive, negative, and zero sequence, in writing the equations for the phase voltages in terms of the reference phase components, the following equations result:

$$V_a = V_{a1} + V_{a2} + V_{a0} \qquad (1)$$

$$V_b = a^2V_{a1} + aV_{a2} + V_{a0} \qquad (2)$$

$$V_c = aV_{a1} + a^2V_{a2} + V_{a0} \qquad (3)$$

Many times it is necessary to express the positive, negative, and zero components in terms of the unbalanced voltages. This can be done by adding the above equations together, and equation (4) below is obtained. Multiplying equation (1), (2), and (3) by 1, a, and a^2, respectively, and adding, expression (5) is obtained. Then multiply by 1, a^2, and a, respectively, and add to obtain equation (6).

$$V_{a0} = 1/3\ (V_a + V_b + V_c) \qquad (4)$$

$$V_{a1} = 1/3\ (V_a + aV_b + a^2V_c) \qquad (5)$$

$$V_{a2} = 1/3\ (V_a + a^2V_b + aV_c) \qquad (6)$$

The same expressions can be shown to exist for the vectors representing the currents in a three-phase system. These will now be set down for later reference.

$$I_a = I_{a1} + I_{a2} + I_{a0} \qquad (7)$$

$$I_b = a^2I_{a1} + aI_{a2} + I_{a0} \qquad (8)$$

$$I_c = aI_{a1} + a^2I_{a2} + I_{a0} \qquad (9)$$

$$I_{ao} = 1/3(I_a + I_b + I_c) \qquad (10)$$

$$I_{a1} = 1/3(I_a + aI_b + a^2I_c) \qquad (11)$$

$$I_{a2} = 1/3(I_a + a^2I_b + aI_c) \qquad (12)$$

These voltage and current vectors with the appropriate subscripts will be used to indicate various voltage and current vectors depending on the relationships to be found. In the following expressions, they will represent the conditions at the fault point.

In the consideration of simultaneous faults, it is known that each fault affects the voltages and currents resulting from the other; therefore, they cannot be treated independently. However, the conditions at each fault point can be set down expressing the fault voltages and currents in terms of their symmetrical components and reference phase.

Using phase a as the reference phase, the currents as those flowing into the fault, and the voltages as those appearing as voltage to ground at the point of fault, the fault equations expressing the relations between the symmetrical components of I_a and V_a will be developed for the conditions to be used in this problem. Consider Fig. 1 for the cases given.

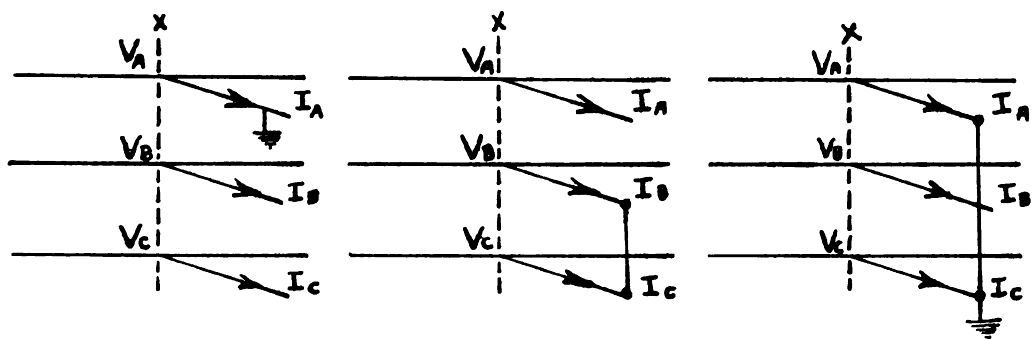

Fig. 1

Fault current and line-to-ground voltage representation at some point X in a system.

Case A. Line-to-Ground Fault on Phase a.

If line a is grounded, $V_a = 0$ and $I_b = I_c = 0$. Substitution of these fault current conditions into equations (10), (11), and (12) will give,

$I_{a0} = 1/3\ I_a$

$I_{a1} = 1/3\ I_a$

$I_{a2} = 1/3\ I_a$

From which

$$I_{a0} = I_{a1} = I_{a2} \qquad (13)$$

Substitution of $V_a = 0$ into equation (1) gives,

$$V_{a1} = (V_{a0} + V_{a2}) \qquad (14)$$

Case B. Line-to-Line Fault Between b and c.

The equations for this fault are

$$V_b = V_c$$

$$I_b = -I_c$$

$$I_a = 0$$

Substitution of the voltage fault conditions into equations (2) and (3) will give,

$$(V_{a0} + a^2 + aV_{a2}) = (V_{a0} + aV_{a1} + a^2V_{a2}$$

$$V_{a1}\ (a^2 - a) = V_{a2}\ (a^2 - a)$$

$$V_{a1} = V_{a2} \qquad (15)$$

If the current conditions are substituted into equations (10), (11), and (12), we have

$$I_{a0} = 0 \qquad (16)$$

$$I_{a1} = 1/3\ I_b\ (a - a^2)$$

$$I_{a2} = 1/3\ I_b\ (a^2 - a)$$

From which

$$I_b = \frac{3\ I_{a1}}{(a - a^2)}$$

$$I_{a2} = I_{a1}\left(\frac{a^2 - a}{a - a^2}\right) = I_{a1}\ \frac{(a - 1)}{(1 - a)}$$

$$I_{a2} = I_{a1}\ \frac{\sqrt{3}\angle 150}{\sqrt{3}\angle -30} = I_{a1}\ \angle 180$$

$$I_{a2} = -I_{a1} \qquad (17)$$

In a similar manner, the expressions for the symmetrical component relations can be obtained for a line-to-line fault between phase a and b. They are

$$I_{a0} = 0 \qquad (18)$$

$$I_{a2} = -a^2\ I_{a1} \qquad (19)$$

$$V_{a2} = a^2\ V_{a1} \qquad (20)$$

Case C. <u>Double Line-to-Ground Fault on Phases a and c.</u>

We again refer to Figure 1 and write immediately that $V_a = V_c = 0$.

