TRANSIENT ANALYSIS OF A G. E.
SERVO-DEMONSTRATION UNIT

Thesis for the Degree of M. S.
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Howard Edmonds Gerlaugh
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THESIS

This is to certify that the

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Transient Analysis of A G.E.

Servo-Demonstration Unit

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Major professor

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TRANSIENT ANALYSIS OF A
G. E. SERVO-DEMONSTRATION UNIT

By

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I. Introduction

Since the development of large sources of power, man has become increasingly interested in its control so that it may be made to perform more useful and exacting work. In particular it is the starting, stopping, and governing of these forces that are included in the field of "control."

Systems which regulate the flow of energy may be classed into four broad classes: open cycle, closed cycle, discontinuous, and continuous. In all of these different types the amount of control energy bears no relation to the power released or governed.

If an electric motor is connected to a source of power and set in motion or stopped by closing or opening a switch, we have a simple discontinuous control. The system is discontinuous in that the current to the motor is completely on or off and there are no intermediate positions possible. If we replaced the switch with a rheostat, the system would become continuous because now the power supplied to the motor has a range values from fully on to off.

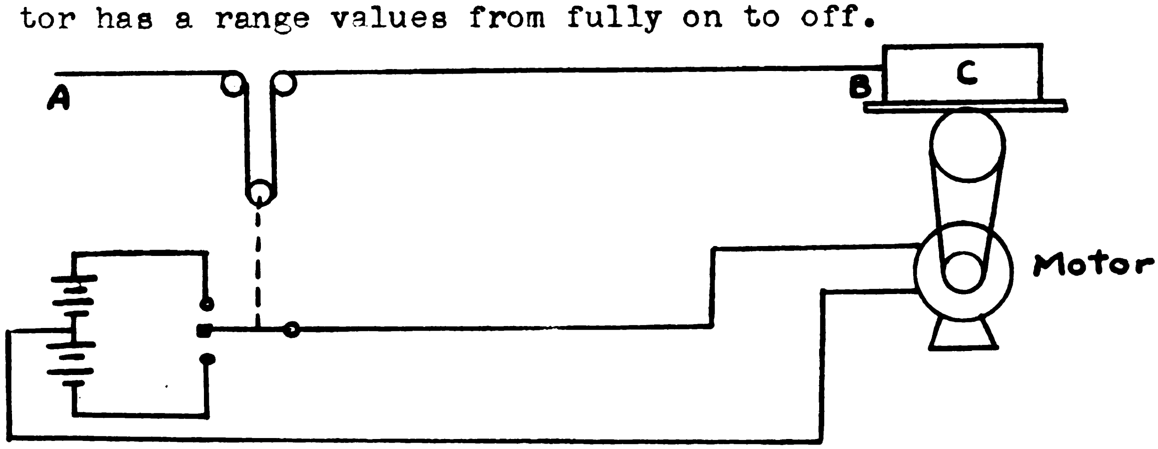
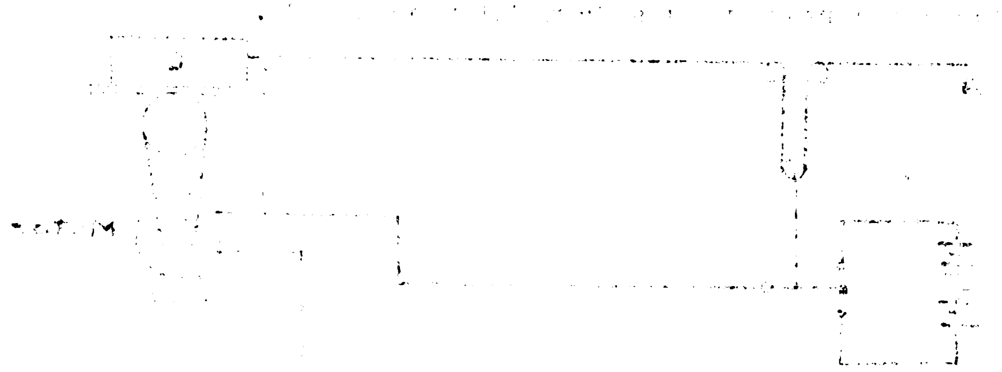


Figure 1

Howard E. Gerlaugh



In the system shown in Figure 1 the switch controlling the direction of rotation of the motor is operated by the difference in position between points A & B and some reference. In this system the point C is made to follow the point A by letting the error or deviation control the rotation of the motor; thus a complete loop or cycle is present. For this reason the control is a closed cycle system and at the same time discontinuous since the switch either turns the motor completely on or off. If a slide wire rheostat, with the sliding arm connected to the differential, were to replace the switch in the above control, it would become closed cycle and continuous.

A servo mechanism consists of an input member, an output member, a differential device which compares the instantaneous positions of the input and output, a controller consisting of an amplifier and output drive motor, and a stabilizing device. These components are so arranged that if the position of the input were $x(t)$ and the output $y(t)$, $x(t)$ would equal $ky(t)$.

It shall be the purpose of this paper to describe a typical servo system, develop a mathematical expression for its performance, determine the constants of the elements, and to compare the calculated characteristics with those determined experimentally.

II. Description of System Analyzed

The analysis which follows was made on a closed cycle, continuous position control servo system. The system was

manufactured by General Electric and is designated as the "Amplidyne Servo Demonstrator". T

The block diagram of the system is shown in Figure 2.

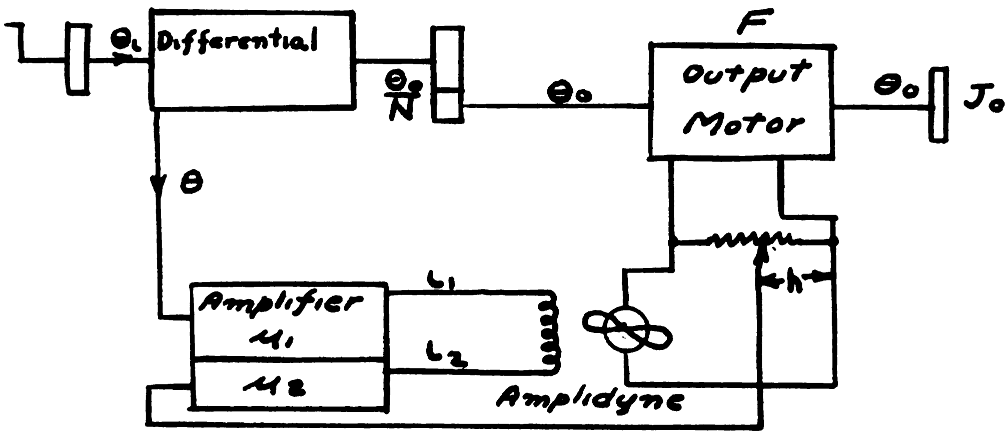


Figure 2.

The input shaft whose angular position is θ_1 is connected to a selsyn generator is converted to three single phase 60 cycle output voltages whose amplitudes are proportional to the cosine of the angle between the magnetic axis of the rotor and stator. The angular position of the output motor shaft θ_0 is connected through a gear train with a gear ratio of 24:1 to a selsyn transformer. The three output voltages of the selsyn generator is connected to the stator of the transformer and the output of the rotor is a 60 cycle voltage whose amplitude is proportional to the sine of the angle between the generator rotor and transformer rotor. The combination of the selsyn generator and transformer are indicated in Figure 2 as a differential whose output signal is proportional to the error θ, where θ is the difference between θ_1 and $\frac{\theta_o}{N}$.

The error voltage is fed into an amplifier of gain u_1 which in turn supplies the field current of the amplidyne.

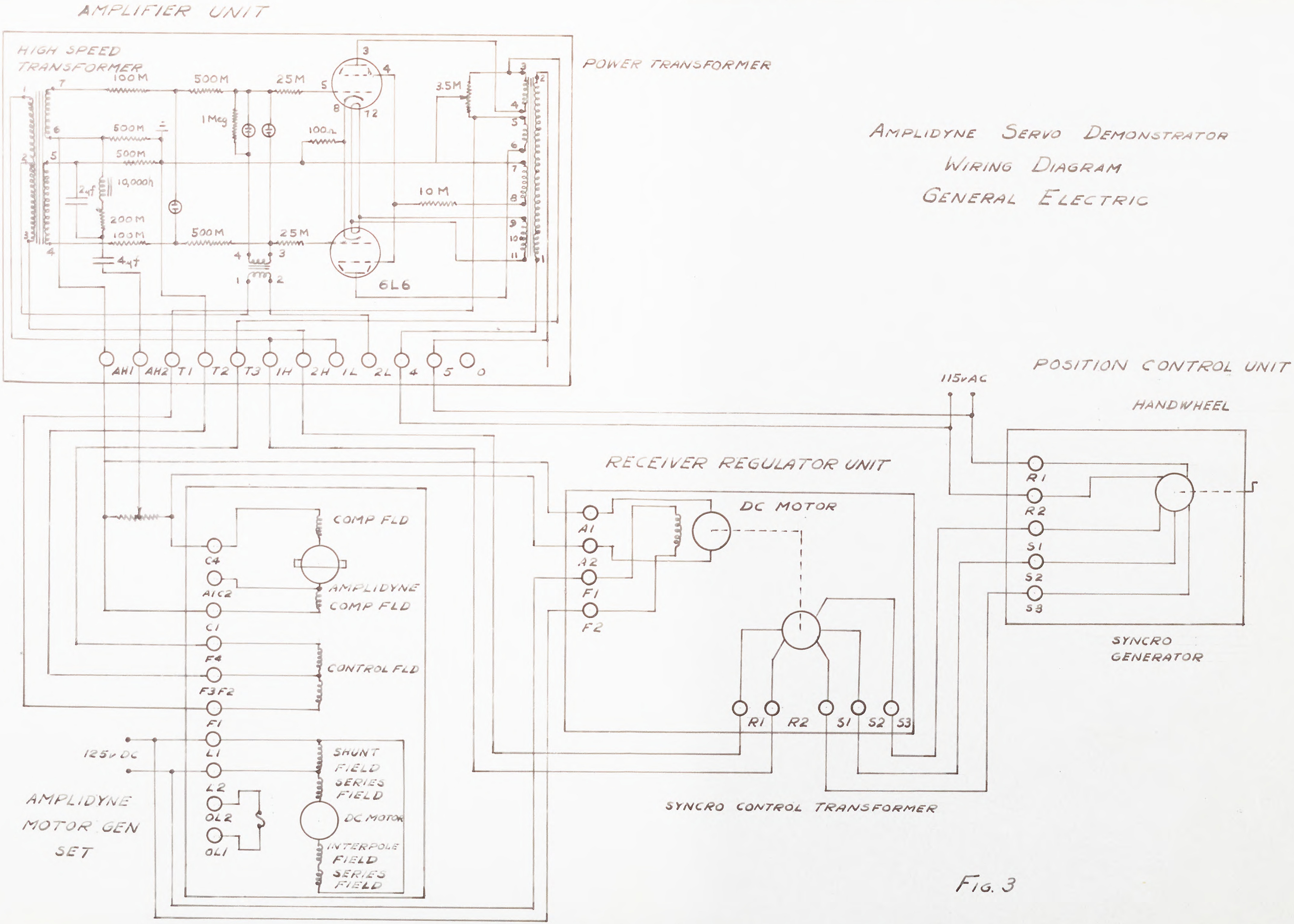

AMPLIDYNE SERVO DEMONSTRATOR
WIRING DIAGRAM
GENERAL ELECTRIC
AMPLIFIER UNIT
HIGH SPEED TRANSFORMER
POWER TRANSFORMER
100M
500M
25M
1 Meg
100Ω
3.5M
500M
500M
10,000h
2μf
200M
100M
4μf
10M
500M
25M
6L6
AH1
AH2
T1
T2
T3
1H
2H
1L
2L
4
5
0
115v AC
POSITION CONTROL UNIT
HANDWHEEL
R1
R2
S1
S2
S3
SYNCRO GENERATOR
RECEIVER REGULATOR UNIT
DC MOTOR
A1
A2
F1
F2
R1
R2
S1
S2
S3
SYNCRO CONTROL TRANSFORMER
COMP FLD
AMPLIDYNE
COMP FLD
CONTROL FLD
C4
A1C2
C1
F4
F3F2
F1
L1
L2
OL2
OL1
125v DC
SHUNT FIELD
SERIES FIELD
DC MOTOR
INTERPOLE FIELD
SERIES FIELD
AMPLIDYNE MOTOR GEN SET

Fig. 3

The complete wiring diagram is given in Figure 3. The amplifier is shown to consist of a pair of 6L6 tubes operating as half wave grid controlled rectifiers, the grid potentials being controlled by the error signal. An error signal of a given A.C. polarity increases the grid potential and output of one tube and decreases the output of the other tube by a like amount. An A.C. voltage of opposite polarity reverses the tube system. The plate voltage on the tubes is supplied from the same 60 cycle source as the selsyns since it serves as a reference to determine the A.C. polarity of the error signal. The neon lamps across the grid circuits of the tube serve to provide overload protection. The feedback circuit of the amplifier returns a portion of the voltage across the armature of the output motor through a differentiating circuit. This provides feedback for the amplifier with a gain of u_2.

The output of the amplidyne is connected to the armature of the motor. The voltage is a function of the error and also of the rate of change in error. When the rate of change of error is maximum the degenerative feedback voltage is largest, decreasing the armature voltage of the motor and preventing the output shaft from overshooting the zero error point as far as it normally would with out feedback. The output angle θ_0 is measured from the motor shaft and is connected through a step down gear train to the selsyn transformer.

The moment of inertia of the output is designated.

J_o and includes that of the motor and associated gear train. The resistor across the terminals of the output motor armature permits adjustment of the feedback voltage. The ratio h represents the ratio of the feedback voltage to the total armature voltage.

In operation the output shaft, where the angle is $\frac{\theta_o}{N}$, is made to follow the input shaft with an angle of θ_i. The accuracy of correspondence during transient and steady state conditions is dependent upon the constants of the system. A quantitative analysis of the system follows.

III. Derivation of Equations.

If the input shaft displacement is made a sinusoidal function of time, the output shaft displacement and the difference between the two or error will also be a sinusoidal function of time with the same frequency but of different amplitudes and phase angles with one another. These functions are shown vectorially in Figure 4.

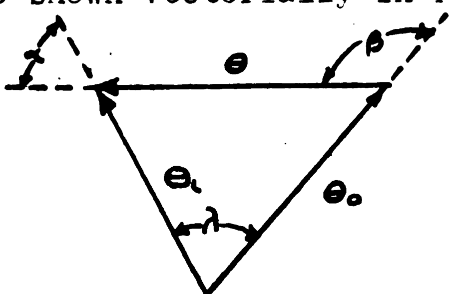


Figure 4

In Figure 4 we see that

$$\theta = \theta_i - \theta_o \tag{1}$$

and that the output vector θ_o lags behind the input vector θ_i by the phase angle λ. α and β are the angles between the error θ and input θ_i vectors and the error θ and output

θ_o vectors respectively. From these geometric considerations, it is seen that the relation between the output displacement and the input displacement can be expressed by the equation

$$\frac{\theta_o}{\theta_i} = Ae^{j\lambda}. \tag{2}$$

Similarly the relation between the output and error is

$$\frac{\theta_o}{\theta} = Be^{j\beta} \tag{3}$$

and the error and input

$$\frac{\theta}{\theta_i} = Ce^{j\alpha}. \tag{4}$$

Using the above relations

$$Be^{j\beta} = \frac{\theta_o}{\theta} = \frac{\theta_i - \theta}{\theta} = \frac{\theta_i}{\theta} - 1 \tag{5}$$

from which

$$\frac{\theta_i}{\theta} = 1 + Be^{j\beta} \longrightarrow \frac{\theta}{\theta_i} = \frac{1}{1+Be^{j\beta}} = Ce^{j\alpha}. \tag{6}$$

Thus knowing $\frac{\theta_o}{\theta}$, it becomes possible to calculate θ/θ_1 , and follows that

$$\frac{\theta_o}{\theta_i} = Ae^{j\lambda} = \frac{\theta_o}{\theta} \times \frac{\theta}{\theta_i} = \frac{Be^{j\beta}}{1+Be^{j\beta}}. \tag{7}$$

The term $Be^{j\beta}$ is given the name of transfer function of a servo system since it relates the ratio between θ_o and θ_1. In the derivation which follows the transfer function will be developed for the G. E. Servo Demonstrator.

