

MEASUREMENTS OF WATER TABLE FLUCTUATIONS IN AN ORGANIC SOIL BY MEANS OF OBSERVATION WELLS AND WATER STAGE RECORDERS

> Thesis for the Degree of M. S. MICHIGAN STATE COLLEGE **Tuvijas Goldoftas** 1951

## This is to certify that the

thesis entitled

"Measurements of Water Table Fluctuations in an Organic Soil by Means of Observation Wells and Water Stage Recorders"

presented by

Tuvijas Goldoftas

has been accepted towards fulfillment of the requirements for

M.S. degree in Agricultural Engineering

Ernest N. Kidder

Date May 18, 1951

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THESIS

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MEASURETHTS OF WATER TABLE FLUCTUATIONS IN AN ORGANIC SOIL BY MEANS OF OBSERVATION WELLS AND WATER STAGE RECORDIES

By

Tuvijas Goldoftas

#### A THESIS

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

MASTAR OF SCIENCE

Department of Agricultural Engineering

**THESIS** 

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#### ACKNOLLEDGMENT

The writer is deeply grateful to Professor E.H. Kidder, of the Department of Agricultural Engineering, for his continued encouragement and assistance during the course of this study. He is indebted to Mr. J.G. Ferris of the United States Geological Survey, Lansing, Michign, for suggesting the problem, loaning the instruments used thereon and for his helpful advice. Gratitude is expressed to Dr. S.G. Bergquist, head of the Department of Geology and Geography, under wines guidance a large portion of this problem was conducted.

Dr. J.F. Davis, of the Department of Soil Science, made the study possible by making available the use of the Michigan State College muck Experimental Farm and showed sustained interest during the entire duration of the problem.

Precipitation data were obtained from Mr. G.A. Crabb Jr., Supervisor of the Michigan Hydrologic Research Project in cooperation with the Michigan Agricultural Experiment Station, East Lansing, Michigan, who in addition gave freely of his time and helpful criticism.

# TABLE OF CONTANYS



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## LIST OF ILLUSTRATIONE



# Table

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## **INTRODUCTION**

The paramount importance of and need for water is well understood and appreciated since without it no form of life can exist. Water constantly supplies the animal and vegetable kingdoms with daily subsistence, provides avenues of transportation, and is a considerable source of modern power.

Water occurs almost everywhere. Its greatest concentration is in the ocean, its greatest scarcity in the desert. Water manifests itself in many forms, examples of which are water vapor; clouds or precipitation in the atmosphere; streams, lakes and oceans on the earth's surface; and sub-surface water in the lithosphere.

Water is of importance to a multitude of individuals such as hydraulic engineers, industrial engineers, hydroelectric engineers, geolOgists, agriculturalists and agricultural engineers. To the latter, the most important type of water is that which is available to plants. The amount of water available to plant roots is a critical factor since upon it depends the very life of every tree, tomato plant, shrub, and the life of man himself.

A.continuous circulation of water to and from.the earth's surface takes place at all times. Water is evaporated from the ocean and large inland bodies of water and rises into the atmosphere, where it condenses and returns to the earth's surface in the form of rain, hail, snow, dew or frost.

Some precipitation falls directly into bodies of water and is re-evaporated. Precipitation which strikes the earth's surface follows different channels of disposal. A part is intercepted by vegetation and man made features (such as buildings) and re-evaporated directly. A portion runs off from the surface of the earth into lakes and streams and is eventually returned to the ocean. A certain amount of this precipitation enters the ground as infiltration. Of this portion a quantity of water is held near the surface by capillary action and is lost by evaporation; some is used by vegetation; a certain amount penetrates the ground through percolation, infiltration, seepage and subsurface flow to find its way back to the streams and ocean by underground channels. Some of this ground-water may percolate to greater depths and reappear later in the form of springs, artesian wells, and geysers. This continuous circulation of water is called the hydrologic cycle, Fig. 1.

The major reasons for the study of water table fluctuations in an organic soil were the following;

- 1. To investigate the natural supply of ground-water in an organic soil in the face of evidence of decreasing water supplies in mineral soils
- 2. To determine whether the method of observation wells (shallow) can successfully be applied in the recording of water table fluctuations on organic soils
- 3. To determine whether the drainage system now in existence is adequate and efficient
- 4. To make recommendations on the findings of the above objectives.