$$I_b = 0$$

From equations (1), (3), and (8), we can write

$V_a - V_c = V_{a1} + V_{a2} + V_{a0} - (aV_{a1} + a^2V_{a2} + V_{a0}$

$= (1 - a)\ V_{a1} + (1 - a^2)\ V_{a2} = 0$

$V_a + V_c = (1 + a)\ V_{a1} + (1 + a^2)\ V_{a2} + 2V_{a0} = 0$

$I_b = a^2I_{a1} + aI_{a2} + I_{a0} = 0$

From which we can write

$$V_{a2} = aV_{a1} \qquad (21)$$

$$V_{a0} = a^2V_{a1} \qquad (22)$$

$$I_{a1} = -a^2I_{a2} - aI_{a0} \qquad (23)$$

The equations (13)--(23) give the relationship for the symmetrical components of I_a and V_a at any point in the circuit for an unsymmetrical fault. For any one fault, three equations can be obtained, so that for a simultaneous fault, six equations can be written which are independent of the system impedances. The other six equations necessary for an analytical solution give the relations between fault currents and voltages for each sequence and therefore depend upon the impedances of the three sequence networks of the system.

The equivalent circuit for replacing a single fault in the positive-sequence network is a single impedance connected in shunt (or series) with the positive-sequence network at the point of fault. The equivalent circuit for a double fault is a three-terminal network. The three terminals are two points of fault and the zero potential bus. The simplest forms of three-terminal networks are the delta and the wye.

After the reduction of the sequence circuits into their equivalent wye form, the equations relating the sequence voltages and currents to the respective sequence impedances can be written.

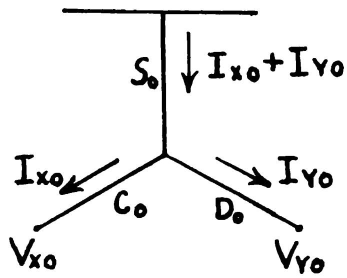

Fig. 2. Zero Sequence Wye

The notation used in the above diagram represents the two faults at point X and Y in the system. The impedances are represented by S_0, C_0, and D_0 and the currents by I_{x0} and I_{y0}. From this equivalent wye, we can write the following equations:

$$V_{x0} = -I_{x0}\,(S_0 + C_0) - I_{y0}\,(S_0) \qquad (24)$$

$$V_{y0} = -I_{x0}\,(S_0) - I_{y0}\,(S_0 + D_0) \qquad (25)$$

Solving these for the currents I_{x0} and I_{y0}

$$I_{x0} = -V_{x0}\,\frac{(D_0 + S_0)}{D_{00}} + V_{y0}\,\frac{S_0}{D_{00}} \qquad (24a)$$

$$I_{y0} = V_{x0}\,\frac{S_0}{D_{00}} - V_{y0}\,\frac{(S_0 + C_0)}{D_{00}} \qquad (25a)$$

$$\text{Where } D_{00} = (S_0 + C_0)(D_0 + S_0) - (S_0)^2$$

$$= S_0D_0 + C_0D_0 + C_0S_0$$

This gives us two more equations necessary to make the analytical solution for a simultaneous fault condition. The other equations necessary are the relations between the positive and negative sequence fault currents and voltages and their impedances.

Similarly for the negative sequence diagram two of the remaining equations can be written.

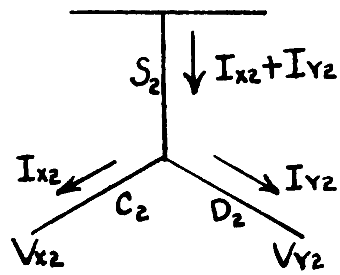

Fig. 3. Negative Sequence Wye.

$$V_{x2} = -I_{x2}\,(C_2 + S_2) - I_{y2}\,(S_2) \qquad (26)$$

$$V_{y2} = -I_{x2}\,(S_2) - I_{y2}\,(S_2 + D_2) \qquad (27)$$

Solving for the currents I_{x2} and I_{y2}

$$I_{x2} = -V_{x2}\,\frac{(S_2 + D_2)}{D_{22}} + V_{y2}\,\frac{S_2}{D_{22}} \qquad (26a)$$

$$I_{y2} = V_{x2}\,\frac{S_2}{D_{22}} - V_{y2}\,\frac{S_2 + C_2}{D_{22}} \qquad (27a)$$

Where $D_{22} = C_2S_2 + D_2S_2 + C_2D_2$

Also, the positive sequence equivalent wye circuit can be drawn and the last two equations written. The equivalent circuit drawn here for the positive sequence is for a system under load with the voltages V_{fx} and V_{fy} being the voltages in the system at the points x and y before the faults occur.

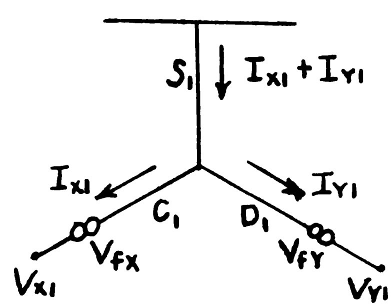

Fig. 4. Positive Sequence Wye.

$$V_{x1} = V_{fx} - I_{x1}\,(C_1 + S_1) - I_{y1}\,(S_1) \qquad (28)$$

$$V_{y1} = V_{fy} - I_{y1}\,(D_1 + S_1) - I_{x1}\,(S_1) \qquad (29)$$

Now for the analytical solution to a problem there are twelve equations and twelve unknowns. Since the solution of these equations is wanted for a line-to-line fault at point x in the system between phase b and c anda line-to-ground fault at point y in the system on phase a, equations (13) and (14) will apply for point y, and equations (15), (16), and (17) will apply for point x. (See Fig. 5.) These equations will be written here now for convenience and with the subscript a omitted since they all refer to phase a. The x or y subscript is added depending on to which fault point they apply.

