PRESTRESSED CONCRETE PRETENSIONING VERSUS POSTTENSIONING

Thesis for the Degree of M. S.

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Steven E. Z. Galezowski

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THESIS

This is to certify that the

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Prestressed Concrete
Pretensioning versus Post-tensioning

presented by

Steven E. Z. Galezewski

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Carl L. Shermer
Major professor

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PRESTRESSED CONCRETE

PRETENSIONING VERSUS POSTTENSIONING

By

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I. INTRODUCTION

Prestressing of concrete is probably the most important development in Civil Engineering in recent years. After two decades of effective life prestressed concrete is revolutionizing an ever wider field of construction, due to its elegance, its soundness, its saving of materials and, where properly used, its economy.

In spite of its great success, neither a code of practice nor design specifications are available in this country. The limited information, suggestions and research findings are scattered in technical papers and books and therefore cannot be used easily by young beginning designers.

The author who at this stage is a beginner but intends to make the field of prestressed concrete his life time career, believes that prestressed concrete construction will continue to grow in importance and in near future will in many applications replace not only ordinary reinforced concrete but also steel and timber. This belief inspired him to study all possible publications on the subject not only in English but also some in French, German and Dutch.

The aims of this paper are, 1) to present the theory of prestressed concrete in as simple a manner as possible and 2) to compare in details both methods of tensioning.

A short history of development and an outline of theory are followed by discussion of pretensioning and posttensioning. The design of identical single span deck bridge by both methods serves as the basis of comparison and gives a typical method of design. Various advantages and disadvantages of each method are pointed out. The design examples presented may be applied equally well to structures other than bridges.

This paper is the author's first step into the science of prestressed concrete, and he has fresh in memory the difficulties encountered while studying the subject. If he succeeds in presenting the principles and design procedures of prestressed concrete in a simple manner understandable to the beginner, his efforts will have been worth-while.

II. GENERAL PRINCIPLES AND PROPERTIES

Prestressing is a technique of construction whereby initial compressive stresses are set up in a member, to resist or annul the tensile stresses produced by the load.

Since concrete is a material with a high compressive strength and a relatively low tensile strength, the advantages of prestressing in concrete construction are almost unlimited. In reinforced concrete the steel takes the stresses that the concrete cannot take and is thus an indispensable part of the structure.

In prestressed concrete, up to the limit of the working load, the steel is not used for reinforcement but only as a means of producing a compressive stress in the concrete. A member made of prestressed concrete is permanently under compression, the stress varying with the load between chosen maxima and minima. As a consequence, there is complete avoidance of cracks under normal loads, and under an overload - providing it is not greater than the elastic limit - the cracks will close again without any deterioration in the structure. Prestressed concrete has a far greater resistance than reinforced concrete to alternating loads, impact loads, vibration and shock, and the permanent compression reduces to a great extent the principal tensions produced by shear forces.

One advantage of prestressing is that under dead load the section may be designed to the minimum concrete stress at the top fibres and the maximum concrete stress at the bottom fibres. When the live load is applied the stresses will be reversed, giving the maximum concrete stress at the top and the minimum concrete stress at the bottom fibres.

With prestressed concrete it is possible to obtain lighter members than with reinforced concrete, and considerable savings of concrete and steel are effected.

III. HISTORY AND DEVELOPMENT

The details of the first work where the tensioning of reinforcement steel was applied to the manufacture of mortar slabs were published in 1886. The steel was tensioned before the concrete was placed and released when it had hardened. The purpose was not, however, to reduce tensile stresses in the concrete but rather to produce simultaneous failure of both steel and concrete. The present basic principle was not understood.

In 1888, use was made of preliminary compressive stresses to increase the load bearing capacity in concrete arches and floors. These stresses were applied by turnbuckles or some such arrangement on tie rods. Between 1896 and 1907 numerous attempts were made to improve reinforced concrete by tensioning the reinforcement. In these early experiments, however, mild steel was used as reinforcement and the importance of high quality concrete was not fully realized, so that in every case the initial prestress was lost almost immediately.

Thus, just after the turn of the century the advantages of prestressing were suspected by the enthusiasts, but they were still unable to make the process really practicable. The chief reasons for this were lack of knowledge of their materials and lack of reliable materials. The small pretensions which were applied were therefore almost swallowed up by shrinkage, creep and plastic flow losses. In the early 1900's the French engineer E. Freyssinet turned his attention to the study of prestressing. In 1908 he carried out tests on a large tie member prestressed with steel wires tensioned after the concrete had set and anchored by wedges in steel plates. These tests, together with observations of other structures under load, led him to suspect the importance of creep and the necessity of reducing its effect by the use of high tensile steel and high quality concrete. However, it was not until nearly twenty years later that he was able to put his theories into practice. By that time - 1928 - Faber and Glanville in England had published the results of their research on creep in concrete which confirmed Freyssinet's own deductions and enabled him to establish his theory of prestressing. The emergence of prestressed concrete as a practical technique dates effectively from this moment.

In America, attempts were made in 1923 to stress the steel after most of the shrinkage had taken place. Hard steel of high elastic limit was used and bond was prevented by coating the wires. One end of the wires was hooked and bonded while the other end was threaded outside the concrete member. The tension was produced by screwing a nut on this threaded portion. Small units such as fence posts and channel slabs were manufactured in this fashion. The application was also made to cylindrical concrete containers: high tensile steel hoops were tensioned by means of turnbuckles and then embedded in concrete. In Germany a bowstring arch bridge was constructed in about 1928 with better quality steel. Tie rods were placed outside the main structure and after the concrete had hardened were tensioned with hydraulic jacks which could be adjusted to compensate for losses in stress.

In America again, an interesting experiment was made in 1930 in the application of heat to prestressing. Bars were embedded in concrete and coated with sulphur. A heavy low voltage current was passed through these bars raising the temperature, melting the sulphur and thus breaking the bond with the concrete. The extension was taken up and anchored, and on cooling the sulphur hardened and remade the bond. Prestressing by heat was used again in 1939 by Freyssinet on the exposed end of the balancing tower of a French hydro-electric scheme. Much attention has been devoted in the last ten years to practical methods of posttensioning, and anchorage systems have been evolved of which the most notable are those developed by Freyssinet and by Gustav Magnel in Belgium.

Hoyer in Germany developed Freyssinet's early pretensioning technique, using thin piano wires, and produced floor beams, sleepers and similar members by this method.

In 1939, F.O. Anderegg working in America, applied prestressing to burnt clay building blocks. High tensile steel ties were threaded through holes in blocks, stressed and grouted in position. The Swiss firm A.C. Stahlton, further developed this application by using indented steel wires to increase the bond between steel and concrete.

Many developments of prestressing have since been made, but they all come under two headings: pretensioning and posttensioning. Each of these two systems has its own special applications in the manufacture of concrete members.

IV. PRETENSIONING

In pretensioning the steel is first stressed and the concrete cast around it. When the concrete has attained sufficient strength, the steel is released and stress is retained by bond with the concrete. The steel is usually in the form of 14 gauge, 12 gauge or 0.2 in diameter wire, the diameter being kept small to increase the bond. Bonding may further be improved by notching the wire. The most usual method of pretensioning is known as the "long line" system by which a number of units may be produced at once. Wires are stretched between anchorages at opposite ends of a long "stretching bed" and the concrete cast round them with spaces or spacers at the desired intervals. When the concrete has hardened sufficiently the stress is released and the wires cut between each unit. Vibration is used to produce high strength concrete, and some special form of curing is often applied to accelerate hardening.

Pretensioning may also be applied to individual units. In this case the wire is stressed and anchored in each mould and the units may be steam cured in an oven. This method has the advantage that a comparatively small factory space is required and a more rapid turnover can be obtained. Another advantage is that if an anchorage slip should occur only one unit would be affected whereas in the case of long line process a number of units might be weakened. The cost of the individual moulds is the only extra expense attached to this method and this may be absorbed in the mass production.

V. POSTTENSIONING

In posttensioning the concrete is cast and allowed to harden before the prestress is applied.

The wires or cables may be placed in position and cast into the concrete, being prevented from bonding by some form of sheath or by other means, or holes may be cast in the concrete and the wires or cables passed through after hardening has taken place. They may then be stressed against the ends of the unit and anchored, and may subsequently be grouted in to protect the steel and give the additional safeguard of bond between the steel and the concrete. With posttensioning there is no limitation on the diameter of the tensioned steel, and the concrete need not be of super high strength, unless high concrete stresses occur; but obviously the bond resistance is reduced with larger steel bars. However, good bond due to grouting greatly improves the properties.

Unlike pretensioning, cables can be curved in posttensioning. This is an advantage because the existence of a vertical component of the prestressing force due to the inclination of the cables vastly reduces the shear stresses.