In the first case the feedback ratio h of the system will be made zero. The relation between the error θ and the total amplidyne field current I, is

$$I_1 = \frac{\mu_1 \theta}{1+pT_1} \tag{8}$$

where u_1 is the gain of the amplifier in ma./radian taken with respect to the output motor shaft, p is the Heaviside operator equal to the first derivative with respect to time,

and T_1 is the time constant of the amplifier field L_1/R_1 with the units of seconds. The product of u_1 and θ give the resultant field current in milliamperes. When the differential equation is solved it has the form

$$I_1 = u_1 \theta \left(1 - e^{-\frac{t}{T_1}}\right). \tag{8a}$$

In equation 8 we assume that the amplification of the amplifier u_1 is constant over range of error found in the system; in the tests which follow this is found to be true.

The torque of a D.C. shunt motor, such as is shown in Figure 3 to be employed in this system, is proportional to the field strength and armature current. In this servo system the field is held constant so that the torque is equal to a constant times the armature current. The output current I_3 of the amplidyne in terms of the field current I_1 may be given by the relation

$$I_3 = \frac{K_1 I_1}{1 + pT_2} \tag{9}$$

where K_1 has the units of amperes/ ma. and T_2 is the time constant of the armature of the amplidyne L_2/R_2. The expression for the torque of the output motor may be immediately written

$$T = K_2 I_3 = \frac{K_2 K_1 I_1}{1 + pT_2} \tag{10}$$

where K_2 is a constant with the units of ft.-lbs./amp.

Let

$$K = K_1 K_2, \tag{11}$$

then

$$T = \frac{K I_1}{1 + pT_2} = \frac{K u_1 \theta}{(1 + pT_1)(1 + pT_2)}. \tag{12}$$

[illegible]

[illegible]

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[illegible]

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[illegible]

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the torque T having the units of ft.-lbs. Also the torque of the motor is related to the moment of inertia and friction of the output system by the equation

$$T = J_o p^2 \theta_o + F p \theta_o \tag{13}$$

J_o having the units of slug-ft.^{2} and the friction coefficient F, the units of ft.-lbs./radian/sec. Equating 12 and 13

$$\frac{K \mu_1 \theta}{(1+pT_1)(1+pT_2)} = (J_o p^2 + F p)\theta_o \tag{13a}$$

and solving for the transfer function $\frac{\theta_o}{\theta}$ we have

$$\frac{\theta_o}{\theta N} = B e^{j\beta} = \frac{K \mu_1}{Np(1+pT_1)(1+pT_2)(J_o p + F)} \tag{14}$$

By dividing 14 by the gear ratio N, measurements of the output displacement may be taken at the selsyn transformer.

For equation 13 to predict accurately the performance of the servo the output current of the amplidyne must be directly proportional to the field current and the torque be a linear function of the armature current. These assumptions are true until saturation takes place in the former, in the field of the amplidyne, and in the latter, in the armature of the motor.

In the case where h has some finite value giving the amplifier degenerative feedback, the equation for the transfer function becomes much more complicated. As the first step, if the voltage across the armature of the motor is taken as E_o as shown in Figure 5, the voltage at the output terminals of the amplifier E due to the feedback hE_o at its input is given by the expression

$$E_2 = \frac{h E_o \mu_2 (1+pRC)}{1+pRC-\mu_2 h p RC} \tag{15}$$

where u_2 is the amplification of the feedback voltage in the amplifier with the units of volts/volt and R and C are the fundamental elements in the differentiating circuit.

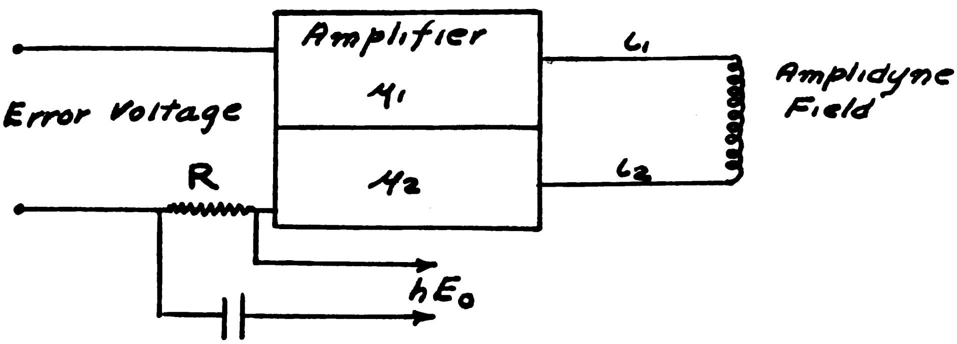


Figure 5

As before the component of the field current due to the error voltage is

$$I_1 = \frac{\mu_1 \theta}{1 + pT_1}. \qquad (8)$$

The component of the amplidyne field currents due to the feedback voltage is

$$I_2 = \frac{E_2}{R_1(1+pT_1)} = \frac{hE_0\mu_2(1+pRC)}{R_1(1+pT_1)[1+pRC(1-\mu_2 h)]}. \qquad (16)$$

The component of the output voltage of the amplidyne due to I, is as in equation 9

$$E_{g1} = \frac{K_1 I_1}{1+pT_2} \qquad (9)$$

and the component due to I_2 is

$$E_{g2} = \frac{K_1 I_2}{1+pT_2} \qquad (17)$$

Using the superposition theorem the amplidyne's total generated voltage Egt may be expressed in the relation

$$E_{gt} = E_{g1} + E_{g2} \qquad (18)$$

The voltage at the output terminals is seen to be

$$E_o = E_{gt} - I_3 R_2 \tag{19}$$

where I_3 is the output current. E_o may also be expressed in terms of the parameters of the motor by

$$E_o = K_3 \phi N_m + I_3 R_a \tag{20}$$

Let

$$K_3 \phi = C_1 \tag{21}$$

and

$$N_m = C_2 p \theta_o \tag{22}$$

so that C_1 will have the units of volts/rad./sec. and C_2 will have no units if the speed of the motor N_m is in radians/second. Rewriting 19 we find that

$$E_o = C_1 C_2 p \theta_o + I_3 R_a \tag{23}$$

The motor torque T may be expressed as

$$T = C_3 I_3 = J_o p^2 \theta_o + F p \theta_o \tag{24}$$

or by equations 18 and 22, solving for I_3 and substituting in 22, the motor torque is

$$T = \frac{C_3 E_{gt} - C_1 C_2 C_3 p \theta_o}{R_a + R_2} \tag{25}$$

If now the right hand side of 23 is solved for I_3 and substituted in 22 and the result used to replace E_o in (16), the component of field current due to feedback is

$$I_2 = \frac{h\left[C_1 C_2 C_3 p \theta_o + T R_a\right] \mu_2 (1 + pRC)}{R_1 (1 + pT_1) C_3 \left[1 + pRC(1 - \mu_2 h)\right]} \tag{26}$$

Combining 8, 9, 26, and 17 the total generated voltage of the amplidyne is

$$E_{gt} = E_{g1} + E_{g2} = \frac{K_1 \mu_1 \theta}{(1+pT_1)(1+pT_2)} + \tag{27}$$

$$\frac{K_1 h \mu_2 (C_1 C_2 C_3 p \theta_0 + T R_a)(1+pRC)}{R_1 C_3 (1+pT_1)(1+pT_2)\left[1+pRC(1-\mu_2 h)\right]}$$

or simplifying

$$E_{gt} = \frac{K_1 \mu_1 \theta R_1 C_3 \left[1+pRC(1-\mu_2 h)\right]}{R_1 C_3 (1+pT_1)(1+pT_2)\left[1+pRC(1-\mu_2 h)\right]} + \frac{K_1 h \mu_2 (C_1 C_2 C_3 p \theta_0 + T R_a)(1+pRC)}{R_1 C_3 (1+pT_1)(1+pT_2)\left[1+pRC(1-\mu_2 h)\right]} \tag{28}$$

substituting the value of T shown on the right side of 24 in 28 and collecting coefficients of θ and θ_0

$$E_{gt} = \frac{\theta \left\{K_1 \mu_1 R_1 C_3 \left[1+pRC(1-\mu_2 h)\right]\right\} ①}{\left\{R_1 C_3 (1+pT_1)(1+pT_2)\left[1+pRC(1-\mu_2 h)\right]\right\} ②} + \frac{\theta_0 \left\{K_1 \mu_2 (1+pRC)\left[C_1 C_2 C_3 p + (J_0 p^2 \theta_0 + Fp)R_a\right]\right\} ③}{\left\{R_1 C_3 (1+pT_1)(1+pT_2)\left[1+pRC(1-\mu_2 h)\right]\right\} ②}. \tag{29}$$

The circled numbers beside the brackets will be used in future expressions to refer to the bracketed quantity on its left. Combining 24, 25, and 29 we have an expression for the transfer function in terms of the system parameters

$$B e^{j\beta} = \frac{\theta_0}{\theta N} = \frac{C_3 \{①\}}{N(J_0 p^2 + Fp) - C_3 \{②\} + C_1 C_2 C_3 p \{③\}}. \tag{30}$$

Equation 30 defines the operation of the system under transient and steady state conditions. After the constants of the above expressions have been determined experimentally, the equation will be solved for a sinusoidal input by substituting

$$p = j\omega \tag{31}$$

and then the transfer function plotted for values of w. By use of equations 6 and 7, $C\varepsilon^{j\alpha}$ and $A\varepsilon^{j\lambda}$ may be plotted

giving the complete representation of the performance of the system.

IV. Experimental Determination of System Constants

In the tests enumerated below the various constants of the system are determined. The tests were made over the same range of values as used in the system for a normal input. Using the resulting curves our assumptions may be proven to hold or be in error, as the case may be, and compensating corrections made.

Test No. 1. The output Inertia of the Motor and Gear Train.

The output inertia of the system is contributed to by the motor armature and two large gears connecting the motor shaft to the output selsyn transformer. The total inertia J_o will be computed with respect to the motor shaft.

To accurately measure the moment of inertia of the irregular armature, a fly wheel whose inertia could be computed was suspended from a piece of long piano wire. It was set to oscillating as a torsional pendulum and its period recorded. The fly wheel was then replaced by the motor armature and its period of oscillation recorded.

Neglecting any damping due to the wire or air resistance, the period of a torsional pendulum is given in seconds by

$$T = 2\pi\sqrt{\frac{I}{K}} \qquad (32)$$

I being the moment of inertia of the system in slug-feet^2 and K the spring constant of the wire. Solving 32 for I and setting up a ratio for two masses using the same wire we find that

$$\frac{I_f}{I_a} = \frac{\frac{T_f^2 K}{4\pi^2}}{\frac{T_a^2 K}{4\pi^2}} = \frac{T_f^2}{T_a^2} \tag{33}$$

and

$$I_a = I_f \frac{T_a^2}{T_f^2} \tag{33a}$$

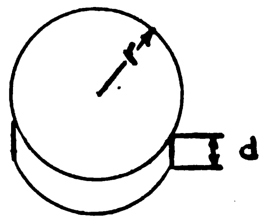


Figure 6

The moment of inertia of the flywheel depicted in Figure 6 is

$$I_a = \frac{1}{2}\frac{W}{g} r^2 = 0.5 \times \frac{3.41}{32.2} \times \left(\frac{2.25}{12}\right)^2 = 1.862 \times 10^{-3} \text{ slug-ft.}^2 \tag{34}$$

The data taken in determining T_f of flywheel is :

Test	No. of oscillations	Time	Period
#1	10	123 sec.	12.3sec.
#2	10	122	12.2
#3	10	124	12.4

T_f average = 12.3 sec.

The data for T_a of motor armature is:

Test	No. of oscillations	Time	Period
#1	10	83.0 sec.	8.3 sec.
#2	10	82.5	8.25
#3	10	83.0	8.3

T_a average = 8.3 sec.

Using equation 34 I_a is found to be

$$I_a = 1.862 \times 10^{-3} \times \frac{8.3^2}{12.4^2} = 848 \times 10^{-6} \text{ slug-ft}^2 \tag{35}$$

The two large gears were initially thought to contribute a larger amount of inertia than is actually the case. Their moment of inertia was calculated since they are of

symmetrical shape and difficult to remove from their assembly. The calculations are given below:

Gear A $\quad r = 2.05$ in. $\quad$ density of steel $a = 0.284$ lb/in^3
$\quad\quad\quad\;\; d = 0.1875$ in.

$$M = \frac{Va}{g} = \frac{\pi r^2 d a}{g} = 0.0216 \text{ slugs} \tag{36}$$

$$I_{GA} = \frac{1}{2} M r^2 = 0.5 \times 0.0216 \times \left(\frac{2.05}{12}\right)^2 = 0.313 \times 10^{-3} \text{ slug-ft}^2 \tag{37}$$

Gear B. $\quad r = 3.03$ in.
$\quad\quad\quad\;\; d = 0.1875$ in.

$$M = 3.14 \times 3.03 \times 0.1875 \times \frac{0.284}{32.2} = 0.0475 \text{ slugs} \tag{38}$$

$$I_{GB} = 0.5 \times 0.0475 \times \left(\frac{3.03}{12}\right)^2 = 1.511 \times 10^{-3} \text{ slug-ft}^2 \tag{39}$$

The moment of inertia of the gear as seen from the motor shaft is

$$I'_{GA} = \frac{I_{GA}}{N^2} \tag{39a}$$

with a similar expression for I'GB. The gear ratio of the train is shown above each pair of

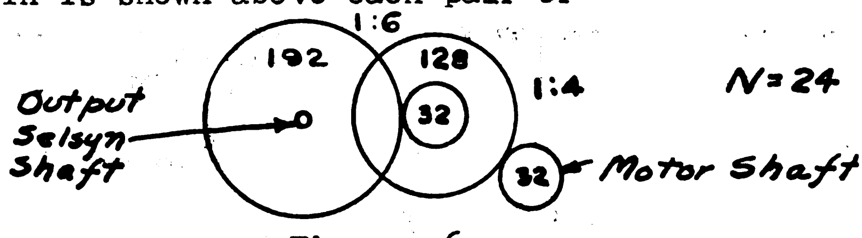


Figure 6

gears and the number inside each gear represents the number of teeth on each gear. The total gear ratio N is the product of the two or 24. The total inertia referred to the motor shaft is

$$J_0 = 848\times10^{-6} + \frac{0.313\times10^{-3}}{4^2} + \frac{1.511\times10^{-3}}{24^2}$$

$$J_0 = (848+22)10^{-6} = 870\times10^{-6}\ \text{slug-ft}^2 \qquad (39\ b)$$

Test No. 2. The Time Constants of the Amplidyne.

In order to determine the time constant of the amplidyne field a Westinghouse oscillograph was used to record the transient current when the circuit was closed. The set-up employed is shown in Figure 7. From the curve of the transient, the length of time for the current to reach 62.3% of its final value was determined. The final current was limited by the resistor to 20 ma. The resistance of one half the field R, was found to be 830 ohms and that of the resistor R_h to be 300 ohms. Using the familiar expression for an RL circuit

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad (40)$$

the inductance of the one half of the field was determined.