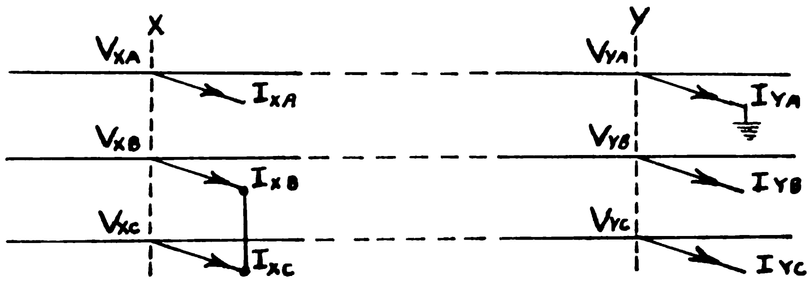

Fig. 5. Fault Conditions on System (Line-to-Line between Phases b and c at one point in system, and Phase a to Ground at another Point).

$$I_{yo} = I_{y1} = I_{y2} \qquad (13a)$$

$$V_{y1} = -(V_{yo} + V_{y2}) \qquad (14a)$$

$$V_{x1} = V_{x2} \qquad (15a)$$

$$I_{xo} = 0 \qquad (16a)$$

$$I_{x2} = -I_{x1} \qquad (17a)$$

From these six equations and equations (24) to (27), the positive and negative components of the voltage and currents are eliminated. If equations (13a), (16a), and (17a) are substituted into equations (24), (25), and (27) for the zero and negative sequence currents, the result is shown below.

$$V_{xo} = 0 - I_{y1} (S_o) \qquad (30)$$

$$V_{yo} = 0 - I_{y1} (D_o + S_o) \qquad (31)$$

$$V_{x2} = I_{x1} (S_2 + C_2) - I_{y1} (S_2) \qquad (32)$$

$$V_{y2} = I_{x1} S_2 - I_{y1} (S_2 + D_2) \qquad (33)$$

These expressions are then substituted into equations (14a) and (15a) and there results an expression for the positive sequence voltages in terms of positive sequence currents and negative and zero sequence impedances.

$$V_{y1} = -(V_{yo} + V_{y2}) = I_{y1} (D_o + S_o) - I_{x1} S_2 + I_{y1} (S_2 D_2)$$

$$V_{x1} = V_{x2} = I_{x1} (S_2 + C_2) - I_{y1} (S_2)$$

Factoring and rearranging gives

$$V_{x1} = I_{x1} (S_2 + C_2) - I_{y1} (S_2) \qquad (34)$$

$$V_{y1} = -I_{x1} (S_2) + I_{y1} (D_o + S_o + D_2 + S_2) \qquad (35)$$

Equations (34) and (35) may now be simplified by being written in this form:

$$\left.\begin{aligned} V_{x1} &= k\, I_{x1} + m\, I_{y1} \\ V_{y1} &= n\, I_{x1} + l\, I_{y1} \end{aligned}\right\} \qquad (36)$$

Where $k = S_2 + C_2$, $m = -S_2$, $n = -S_2$, and $l = (D_o + S_o + D_2 + S_2)$

If the values of V_{x1} and V_{y1} from (36) are now substituted into equations (28) and (29), the following expression results:

$$V_{xf} = I_{x1} (C_1 + S_1) + I_{y1} (S_1) + k\, I_{x1} + m\, I_{y1}$$

$$V_{yf} = I_{x1} (S_1) + I_{y1} (D_1 + S_1) + n\, I_{x1} + l\, I_{y1}$$

Solving these for I_{x1} and I_{y1},

$$\left.\begin{aligned} I_{x1} &= V_{xf} \frac{(D_1 + S_1 + l)}{\Delta_o} - \frac{V_{yf} (S_1 + m)}{\Delta_o} \\ I_{y1} &= \frac{-V_{xf} (S_1 + n)}{\Delta_o} + \frac{V_{yf} (C_1 + S_1 + k_1)}{\Delta_o} \end{aligned}\right\} \qquad (37)$$

Where $\Delta_o = (C_1 + S_1 + k)(D_1 + S_1 + 1) - (S_1 + n)(S_1 + m)$

The expression (37) can now be used for calculating the fault currents in the positive sequence at the two fault points. Once these are determined, the voltages V_{x1} and V_{y1} are determined from equation (36). The values of V_{xo}, V_{yo}, V_{x2}, and V_{y2} are then found from equations (30)--(33). There remains only I_{xo}, I_{yo}, I_{x2}, I_{y2}, which can be calculated from equations (24a)--(27a). This then gives all of the six fault voltages and the six fault currents. Knowing these will enable the calculations to be made of the voltages and current at any point in the circuit due to the faults.

The values of k, m, n, and l will now be computed for another simultaneous fault condition(see Fig. 6).

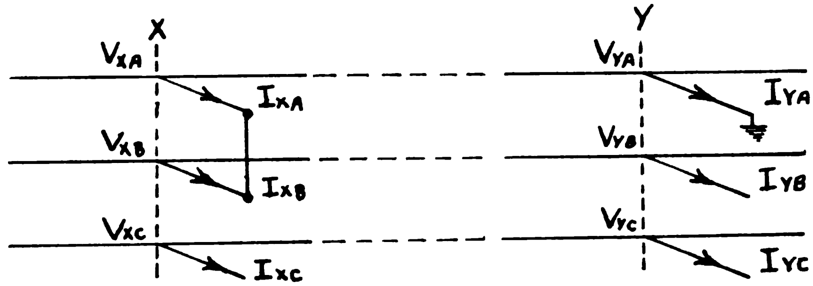

Fig. 6. Simultaneous Fault Condition on a Power System (Line-to-Line Fault Between Line a and b at one point in system, with Phase a to Ground at another Point in System).

The equations for the relation between the sequence voltages and currents at point x and y are (13), (14), (18), (19), and (20).

$$I_{yo} = I_{y1} = I_{y2} \qquad (13a)$$

$$V_{y1} = -(V_{yo} + V_{y2}) \qquad (14a)$$

$$I_{xo} = 0 \qquad (18a)$$

$$I_{x2} = -a^2 I_{x1} \qquad (19a)$$

$$V_{x2} = a^2 V_{x1} \qquad (20a)$$

Again the subscript a is omitted to avoid writing a triple subscript. The subscript x or y has been added to determine which equations express the conditions at which fault point.

