ESTABLISHING AND MEASURING A BASE LINE

THESIS FOR THE DEGREE OF B. S.

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1934

Surveying
Title: Base line

Establishing and Measuring
A Base Line

A Thesis Submitted to
The Faculty of
MICHIGAN STATE COLLEGE
of
AGRICULTURE AND APPLIED SCIENCE

By

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Candidates for the Degree of

Bachelor of Science

June 1934

THESIS

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FOREWORD

We chose the subject of "Establishing a Base Line" for our thesis because we wished to learn more about United States Coast & Geodetic Survey methods of obtaining their great accuracy in measuring distances. We first met with this subject in a Geodesy class at Michigan State College. We believe that the practice obtained in performing the work necessary for the completion of this thesis has given us experience that we otherwise would not have obtained.

We wish to thank the Civil Engineering Department for supplying the necessary equipment, to thank Professor C.M. Cade for his aid, advice and criticism, and to thank Mr. F.F. VanAtta and Mr. D.H. Barnes for the elevations with which they provided us, in connection with their thesis.

We believe that the college will derive some benefit from this thesis because the base line is permanent and can be used in connection with courses given in Geodesy and Astronomy.

Signed

Howard Girardin

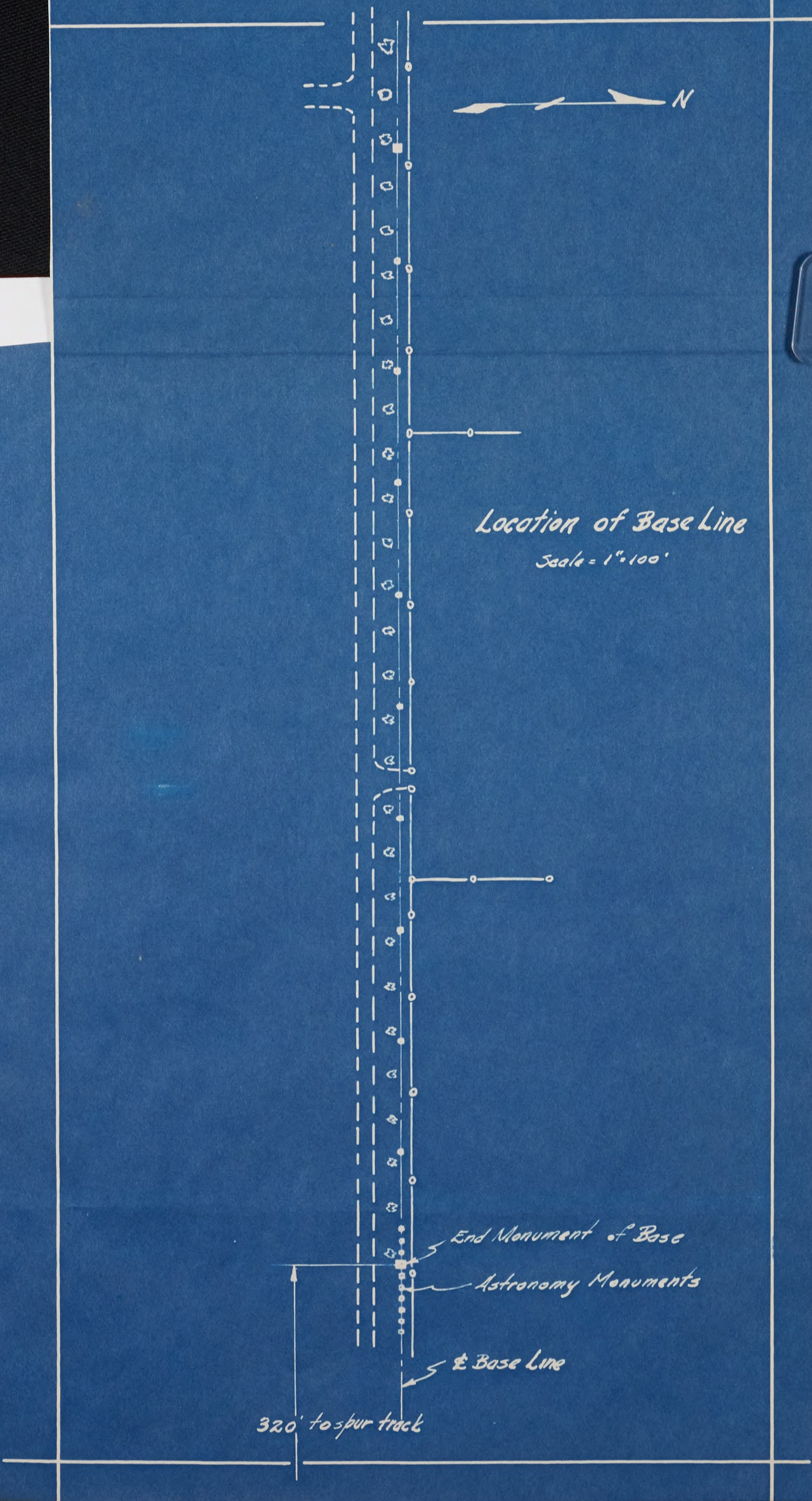
Fred Wheeler

Louis Stachel

Establishing and Measuring
A Base Line

A base line is the preliminary step in triangulation work and therefore is a very important part. The length of the base line must be determined by at least two complete measurements with standardized tapes, with a resultant accuracy of a probable error of 1 part in 1 million. This measurement is used in triangulation work to compute the sides of the original triangle by the method of "Least Squares". Then the computed sides are used to determine the sides of other triangles. This procedure continues and it can be seen that a slight error in the measurement of the base line will accumulate as the work progresses.

The base line is located on the north side of the road running East and west from Farm Lane, in front of the Beef Cattle Barn, to South Harrison Road in front of the Michigan State Police Barracks. It begins about 300 ft. west of the spur track and extends westward for approximately 1000 ft. A sketch showing the location of the base line in reference to the fence and road is on the following page.


N
Location of Base Line
Scale = 1" = 100'
End Monument of Base
Astronomy Monuments
℄ Base Line
320' to spur track