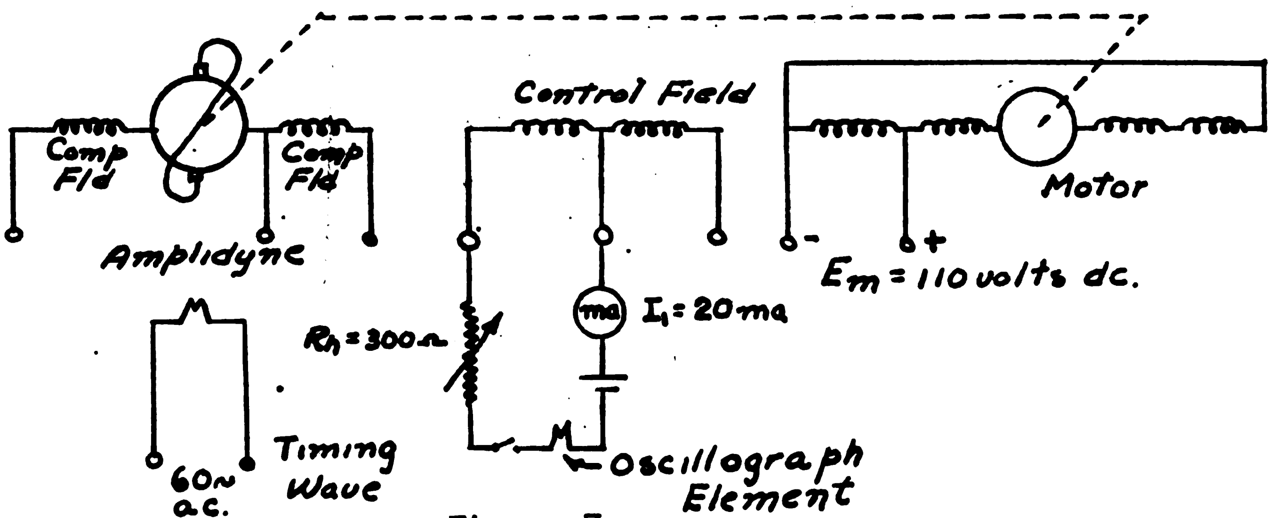
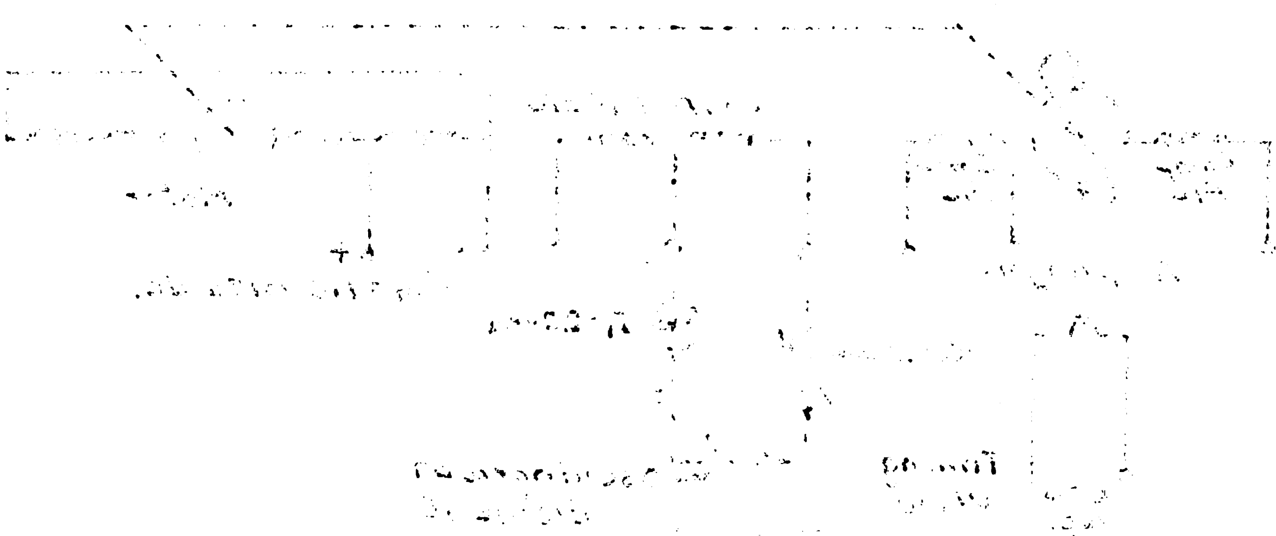


Figure 7

A series of three oscillographs were taken, one of which is shown in Figure 8. The average of the three time constants was found to be $T_1' = 0.0262$ seconds since



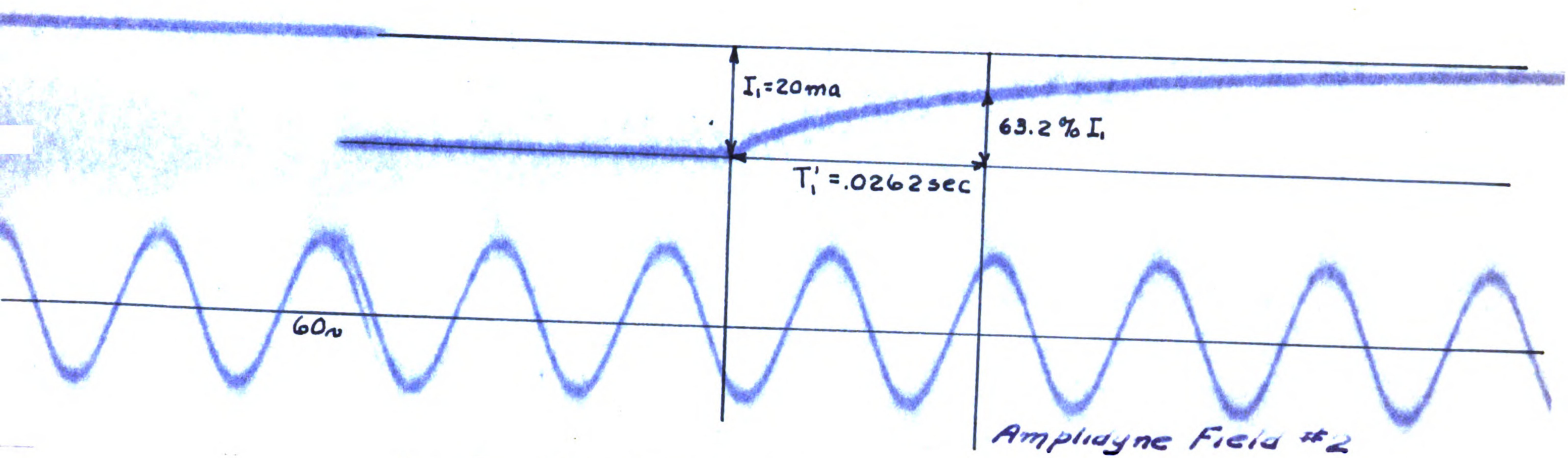


Fig 8

$$T_1' = \frac{L_1}{R_T} = 0.0262 = \frac{L_1}{830 + 300}$$

$$L_1 = 29.6 \text{ henries} \qquad (40a)$$

The time constant of the field alone is found by

$$\frac{\frac{L_1}{R_T}}{\frac{L_1}{R_1}} = \frac{T_1'}{T_1} \longrightarrow T_1 = \frac{1130}{830} \times 0.0262 = 0.0357 \text{ sec.} \qquad (40b)$$

In Figure 9 is shown the circuit diagram followed in obtaining the oscillograms of the amplidyne armature current when the field is suddenly applied.

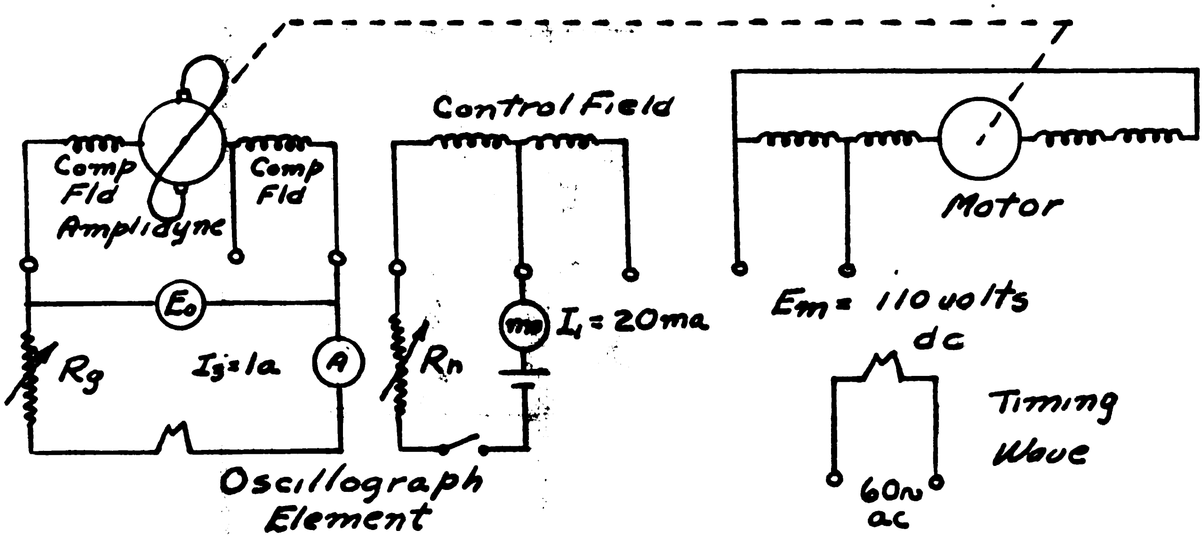
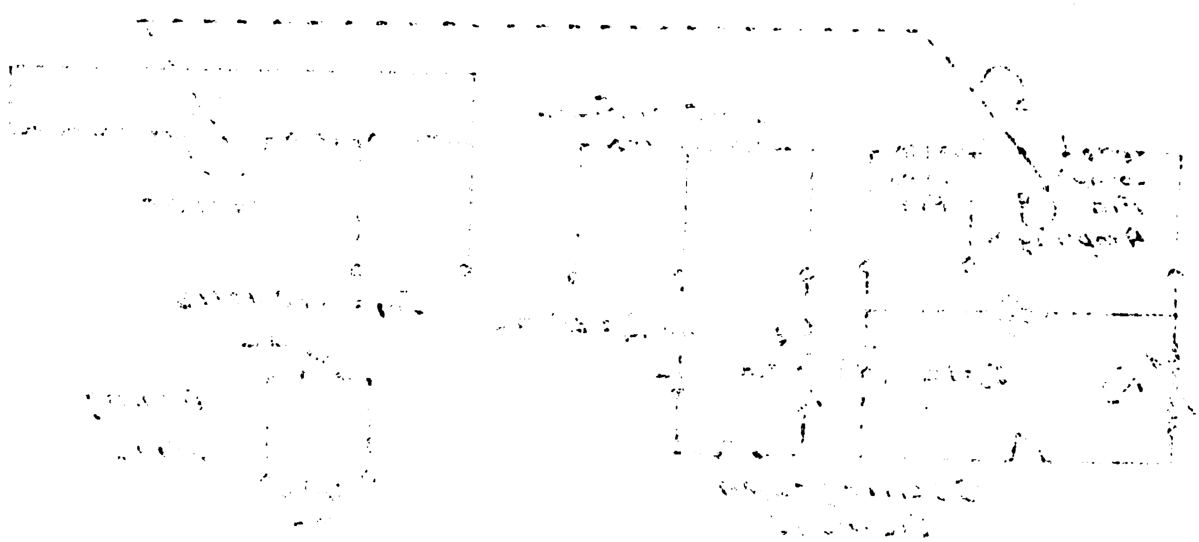


Figure 9

The resistors R_h and R_g were set to give an I_1 of 20 ma. and I_3 of 1 ampere respectively. When measured R_h was found to be 300 ohms and R_g, 260 ohms. The resistance of the armature R_2 measured by the volt-ammeter method while the motor was running had the value of 54 ohms.

A series of six oscillographs were made, two of which were disgarded because the motor voltage E_m varied from 110 volts. The time to rise to a value of armature current equal to 0.634 amperes was scaled from each one and found to have a value of 0.0333 seconds. A typical oscillograph is shown in Figure 10.



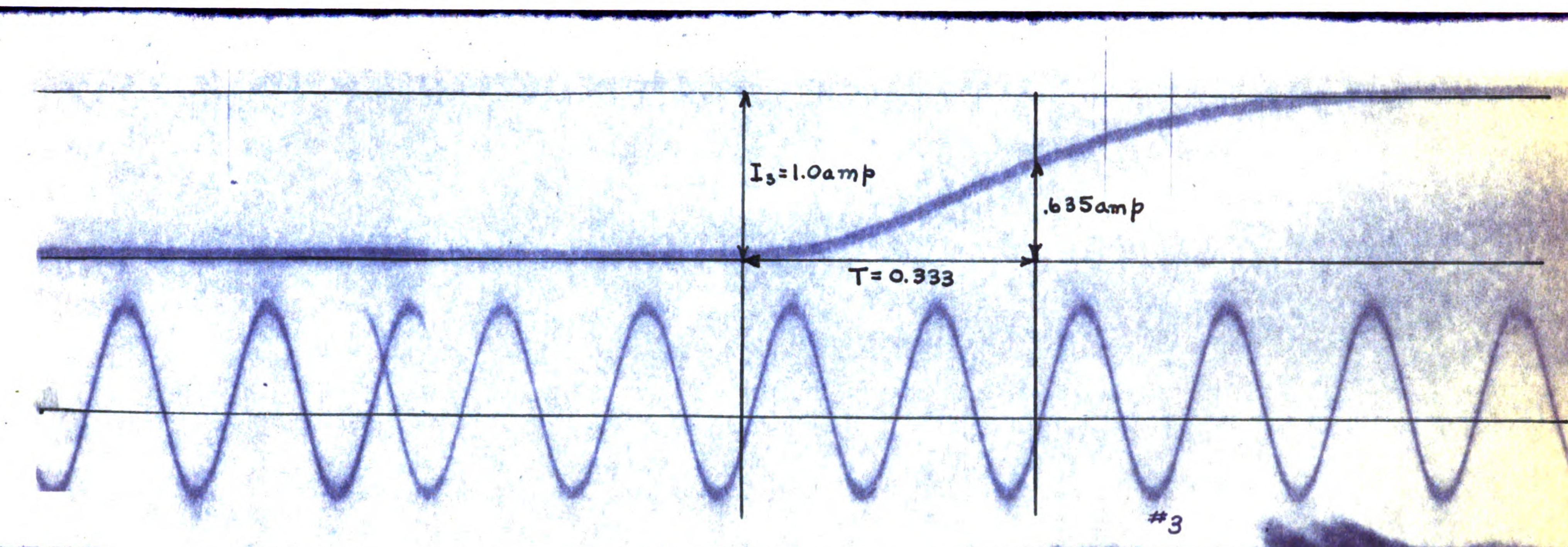


FIG. 10

The current I_3 follows the equation

$$I_3 = \frac{KE_1}{R_1R_2}\left[1 + \frac{e^{-\frac{R_1}{L_1}t}}{\left(\frac{L_2R_1}{R_2L_1} - 1\right)} + \frac{e^{-\frac{R_2}{L_2}t}}{\left(\frac{L_1R_2}{R_1L_2} - 1\right)}\right] \quad (41)$$

When the values taken from the oscillograph and the above circuit constants are substituted in 41, the resulting transcendental equation may be solved by trial and error for L_2. The value found for L_2 is 1.9 henries and the equation 41 appears as

$$I_3 = 1\left[1 + \frac{e^{-\frac{.0333}{.0262}}}{\left(\frac{1.9}{.0262 \times 314} - 1\right)} + \frac{e^{-\frac{314 \times .0333}{1.9}}}{\left(\frac{.0262 \times 314}{1.9} - 1\right)}\right] \quad (42)$$

With Rg in the circuit the time constant is

$$T_2' = \frac{L_2}{R_2} = \frac{1.9}{314} = 0.00605 \text{ sec.} \quad (43)$$

and of the armature alone

$$T_2 = \frac{1.9}{54} = 0.0352 \text{ sec.} \quad (44)$$

Test No. 3. Speed-Torque Curve of Output Motor with Input From Lab Supply

In this test the voltage across the motor armature was obtained from the lab outlet as shown in Figure 11. The motor was coupled mechanically to a small one-eighth horsepower generator with adjustable shunt field which served as a dynamometer. The motor was loaded by increasing the generator field while its output was dissipated in resistors. The torque of the motor was measured by a spring balance connected to the generator and the speed of the motor by a stroboscope. The resistance of the armature R_a is 23 ohms.

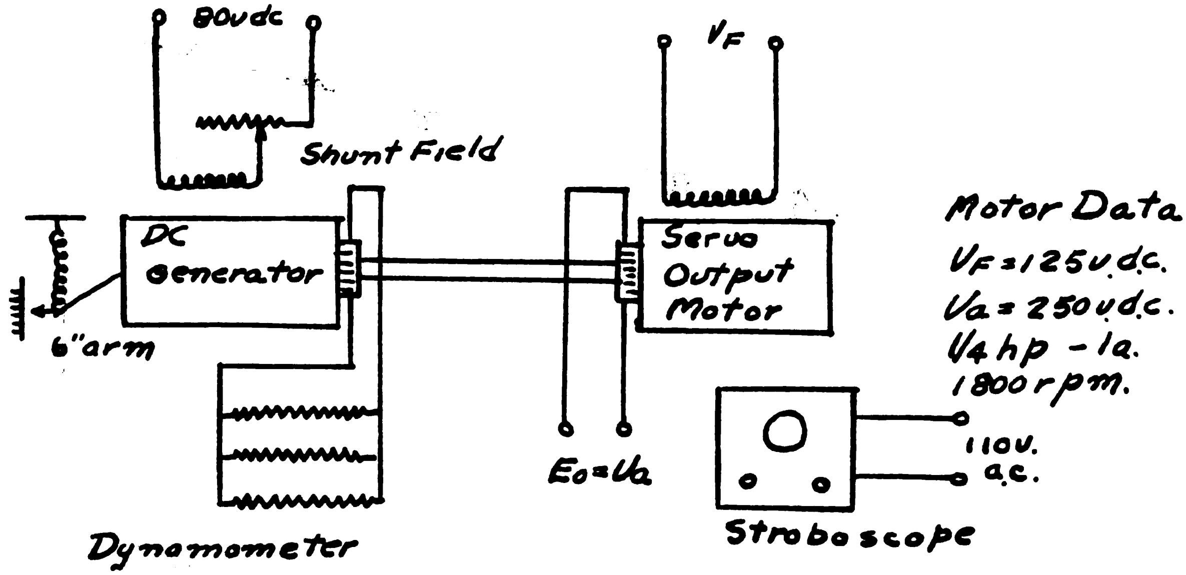


Figure 11

The data taken in the test is as follows:

Part A E_o = Va = 250 v. V_f = 109v.