Using equations (24)--(27) and substituting equations (13a), (18a), and (19a) into them for the zero and negative components of current, the result is

$$\left.\begin{aligned} V_{xo} &= 0 - I_{y1}\,(S_0) \\ V_{yo} &= 0 - I_{y1}\,(D_0 + S_0) \\ V_{x2} &= a^2 I_{x1}\,(C_2 + S_2) - I_{y1}\,(S_2) \\ V_{y2} &= a^2\, I_{x1}\,(S_2) - I_{y1}\,(D_2 + S_2) \end{aligned}\right\} \qquad (38)$$

Substituting these values of (38) into equations (14a) and (15a), the following expressions result:

$$a^2 V_{x1} = a^2 I_{x1}\,(C_2 + S_2) - I_{y1}\,(S_2)$$

$$V_{y1} = -a^2 I_{x1}\,(S_2) + I_{y1}\,(D_0 + S_0 + D_2 + S_2)$$

Rearranging and factoring gives,

$$V_{x1} = I_{x1}\,(C_2 + S_2) - I_{y1}\,(aS_2)$$

$$V_{y1} = -I_{x1}\,(a^2 S_2) + I_{y1}\,(D_0 + S_0 + D_2 + S_2)$$

Rewriting the above, they become,

$$\left.\begin{aligned} V_{x1} &= k\, I_{x1} + m\, I_{y1} \\ V_{y1} &= n\, I_{x1} + l\, I_{y1} \end{aligned}\right\} \qquad (39)$$

Where $k = C_2 + S_2$, $m = -aS_2$, $n = -a^2 S_2$, and

$l = (D_0 + S_0 + D_2 + S_2)$

The equations from (39) can then be substituted into the equations for the positive sequence equivalent wye and the expressions for the positive sequence fault currents determined like (37). The equations in (37) are valid for any one system for any variety of faults which are located at the same points. Just the values of k, m, n, and l need be determined for each condition. If the faults occur at different points every time, then new values of the equivalent wye circuits must be computed also.

A problem will now be solved illustrating the use of these equations for a short circuit on phases b and c with phase a to ground.

Part II

The problem which is to be solved by the method of symmetrical components was suggested to Dr. J. A. Strelzoff by Consumers Power Company. It is a typical problem involving a portion of a large distribution system with small generating units connected into the system through the distribution. It has been separated from the rest of the system with the equivalent voltage and impedance values of the system determined at the point of separation.

In these types of systems, there must be adequate protection provided these small generating units for the different types of faults which may occur. Many of these conditions are now being neglected due to the cost of obtaining the use of network analyzers and of setting the problems up. Therefore, the analytical solutions to simultaneous fault conditions on portions of large systems are very much desired. It is not necessarily a case of obtaining the exact solution for each current in each branch due to the faults but more to find the trend of the currents for many fault conditions and then setting the protective equipment from this overall data.

The problem worked here is for phase B and C short circuited together at some point with phase A shorted directly to ground at some point in the system isolated from the former fault point.

The one-line diagram is shown in Fig. 7. Below the figure are the impedance values of the loads, transmission lines, and generators figured on one base KVA. Figure eight then shows the one-line diagram with the values of the impedances included at each load.

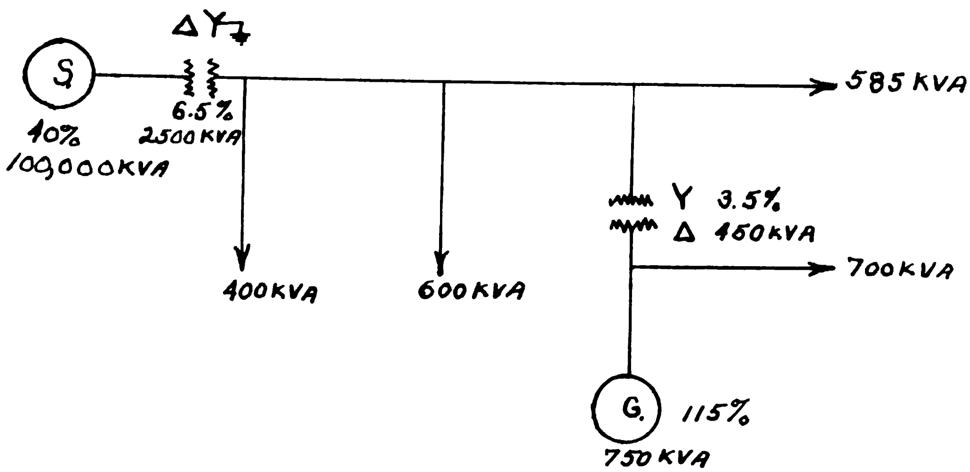

Fig. 7
One-line diagram of power system.

$$S = \frac{40\% \, (7.5)^2 \times 10}{100,000 \text{ KVA}} = j\,2.25$$

$$T_1 = \frac{6.5\% \, (7.5)^2 \cdot 10 \text{ KV}}{2500 \text{ KVA}} = j\,1.46$$

