

THE THRUST OF MATHEMATICS IN DEVELOPMENT: TRENDS AND PROSPECTS*

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THROUGHOUT HISTORY, AS mankind grappled with nature in the process of development, many new disciplines emerged and greater interdependence among all activities of developmental significance ensued. As far as we can see, this trend will continue — and continue at an increasing pace — possibly indefinitely. In most of these developments the use of mathematics is both apparent and increasingly necessary for their advancement. In fact, today it is correct to say that in every activity — and I must emphasize activity *in whatever form* — mathematics is applicable. In support of this assertion let me cite two cases that are easy to comprehend and have been encountered by most people in one form or other before.

Firstly, there is the concept of 'force' which can be mathematically characterized by an equation involving a general function (i.e. a differential equation). For a specific phenomenon the general function can be explicitly derived from the conditions peculiar to that phenomenon and thus the equation would then quantify the force involved. Turning to real life, we know that some form of force exists in every activity: it may be air pressure; blood pressure; electric current; heat intensity; effect of light or rain on plant growth; the force of inflation on consumer expenditure or on the exchange rate; the force of the public sector borrowing requirement on interest rates; the force of rural development programmes on government expenditure — it may even be the force of death arising from the disease AIDS. Each of these has a corresponding mathematical formulation that is a special case of the general equation of force. To put this in other words, one may say that the general equation of force is a dynamic model whose parameters can be found for specific occurrences. The parameters would depend on the history of, and other factors contributing to, the state of the occurrence. In particular, the historical trends of the parameters would constitute a basis for predicting their future form. Let me also add that in some cases such parameters can be difficult to find, largely because of current human inability to comprehend fully the factors involved. However, even in such cases, approximate values for the parameters can often be found and the equation can be employed for general guidance.

I now turn to a second case illustrating why mathematics is penetrating

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virtually all disciplines and is closely linked with their development. In appraising most activities, one observes, sometimes collecting a sub-sample of the result and testing it or performing some other simulation of the experience. Thus empirical studies are widely employed in projects and, with the coming of computers, almost every development programme will incorporate such studies in one form or another. Mathematics, or more specifically the arm of mathematics called statistics, is the main tool employed in synthesizing the results based on empirical studies. Directly and indirectly, other areas of mathematics also contribute (one can cite an area such as Probability Theory, the applied arm of which is statistics).

Undoubtedly, the major developmental thrust of mathematics falls into the area of Science and Technology, and a great deal of the stock-in-trade of applied mathematics awaits delivery to that area. Furthermore, much of the development of our time and perspective is closely tied up with Science and Technology. It is, therefore, imperative to focus first on the role of mathematics in Science and Technology, and then turn to the other complementary areas of development. But before this the role of Science and Technology in development needs clarification. To this end, I prefer to look at Science and Technology as consisting of four main areas: Basic Sciences — which include subjects like Biology, Chemistry, Physics, Geology, Mathematics and their offshoots like Biophysics, Theoretical Physics and Biochemistry; Applied Sciences — which include Computer Science, Agriculture, Engineering and Medicine; Technology Development — which deals with the adaptation of existing scientific and (imported) technological results for prospective use in industry and may also encompass part of the work done under Engineering (including Computer Engineering); and, finally, Technology Application — which deals with the use of developed technologies in industry and the commercialization of products and other results discovered.

In most cases the results of Science and Technology employed in development have their roots in the Basic Sciences and are gradually turned into some practical use via the Applied Sciences, through Technology Development, finally reaching Technology Application. It is also pertinent to note that countries that are deficient in any one of these four areas often find themselves unable to utilize fully the results of Science and Technology. Indeed, scientists in some developing countries have made useful discoveries at the levels of the Basic and Applied Sciences — I have fields like traditional medicine and the biochemistry of various plants in mind — but their countries cannot fully utilize the discoveries because they do not have the relevant infrastructure needed for the implementation process (at the level of Technology Development). Eventually, the results find their way to the developed countries, where they are processed and, in most cases, generate enormous profits partly through export to the developing world — their countries of origin. My message here is that for a country that is weak at the Technological (Development and/or Application) level most (Basic and

Applied) Scientific Research may not be of direct benefit to the country's development.

Yet the science of today is the root of the technology of tomorrow. Thus, technological advancement cannot be sustained without recourse to science. Indeed, countries that ignore (Basic and/or Applied) Scientific Research would soon find their long-term development programmes strangled in one form or another. What is desirable is development in all areas of Science and Technology with the level of emphasis for each specific area of Science and Technology determined by the peculiarities of the particular country. Of course, like all issues in real life, the dividing line between these areas of Science and Technology is thin.