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CYCLE HYDROLOGIC THE<br>T  $\mathbf{I}$ FIGURE

To the best knowledge of the writer, similar studies have not been made on an organic soil in the past. Drainage of muck is an important and difficult task because of the danger of overdrainage, a condition which may prove to be disastrous to the owner of the land.

The method used in this study consisted of a series of observation wells installed between three tile lines. Two automatic water stage recorders were used to give a continuous record of the water table level fluctuations in two of the wells. The wells - 13 in total consisted of twelve-5 foot wooden wells and one-13 foot metal well. Recordings of the fluctuations of the water table level at the Michigan State College Muck Experimental Farm began on September 1. 1949 and ended March 30, 1950. During this period of time data were gathered on: daily fluctuations of the ground-water level (or water table) in the metal and one of the wooden wells equipped with automatic water stage recorders, Figs. 8 and 9; daily precipitation; daily temperatures; weekly tape measurements of the water table level in the remaining wooden wells. The continuous hydrographs were combined into one composite hydrograph of the entire duration of the problem in Fig.  $15.$ 

#### REVIEW OF LITERATURE

## King

Some of the earliest observations of water table fluctuations were made by King (1) in 1888 near the University of Wisconsin campus. Noting diurnal fluctuations in certain shallow wells, King attributed these variations to changes in temperature.

#### **Veatch**

In 1903, Veatch (6) conducted "a few observations" on Long Island, New'York, and compiled a detailed publication on his study. His primary contribution was a classification of the fluctuations of water levels in wells.

"A. fluctuations due to natural causes.

- I. Rainfall and evaporation
	- l. Fluctuations not depending on single showers
		- a. Regular annual fluctuations
		- b. Irregular secular changes
	- 2. fluctuations produced by single showers
		- a. By transmission of pressure without any actual addition to the ground water
		- b. By the actual addition of rain to the ground water
- II. Barometric changes
- III. Thermometric changes
	- 1. fluctuation directly related to temperature
	- 8. Fluctuation inversely related to temperature
		- a. At the surface of the ground water table, directly through temperature changes
			- b. In deeper zones, by pressure changes produced by fluctuations of the preceding class
- IV. Fluctuations produced by adjacent bodies of surface water: rivers, lakes, the ocean
	- 1. By changes in rate of ground water discharge
	- 2. By seepage
	- 3. By plastic deformation due to various loads
- V. Fluctuations due to geologic changes
- B. Fluctuations due to human agencies
	- 1. Settlement, deforestration, cultivation, drainage
	- 2. Irrigation
	- 3. Dame
	- 4. Underground water supply developments
	- 5. Unequal loading
- 0. Fluctuations due to indeterminate causes."

## Lee

In 1912, Lee concluded that not only could evaporation be measured, but that it acted as an accurate indicator of available ground-water supply in valleys of the closed basin type (2).

## Smith

In 1916, Smith noted daily fluctuations resembling those observed by King (5). He reasoned that the daily decline of the water table was due to withdrawal by trees of ground-water from the zone of saturation.

# White

An experiment conducted in the Escalante Valley near Milford, Utah, was carried out by White (7) from 1925 until 1927. This region occupies an area of about 1,000 square miles and is situated between the ranges of the Mineral and the Iron Mountains on the east side and the San Francisco Mountain on the west side. During the Pleistocene time it was part of former Lake Bonneville. The soils encountered during the study were (1) deep loam, alkali free; (2) gravelly soil over clay base, almost alkali free; (3) heavy alkali soils from which salts can be leached by washing; (4) highly alkaline soils, unsuitable for agrioulture, and (5) sandy soils, unsuitable for farming.

White discusses in detail the daily fluctuations of the water table in relation to the following types of land coverings: alfalfa. salt grass, greasewood, rabbit brush, shad scale, pickleweed, willows, meadow grass, sage brush and cleared fields, before and after cutting plants.

This study proved convincingly that diurnal fluctuations of the watar table are caused by growing vegetation, drawing its supply from shallow ground-water. White advanced the following formula for the determination of the amount of water absorbed by plants and lost through evaporation:

# $q = y(24 r - s)$

where  $(q)$  is the depth of water withdrawn in inches,  $(y)$  is the specific yield of the soil,  $(r)$  is the hourly rate of rise of the water table from midnight to 4 a.m. in inches, and (s) is the not fall or rise of the water table during the 24 hour period, in inches.