$$\text{Load } 1 = \frac{100\% \, (7.5)^2 \times 10}{400 \text{ KVA}} = 141$$

$$\text{Load } 2 = \frac{100\% \, (7.5)^2 \times 10}{600 \text{ KVA}} = 94$$

$$\text{Load } 3 = \frac{100\% \, (7.5)^2 \times 10}{585 \text{ KVA}} = 96$$

$$\text{Load } 4 = \frac{100\% \, (7.5)^2 \times 10}{700 \text{ KVA}} = 81$$

$$T_2 = \frac{3.5\% \, (7.5)^2 \times 10}{450 \text{ KVA}} = j\,4.36$$

$$\text{Gen.} = \frac{115\% \, (7.5)^2 \times 10}{750 \text{ KVA}} = j\,86$$

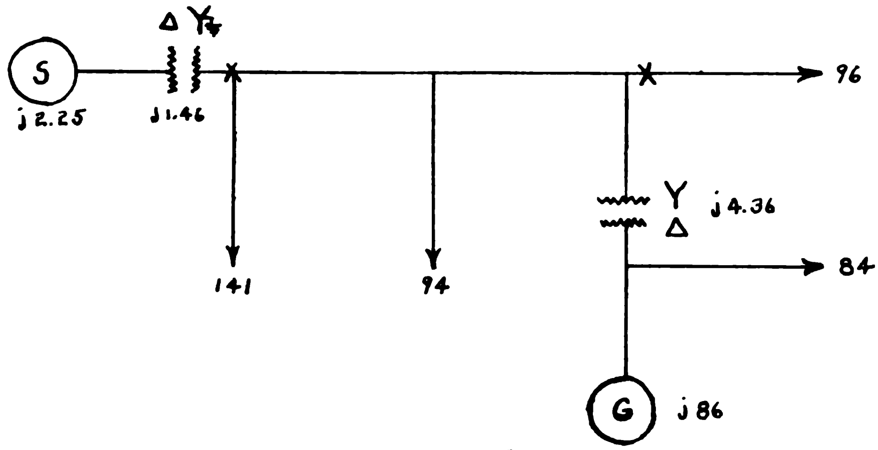

Fig. 8
System one-line diagram showing impedances of components.

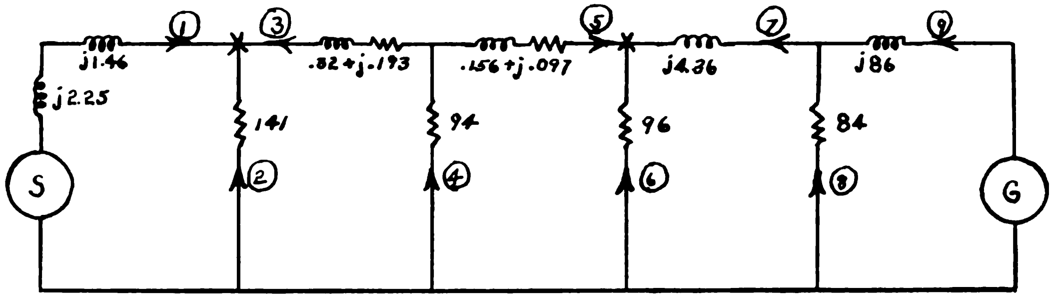

Fig. 9 Positive Sequence Diagram.

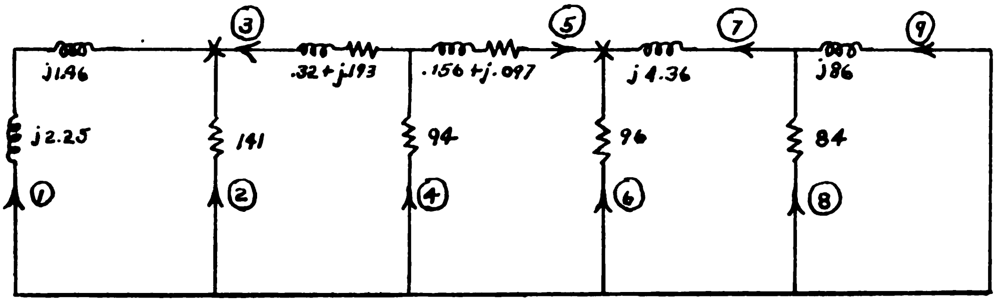

Fig. 10 Negative Sequence Diagram.

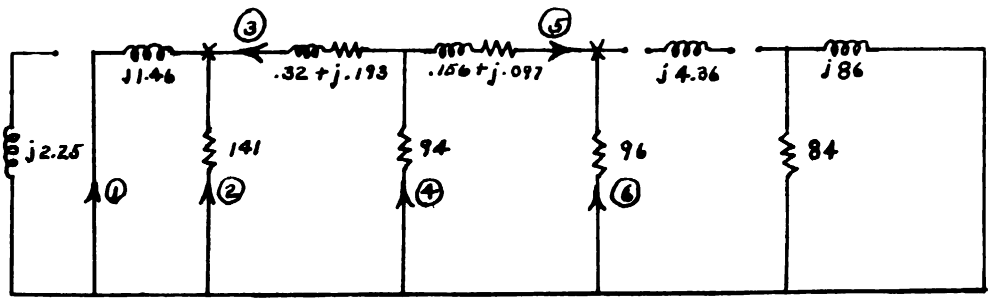

Fig. 11 Zero Sequence Diagram.

Figure 8 is then followed by figures 9, 10, and 11, which are one-line diagrams of the positive, negative, and zero sequence respectively. The impedance values of each branch are indicated in each diagram. The numbers in circles represent the number of the branch in the circuit. These will be used in representing the branch currents throughout the remainder of the computations. The two points of fault are represented by the x's on figures 8, 9, 10 and 11.

Before the fault voltages and currents can be computed, it will be noticed from equations (37) that V_{xf} and V_{yf} will have to be determined as well as the values of S_1, S_2, S_0, D_1, D_2, D_0, C_1, C_2, and C_0. V_{xf} and V_{yf} are the voltage values occurring at the points of fault before the faults occur. Therefore, these are determined from the one-line diagram of the system operating normally with balanced loads.

Figure 12 shows the system with points x and y, the impedance, and generator values all indicated. The wye section between points x and y have been changed to a pi for further ease in computations.

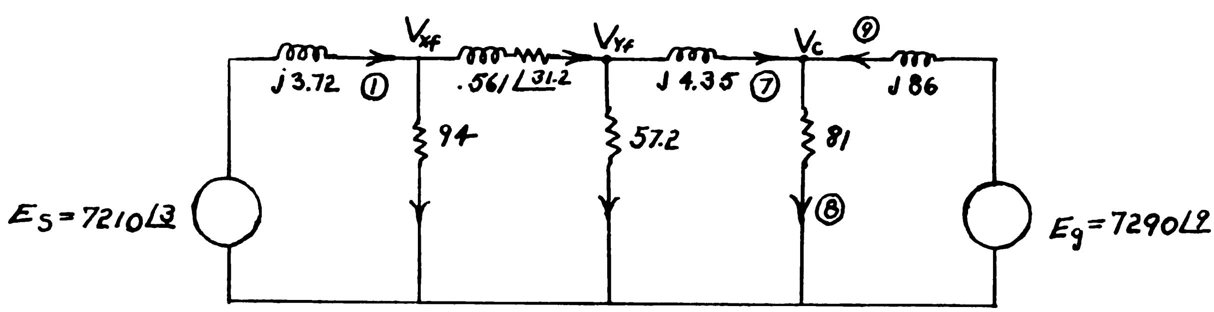

Fig. 12
System under normal operating conditions.