Let me now turn to the thrust of Mathematics in each of the four main areas of Science and Technology that I have cited. Firstly, I would like to look at the Basic Sciences. As Mathematics is itself one of the Basic Sciences, I will start by looking briefly at the situation within Mathematics. The branches of Mathematics that a mathematician commonly looks at and often treats as distinct fields — and which are the ones employed in mathematical reviews — amount to a total exceeding eighty. However, the more familiar divisions of Mathematics, that should also be familiar to the general reader, are Pure Mathematics and Applied Mathematics. Pure Mathematics embraces aspects of the theory of mathematics and, at research level, the creation of new mathematics. On the other hand, Applied Mathematics (including Statistics) deals with the extensive and intensive application of mathematical results to physical and practical situations and the formulation of these in mathematical terms. Strictly, the techniques employed are not new but rather are borrowed from Pure Mathematics and adapted accordingly as the physical situation at hand demands. The dividing line between the two branches is faint and blurred by various aspects of Mathematics of interest both to Pure and Applied mathematicians. These may be viewed as aspects of Pure Mathematics undergoing reformulation for absorption into Applied Mathematics.

Both Pure and Applied Mathematics have expanded, especially over the last few decades. Recent major trends in the composition of the subject include the increasing interaction between Pure and Applied Mathematics, with some (even abstract) areas of Pure Mathematics being quickly turned into applicable form, thus increasing the Applied Mathematics stock. Let me underline one feature of the pattern of mathematical expansion. As I have indicated earlier, both Pure and Applied Mathematics have expanded to make up more than eighty somewhat distinct sub-fields of Mathematics. What fascinates me is the way this has happened: In Pure Mathematics the expansion is largely vertical; although various sub-branches have emerged and signs of other offshoots to come are apparent, the trend is in the vertical direction. This has serious implications for future research in Pure Mathematics, for the top of the tree (of Pure Mathematics)

is moving further and further away from the ground; even if one tries climbing along one of the branches, the branches too have their ends vertically directed and are growing fast. This gives pressure for more and more specialization and, *ipso facto*, the emergence of more vertically growing sub-branches of Pure Mathematics. On the other hand, the expansion of Applied Mathematics has been largely horizontal, through interaction with other disciplines. As will be apparent later, this phenomenon has been brought about mainly by the practical realities of life.

You will recall that at the beginning I mentioned that the aspect of force can be studied in mathematical terms. But then force is also the starting point of Physics, the approach being largely experimental. The alliance of Mathematics and Physics, therefore, dates back to the birth of Physics. As is the case with many developments, theory leads practice, and in Physics the theory not only accelerated great discoveries, paving the way for experimentation, but did so with a flavour that is heavily mathematical. This led to the spin-off Theoretical Physics, now standing as a distinct discipline. Others, perhaps with a mathematical bias, prefer to call it Mathematical Physics and some even view it as one of the many sub-branches of Applied Mathematics. In a related manner, the interaction between Mathematics and Biology has brought about another spin-off, Bio-mathematics. The use of statistics in the classification of species and the rapid growth of Mathematical Modelling, which is invading areas such as Ecology, particularly population dynamics, are wetting the appetites of many mathematicians. In fact Mathematical Modelling touches on some aspects of virtually all the Basic Sciences — Chemistry, Biochemistry and Geology included — and helps to reveal the interdependence among these sciences.

It is pertinent to note that new developments among the Basic Sciences make mathematicians very inquisitive not just because such developments expand the territory for mathematical applications but also because they often contribute towards mathematical innovation — both Pure and Applied. Thus the interaction of Mathematics and the other Basic Sciences reveals a two-way process enriching both sides.

Such a two-way process seems to be either absent or minimal between Mathematics and the Applied Sciences. Here — barring Computer Science, which has only recently emerged, and viewing Applied Sciences as distinct from Technology — the situation seems to be static. Even at university level the mathematical input in subjects like Engineering and Agriculture tends to consist of standard results that have been with us for quite some time. (To some extent the input of Physics in Engineering shows a similar trend.) However, lately there seems to have emerged a process of change in the approach employed in the Applied Sciences whereby the intrinsic outlook of the division is being replaced by two thrusts: One calls for greater specialization, not just relying on the input of

a Basic Science but also independently grappling to understand relevant aspects of nature, thus attaining a form much like that of a Basic Science; such specialization is on the increase in Medicine and Agriculture. The other thrust is that leaning towards Technology Development; indeed, most universities have now created Technology faculties in which the main departments relate to Engineering. Where an Applied Science reveals a leaning towards the Basic Sciences or Technology Development the mathematical picture in that discipline shows a similar bias. I shall discuss the situation between Computer Science and Mathematics, which I consider one of the rewarding developments of this century, at a convenient point later.