### meinser

In his discussion of water level fluctuations in wells, Meinzer (3) reviews briefly the tremendous amount of work done by White. Daily and seasonal fluctuation curves are mentioned, followed by estimates of consumptive use, based on seasonal fluctuations as developed by Harding.

## rerris

The chapter on "Ground-Water" in Hydrology by Wisler and Brater (8) defines the quantitative approach to hydrology from a technical and mathematical viewpoint. It deals with the porosity of materials,

mechanics of interstitial flow, permeability and transmissibility, ground-water hydraulics, and the mathematical derivations of the nonequilibrium and the equilibrium.formnlae. In addition to describing water level changes, caused by natural agencies, Ferris enters into detailed discussion of fluctuations due to pumping.

## Merrie

This study deals with organic soils. The author presents the results he obtained from controlled drainage (5). "Controlled drainage in agriculture is essentially control of the ground-water level by regulation of the flow of water in the drainage system." The author further discusses measurements of ground-water movement in an organic soil by observation of the level changes in ditches surrounding the plots.

Morris emphasizes the fact that muck soils must be under-drained if adequate water supplies are to be made available to the crops. Bub-irrigation is not feasible as it may require up to 12 days to establish the proper amount of available water midpoint between tile lines (50 feet apart). It is suggested that cover crops be used to prevent wind erosion rather than ground water fluctuation control. In his final statement, Morris lists the advantages and disadvantages of sub-irrigation.

#### EXPERIMENTAL WORK

#### Location of the Area

The Michigan State College Muck Experimental Farm, site of this study, is located in T 5 N, R l W, Bath Township, Clinton County, Michigan, Figs. 2 and 3. According to the Clinton County Soil Survey map of 1942 (9), the muck farm is part of the Corey marsh which is overlain primarily by Carlisle muck.

#### Description of the Area

The field where the problem was conducted is rectangular in shape and measures approximately 2250 feet by 800 feet. Its long side is oriented north and south, Fig. 4.

.A road divides the field into two unequal units. The southern part is larger and contains two north-south alleys, each 50 feet wide and divided into two driveways by double rows of young evergreen trees. The field is tiled in an east-west direction. The depth to the tile line at the east end is 48 inches, and at the west end 56 inches. This slope of eight inches in 800 feet thus creates a grade of one inch per 100 feet in the laterals. Along the west edge of the field, and draining northward, lies a main which drains the water from the laterals into a sump equipped with a reversible pump.

#### The weather

One of the most influential factors to affect this investigation was the exceptionally mild weather experienced in the Lansing, Michigan area during the winter season of 1949-1950. It deserves particular

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FIGURE 4 STATE COLLEGE **DIAGRAM** OF THE **MICHIGAN** FARM SHOWING LOCATION MUCK EXPERIMENTAL AND OBSERVATION OF BORINGS WELLS. WELLS  $- - - =$  TILE LINES C.W. = OBSERVATION

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mention if the reader is to avoid erroneous interpretations and misleading conclusions. The temperatures are indicated in Fig. 16. The fluctuations of the water table which were observed would most likely not have occurred had the ground been frozen as normal winter weather in this region would warrant. Unseasonably warm weather, however, prevented the ground from freezing at depth ( a continuous freezing temperature not beginning until Jamary 13, 1950) and allowed precipitation to infiltrate and percolate into the soil. This resulted in water table fluctuations somewhat comparable to spring season conditions.

#### Data Period

Recording of the water table fluctuations began on September 1, 1949 for the metal wall and on December 15, 1949 for the wooden wells. Clock failure resulted in some missing data. All recording of data was discontinued on March 50, 1950. During periods of missing data the readings shown were obtained by measuring the water table level with a steel tape.

Rigorous weather during the last few weeks of the investigation seemed to have little effect upon the functioning of the recorders. A protective cover of unilch around each well actually prevented the water from freezing in the wells.

## Borings and Samples

The purpose of this initial work was twofold: (1) to establish a cross section of the soil profile and (2) to determine the most suitable site for installing an observation well which would rest in sand. The borings were made with a single cup peat rod, Fig. 5.



Figure 5 - Peat rod used in obtaining core samples

The bottom section of the rod consists of a brass cylinder with a bevelled cutting edge for easier ground penetration. The cylinder houses a brass piston, eight inches long and one inch in diameter. The extension sections are of steel, 3/8 of an inch in diameter and 24 inches long.