N-rpm	F -oz.	T- ft.lb.
1960	0.5	0.0156
1940	4.0	0.125
1900	8.0	0.250
1870	12.0	0.375
1840	16.0	0.500
1800	20.0	0.625
1780	24.0	0.750

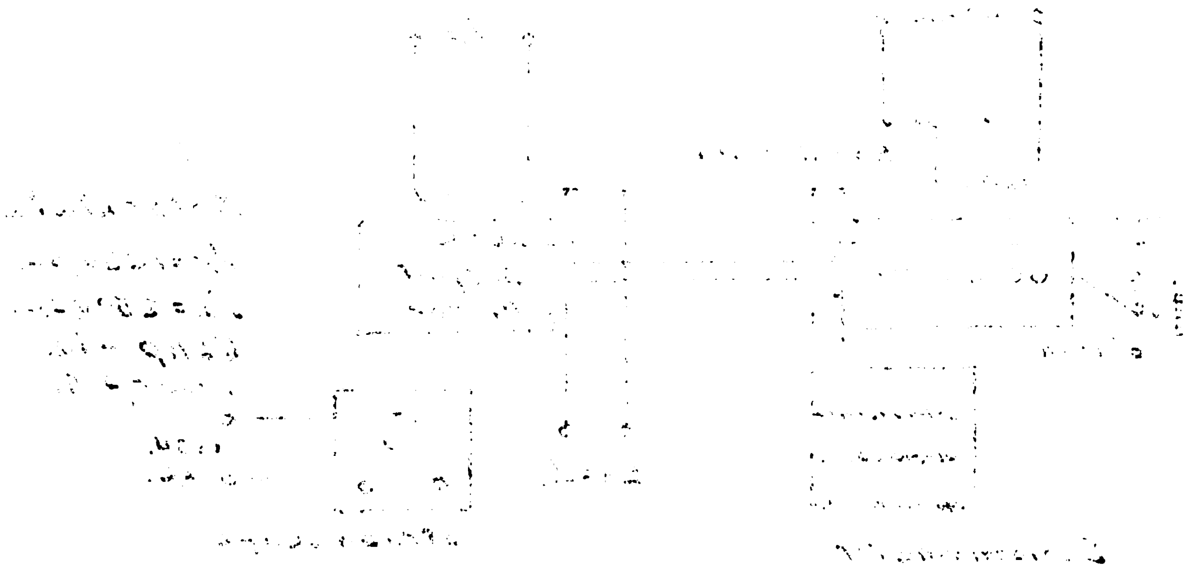
Part B E_o = Va = 180v. V_f = 109v.

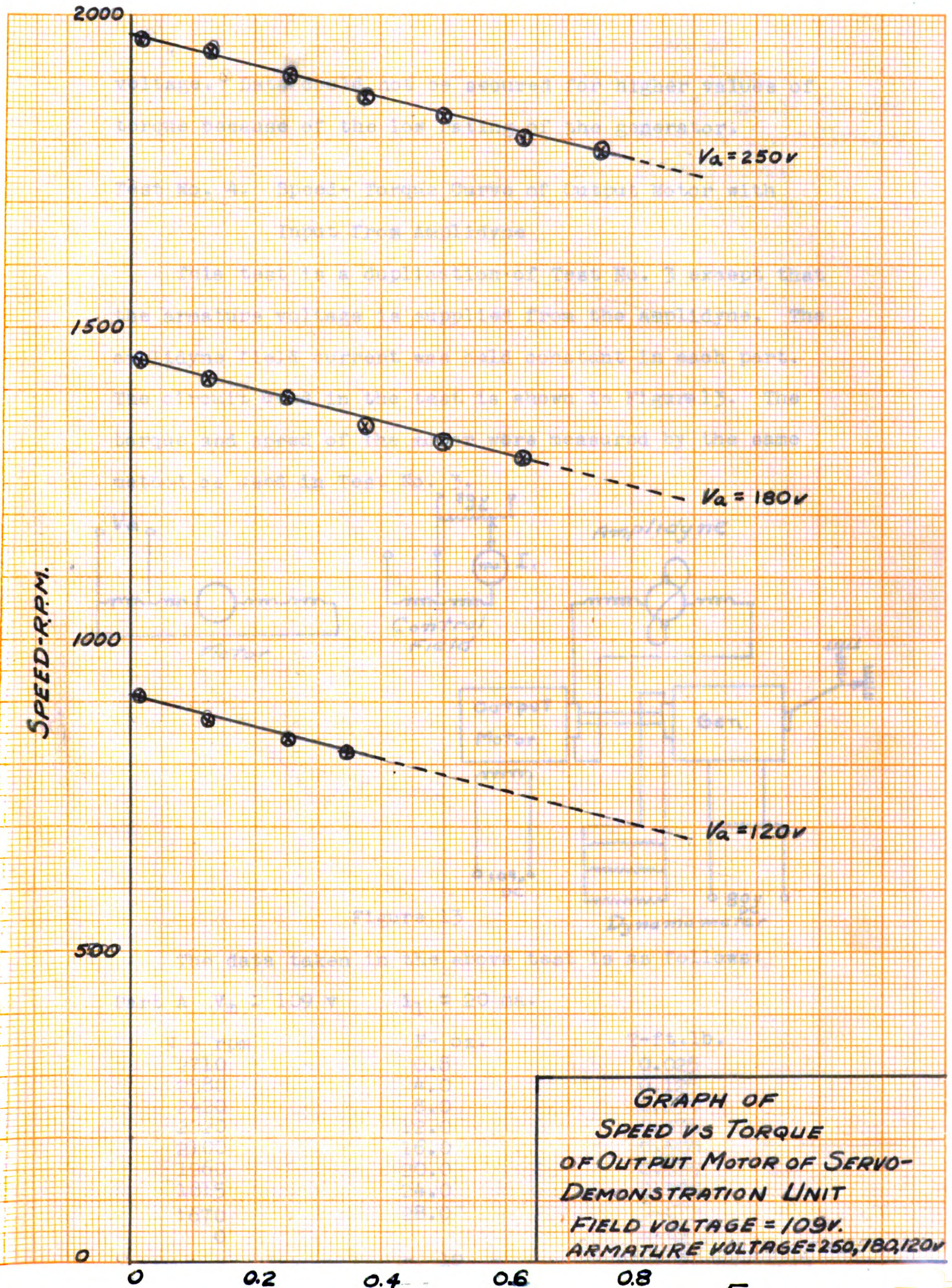
1445	0.5	0.0156
1420	4.0	0.125
1385	8.0	0.250
1342	12.0	0.375
1320	16.0	0.500
1290	20.0	0.625

Part C E_o = Va = 120v V_f = 109v

910	0.5	0.0156
870	4.0	0.125
840	8.0	0.250
820	11.0	0.344

and is plotted in Figure 12. Of interest is the observation that the speed torque relationship is linear since curves are straight lines. The curves are parallel and their distance from one another are proportional to the armature





GRAPH OF
SPEED VS TORQUE
OF OUTPUT MOTOR OF SERVO-
DEMONSTRATION UNIT
FIELD VOLTAGE = 109V.
ARMATURE VOLTAGE=250,180,120V
SPEED-R.P.M.
2000
1500
1000
500
0
0
0.2
0.4
0.6
0.8
Va=250v
Va = 180v
Va =120v

voltage. Data could not be secured for higher values of torque because of the low rating of the generator.

Test No. 4. Speed- Torque Curve of Output Motor with Input from Amplidyne

This test is a duplication of Test No. 3 except that the armature voltage is supplied from the amplidyne. The amplidyne field current was held constant in each part. The circuit used in the test is shown in Figure 13. The torque and speed of the motor were measured by the same method as used in Test No. 3.

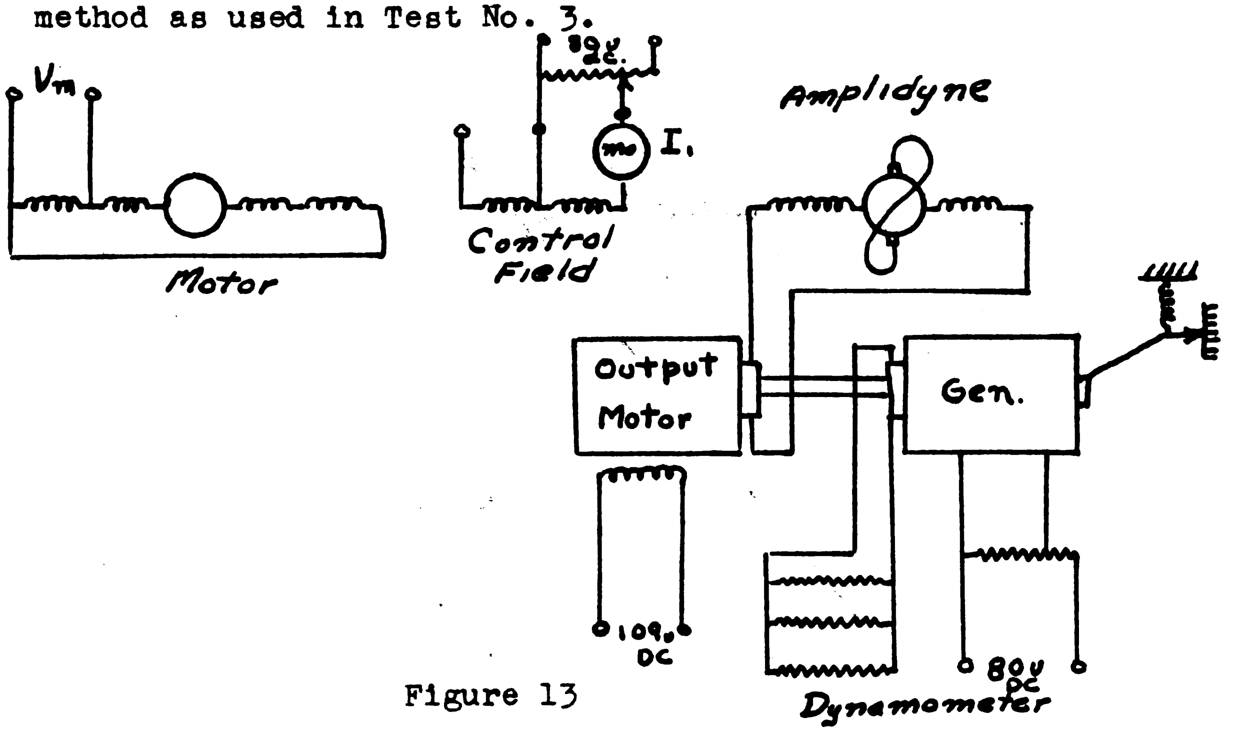
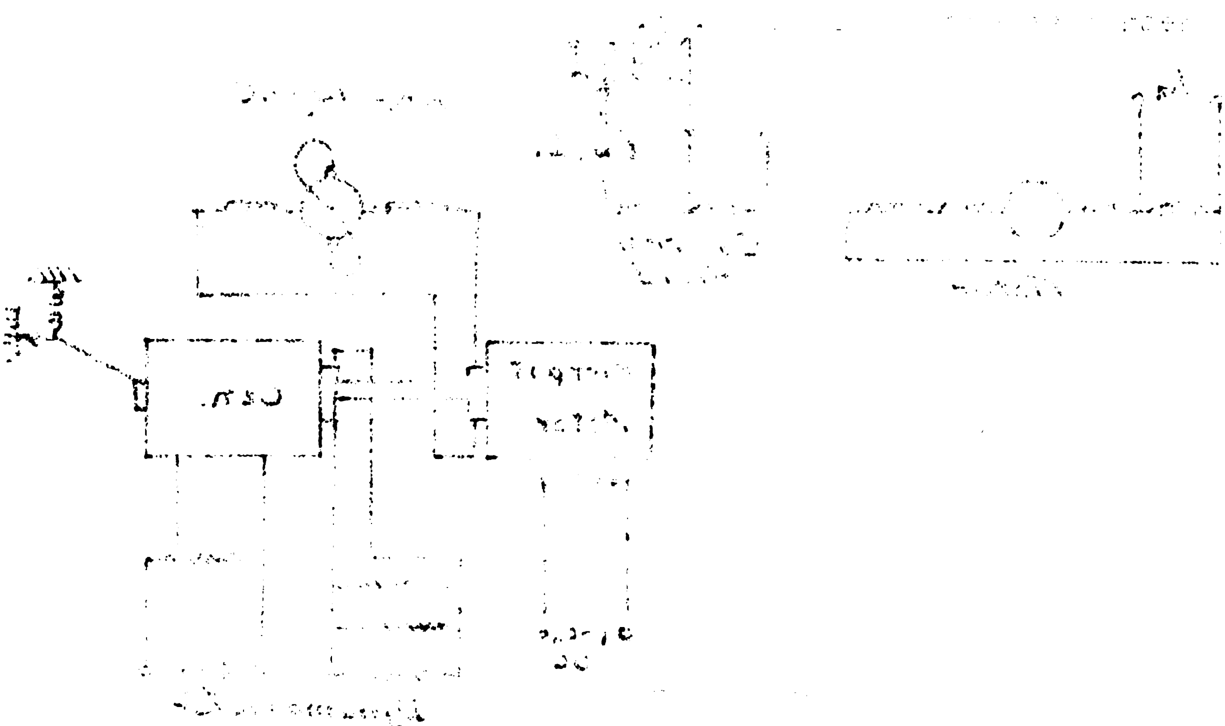


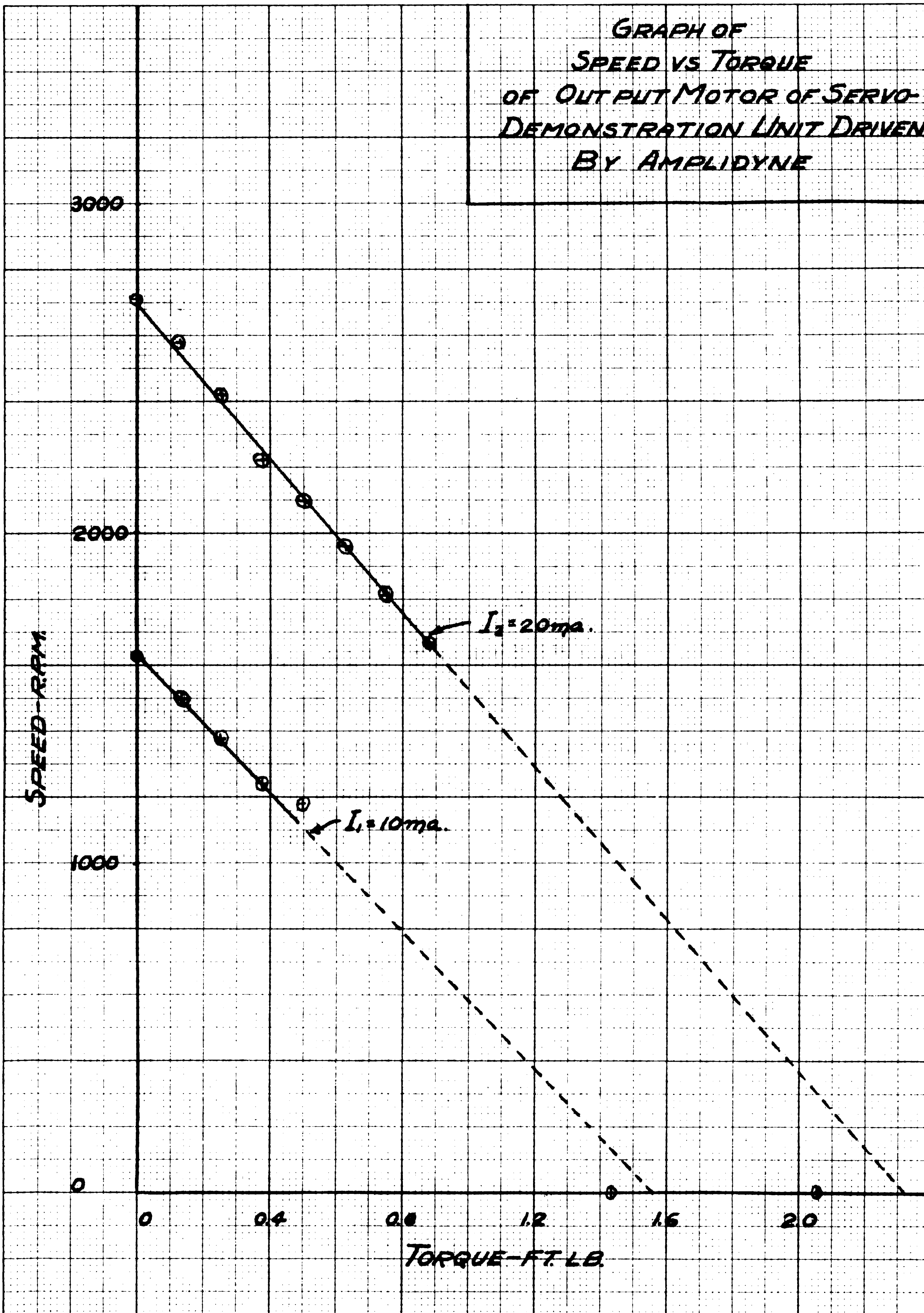
Figure 13

The data taken in the above test is as follows:

Part A $V_m = 109$ v $i_1 = 20$ ma.