The node voltage values shown will now be computed giving the values of V_{xf} and V_{yf} to be used in equation (37). These voltage values along with voltage V_c will then be used to compute the branch currents under normal operation. The node voltage equations are:

$$V_{xf}\,(Y_{11}) - V_{yf}\,(Y_{12}) - V_c\,(Y_{13}) = 7210 \angle \underline{3}\ (-j.269)$$

$$-V_{xf}\,(Y_{21}) + V_{yf}\,(Y_{22}) - V_c\,(Y_{23}) = 0$$

$$-V_{xf}\,(Y_{31}) - V_{yf}\,(Y_{32}) + V_c\,(Y_{33}) = 7290 \angle \underline{9}\ (-j.01162)$$

The admittance values being:

$Y_{11} = 1.945 \angle \underline{-37.8}$

$Y_{12} = Y_{21} = 1.782 \angle \underline{-31.2}$

$Y_{13} = Y_{31} = 0$

$Y_{22} = 1.93 \angle \underline{-36.8}$

$Y_{23} = Y_{32} = .23 \angle \underline{-90}$

$Y_{33} = .242 \angle \underline{-87.1}$

These equations are then solved by the method of determinants, giving the values of V_{xf}, V_{yf}, and V_c.

$V_{xf} = 7030 \angle \underline{-5.3}$

$V_{yf} = 6960 \angle \underline{-5.7}$

$V_c = 6900 \angle \underline{-8}$

These node voltage values are then used to compute the currents in the system when it is operating under normal conditions. Upon changing the pi circuit back to the original wye and numbering the branches as they are in figure 13, the currents were computed and tabulated in Table I. These current values will be used later after all the fault currents have been computed. By superposition these currents will be combined to give the total currents which occur when the system is in operation and these faults take place.

Table I
BRANCH CURRENTS UNDER NORMAL CONDITIONS

Branch	Currents
I_1	276-j4.63
I_2	50-j4.63
I_3	224-j43.6
I_4	89.4-j23.0
I_5	135-j20.7
I_6	72.2-j7.2
I_7	63.2-j14.23
I_8	84.3-j11.9
I_9	24.3-j3.50

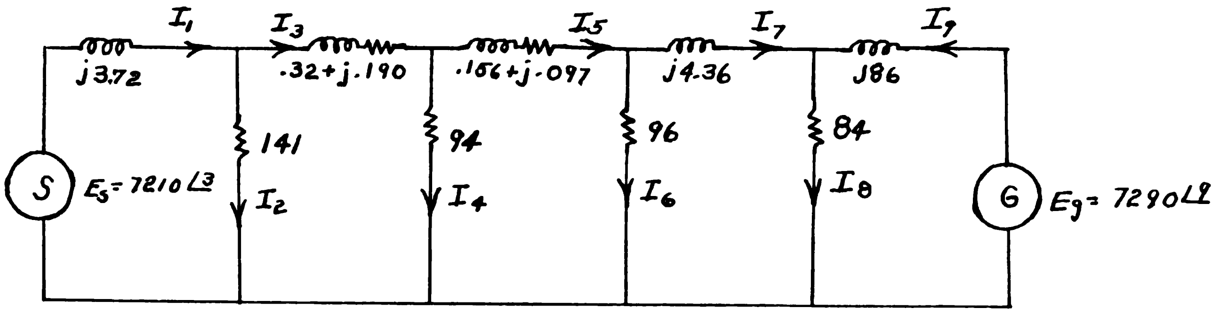

Fig. 13
System under normal conditions, showing the current directions in each branch.

The next phase of the problem is to determine the impedance values of the equivalent wyes of the positive, negative and zero sequence circuits.

For the zero sequence circuit, branch 1 and 2 were first paralleled. Then the wye circuit between point x and y was changed to a pi (see Fig. 11) and the parallel legs of the pi were combined with the combined value of branch 1 and branch 2 and also with branch 6. This pi circuit was then changed to an equivalent wye, thus giving the values of C_0, D_0, and S_0.

The positive and negative sequence circuits are identical except for the generators in the positive sequence circuit. Therefore, the equivalent wye circuit will have the same impedance values when the values of D_1, C_1, and S_1 and D_2, C_2, and S_2 are computed.

From figure 9 or 10 it can be seen that after changing the wye section between points x and y in the circuit to a pi section, there remains only series and parallel combinations of impedances to calculate. After these combinations have been made, retaining points x and y, there remains only to change the total pi combination to an equivalent wye circuit, thus giving the values of C_1, D_1, S_1, C_2, D_2, and S_2. These values are now tabulated in Table II.

Table II

Branch	Impedance Value
C_o	$-.0068+j.0124$
D_o	$.482+j.295$
S_o	$.069+j1.445$
$C_1=C_2$	$-.0025+j.0594$
$D_1=D_2$	$.469+j.229$
$S_1=S_2$	$.484+j3.436$

The values of k, l, m, and n can now be computed. They appear in equations (36), which are used to compute the positive sequence voltages.

$$k = C_2 + S_2 = (-.0025 + j\,.0594) + (.484 + j\,3.436)$$

$$= .4815 + j\,3.4954$$

$$m = n = -S_2 = -.484 - j\,3.436$$

$$l = D_0 + S_0 + D_2 + S_2$$

$$= (.482 + j\,.295) + (.069 + j\,1.445) + (.469 + j\,.229) +$$

$$(.484 + j\,3.436)$$

$$l = (1.504 + j\,5.405)$$

From equation (37) it can be seen that it is only necessary to obtain the value of Δ and then the fault currents and voltages can be computed.