Samples of cores were collected every one half foot of vertical penetration, and carefully examined. The following strata were encountered in successive order: muck, peat, marl, clay and sand. After borings A,B,C, and D were completed, Fig. 4, it became apparent that the northern part of the field presented a sand level deeper than was possible to reach with the available 13 foot metal wall casing, Table l. The southern portion of the field, however, appeared to present possibilities. Since one of the prerequisites for the installation of the metal well was to have its bottom rest in sand, it became at Once apparent that the northern unit would be impractical in view of the fact that the metal casing was only 13.56 feet long. It appears that, if the site of the muck land was a pond or small lake in the geologic past, the shore must have been in a direction corresponding to the southern end of the field, while the northern unit points to what may have been the center of the body of water. It was on the basis of these preliminary borings that the study was confined to the southern section of the muck farm. By taking samples every half foot, the cross sections indicated by borings 1 to 8, Table 1, were obtained as shown in Figs. 4 and 6. This represents a profile of the land in an eastwest direction over a distance of approximately 700 feet.

16						
TABLE 1 SECTIONS OF WELLS						
Well	$\sim$ Materials - Depth in feet below the ground surface					
	Muck	Peat	Marl	Clay	Sand	
$\mathbf{1}$ $\boldsymbol{z}$	$0 - 6$ $0 - 7$	$6 - 8$ $7 - 10$	$8 - 10$ $10 - 12$	$10 - 20$ $12 - 20$	$20 +$ $20 +$	
$\mathbf 3$	$0 - 5$	$5 - 8$	Trace	$8 - 11\frac{1}{2}$	$11\frac{1}{2}$ +	
$\blacktriangleleft$	$0 - 6$	$6 - 8$	Trace	$8 - 17\frac{1}{2}$	$17\frac{1}{2}$ +	
$\bf{5}$	$0 - 11$	$11 - 12$	$12 - 14$	$14 - 29$	$29 +$	
6	$0 - 7$	$7 - 8$	$8 - 10$	$10 - 27$	$27 +$	
$\boldsymbol{7}$	0- $6\frac{1}{2}$	$6\frac{1}{2} - 8$	$8 - 8\frac{1}{2}$	$8\frac{1}{2}$ -27 $\frac{1}{2}$	$27\frac{1}{2}$ +	
8 $\pmb{\mathtt{A}}$	$0 - 6$ $0 - 6$	$6 - 7\frac{1}{2}$ $6 - 8$	$7\frac{1}{2} - 8$ $8 - 9$	$8 - 23$ $9 - 22$	$23+$ $22 +$	
$\, {\bf B}$	$0 - 6$	$6 - 8$	$8 - 10$	$10 - 23$	$23+$	
$\mathbf C$	$0 - 8$	$8 - 12$	$12 - 18$	$18 - 47 +$	$\boldsymbol{\mathcal{P}}$	
D	$0 - 7$	$7 - 11$	$11 - 22$	$22 - 42 +$	$\pmb{?}$	

TABLE 1 SECTIONS OF WELLS 16<br>TABLE 1 SECTIONS OF WELLS

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#### METAL'HELL

#### Description

The metal well consisted of a galvanized sheet metal pipe, eight inches in diameter and 13.56 feet in length, Fig. <sup>7</sup> . The bottom two feet were drilled with 3/32-inch holes, averaging 10 holes per square inch. The purpose of these perforations was to allow a free water movement to take place through the bottom of the pipe.

### Installation

The hole for the long metal observation well was dug on August 31, 1949 with an eight-inch posthole digger nine feet long. Additional depth was secured by digging a hole in the ground five feet deep with a shovel, then by standing in the bottom of the hole and Operating the posthole digger, enough additional depth was obtained to permit the metal casing to be lowered to a depth Of 12.3 feet from the surface of the ground. A 1.2 foot length Of pipe was left protruding above the soil surface.

## Installation of Water Stage Recorder and Corrections

On September 1, 1949, a Stevens type-F water stage recorder was installed atop the protruding section of the metal well. The copper float used was seven inches in diameter; the recorder was housed in a protective shelter of weather-treated plywood, Figs. 8 and 9.