In order to make the base line permanent the monuments were constructed of concrete. The monument at each end of the base is 16 in. square at the bottom and is placed about 12 in. under the surface of the ground and projects about 8 in. above the surface. They are reinforced with an iron pipe the total length of the monument. Securely fastened in the top of the pipe is a 3 in. diameter, standard U.S.C.& G.S. brass plate. Each end monument is also fitted with a brass plate providing spaces for Latitude, Longitude and Elevation of the mark in the center of the 3 in. diameter brass plate. The monuments placed in between the two large monuments are the standard U.S.C. & G.S. monuments, placed at intervals of approximately 100 ft. These monuments are 11 in. square at the bottom and taper down to 7 in. square at the top, being 4 ft. 6 in. high. They are placed in the ground with from 8 to 18 in. of the monument protruding above the surface, depending upon the topography of the ground between stations. Each monument is reinforced with an iron

End Monument

pipe in which a brass plate is secured, as in the end monuments.

At the east end of the base smaller monuments are placed at intervals of 10 ft., to be used in connection with the field work of the astronomy classes given by the Civil Engineering Department. These monuments are 6 in. square and 3 ft. long containing a brass plate and reinforced in the same manner as the other monuments.

After the sites of the terminal stations were selected, the first step was to place the transit on one end of the line and sight on a range pole of target accurately placed on the other end of the line. The stakes to mark the points to place the monuments were accurately set on line at intervals of approximately 100 ft. The line was then cleared of brush so that the tape would hang free when under tension.

The intermediate and astronomical monuments were made in the concrete laboratory, making use of the forms provided by the U.S.C. & G.S. They were made of concrete of a 1:4:2 mix and of a consistancy so that the mixture could be easily tamped. After setting for a week the monuments were transported to the site of the base line. The end monuments were poured in the field, using a standard form provided by the U.S.C. & G.S. After secondary stakes were set and strings stretched to mark the point already marked on the preliminary stakes, holes were dug and the concrete monuments placed directly under the intersection of the strings. The monuments provided for the astronomy class were placed in the ground directly on line and approximately 10 ft. apart. A mark was scratched on the brass plate directly on line to be of aid in lining the tape. Also a mark was scratched perpendicular to the first one as a reference when the distance was measured.

The measuring of the base was done with tapes which were standardized at the Bureau of Standards at Washington D.C. All measurements of the line were run once forward and once backward with each tape and under varying weather conditions. Tape-stretchers and a standardized scale were employed to get the required tension on each tape for every measurement. Dividers and a steel scale, divided into 1/100 of an inch, were used to obtain the slight variations over or under 100.000 ft.