$$\Delta = (C_1 + S_1 + k)\,(D_1 + S_1 + l) - (S_1 + n)\,(S_1 + m)$$

$$= (.9680 + j\,6.991)\,(2.457 + j\,9.070)$$

$$= (-61.04 + j\,25.96)$$

Knowing these values, the positive sequence fault currents can be computed at point x and y. Using equation (37), where $m = -S_2$ and $S_1 = S_2$, it is seen that they reduce to,

$$I_{x1} = V_{xf}\,\frac{(D_1 + S_1 + l)}{\Delta}$$

$$I_{y1} = V_{yf}\,\frac{(C_1 + S_1 + k_1)}{\Delta}$$

$$I_{x1} = 7030\,\angle{-5.3}\,\left[\frac{2.437 + j\,9.07}{-61.04 + j\,25.96}\right] = (45.2 - j\,998)$$

$$I_{y1} = 6960\,\angle{-5.7}\,\left[\frac{.9680 + j\,6.9908}{-61.04 + j\,25.96}\right] = (-254 - j\,693)$$

Knowing the positive sequence fault currents, from equations (36) it can be seen that the positive sequence fault voltages may be determined.

$$\left.\begin{aligned} V_{x1} &= kI_{x1} + mI_{y1} \\ V_{y1} &= nI_{x1} + lI_{y1} \end{aligned}\right\} \qquad (36)$$

Substituting the known values of k, m, n, l, I_{x1}, and I_{y1}, the values of V_{x1} and V_y are easily found.

$$V_{x1} = (.4815 + j\ 3.4954)\ (45.2 - j\ 998) + (-.484 - j\ 3.436)\ (-254 - j\ 693)$$

$$V_{x1} = 1253 + j\ 886$$

$$V_{y1} = (-.484 + j\ .295)\ (45.2 - j\ 998) + (1.504 + j\ 5.405)(-254 - j\ 693)$$

$$V_{y1} = -89 - j\ 2087.6$$

The remaining values of fault currents I_{x2}, I_{xo}, I_{y2}, and I_{yo} may be determined from equations(13a), (16a), and (17a).

$$I_{yo} = I_{y1} = I_{y2} \qquad (13a)$$

$$I_{xo} = 0 \qquad (16a)$$

$$I_{x2} = -I_{x1} \qquad (17a)$$

Therefore,

$$I_{yo} = I_{y2} = -254 - j\ 693$$

$$I_{xo} = 0$$

$$I_{x2} = -45.2 + j\ 998$$

The values of fault voltages V_{x2}, V_{xo}, V_{y2}, and V_{yo} may be calculated from equations (30), (31), (14a), and (15a).

From equation (30) and (31),

$$V_{xo} = -I_{y1}\ (S_o) = -(741.24\ \underline{/-110.9})(1.445\ \underline{/87.26})$$

$$V_{xo} = -981 + j\ 429.5$$

$$V_{yo} = -I_{y1}\ (D_o + S_o) = -(-254 - j\ 693)(.551 + j\ 1.74)$$

$$V_{yo} = -1065.8 + j\ 823.8$$

The equation (14a) will give,

$V_{y2} = -(V_{y1} + V_{yo})$

Also V_{y2} may be determined from equation (33) with both equations yielding the same results

$V_{y2} = I_{x1}\ (S_2) - I_{y1}\ (S_2 + D_2)$ (33)

Substituting the known values of I_{x1}, I_{y1}, S_2, and D_2 into equation (33) will give,

$V_{y2} = 1154.4 + j\ 1263.8$

Also equation (32) or (15a) may be used to determine the value of V_{x2}.

$V_{x2} = V_{x1}$ (15a)

$V_{x2} = 1253.1 + j\ 885.8$

These twelve values of fault voltages and currents will now be tabulated in a table so as to be readily available:

Table III

Voltages & Currents	Value
V_{x1}	$1253 + j\ 886$
V_{x2}	$1253 + j\ 886$
V_{xo}	$-981 + j\ 429.5$
V_{y1}	$-89 - j\ 2087.6$
V_{y2}	$1154 + j\ 1263.8$
V_{yo}	$-1065.8 + j\ 823.8$
I_{x1}	$45.2 - j\ 998$
I_{x2}	$-45.2 + j\ 998$
I_{xo}	0
I_{y1}	$-254 - j\ 693$
I_{y2}	$-254 - j\ 693$
I_{yo}	$-254 - j\ 693$

These fault voltage and current values are now used with their respective sequence circuit (figures 9, 10 and 11) to determine the fault currents throughout each circuit. The negative and zero sequence circuits are relatively easy because no generated voltages appear in them. The changes in the positive sequence system currents resulting from the faults can be determined from the positive-sequence fault currents and voltages and the positive sequence network with the internal generated voltages equated to zero. The currents due to the faults superimposed upon the load currents before the fault give the total positive sequence currents.

Because V_{xo} is equal to zero the zero sequence currents can be determined by determining the equivalent impedance of the circuit to the left of fault point y by series and parallel impedance combinations. Then finding currents I_5 and I_6 the circuit can be expanded out again giving the current values of the remaining branches.

For the positive and negative sequence circuits the two points of fault each have voltage values occurring and therefore the currents may be found as a result of these fault voltage values.

Adding the values of current found in the branches at each fault point gives a check on the values of fault current flowing from the circuit at point x and y. These values check within the allowable error for such computations.

These values of currents are all in reference to phase A and are tabulated in Table IV.

Table IV
Currents in Phase A

Branch Current	Sequence Positive	Negative	Zero
I_1	-238+j337	-238+j337	-294-j672
I_2	-8.96-j5.75	-8.96-j5.75	7.01-j3.075
I_3	292-j1329	201.8+j666.3	278+j684.5
I_4	12.16-j2066	-12.67-j12.1	11.1-j7.34
I_5	-280-j737	-214-j679	-276-j688
I_6	.947+j21.7	-12-j13.17	11.08-j8.8
I_7	25.3+j22.3	-27.6-j.67	0
I_8	2.35+j24.4	-14.32-j14.1	0
I_9	23-j2.22	-13.3+j13.5	0

The values of currents for phase B and C can now be determined from the values of the symmetrical components of phase A. These can be determined from equations 1, 2, and 3. It can be seen from these equations that the positive and negative sequence values of currents for phase B can be calculated by multiplying the positive and negative sequence values of current of phase A by the vector operators of a^2 and a, respectively. For phase C the positive and negative sequence values of current of phase A must be multiplied by a and a^2, respectively. These values will now be tabulated in Tables V and VI.