Considerable difficulty in accurately recording the fluctuations of the water table was encountered at first due to probable plugging at the bottom of the observation well. This condition resulted in interference of free movement of water into and out of the well, probably



FIGURE 7 - CROSS SECTION OF LONG WELL



Figure 8 - Stevens type-F recorder in Operation over long well, recorder shelter tilted back



Figure 9 - Stevens type-F recorder showing (1) drum, (2) pen, (3) chart, (4) pulley for beaded cable, and (5) clock

yielding erroneous data. Three attempts were made to clear the well of the clay and silt particles which had accumulated at the bottom 'of the well and plugged the drilled holes. These attempts included stirring the fine materials with a homemade plunger and immediately pumping the water with the particles in suspension out of the well by means of a pitcher pump. A drive point was finally driven in on September 22, 1949, which projects 18 inches below the bottom of the well. Then, by introducing a small diameter pipe inside the drive point pipe, silt and clay were pumped out and favorable results were obtained subsequently, Figs. 10 and 11. Free water movement was shown by a significant rise of the water table of 1.13 feet within a period of 36 hours, Fig. 11. The precipitation for the same period - December 20-22, 1949 - as recorded on a standard weather Bureau rain gage was 3.01 inches.

#### WOODEN WELL

#### Description

The 12 wooden wells installed consisted of square wooden casings, three inches on the inside, and five feet in length. Each well was equipped with a flange, located one foot from the top of the well. which rested on top of the ground surface and prevented the wells from subsiding into the muck. A small wooden lid covered the top opening of each well. The wooden wells, one buried in the ground and another standing a ongside a standard long handle shovel are shown in Fig. 12.





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FIGURE II - HYDROGRAPH OF METAL AND WOODEN WELLS FOR WEEK OF DECEMBER 15 - 22, 1949





Figure 12 - Short wooden well. At left in nosition, and at right comnared to standard shovel

### Installation

On November 10, 1949, these 12 wooden wells were installed between three tile lines as shown in Fig. 13. The seemingly erratic spacing between wells was caused by the presence of evergreens between which the wells were placed. The purpose of this row of wooden wells was twofold: to determine the water table curve between tile lines, and to find a second recorder site to attemnt to establish any correlation or similarity in fluctuation pattern behavior between the long metal and shallow wooden wells.

### Installation of Water Stage Recorder

On December 15, 1949, another Stevens type-F water stage recorder was installed on a wooden well. The chief difference between the two recorders was the size of the float used for the metal and wooden wells. Contrasted to the 7-inch diameter copper float used in the metal well, the short well accommodated but a  $2\frac{1}{6}$ -inch diameter homemade float. The difference in diameter sizes between the float resulted in less sensitive readings in the wooden well. This is readily manifested in the stairstep appearance of the hydrograph curves in the short wooden well, Fig. 11.





#### RESULTS AND DISCUSSION

Although a number of studies similar in character to the one performed are on record, there is no evidence of any such work on organic soils. Almost all literature encountered dealt with mineral soils.

muck is the product of decomposition of peat, which is a material of organic origin that consists primarily of an accumulation of plant remains in various stages of decomposition. Peat usually develops in humid regions, near inland bodies of water, sich as lakes or ponds, Fig. 14.

The data collection period lasted from September 1, 1949 to March 30, 1950, for a total of seven months. This particular time of year was selected in order to include, within a short period of time, as many different seasons as possible and a diversity of ground coverings. The seasons included the late summer of 1949, the fall of 1949 and the winter of 1949-1950.

Such a choice resulted in a variety of soil cover conditions. The ground was covered with vegetation during the summer and early fall of 1949; it was bare and not frozen during the late fall and early winter of 1949, and it was but partially frozen during the winter of 1950.

The Michigan State College Muck Experimental Farm raised a variety of crops during the growing season of 1949. Some of these were barley, cabbage, beets, carrots, celery, corn, lettuce, mint, onions, potatoes,



After Dachnowski

Figure  $14$  - Diagrammatic representation of the formation of organic deposits.

- A Accumulation of aquatic vegetation on open water (ow) bordered by marsh growth on the left.
- B Filling in of the open water by plant debris and closing in of marsh vegetation.
- C The swamp forest has taken the place of both aquatic and marsh vegetation.

sugar beets, tomatoes, and wheat. Such a diversity of crops makes it difficult to establish which crops are responsible for the various water table fluctuations. Due to the limited time for this study and the impossibility of determining the effect of each crop on the water table, the variation of crops are omitted in the discussion.

## Fluctuations of the Water Table due to Precipitation

Several abrupt rises in the water table level were observed, Fig. 15. In many instances a considerable amount of precipitation occurred the day or days immediately preceding such sharp rises. These rises most obvious during the following Specific periods of time are:



It is important to note that the hydrographs of both the deep and shallow wells (13.56 feet and five feet deep respectively) followed similar water table fluctuation patterns. It seems therefore that the difference in depth to the water table and the reported layer of clay which separated the bottoms of both wells had little influence, if any, on the response of each well to rainfall penetration.