VI. THEORY AND STRESS ANALYSIS

Stresses

To start with, two assumptions will be made:

1. Plane transverse sections of the beam remain plane and normal to the longitudinal axis when the beam is bent.
2. The material of the beam obeys Hooke's law

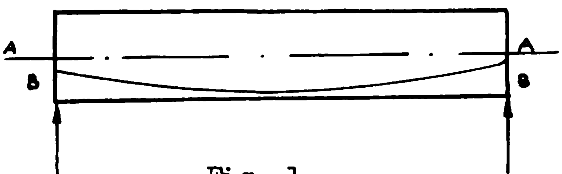


Fig. 1

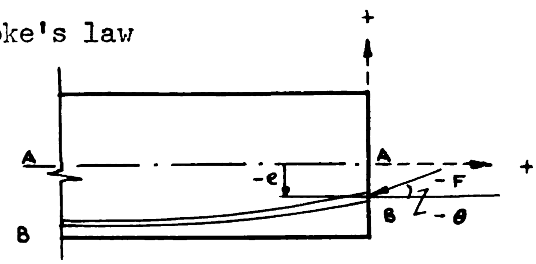


Fig. 2

In Fig. 1 is shown a beam in which A.A. is the line of centroids and BB is the line of the prestressing cable, in which there exists a tensile force F. It is assumed that the horizontal component (H) of the cable force (F) is constant throughout the length of the beam. At any section of the beam, the forces in the beam and in the cable must be in equilibrium and it is therefore possible to equate forces and moments at any section. From Fig. 2 we can write:

$$H = F \cos \theta \quad . . . . . \quad (1)$$

Equating forces in the direction of AA it is seen that the reaction -H of the cable upon the concrete will produce a compressive stress in the concrete given by:

$$f_1 = -\frac{H}{A} \quad . . . . . . \quad (2)$$

where A is the area of the section of the beam.

Equating forces in the vertical direction it is seen that there will be a shear force S over the section of the beam due to the cable reaction given by:

$$S = F \sin \theta = H \tan \theta \quad \ldots\ldots (3)$$

From (1), since the reaction -H from the prestressing cable is not applied along the line of centroids AA but is eccentric by the amount -e it will produce a bending moment on the section given by

$$M = He \quad \ldots\ldots\ldots\ldots\ldots (4)$$

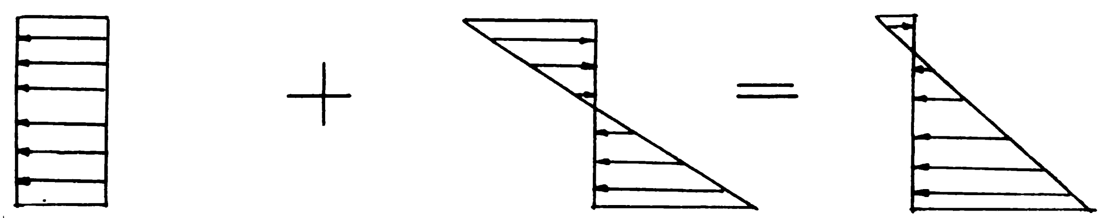
This bending moment will set up stresses in the beam, the values being given by the standard formula:

$$f_2 = \frac{My}{I} = \frac{Hey}{I} \quad \ldots\ldots\ldots\ldots (5)$$

The algebraic sum of these two stress systems gives an expression for the total stress on the section when the beam is in the unloaded state.

Thus $$f_p = f_2 + f_1 = H \left[\frac{ey}{I} - \frac{1}{A}\right] \ldots\ldots (6)$$

This is shown in the stress diagrams of Fig. 3.



Direct stress   Bending stress   Prestress

Fig. 3

Under working conditions a beam will have to be safe when it is within the range of conditions between dead load only and dead load plus maximum live load. It is necessary, therefore, to investigate the stress distributions of these two states.

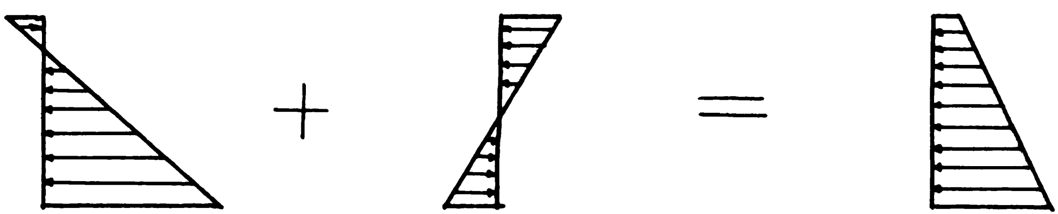
In the dead load state, that is when the beam is acted upon only by its weight, there will exist at the section considered a bending moment of value $-M_d$ which will cause a stress distribution.

$$f_d = -\frac{M_d y}{I} \quad \ldots\ldots\ldots\ldots \quad (7)$$

This is to be added algebraically to the unloaded prestress expression (6) to obtain the total stress.

That is $$f = H\left[\frac{ey}{I} - \frac{1}{A}\right] - \frac{M_d y}{I} \quad \ldots\ldots\ldots \quad (8)$$

The addition of stresses is shown in Fig. 4.



Prestress | Dead Load | Prestress + Dead Load

Fig. 4

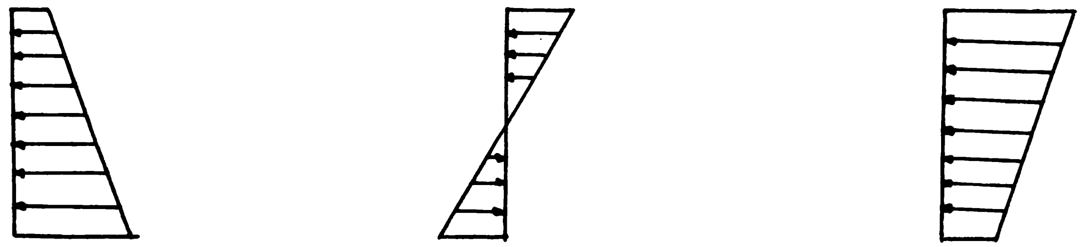
Further, the addition of a live load to the beam causes an additional bending moment $-M_L$ resulting in additional stresses.

$$f_L = -\frac{M_L y}{I} \quad \ldots\ldots\ldots\ldots \quad (9)$$

Adding this expression to that of (8) gives an expression for the stress at the upper limit of the range of loading conditions.

$$f = H\left[\frac{ey}{I} - \frac{1}{A}\right] - \frac{y}{I}\left[M_d + M_L\right] \quad \ldots \quad (10)$$

The result is shown in Fig. 5.



Prestress
+ Dead Load

Live Load

Prestress
+ Dead Load
+ Live Load

Fig. 5

The theory developed is applicable to all sections of a beam for any distribution of loading and for uniform or variable sections along the length of the beam.

The critical section of a prestressed beam, as in fact with other types of beam, will be generally that at the point of maximum bending moment. When designing a beam, therefore, the section of maximum bending moment must be considered first.

By inserting the values for the dead load and the live load bending moments at the critical section into the expressions (8) and (10) we obtain the stress distribution in terms of the unknown beam characteristics and, by noting what are the limiting stress values which can be tolerated in the concrete, these required beam characteristics may be found. The section of the beam will be used most efficiently if the maximum and minimum stresses in both limiting cases of loading are in fact the maximum and minimum allowable stresses.

When designing for the point of maximum bending moments, the limiting stress diagrams will be as shown in Fig. 6.

Prestress
+ Dead load

Prestress
+ Dead load
+ Live load

Fig. 6

<u>Symmetrical section</u>

It is readily seen from Fig. 6 that if the maximum working stress is $-f_c$ then the average stress on the section is $-1/2\ f_c$ and this is produced by the prestressing reaction $-H$ so that:

$$H = \tfrac{1}{2} f_c A \quad \ldots\ldots\ldots\ldots\ldots (11)$$

Under the condition of dead load only, Fig. 6 shows that the stress is zero at $y = +\frac{d}{2}$ at the top fibre, where "d" is the depth of the beam. Thus this may be included into expression (9) giving:

$$0 = H\left[\frac{ed}{2I} - \frac{1}{A}\right] - \frac{M_d\, d}{2I} \quad \ldots\ldots (12)$$

Similarly under the condition of dead load plus maximum live load the stress becomes zero at $y = -\frac{d}{2}$ at the bottom fibre. Thus expression (10) becomes:

$$0 = -H\left[\frac{ed}{2I} + \frac{1}{A}\right] + \frac{d}{2I}\left[M_d + M_L\right] \quad . \ (13)$$

Adding these two above expressions we have:

$$\frac{2H}{A} = \frac{M_L d}{2\ I} \quad \ldots\ldots\ldots\ldots\ldots (14)$$

and substituting the above expression (11) for H, (14) becomes:

$$f_c = \frac{M_L}{Z} \quad \ldots\ldots\ldots\ldots \quad (15)$$

If therefore, the shape of the section to be used is known, the section modulus Z will fix the dimensions of the section. It will further be noticed that the size of the beam section is dependent only upon the concrete stress and the live load bending moment. Also, theoretically the dead load does not influence the beam size, but only the cable eccentricity.