N - rpm	F- oz.	T-ft.lb.
2710	0.8	0.025
2580	4.0	0.125
2420	8.0	0.250
2220	12.0	0.375
2100	16.0	0.500
1960	20.0	0.625
1815	24.0	0.750
1670	28.0	0.875
0		2.05





GRAPH OF
SPEED VS TORQUE
OF OUTPUT MOTOR OF SERVO-
DEMONSTRATION UNIT DRIVEN
BY AMPLIDYNE
3000
2000
1000
0
SPEED-R.P.M.
I_2 = 20ma.
I_1 = 10ma.
0
0.4
0.8
1.2
1.6
2.0
TORQUE-FT. LB.

Part B $V_m = 109$ v $i_1 = 10$ ma.

N- rpm	F - oz	T - ft. lb.
1620	0.4	0.0125
1500	4.0	0.125
1375	8.0	0.250
1245	12.0	0.375
1185	16.0	0.500
0		1.425

Figure 14 shows the curves plotting the above data. The points showing the blocked rotor torque are to the left of the values indicated by the extended dotted lines because of the saturation in the motor armature. There is a greater decrease in the blocked rotor torque at 20 ma. because of the saturation in the amplidyne field.

From these curves the K of equation 11 was determined and is found to be

$$K = \frac{1.56}{10} = 0.156 \text{ ft. lb.}/\text{ma.} \tag{45}$$

In equation 13 and 24 the motor torque is equal to the force due to inertia and the frictional force which is proportional to the speed of the motor shaft. The torque necessary to overcome the friction is given by

$$T = Fp\theta_o \tag{46}$$

or

$$F = \frac{T}{p\theta_o} \tag{47}$$

This is the inverse of the slope of the motor's speed-torque curve; the friction coefficient becoming

$$F = \frac{1.56}{\frac{1630}{60} \times 6.28} = 9.15 \times 10^{-3} \text{ ft. lb./rad/sec.} \tag{48}$$

Test No. 5   Saturation Curve of Amplidyne

A saturation curve of the amplidyne was run in order to determine if there was a linear relationship between the field current and terminal voltage and to determine the constant K_1 of equation 9.

The circuit is shown in Figure 15 and the data taken in the test follows.

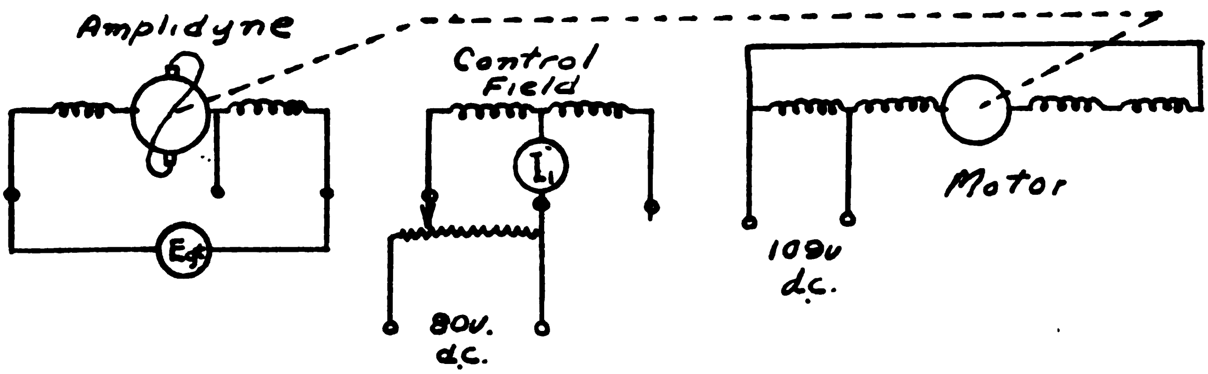


Figure 15

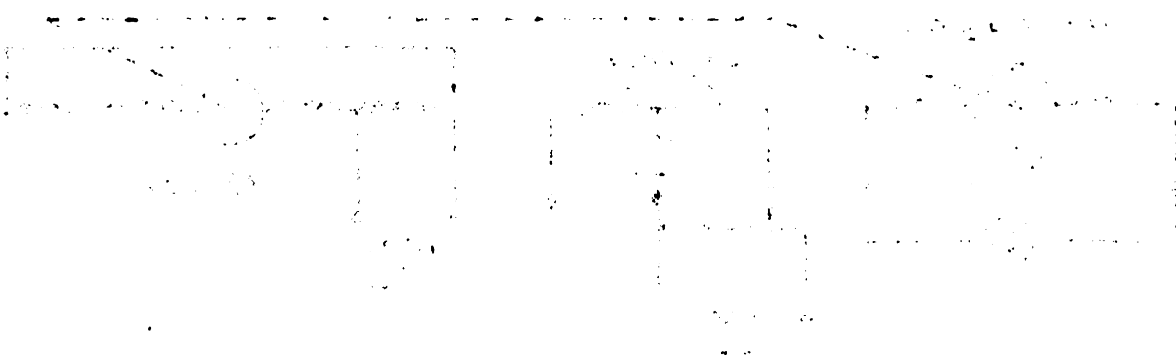
i_1 - ma	Egt - volts
0	5
2	32
4	69
6	109
8	151
10	189
12	229
14	266
16	298
18	326
20	348

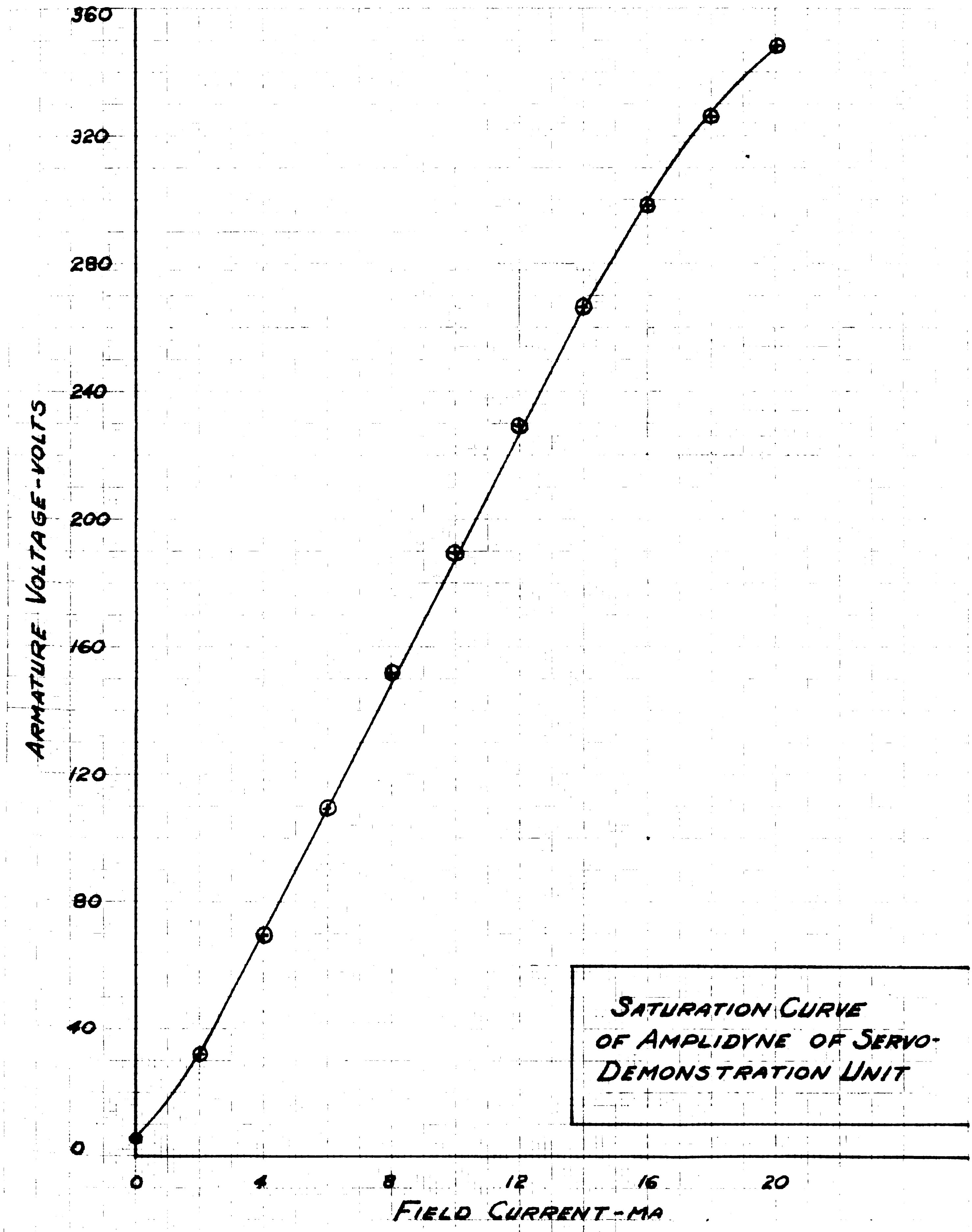
The data is plotted in Figure 16 and K_1 is determined to be

$$K_1 = \frac{200}{10.4} = 19.22 \text{ volts/ma.} \qquad (49)$$

The voltage is shown to be linear up to about 17 ma.

   Howard E. Gerlaugh





360
320
280
240
200
160
120
80
40
0
ARMATURE VOLTAGE-VOLTS
0
4
8
12
16
20
FIELD CURRENT-MA
SATURATION CURVE
OF AMPLIDYNE OF SERVO-
DEMONSTRATION UNIT

Test No.6. Amplidyne Output Voltage vs Current

In studying further the characteristics of the amplidyne a current vs. voltage curve was taken. The circuit diagram is shown in Figure 17 followed by the test data.

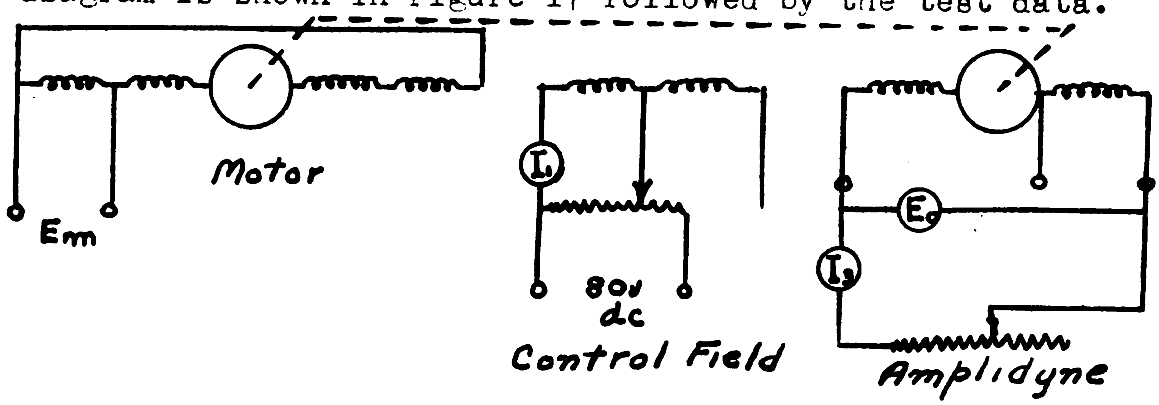


Figure 17

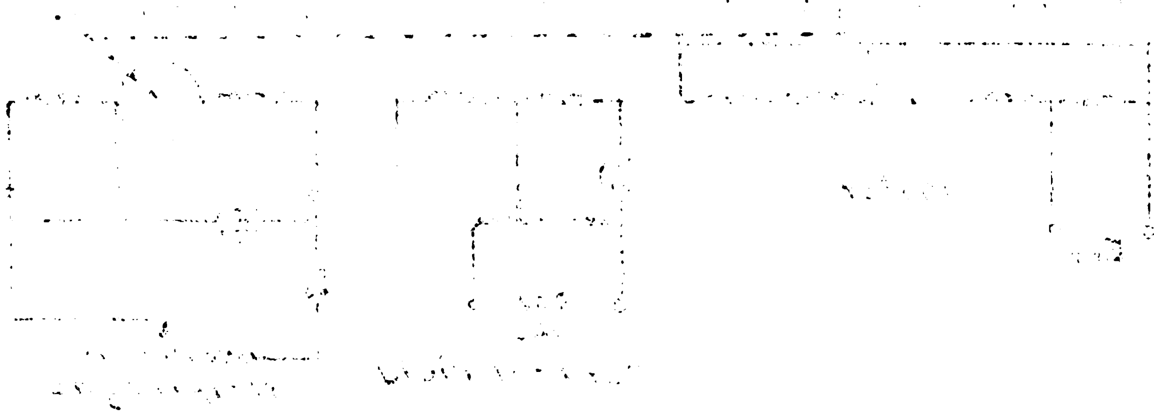
Part A $E_m = 109v$ $i_1 = 10$ ma.

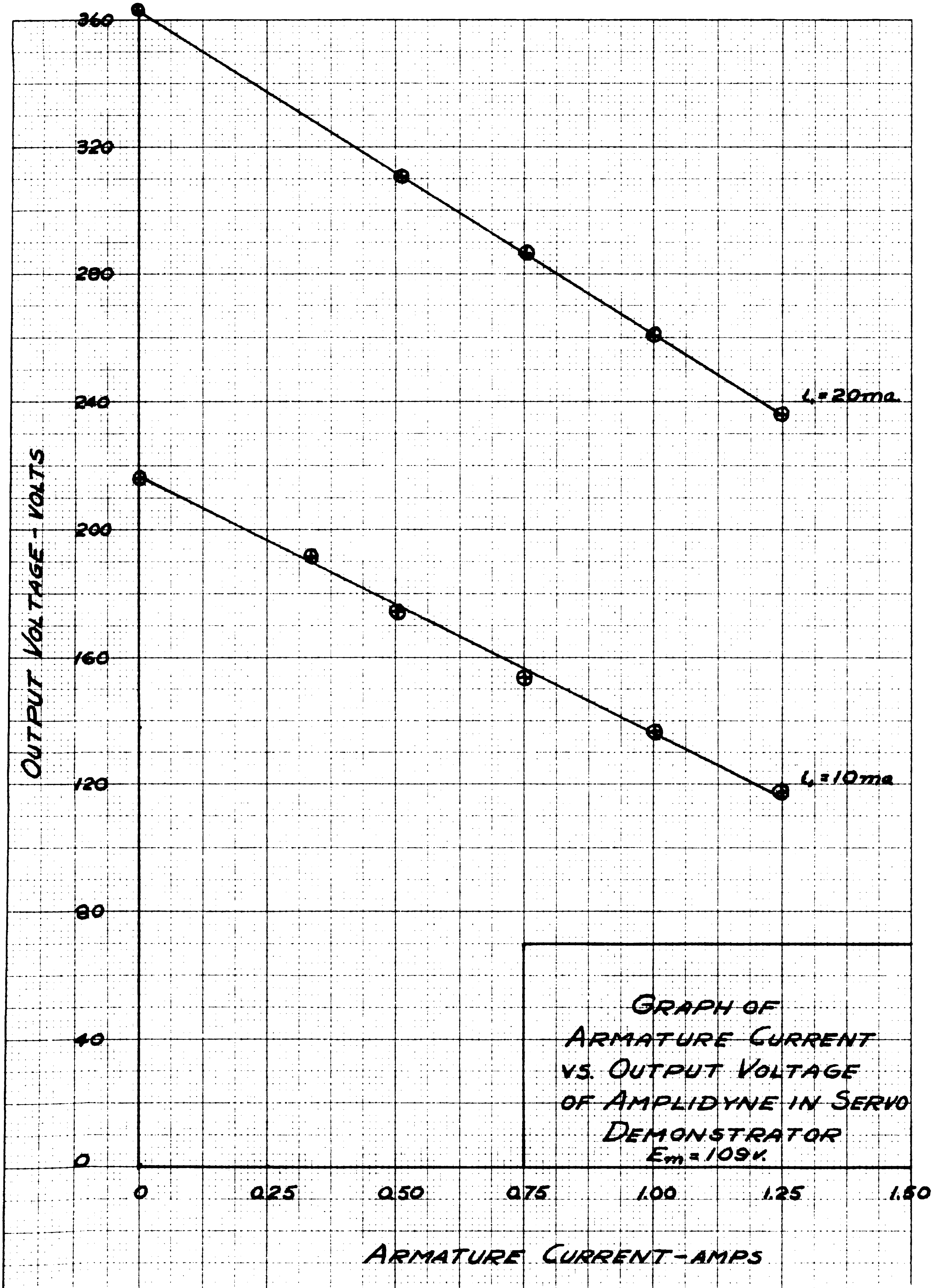
E_o - volts	i_2 - amps
216	0
191	0.33
175	0.50
154	0.75
137	1.00
118	1.25

Part B $E_m = 109v$ $i_1 = 20$ ma.