The rear tape-man stretched the tape with a tape-stretcher and accurately placed the zero of the tape on the mark on the center of the brass plate at the zero station. The front man stretched the tape to the required tension and kept it thus until the

Rear Tape-man

third man had measured the slight deviation from exactly 100.000 ft., by means of the dividers. When the third man had measured it with the dividers, he then used the scale and found out how many 1/100 of an inch it was over or under 100.000 ft. He recorded this as (+) when over and (-) when under and while he did this the front man read and recorded the temperature on a standardized thermometer which was supported on the tape so that the exact temperature of the tape could be known. This procedure was repeated between each station until the end of the line was reached. Then the tape was turned around and another measurement was made in the same manner going back to the zero station.

Front tape-man and third man

The lengths of the tapes and the pounds of tension to apply varied with each tape. This information along with the temperature at which the tape was standardized was sent from Washington, D.C. when the tapes were returned after they were standardized.

In finding the latitude of one end of the base line, and the aximuth of the line, observations on Polaris were taken. These observations were taken at night along with the astronomy class. A theodolite, a chronometer and a stop watch were used in taking the readings. The theodolite was set over the point on one end monument and the plates set on zero. The cross hairs were sighted on the Capitol National Bank tower in Lansing and an angle of approximately 90 degrees turned off. The telescope was elevated to an angle of about 42 degrees and when this was done, Polaris could be seen in the telescope lens. The cross hairs were set horizontally and vertically on Polaris and the stop watch syncronized with the chronometer. When the instrument-man had the cross hairs exactly on the star he called "take" and the stop watch was stopped. The time to a tenth of a second was noted and recorded. The horizontal and vertical angles were read and recorded. Three readings were taken in this manner and then the cross hairs were sighted again on the tower and the horizontal angle read to serve as a check on the readings. The telescope was inverted and the same procedure followed to take three more readings with the telescope in this position to take care of any error in the instrument. After these were taken the cross hairs hairs again centered on the tower and the mark checked. This completed the observations.

The same observations were used to find the azimuth of the line as those to find the Latitude of the point. The azimuth of the star was computed and the horizontal angle that was read on the theodolite when each reading was made was added to the azimuth of the star. This yielded the azimuth of the imaginary line from the point to the tower. The angle from the tower to the base line was turned off and added to the azimuth of the imaginary line from the point to the tower. This addition yielded the required azimuth of the line proper.

Conclusion

In concluding this thesis we hope to have conveyed to the reader a definite and clear idea of how a base line, suitable for U.S.C. & G.S. work is measured and how the Latitude of a point and the azimuth of a line may be computed from a series of observations on a pole star. Upon examining the data it is very plainly seen that the best results in measuring were obtained when the temperature remained nearly constant because the variation due to a few degrees change in the temperature of the tape causes a considerable error in the measurement of the line. Therefore the best time for measuring a line is on a cloudy, misty day or else at night.

The following data was obtained in measuring the base line with tape No. B.S. 4413 on a cloudy day.

Supported at 0 & 100 ft. at 68°F, 10 lbs. tension

Length = 99.940 ft.

Sta. No.	Temperature Degrees F.	Dist.(+)or(-) ft.	Diff. in elev. ft.	Sta. No.	Elev.
0-1	70	+0.07917	0.040	0	849.1998
1-2	69	-0.03500	0.133	1	849.2398
2-3	69	+0.04250	1.585	2	849.3727
3-4	72	+0.00917	1.482	3	847.7877
4-5	72	-0.02917	0.089	4	846.3057
5-6	71	+0.01083	0.255	5	846.2163
6-7	70	+0.08250	0.585	6	846.4713
7-8	69	-0.04083	2.568	7	847.0563
8-9	70	-0.00583	0.758	8	849.6243
9-10	70	-0.01833	0.505	9	850.3823
				10	849.8873
				10	849.8873
10-9	70	-0.01833	0.505	9	850.3823
9-8	70	-0.00583	0.758	8	849.6243
8-7	70	-0.03917	2.568	7	847.0563
7-6	70	+0.08167	0.585	6	846.4713
6-5	70	+0.01167	0.255	5	846.2163
5-4	70	-0.03000	0.089	4	846.3057
4-3	69	+0.01167	1.482	3	847.7877
3-2	67	+0.04417	1.585	2	849.3727
2-1	66	-0.03833	0.133	1	849.2398
1-0	68	+0.08167	0.040	0	849.1998

By using the data on page 16, the following results were obtained.