Rearranging expression (12) gives:

$$e = \frac{M_d}{H} + \frac{2I}{Ad} \quad \ldots\ldots\ldots\ldots \quad (16)$$

whilst rearranging expression (14) gives:

$$\frac{M_L}{H} = \frac{4I}{Ad} \quad \ldots\ldots\ldots\ldots \quad (17)$$

therefore this latter expression (17) may be inserted into the former (16) giving:

$$e = \frac{M_d}{H} + \frac{M_L}{2H} \quad \ldots\ldots\ldots\ldots \quad (18)$$

The expressions (11), (15) and (18) enable the values of the horizontal component of tension H, the maximum cable eccentricity -e and the beam section dimensions to be readily calculated from the known moment and stress values.

Unsymmetrical sections

There exist two distinct groups of unsymmetrical beams, owing to the fact that prestressed beams possess direction sense. The distinction lies in the position of the centroid.

Case 1

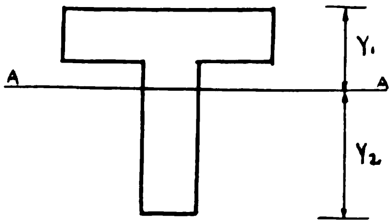


Fig. 7.

Consider first the case of an unsymmetrical section with the centroid displaced towards the upper fibre as shown in Fig. 7. In this case bending will cause a greater stress variation at the lower fibre and, since the greatest permissible stress range is from full working stress to zero stress, this must occur here.

The form of the stress diagrams will thus be as in Fig. 8.

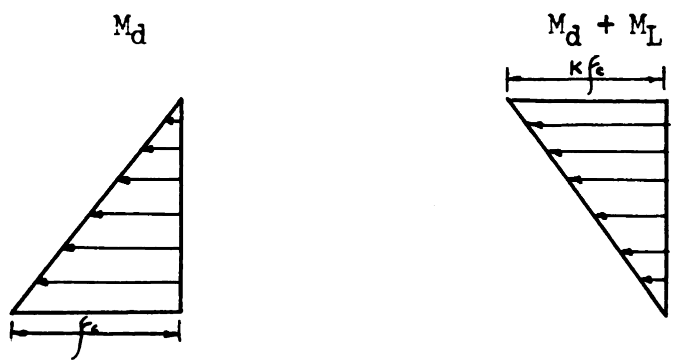


Fig. 8.

Using these limits we can substitute in the expressions (8) and (10).

From (8)

$$0 = H\left[\frac{ey_1}{I} - \frac{1}{A}\right] - \frac{M_d y_1}{I} \quad \ldots\ldots\ldots\ldots \quad (19)$$

$$-f_c = -H\left[\frac{ey_2}{I} + \frac{1}{A}\right] + \frac{M_d y_2}{I} \quad \ldots\ldots\ldots\ldots \quad (20)$$

From (10)

$$0 = -H\left[\frac{ey_2}{I} + \frac{1}{A}\right] + \frac{y_2}{I}\left[M_d + M_L\right] \quad \ldots\ldots\ldots \quad (21)$$

$$- Kf_c = H\left[\frac{ey_1}{I} - \frac{1}{A}\right] - \frac{y_1}{I}\left[M_d + M_L\right] \quad \ldots \quad (22)$$

where (20) and (21) give

$$f_c = \frac{y_2}{I} M_L \quad \ldots\ldots\ldots\ldots\ldots \quad (23)$$

and (19) and (22) give

$$Kf_c = \frac{y_1}{I} M_L \quad \ldots\ldots\ldots\ldots\ldots \quad (24)$$

whence

$$K = \frac{y_1}{y_2} \quad \ldots\ldots\ldots\ldots\ldots \quad (25)$$

Here we see again that the beam size is determined by the live load bending moment alone.

Multiplying (19) and (20) by y_2 and y_1 respectively and adding yields an expression for H.

$$H = \frac{Af_c y_1}{d} \quad \ldots\ldots\ldots\ldots\ldots \quad (26)$$

where $d = y_1 + y_2$ = beam depth.

Subtracting (21) from (19) yields an expression for the cable eccentricity which is easily cast into the form.

$$e = \frac{M_d}{H} + \frac{y_2}{d}\frac{M_L}{H} \quad \ldots\ldots\ldots\ldots\ldots \quad (27)$$

Case 2

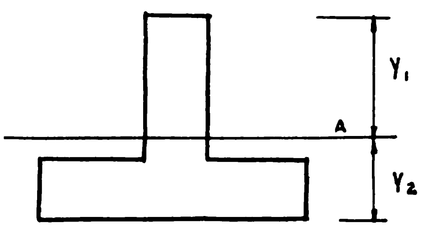


Fig. 9.

In the other case of non-symmetrical section where the centroid is displaced downwards (fig. 9) $y_1 > y_2$.

From the equations (8) and (10) we again obtain expressions for M_L, H, and e.

$$f_c = \frac{y_1 M_L}{I} \quad \ldots\ldots\ldots\ldots\ldots (28)$$

$$H = \frac{A y_2 f_c}{d} \quad \ldots\ldots\ldots\ldots (29)$$

$$e = \frac{M_d}{H} + \frac{y_2}{d}\,\frac{M_L}{H} \quad \ldots\ldots\ldots (30)$$

In both cases the expressions reduce to (11), (15) and (18) when the section becomes symmetrical.

Shear

The shear stress at any point is given by the expression

$$f_s = \frac{S}{Ib}\int_{y}^{y_1} yb\,dy \quad \ldots\ldots\ldots (31)$$

which reduces to the simple expression

$$f_s = \frac{3S}{2A} \quad \ldots\ldots\ldots\ldots\ldots (32)$$

for the maximum stress in a rectangular beam.

If f_s represents the shear stress, f the value of the longitudinal compressive stress, and f_T the principal tensile stress due to these stresses, then the usual statical analysis yields the well known formula

$$f_T = -\frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2} \quad \ldots (33)$$

According to Professor A. L. L. Baker special shear reinforcement is unnecessary provided that the principal tensile stress at failure does not exceed the concrete tensile strength.

Losses in prestress

Five kinds of losses in prestress usually occur:

1. Elastic compression of concrete caused by pretensioned bonded wires.
2. Shrinkage of concrete.
3. Creep of concrete.
4. Creep of steel wires.
5. Anchorage slip.

Little is known of these effects, particularly in relation to the improved materials in recent use. It is usual to allow a certain percentage of the initial prestress to cover these losses.

In case of pretensioning Kurt Billig of England suggests loss of 30,000 p.s.i. while Gustav Magnel of Belgium recommends to use a loss of 20 percent of initial prestress.

With posttensioning Billig's figure is 15,000 p.s.i. while Magnel allows 16 percent of initial prestress.

VII. DESIGN OF PRESTRESSED CONCRETE BRIDGE DECK

In the following pages the procedures suitable for the design of pretensioned and posttensioned prestressed reinforced concrete single span bridge deck will be presented. The designs do not give the best or the most economical solution but merely show the way to deal with the problem. Method used is approximate but its accuracy should suffice for most problems encountered in ordinary practice.

Description of Deck

The deck structure considered is simply supported and has a 66 ft. span. The highway is 26 ft. wide (two lanes) and is flanked by 2 ft. wide curb walks. The bridge deck is composed of 15, I-shaped girders. The bottom flange is 2 ft. wide and adjacent bottom flanges are placed close together. Width of top flanges is reduced and the deck is made to act integrally by filling the gaps between top flanges with carefully placed and vibrated high strength concrete. Sides of the top flanges are at an angle (see cross section) to further insure integral action and to make sure that no separation can occur in the joints. The deck slab is finally stressed laterally by tie rods with end bearing plates and nuts providing anchorage. The live load is H 15-44 and design requirements adhere in general to Standard Specifications for Highway Bridges, Fifth Edition, 1949 adopted by the American Association of State Highway Officials.

A.A.S.H.O. specifications which were written before consideration was given to prestressed concrete do not include requirements specifically intended for that type of construction. Therefore those requirements are taken from a proposal for a draft code of Practice for Prestressed Reinforced Concrete by Kurt Billig of England.

Specifications

Load - H 15-44

Roadway - 26 ft. (two lanes) with two 2 ft. curbs and overall width of 30 ft.

Depth of joist, not more than 26 inches.

Design for dead load, live load and impact.

Impact factor = $\frac{50}{66+125}$ = .26 (p.135 A.A.S.H.O.)

Maximum live load moment per lane (p.238 A.A.S.H.O.)
484.1 x 12 000 = 5 810 000 in lb.

Maximum live load shear per lane (p.238 A.A.S.H.O.)
35.3 x 1 000 = 35 300 lbs.

Maximum deflection allowed for live load plus impact.
1/800 times span (p.168 A.A.S.H.O.)