Table V

Currents in Phase B

Branch	Sequence Positive	Sequence Negative	Sequence Zero
I_1	410.8+j37.6	-173.27-j374.9	-294-j671.5
I_2	-945+j4.89	9.46-j4.89	7.01-j3.075
I_3	-1297+j411.8	-677.87-j158.4	278+j684.5
I_4	-1795+j1022	16.81-j4.92	11.1-j7.34
I_5	-498+j510.8	695+j153.9	-276.5-j688.5
I_6	18.35-j11.69	17.4-j3.81	11.08-j8.8
I_7	6.603-j33.1	14.37-j23.55	0
I_8	19.96-j14.24	19.37-j5.35	0
I_9	-13.42-j18.8	-5.04-j18.26	0

Table VI

Currents in Phase C

Branch	Positive	Sequence Negative	Zero
I_1	-172.8-j374.6	411.3+j37.35	-294.2-j671.5
I_2	3.46-j4.89	-.5+j10.63	7.01-j3.08
I_3	1005+j917.7	476.1-j507.4	278+j684.5
I_4	1783+j1043	-4.15+j17.02	11.1-j7.34
I_5	778.2+j126	-481+j525.3	-276.5-j688.5
I_6	-19.3-j10.1	-5.4+j16.97	11.08-j8.8
I_7	-31.93+j10.81	13.21+j24.21	0
I_8	-22.3-j10.2	-5.05+j19.5	0
I_9	-9.58+j21.03	18.34+j4.76	0

The current directions for the sequence networks are as shown in figures 9, 10, and 11. Each branch current is in the same direction in each sequence. By superposition all currents may now be imposed on one diagram (Fig. 14) giving the total current in each branch and phase. They were summed up in the direction shown on the sequence networks. The positive, negative, and zero sequence fault currents are superimposed together with the original currents flowing before the fault occurred.

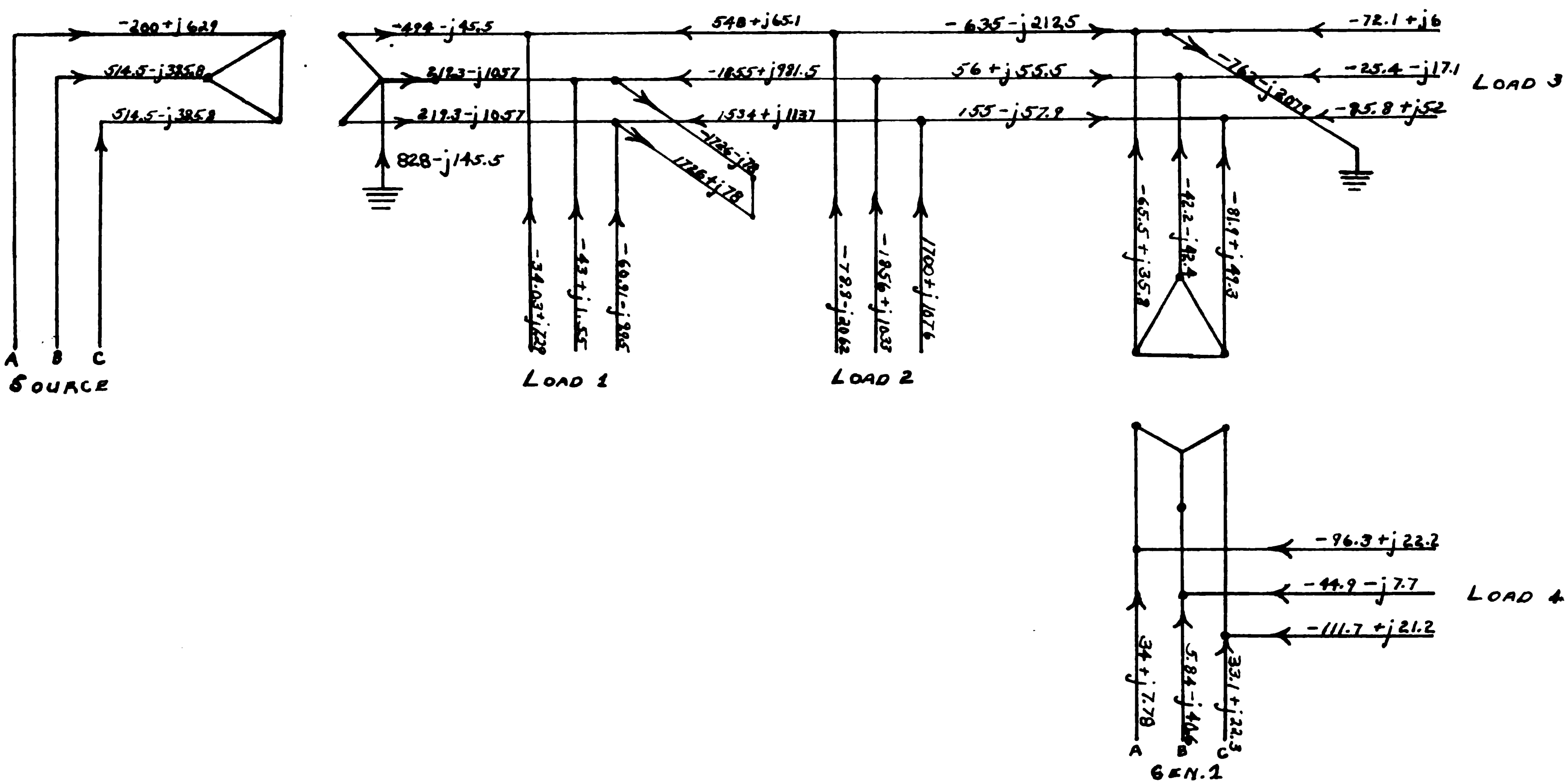

Fig. 14
Diagram of System showing Phase Currents

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