Turning to the area of Technology Development, the scenery largely contains a lot of mathematical results awaiting use. I see the role of a mathematician in this area largely as formulating practical problems in mathematical terms, then either solving the mathematical problems or identifying their solutions among the widely available mathematical results, and, finally, interpreting the mathematical results in terms of the physical situations at hand — thus enabling the technologist to effect implementation. All this can often be done without calling for original mathematical research but rather by extracting the relevant results and techniques from the large stock of mathematics already available in the literature. Sometimes spin-offs occur whereby a mathematical solution obtained not only answers the physical problem at hand but is also widely applicable to other issues of a different physical form. In some cases the practical problem may look complicated yet the mathematical solution may be readily available. We illustrate such a case in Example 1.

It seems quite evident that the solution to this problem could be applied to many real-life issues including aspects of safety control. (The mathematical solution uses ideas commonly employed in the area of Mathematics called Group Theory and can be studied at university undergraduate level). Other variations, like tying a million rings in such a way that if any two of the rings break the remainder will fall in tied pairs, are also possible.

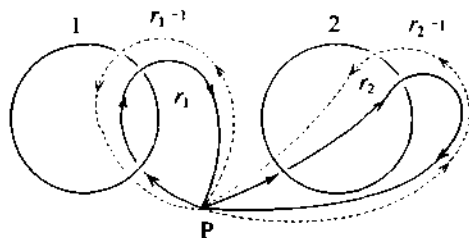
As I noted earlier, a peculiar feature of Mathematics for Technology Development — which fascinates many technologists especially now with the emergence of high-technology disciplines like microelectronics — is the large stock of mathematical results that may be applicable. Thus the thrust of Mathematics in Technology Development is vigorous, not so much because of the high turnover in original mathematics but rather because of the increasing relevance to results already on the market — sometimes picking very elementary mathematics, sometimes stumbling on theorems that had been shelved in the archives since the last century and occasionally encountering some recent mathematical theorems, all with concrete applications.

Take, for instance, the so-called four-colour conjecture which states that for

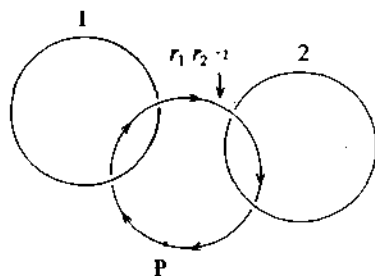
Example 1

Consider a case where there is a need to tie together many, say one million, rings (made of flexible material) in such a manner that if any one of the rings breaks the remainder will all fall apart. How should the rings be tied?

To obtain the mathematical solution we proceed as follows: Let the rings be labelled 1, 2, 3, ... and position them in a horizontal plane. Define the rings $r_1, r_2, r_3, \dots$ as follows: Let P be a fixed point to be used as a reference point in the sense that when a ring passes through P we say the ring starts and ends at P. Let r_1 be a ring that starts at P and passes through Ring 1 from below coming out above to end at P. Ring r_1^{-1} (called the inverse of r_1) is the ring that reverses the process of r_1 , that means it passes through Ring 1 from above and it starts and ends at P. Similarly one defines the rings $r_2, r_2^{-1}, r_3, r_3^{-1}, \dots$ with respect to Rings 2, 3, ...



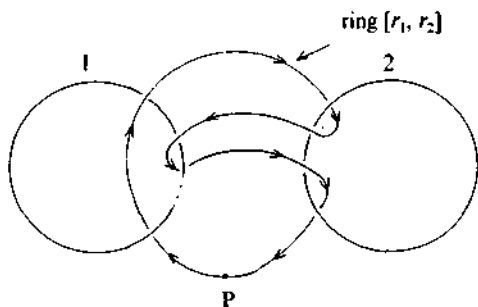
Next we define ring $r_1 r_2^{-1}$: this starts at P, passes through Ring 1 from below coming out above and then passes through Ring 2 from above to come out below and end at P. Similarly we can define $r_1 r_2$ and, *ipso facto*, products of any of the rings $r_1, r_1^{-1}, r_2, r_2^{-1}, r_3, r_3^{-1}, \dots$