Concrete strength at time girder is subjected to prestress
5 000 p.s.i. (n = 6)

Allowable concrete stress in extreme fiber in compression
2 000 p.s.i.

No tensile concrete stresses are allowed anywhere on the cross section of the prestressed joist under any combination of design loads.

For check on cracking load, allowable concrete stress in extreme fiber in tension : 700 p.s.i.

Ultimate load moment not less than 2.5 (D.L. + L.L.)

First crack moment 1.5 (D.L. + L.L.)

Tensile strength of steel wire : 250 000 p.s.i.

Allowable initial steel stress : 0.6 x 250 000 = 150 000 p.s.i. (Magnel)

The wire diameter shall be 0.2 in. with a nominal cross-sectional area of 0.031 in^2.

Loss in prestress (pretensioned) due to all causes 20 percent of initial prestress.

Loss in prestress (posttensioned) shrinkage and creep 16 percent of initial prestress.

Diaphragms

Depth - same as depth of the girder

Width - 6 inches

Location - at center, quarter points and ends (5 per girder)

Cables - provide two cables in each diaphragm, one 7" below the top, the other 5" above the bottom of the girder.

Area of each cable - 0.37 sq. inch (12-0.2 in. wires)

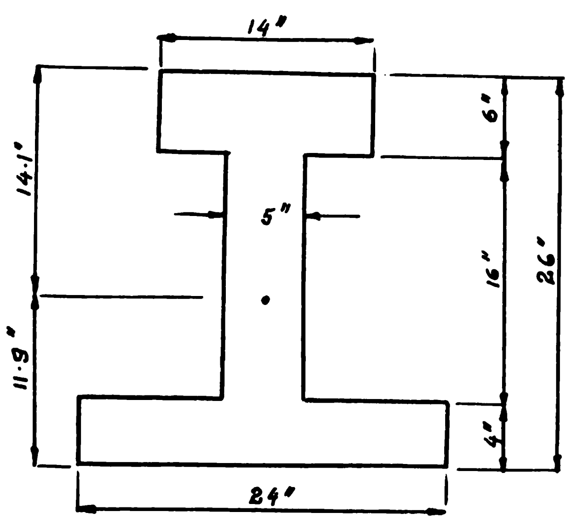
Prestress force in each cable 60 000 lb. initially.

A. Pretensioned Bridge Deck

Dimensions and properties of cross-section.

The most suitable layout cannot be computed solely by application of equations but must largely be based on experience and judgement. In our case it was assumed that shallow girders (d = 26") were required and the section shown in Fig. 10 was adopted.

Area	Statical Moment	Moment of Inertia
6 x 9 = 54	54 x 3 = 162	$54\left(\frac{6^2}{12} + 11.1^2\right) = 6\ 800$
5 x 26 = 130	130 x 13 = 1 690	$130\left(\frac{26^2}{12} + 1.1^2\right) = 7\ 500$
4 x 19 = 76	76 x 24 = 1 824	$76\left(\frac{4^2}{12} + 9.9^2\right) = 7\ 550$
A = 260	3 676	$Ar^2 = I = 21\ 850\ in^4$



$$y_1 = \frac{3\ 676}{260} = 14.1 \text{ in.}$$

$$y_2 = 26 - 14.1 = 11.9 \text{ in.}$$

$$r^2 = \frac{I}{A} = \frac{21\ 850}{260} = 84$$

$$\text{weight of girder} = \frac{150 \times 260}{144} = 270 \text{ lbs}$$

Fig. 10

Design Loads and Moments

$$M_g = \frac{66^2 x270x12}{8} = \underline{\underline{1\ 765\ 000}} \text{ in lb.}$$

cast in place concrete weighs $\frac{60x150}{144}$ = 63 lbs. per ft. of girder

$$M_{c.c.} = \frac{63x66^2 x12}{8} = \underline{\underline{411\ 000}} \text{ in lb.}$$

the curb and railing weigh 200 lb. per ft.

$$M_c = \frac{2x200x66^2 x12}{8} = 2\ 610\ 000 \text{ in lb.}$$

Curb and rails are placed after the deck has been concreted, therefore for simplification M_c is added to the live load moment

M_{LL} = 2x1.26x5 810 000 = 14 630 000 in lb.
M_c = 2 610 000 in lb.

M_{LL} = 17 240 000 in lb.

Live load moment per girder = $\frac{17\ 240\ 000}{15}$ = 1 148 000 in lb.

Total load moment per girder = $M_g + M_{cc} + M_{LL}$

1 765 000
411 000
1 148 000

M_t = 3 324 000 in lb.

Eccentricity

At support the gravity load moment is zero using (6) and equating it to zero (no tension allowed) we obtain: $e = \frac{I}{y_1 A} = \frac{r^2}{y_1}$

in our case $e = \frac{84}{14.1}$ = 5.96 inches

Stress checked for total design load moment.

$$\text{top} \quad f_c = \frac{M_t}{I} y_1 = \frac{3\ 324\ 000 \text{x} 14.1}{21\ 850} = 2\ 140 \text{ p.s.i.}$$

$$\text{bottom } f_c = \frac{M_t}{I} y_2 = \frac{3\ 324\ 000 \text{x} 11.9}{21\ 850} = 1\ 820 \text{ p.s.i.}$$

Section is fully developed. (Higher than allowable stress in top fiber will be reduced when higher I of composite section is substituted).

Determination of wire areas

Total design load moment M_t acting together with prestress force H applied with eccentricity 5.96 in. gives the critical stress at the bottom fiber. Using (10) and setting the bottom fiber stress equal to zero, which is the limiting value in the design specifications, gives:

$$H = \frac{M_t}{\frac{r^2}{y_2} + e} = \frac{3\ 324\ 000}{\frac{84}{11.9} + 5.96} = \mathbf{255\ 000\ lbs.}$$

allowing 20 percent for losses - initial prestress = $\frac{255\ 000}{0.8}$ = 319 000 lbs.

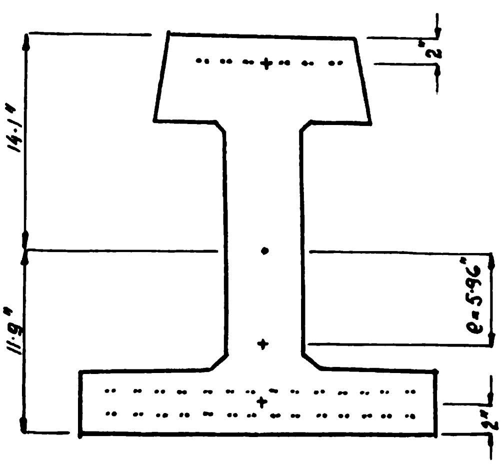


Fig. 11

$$\text{wire area} = \frac{319\ 000}{150\ 000} = 2.12 \text{ sq.in.}$$

$$\text{top wires area} = \frac{2.12}{22}\ 3.94 = 0.38 \text{ sq.in.}$$

$$\text{bottom wires area} = \frac{2.12}{22}\ 18.06 = 1.74 \text{ sq.in.}$$

Use 12 wires at top - 12x0.031 = 0.37 sq. in.

56 wires at bottom - 56x0.031 = 1.74 sq. in.

For arrangement of wires, see Fig. 11.

Investigation of stresses

For investigation of stresses transformed section could be used, giving more exact results. This refinement is not necessary however, because the use of gross section gives the discrepancy which is insignificant and a slight error is on the safe side.

Prestress

top -

initial $f_p = 319\ 000 \left[\frac{5.96x14.1}{21\ 850} - \frac{1}{260} = \right] 0$

final $f_p = 255\ 000 \left[\frac{5.96x14.1}{21\ 850} - \frac{1}{260} = \right] 0$

bottom -

initial $f_p = 319\ 000 \left[-\frac{5.96x11.9}{21\ 850} - \frac{1}{260} = \right] 2\ 260$ p.s.i. (comp)

final $f_p = 255\ 000 \left[-\frac{5.96x11.9}{21\ 850} - \frac{1}{260} = \right] 1\ 810$ p.s.i. (comp)

Prestress combined with D.L. of girder

top - $f_g = \frac{1\ 765\ 000x14.1}{21\ 850} = 1\ 140$ p.s.i. (comp)

bottom - $f_g = \frac{1\ 765\ 000x11.9}{21\ 850} = 965$ p.s.i. (ten)

Combined stresses

top - initial and final $f = 0 + 1\ 140 = 1\ 140$ p.s.i. (comp)

bottom - initial $f = 2\ 260 - 965 = 1\ 295$ p.s.i. (comp)

bottom - final $f = 1\ 810 - 965 = 845$ p.s.i. (comp)

Prestress combined with D.L. of girder and cast in place concrete

$$\text{top} - \quad f_{c.c.} = \frac{411\ 000 \times 14.1}{21\ 850} = 265 \text{ p.s.i. (comp)}$$

$$\text{bottom} - f_{c.c.} = \frac{411\ 000 \times 11.9}{21\ 850} = 224 \text{ p.s.i. (ten)}$$