In contrast the following are several heavy precipitations which had little effect on the water table.

> 1. September 19, 1949 - 0.96 inches (rain) 2. October 6, 1949 - 0.72 inches (rain) 3. October 11, 1949 - 0.77 inches (rain)  $4.$  November 24, 1949 - 0.68 inches (snow)

The section of soils above the water table (the zone of aeration) is subdivided into three sections. In descending order they are the belt of soil water, the intermediate belt and the capillary fringe, which is located immediately above the water table.

In order for a particle of water to reach the water table and thus raise its level, the particle must first satisfy the water requirements of the aforementioned belts. When precipitation takes place and reaches the surface of the ground, it begins to infiltrate into the t0p layers of the soil. When the soil is saturated to the point of adequately supplying the needs of the existing root systems, the excess water then slowly proceeds to the intermediate belt. After covering or coating each individual particle with a thin film of water, the water front continues its downward motion to the capillary fringe. The water present in this area is there by virtue of the capillary pull of small "tubes" drawing water from the water table. When the water front reaches the capillary fringe it saturates it thus causing the water table to rise. The same process takes place in reverse order during droughts, when instead of adding to the water table, it is being taxed continuously for supply of moisture.

The rain of 0.92 inches on September 19, 1949 raised the water table only 0.20 feet. This may be explained by the warm temperature which prevailed during that part of the season causing increases in evapo-transpiration. The ground was still in crop at the time, <sup>a</sup> factor which was instrumental in preventing much rainwater from reaching the ground-water level. The examples of rapid water table

rises as described on page 29 were all under bare ground conditions. The last crops were harvested from the farm about the first half of October, 1949.

.Although intense precipitation often causes the water table level to rise sharply, as explained above, instances of no reaction due to precipitation have also been mentioned. Because of such inconsistencies, a brief discussion is necessary to clarify such anomolies.

After precipitation has fallen, numerous variables affect its dieposal. The rise of the water table will depend upon the quantity of precipitation that reaches the zone of saturation (crowned by the water table). The amount of water that may percolate to the water table level depends on (1) the amount of surface runoff that will take place; (2) the amount of evaporation from the ground surface; (3) the evapo-transpiration of the vegetation; (4) the amount of hygroscOpic and capillary water in the soil; (5) the soil and air temperatures, (6) the amount of time required for the precipitation to reach the water table level, depending of course on the permeability of the soil, and the nature of its cover, and (7) the frequency of precipitation.

## Diurnal Fluctuations of the Water Table due to Evapo-transpiration

Daily fluctuations of the water table, especially those Occurring successively in more or less equal increments, are rnanifested by a continuous lowering of the water table and are eattributed to evapo-transpirative consumption. Such fluctuations

can be observed through,the growing season during a rainless, warm. period of time. This occurred during the week of November 3 to November 10, 1949, Fig. 10. Except for 0.09 inches of rain on November 4, the week can be considered to have experienced no precipitation. A gradual lowering of the water table took place from day to day. The "sharper" decline occurred during the hours of 11:00 a.m. and 2:00 p.m. The period of recovery as discussed by White (7) and Ferris (8) does not appear distinctly. In the hydrograph shown, the period of recovery is "hidden" for all practical purposes by a straight line. An attempt was made to "unmask" the real shape of the recovery curve, but due to the small scale of the hydrograph the task had to be abandoned as not only impractical, but highly erroneous. Considering the lack of similar data on organic soils, it may be concluded that in such soils a period of recovery as observed in mineral soils does not normally occur, but that the recovery consists in a stabilization of the ground-water level.

# Characteristics of the Metal and Wooden Wells Water Table Profiles

It is apparent that the water level in the wooden well lagged that of the metal well, Fig. 15, consistently throughout the duration of the study. The explanation for this difference in water level may be interpreted as follows: Since the metal well (13.56 feet) is standing in sand, Fig. 7, which in turn may be an aquifer, the water in this water-bearing formation, if it has an area of intake some distance from the muck farm, is subject to a certain amount of hydrostatic pressure, which is defined by Meinzer (26) as being

"due to the weight of water at higher levels in the same zone of saturation." 'This condition is plausible in view of the fact that a well drilled on the muck farm in the Suring of 1950 yielded an artesian flowing well at a depth of 70 feet. The flow was not abundant, but it was continuous and amounted to the continuous discharge of a garden hose.