Sta. No.	Meas. dist. ft.	Temp. corr.	True inc. length	Grade corr. (-)	True hor. length
0-1	100.01917	+0.00129	100.02046	0.00000	100.02046
1-2	99.91500	+0.00065	99.91565	0.00009	99.91556
2-3	99.98250	+0.00065	99.98315	0.01256	99.97059
3-4	99.94917	+0.00258	99.95175	0.01099	99.94076
4-5	99.91083	+0.00258	99.91341	0.00000	99.91341
5-6	99.95083	+0.00194	99.95277	0.00033	99.95244
6-7	100.02250	+0.00129	100.02379	0.00172	100.02207
7-8	99.89917	+0.00065	99.89982	0.03302	99.86680
8-9	99.93417	+0.00129	99.93546	0.00287	99.93259
9-10	99.92167	+0.00129	99.92296	0.00127	99.92169
10-9	99.92167	+0.00129	99.92296	0.00127	99.92169
9-8	99.93417	+0.00129	99.93546	0.00287	99.93259
8-7	99.90083	+0.00129	99.90212	0.03301	99.86911
7-6	100.02167	+0.00129	100.02296	0.00173	100.02123
6-5	99.95167	+0.00129	99.95296	0.00033	99.95263
5-4	99.91000	+0.00129	99.91129	0.00000	99.91129
4-3	99.95167	+0.00065	99.95232	0.01098	99.94134
3-2	99.98407	-0.00065	99.98342	0.01257	99.97085
2-1	99.90167	-0.00129	99.90038	0.00009	99.90029
1-0	100.02167	0.00000	100.02167	0.00000	100.02167

The following data was obtained in measuring the base line with tape No. G2155 on a cloudy, misty day.

Supported at 0 & 100 ft. at 68°F, 22 lbs. tension

Length = 99.998 ft.

Sta. No.	Temperature Degrees F.	Dist.(+)or(-) ft.	Diff. in elev. ft.	Sta. No.	Elev.
0-1	52	+0.08249	0.040	0	849.1998
1-2	53	-0.03580	0.133	1	849.2398
2-3	54	+0.03540	1.585	2	849.3727
3-4	54	-0.00299	1.482	3	847.7877
4-5	54	-0.07583	0.089	4	846.3057
5-6	54	+0.00458	0.255	5	846.2163
6-7	54	+0.08291	0.585	6	846.4713
7-8	54	-0.08333	2.568	7	847.0563
8-9	56	-0.00166	0.758	8	849.6243
9-10	58	-0.02166	0.505	9	850.3823
				10	849.8873
				10	849.8873
10-9	51	-0.02416	0.505	9	850.3823
9-8	51	-0.00166	0.758	8	849.6243
8-7	51	-0.08208	2.568	7	847.0563
7-6	52	+0.08541	0.585	6	846.4713
6-5	52	+0.00499	0.255	5	846.2163
5-4	52	-0.07374	0.089	4	846.3057
4-3	52	-0.00208	1.482	3	847.7877
3-2	52	+0.03624	1.585	2	849.3727
2-1	52	-0.03416	0.133	1	849.2398
1-0	52	+0.08333	0.040	0	849.1998

By using the data on page 12, the following results were obtained.

Sta. No.	Meas. dist. ft.	Temp. corr. (-)	True inc. length	Grade corr. (-)	True hor. length
0-1	100.08049	0.01032	100.07017	0.00000	100.07017
1-2	99.96220	0.00968	99.95253	0.00008	99.95244
2-3	100.03340	0.00903	100.02437	0.01258	100.97499
3-4	99.99501	0.00903	99.98598	0.01099	99.97499
4-5	99.92217	0.00903	99.91314	0.00000	99.91314
5-6	100.00258	0.00903	99.99355	0.00032	99.99323
6-7	100.08091	0.00903	100.07188	0.00171	100.07017
7-8	99.91467	0.00903	99.90564	0.03303	99.87261
8-9	99.99634	0.00774	99.98860	0.00286	99.98574
9-10	99.97634	0.00645	99.96989	0.00128	99.96861
10-9	99.97384	0.01097	99.96287	0.00127	99.96160
9-8	99.99634	0.01097	99.98537	0.00285	99.98252
8-7	99.91592	0.01097	99.90495	0.03302	99.87193
7-6	100.08341	0.01032	100.07309	0.00173	100.07136
6-5	100.00299	0.01032	99.99267	0.00032	99.99235
5-4	99.92426	0.01032	99.91394	0.00000	99.91394
4-3	99.99592	0.01032	99.98560	0.01094	99.97466
3-2	100.03424	0.01032	100.02392	0.01256	100.01136
2-1	99.96384	0.01032	99.95352	0.00009	99.95343
1-0	100.08133	0.01032	100.07101	0.00000	100.07101

The following data was obtained in measuring the base line with tape No. G2155 at night.