Combined stresses

top - initial and final $f = 0 + 1\ 140 + 265 = 1\ 405$ p.s.i. (comp)

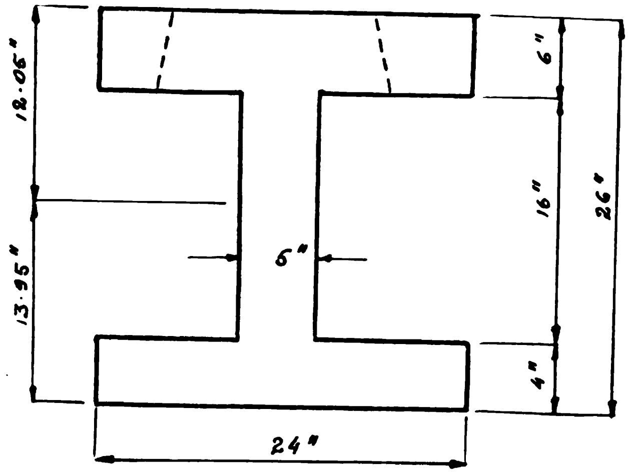
bottom - initial $f = 2\ 260 - 965 - 224 = 1\ 071$ p.s.i. (comp)

bottom - final $f = 1\ 810 - 965 - 224 = 621$ p.s.i. (comp)

Moment of inertia of composite section

After the cast in place concrete has gained sufficient strength and the lateral tie rods have been tightened, the entire concrete deck acts as an integral unit and a new moment of inertia is used to determine the stresses due to live load. (See Fig.12)

Area	Statical Moment	Moment of Inertia
6 x 19 = 114	114 x 3 = 342	$114 \left(\frac{6^2}{12} + 9.05^2 \right) = 9\ 680$
5 x 26 = 130	130 x 13 = 1 690	$130 \left(\frac{26^2}{12} + 0.95^2 \right) = 7\ 340$
4 x 19 = 76	76 x 24 = 1 824	$76 \left(\frac{4^2}{12} + 11.95^2 \right) = 10\ 940$
A = 320	3 856	I = 27 960 in^4



$$y_1 = \frac{3\ 856}{320} = 12.05 \text{ in.}$$

$$y_2 = 26 - 12.05 = 13.95 \text{ in.}$$

Fig. 12

Prestress combined with D.L. of girder, cast in place concrete and L.L.

top - $f_{L.L.} = \frac{1\ 148\ 000 \times 12.05}{27\ 960} = 495$ p.s.i. (comp)

bottom $f_{L.L.} = \frac{1\ 148\ 000 \times 13.95}{27\ 960} = 573$ p.s.i. (ten)

Combined stresses

top - initial and final $f = 0 + 1\ 140 + 265 + 495 = 1\ 900$ p.s.i. (comp)

bottom - initial $f = 2\ 260 - 965 - 224 - 573 = 498$ p.s.i. (comp)

bottom - final $f = 1\ 810 - 965 - 224 - 573 = 48$ p.s.i. (comp)

Exterior girder

Interior and exterior girders are alike. Stresses in an exterior girder due to prestress and girder weight are the same. Exterior girder is assumed however to carry the total weight of curb and railing instead of wheel loads.

Investigating stresses for design load:

$$M_c = \frac{200 \times 66^2 \times 12}{8} = 1\ 305\ 000 \text{ in lb.}$$

top - $f = \dfrac{1\ 305\ 000 \times 12.05}{27\ 960} = 562$ p.s.i. (comp)

bottom - $f = \dfrac{1\ 305\ 000 \times 13.95}{27\ 960} = 650$ p.s.i. (ten)

Combined stresses

top - initial and final $f = 0 + 1\ 140 + 265 + 562 = 1\ 967$ p.s.i. (comp)

bottom - initial $f = 2\ 260 - 965 - 224 - 650 = 421$ p.s.i.(comp)

bottom - final $f = 1\ 810 - 965 - 224 - 650 = 29$ p.s.i.(ten)

Stress distribution curves

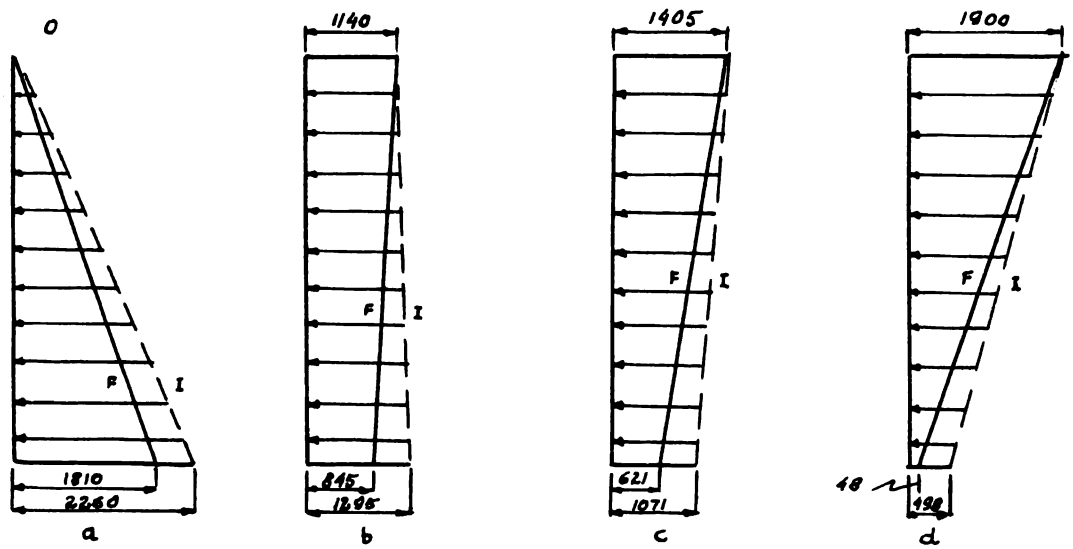


Fig. 13

a) Prestress alone

b) Prestress plus dead load of joist

c) Prestress plus total dead load

d) Prestress plus total dead and live load

I = initial F = final

Moment at first crack

$f_{cr} = 700 + 48 = 748$ p.s.i.

$M_{cr} = \dfrac{748\text{x}27\ 960}{13.95} = 1\ 496\ 000$ in lb.

M_t at working load M = 3 324 000 in lb.

M_t at cracking load M= 3 324 000 + 1 496 000 = 4 820 000 in lb.

Ratio of the two moments = $\dfrac{4\ 820\ 000}{3\ 324\ 000} = 1.45$

which is close to 1.5 called for in Swiss and English specifications.

Moment at ultimate load

$f_s = 250\ 000$ p.s.i.

$f'_c = 5\ 000$ p.s.i.

Plasticity ratio $\beta = \dfrac{1}{1 + \left(\dfrac{f'_c}{4\ 000}\right)^2} = \dfrac{1}{1 + \left(\dfrac{5\ 000}{4\ 000}\right)^2} = 0.39$

Steel ratio $p = \dfrac{A_s}{bd} = \dfrac{1.74}{14\text{x}23.5} = 0.00527$

Neutral axis ratio $k = \dfrac{2\ pf_s}{(1+\beta)f'_c} = \dfrac{2\text{x}0.00527\text{x}250\ 000}{(1+0.39)\ 5\ 000} = 0.380$

Moment arm ratio $j = 1 - \dfrac{1+\beta+\beta^2}{3(1+\beta)}k = 1\text{-}0.370\text{x}0.380 = 0.860$

Ultimate moment $M_u = A_s f_s jd = 1.74\text{x}250\ 000\text{x}0.860\text{x}23.5 = 8\ 800\ 000$ in lb.

Ratio = $\dfrac{M_u}{M_{d.L+L.L.}} = \dfrac{8\ 800\ 000}{3\ 324\ 000} = 2.65$

or $M_{ultimate} = 2.65$ times $M_{d.L.+L.L.}$

Deflection

Allowable L.L.+ Impact deflection $\Delta = \dfrac{66\text{x}12}{800} = 0.99$

Assume the modulus of elasticity of concrete $E_c = 5x10^6$ p.s.i.

Actual L.L.+ impact deflection $\Delta = \frac{5x1\ 148\ 000(66x12)^2}{48x5\ 000\ 000x27\ 960} = 0.54$

$0.54 < 0.99$

<u>Shear</u>

Shear stress is maximum at the centroid and is given by

$f_s = \frac{SQ}{bI}$ b = 5 in. I = 21 850 in^4

$Q = 6 \times 9\ (14.1 - \frac{6}{2}) + \frac{5x14.1^2}{2} = 600 + 496 = 1\ 096\ \ in^3$

live load shear = 35 300 lb.

curbs and rails shear = 200 x 33 = 6 600 lb.