Another explanation may rest with the presence of the impermeable layer of clay which separates both water levels. If the clay is completely impermeable, then it may act as a pressure membrane upon the lower formations. In this case any change in barometric pressure or loading of the upper formations with water would compress or load the surface layers which in turn would press against the clay and would subsequently raise the level in the metal observation well. It thus becomes evident that although hydrostatic pressure will cause a differential of water level in a deeper well, the end result of the water table behavior can be obtained from a shorter observation well. There appears to be no need in having the bottom of the well rest in sand. This statement is based solely upon the experience gathered from this one particular experiment in an organic soil. One must not consider this as final as many duplicate experiments would be required before a definite statement can be made. In experimental work of this nature, where the number of unknowns is great and where any amount of control is difficult to obtain great care must be exercised not to reach hasty conclusions.

## water Table Curves During Data Period

The fluctuations of the water table profile for the seven months period from.September l, 1949 until March 30, 1950 are showing in Fig. 15. This curve is a composite hydrograph made from the weekly data charts gathered during the study. In order to draw a curve for the whole period, it was decided to select the noon hour as the representative time of the day, each day. The precipitation data appearing in Fig. 15 were not obtained directly from the muck farm, but from a location approximately one mile to the southwest.

Rainfall is represented by plain bars, Fig. 15. Snow is denoted by bars crowned with a small circle and a combination of snow and rain by black circles. The variation in precipitation pattern caused a considerable difference in the water table behavior as will be discussed later. When the clocks failed to operate properly (due to failure to rewind, need for lubrication or low temperatures in the later stages of the study) the water table curve is represented by a straight, dotted line. The readings appearing at the beginning and end of such weeks were secured by measuring the water table level in the well with a steel tape.

### Tbmperature Curves

The four temperature curves which appear in Fig. 16 were plotted from data obtained from automatically recording hygro-thermographs, located at the Michigan State College muck Experimental Farm. The various leads connected to the hygro-thermograph were installed in such a manner as to record temperatures at the following elevations and depths: eight inches above the surface of the ground; at the surface of the ground;







![](_page_47_Picture_0.jpeg)

three and one-half inches below the surface and seven inches below the ground surface. In order to maintain consistency, the noon temmeratures were selected for plotting the curves. It is apparent, upon examination of Fig. 16 that temperature changes are greatly reduced and often negligible at a depth or seven inches below the surface of the ground. This is in apparent contrast with the many, sensitive temperature fluctuations registered at the ground surface and at eight inches above the surface. As will be observed later, this had much influence on the fluctuations of the water table level.

#### Fluctuations of the Water Table Level Between Tile Lines

After the wooden wells were installed on November 10, 1949, the levels of the water table were measured weekly with a steel tape and recorded. These fluctuations appear in Figs. 17-21. The water levels plotted indicate rises and falls for the dates adjoining each curve. Since drainage is a question of paramount importance in organic soils, an analysis or the draw-down curves follows. The amount of draw-down for each week corresponds directly with the rise and fall of the water table curves. With the apparent exception of November 17, 1949, the water table between tile lines rose in the well closest to the tile, then levelled off and lowered again in the proximity of the next tile line.

The only "ideal" curve appears on December 15, 1949. According to Fig. 18 (December 15, 1949) the water level was declining from its first sharp winter peak, and the temperature on that day was  $20'$ F. such a curve being singular in its occurrence is here considered the

3'7

![](_page_49_Picture_0.jpeg)

![](_page_50_Picture_18.jpeg)

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![](_page_51_Picture_7.jpeg)

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![](_page_52_Figure_0.jpeg)

![](_page_53_Picture_15.jpeg)

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exception rather than the rule.

Most all other curves present a peculiarity which must be discussed. After rising from the tile line proxinity, all curves show a slight lowering, or "bend" downward midway between the tile lines.

It is known that the drainage efficiency of a tile system is smallest at a point midway between two lines, and often the effect is nil. This condition would seem to warrant the conclusion that spacing of tiles 60 feet apart in muck is an inadequate distance. It must, however, be remembered that at no time during the study was the land flooded, and the highest point the water table reached on a shallow well was 1.02 feet below the ground surface. This happened as a result of one inch of precipitation on March 25 and 26, 1950. Another factor to be considered is that the vertical profile of the curves pictures such a bend of the water table in an exaggerated manner.