E_o - volts	i_2 - amps
362	0
310	0.52
286	0.75
260	1.00
236	1.25

In the curves plotted in Figure 18, we see that there is not quite a linear function between the field current and armature current because of the saturation in the field at 20 ma. The curves are not exactly parallel; whether this is due to the fact that the machine was cold during the 10 ma run or to plotting is not known.





360
320
280
240
200
160
120
80
40
0
OUTPUT VOLTAGE - VOLTS
0
0.25
0.50
0.75
1.00
1.25
1.50
ARMATURE CURRENT-AMPS
i = 20ma
i = 10ma
GRAPH OF
ARMATURE CURRENT
VS. OUTPUT VOLTAGE
OF AMPLIDYNE IN SERVO
DEMONSTRATOR
Em = 109V.

FIG. 18

Test No. 7. Curve of Armature Current vs. Torque

In order to determine the constant C_3 in equation 24 a curve of armature current vs. torque of the output motor was made. Also a check as to linearity of the torque and armature current can be made. The same equipment was employed as in Test No 3 with the addition of an ammeter in the motor circuit to measure the armature current. The data taken in the test is plotted in Figure 19. The armature voltage was taken from the lab supply and held constant at the values indicated.

Part A E_m = 108v E_f = 108 v

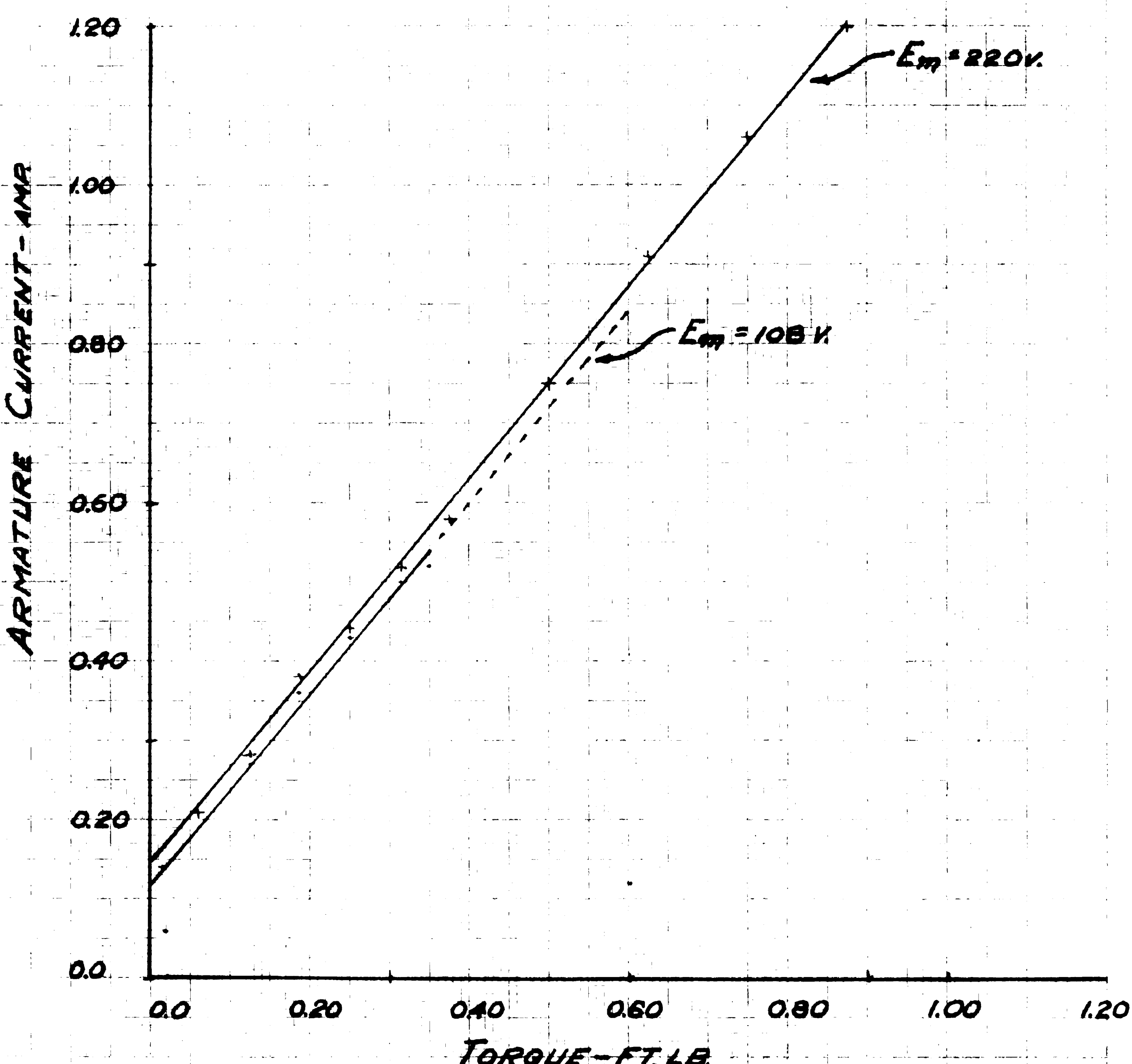
i_3 - amps	F- oz	T-ft. lbs.
0.14	0.6	0.0188
0.20	2.2	0.0688
0.27	4.0	0.125
0.36	6.0	0.1875
0.43	8.0	0.250
0.50	10.0	0.312
0.52	11.2	0.350

Part B E_m = 220v E_f = 108 v

i_3 - amps	F - oz	T- ft. lb.
0.14	0.5	0.0156
0.21	2.0	0.0625
0.28	4.0	0.125
0.38	6.0	0.1875
0.44	8.0	0.250
0.52	10.0	0.312
0.58	12.0	0.375
0.75	16.0	0.500
0.91	20.0	0.625
1.06	24.0	0.750
1.20	28.0	0.875

The constant C_3 which is the inverse of the slope of the curve is found to be

$$C_3 = \frac{0.60}{0.72} = 0.834 \text{ ft. lb./amp.}$$



GRAPH OF
ARMATURE CURRENT
VS TORQUE FOR SERVO
OUTPUT MOTOR

Test No. 8 . Gain of Amplifier Without Feedback

In this test we will determine the relation between the angular error and the resultant field current at the output of the amplifier. The rotor of the selsyn transformer on the output shaft was blocked, making the angular displacement of the selsyn generator on the input shaft the error. Figure 20 shows the arrangement of the equipment in making the test. The feedback voltage was zero.

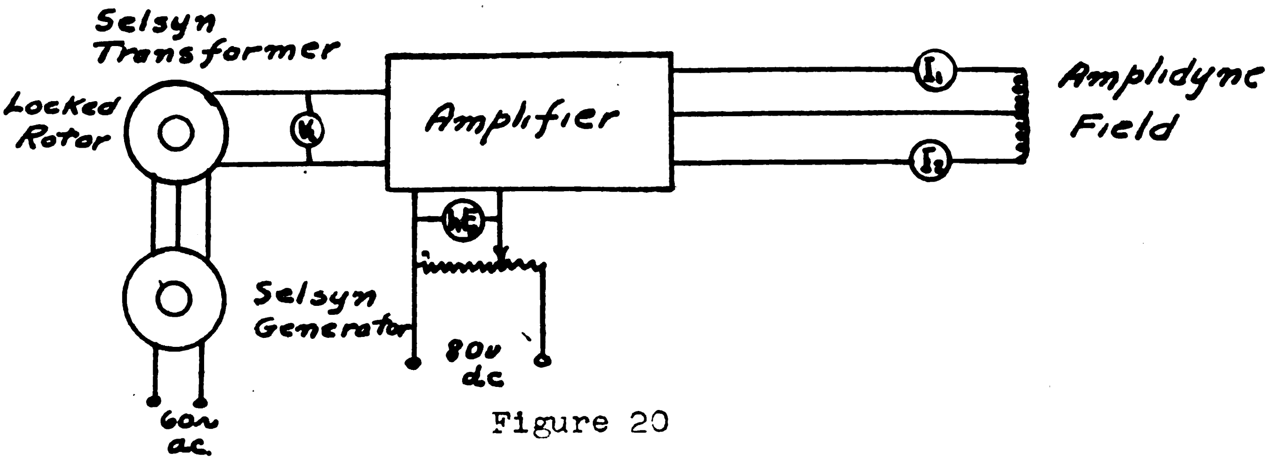


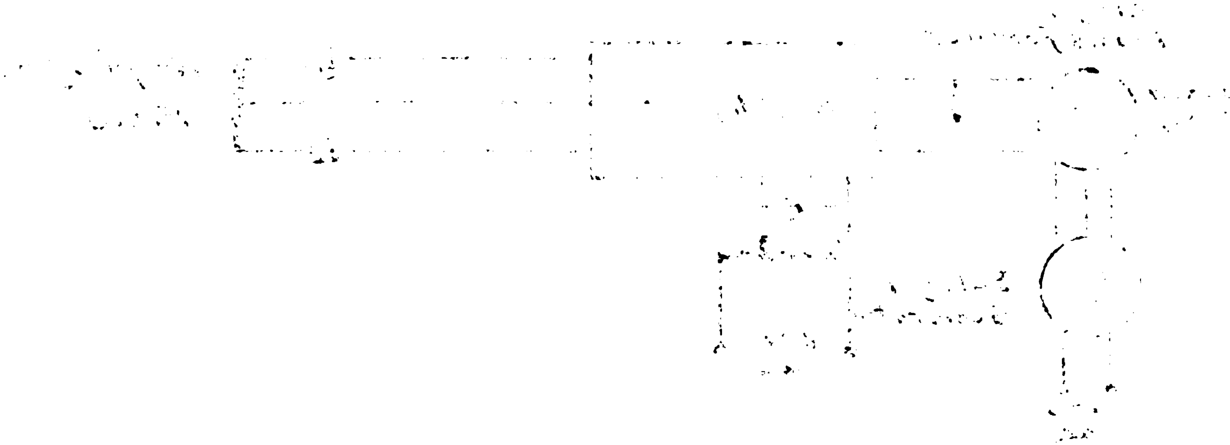
Figure 20

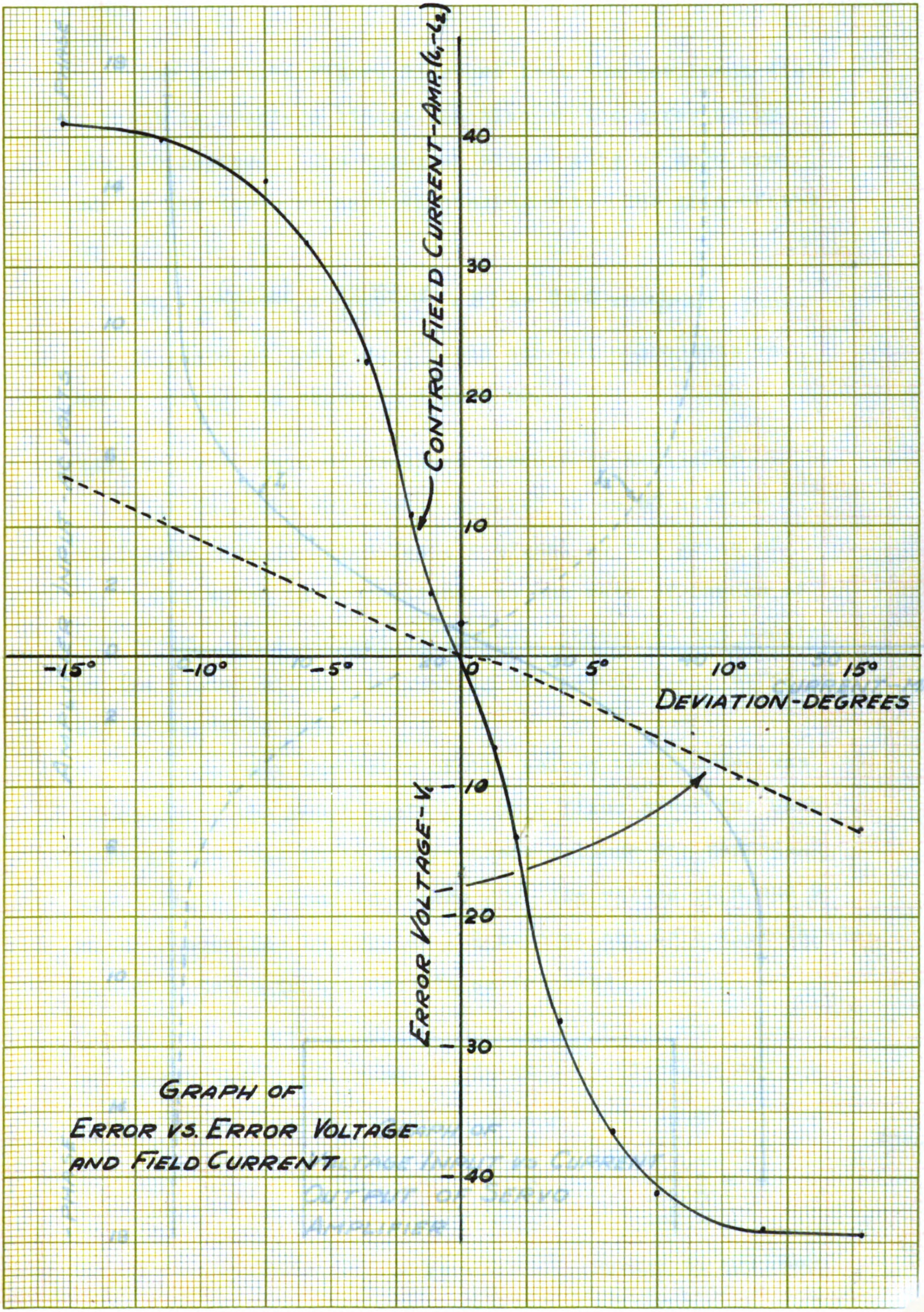
θ - teeth	V_1 -volts,ac	i_1-ma	i_2-ma	θ-degree	(i_1-i_2)-ma
-10	17	0	41.0	-19°	-41.0
-8	13.8	0	41.0	-15.2	-41.0
-6	10.6	1.0	40.5	-11.4	-39.5
-4	7.1	2.4	39.0	-7.6	-36.6
-3	5.2	5.4	37.3	-5.7	-31.9
-2	3.4	10.7	33.4	-3.8	-22.7
-1	1.7	16.8	27.7	-1.9	-10.9
-½	0.9	20.0	24.8	-0.95	-4.8
0	0.08	23.9	21.3	0.0	2.6
½	0.23	26.1	19.2	0.95	6.9
1	1.04	29.8	16.0	1.9	13.8
2	2.8	36.4	9.2	3.8	27.2
3	4.4	41.0	4.6	5.7	36.4
4	6.0	43.5	2.4	7.6	41.1
6	9.5	45.0	1.0	11.4	44.0
8	13.2	45.0	0.5	15.2	44.5
10	16.3	45.0	0.0	19.0	45.0

Equilibrium position:

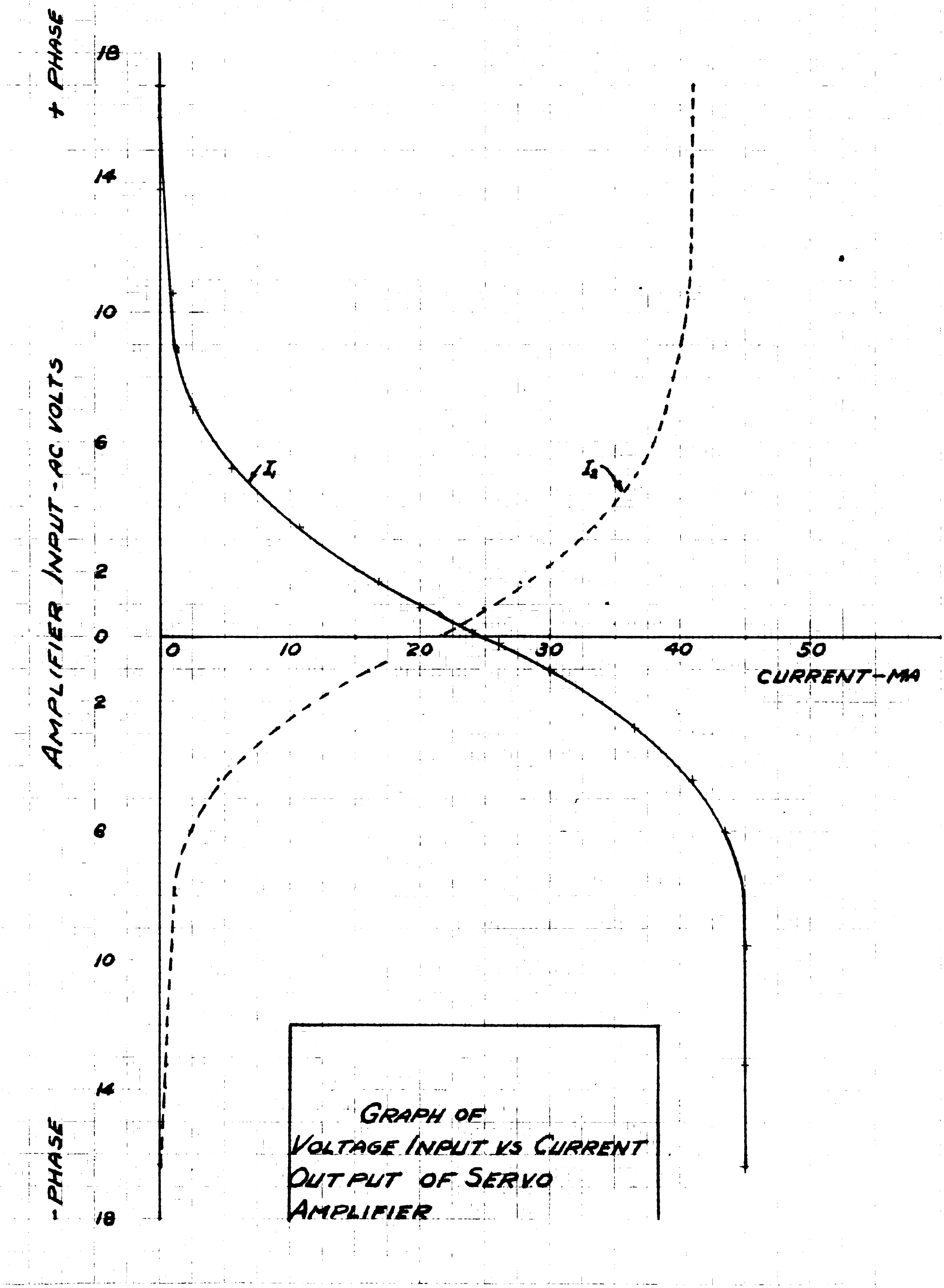
V_1 = 0.32v ; i_1 = 24.2 ma; i_2 = 23 ma.

190 teeth in gear.





CONTROL FIELD CURRENT-AMP. $(i_1 - i_2)$
40
30
20
10
-15°
-10°
-5°
0
5°
10°
15°
DEVIATION-DEGREES
ERROR VOLTAGE-V_1
-10
-20
-30
-40
GRAPH OF
ERROR VS. ERROR VOLTAGE
AND FIELD CURRENT



GRAPH OF
VOLTAGE INPUT VS CURRENT
OUTPUT OF SERVO
AMPLIFIER

The curve in Figure 21 shows that the resultant field current is nearly a linear function of the error up to about 5 degrees. The humps on either side of zero are due to the error voltage flattening out. The gain of the amplifier u_1 is

$$\mu_1 = \frac{\Delta(i_1 - i_2)}{\Delta\theta} = \frac{22.5+28}{3.7+3.75} \times 57.3 = 388 \text{ ma/rad.} \quad (50)$$

A graph of individual field current components vs. error voltage is shown in Figure 22.

Test No. 9. Gain of Feedback Circuit of Amplifier

The feedback voltage is applied to an RC differentiating circuit which feeds a signal to the grids of the tubes which is proportional to the rate of change of the error. This circuit effectively increases the damping of the system without increasing the steady state error to any appreciable extent.

In the test the rotor of the selsyn transformer was blocked and the input shaft set at a fixed error for each part of the test while the feedback voltage was varied. The circuit is shown in Figure 20, The values of R and C used in equation 15 are: R= 1.2 megohms and C= 2 microfarads. The data taken in the test is as follows:

Part A $\theta = 0$

hE_o- volts	i_1 - ma	i_2-ma	(i_1-i_2)- ma.
0	25.1	23.5	1.6
5.2	21.2	27.1	-5.9
10	18.1	29.8	-11.7
15	15.0	32.4	-17.4
20	12.0	34.2	-22.2
24.9	10.0	35.9	-25.9
30	7.9	37.2	-29.3

Part B $\theta = 2.5^{o}$

hE_o -volts	i_1-ma	i_2-ma	(i_1-i_2)-ma
0	41.0	5.0	36
5	40.1	6.1	34
10	39.5	7.8	31.7
15	38.0	10.0	28.0
20	36.8	12.2	24.6
25	34.9	14.8	20.1
30	31.9	18.2	13.7

Part C $\theta = -2.5^{o}$

hE_o-volts	i_1-ma.	i_2-ma.	(i_1-i_2)-ma.
0	8.2	35.8	-27.6
5	6.2	36.6	-30.4
10	4.9	37.2	-32.3
15	3.6	38.1	-34.5
20	2.7	38.5	-35.8
25	2.0	38.9	-36.9
30	1.5	39.2	-37.7

The curves in Figure 23 are not parallel because of the fact that the tubes draw appreciable grid current and the input transformer becomes saturated at higher values of feedback voltage. In the determination of the gain u_2 in equation 15, an average value was determined to be

$$\mu_2 = \frac{\Delta(i_1-i_2)}{\Delta hE_o} R_1 = \frac{16 \times .830}{28.15} = 0.462 \text{ volts/volt} \quad (51)$$

IV. Calculations and Performance Curves.

In the last section the constants of the servo system with and without feedback were determined. As the next step we will make the substitutions and draw a polar plot or Nyquist diagram of the transfer function. A sinusoidal input will be assumed and it follows that the error and output will also be sinusoidal functions of the same frequency but differing in phase and amplitude. Since this is to be a sinusoidal analysis, the differential equations will be solved by making

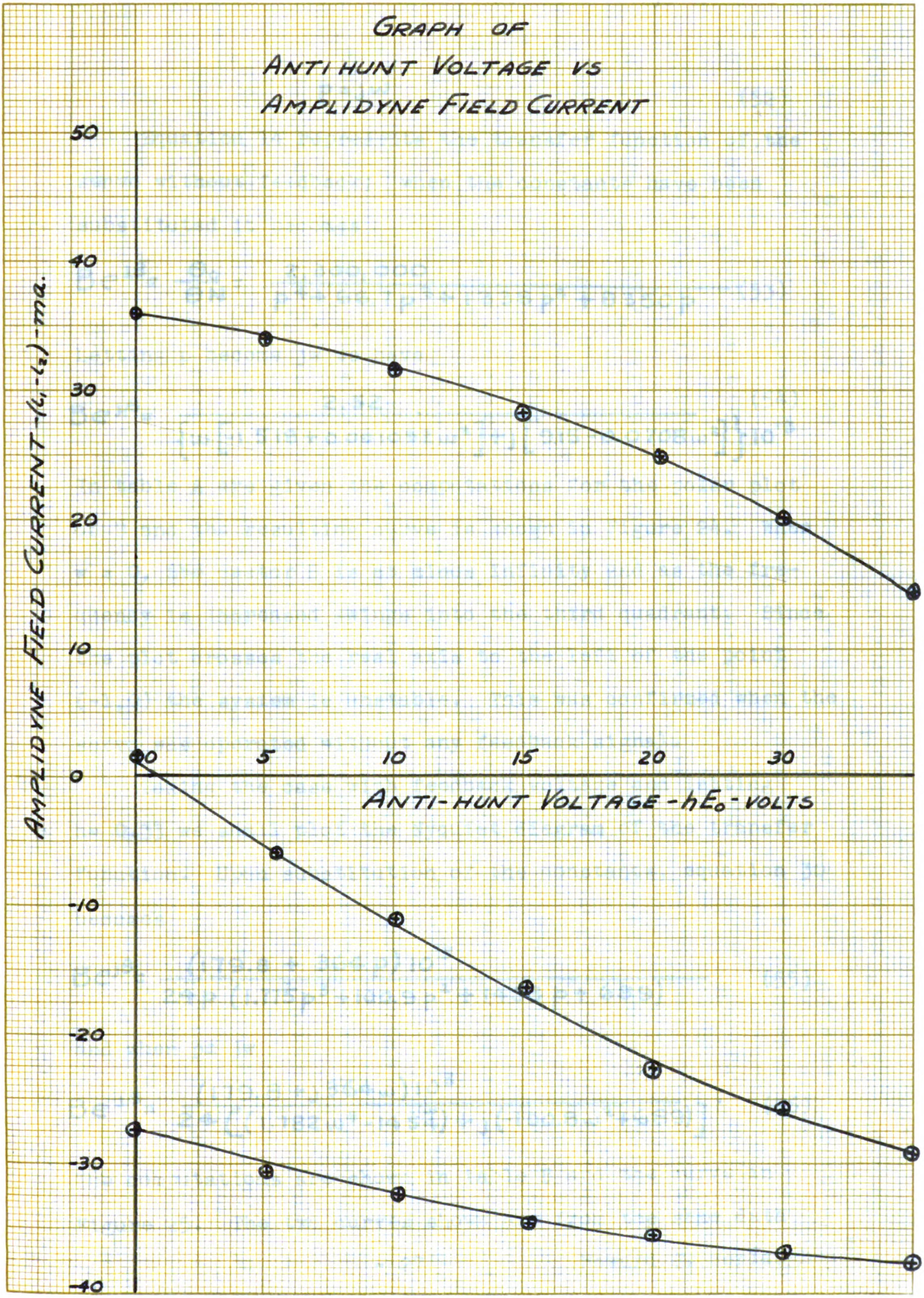

GRAPH OF
ANTI HUNT VOLTAGE VS
AMPLIDYNE FIELD CURRENT
AMPLIDYNE FIELD CURRENT - $(i_1 - i_2)$ - ma.
ANTI-HUNT VOLTAGE - hE_0 - VOLTS
50
40
30
20
10
0
-10
-20
-30
-40
0
5
10
15
20
30

FIG. 23

$$p = jw \tag{52}$$

Equation 14 represents the transfer function of the servo without feedback; when the constants have been substituted it becomes

$$Be^{j\beta} = \frac{\theta_o}{\theta N} = \frac{2,300,000}{p^4 + 66.7p^3 + 1338p^2 + 8350p} \tag{53}$$

Letting p become jw we have

$$Be^{j\beta} = \frac{2.52}{\left\{w\left[-1.518 + 0.001092w^2\right] + j\left[9.15 - 0.0708w^2\right]\right\}10^{-3}} \tag{54}$$

In table A are given the computations for the polar plot of B and the resulting curve is shown in Figure 24. When w = 0, the vector B is at minus infinity and as the frequency is increased swings into the third quadrant. Since the plot crosses the real axis to the left of the point (-1,0) the system is unstable. This was confirmed when the servo was operated without any feedback signal.

Taking the case with feedback voltage and letting h= 0.33 we shall plot the Nyquist diagram of the transfer function. Upon substitution of the constants, equation 30 becomes

$$Be^{j\beta} = \frac{(179.8 + 364p)10^3}{24p(1.775p^3 + 100.9p^2 + 1453p + 689)} \tag{55}$$

and when p= jw

$$Be^{j\beta} = \frac{(179.8 + j364w)10^3}{24\left[(1.782w^3 - 1454w) + j(-100.8w^2 + 689)\right]} \tag{56}$$

The computations are shown in table B and the curve in Figure 25. The two curves shown are from the same data

TABLE A COMPUTATION OF $\frac{\theta_o}{\theta_N}$ WITHOUT ANTIHUNT CIRCUIT

ω Rad/sec	A	B	A^2	B^2	A^2+B^2	$\frac{K_1 \mu_1}{\omega(A^2+B^2)}$	$R\angle\theta$	$B = \frac{\theta_o}{\theta_N}$
0	0	9.15×10^{-3}	0	83.7×10^{-6}	83.7×10^{-6}	∞	$9.15\times10^{-3}\angle-90^\circ$	$\infty\angle-90^\circ$
1	-1.517×10^{-3}	9.08×10^{-3}	2.3×10^{-6}	82.4×10^{-6}	84.7×10^{-6}	29,800	$9.20\times10^{-3}\angle-99.5^\circ$	$27.4\angle-99.5^\circ$
5	-7.47×10^{-3}	7.38×10^{-3}	55.8×10^{-6}	54.4×10^{-6}	110.2×10^{-6}	4590	$10.5\times10^{-3}\angle-135.3^\circ$	$48.3\angle-135.3^\circ$
7	-10.27×10^{-3}	5.68×10^{-3}	105.5×10^{-6}	32.2×10^{-6}	137.7×10^{-6}	2620	$11.7\times10^{-3}\angle-151^\circ$	$30.6\angle-151^\circ$
10	-14.09×10^{-3}	2.07×10^{-3}	198.5×10^{-6}	4.28×10^{-6}	202.8×10^{-6}	1240	$14.23\times10^{-3}\angle-164.6^\circ$	$17.64\angle-164.6^\circ$
11	-15.24×10^{-3}	0.59×10^{-3}	232.5×10^{-6}	$.348\times10^{-6}$	232.8×10^{-6}	985	$15.24\times10^{-3}\angle-177.8^\circ$	$15.0\angle-177.8^\circ$
13	-17.34×10^{-3}	-2.84×10^{-3}	301×10^{-6}	8.06×10^{-6}	309×10^{-6}	629	$17.52\times10^{-3}\angle-189.1^\circ$	$11.0\angle-189.1^\circ$
17	-20.45×10^{-3}	-11.30×10^{-3}	419×10^{-6}	127.8×10^{-6}	547×10^{-6}	272	$23.3\times10^{-3}\angle-209^\circ$	$6.33\angle-209^\circ$
20	-21.6×10^{-3}	-19.2×10^{-3}	466×10^{-6}	369×10^{-6}	835×10^{-6}	150.8	$28.9\times10^{-3}\angle-221.7^\circ$	$4.37\angle-221.7^\circ$
25	-20.9×10^{-3}	-35.0×10^{-3}	436×10^{-6}	1225×10^{-6}	1761×10^{-6}	26.0 *	$40.7\times10^{-3}\angle-239.1^\circ$	$2.33\angle-239.1^\circ$
30	-16.0×10^{-3}	-54.5×10^{-3}	254×10^{-6}	2970×10^{-6}	3224×10^{-6}	26.0	$56.7\times10^{-3}\angle-253.6^\circ$	$1.475\angle-253.6^\circ$
40	$+9.29\times10^{-3}$	-104.1×10^{-3}	86.3×10^{-6}	10.94×10^{-3}	10.94×10^{-6}	5.76	$104.3\times10^{-3}\angle+84.9^\circ$	$0.601\angle84.9^\circ$
50	$+60.5\times10^{-3}$	-168×10^{-3}	3660×10^{-3}	28.2×10^{-3}	31.86×10^{-3}	1.585	$180.3\times10^{-3}\angle70.4^\circ$	$0.248\angle70.4^\circ$
60	145.4×10^{-3}	-246×10^{-3}	21.2×10^{-3}	60.5×10^{-3}	80.7×10^{-3}	0.514	$286\times10^{-3}\angle59.4^\circ$	$0.147\angle59.4^\circ$
80	438×10^{-3}	-444×10^{-3}	192×10^{-3}	197×10^{-3}	389×10^{-3}	0.0812	$757\times10^{-3}\angle45.4^\circ$	$0.0614\angle45.4^\circ$
100	940×10^{-3}	-669×10^{-3}	883×10^{-3}	488×10^{-3}	1371×10^{-3}	0.01835	$1171\times10^{-3}\angle36.6^\circ$	$0.0216\angle36.6$