Now we are in a position to give the solution to our problem: To tie three rings, 1, 2, 3, such that if one breaks the remaining two will fall apart, Ring 3 may be constructed using the formula

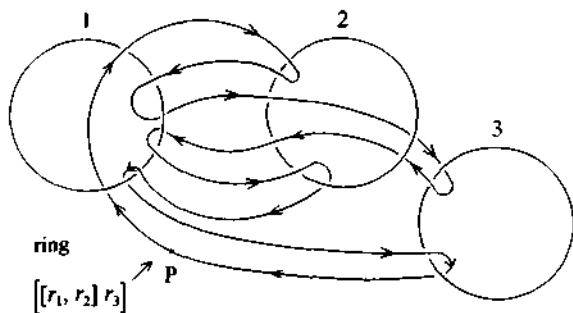
$$[r_1, r_2] = r_1 r_2 r_1^{-1} r_2^{-1} \quad (3)$$

Example 1 (cont)



To tie four rings, 1, 2, 3, 4, such that if one breaks the remaining three will fall apart, Ring 4 may be constructed using the formula

$$[[r_1, r_2] r_3] = [r_1, r_2] r_3 [r_1, r_2]^{-1} r_3^{-1} \quad (4)$$



Proceeding in this manner we see that to tie one million rings, 1, 2, 3, ..., 1 000 000, in such a way that if one breaks the remainder will all fall apart, we may construct the 1 000 000th ring using the formula

$$[[\dots [r_1, r_2] r_3], \dots, r_{999\,998}, r_{999\,999}] \quad (1\,000\,000)$$

which, with the aid of a computer, can be done easily.

From the formulae (3), (4), ..., (1 000 000) we see that if one removes r_1, r_1^{-1} or r_2, r_2^{-1} or ... or $r_{999\,999}, r_{999\,999}^{-1}$ — which corresponds to breaking Ring 1, 2, 3, ..., 999 999, respectively — the formula collapses to 1 — which corresponds to all the remaining rings falling apart.

This completes our solution.

any real or imaginary map of any size, the minimum number of colours to colour the countries so that no two adjoining countries have the same colour is four. The conjecture was well known to mathematicians even during the last century but was only solved, with the partial aid of a computer, in the USA around 1977. This result should be of great interest to, for instance, designers and artists — especially where paint may be in short supply — as a minimum of four different kinds of paint can always be employed to give distinct patterns. The result can also be applied to some military matters. In short, the result simply says that the number 4 has a special role in many cases where a minimal number of items is required for a defined purposes and the message is that it be recalled in developmental issues of that form.

Though the Mathematics employed in Technology Application occasionally grows like that for Technology Development, especially on aspects of Transportation and Communications Studies the main trend concerns the construction of simple models and numerical tables for use by the extension services, the public, the commercial sector and other bodies who need not necessarily have the mathematical know-how but are engaged in activities where Mathematics is applicable. For instance, many life assurers only know how to calculate the premium of a life or endowment policy by using ready-made tables for the purpose but do not know how the tables are derived; such is also the case with building societies when calculating instalments, for a house mortgage, for example. Behind these people are mathematicians engaged in the translation of the theory on life expectation — with the desired rates of return on the investments and the main risk factors incorporated — to figures given in the tables and other simple models. The extent to which such mathematical activity will increase is largely dependent on the pace of Technology Development; in particular, the emergence of the computer has inspired the construction of many mathematical models employed in various aspects of Technology Application. It is also pertinent to note that here one is mostly dealing not only with mathematically well-established results but with results whose relevance for specific cases has already been noted in the course of Technology Development and/or Applied Scientific study.

For the time being, this completes my analysis of Mathematics and Science and Technology *per se*, with the exception of the linkage between Mathematics and Computer Science which I am deferring to a more convenient point.

Both from the stand-point of Mathematics and generally, if I were asked to cite in order of priority the three most critical requirements for prospective development, I would say that, firstly, it is the ability to predict oncoming natural phenomena — like rain, earthquakes, biological phenomena and droughts — long in advance. Secondly, I would say it is again the ability to predict oncoming natural phenomena long in advance and, thirdly, well, it is still the ability to

predict oncoming natural phenomena long in advance. For instance it is possible, with a good degree of confidence, to predict the weather for the next twelve hours or so, which seems to be the furthest meteorologists today can take it. Further ahead, more variables enter the picture and the result becomes increasingly unpredictable. So far, economists are probably the main professional group deserving praise for embarking on long-term forecasts of issues pertinent to their discipline for many decades. However, they are also the best known for erroneous predictions. This, in my view, makes economics one of the main areas of our lifetime in need of intensive cultivation both in the theoretical arena and on the applied side. It is in such a cultivation that I see Mathematics as one of the main tools to be used.