Considering these facts, it seems reasonable to assume that 60-foot spacing of tile lines on muck soils is satisfactory. There are no doubt instances where closer tiling is indiepensible in organic soils, but from the observations carried out, the present tiling distance appears to be wholly adequate.

An important observation to be made from this study is the rate at which the water table level dropped at a location in the proximity to a tile line and at a point midway between two lines. This rate of dron can be determined by establishing the fluctuations of the metal and the wooden wells which were located seven feet south of the foremost

northern tile line under study and 32 feet south from the same tile line respectively.

From December 20-22, 1949, the water table rose 1.11 feet and 0.96 feet in the metal and wooden wells reapectively due to 3.01 inches of precipitation (rain). The drop of the water table occurred as follows during 24-hour periods:

![](_page_55_Picture_421.jpeg)

From January 2-4, 1950, the water table rose 0.40 feet and 0.35 feet in the metal and wooden wells respectively as the result of a precipitation of 1.07 inches (rain). The lowering of the water table in 24-hour periods occurred as follows:

![](_page_55_Picture_422.jpeg)

On January 14, 1950, the water table rose 0.32 feet and 0.37 feet in the metal and wooden wells respectively as a result of 0.93 inches of precipitation (rain). The ensuing dr0p of the water table was as follows:

![](_page_56_Picture_447.jpeg)

From.January 24-26, 1950, the water rose 0.69 feet and 0.77 feet in the metal and wooden wells respectively as the result of 1.04 inches of precipitation (rain and snow). The drop of the water table occurred as follows:

![](_page_56_Picture_448.jpeg)

With but few exceptions, the rate of fall of the water table was more rapid at midpoint between the tile lines than in the proximity of a line. The sharp rise on December 20-22, 1949 caused by the large precipitation of 301 inches is the only one which dronped ranidly.

The other three drops occurred in a much slower manner, 0.10 feet or less in the wooden well, except on January 31, 1950 and February 2, 1950 when the drOp per 24-hours was 0.12 feet.

The average drop per 24-hours in the wooden well for the last three water table rises was 0.061 feet or 0.74 inches. It is obvious that the rate of fall diminishes toward the end of the period of water table drOp. In contrast to the 0.061 value mentioned above, the average rate of drop per 24-hours from December 23-25, 1949 was 0.20 feet or 2.48 inches. This impressive difference is due to the fact that when the December rise and drop occurred the soil was moderately wet, and the large precipitation caused a rapid rise of the water table. Good drainage lowered the level rapidly. In the latter instances mentioned, the soil was fairly well saturated so that the moderate precipitations raised the water table level appreciably but the drop was slowed down considerably. The saturated condition of the soil prevented drainage to be thorough from one precipitation to another in view of the fact that these occurred almost weekly not permitting thorough drainage to take place. The result was a slower lowering of the water table than in the case of the Decanber fluctuation.

#### **CONCLUSIONS**

It is hazardous to attempt to draw definite conclusions from this brief study of water table fluctuations in organic soils. A great deal more research is mandatory before final recommendations can be advanced. The following are offered as tentative conclusions.

1. Under conditions of the soil being at field capacity the water table is usually sensitive to precipitation, any amount of which makes the ground-water level fluctuate. Any deviation from field capacity due to such variables as temperature, depleted moisture in the soil, and the existing vegetation may prevent such sharp rises from taking place.

2. During warm, rainless periods, a depletion of water from the ground-water reservoir takes place. This phenomenon is due to direct evaporation from the soil surface or to transpiration of water by plants or both.

3. The difference in the basal level of the wells resulted in a differential level of the water in the metal and wooden wells. This could have been the result of the action of hydrostatic or barometric pressures or both.

4. Deep wells, although more accurate, are not indispensable for good recording of the water table fluctuations from an agricultural standpoint. Shallow wooden wells will provide the necessary information regarding the water table level fluctuations in the root zone.

5. The average rate of dron of the ground-water level midway between two tile lines was computed to be 0.061 feet or 0.74 inches per 24 hours. The maximum rate of drop was 0.20 feet or 2.48 inches in 24 hours.

6. The air and soil temperatures have marked effects on the behavior of the water table.

7. A tile spacing of 60 feet provided satisfactory drainage.

8. The combination of observations wells and automatic water stage recorders is a convenient and efficient method for studying the fluctuations of the ground-water level.

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