41 900 lb.

live load shear per girder = $\frac{41\ 900}{7.5}$ = 5 587 lb.

dead load shear per girder 33 (270+63) = 10 989 lb.

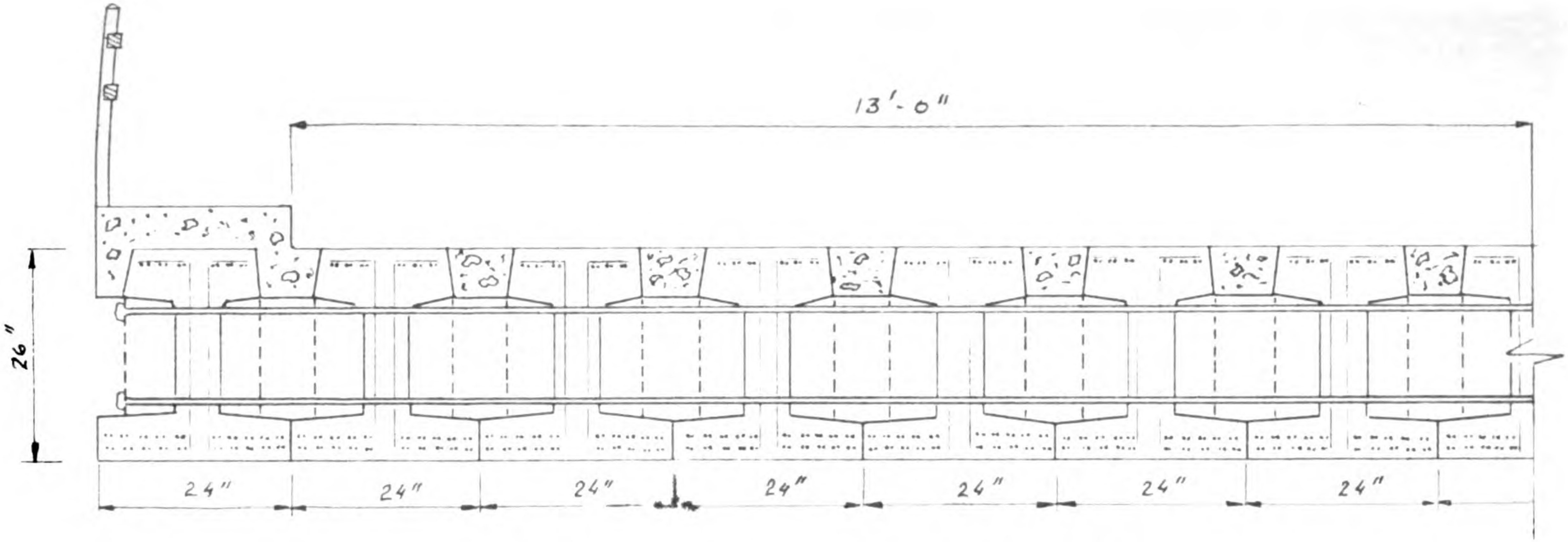
total end shear per girder S = 16 580 lb.

$f_s = \frac{16\ 580x1\ 096}{5x21\ 850} = 166$ p.s.i.

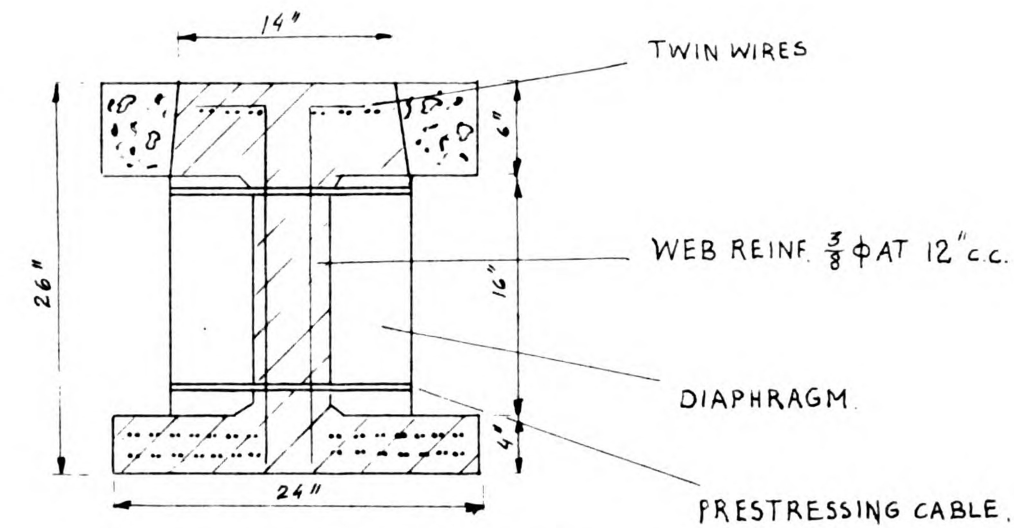
Horizontal compressive stress $f = \frac{255\ 000}{260} = 980$ p.s.i.

The stresses f_s and f produce a principal tensile stress which at the centroid at the support is given by:

$$f_T = -\frac{f}{2} + \sqrt{(\frac{f}{2})^2 + f_s^2} = -\frac{980}{2} + \sqrt{(\frac{980}{2})^2 + 166^2} = 27 \text{ p.s.i.}$$



CROSS SECTION THROUGH HALF OF THE DECK
66 FT. SPAN
PRETENSIONED.



CROSS SECTION THROUGH GIRDER

B. Posttensioned Bridge Deck

The design procedure for posttensioning is essentially the same as for pretensioning, but since the first method allows the use of curved cables which offer certain advantages, this will be illustrated in the next example.

Dimensions and properties of cross section

For better comparison, the same section is used as in pretensioning.

$A = 260$ sq. in.

$y_1 = 14.1$ in.

$y_2 = 11.9$ in.

$I = 21\ 850\ in^4$

Design Loads and Moments

Same as in pretensioning.

$M_g = 1\ 765\ 000$ in lb.

$M_{c.c.} = 411\ 000$ in lb.

$M_{L.L.} = 1\ 148\ 000$ in lb.

$M_t = 3\ 324\ 000$ in lb.

Eccentricity at mid Span

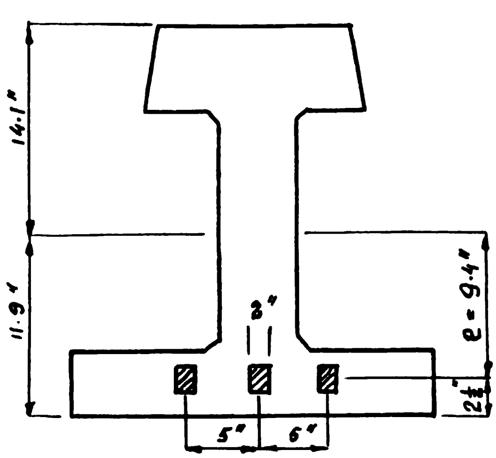


Fig.14

At mid span all cables are at the bottom (see Fig.14) and their center of gravity is assumed to be $2\frac{1}{2}$ inches above the girder soffit.

$$e = 11.9 - 2.5 = 9.4 \text{ in.}$$

Determination of wire areas

As before

$$H = \frac{M_t}{\frac{r^2}{y_2} + e} = \frac{3\ 324\ 000}{\frac{84}{11.9} + 9.4} = 202,200 \text{ lbs.}$$

allowing 16 percent for losses - initial prestress =

$$\frac{202\ 200}{0.84} = 240\ 800 \text{ lbs.}$$

$$\text{wire area} = \frac{240\ 800}{150\ 000} = 1.605 \text{ sq.in.}$$

Magnel cables will be used (0.2 in wires placed in layers of four with 3/16 in clear spacing).

$$\text{No. of wires required} = \frac{1.605}{0.031} = 52$$

Use three cables 20 wires each (total 60 wires).

Eccentricity at support

Two cables will be kept straight and only one will be curved.

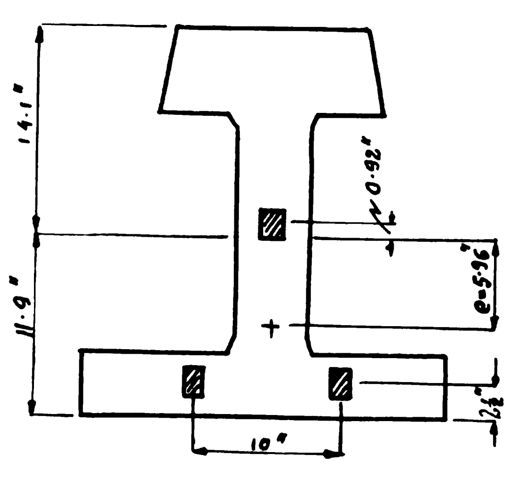


Fig.15

At support, in order to keep stress in the top fiber equal to zero

$$e = \frac{r^2}{y_1} = \frac{84}{14.1} = 5.96 \text{ inches}$$

position of curved cable:

$$-3x5.96 = -2x9.4 + 1\ X$$

X = +0.92 above the centroid of the section.