The entry of Mathematics into economics received its first celebrated welcome not so much from Karl Marx's quantification of surplus labour but rather from Leontief's input-output table (1919) which still remains possibly the most widely known and widely employed mathematical model in macro-economic planning. The model is given for a particular year in matrix form, and each entry has a numerical value indicating the input from one economic sector, say, i , into another, say, j . For convenience let $a(i, j)$ denote such a numerical entry. Using elementary mathematics (on matrix algebra) the model can be projected to give an estimate of inter-sectoral linkages of a given future year. I see two ways in which Mathematics can be further employed to improve the reliability of such a model. Firstly, production functions, incorporating various background factors, can be devised and employed in deriving the numerical entries $a(i, j)$. For instance, in Zimbabwe, where the agricultural sector has often been depressed by droughts, the amount of rain could be incorporated as one of the variables of production functions depicting inputs from the agricultural sector. Secondly, both the form of such production factors and the compilation of sectoral data will need to be attained in a manner that either does not allow inclusion of structural errors or minimizes such errors with increasing data. This point is illustrated in Example 2.

It is apparent that both the idea of 'force' and 'empirical study' cited at the beginning are incorporated in the construction of production functions. Also the model can be suitably adjusted and applied to other macro-economic questions — some even apply it in forecasting future energy consumption requirements of the various sectors of the national economy.

Other models of different mathematical formulation are employed in various fields of economics. Even institutional investors — though hesitant — often get a glimpse of models with a view to selecting investment portfolios that give maximal returns at certain levels of risk. In the field of financial accounting, as companies expand in cross-border activities and currency fluctuations widen, coupled with price inflation of most commodities not falling below a double-digit

Example 2

Suppose we have several squares and for each square we are given estimates, x and y , of the length of a side. If data giving the area of each square, A , is needed, should we use the formula

$$A = \frac{x^2 + y^2}{2} \quad (1)$$

$$\text{or} \quad A = \left(\frac{x + y}{2} \right)^2 \quad (2)$$

Both (1) and (2) are correct formulae for the area of a square; however, formula (1) contains a bigger structural error than formula (2). Thus, formula (2) would give a better result. The actual mathematical proof comes from an area called Expectation Theory.* From the latter field we in fact learn that the ideal formula to use is

$$A = xy$$

which does not contain a structural error. Also, generalizations of (1) and (2) in the case where n estimates of the length of a square are given give a formula corresponding to (2), whose structural error tends to zero as n increases, while the formula corresponding to (1) retains a structural error that does not change regardless of the size of n .

* With the usual statistical notation for mean, μ , and variance, σ^2 , of random variables x and y , and with E denoting expectation, we get

$$E(\bar{A}) = E\left(\frac{x^2 + y^2}{2}\right) = \frac{E(x^2) + E(y^2)}{2} = \frac{2\mu^2 + 2\sigma^2}{2} = \mu^2 + \sigma^2$$

$$= A + \sigma^2 \text{ giving error of } \sigma^2$$

$$E(\bar{A}) = E\left(\left(\frac{x + y}{2}\right)^2\right) = \frac{E(x^2 + y^2) + E(2xy)}{4} = \frac{(2\mu^2 + \sigma^2)}{2} = \mu^2 + \frac{\sigma^2}{2}$$

$$= A + \frac{\sigma^2}{2} \text{ giving error of } \frac{\sigma^2}{2}$$

$$E(\bar{A}) = E(xy) = \mu^2 = A \text{ which has no error term.}$$

annualized percentage, I foresee current cost-accounting practice — which revalues assets, stock, credits and debits in line with price changes — gaining popularity, and mathematical models (on current cost accounting) being devised for use at least in planning the budget.

Lately, mathematical applications have extended even to social sciences such as sociology and international law, for instance, in analysing international disputes and devising negotiating strategies and in the general aspects of management decision-making. The techniques commonly employed in these areas come from a field called Discrete Mathematics including one called Graph Theory. However, I have a feeling that a concrete mathematical breakthrough in such studies is hampered by the heavily subjective nature of strategies employed which often do not agree with mathematical deductive reasoning. Despite this, where the studies focus on prospective events — I have in mind an area such as marketing — some useful guidelines (to the events) can be achieved.