* CORRECTION: 57.4

[illegible]	N	T	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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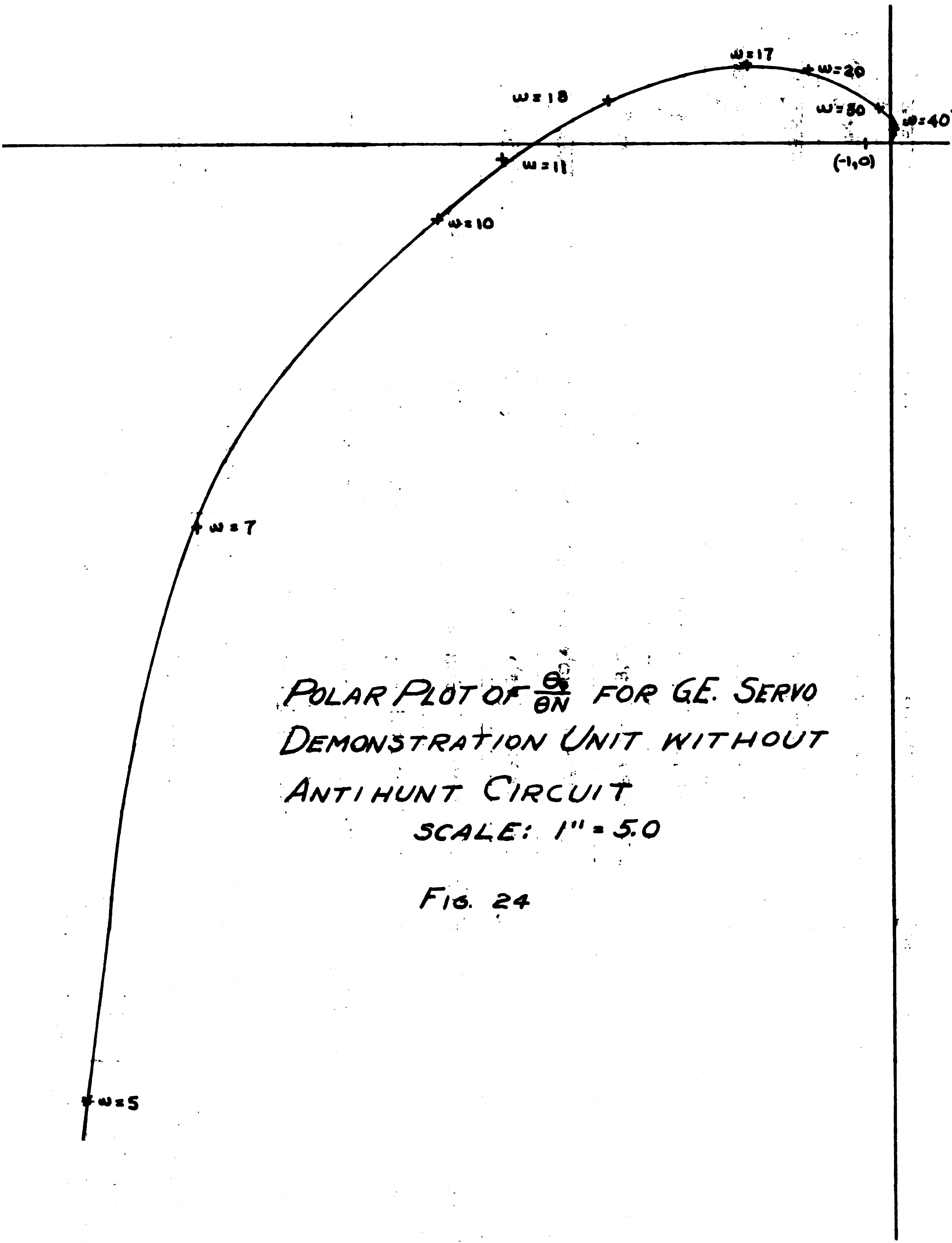
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0	0	810×10^{-4}	0	[illegible]	[illegible]	∞	[illegible]	[illegible]

TABLE 4 COMPARISON OF [illegible]



POLAR PLOT OF $\frac{\theta_o}{\theta_N}$ FOR G.E. SERVO DEMONSTRATION UNIT WITHOUT ANTIHUNT CIRCUIT

SCALE: 1" = 5.0

FIG. 24

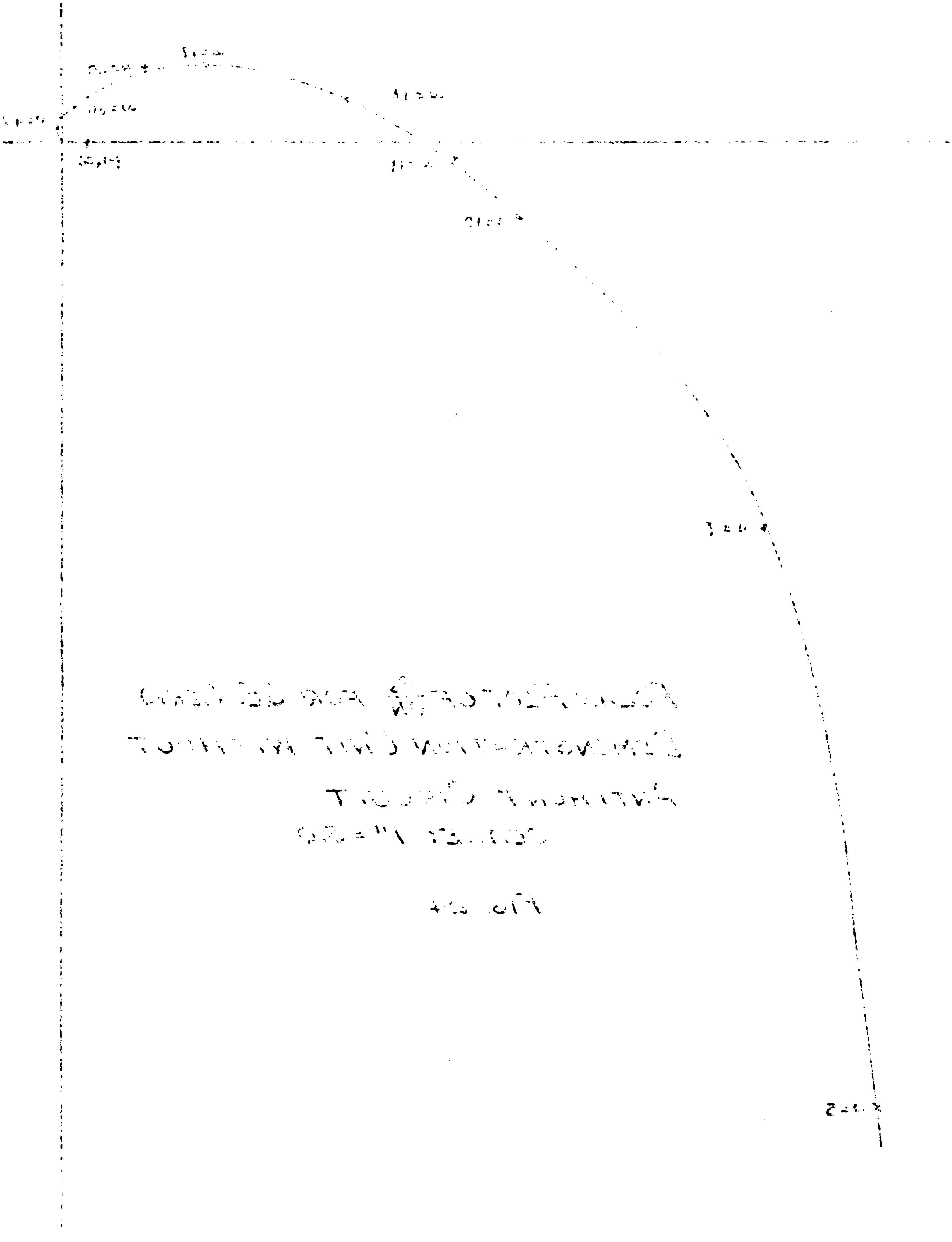
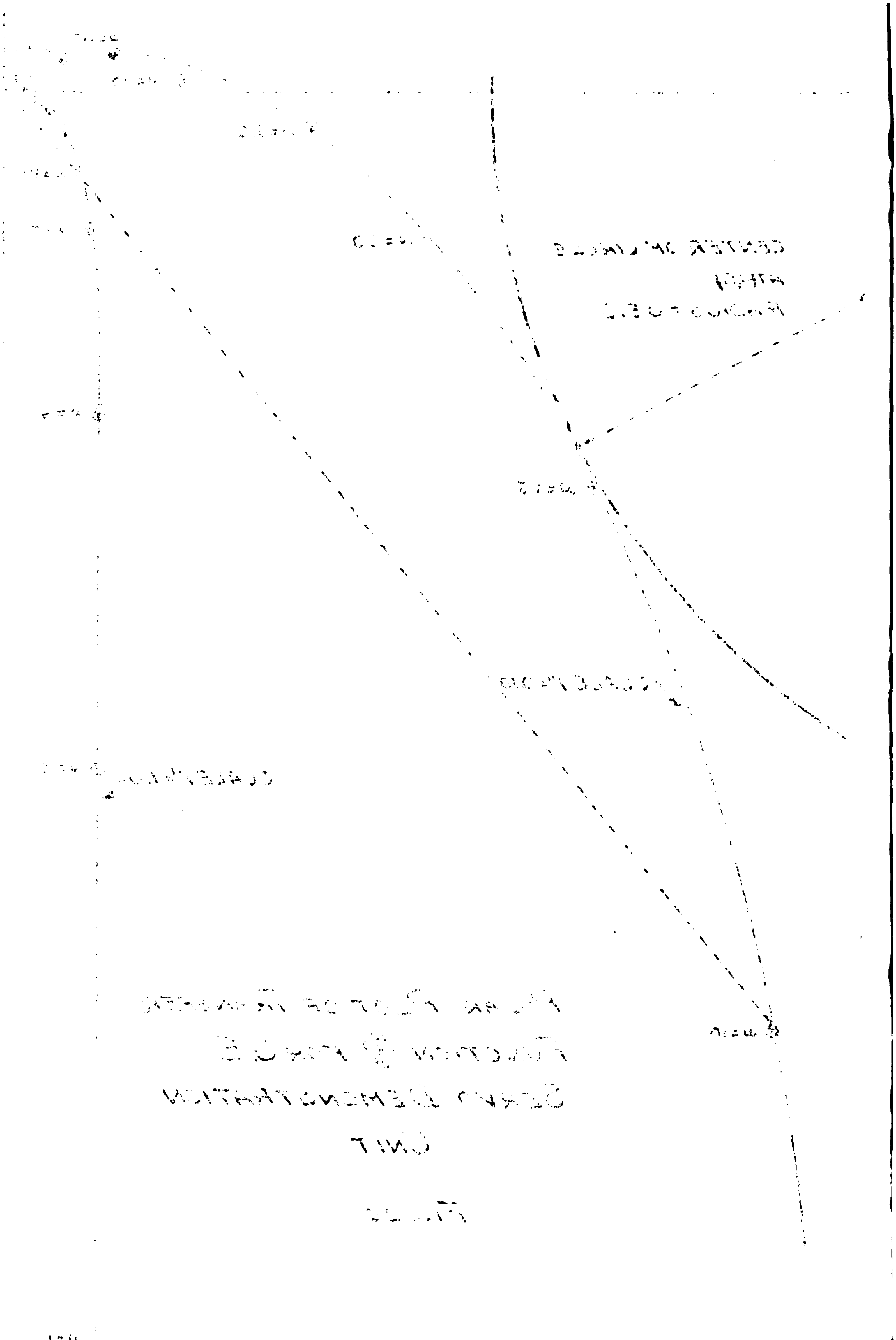


TABLE B COMPUTATION OF $\frac{\theta_o}{\theta_N}$ WITH ANTIHUNT CIRCUIT

ω RAD/SEC	A+jB	$M\angle\alpha$	C+jD	C+jD	$N\angle\beta$	$\frac{\theta_o}{\theta_N} = \frac{10^8 M\angle\alpha}{N\angle\beta\ 24}$
0	179.8+j0	$179.8\angle 0$	0 +j 689	0+j689	$689\angle 90°$	$\infty\angle -90°$
1	179.8+j864	$406\angle 63.9°$	(1.782-1454)+j(-100.8+689)	-1452+j588	$1564\angle 157.9°$	$10.88\angle -94.0°$
2	179.8+j728	$748\angle 76.2°$	(14.26-2908)+j(403+689)	-2894+j286	$2894\angle 174.3°$	$5.43\angle -98.1°$
4	179.8+j1456	$1465\angle 83°$	(114.2-5830)+j(-1612+689)	-5716-j923	$5780\angle 189.2°$	$2.66\angle -106.2°$
8	179.8+j2912	$2930\angle 86.5°$	(915-11640)+j(-6470+689)	-10,720-j5780	$12,200\angle 208.3°$	$1.25\angle -121.8°$
10	179.8+j3640	$3640\angle 87.2°$	(1782-14540)+j(-10,080+689)	-12,760-j9390	$15,810\angle 216.4°$	$0.967\angle -129.2°$
15	179.8+j5460	$5460\angle 88.0°$	(6330-21,800)+j(-22,700+689)	-15,770-22,000	$27,000\angle 234.3°$	$0.567\angle -146.3°$
20	179.8+j7280	$7280\angle 88.6°$	(14,280-29,080)+j(-40,400+690)	-14,800-j39,700	$42,300\angle 249.5°$	$0.358\angle -160.9°$
25	179.8+j9100	$9100\angle 89.0°$	(27,900-36,400)+j(-63,200+700)	-8500-j62,500	$63,000\angle 262.3°$	$0.242\angle -173.3°$
30	179.8+j10,920	$10,920\angle 89.04°$	(48,100-43,700)+j(-90,800+700)	+4400-j90,100	$90,100\angle 272.8°$	$0.136\angle -183.8°$
40	180+j14560	$14560\angle 90°$	(14,000-58,200)+j(-161,400+700)	+55,800-j160,700	$169,500\angle -70.8°$	$.0895\angle 160.8°$
50	180+j18,200	$18,200\angle 90°$	(223,000-72,700)+j(-252,000+700)	+150,300-j251,300	$293,000\angle -59.1°$	$.0518\angle 149.1°$
70	180+j25,500	$25,500\angle 90°$	(611,000-101,800)+j(-495,000+700)	509,200-j494,000	$708,000\angle -44.2°$	$.0215\angle 134.2°$
100	180+j36,400	$36,400\angle 90°$	(1,782,000-145,000)+j(-1,008,000+700)	1,637,000-j1,007,000	$1,920,000\angle -31.6°$	$.0079\angle 121.6°$



but are drawn to different scales in order to bring out its general shape and to see its characteristics when crossing the real axis. It crosses at the point (-0.196,0) and the servo is therefore stable.

The radius of the circle with center at the point (-1,0) and tangent to the transfer locus is a measure of the stability of the system and its dampening constant. It is interesting to note that no matter what type of damping the system employs, all systems with the same damping coefficient will have the stability circle tangent at the same point. With a radius of 0.615, the damping coefficient is 0.45.

Some period of time was spent in trying to obtain the curve in Figure 25 experimentally. The loop was opened by blocking the rotor of the selsyn transformer on the output shaft and disconnecting it mechanically from the output motor. Selsyn generators were connected to the input and output shafts so as to obtain an A.C. voltage proportional to their respective angular displacement.

The input shaft was then driven back and forth about the zero reference point sinusoidally. This was done by mounting a long rod eccentrically on the shaft of a drive motor and connecting the other end of the rod to the outer rim of the flywheel geared to the input shaft . As the motor revolved the input was moved with a sinusoidal motion of about 2.5 degrees amplitude and with the same number of cycles per second as the motor made revolutions per second.

The speed of the motor was varied for each test and an oscillograph used to photograph the voltages from the input and output selsyns.

The amplitude of the error was constant and the amplitude and phase of the output could be then measured from the oscillographs. Using this data the polar plot of the transfer function could be made.

The difficulty encountered in the above procedure was due to the fact that the output shaft of the servo did not oscillate about one point but slowly "crept". This was caused by not having the input vibrating about the exact zero error point of the system. This point is very critical and changes with small fluctuations in line voltage and operating temperature.

Further investigations into experimentally obtaining this polar plot of the transfer function might use the output voltage of the amplidyne and convert it into the output displacement by certain derived expressions.

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