See Fig.15.

Investigation of stresses at mid span

Prestress

top -

initial $f_p = 240\ 800 \left[\frac{9.4 x 14.1}{21\ 850} - \frac{1}{260} \right] = 532$ p.s.i. (ten)

final $f_p = 202\ 200 \left[\frac{9.4 x 14.1}{21\ 850} - \frac{1}{260} \right] = 446$ p.s.i. (ten)

bottom -

initial $f_p = 240\ 800 \left[-\frac{9.4 x 11.9}{21\ 850} - \frac{1}{260} \right] = 2\ 160$ p.s.i. (comp)

final $f_p = 202\ 200 \left[-\frac{9.4 x 11.9}{21\ 850} - \frac{1}{260} \right] = 1\ 815$ p.s.i. (comp)

Prestress combined with D.L. of girder

top - $f_g = 1\ 140$ p.s.i. (comp)

bottom - $f_g = 965$ p.s.i. (ten)

Combined stresses

top - initial $f = -532 + 1\ 140 = 608$ p.s.i. (comp)

final $f = -446 + 1\ 140 = 694$ p.s.i. (comp)

bottom- initial $f = +2\ 160 - 965 = 1\ 195$ p.s.i. (comp)

final $f = +1\ 815 - 965 = 850$ p.s.i. (comp)

Prestress combined with D.L. of girder and cast in place concrete

top - $f_{c.c.} = 265$ p.s.i. (comp)

bottom - $f_{c.c.} = 224$ p.s.i. (ten)

Combined stresses

top - initial $f = 608 + 265 = 873$ p.s.i. (comp)

final $f = 694 + 265 = 959$ p.s.i. (comp)

bottom- initial $f = 1\ 195 - 224 = 971$ p.s.i. (comp)

final $f = 850 - 224 = 626$ p.s.i. (comp)

<u>Prestress combined with D.L. of girder, cast in place concrete and L.L.</u>

top - f_{LL} = 495 p.s.i. (comp)

bottom - f_{LL} = 573 p.s.i. (ten)

<u>Combined stresses</u>

top - initial f = 873 + 495 = 1 368 p.s.i. (comp)

final f = 959 + 495 = 1 454 p.s.i. (comp)

bottom - initial f = 971 - 573 = 398 p.s.i. (comp)

final f = 626 - 573 = 53 p.s.i. (comp)

<u>Exterior girder</u> (check for stresses)

top - f_{LL} = 562 p.s.i. (comp)

bottom - f_{LL} = 650 p.s.i. (ten)

<u>Combined stresses</u>

top - initial f = 873 + 562 = 1 435 p.s.i. (comp)

final f = 959 + 562 = 1 521 p.s.i. (comp)

bottom - initial f = 971 - 650 = 321 p.s.i. (comp)

final f = 626 - 650 = 24 p.s.i. (ten)

<u>Stress distribution curves</u>

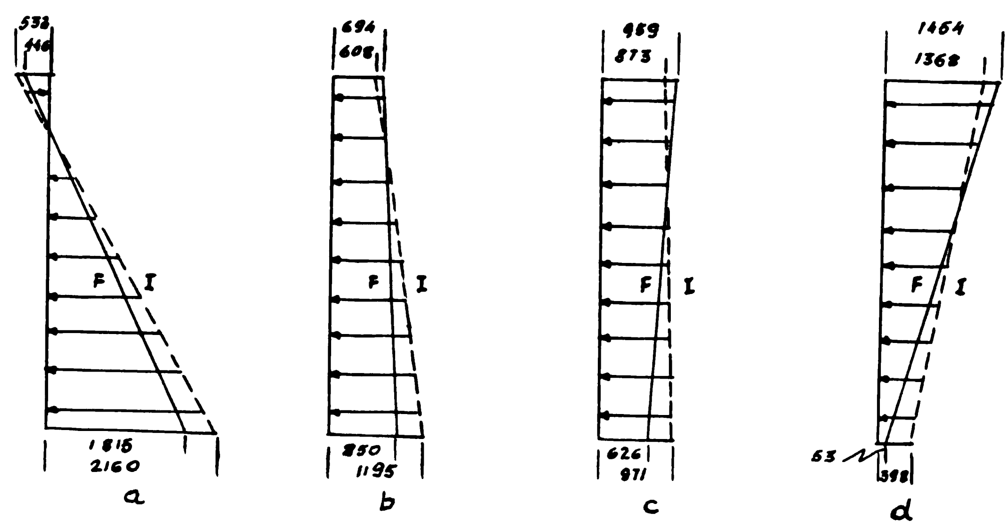


Fig. 16

(Fig.16)

a) Prestress alone

b) Prestress plus dead load of joist

c) Prestress plus total dead load

d) Prestress plus total dead and live load

I = initial F = final

Moment at first crack

$f_{cr} = 700 + 53 = 753$ p.s.i.

$M_{cr} = \frac{753 \times 27\ 960}{13.95} = 1\ 510\ 000$ in lb.

M_t at working load M = 3 324 000 in lb.

M_t at cracking load M = 3 324 000 + 1 510 000 = 4 834 000 in lbs.

Ratio of the two moments = $\frac{4\ 834\ 000}{3\ 324\ 000} = 1.46$

which is close to 1.5 called for in Swiss and English specifications.

Moment at ultimate load

Plasticity ratio $\beta = 0.39$

Steel ratio $p = \frac{A_s}{bd}\ \frac{60 \times 0.031}{14 \times 23.5} = 0.00565$

Neutral axis ratio $k = \frac{2\ pf_s}{(1+\beta)f'_c} = \frac{2 \times 0.00565 \times 250\ 000}{(1+0.39)\ 5\ 000} = 0.406$

Moment arm ratio $j = 1 - \frac{1+\beta+\beta^2}{3(1+\beta)}\ k = 1 - 0.37 \times 0.406 = 0.85$

Ultimate Moment $M_u = A_s f_s jd = 1.86 \times 250\ 000 \times 0.85 \times 23.5 = 9\ 270\ 000$ in lb.

Ratio $= \frac{M_u}{M_{DL+L.L.}} = \frac{9\ 270\ 000}{3\ 324\ 000} = 2.8$

or $M_{ultimate} = 2.8$ times $M_{D.L+L.L.}$

Shear

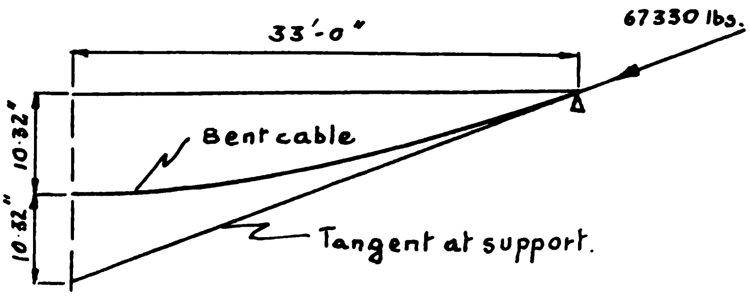


Fig. 17

$I = 21\ 850\ in^4$

$b = 5$ in.

$Q = 1\ 096\ in^3$

$S = 16\ 580$ lb.

In this case the vertical component of the tension in the bent cable which is directed downward will reduce the end shear due to vertical loading which is acting upward.

Permanent tension in single cable = $\frac{202\ 200}{3} = 67\ 330$ lb.