In all these models the mathematics used is standard and widely known and the computer is used in analysing data and testing the models. I now want to look at the relationship between Mathematics and Computer Science. The latter as a discipline is relatively new, as I stated before, and its whole existence is centred on the emergence of the computer. It is, therefore, relevant to look at Mathematics *vis-à-vis* the computer (and its capabilities). Many people have often wondered whether certain inventions can now be made more easily using a computer in place of Mathematics and whether the outlook of Mathematics is going to change because of computer usage. To answer these speculations let me start by reiterating that Mathematics *per se* remains solidly a Basic Science subject with tentacles — namely its applications — spreading virtually over all the other disciplines. Viewed from a mathematical perspective, the emergence of the computer enabling the transmission of mathematical results from the Basic Science level to users on the ground is the natural outcome of Technology Development. Over the next few years those mathematical results with numerically based solutions, or with solutions comprehensible in related numerical forms, will be delivered to real-life use with the aid of the Mathematics agency, namely the computer. My guess is that within twenty years, the computer will be more common all over the world than the hi-fi (music) system is today. Unfortunately, the scope of mathematical results that the computer can perceive is still very small and hardly covers a significant part of advanced Pure Mathematics. There are many mathematical results, especially those of this century, that society cannot use in real life simply because there does not yet exist a relevant technological capability to usher in their usage. I am optimistic, however, that in the near future — possibly within five decades — we will witness a second stage of higher technology emerging; some kind of super high-technology that would include the invention of super-computers or some other

related technology capable of employing a wide range of abstract mathematical results in real-life issues. Such a technology would have a remarkable multiplier effect on the pace of development.

For most mathematical (especially pure) research, barring cases of the type discussed above, the computer is rarely used, and my view is that this situation is likely to remain like that until, possibly, some remarkable development of the type I have called super high-technology happens. In short, the computer remains a tool, albeit an intelligent one, to be used in various areas, particularly those in which mathematics is applicable.

I have discussed how the mathematical perspective differs between the main divisions of Science and Technology, in particular, that the relationship between Mathematics and the other Basic Sciences reveals a two-way process with each side having an input into the other, especially at the level of research. On the other hand, Mathematics relates to Technology in a largely one-way process, depicting the application of Mathematics in technological issues. Technology plays a critical role in enabling us to realize the benefits of nature and remains equally crucial in solving problems of our own making. This reinforces our confidence; as Karl Marx once stated, the only problems which mankind is capable of creating are those which mankind can solve. Indeed, Mathematics not only provides a concrete framework for analysing technical issues but even the kind of mathematical techniques employed are often available within the armoury of results established long before.

I also reflected on the increasing role of Mathematics in socio-economic studies. In particular, I have noted the good prospects of greater intimacy between Mathematics and Economics. In this connection I wish to underline the following point: In my view, the future of Economics as a discipline with particular relevance to development will depend on its ability to interpret and forecast practical economic scenarios. In that endeavour I see Mathematics as Economics' key partner, especially in deriving models depicting such scenarios.

I also underlined that, equipped with Mathematics, various phenomena can be understood much better and one tends to face issues from an aggressive rather than a defensive stance. As Julius Caesar said, 'The things that frighten us look at our backs but when they see our faces, they vanish'. Indeed, there are several real-life problems that, *prima facie*, are difficult to comprehend, yet, once looked at squarely and formulated into mathematical terms, are not only more easily understandable, but often their solutions readily follow.

I also discussed why and how Mathematics is rapidly expanding — with the trend for Pure Mathematics being largely vertical while that for Applied Mathematics is mainly horizontal. Since Pure Mathematics is the pivot of all Mathematics, the future of Mathematics will also depend on our ability to devise

approaches that will enable the human mind to comprehend a lot more mathematical results in line with the (vertical) growth in Pure Mathematics.

Finally, I have in some way underscored the following point: the dialectics of development reveal that all things and all activities are interlinked and constantly changing, and what we call development is simply a manifestation of how mankind manages such linkages. From a mathematical perspective, not only do we find (mathematically) quantifiable phenomena like force apparent in all activities of whatever form, but even mathematical models derived for specific activities often turn out to be applicable to many other situations. Mathematics, therefore, provides a way of perceiving the totality of such linkages as a united process — a process which mankind will continue to strive to come to grips with in its struggle for a better life. Such signifies the Thrust of Mathematics in Development: Its Trends and Prospects.