Supported at 0 & 100 ft. at 68°F, 22 lbs. tension

Length = 99.998 ft.

Sta. No.	Temperature Degrees F.	Dist.(+)or(-) ft.	Diff. in elev. ft.	Sta. No.	Elev.
0-1	60	+0.08416	0.040	0	849.1998
1-2	60	-0.03583	0.133	1	849.2398
2-3	60	+0.03583	1.585	2	849.3727
3-4	60	-0.03333	1.482	3	847.7877
4-5	60	-0.07750	0.089	4	846.3057
5-6	60	+0.00416	0.255	5	846.2163
6-7	60	+0.09583	0.585	6	846.4713
7-8	59	-0.08416	2.568	7	847.0563
8-9	59	-0.00083	0.758	8	849.6243
9-10	59	-0.01166	0.505	9	850.3823
				10	849.8873
				10	849.8873
10-9	58	-0.01450	0.505	9	850.3823
9-8	58	-0.00166	0.758	8	849.6243
8-7	58	-0.08583	2.563	7	847.0563
7-6	58	+0.09333	0.585	6	846.4713
6-5	58	+0.00166	0.255	5	846.2163
5-4	58	-0.07250	0.089	4	846.3057
4-3	58	-0.00250	1.482	3	847.7877
3-2	58	+0.03416	1.585	2	849.3727
2-1	58	-0.03500	0.133	1	849.2398
1-0	58	+0.08442	0.040	0	849.1998

By using the data on page 14, the following results were obtained.

Sta. No.	Meas. dist. ft.	Temp. corr. (-)	True inc. length	Grade corr. (-)	True hor. length
0-1	100.08216	0.00516	100.07700	0.00000	100.07700
1-2	99.96217	0.00516	99.95701	0.00007	99.95694
2-3	100.03383	0.00516	100.02867	0.01257	100.01610
3-4	99.96466	0.00516	99.95950	0.01096	99.94854
4-5	99.92050	0.00516	99.91534	0.00000	99.91534
5-6	100.00216	0.00516	99.99700	0.00034	99.99666
6-7	100.09383	0.00516	100.08867	0.00173	100.08694
7-8	99.91384	0.00581	99.90803	0.03302	99.87501
8-9	99.99717	0.00581	99.99126	0.00287	99.98839
9-10	99.98634	0.00581	99.98053	0.00127	99.97925
10-9	99.98350	0.00645	99.97705	0.00125	99.97580
9-8	99.99634	0.00645	99.98989	0.00286	99.98703
8-7	99.91217	0.00645	99.90572	0.03301	99.87271
7-6	100.09133	0.00645	100.08488	0.00172	100.08316
6-5	99.99966	0.00645	99.99321	0.00032	99.99289
5-4	99.92550	0.00645	99.91905	0.00000	99.91905
4-3	99.99550	0.00645	99.98905	0.01099	99.97806
3-2	100.03217	0.00645	100.02572	0.01255	100.01317
2-1	99.96300	0.00645	99.95655	0.00009	99.95646
1-0	100.08241	0.00645	100.07596	0.00000	100.07596

By using the data on page 18, the following results were obtained.

Sta. No.	Meas. dist. ft.	Temp. corr.	True inc. length	Grade corr. (-)	True hor. length
0-1	100.02667	+0.00258	100.02925	0.00000	100.02925
1-2	99.91167	+0.00129	99.91296	0.00008	99.91288
2-3	99.98167	+0.00129	99.98276	0.01256	99.97020
3-4	99.95583	+0.00129	99.95712	0.01098	99.94614
4-5	99.91250	+0.00129	99.91379	0.00000	99.91379
5-6	99.95667	+0.00065	99.95732	0.00033	99.95699
6-7	100.02667	+0.00065	100.02732	0.00171	100.02561
7-8	99.90676	0.00000	99.90676	0.03302	99.87365
8-9	99.94167	0.00000	99.94167	0.00286	99.93881
9-10	99.92833	0.00000	99.92833	0.00127	99.92706
10-9	99.92750	0.00000	99.92750	0.00130	99.92620
9-8	99.94333	0.00000	99.94333	0.00283	99.94050
8-7	99.90167	0.00000	99.90167	0.03301	99.86866
7-6	100.02750	0.00000	100.02750	0.00171	100.02579
6-5	99.95917	-0.00129	99.95788	0.00033	99.95755
5-4	99.91500	-0.00129	99.91371	0.00000	99.91371
4-3	99.95583	-0.00065	99.95518	0.01096	99.94422
3-2	99.98500	-0.00065	99.98435	0.01256	99.97179
2-1	99.91033	-0.00065	99.91018	0.00008	99.91010
1-0	100.02500	-0.00065	100.02435	0.00000	100.02435