$$\text{slope} = \frac{2 \times 10.32}{12 \times 33} = .0575$$

$$S_1 = 67\ 330 \times .0575 = 3\ 880 \text{ lbs}$$

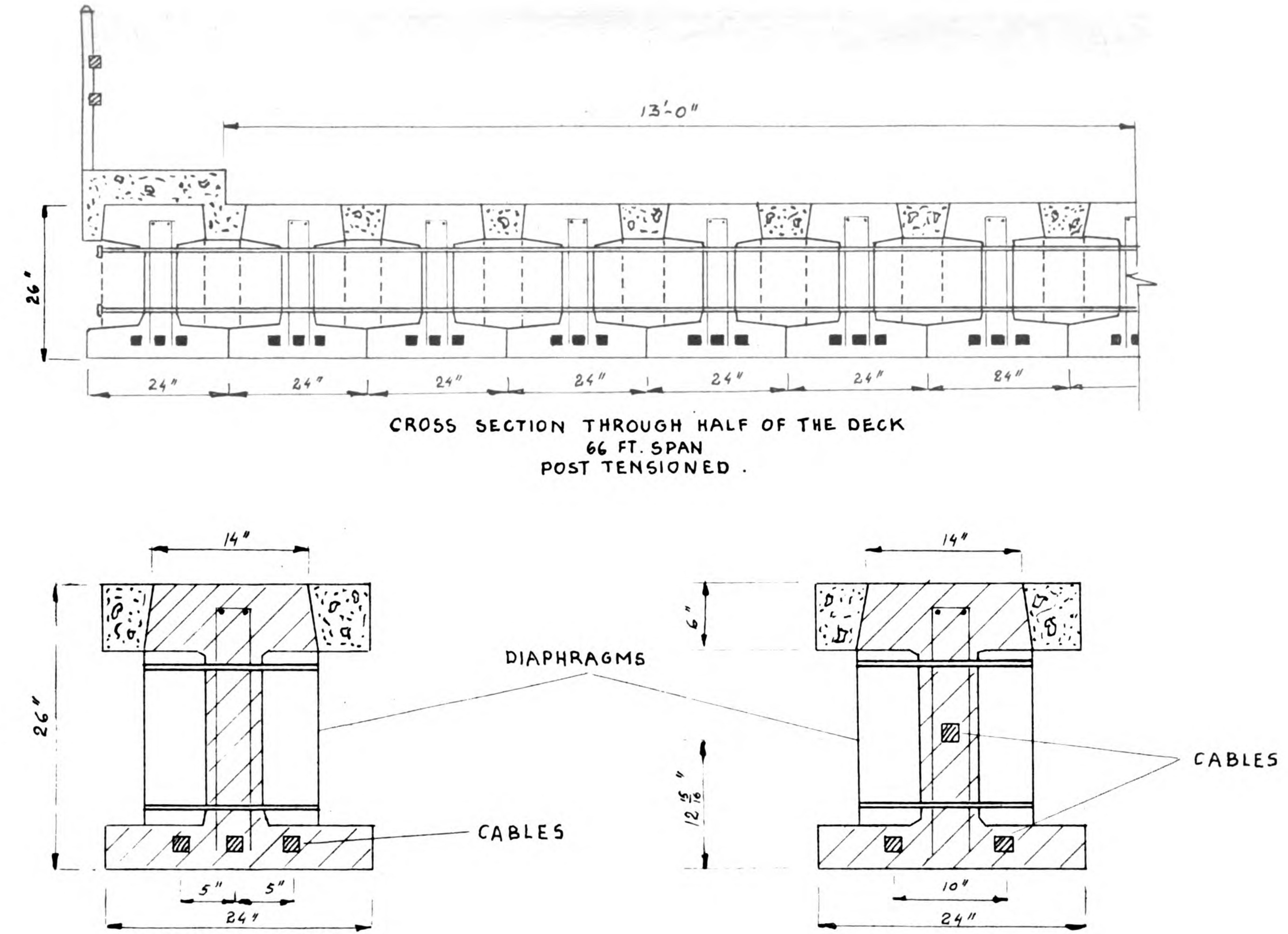
$$f_s = \frac{(S - S_1)\ Q}{bI} = \frac{12\ 700 \times 1\ 096}{5 \times 21\ 850} = 127 \text{ p.s.i.}$$

$$\text{Horizontal compressive stress } f = \frac{202\ 200}{260} = 780 \text{ p.s.i.}$$

The stresses f_s and f produce a principal tensile stress given by:

$$f_T = -\frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2} = -\frac{780}{2} + \sqrt{\left(\frac{780}{2}\right)^2 + 127^2} = 20 \text{ p.s.i.}$$

Comparing with pretensioning, this is a 26 percent reduction in shear stress.



CROSS SECTION THROUGH HALF OF THE DECK
66 FT. SPAN
POST TENSIONED.

CROSS SECTION THROUGH GIRDER AT CENTER

CROSS SECTION THROUGH GIRDER AT SUPPORT.

VIII. DISCUSSION AND CONCLUSIONS

Prestressed Concrete has many advantages as compared with ordinary reinforced concrete. Some of them are:

1. Entire absence of permanent cracks
2. Reduction in depth of section which is important in floor and bridge construction.
3. Reduction of the weight of structure
4. Increased economy
   a) Saving of steel
   b) Saving of concrete
   c) Reduction in maintenance cost
   d) Increased life of structure
5. Low shearing stresses
6. Elimination of excessive deflections

There are certain advantages and disadvantages in using either method of tensioning.

The pretensioning is most suitable for units of small cross section which could not easily accomodate the comparatively bulky posttensioning cable. The system is also well adapted to the mass production of large members of similar units, such as railway sleepers, floor joists, beams, poles, piles, etc., when it is found to be very economical. It has however, certain disadvantages which make its use more limited than that of the other method in the case of very large members. The wires must be straight, so that the shear-resisting properties of curved-up cables are not enjoyed. With this method loss of prestress could occur from shrinkage of the concrete as well as elastic deformation and creep in the steel and the concrete.

One fundamental advantage of posttensioning is that, as the reaction from stressing the wires is taken on the concrete, there is no loss of stress due to elastic deformation, as with pretensioning. Furthermore as the concrete has hardened, the shrinkage in the concrete has already taken place, leaving only creep losses in the steel and concrete.

Another advantage of posttensioning is that the wires may be bent upwards towards the support, giving an active vertical component of the prestressing force acting against the shear force and enabling high shear loads to be taken. On the basis of our design we notice that comparing with pretensioning a reduction of prestress force and steel area and an increase in ultimate moment results when the wires are posttensioned. It is also seen that the bent cable causes a substantial reduction in the total vertical shear on the section. The main disadvantage of posttensioning is the limitation to size. If the members are too small the cost of prestressing may not be worth-while, for in every case anchorages are required at each end of the cable and jacking costs are unaffected by the length of cable. Therefore, for short spans pretensioning is believed to be more economical, but as the spans become longer, the conditions tend to favor posttensioning.

APPENDIX A

Notation

A = Area of concrete

A_s = Area of steel

b = Width of beam

d = Depth of beam

E = Elastic modulus with various subscripts

e = Effective cable eccentricity

F = Cable force

f = Stress, with various subscripts

H = Horizontal component of cable force

I = Second moment of area of section

j = Moment arm ratio

K = Stress ratio

k = Neutral axis ratio

l = Beam length

M = Bending moment, with various subscripts

p = Steel ratio

r = radius of gyration

S = Shear force with various subscripts

y = Distance from centroid, with various subscripts

Z = Section modulus

θ = Cable inclination

β = Plasticity ratio

APPENDIX B

Summary of Proposed Design Specifications for Prestressed Concrete

		Kurt Billig England (1)	Ritter & Lardy Switzerland (2)	Gustave Magnel Belgium (3)
1.	Min. cyl. strength at release of prestress	$2/3f'_c$ = 3200 psi.	$2/3f'_c$ = 4400 psi.	f''_c
2.	Min. cyl. strength at application of load	f'_c = 4 800 psi.	f'_c= 6 600 psi.	f'_c = 5 100 psi.
3.	Allow. compr. stress at release of prestress	$0.4f'_c$	$0.4f'_c$	$0.45\ f''_c$
4.	Allow. compr. stress at application of load	$0.4f'_c$	$0.4f'_c$	$0.33f'_c$
5.	Allow. tensile stress without special reinf.	10 percent of allow.compr. stress for special cases	-	10 percent of allow.compr.stress
6.	Allow. tensile stress with special reinf.		-	20 percent of allow.compr.stress
7.	Modulus of rupture	$0.18f'_c$	$0.15f'_c$	-
8.	Modulus of elasticity	$4x10^6$ psi.	$5.7x10^6$ psi.	-
9.	Max. coeff.of shrinkage	0.0003 in./in.	0.0005 in./in.	-
10.	Max. w/c ratio	0.45 (5 gal. per sack)	-	-
11.	Max. slump	1 in.	-	-
12.	Shear and bond	$0.04f'_c$ but 250 psi.	-	-
13.	Allow. principal tensile stress at N.A.	-	115 psi.	10 percent of allow.compr.stress
14.	Length of ordinary moist curing	8 days	-	-
15.	Ultimate strength of wire	f_s = 200 000-250 000 psi.	f_s = 215 000 psi.	-
16.	Yield point 0.2% residual strain	$f_y = 0.7f_s$	$f_y = 0.8f_s$	f_y
17.	Max. initial prestress in bonded wire	$0.85f_y$ or $0.70f_s$	$0.85f_y$ or $0.70f_s$	-
18.	Max. initial prestress in bondless wire	$0.75f_y$ or $0.60f_s$	-	$0.8f_y$ or $0.6f_s$
19.	Ultimate elongation of steel	5 percent	-	-
20.	Loss in prestress, bonded wire, shrinkage and creep	20 000 psi.	-	16 percent of initial prestress
21.	Loss in prestress, bonded wire, due to all causes	30 000 psi.	-	20 percent of initial prestress
22.	Loss in prestress, posttensioned,shrinkage and creep	15 000 psi.	-	16 percent of initial prestress
23.	Ultimate moment	= 2.5 (DL+LL)	= 2.0(DL+LL) = 2.5 (LL)	-
24.	Cracking moment	= 1.5 (DL+LL)	= 1.5(DL+LL)	-

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