The values on pages 13 and 15 were given a weight of 2 and the values on pages 11,17 and 19 were given a weight of 1. Then the arithmetical mean was taken and the following results obtained.

Sta. No.	Mean hor. length ft.
0-1	100.05676
1-2	99.94012
2-3	100.00007
3-4	99.96185
4-5	99.91436
5-6	99.98225
6-7	100.06108
7-8	99.87158
8-9	99.97076
9-10	99.95526

Total length = 999.71341

Sample Calculations for Base Line

Data taken from page 10.

Tape G2155   Length = 99.998 ft. at 68°F and 22 lbs tension

Measurement made from Sta. 1 to Sta. 2.

Temperature = 54°F.

Increment under 100.00' = 0.04583'

Meas. length = 99.998 - 0.04583 = 99.95217

Temp. corr. = 0.00000645 X 100 X 14 = -0.00903'

True inclined length = 99.95217 - 0.00903 = 99.94314'

Grade correction = $-\left(L - \sqrt{L^2 - H^2}\right)$

where L = length and H = diff. in elev.

Grade correction = $-\left(99.94314 - \sqrt{99.94314^2 - 0.133^2}\right)$

= -0.00009'

True hor. length = 99.94314 - 0.00009 = 99.94305'

Data and Calculations for Obtaining

Latitude and Azimuth of Line

Telescope	Watch time h	m	s	Vertical angle	Horizontal angle
Reversed	20	39	48.6	41° 52' 00"	81° 22' 40"
"	20	49	19.0	41° 51' 10"	81° 26' 20"
"	20	52	21.7	41° 50' 40"	81° 27' 30"
Direct	21	02	36.0	41° 53' 40"	81° 30' 00"
"	21	05	33.1	41° 54' 40"	81° 31' 10"
"	21	08	48.6	41° 55' 00"	81° 32' 20"
Average	20	56	20.53	41° 52' 51.67"	81° 28' 20"

		h	m	s
Average of watch readings	=	20	56	20.53
Known watch and clock corr.	=			+18.00
Local civil time (T)	=	20	56	38.53

Solar time (θ) = 11h 31m 56.27s or 174° 9' 0.45"

Coordinates of Polaris δ = 88° 56' 58.32"; α = 1h 37m 64.03s

Hour angle (t) = $\theta - \alpha$ = 9h 53m 52.24s or 148° 28' 3.6"

Reduction

θ = 11h 31m 56.27s

α = 9h 53m 52.24s

t = 12h 49m 3.6s

h' = 41° 52' 51.67"

Z' = 48° 7' 8.33"

r = 0° 1' 8.64"

Z = 48° 8' 16.97"

Reduction Con'd

$\tan F = \cot\delta \cos t$

$\cot\delta = 0.01834$

$\cos t = -0.85248$

$\tan F = -0.01563$

$F = -0° 52' 43.44''$

$\sin(\phi + F) = \cos F \cos Z \csc$

$\cos F = 0.000883$

$\cos Z = 0.667338$

$\csc\delta = 1.000170$

$\sin(\phi + F) = 0.6673734$

$(\phi + F) = 41° 51' 51.82''$

$\phi = 42° 44' 35.26''$

$$\text{Sin } A = \frac{\sin t \cos\delta}{\sin Z} = \frac{0.52297 \times 0.01833}{0.74475} = 0.0128714$$

Azimuth of star (A) = 0° 44' 20.00"

Azimuth to tower = 97° 51' 40.00"

Angle from tower to line = 7° 55' 7"

Azimuth of line = 97° 51' 40.00" - 7° 55' 7"

" " " = 89° 56' 33.00"